TIME SERIES

- It is used to make estimations for the future

  Example: A businessman finds out his likely sales for the upcoming 5 years so that his production does not hamper the sales.

- Estimating the future based on the data gathered from the past.
- Time series deals with statistical data which are collected, observed or recorded for successive intervals of time.
- Observing the numerical data at the different point of time, the set of observations is called the time series.

What is Time Series?

Time series analysis is **a specific way of analysing a sequence of data points collected over an interval of time**. In time series analysis, analysts record data points at consistent intervals over a set period of time rather than just recording the data points intermittently or randomly. In other words we can say that Time series is a set of statistical data arranged in a chronological order.

COMPONENTS OF TIME SERIES:

If the time-series data contains observations of just single variable such as demand as a product of time then it can be termed as univariate time-series data.

If the data consists of more than one variable such as demand as product of time, price as time, amount of money spent as the promotion of time etc. then the time-series is termed as multivariate time-series.

A time series is broken into major four components:

1. **Trend component (T_t):** The trend component is the consistent long term upward or downward movement of the data.
2. **Seasonal component (S_t):** Seasonal trends are the repetitive upward or downward movements from the trend that occurs within a calendar year at fixed intervals such as seasons, months, quarters, months etc. The upward or downward fluctuations may be caused by the festivals, customs within the society, business practices such as End of season Sale etc.
3. **Cyclical component (C_t):** Cyclical component is a fluctuation around the trend line at random interval that happens due to macro-economic changes such as recession, unemployment etc. Cyclical fluctuations have repetitive pattern with time between repetitions of more than a year. Whereas in the case of seasonality, the fluctuations are observed within a calendar year which are driven by festivals and customs that exist in a society. A major difference between cyclical and the seasonal trend is that seasonal changes occurs at the fixed time in the year, whereas the cyclical fluctuations occurs randomly. That is the reason why the periodicity of the seasonal fluctuations is constant whereas the periodicity of the cyclical fluctuations are not constant. Cyclic component represent consistently recurring rise and decline in the activity.

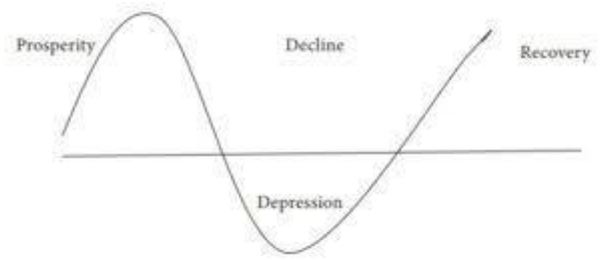
The four phases of Cyclic component are:

- Prosperity

- Decline

- Depression

- Improvement



Four phases of cyclic component

4. **Irregular component (I_t):** Irregular component is the white noise or random uncorrelated changes that follow a normal distribution with mean value 0 and constant variance.

UTILITY OF TIME SERIES:

- It helps in understanding the past behaviours.
- It helps in predicting the future outcomes.
- It helps in evaluating the current accomplishments.
- It facilitates comparison.

DECOMPOSITION OF TIME SERIES:

In the traditional or classical time series analysis there is an assumption that there is a multiplicative relationship between the four components. Any particular value in a series is the product of factor that can be attributed the various components.

$$\boxed{Y = T_t * S_t * C_t * I_t}$$

Multiplicative models are more common and are a better fit for most of the datasets.

The additive model assumes that the seasonal and cyclical components are independent of the trend component. Additive models are not very common, since in many cases the seasonal component may not be independent of the trend.

$$Y = T_t + S_t + C_t + I_t$$

Measurements of Trends:

Following are the methods by which we can measure the trend.

(i) Freehand or Graphic Method.

(ii) Method of Semi-Averages.

(iii) Method of Moving Averages.

(iv) Method of Least Squares.

(i) Freehand or Graphic Method.

It is the simplest and most flexible method for estimating a trend. We will see the working procedure of this method.

Procedure:

(a) Plot the time series data on a graph.

(b) Draw a freehand smooth curve joining the plotted points.

(c) Examine the direction of the trend based on the plotted points.

