Неподвижные точки

```
syms u r
r_min = 0; r_max = 3*sqrt(3)/2;
R = [r_min r_max];
answ = solve (u == r*sqrt(u)*(1-u), u);
```

```
Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the
 solution, specify the 'ReturnConditions' value as 'true'.
```

```
syms stat_p1(r) stat_p2(r)
stat_p1 = answ(2)
```

```
stat_p1 =
```

$$\frac{\left(\sqrt{4\,r^2+1}-1\right)^2}{4\,r^2}$$

```
fplot(stat_p1,R); hold on;
plotstyle1('$r$','$u^*$',['$$ u^* = ' latex(stat_p1) '$$'],'east');
hold off;
exportgraphics(gcf,'pictures\ds2_stp.pdf');
```

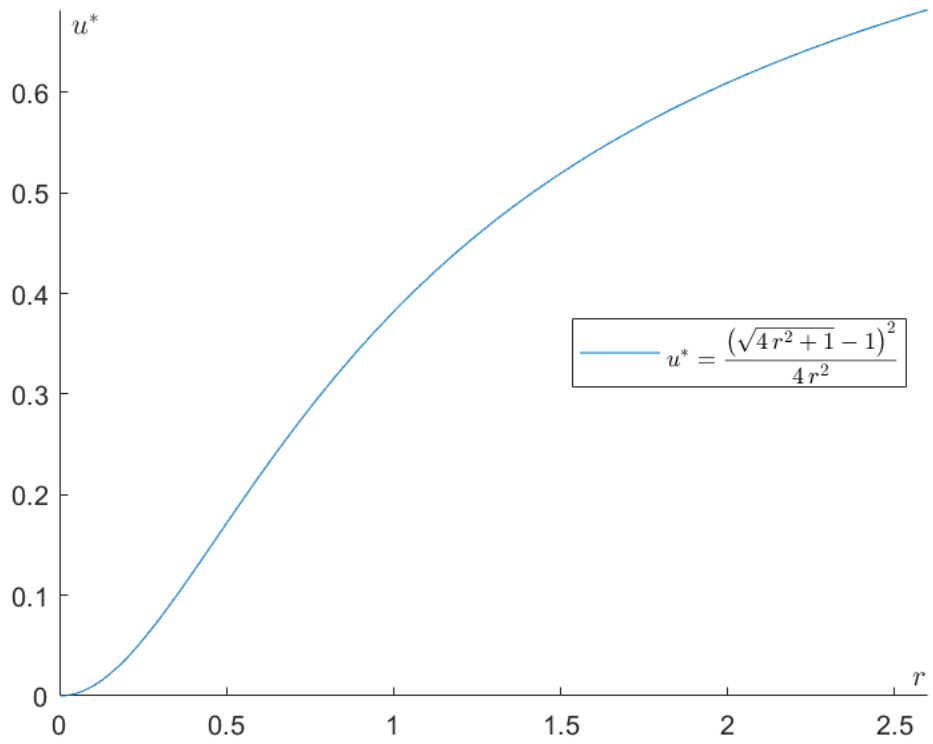


График первой неподвижной точки

```
stat_p2 = answ(3)
```

```
stat_p2 =
```

$$\frac{\left(\sqrt{4\,r^2+1}+1\right)^2}{4\,r^2}$$

```
fplot(stat_p2,[0.1 r_max]); hold on;
ax = gca; ax.YLim = R;
plotstyle1('$r$','$u^*$',['$$ u^* = ' latex(stat_p2) '$$'],'northeast');
hold off;
```

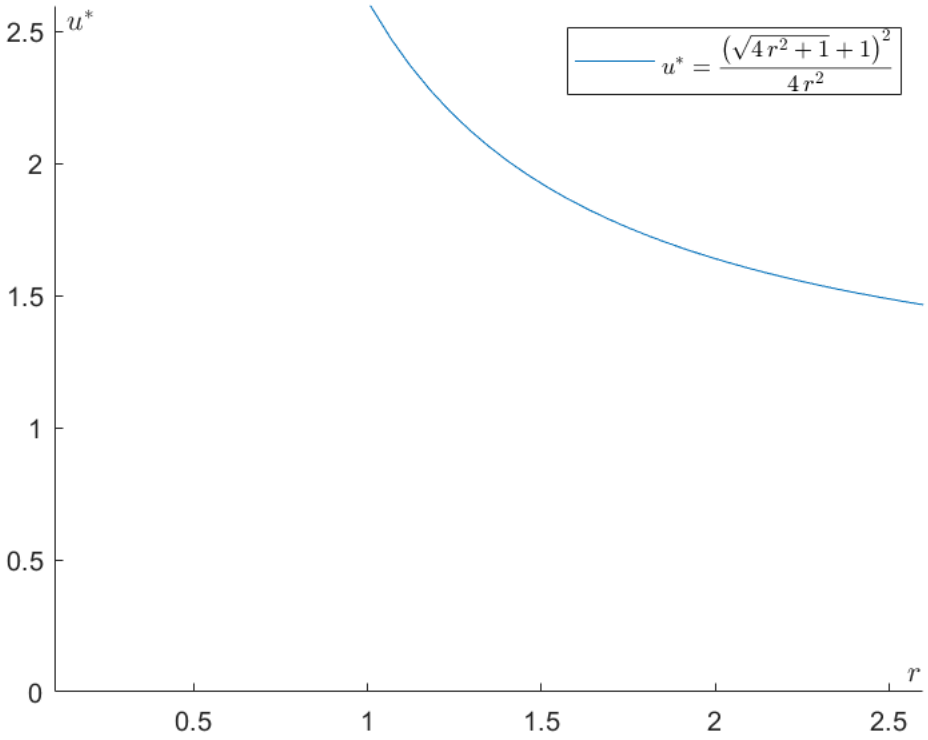


График второй неподвижной точки

Собственные значения

```
syms r u
r_min = 0; r_max = 3*sqrt(3)/2;
R = [r_min r_max];
j12 = -(sqrt(4*r^2 + 1)-1)^2/(4*r);
J = [1/2 j12; 1 0]
```

```
J =
```

