

Day - 20

Module - vi :- Intro to graphs graph types,

term terminologies, Representation of graph graph,  
Graph traversal - BFS, DFS, Spanning tree, Minimum  
spanning tree, shortest Path algorithms.

Def hashing - Functions, Load factor and  
collision, (open addressing (linear probing)).

Chaining method.

Module - V :- Binary trees :- Intro, types,  
terminology, code, Representation.

BST :- Intro, representation, Code

Sequential representation B-in tree

B tree, B+ tree

Module - iii :- Comparison linear binary, Big  
 $O$  notation -

Selection, Bubble :- Define, how it works.

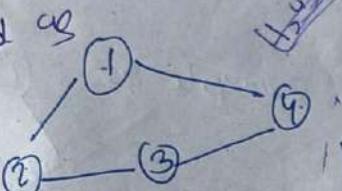
## Module 7 :-

Intro to graph - Graph is a non linear data structure which is made of vertices and edges. It's used to represent relationships between two objects and it has many real life implementations like :- Website backlinks, telephone network, social media network.

Graph is represented as

$$G = (V, E)$$

where  $G$  is the graph which consists of  $|V|$  vertices and  $|E|$  edges

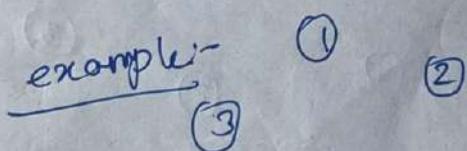


$$|V| = \{1, 2, 3, 4\}$$

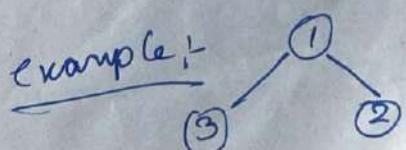
$$E = \{(1, 2), (1, 4), (2, 3), (2, 4), (3, 2), (3, 4), (4, 1), (4, 3)\}$$

Graph types :- There are general types of graphs :-

- Vertex / vertices :- It's an entity which holds multiple data / information in it.



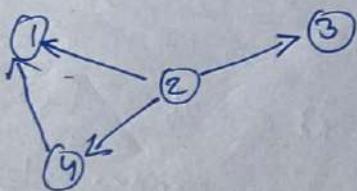
- Edge :- Edge is a connecting point between two vertices and it may



- Direct edges :- An edge which has direction known as

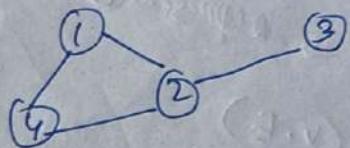
direct edge .

ex:-



- Undirect edge :- An edge ~~which~~ which doesn't have any direction on its bidercted .

example :-



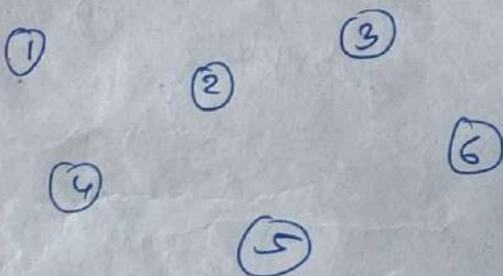
- Trivial graph :- A graph containing no edges and one vertex

example :-



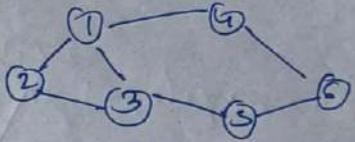
- Null Graph :- Gets like a trivial graph if several vertices and no edges .

example -



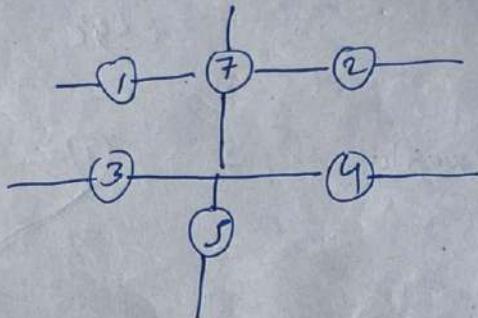
- Finite graph :- If a graph's nodes and edges are limited in a number.

example :-



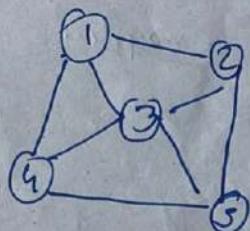
- Infinite graph :- If a graph's ~~has~~ vertices or edge infinite.

example :-



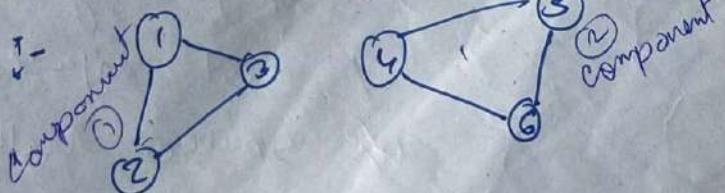
- Connected graph :- If ~~an edge~~ there's always an edge between two vertices for travel it's known as connected graph.

example :-



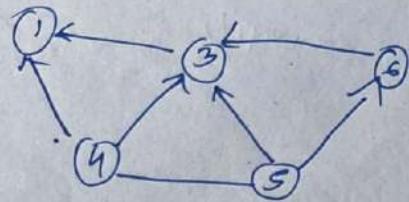
- Disconnected graph :- When there's no edge b/w two vertices in a gm.

example :-



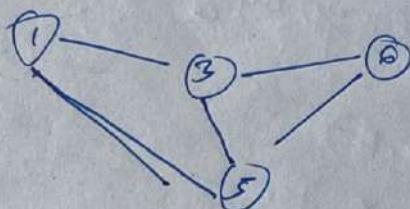
• Direct graph :- A graph's all the edges are direct.

example:-



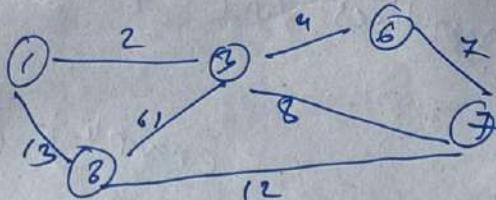
• Undirected graph:- A graph's all the edges are undirected.

example:-



• Weighted graph:- If there's a cost value assigned to edges of a graph +

example:-



Representation of a graph:- Graph can be represented in several ways.

1) Adjacency List

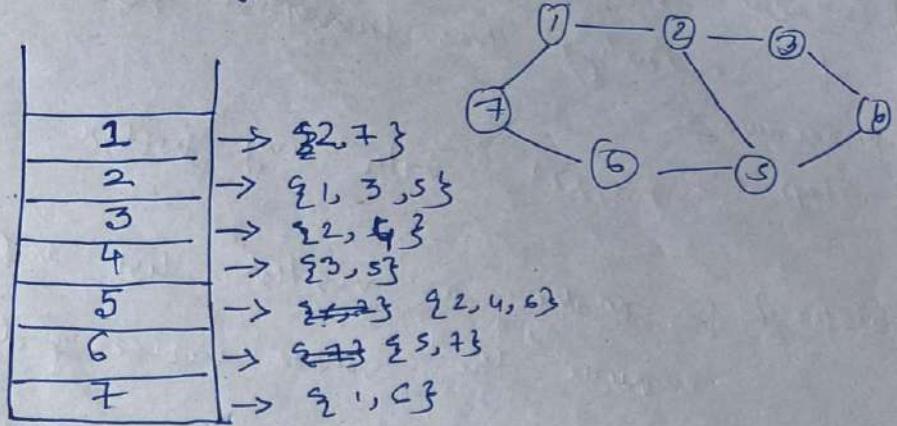
2) Adjacency Matrix

3) Cost adjacency list

4) Sparse Matrix

5) Cost adjacency matrix

## 10 Adjacency list



Adjacency matrix

	1	2	3	4	5	6	7
1	0	1	0	0	0	0	1
2	1	0	1	0	1	0	0
3	0	1	0	1	0	0	0
4	0	0	1	0	0	1	0
5	0	1	0	1	0	1	0
6	0	0	0	0	1	0	1
7	1	0	0	0	0	1	0

[if  $a_{ij} \neq 0$   
then there's an  
edge b/w  
then 1  
otherwise  
0]

- Graph traversal :- It's a method to traverse one vertex to another vertex through an connected edge between them for a particular reason.
- There are two types of graph traversal

BFS, DFS :-

- DFS :- It's a well known traversal technique for traversing a graph

Queue ADT is implemented by the time of executing this traversal technique.

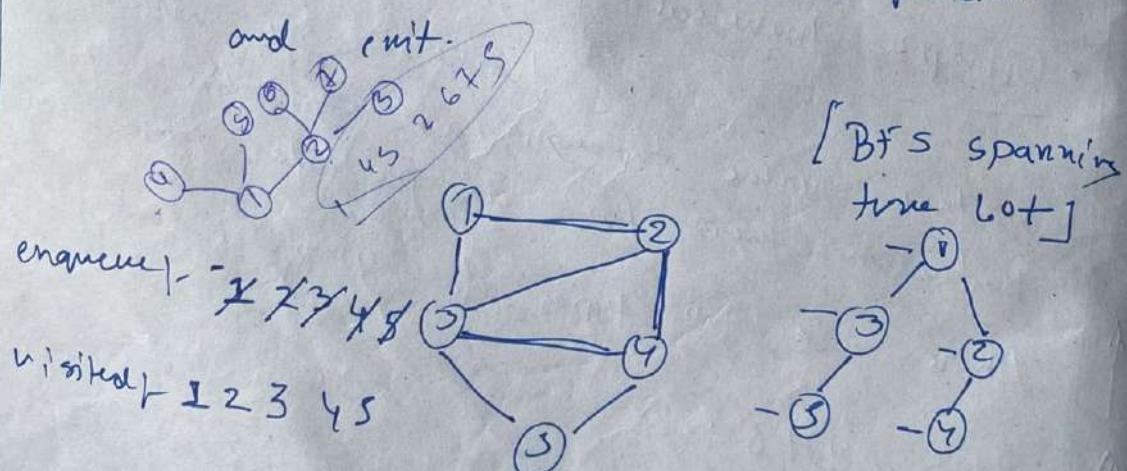
procedure:-

Step 1:- Start with any vertex.

Step 2:- 1- Make a visited list and enqueue the source vertex and mark it visited afterwards enqueue all the adjacent vertices of that source vertex and ~~dequeue~~ mark them visited.

Step 3:- Make this process Repeat unless all the vertices are visited.

Step 4:-  $\oplus$  Return the BFS to main function



• BFS traversal is not unique!

~~DFS~~ - It's a graph traversal technique.

Stack ADT is initialised while executing ~~tree~~ this traversal.

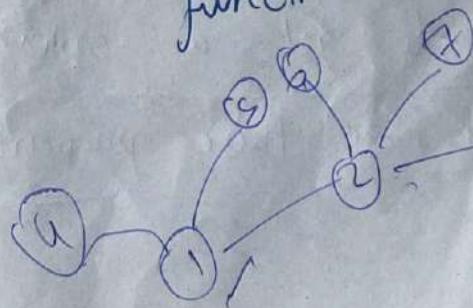
Procedure :-

Step 1- Start with any vertex.

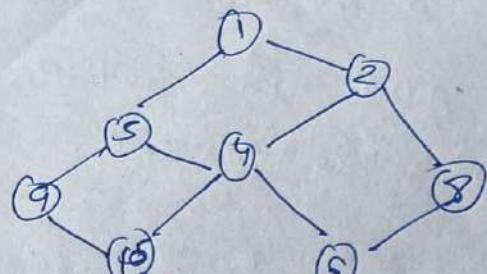
Step 2- Initialise a stack and travel a source ~~vertex~~ vertex and ~~not~~ push it onto the stack and pop it and if any one of its adjacent vertex has not been visited push it onto the stack and if all the adjacent vertices of the source vertex is traversed then back track and travel the remaining ones.

Step 3- Repeat ~~the~~ Step 2 unless and until all the vertices are visited and receive the dfs result.

Step 4- Return the dfs from the main function and exit.

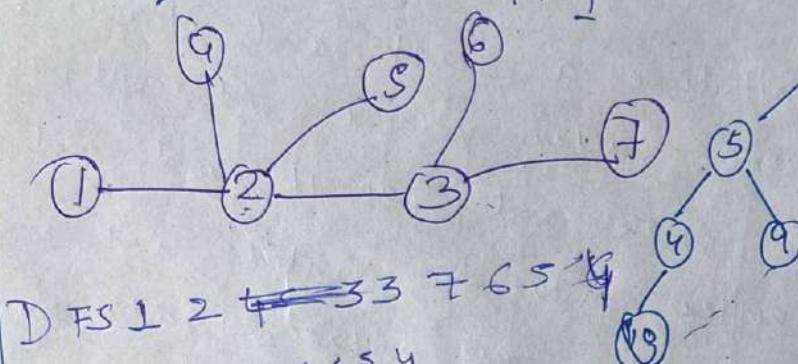
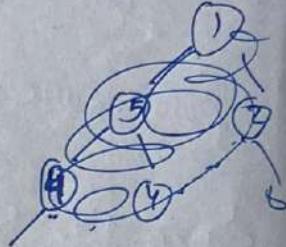


DFS: 1 2 3 4 5 6 7  
BFS: 1 2 3 4 5 6 7



Stack [1, 2, 8, 6, 7, 4, 5, 9, 10]

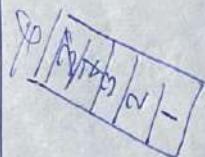
Result [1 2 8 6 7 4 5 9 10]



DFS 1 2 3 5 6 7 4 9 8 1

Result :- 1 2 3 5 6 7 4

DFS spanning tree



Inorder

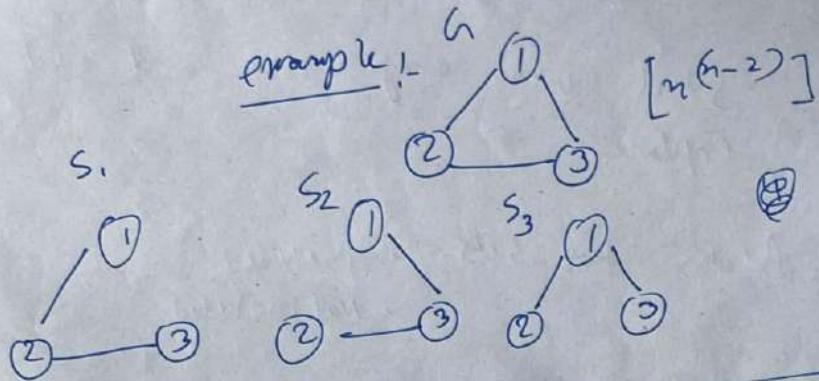
Preorder

Result :-

[10, 4, 5, 9, 1,  
2, 8, 5, 7]

- Spanning tree ? A subgraph of a graph "G" is known as spanning tree. for a "n" no. of "G" graph  $n(n-1)$  Spanning trees are possible.  $S \subseteq G$  where  $|V|$  vertices and  $|V|-1$  edge must be present on the subgraph "S". In order to get a spanning tree a graph must have these properties.