(d) Draw a straight line which will pass through the maximum number of plotted points.

Example 9.1

Fit a trend line by the method of freehand method for the given data.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Sales	30	46	25	59	40	60	38	65

Solution:

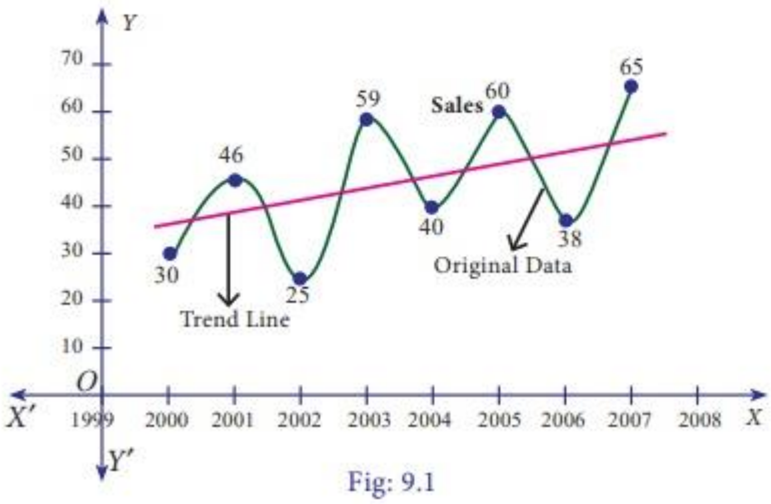


Fig: 9.1

Note

The trend drawn by the freehand method can be extended to predict the future values of the given data. However, this method is subjective in nature, predictions obtained by this method depends on the personal bias and judgement of the investigator handling the data.

MERITS:

1. It is simple method of estimating trend which requires no mathematical calculations.

2. It is a flexible method as compared to rigid mathematical trends and, therefore, a better representative of the trend of the data.

3. This method can be used even if trend is not linear.

4. If the observations are relatively stable, the trend can easily be approximated by this method.

5. Being a non mathematical method, it can be applied even by a common man.

DEMERITS:

1. It is subjective method. The values of trend, obtained by different statisticians would be different and hence, not reliable.

2. Predictions made on the basis of this method are of little value.

(ii) Method of Semi-Averages

In this method, the semi-averages are calculated to find out the trend values. Now, we will see the working procedure of this method.

Procedure:

(i) The data is divided into two equal parts. In case of odd number of data, two equal parts can be made simply by omitting the middle year.

(ii) The average of each part is calculated, thus we get two points.

(iii) Each point is plotted at the mid-point (year) of each half.

(iv) Join the two points by a straight line.

(v) The straight line can be extended on either side.

(vi) This line is the trend line by the methods of semi-averages.

MERITS:

1. It is simple method of measuring trend.

2. It is an objective method because anyone applying this to a given data would get identical trend value.

DEMERITS:

1. This method can give only linear trend of the data irrespective of whether it exists or not.

2. This is only a crude method of measuring trend, since we do not know whether the effects of other components are completely eliminated or not.

Example 9.2

Fit a trend line by the method of semi-averages for the given data.

Year	2000	2001	2002	2003	2004	2005	2006
Production	105	115	120	100	110	125	135

Solution:

Since the number of years is odd(seven), we will leave the middle year's production value and obtain the averages of first three years and last three years.

Year	Production	Average
2000	105	
2001	115	$\frac{105+115+120}{3}=113.33$
2002	120	
2003	100 (left out)	
2004	110	
2005	125	$\frac{110+125+135}{3}=123.33$
2006	135	

Table 9.1

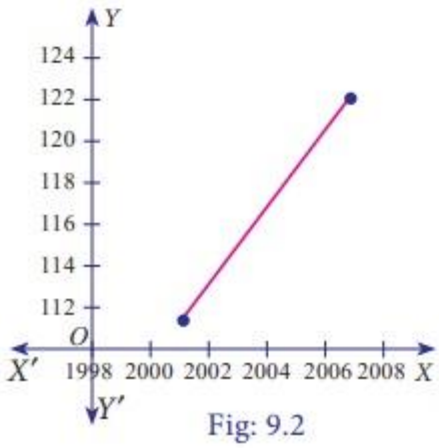


Fig: 9.2

Example 9.3

Fit a trend line by the method of semi-averages for the given data.