$$\begin{pmatrix} \frac{1}{2} & -\frac{\left(\sqrt{4\,r^2+1}-1\right)^2}{4\,r} \\ 1 & 0 \end{pmatrix}$$

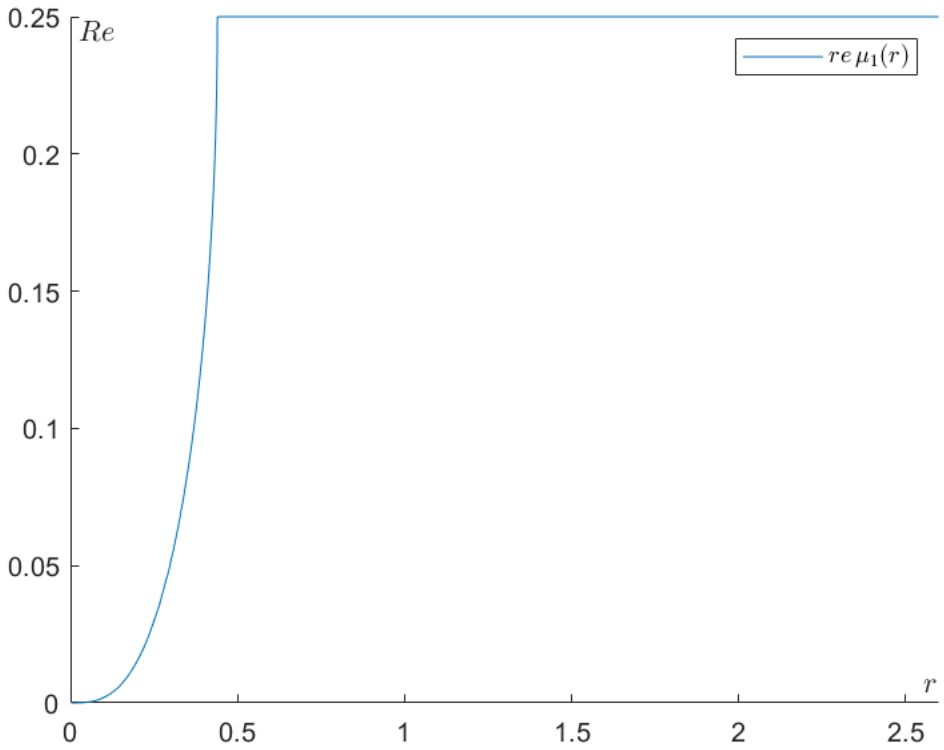
```
lambda = eig(J)
```

```
lambda =
```

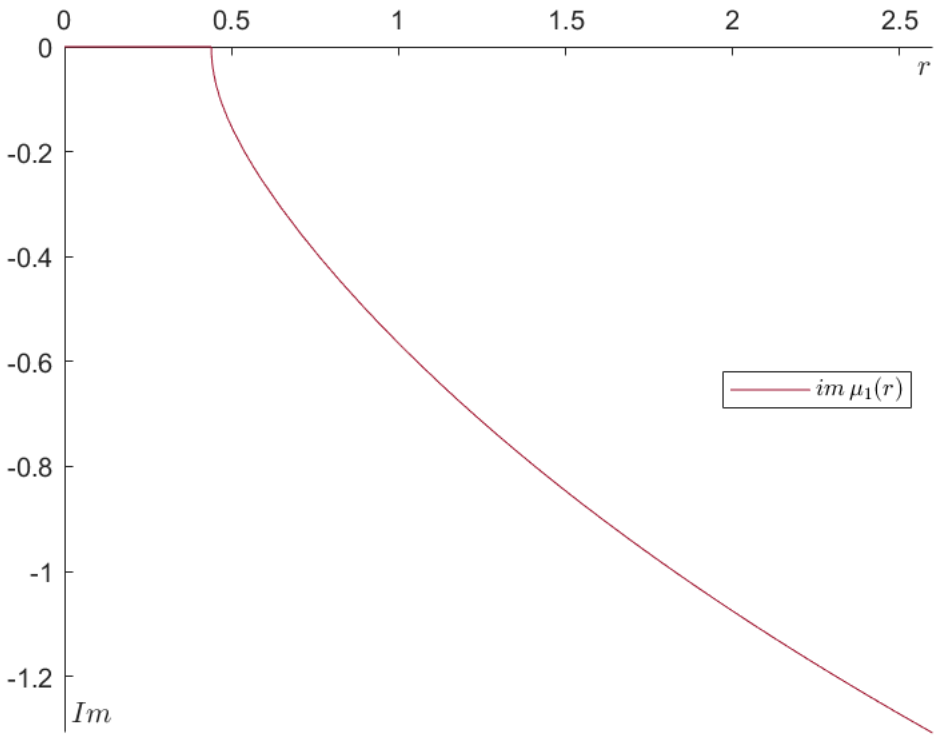
$$\begin{pmatrix} \frac{1}{4} - \frac{\sqrt{\frac{r - 16\,r^2 + 8\,\sqrt{4\,r^2+1} - 8}{4\,r}}}{2} \\ \frac{\sqrt{\frac{r - 16\,r^2 + 8\,\sqrt{4\,r^2+1} - 8}{4\,r}}}{2} + \frac{1}{4} \end{pmatrix}$$