- Connected :- There's always an edge b/w two vertices
- complete :- An undirected graph.
- Acyclic Graph :- There's no cycle in a graph -



- Minimum-Cost Spanning trees :- A minimum cost spanning tree can be constructed for weighted, undirected, connected graphs. Where the some or cost values are given on a graph's edge, the cost value denotes storage, time, or cost, space anything. Since there are  $n^{(n-2)}$  spanning trees a graph can have searching for minimum cost spanning trees are ~~too~~ time taken. So we use two greedy methods to find mst of a graph.

Dinic's  
Algorithm

Prim's Algo is based on the well known algorithm to finding MST of a graph in an efficient way. The T.C. of this algorithms is  $O(V^2)$

### Procedure

Step 1:- Start with any vertex and start explore

Step 2:- Make two sets • included  
• Not included

and the source vertex into included mst and all the remaining ones ~~or~~ in not included mst.

Step 3:- Start exploring and comparing from the source vertex to • the adjacent vertices, if any ~~vertices~~ adjacent ~~edges~~ ~~edges~~ edge's cost is • minimum then simply add it to the included mst set. Repeat this unless and until all the vertices are explored and not included set ~~or~~ contains  $\emptyset$  value.

Step 4 - Sum all the cost & mark exit.

Try not to create a cycle

$$E \rightarrow C = 2$$

$$E \rightarrow F = 3$$

$$C \rightarrow A = 4$$

$$C \rightarrow B = 2$$

$$C \rightarrow D = 3$$

$$C \rightarrow F = 4$$

$$E \rightarrow F = 3$$

$$B \rightarrow A = 4$$

$$C \rightarrow A = 4$$

$$C \rightarrow D = 3$$

$$C \rightarrow F = 4$$

$$E \rightarrow F = 3$$

$$V = \{A, B, C, D, E, F\}$$

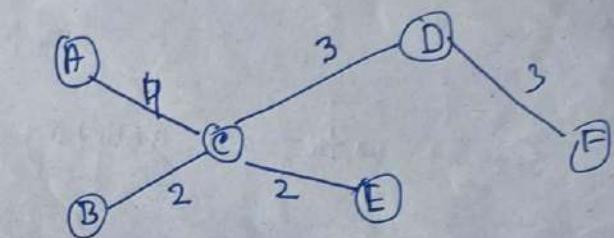
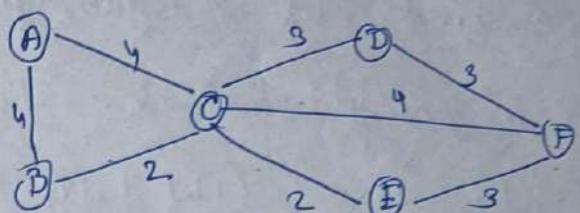
$$V_{\Sigma} = \{A, B, D, E, F\}$$

$$V = \{A, D, F\}$$

$$V = \{A, F\}$$

$$V = \{\}$$

$$V = \{\}$$



$$\text{MST} \vdash 4 + 2 + 2 + 3 + 3 \\ = 14 \quad (\text{Any})$$

$$A = \{E\}$$

$$A = \{C, E\}$$

$$A = \{B, C, E\}$$

$$A = \{B, C, D, E\}$$

$$A = \{B, C, D, E, F\}$$

$$A = \{A, B, C, D, E, F\}$$

Dijkstra Algorithm :- This algorithm is used for finding

Single Source Shortest Path of a graph. It was conceived by Edsger W. Dijkstra in 1956. It's a greedy method. Time Complexity of this algo is  $O(|E| + |V| \log |V|)$

Procedure :-

Step 1:- Start with an arbitrary vertex

Step 2:- Make a table where left hand side source vertex and Destination vertex in right hand side.

Step 3:- Set the value of current vertex to "0" and remaining ones to " $\infty$ ".

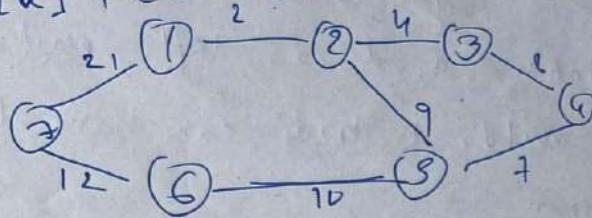
Step 4:- Every time compare if a adjacent vertex edge is smallest and pick it and try to reach destination as minimum cost as can be, ~~destination~~ - ~~if~~ ~~6~~

Step 5:- Make these process repeat unless and until all the ~~vertex~~ the reached by point is found.

Step 61- Return the shortest path to main function and exit.

Relaxation

if  $(d[u] + c(u,v)) < d[v]$  then  
 $d[v] = d[u] + c(u,v)$



$$S \rightarrow 1 = 9 + 2 \\ = 11$$

$$S \rightarrow 3 = 9 + 4$$

= 13

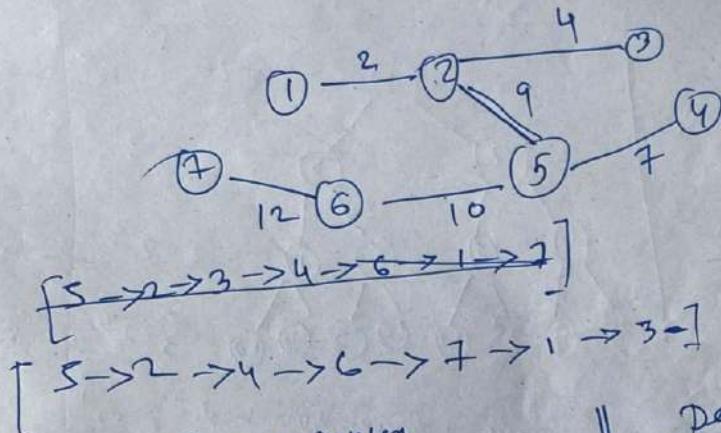
$$S \rightarrow 4 = 7$$

$$S \rightarrow 5 = 10$$

$$S \rightarrow 6 = 7 \\ = 10 + 12$$

= 22

$$S \rightarrow 2 \rightarrow 1 \\ = 9 + 2 + 21 \\ = 23$$



Source vertex

5  
0  
5, 2

S, 2, 4

S, 2, 4, 6

S, 2, 4, 6, 1

S, 2, 3, 4, 6, 1

S, 2, 3, 4, 6, 1, 7

Destination vertex

1, 2, 3, 4, 7, 6  
 $\infty \quad \infty \quad \infty \quad \infty \quad \infty \quad \infty$

$\infty \quad 9 \quad \infty \quad \infty \quad \infty \quad \infty$

$\infty \quad 9 \quad \infty \quad 7 \quad \infty \quad \infty$

$\infty \quad 9 \quad \infty \quad 7 \quad \infty \quad 10$

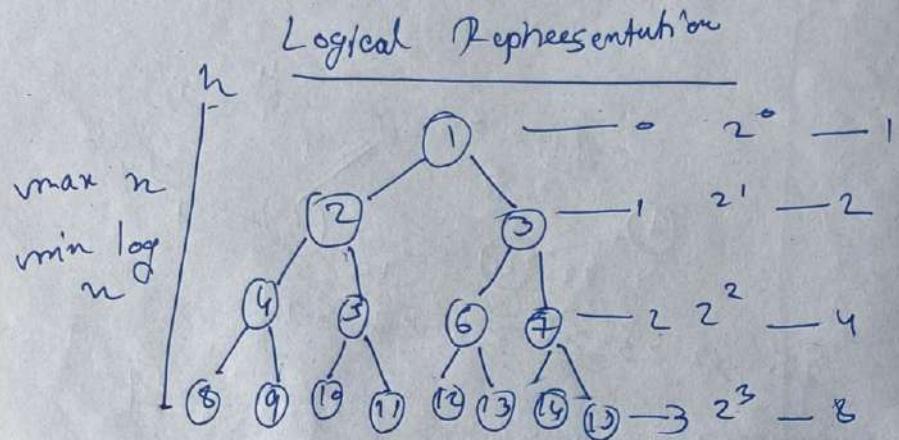
11, 9,  $\infty$ , 7,  $\infty$ , 10

11, 9, 13, 7,  $\infty$ , 10

11, 9, 13, 7,  $\infty$ , 10

Module 2  $\rightarrow$  VI

Binary trees: It's a non linear hierarchical data structure which have a data section and a left child and a right child. A node can have at most two children.



A tree can have maximum node  $= 2^h$  and maximum child  $= 2^{(h+1)}$ ,  
min child  $= \underline{2^{(h+1)}} h+1$

Terminology:

- (1) Node
- (2) Edge
- (3) Root
- (4) child
- (5) Leaf
- (6) height
- (7) parents
- (8) siblings
- (9) Interval
- (10) Non interval

[node possible binary trees with  $n$  nodes]

$$\Theta(n!(2^n - n))$$

and 4 nodes

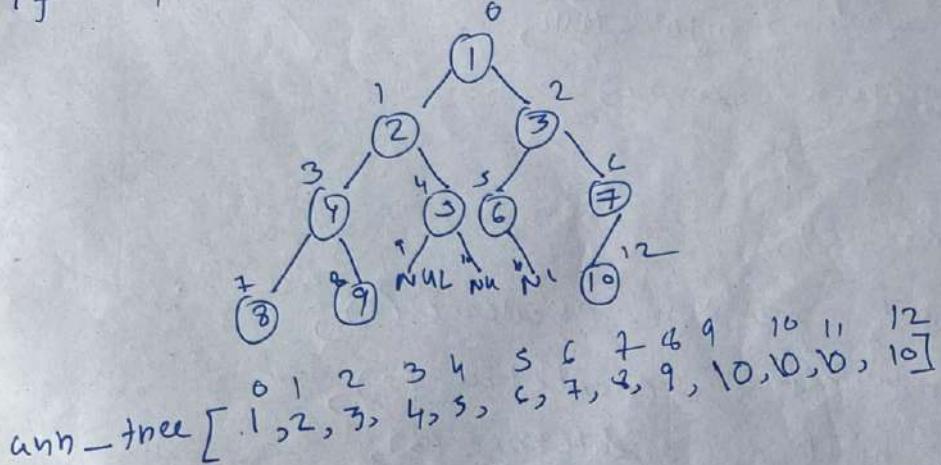
$$14(2n!)/$$

$$(n)! \times (n+1)!$$

## • Representation I

### ① Array Representation / Sequential

Sequential representation is an array representation of a binary tree where the root node is in '0' th index and left child  $2^{i+1}$  and right child  $2^{i+2}$ . If the array's index starts with "0"



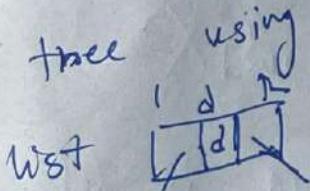
root +  $\frac{1}{2}$

left +  $2^{i+1}$

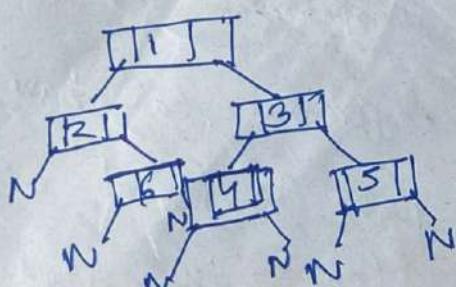
right +  $2^{i+2}$

② Linked Representation :- A common representation and implementation of a

tree using linked list. We use doubly linked



example



code:-

```
Struct node *  
{  
    int data;  
    Struct node * left;  
    Struct node * right;  
}
```

Struct node \* createtree (int data)  
{

```
Struct node * newnode;  
newnode = malloc (sizeof (Struct node));
```

newnode -> data = data;

newnode -> left = NULL;

newnode -> right = NULL;

return newnode;

}

main () {

```
Struct node * n = createtree (1);
```

n -> left = createtree (2);

n -> right = createtree (3);

free (n);

return 0;

}

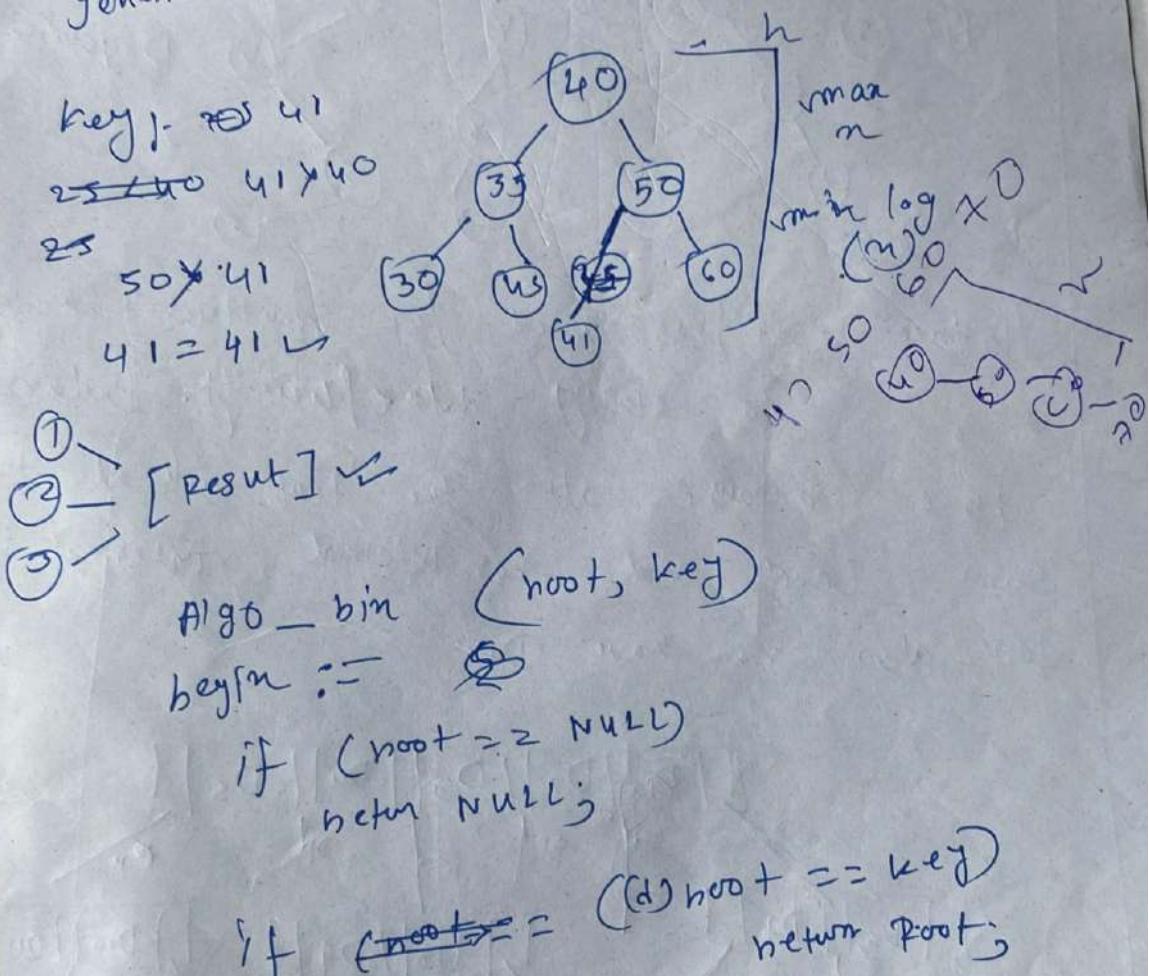
- Bst:- It's special kind of tree structure used to searching purpose. Searching can be done in  $\log(n)$  time complexity.