Year	1990	1991	1992	1993	1994	1995	1996	1997
Sales	15	11	20	10	15	25	35	30

Solution:

Since the number of years is even(eight), we can equally divide the given data it two equal parts and obtain the averages of first four years and last four years.

Year	Production	Average
1990	15	$\frac{15+11+20+10}{4}=14$
1991	11	
1992	20	
1993	10	
1994	15	$\frac{15+25+35+30}{4}=26.25$
1995	25	
1996	35	
1997	30	

Table 9.2

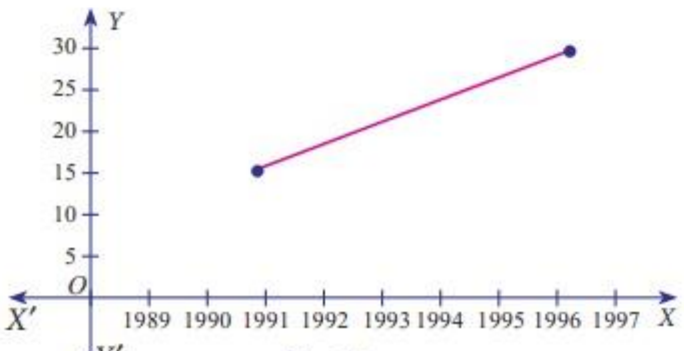


Fig: 9.3

Note

(i) The future values can be predicted.

(ii) The trend values obtained by this method and the predicted values are not precise.

(iii) Method of Moving Averages

Moving Averages Method gives a trend with a fair degree of accuracy. In this method, we take arithmetic mean of the values for a certain time span. The time span can be three-years, four -years, five- years and so on depending on the data set and our interest. We will see the working procedure of this method.

Procedure:

(i) Decide the period of moving averages (three- years, four -years).

(ii) In case of odd years, averages can be obtained by calculating,

$$\frac{a+b+c}{3},\frac{b+c+d}{3},\frac{c+d+e}{3},\frac{d+e+f}{3},\ldots\ldots$$

(iii) If the moving average is an odd number, there is no problem of centering it, the average value will be centered besides the second year for every three years.

(iv) In case of even years, averages can be obtained by calculating,

$$\frac{a+b+c+d}{4},\frac{b+c+d+e}{4},\frac{c+d+e+f}{4},\frac{d+e+f+g}{4},\ldots$$

(v) If the moving average is an even number, the average of first four values will be placed between 2nd and 3rd year, similarly the average of the second four values will be placed between 3rd and 4th year. These two averages will be again averaged and placed in the 3rd year. This continues for rest of the values in the problem. This process is called as centering of the averages.

MERITS:

1. This method is easy to understand and easy to use because there are no mathematical complexities involved.

2. It is an objective method in the sense that anybody working on a problem with the method will get the same trend values. It is in this respect better than the free hand curve method.

3. It is a flexible method in the sense that if a few more observations are added, the entire calculations are not changed. This not with the case of semi-average method.

4. When the period of oscillatory movements is equal to the period of moving average, these movements are completely eliminated.

5. By the indirect use of this method, it is also possible to isolate seasonal, cyclical and random components.

DEMERITS:

1. It is not possible to calculate trend values for all the items of the series. Some information is always lost at its ends.

2. This method can determine accurate values of trend only if the oscillatory and the random fluctuations are uniform in terms of period and amplitude and the trend is, at least, approximately linear. However, these conditions are rarely met in practice. When the trend is not linear, the moving averages will not give correct values of the trend.

3. The trend values obtained by moving averages may not follow any mathematical pattern i.e. fails in setting up a functional relationship between the values of X(time) and Y(values) and thus, cannot be used for forecasting which perhaps is the main task of any time series analysis.

4. The selection of period of moving average is a difficult task and a great deal of care is needed to determine it.

5. Like arithmetic mean, the moving averages are too much affected by extreme values.

Example 9.4

Calculate three-yearly moving averages of number of students studying in a higher secondary school in a particular village from the following data.