Первый мультипликатор

```
mu_1 = lambda(1);

fplot(real(mu_1),R); hold on;
plotstyle1('$r$','$Re$','$re\,\mu_1(r)$','northeast');
hold off;
exportgraphics(gcf,'pictures\ds2_mu1_re.pdf');
```




```
fplot(imag(mu_1),R,'Color','#A2142F'); hold on;
plotstyle1('$r$','$Im$','$ im\,\mu_1(r) $','east');
ax = gca;
ax.YLabel.Position(2) = ax.YLim(1);
ax.XLabel.Position(2) = ax.YLim(2) - 0.075;
hold off;
exportgraphics(gcf,'pictures\ds2_mu1_im.pdf');
```



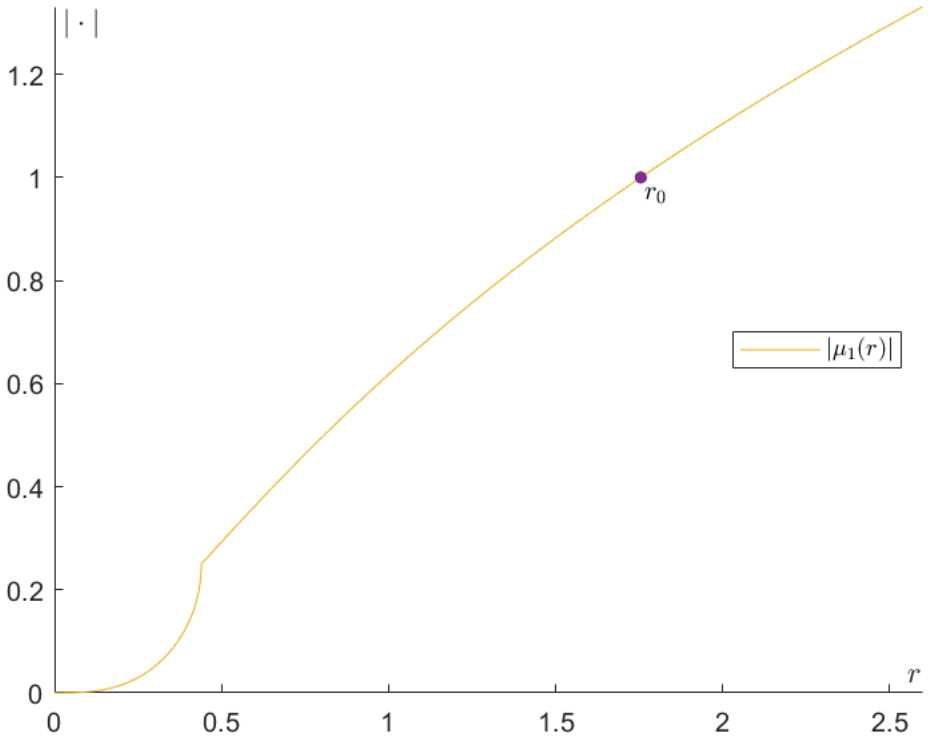

```
solve(abs(mu_1)== 1, r) %не решается символьно
```

```
Warning: Unable to solve symbolically. Returning a numeric solution using vpasolve.
ans = −2.306443932358772372036249693149
```

```
num_mu_1 = matlabFunction(mu_1);
r1_0 = fminbnd(@(r) (abs(num_mu_1(r))-1)^2, 0,r_max) %точка перехода модуля собственного значен
```

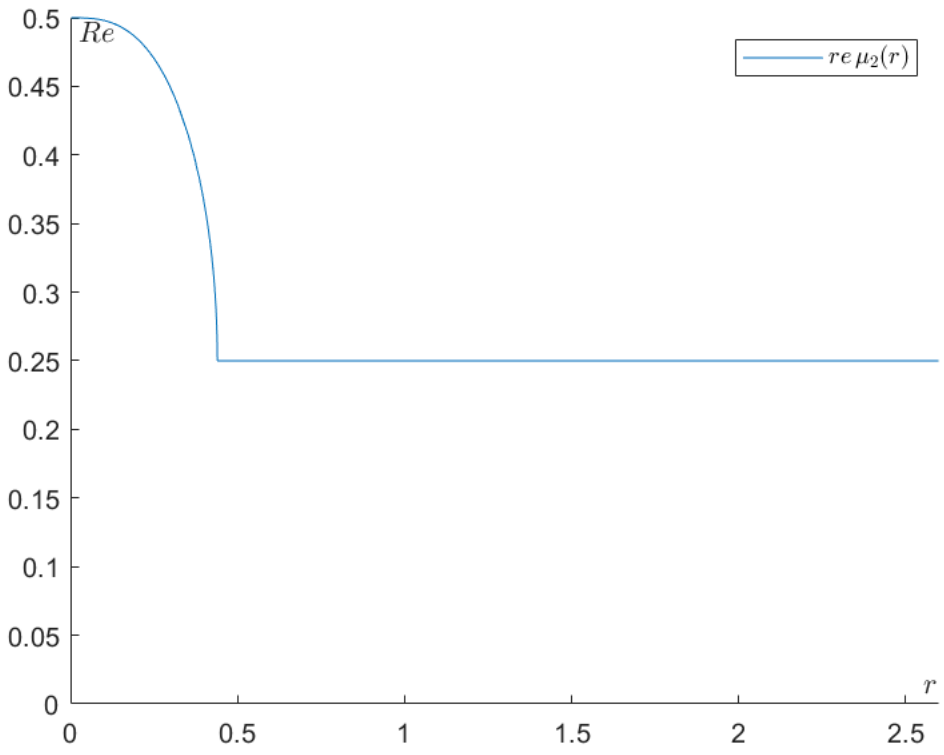
```
r1_0 = 1.7549
```

```
fplot(abs(mu_1),R,'Color','#EDB120'); hold on;
plotstyle1('$r$','$|\cdot|$','$|\mu_1(r)|$','east');
plot(r1_0,1,'Marker','.',"MarkerSize", 16,'Color','#7E2F8E');
text(r1_0+0.01,1-0.03,'$r_0$','Interpreter',"latex");
hold off;
exportgraphics(gcf,'pictures\ds2_mu1_abs.pdf');
```