Properties:-

- A node can have at max two child
- For "n" no of left ~~no~~ child is less than the root node & right. only 1 d is

is greater than the root node.

- Searching time depend on the height of the BST.
- Max height of a BST can have " $n$ ".  
time and min can have  $\log(n)$  time.  
freeness
- From " $n$ " no of ~~BST~~  $n!$  BST can be generated. [Duplicates are not allowed].

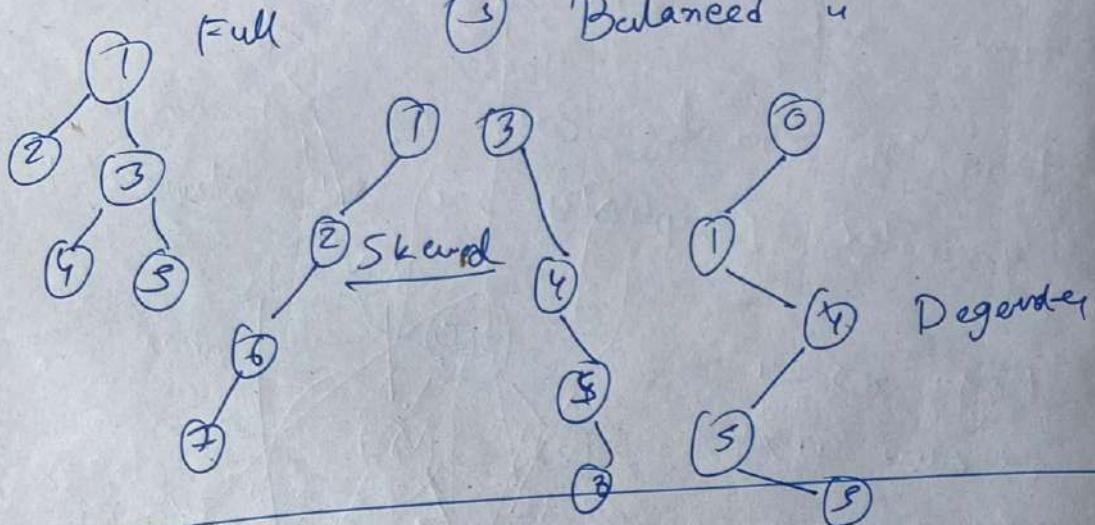


else if ( $(d) root > \text{key}$ )  
return ( $root \rightarrow \text{left}, \text{key}$ )

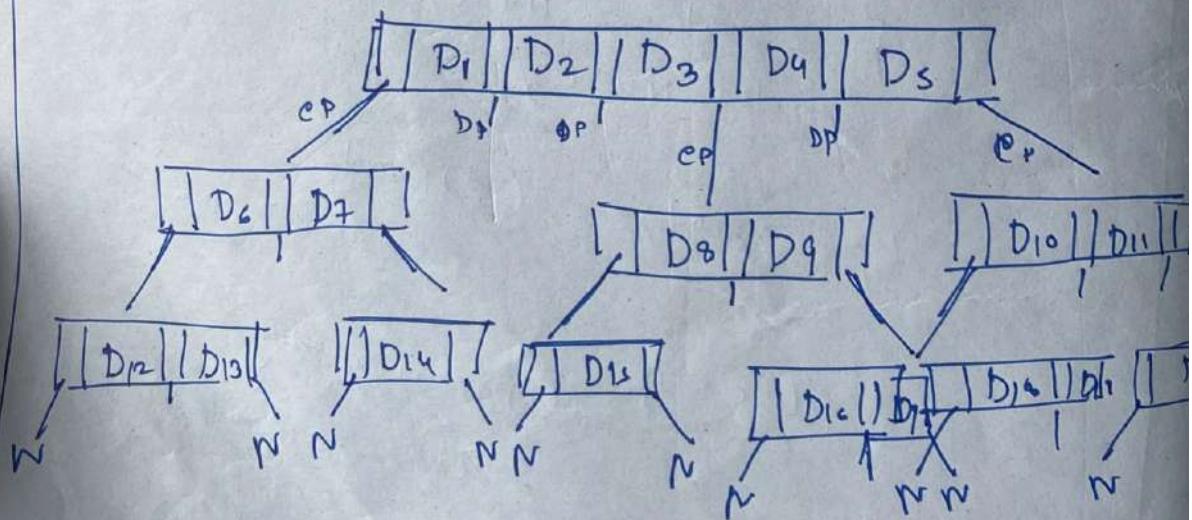
else return ( $root \rightarrow \text{right}, \text{key}$ );

end :-

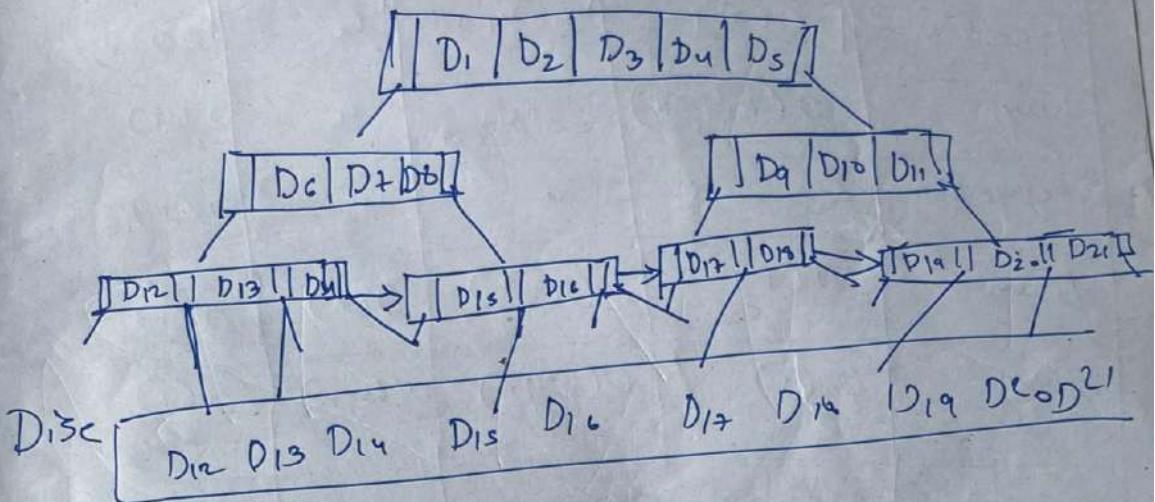
- To Types of trees
- ① Full binary
  - ② Perfect "
  - ③ Skewed "
  - ④ Degenerate "
  - ⑤ Balanced "



B-tree :- It's used for multilevel indexing to store data efficiently. Balanced tree because all the leaf are at the same level and non internal, internal has DP also can have DP



B + tree Same as B tree But only leaf nodes can have DP which points to a disc sector in a secondary memory and all the leaf nodes are connected with a next node pointer like a linked list



### B tree

1) Pointers can be found at any node

2) Since pointers can be found at any node search time minimizes

3) Insertion is difficult

4) Deletion is time taken

5) Tree can have minimum time  $O(n)$

6) No linked list

### B + tree

1) only at leaf node

2) only in leaf node minimum time

3) insertion is easy

4) deletion is easy

5) minimum time  $\log(n)$

6) linked list

	Time Complexity	Space Complexity	
	best	worst	s.e
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Bubble	"	"	"
Insertion	$O(n^2)$	"	"
Heap	$O(N \log N)$	"	"
Merge Sort	"	"	$O(N)$
Linear	$O(1)$	$O(n^2)$	$O(n)$
Binary	$O(1)$	$O(\log n)$	$O(\log n)$
Dijkstra	$O( V  +  E  \log  V )$		
Prims	$O( V  \log  V  +  E  \log  V ) = O( E  \log  V )$		
Kruskal	$O( E  \log  E )$		

### Linear

- 1) It's a sequential search technique

$$\text{if } O(n)$$

### Binary

2) Every time it eliminates the half of the array at each iteration and compare the mid element with the key

$$2) O(\log n)$$

3) Works for small dataset | 3) Big dataset.

4) Deals with sorted and  
unsorted both data set | 4) Only sorted

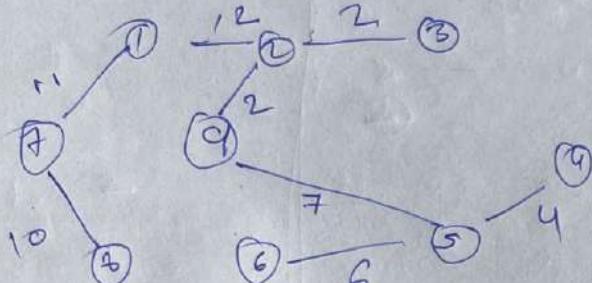
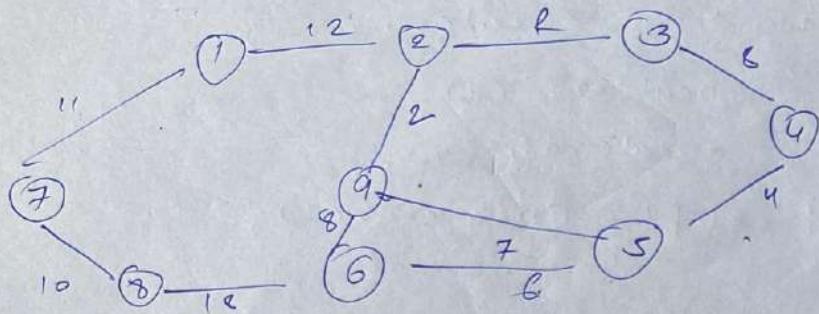
5) for ( $i=1$  to  $n$ ) | 5)  $mid = (i+1)/2$   
if ( $a[i] == x$ )  
return  $i$ ; | if ( $i == j$ )  
return  $-1$ ; |  
~~else~~  $j = mid + 1$  | if ( $a[mid] == x$ )  
return  $mid$ ; |  
else if ( $a[mid] > x$ )  
 $j = mid + 1$  | fun call(); |  
else  $i = mid + 1$  | fun call(); |

Tree, SK

#include <stdio.h>  
#include <stdlib.h>  
#include <assert.h>

struct node {  
 int data;  
 struct node \*left;  
 struct node \*right;  
};

struct node \*root = NULL;



$$MST \quad 6 + 4 + 7 + 2 + 2 + 12 + 11 = 44$$

$$V = \{1, 2, 3, 4, 5, 6, 7, 9\}$$

$$A = \{6, 7\}$$

$$V = \{1, 2, 3, 4, 9, 8, 7\}$$

$$A = \{5\}$$

$$V = \{1, 2, 3, 8, 7\}$$

$$A = \{4, 5, 6\}$$

$$V = \{1, 8, 7\}$$

$$A = \{2, 3, 4, 5, 6, 9\}$$

$$V = \{8\}$$

$$A = \{1, 2, 3, 4, 5, 6, 9\}$$

$$V = \{8\}$$

$$A = \{1, 2, 3, 4, 5, 6, 7, 9\}$$

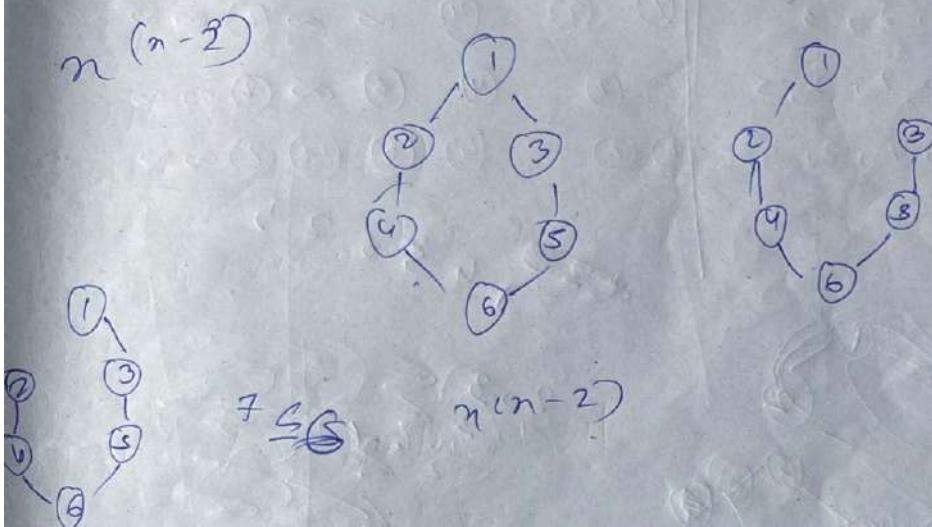
$7 \rightarrow 7 = 11$   
 $6 \rightarrow 8 = 16$   
 $6 \rightarrow 8 = 18$   
 $7 \rightarrow 8 = 10$