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Number of students	332	317	357	392	402	405	410	427	435	438

Solution:

Computation of three- yearly moving averages.

Year	Number of students	3- yearly moving Total	3- yearly moving Averages
1995	332	---	---
1996	317	1006	335.33
1997	357	1066	355.33
1998	392	1151	383.67
1999	402	1199	399.67
2000	405	1217	405.67
2001	410	1242	414.00
2002	427	1272	424.00
2003	435	1300	433.33
2004	438	---	---

Table 9.3

Example 9.5

Calculate four-yearly moving averages of number of students studying in a higher secondary school in a particular city from the following data.

Year	2001	2002	2003	2004	2005	2006	2007	2008
Sales	124	120	135	140	145	158	162	170

Solution:

Computation of four- yearly moving averages.

Year	Sales	4-yearly centered moving total	4-yearly moving Average	4-yearly centered moving Average
2001	124	--	--	--
2002	120	--	--	--
		519	129.75	
2003	135	--		132.37
		540	135.00	
2004	140	--		139.75
		578	144.50	
2005	145	--		147.87
		605	151.25	
2006	158	--		155.00
		635	158.75	
2008	162	--		162.50
		665	166.25	
2007	170	--	--	-
2008	175	--	--	-

Table 9.4

Note

The calculated 4-yearly centered moving average belongs to the particular year present in that row eg; 132.37 belongs to the year 2003.

(iv) Method of Least Squares

The line of best fit is a line from which the sum of the deviations of various points is zero. This is the best method for obtaining the trend values. It gives a convenient basis for calculating the line of best fit for the time series. It is a mathematical method for measuring trend. Further the sum of the squares of these deviations would be least when compared with other fitting methods. So, this method is known as the Method of Least Squares and satisfies the following conditions:

(i) The sum of the deviations of the actual values of Y and $\hat{Y}$ (estimated value of Y) is Zero. that is $\Sigma(Y-\hat{Y}) = 0$.

(ii) The sum of squares of the deviations of the actual values of Y and $\hat{Y}$ (estimated value of Y) is least. that is $\Sigma(Y-\hat{Y})^2$ is least ;

Procedure:

(i) The straight line trend is represented by the equation $Y = a + bX$ …(1)

where Y is the actual value, X is time, a, b are constants

(ii) The constants 'a' and 'b' are estimated by solving the following two normal

Equations $\Sigma Y = n\,a + b\,\Sigma X$ ...(2)

$\Sigma XY = a\,\Sigma X + b\,\Sigma X^2$ ...(3)

Where 'n' = number of years given in the data.

(iii) By taking the mid-point of the time as the origin, we get $\Sigma X = 0$

(iv) When $\Sigma X = 0$, the two normal equations reduces to

$$\Sigma Y = n\,a + b\,(0) \quad ; \; a = \frac{\sum Y}{n} = \bar{Y}$$

$$\Sigma XY = a(0) + b\,\Sigma X^2 \;\; ; \; b = \frac{\sum XY}{\sum X^2}$$

The constant 'a' gives the mean of Y and 'b' gives the rate of change (slope).

(v) By substituting the values of 'a' and 'b' in the trend equation (1), we get the Line of Best Fit.

MERITS:

1. Given the mathematical form of the trend to be fitted, the least squares method is an objective method.

2. Unlike the moving average method, it is possible to compute trend values for all the periods and predict the value for a period lying outside the observed data.

3. The results of the method of least squares are most satisfactory because the fitted trend satisfies the two most important properties, i.e. (1) $\sum(Y0 - Yt) = 0$ and (2) $\sum(Y0 - Yt)2$ is minimum. Here Y0 denotes the observed values and Yt denotes the calculated trend value. The first property implies that the position of fitted trend equation is such that the sum of deviations of observations above and below this equal to zero. The second property implies that the sums of squares of deviations of observations, about the trend equations, are minimum.

DEMERITS:

1. As compared with the moving average method, it is cumbersome method.

2. It is not flexible like the moving average method. If some observations are added, then the entire calculations are to be done once again.