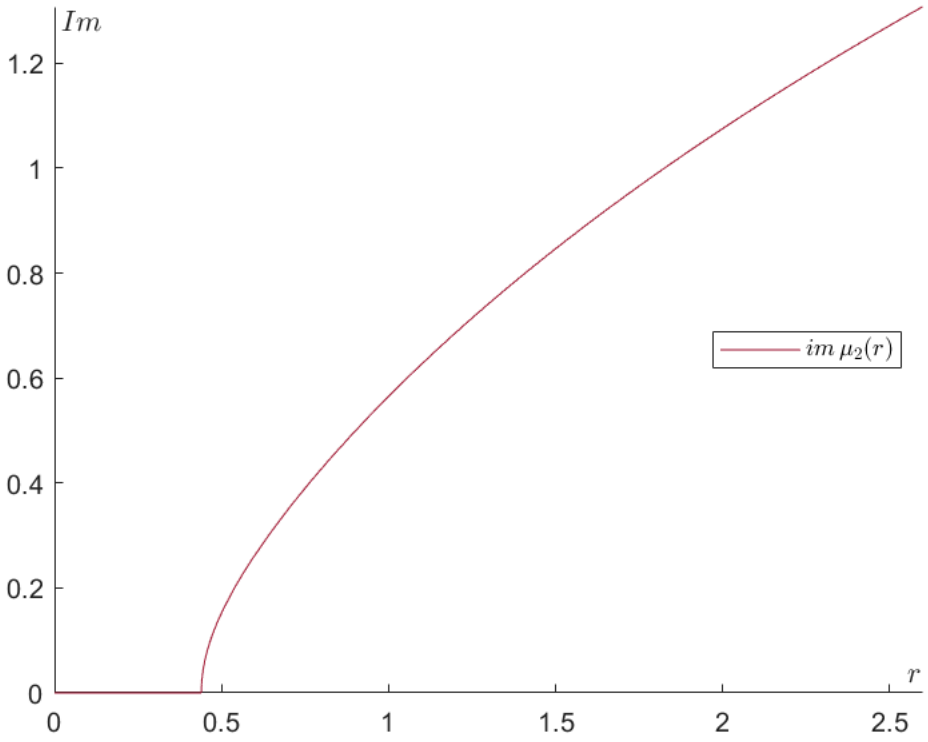


Второй мультипликатор

```
mu_2 = lambda(2);

fplot(real(mu_2),R); hold on;
ax = gca; ax.YLim(1) = 0;
plotstyle1('$r$','$Re$','$re\,\mu_2(r)$','northeast');
hold off;
exportgraphics(gcf,'pictures\ds2_mu2_re.pdf');
```




```
fplot(imag(mu_2),R,'Color','#A2142F'); hold on;
plotstyle1('$r$','$Im$','$ im\,\mu_2(r) $','east');
hold off;
exportgraphics(gcf,'pictures\ds2_mu2_im.pdf');
```