$6 \rightarrow 7 = 1$   
 $6 \rightarrow 8 = 18$   
 $5 \rightarrow 4 = 4$   
 $6 \rightarrow 9 = 28$   
 $6 \rightarrow 8 = 28$   
 $4 \rightarrow 3 = 28$   
 $5 \rightarrow 9 = 27$   
 $6 \rightarrow 9 = 28$   
 $6 \rightarrow 8 = 18$   
 $4 \rightarrow 3 = 28$   
~~5~~  
 $6 \rightarrow 8 = 18$   
 $9 \rightarrow 2 = 22$   
 $4 \rightarrow 3 = 28$   
 $6 \rightarrow 8 = 18$   
 $2 \rightarrow 1 = 12$   
 $2 \rightarrow 3 = 2$   
 $2 \rightarrow 12 = 12$   
 $6 \rightarrow 8 = 18$

Spanning tree: Spanning tree is a Subgraph  
of a graph

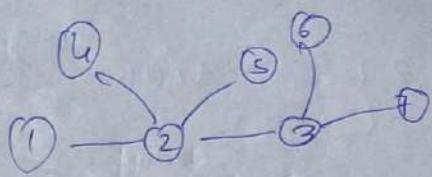
$$S \subseteq G$$

$S$  is a subgraph of " $G$ " with  $n$  vertices must be present in " $S$ " and  $|V|-1$  edges, in order to have a spanning tree a graph must have - Connected  
- Complete  
- Acyclic  
-  $|V|$   
-  $|V|-1$



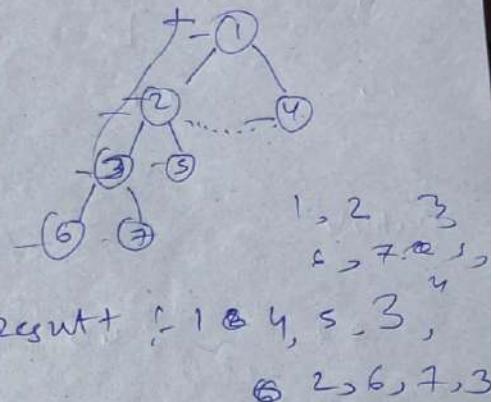
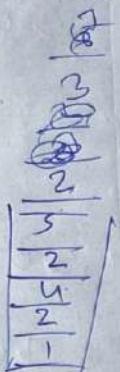
Dijkstra :- It's a method for finding single source shortest path for a graph " $G$ ". The way conceived by the Dutch computer Scientist Edsger W. Dijkstra in 1956, the complexity of Dijkstra is  $O(V^2)$  ( $|V| \times |V|$ )  
it can be improved to  $O(|V| + |E| \log |V|)$   
 $O(M + |E| \cdot \log |V|)$

BFS :- Breadth First Search



⑧  $\exists x \forall y \forall z$

Result 1 2 3 4 5 6 7

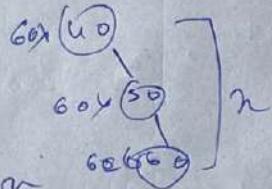


DFS1 - S7k

Bst

40 50 60 40 60 50

key } 60



60 40 50

$$\text{[GDP] } \text{---} \text{[ADP]} ]_n$$

6. 50 60 40

50 60 70 ] n

```

graph TD
    n[n] --- 60((60))
    n --- 50((50))
    n --- 40((40))

```

- Selection

22      40      14      30      66

14      40      22      30      66  
14      22      40      30      66

Bubbly

22      40      14      30      66  
22      14      40      30      66  
22      14      30      40      66

5                  4                  3                  2      1  
4                  5                  3                  2      1  
4                  3                  5                  2      1  
4                  3                  2                  2      1  
4                  3                  2                  5      1  
4                  3                  2                  1      5 P-1  
3                  4                  2                  1      5  
3                  2                  4                  1      5  
3                  2                  1                  4      5 2-2  
2                  3                  1                  4      5  
2                  1                  3                  4      5 P-3

## Binary Search

	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
5	1	2	3	4

key = 4  
 $0 + 4$   
 $= \frac{4}{2} 2$

1) mid = 2 = 2 < 4 ✓

2) mid = 3 = 2 < 4 ✓

3) mid > 4 ✓  
 mid = 4 ↴

4) mid = 4 ↴

$$\frac{3+4}{2} = \frac{7}{2}$$

3

③ → ④



T-C

Selection

$O(n)$   $O(n)$   $O(n^2)$   $O(1)$

Bubble

$O(n)$   $O(n)$   $O(n)$   $O(1)$

Insertion

$O(n)$   $O(n)$   $O(n)$   $O(1)$

Linear

$O(1)$   $O(n)$   $O(n)$   $O(1)$

Binary

$O(1)$   ~~$O(n)$~~   $O(\log n)$   $O(1 \log n)$   $O(1)$

Heap

$O(n \log n)$   $n(\log n)$   $O(n \log n)$   $O(n \log n)$

Quicksort

u u u  $O(n)$

Dijkstra

$O((V+E) + \log(V))$   $O(V^2)$

Prim

$O((V+E) + \log(V))$   $O(V^2)$



2) multi  $(A[2], B[2], n) \in$   
 for  $i=0; i < n-1; i++) \in$   
 $n$   
 $\frac{2x}{n+1} - \text{for } (i=0; i < n-1; i++) \in (n)$   
 $(n+1) * n \left\{ \begin{array}{l} \text{for } (j=0; j < n; j++) \in \\ c[i][j] = a[i][k] + b[k][j] \end{array} \right.$   
 ~~$n \times n$~~   
 ~~$0(n^2) + 2n + 1$~~   
 ~~$(n+1) * n$~~   
 $n \times m \times n \quad c[i][j] = a[i][j] + b[j][i]$   
 $2n^3 + 3n^2 + 2n + 1$   
 $O(2n^3)$   
 $O(n^2)$

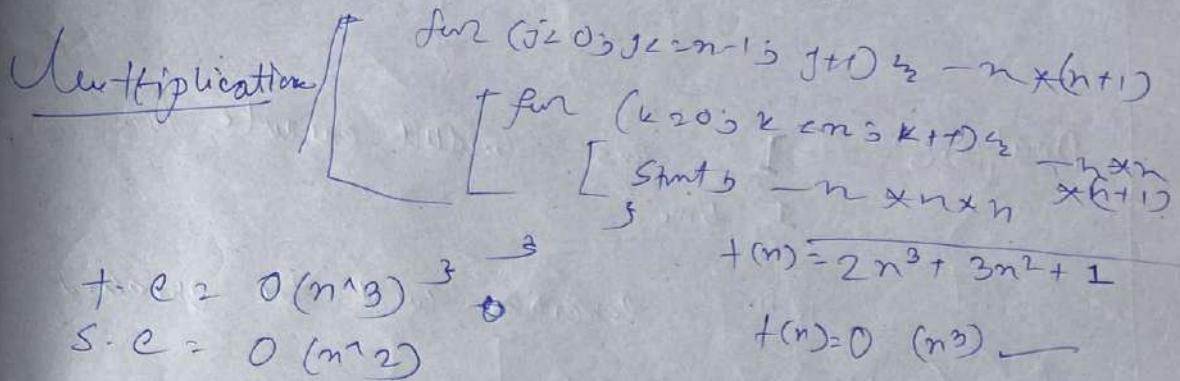
<u>S.C.</u>	
i	- 1
j	- 1
#	- n^2
c	- n^2
b	- n^2
m	- 1
	( ) (n^2)

for  $i = 1$  to  $n$  do  $\Sigma$   $O(n^2)$

for  $(j = 1 \text{ to } 1) \quad j_3$   
 $\frac{i}{2} \quad \frac{j}{1} \quad \text{en}$   
 $i < 10$   
 $\cancel{2} \leftarrow$   
 $122v$   
 $2 < 20$   
 $\cancel{1} \leftarrow$   
 $32$   
 $\cancel{2} \leftarrow$   
 $32$   
 $\cancel{3} \leftarrow$   
 $49$

Time Complexity!

2) for ( $i=0, i < n, i++$ )  $\in \Theta(n)$



2) for ( $i=0, i < n, i++$ )  $\in \Theta(n)$

for ( $j=0, j < n, j+1$ )  $\in \Theta(n)$

$C[i][j] = A[i][j] + B[i][j] \in \Theta(1)$

S.C

$A - n^2$	$B - n^2$	$C - n^2$
$i - n$	$j - n$	$k - n$
$\sum_{i=0}^{n-1} n = n^2$	$\sum_{j=0}^{n-1} n = n^2$	$\sum_{k=0}^{n-1} n = n^2$

$t.c = O(n^2)$

S.C

$A - n^2$	$B - n^2$	$C - n^2$
$i - n$	$j - n$	$k - n$
$\sum_{i=0}^{n-1} n = n^2$	$\sum_{j=0}^{n-1} n = n^2$	$\sum_{k=0}^{n-1} n = n^2$

$t.c = O(n^2)$

$S.C = O(n^2)$

3) for ( $i=0, i < n, i++$ )  $\in \Theta(n)$

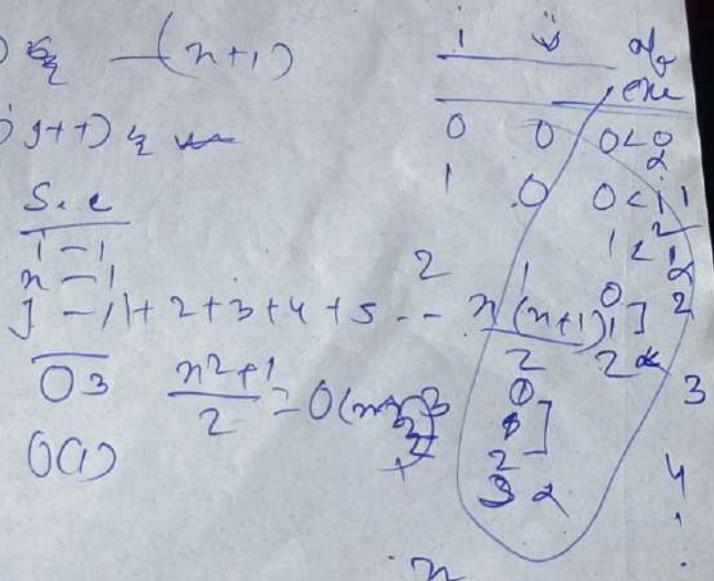
for ( $j=0, j < i, j+1$ )  $\in \Theta(i)$

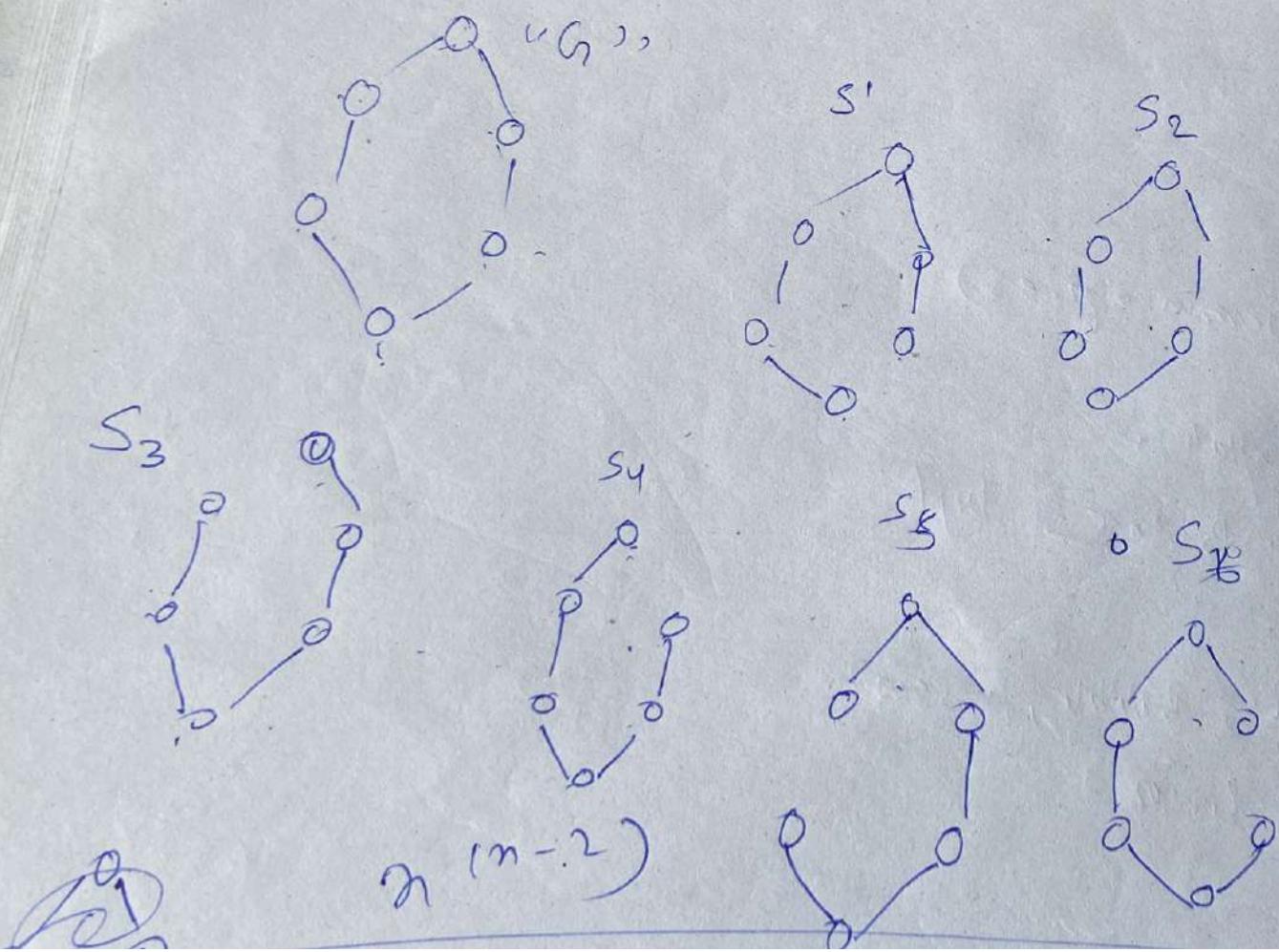
stmt;  $\in \Theta(i)$

$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$

$S.C = O(1)$

$T.C = O(n^2)$ .