3. It can predict or estimate values only in the immediate future or the past.

4. The computation of trend values, on the basis of this method, doesn't take into account the other components of a time series and hence not reliable.

5. Since the choice of a particular trend is arbitrary, the method is not, strictly, objective.

6. This method cannot be used to fit growth curves, the pattern followed by the most of the economic and business time series.

Example 9.6

Given below are the data relating to the production of sugarcane in a district.

Fit a straight line trend by the method of least squares and tabulate the trend values.

Year	2000	2001	2002	2003	2004	2005	2006
Prod. of Sugarcane	40	45	46	42	47	50	46

Solution:

Computation of trend values by the method of least squares (ODD Years).

Year(x)	Production of Sugarcane(Y)	$X=(x-2003)$	X^2	XY	Trend values(Yt)
2000	40	−3	9	−120	42.04
2001	45	−2	4	−90	43.07
2002	46	−1	1	−46	44.11
2003	42	0	0	0	45.14
2004	47	1	1	47	46.18
2005	50	2	4	100	47.22
2006	46	3	9	138	48.25
$N=7$	$\Sigma Y = 316$	$\Sigma X = 0$	$\Sigma X^2 = 28$	$\Sigma XY = 29$	$\Sigma Yt = 316$

Table 9.5

$$a=\frac{\sum Y}{n}=\frac{316}{7}=45.143 \ ; \ b=\frac{\sum XY}{\sum X^2}=\frac{29}{28}=1.036$$

Therefore, the required equation of the straight line trend is given by

Y = a+bX;

Y = 45.143 + 1.036 (x-2003)

The trend values can be obtained by

When X = 2000 , Yt = 45.143 + 1.036(2000–2003) = 42.035

When X = 2001, Yt = 45.143 + 1.036(2001–2003) = 43.071,

similarly other values can be obtained.

Example 9.7

Given below are the data relating to the sales of a product in a district.

Fit a straight line trend by the method of least squares and tabulate the trend values.

Year	1995	1996	1997	1998	1999	2000	2001	2002
Sales	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

Solution:

Computation of trend values by the method of least squares.

In case of EVEN number of years, let us consider

$$X = \frac{(x - Arithimetic\ mean\ of\ two\ middle\ years)}{0.5}$$

Year(x)	Sales(Y)	$X = \frac{(x - 1998.5)}{0.5}$	XY	X^2	Trend values(Y_t)
1995	6.7	−7	46.9	49	5.6166
1996	5.3	−5	26.5	25	5.7190
1997	4.3	−3	12.9	9	5.8214
1998	6.1	−1	6.1	1	5.9238
1999	5.6	7	39.2	49	6.0261
2000	7.9	5	39.5	25	6.1285
2001	5.8	3	17.4	9	6.2309
2002	6.1	1	6.1	1	6.3333
N=8	47.8	$\Sigma X = 0$	194.6	168	

Table 9.6

$$a = \frac{\sum Y}{n} = \frac{47.8}{8} = 5.975 \quad ; \; b = \frac{\sum XY}{\sum X^2} = \frac{4.3}{42} = 0.10238$$

Therefore, the required equation of the straight line trend is given by

$$Y = a + bX \quad ; \; Y = 5.975 + 0.10238\, X.$$

$$\text{When } X = 1995\,,\; Y_t = 5.975 + 0.10238\left(\frac{1995 - 1998.5}{0.5}\right) = 5.6166$$

$$\text{When } X = 1996\,,\; Y_t = 5.975 + 0.10238\left(\frac{1996 - 1998.5}{0.5}\right) = 5.7190$$

similarly other values can be obtained.

Note

(i) Future forecasts made by this method are based only on trend values.

(ii) The predicted values are more reliable in this method than the other methods.

(v) Methods of measuring Seasonal Variations By Simple Averages :

Seasonal Variations can be measured by the method of simple average. The data should be available in season wise likely weeks, months, quarters.

Method of Simple Averages:

This is the simplest and easiest method for studying Seasonal Variations. The procedure of simple average method is outlined below.

Procedure:

(i) Arrange the data by months, quarters or years according to the data given.