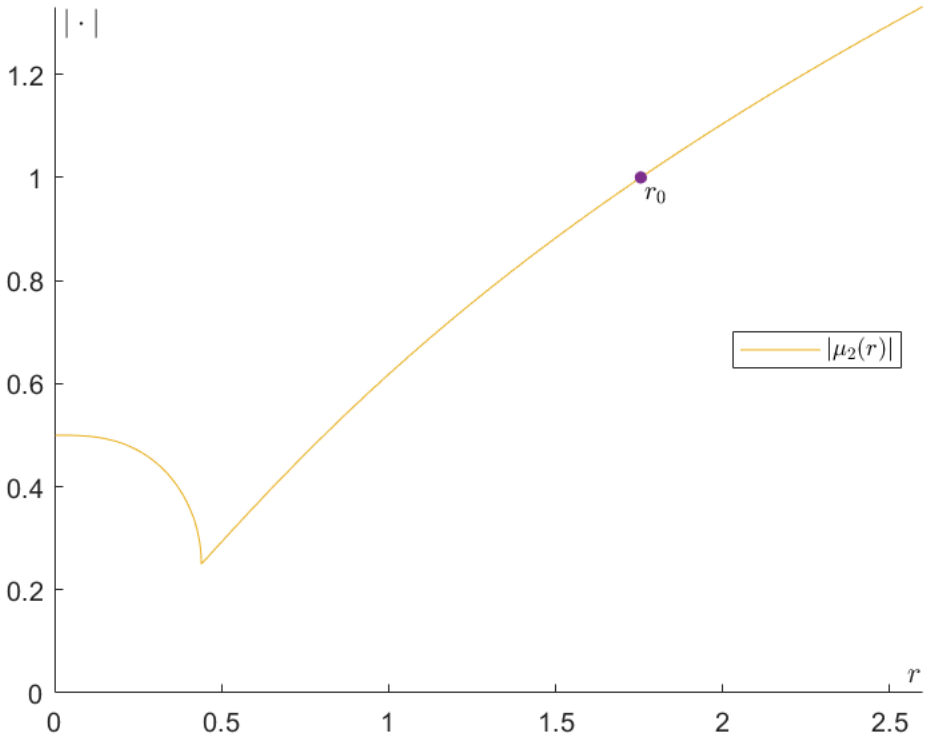

```
solve(abs(mu_2)== 1, r) %не решается символьно
```

```
Warning: Unable to solve symbolically. Returning a numeric solution using vpasolve.
ans = −1.157298106138375990825055520004 8
```

```
num_mu_2 = matlabFunction(mu_2);
r2_0 = fminbnd(@(r) (abs(num_mu_2(r))-1)^2, 0,r_max) %точка перехода модуля собственного значен
```

```
r2_0 = 1.7549
```

```
fplot(abs(mu_2),R,'Color','#EDB120'); hold on;
ax = gca; ax.YLim(1) = 0;
plotstyle1('$r$','$|\cdot|$','$|\mu_2(r)|$','east');
plot(r2_0,1,'Marker','.',"MarkerSize", 16,'Color','#7E2F8E');
text(r2_0+0.01,1-0.03,'$r_0$','Interpreter',"latex");
hold off;
exportgraphics(gcf,'pictures\ds2_mu2_abs.pdf');
```



Бифуркация Неймарка-Сакера (условия невырожденности)

```
r0 = r2_0
```

```
r0 = 1.7549
```

```
mu_1 = lambda(1);
vpa(subs(diff(abs(mu_1),r),r,r0))
```

```
ans = −0.44106668387140510210644703536411 − 0.11388361352121636481411802856467 i
```

```
theta0 = vpa(angle(subs(mu_1,r,r0)))
```

```
theta0 = −1.3181146147283507747748111385385
```

```
k = 1:4;
exp(i*k*theta0)
```

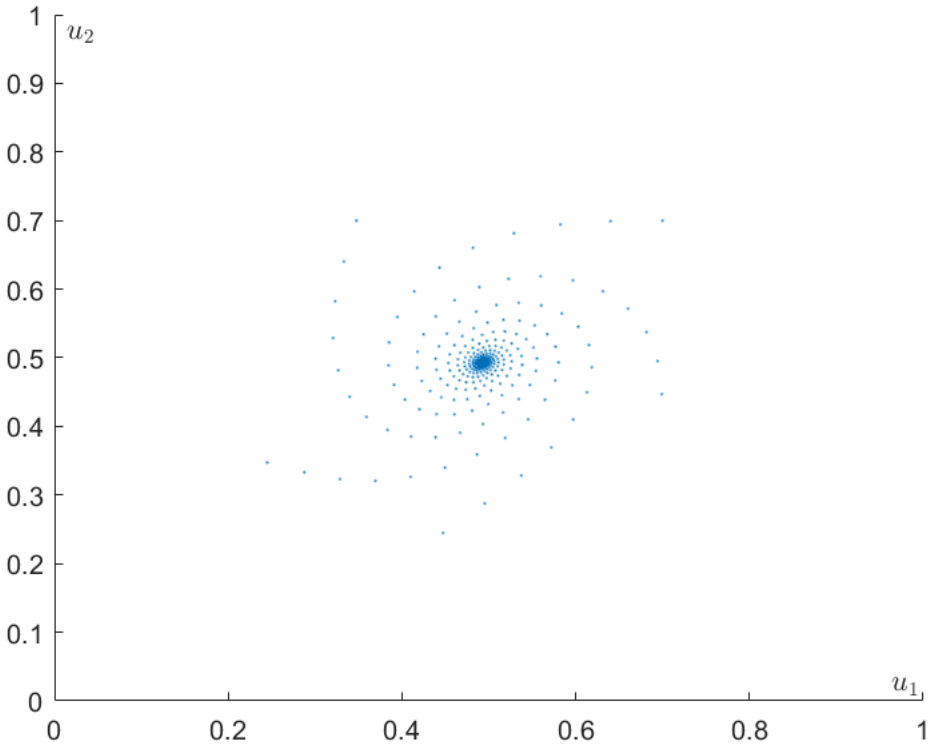
```
ans = (0.25000141066078419907412358652097 − 0.96824547231970981038197238123025 i   −0.8749985893
```

Бифуркация Неймарка-Сакера (иллюстрации)