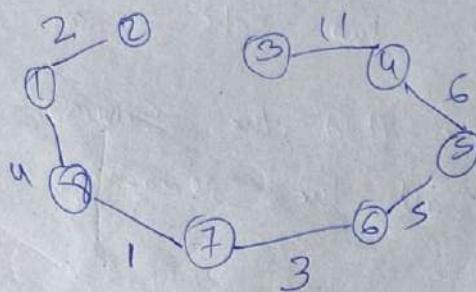
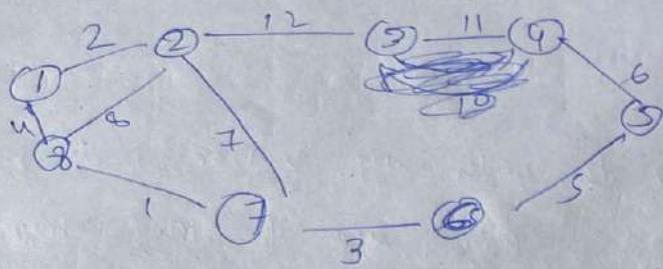
Prims:- It's a gr  
 Cost spa  
 Efficient and

Procedure:-

Step 1:- Start  
 minimum

Step 2:- Now

Step 6:- Return the MST from main function and exit.



$$\begin{aligned} 7 \rightarrow 2 &= 7 \\ 7 \rightarrow 6 &= 3 \\ 7 \rightarrow 4 &= 1 \end{aligned}$$

$$\begin{aligned} 7 \rightarrow 2 &= 7 \\ 7 \rightarrow 6 &= 3 \\ 2 \rightarrow 1 &= 4 \\ 3 \rightarrow 2 &= 8 \end{aligned}$$

$$\begin{aligned} 6 \rightarrow 5 &= 5 \\ 7 \rightarrow 2 &= 7 \\ 8 \rightarrow 1 &= 4 \\ 3 \rightarrow 2 &= 8 \\ 1 \rightarrow 2 &= 2 \\ 6 \rightarrow 3 &= 5 \\ 7 \rightarrow 2 &= 7 \\ 8 \rightarrow 2 &= 8 \end{aligned}$$

$$1+3+4+2+5+6+11 = 32 \text{ (Ans)}$$

$$V = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$\begin{aligned} 2 \rightarrow 3 &= 12 \\ 6 \rightarrow 5 &= 5 \end{aligned}$$

$$\begin{aligned} 2 \rightarrow 3 &= 12 \\ 5 \rightarrow 4 &= 6 \end{aligned}$$

$$V_2 = \{1, 2, 3, 4, 5\}$$

$$A = \{7, 8\}$$

$$\begin{aligned} 2 \rightarrow 3 &= 12 \\ 9 \rightarrow 3 &= 11 \end{aligned}$$

$$V = \{1, 2, 3, 4, 5\}$$

$$A = \{1, 6, 7, 8\}$$

$$V_2 = \{4, 3, 2\}$$

$$A = \{1, 2, 6, 7, 8\}$$

$$V_2 = \{4, 3\}$$

$$A = \{1, 2, 5, 6, 7, 8\}$$

$$V_2 = \{3\}$$

$$A = \{1, 2, 4, 5, 6, 7, 8\}$$

$$V_2 = \emptyset$$

$$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

## ~~Graph~~ Graph + terminologies

①  $G = (V, E)$

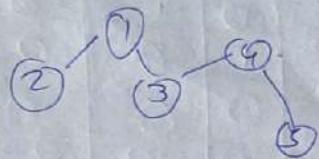
② vertex -

③ Edge -

④ Direct edge -

⑤ weighted

⑥ unweighted



$V = \{1, 2, 3, 4, 5\}$

$E = \{(1,2), (1,3), (2,4), (3,4)\}$

$(3,4), (4,5), (4,5), (5,4)$

3

## Graph types

① ~~Finite graph~~ Finite graph

② infinite graph

③ null graph

④ trivial graph

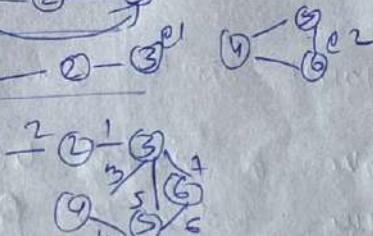
⑤ Direct

⑥ Undirected

⑦ connected

⑧ Disconnected

⑨ Weighted



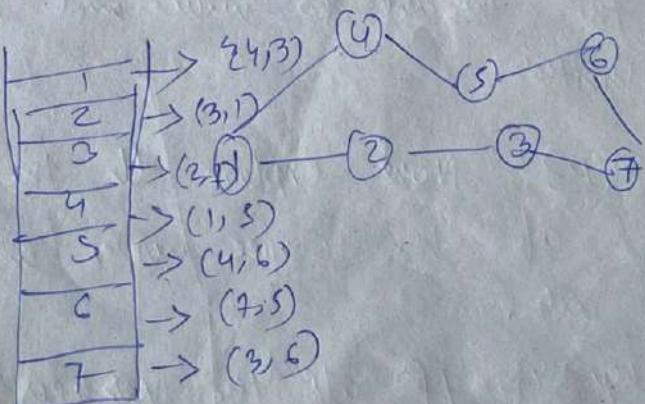
## Graph Representation

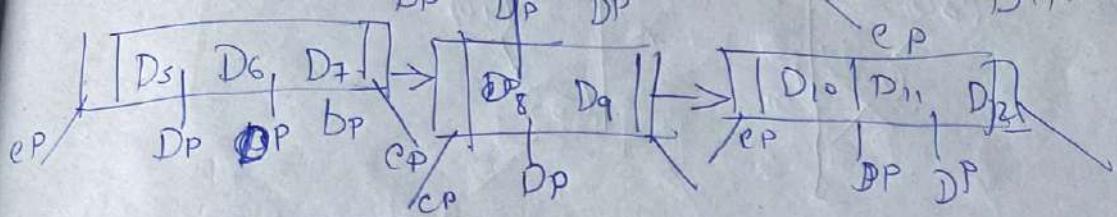
① Adj list

② u matrix

③ cost adj u

④ u matrix





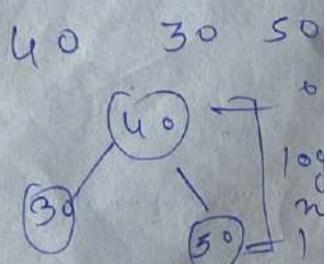
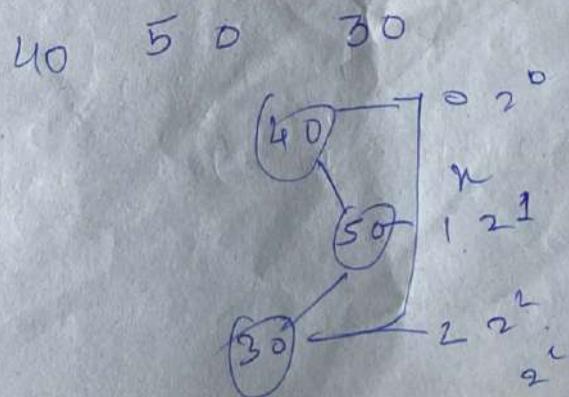
Bst:- It's special kind of Binary tree which is mainly used for searching purpose

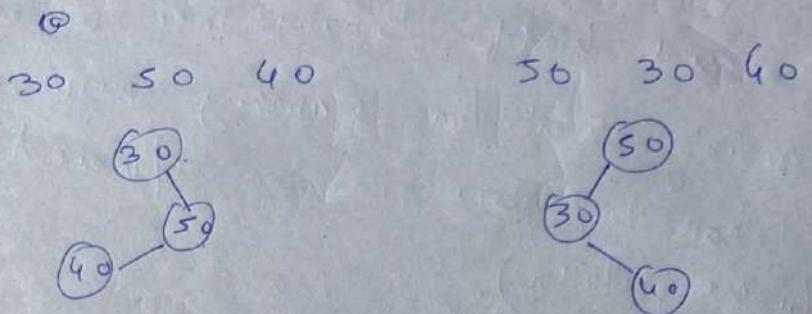
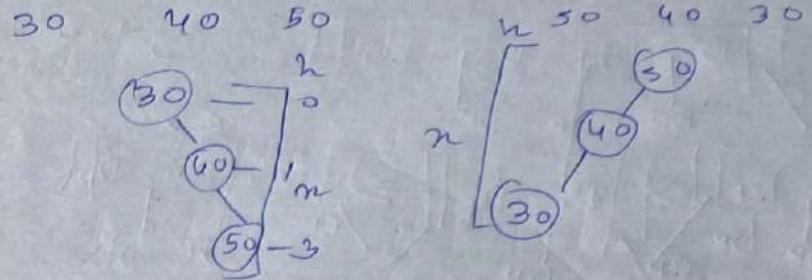
④ Searching can be done in an efficient and minimum time  $O(\log n)$ .

Properties :-

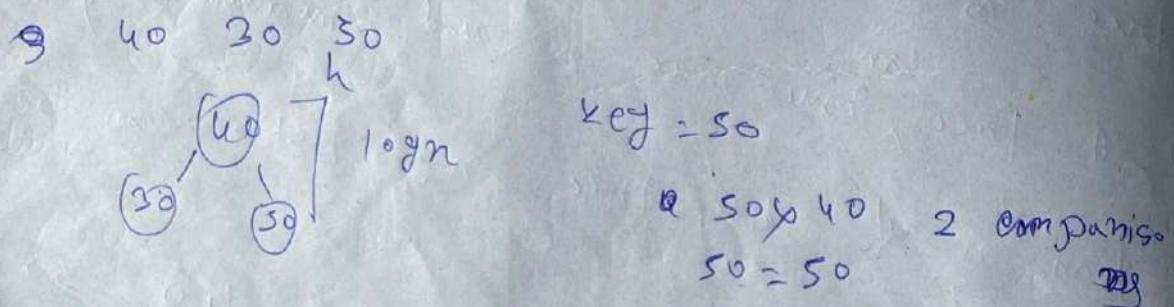
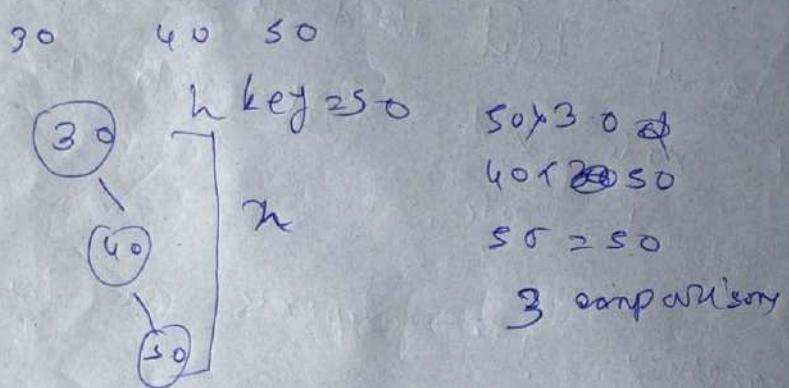
- 1) A Bst can have at most two child.
- 2) No of left child is less than the root node and right child is greater than the root node.
- 3) Searching time depends on the height of tree
- 4) For a tree, max height is  $\log n$  and minimum  $(\log n)$
- 5) For  $n$  no of tree  $n!$  Bsts can be generated

example:-

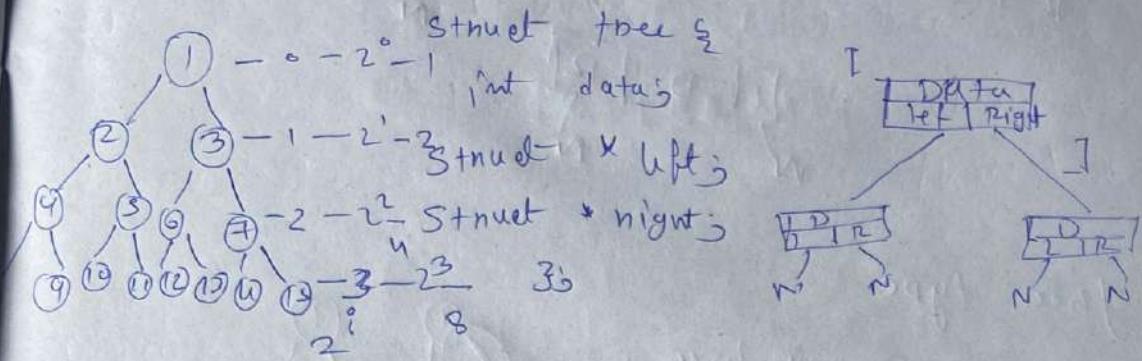




For different set of key we can generate different size of Binary Search tree



- A tree structure made of nodes and edges
- In C lang we define it as a structure



- Non linear hierarchical data structure  
max node:  $2^n$   
max child:  $2^{n+1} - 1$   
min u =  $2^{n+1}$

