H.W. 8

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Image:

1	1	9
1	1	9
9	9	9

Data

RGB

1 1 1
1 0 1
0 1 1
9 9 8
9 9 9
8 9 8

Q1

F.G = I(2,2) = 1

B.G = I(3,3) = 9

Edge weights b/w sink and pixels = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Edge weights b/w source and pixels = $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Q2

weights b/w 'p' and 'q' = $\exp\left(-(I_p - I_q)^2\right)$

weights around centre = $\begin{bmatrix} 1 & 1 & e^{-64} \\ 1 & 1 & e^{-64} \\ e^{-64} & e^{-64} & e^{-64} \end{bmatrix}$

Data.

Q3

R	G	B	
1	1	1	→ cluster 1
1	0	1	
0	1	1	
9	9	8	→ cluster 2
9	9	9	
8	9	8	

GMM Components.

① Cluster 1.

(μ) mean = $[0.66, 0.66, 1]$

covariance Matrix $(\Sigma) = (X - \bar{X})^T (X - \bar{X}) / n-1$

$$A = X - \bar{X} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 2/3 & 2/3 & 1 \\ 2/3 & 2/3 & 1 \\ 2/3 & 2/3 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 0 \\ 1/3 & -2/3 & 0 \\ -2/3 & 1/3 & 0 \end{bmatrix}$$

$\therefore \Sigma = A^T A / 2$ as $n = 2$

$$\Sigma = \begin{bmatrix} 0.33 & -0.167 & 0 \\ -0.167 & 0.33 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

weight $(w) = 9/18 = 1/2$

5) Cluster 2

(i) mean = $\begin{bmatrix} 8.667 & 9 & 8.33 \end{bmatrix}$

cov matrix $(\Sigma) = (x - \bar{x})^T (x - \bar{x}) / n-1$

$$A = \begin{bmatrix} 9 & 9 & 8 \\ 9 & 9 & 9 \\ 8 & 9 & 8 \end{bmatrix} - \begin{bmatrix} 8.667 & 9 & 8.33 \\ 8.667 & 9 & 8.33 \\ 8.667 & 9 & 8.33 \end{bmatrix}$$

$$= \begin{bmatrix} 0.33 & 0 & -0.33 \\ 0.33 & 0 & 0.667 \\ -0.667 & 0 & 0.33 \end{bmatrix}$$

$\Sigma = A^T A / 2$

$$\Sigma = \begin{bmatrix} 0.33 & 0 & 0.1667 \\ 0 & 0 & 0. \\ 0.1667 & 0 & 0.33 \end{bmatrix}$$

weight $(w) = 9/18 = 1/2$