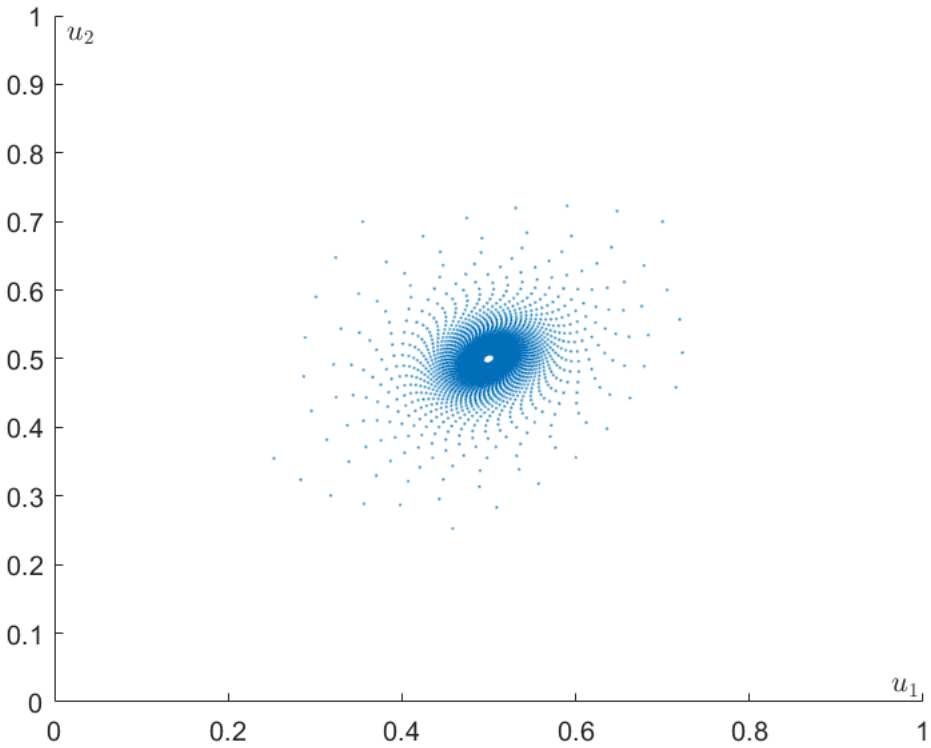
```
r0 = 1.7549;
u0 = [0.7 0.7]';
rl0 = r0 - 0.3
```

```
rl0 = 1.4549
```

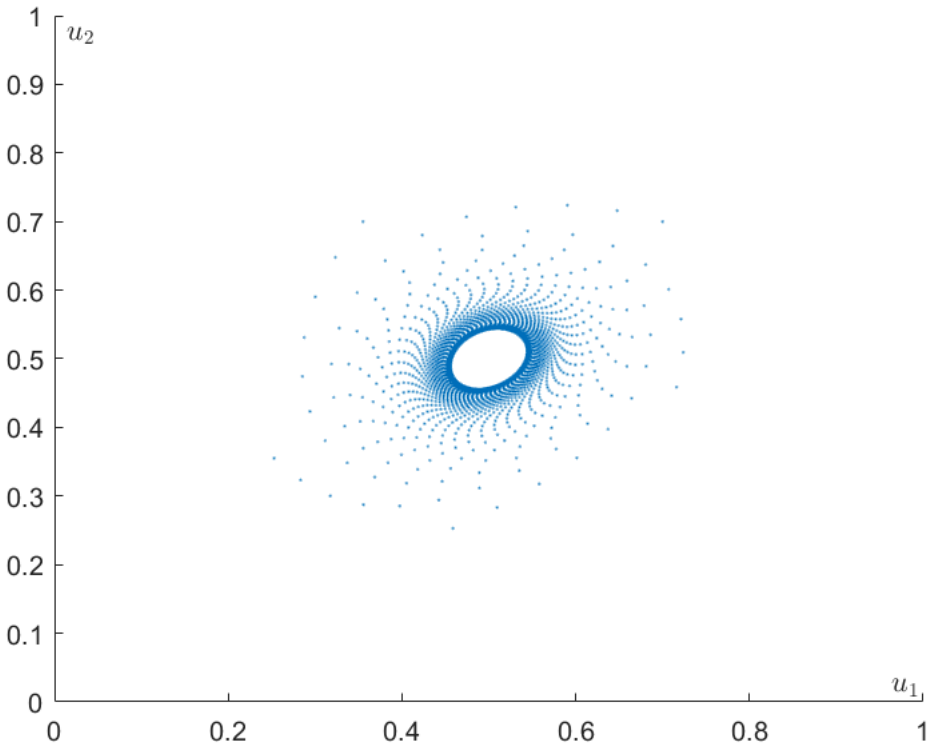
```
r_point = r0 - 0.37;
r_point_lost = r0 - 0.341;
r_small_curve = r0 - 0.34;
r_med_curve = r0 - 0.33;
r_big_curve = r0 - 0.29;
r_dissipated_curve = r0 - 0.281;

plotNS(r_point,u0,10000);
plotstyle2('$u_1$','$u_2$');
exportgraphics(gcf,'pictures\ds2_NS1.png');
```