## Jenninologies

- Parent
- Child :-
- Parent,
- Sibling
- Ancestor
- Descendants

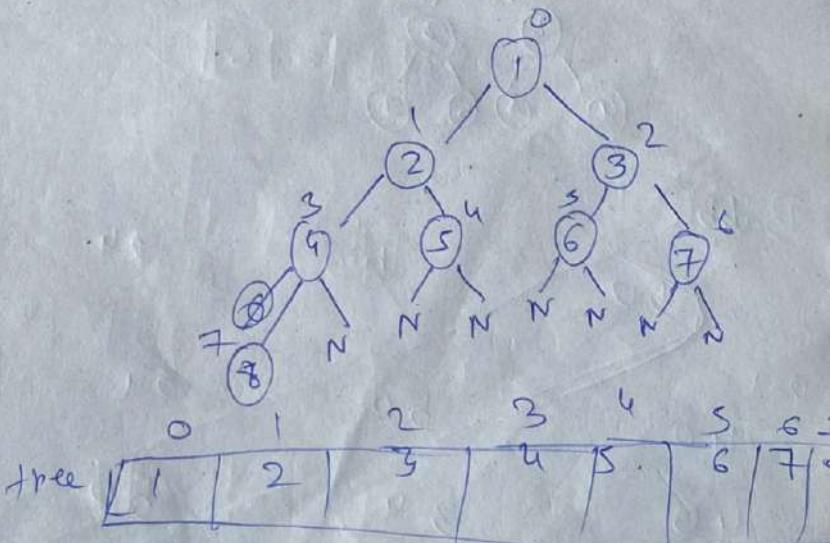
leaf  
edge  
node  
height  
level

Sequential Array representation of a tree

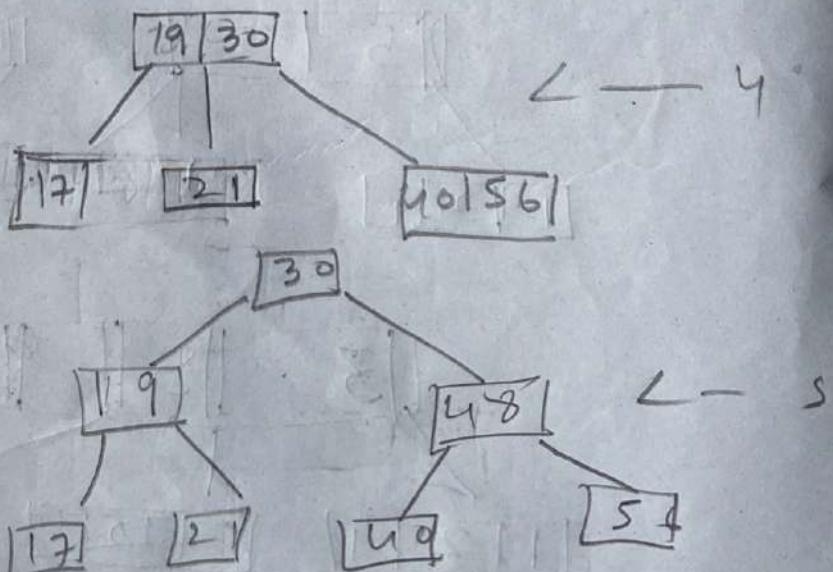
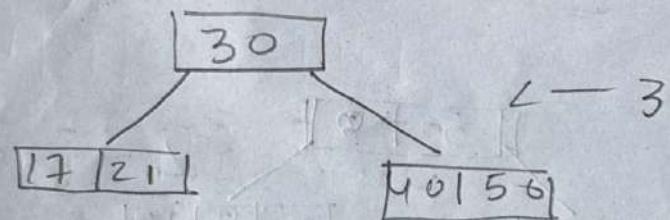
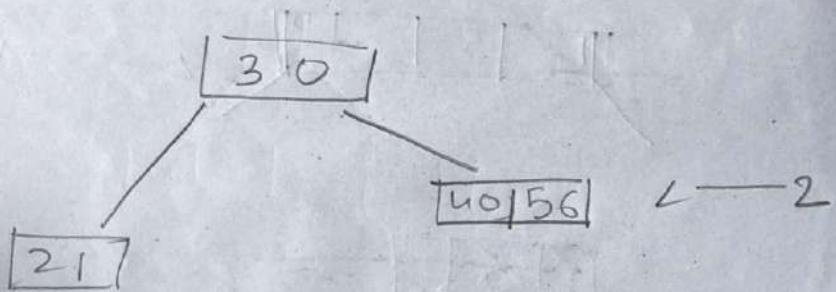
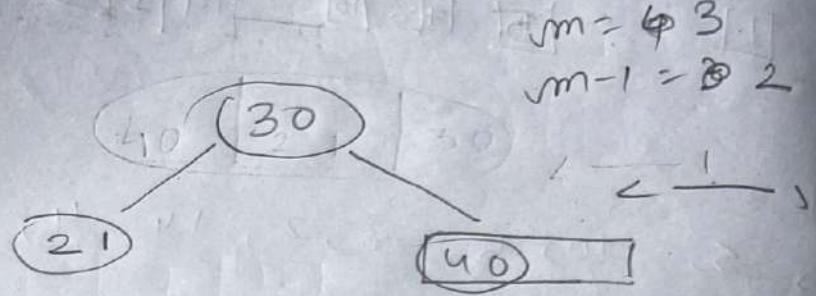
$$\text{root} = 0^{\text{th}}$$

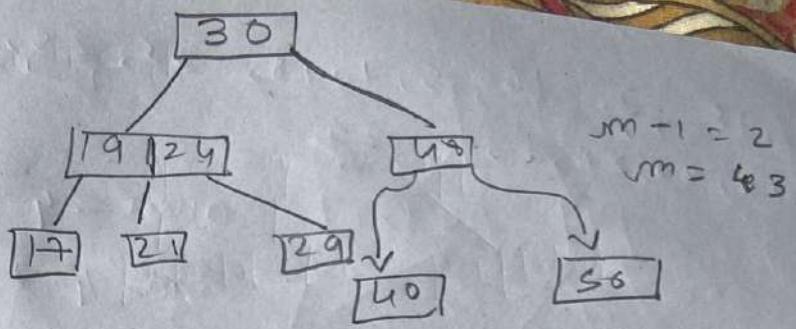
$$1\text{-child} = 2^i + 1$$

$$2\text{-child} = 2^i + 2$$

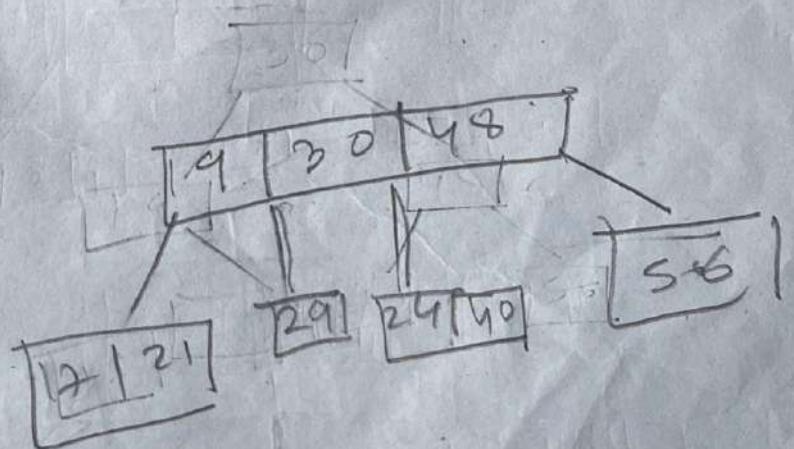
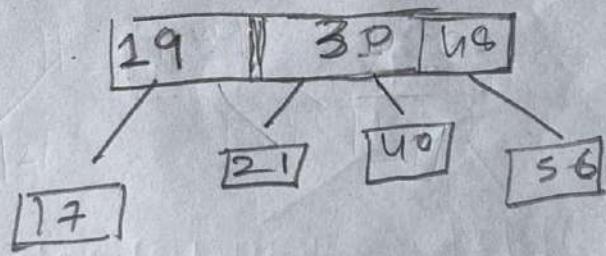
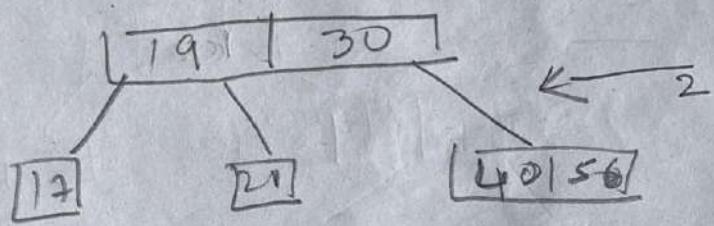
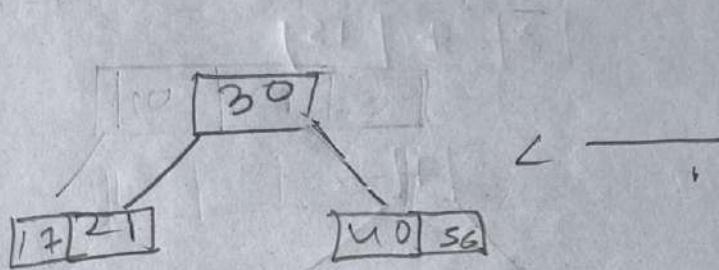