(ii) Find the sum of the each months, quarters or year.

(iii) Find the average of each months, quarters or year.

(iv) Find the average of averages, and it is called Grand Average (G)

(v) Compute Seasonal Index for every season (i.e) months, quarters or year is given by

$$\text{Seasonal Index (S.I)} = \frac{\textit{Seasonal Average}}{\textit{Grand average}} \times 100$$

(vi) If the data is given in months

$$\text{S.I for Jan} = \frac{\textit{Monthly Average (for Jan)}}{\textit{Grand average}} \times 100$$

$$\text{S.I for Feb} = \frac{\textit{Monthly Average (for Feb)}}{\textit{Grand average}} \times 100$$

Similarly we can calculate SI for all other months.

(vii) If the data is given in quarter

$$\text{S.I for I Quarter} = \frac{\textit{Average of I quarter}}{\textit{Grand average}} \times 100$$

$$\text{S.I for II Quarter} = \frac{\textit{Average of II quarter}}{\textit{Grand average}} \times 100$$

$$\text{S.I for III Quarter} = \frac{\textit{Average of III quarter}}{\textit{Grand average}} \times 100$$

$$\text{S.I for IV Quarter} = \frac{\textit{Average of IV quarter}}{\textit{Grand average}} \times 100$$

Example 9.8

Calculate the seasonal index for the monthly sales of a product using the method of simple averages.

Months / Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
2001	15	41	25	31	29	47	41	19	35	38	40	30
2002	20	21	27	19	17	25	29	31	35	39	30	44
2003	18	16	20	28	24	25	30	34	30	38	37	39

Solution:

Computation of seasonal Indices by method of simple averages.

months / Years	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
2001	15	41	25	31	29	47	41	19	35	38	40	30
2002	20	21	27	19	17	25	29	31	35	39	30	44
2003	18	16	20	28	24	25	30	34	30	38	37	39
Monthly Total	53	78	72	78	70	97	100	84	100	115	107	113
Monthly Averages	17.67	26	24	26	23.33	32.33	33.33	28	33.33	38.33	35.67	37.67

Table 9.7

$$\text{S.I for Jan} = \frac{\textit{Monthly Average (for Jan)}}{\textit{Grand average}} \times 100$$

$$\text{Grand Average} = \frac{355.582}{12} = 29.63$$

$$\text{S.I for Jan} = \frac{17.666}{29.361} X100 = 59.62 \,; \quad \text{S.I for Feb} = \frac{26}{29.361} X100 = 87.77 \,;$$

Similarly other seasonal index values can be obtained.

Months	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
Seasonal Index	59.62	87.77	80.99	87.77	78.74	109.12	112.49	94.49	112.49	129.36	120.36	126.89

Example 9.9

Calculate the seasonal index for the quarterly production of a product using the method of simple averages.

Year	I Quarter	II Quarter	III Quarter	IV Quarter
2005	255	351	425	400
2006	269	310	396	410
2007	291	332	358	395
2008	198	289	310	357
2009	200	290	331	359
2010	250	300	350	400

Solution:

Computation of Seasonal Index by the method of simple averages.

Year	I Quarter	II Quarter	III Quarter	IV Quarter
2005	255	351	425	400
2006	269	310	396	410
2007	291	332	358	395
2008	198	289	310	357
2009	200	290	331	359
2010	250	300	350	400
Quarterly Total	1463	1872	2170	2321
Quarterly Averages	243.83	312	361.67	386.83

Table 9.8

$$\text{S.I for I Quarter} = \frac{Average\, of\, I\, quarter}{Grand\, average} \times 100$$

$$\text{Grand Average} = \frac{1304.333}{4} = 326.0833$$

$$\text{S.I for I Q} = \frac{243.8333}{326.0833} \times 100 = 74.77\,;$$

$$\text{S.I for II Q} = \frac{312}{326.0833} \times 100 = 95.68\,;$$

$$\text{S.I for III Q} = \frac{361.6667}{326.0833} \times 100 = 110.91\,;$$

$$\text{S.I for IV Q} = \frac{386.833}{326.0833} \times 100 = 118.63$$