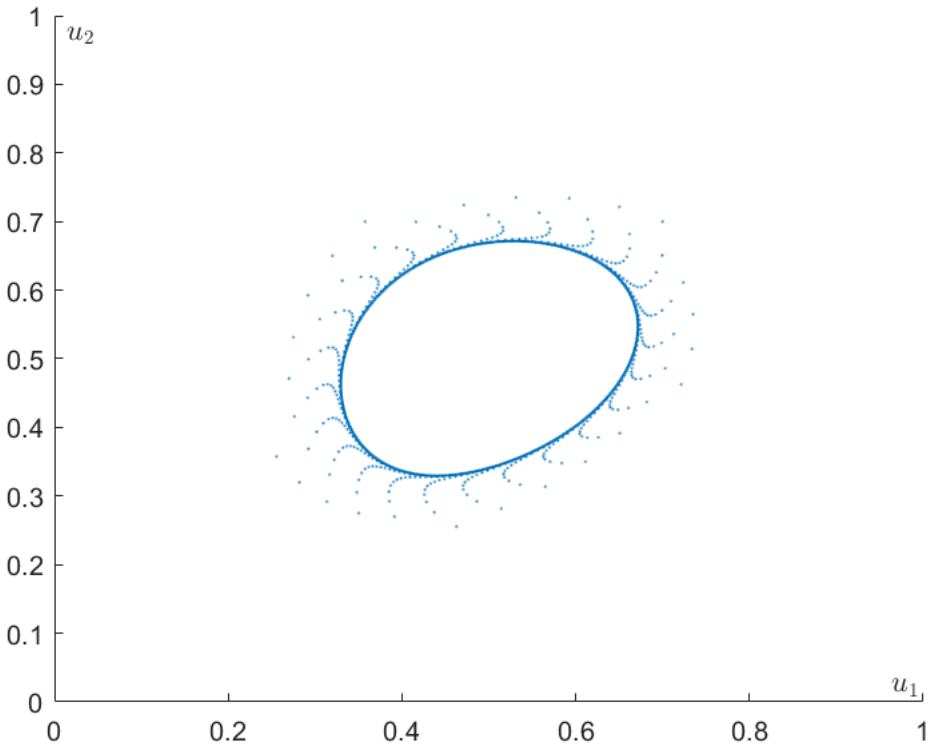

```
plotNS(r_point_lost,u0,10000);
plotstyle2('$u_1$','$u_2$');
exportgraphics(gcf,'pictures\ds2_NS2.png');
```



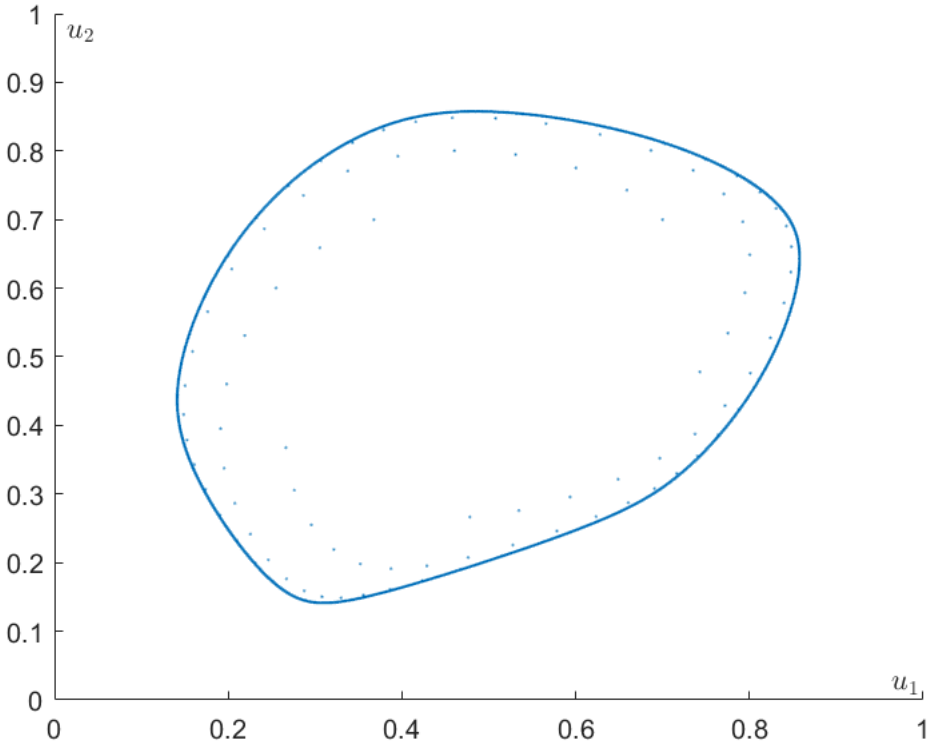

```
plotNS(r_small_curve,u0,10000);
plotstyle2('$u_1$','$u_2$');
exportgraphics(gcf,'pictures\ds2_NS3.png');
```




```
plotNS(r_med_curve,u0,10000);
plotstyle2('$u_1$','$u_2$');
exportgraphics(gcf,'pictures\ds2_NS4.png');
```




```
plotNS(r_big_curve,u0,10000);
plotstyle2('$u_1$','$u_2$');
exportgraphics(gcf,'pictures\ds2_NS5.png');
```




```
plotNS(r_dissipated_curve,u0,10000);
plotstyle2('$u_1$','$u_2$');
exportgraphics(gcf,'pictures\ds2_NS6.png');
```