40, 21, 30, 56, 17, 19, 48, 29, 24





Order - 4



$\overbrace{7, 15, 3, 16, 5, 1, 18, 35, 24, 39}$   
 17, 14, 20, 22, 25, 27

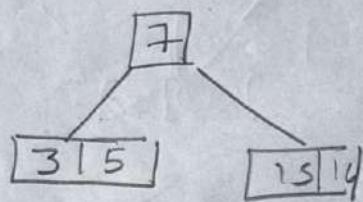
Order = 4

$m = 4$

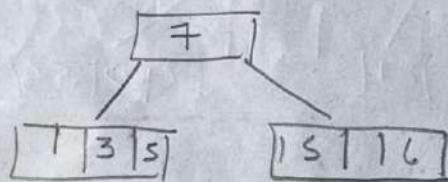
$m - key = 3$

3	7	15
---	---	----

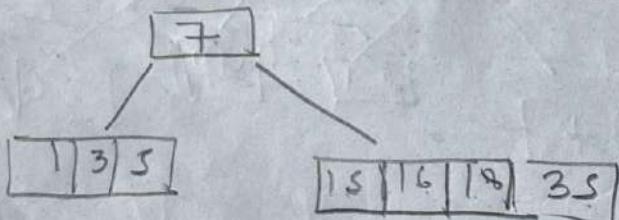
5 →  
16 →



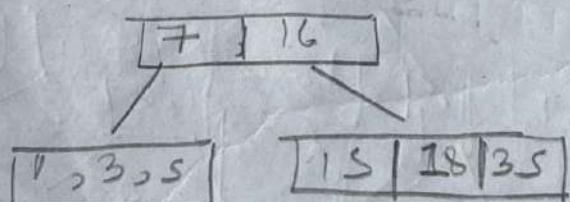
1 →



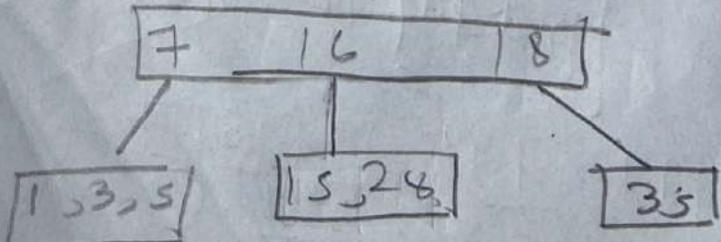
18



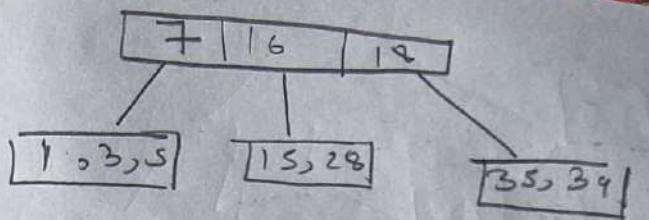
35



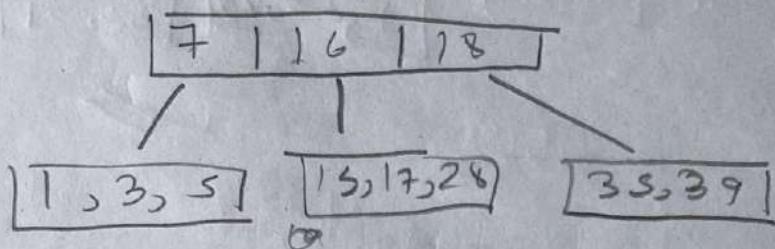
24



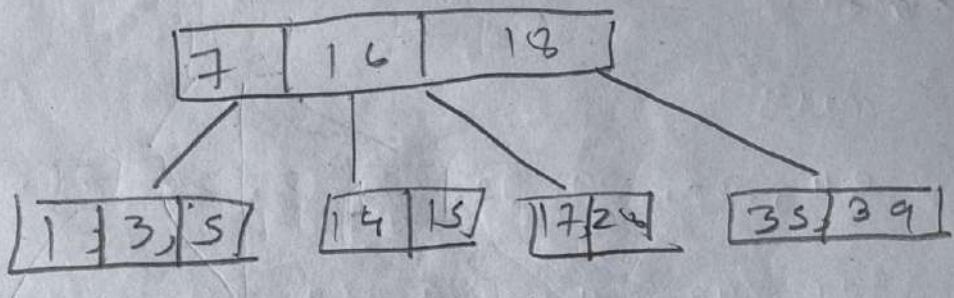
39



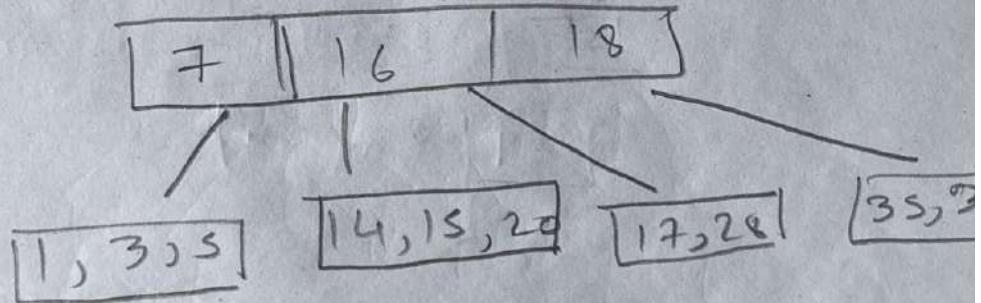
17



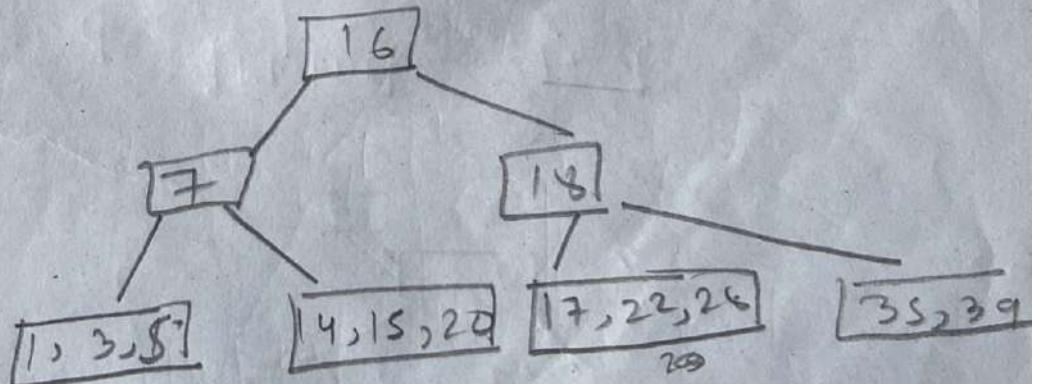
14



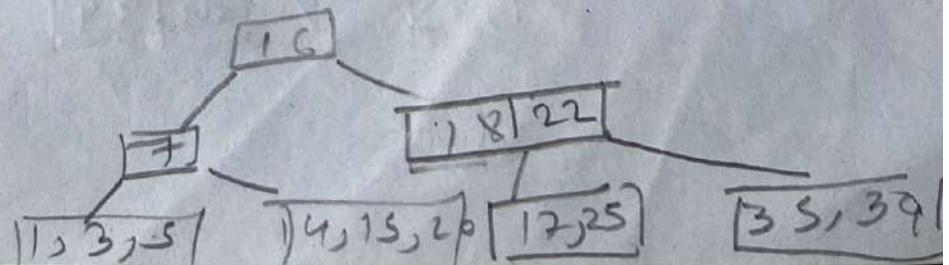
20



22

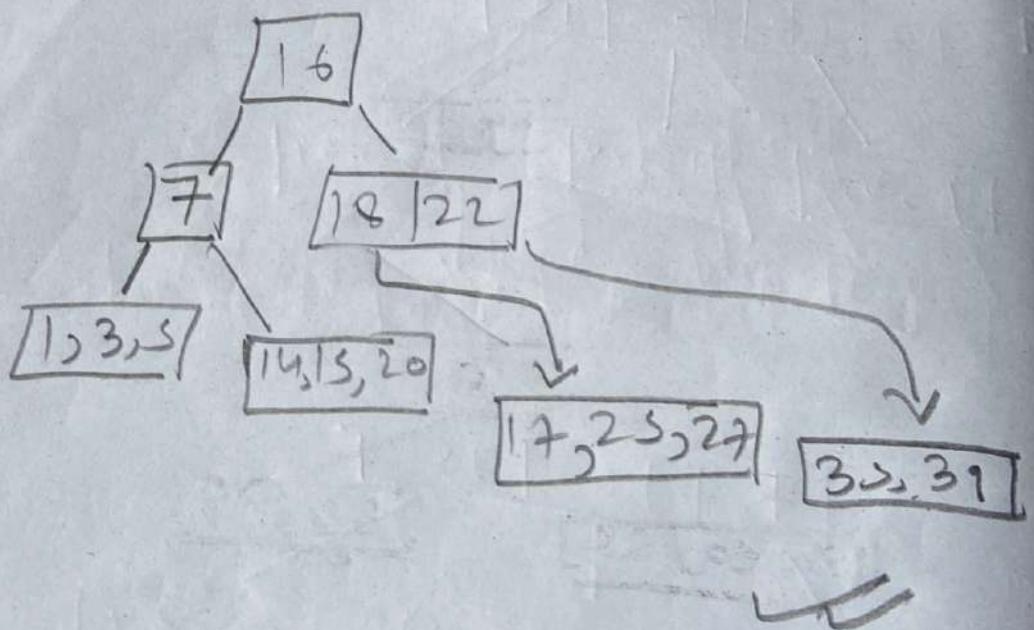


5





27

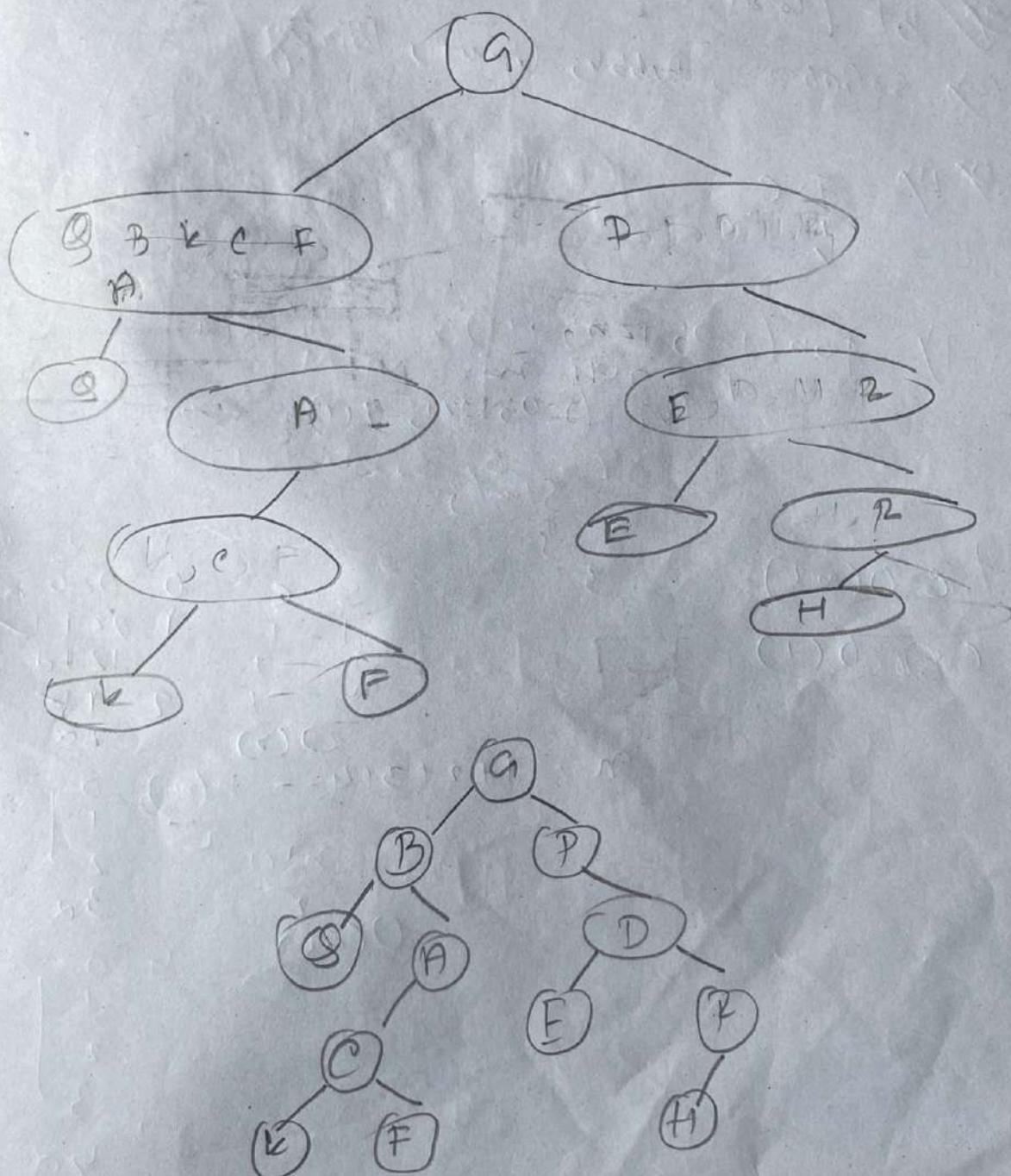


Construct a B tree of order 4 wif

Q) Consider the following sequence of binary tree traversal

Inorder :- Q, B, K, C, F, A, G, P, E, D, H, R

Preorder :- G, B, Q, A, C, K, F, P, D, E, R, H



$$\frac{T \cdot C}{V}$$

1)  $\text{for } (i=0; i < n; i++) \{ \dots \}$  —————  $n+1$   
 $\quad \quad \quad \text{for } (j=0; j < i; j++) \{ \dots \}$

t.c o(n<sup>2</sup>)

C B O C I

Stuntj  
3

$$n = 1 + 2 +$$

$S \cdot e$	$\frac{1}{0} \quad \frac{0}{0 < 0}$	$\frac{i}{0 < 1} \quad \frac{1}{1212}$	$\frac{0}{1 \sim} \quad \frac{2}{2\alpha} \quad 2$
$\frac{1}{n} = 1$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$
$\frac{1}{m} = 1$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$
$\frac{1}{O(n)}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$3+4 \sim$	$n(m+3)$	$0 \quad 1 \quad 2$	$3$
$\frac{2m^2+1}{2}$	$\frac{2}{2}$	$3\alpha$	
$O(m^2)$		$0 \quad 1 \quad 2 \quad 3$	$4$

for ( $i=0$ ;  $i < n$ ;  $i = i \times 3$ )  $\{$

Start  $i$  ——————  $(\log_3 n)$  ——————  $\frac{i}{0}$

$3^k = ?$

$3^1 = 3^1 \times 2 = 2$
$3^2 = 3^2 \times 2 = 2^2$
$3^3 = 3^3 \times 2 = 2^3$
$3^4 = 3^4 \times 2 = 2^4$

assume the upper mentioned

loop will execute as long as  $3^k \leq n$

then it will terminate as long as

$$3^k = n$$

$$\text{Since } 3^k = 2^k$$

$$3^k > n$$

$$\log_3 k = n$$

$$k = \log_3 n$$

$$O(\log_3 n) + c$$

$$O(1) + \frac{n-1}{O(2)}$$

for ( $i=0$ ;  $i < n$ ;  $i = i \times 2$ )  $\{$   $\frac{n}{2^k}$   $\}$   $\frac{n}{2^k}$   $\rightarrow O(n/2^k)$

$$O(n/2^k)$$

Dominating  $O(n)$  ✓

int P=0

for ( $i=0$ ;  $i < n$ ;  $i = i \times 2$ )  $\{$   $P = P + 1$   $\}$

for ( $j=0$ ;  $P > j$ ;  $j = j \times 2$ )  $\{$

Stu

$\frac{1}{\log P}$

$P \cdot n \cdot O(\log \log n)_3$

$$\frac{2 \cdot \log n \cdot P}{\log n \times \log P}$$

for ( $i=0$ ;  $i < \text{bound}$ ;  $i++$ )  $\in -n^4$

for ( $j=0$ ;  $j < n$ ;  $j+=3$ )  $\in -n \times \binom{n+1}{2}$

$\sum i_j = n \times (n$

    for ( $j=1$ ;  $j < n$ ;  $j+=2$ )  $\in -n \times (n) \times$

$\sum j_j = j_j = n \times n \times \log_2 n$   
         $= n \times n \times \log n$

$$3n \log n + 3n^2 + 2n + 1$$

$$3n \log n$$

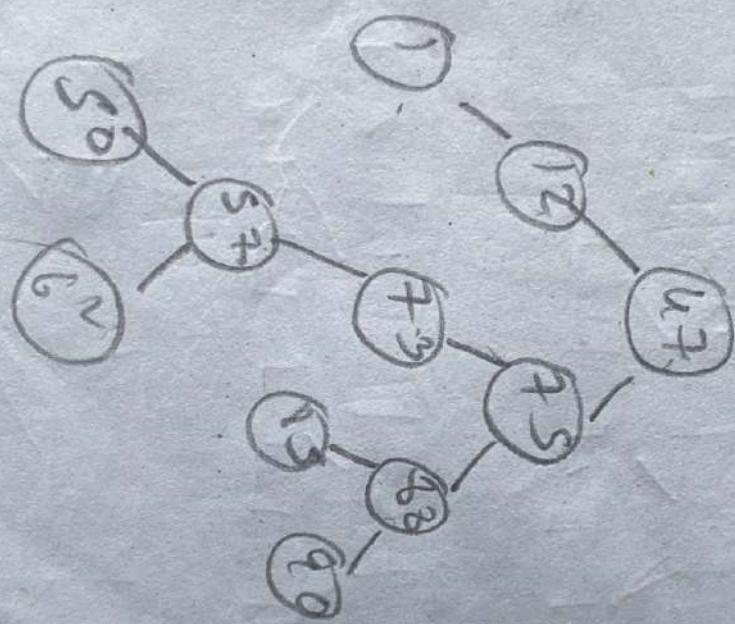
$$\approx O(n \log_2 n)$$

of ansu 4:- 5, 3, 21, 9, 1, 13, 2, 7, 10

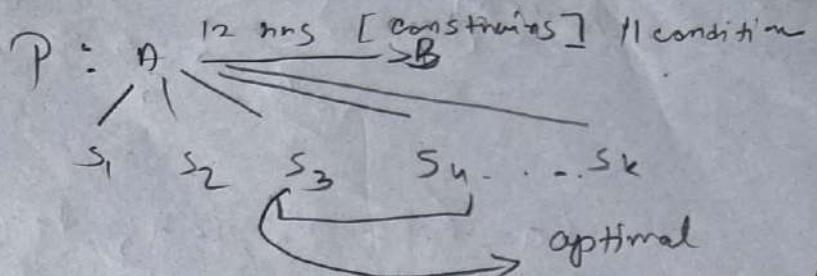
Bst

62

42, 12, 75, 23, 96, 23, 82, 1, 25, 30



Greedy



Learn & every solution is adaptive for the given condition But the feasible solution's are ( $s_3$  and  $s_4$ ) since there could be one optimal and ~~Best~~ Best solution for solving a problem In this case I chose  $s_3$  ( $s_3$ ) as an optimal solution and all the solutions can able to solve the problem but being sticking on a particular constraint so two solutions are chosen  $s_3$  and  $s_4$  and after encosing them I chose the minimum cost optimal feasible and adaptive solution out of them is  $s_3$ . So for this particular problem I should take solution.

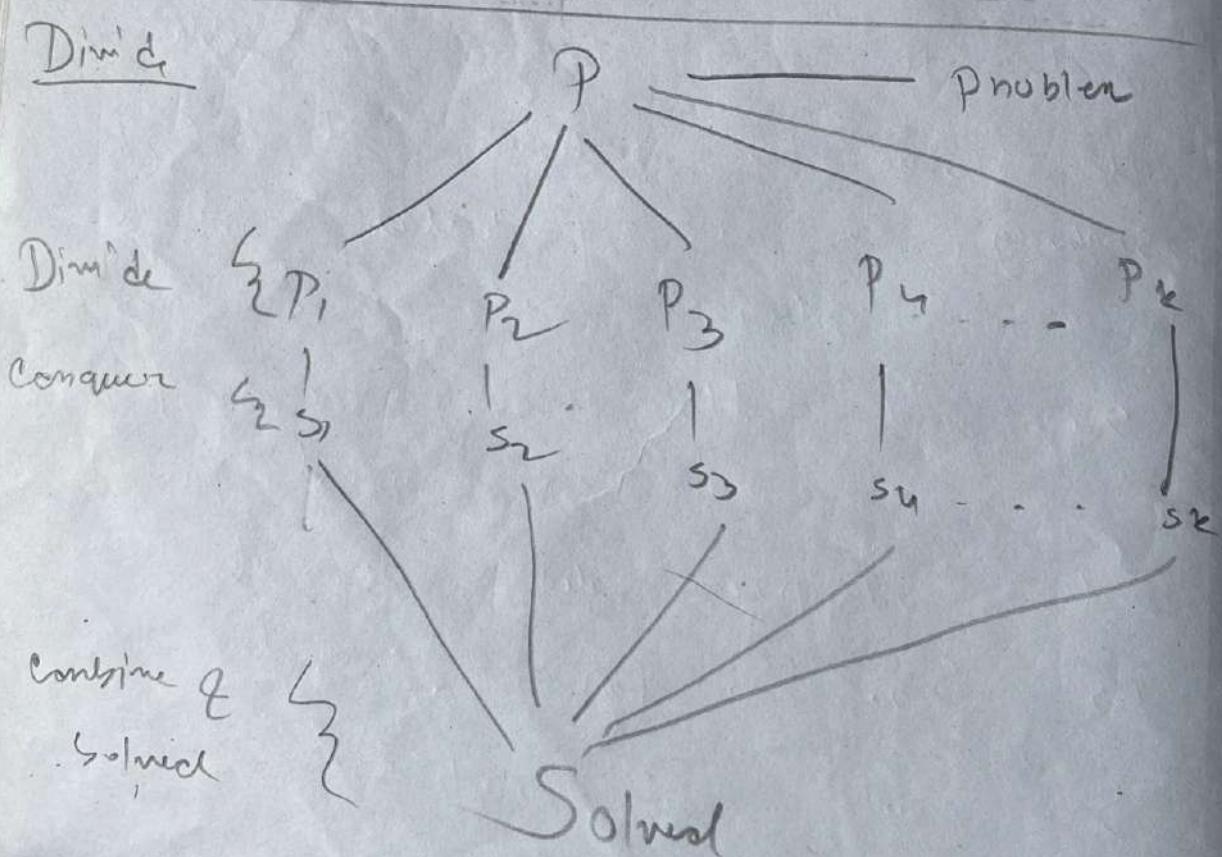
Crossed by  $(a, n)$   $a \boxed{k_1 | k_2 | k_3 | k_4} \downarrow$   
 $\underbrace{\quad}_{\mathcal{Z}}$

for ( $i = 0$ ;  $i < n$ ;  $i++$ )  $\{\}$

$$x = \text{add}(k),$$

If  $b$  is feasible &  
true

$$x = k + x_j$$



algo Dac (P) ↴

if (P small) ↴

S (P)  
↳

else ↴

Divide  $P_1, P_2, P_3, P_4, P_k$ ;

apply  $Dac(P_1, P_2, P_3, P_4, \dots, P_k)$

combine  $(Dac(P_1) + (P_2) + (P_3) + (P_k))$ ,

↳

↳

Analysis: If a problem P is given

& the problem is big convert

solved it directly. Bourzouk's problem

into smaller p ↴ subproblems and

find the solution for each subproblem

↳ and recursively solved if true

and all the subproblems "k" and

combine them and make big

problem solved. Algorithm is recursive

for every subproblem must be <sup>some</sup> ↴ ~~easy~~

nature if the problem is to sort

envy.

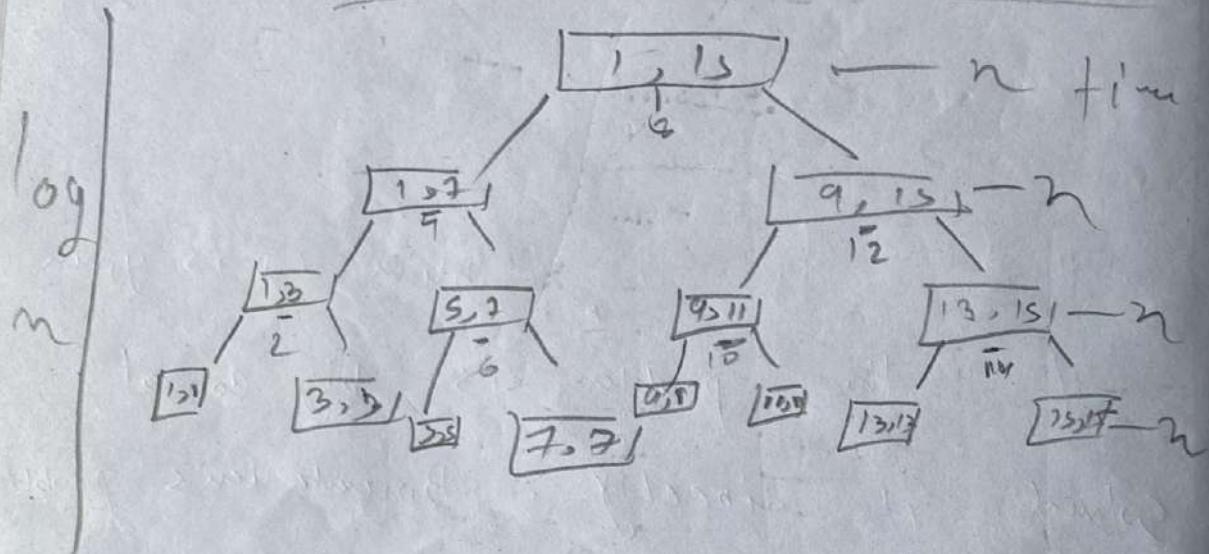
$$\frac{1+15}{2} = \frac{16}{2} = 8$$

~~Q~~ which sort

$$\frac{1+15}{2} = \frac{16}{2} = 8$$

[1] . . . . . [15]

Suppose the partition way come at element ~~mid~~



The time complexity is depend on tree's particular constraint it's  $\Theta(n \log n)$   
 height  $\log n$  (0  $\log n$ )  
 $O(n \log n)$

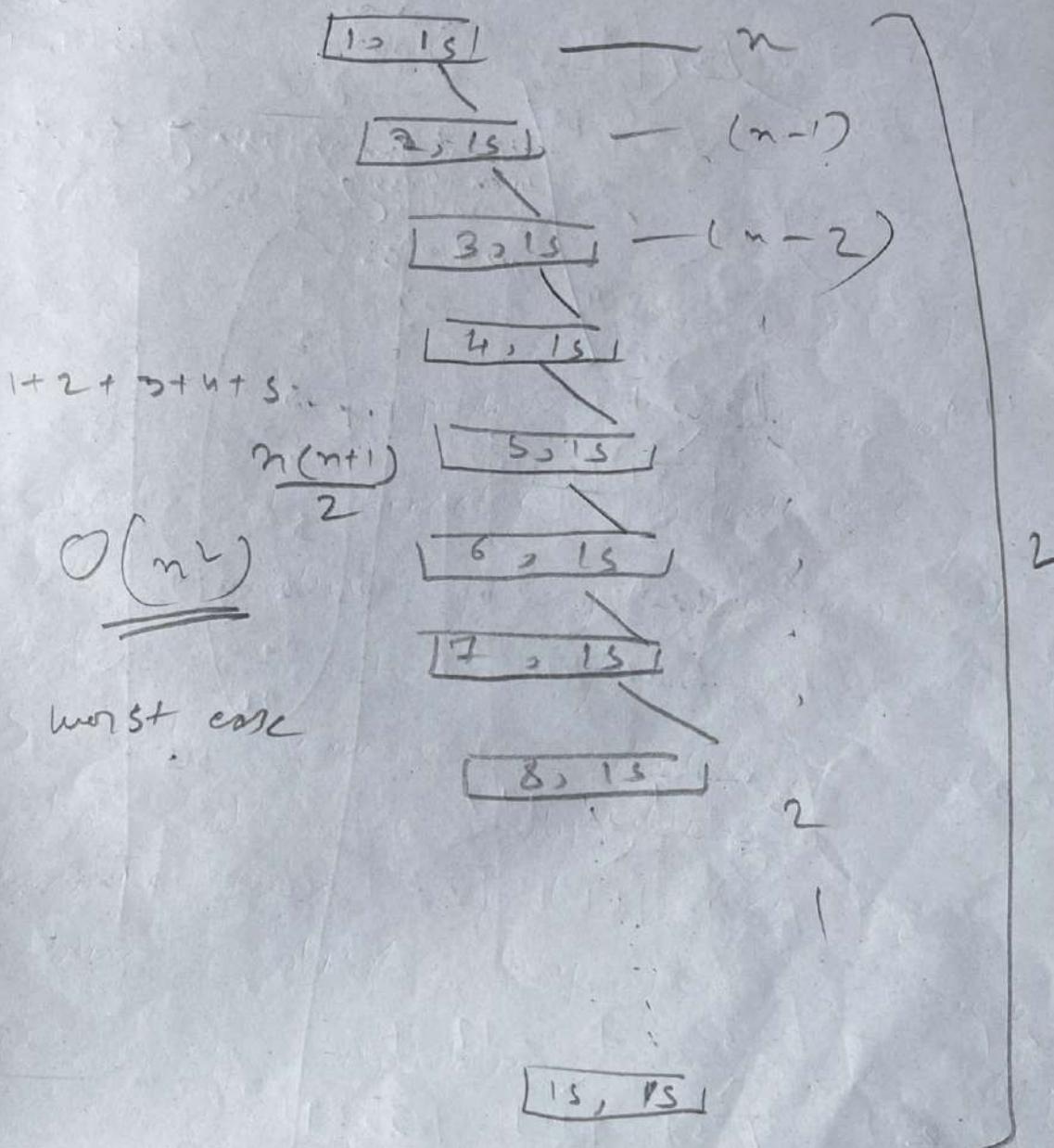
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[best case  
average case]

Worst

'Suppose for partition  
way done at begin

[0] - - - - - [IS]



open chaining  
closed chaining

↓  
23, 29, 31

71 Closed chaining hashing       $f(n) = n$   
open addressing  
Linear - O(n) of space       $f(n) \leq \text{size}$   
Quadratic -

[11 117]

[11117 119]

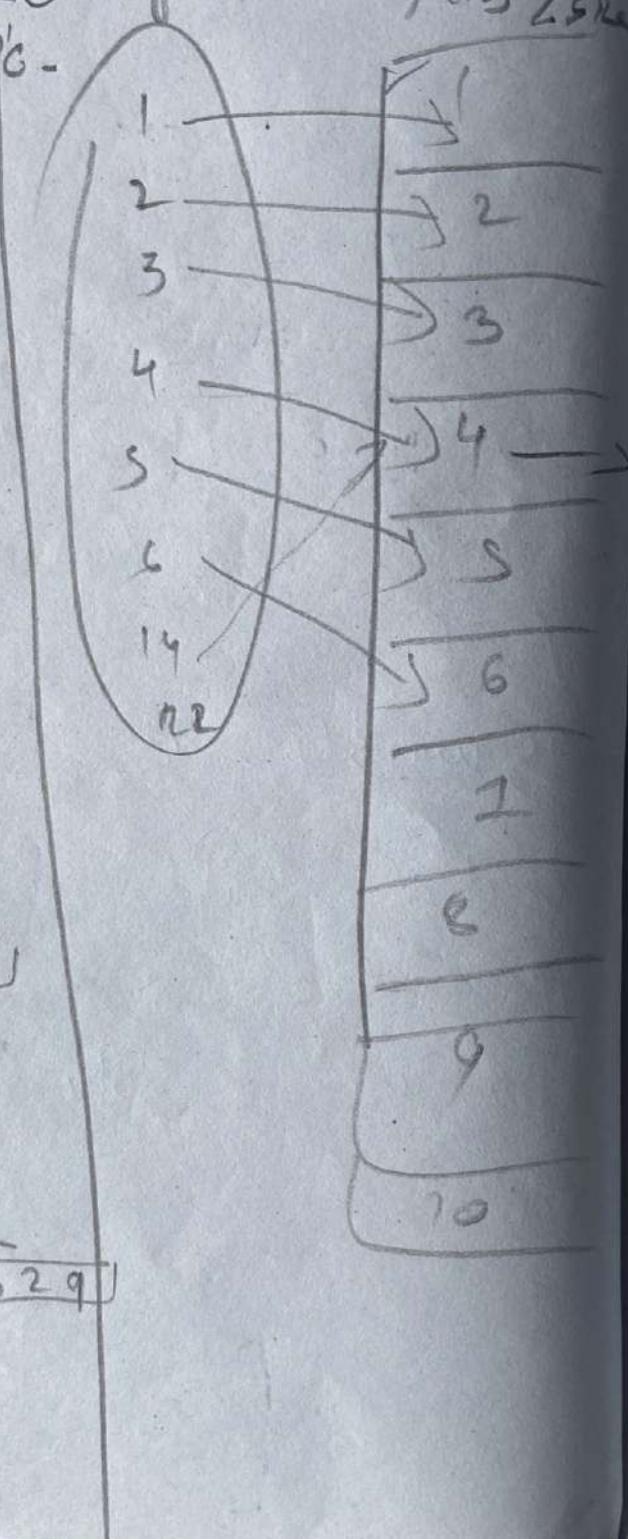
171

23

171

11

[23 229]



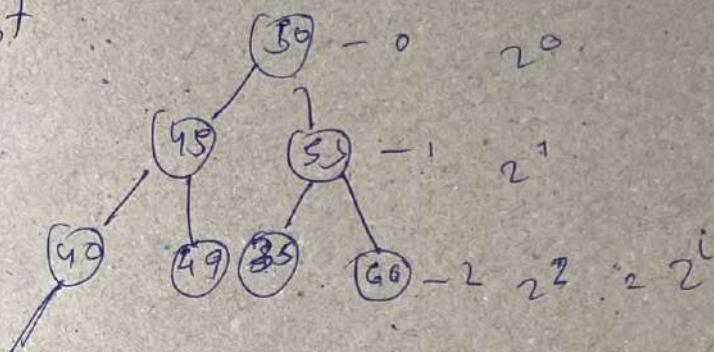
$$f(14) \\ = 4$$

In:- B,

Out:- B  
closing  
= 0

③

Bst



Code:-

struct node \* search (struct node \* root, int key)

{  
if (root == NULL) {  
 return NULL;  
}

if (root->d == key) {  
 return root;  
}

else if (root->key) {  
 return search (root->left, key);  
}

else {  
 return search (root->right, key);  
}

}