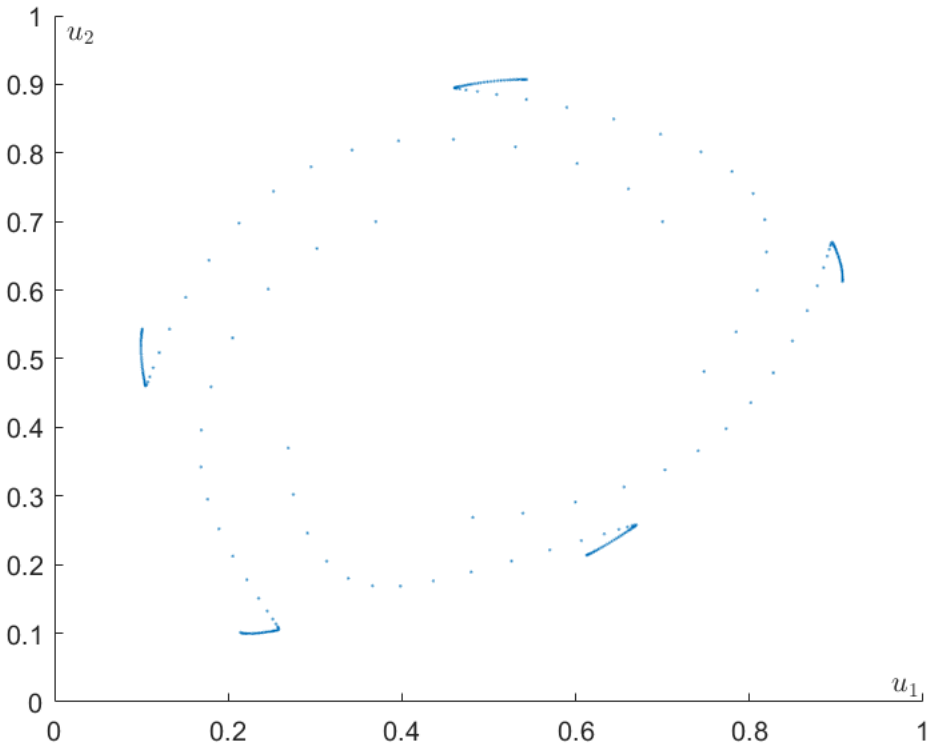
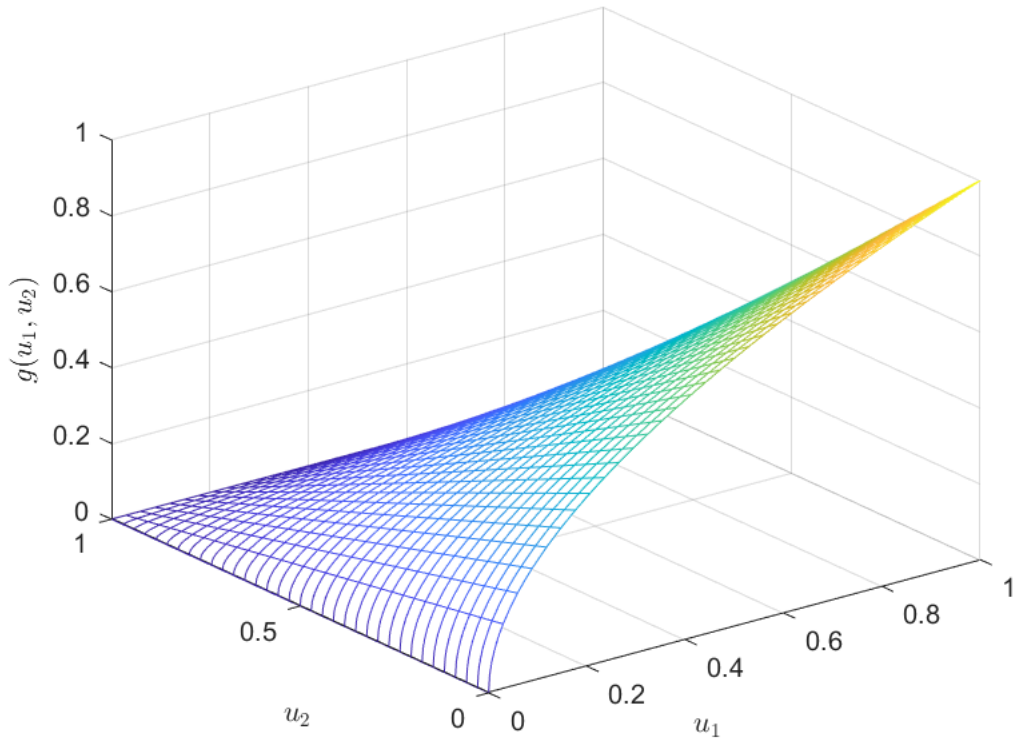


График g(u_1,u_2)

```
fmesh(@(u1,u2) sqrt(u1).*(1-u2), [0 1])
ax = gca;
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex'; ax.ZLabel.Interpreter = '
    ax.YLabel.Rotation = 0;
    ax.XLabel.String = '$u_1$'; ax.YLabel.String = '$u_2$'; ax.ZLabel.String = '$g(u_1,u_2)$';
exportgraphics(gcf,'pictures\ds2_g.png');
```



Вспомогательные функции

```
function res = ds2(u,r)
    res = [r*sqrt(u(1))*(1-u(2));
            u(1)];
end

function U = iterate(ds, times, u0)
    U = zeros(2,times+1);
    U(:,1) = u0;

    for it = 1:times
        U(:,it+1) = ds(U(:,it));
    end
end

function plotstyle1(x_label, y_label, line_label, lgd_location)
    ax = gca;
    X = ax.XLim; Y = ax.YLim;
    height = abs(Y(2)-Y(1)); width = abs(X(2)-X(1));
    ax.XAxisLocation = 'origin'; ax.YAxisLocation = 'origin'; ax.Box = "off";
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
    ax.XLabel.Position = [X(2)-width*0.01 Y(1)+height*0.05]; ax.YLabel.Position = [X(1)+width*0
    ax.YLabel.Rotation = 0;
    ax.XLabel.String = x_label; ax.YLabel.String = y_label;
    legend(line_label,'Interpreter','latex','AutoUpdate','off','Location',lgd_location);
end
```

```
function plotstyle2(x_label, y_label)
    ax = gca;
    X = ax.XLim; Y = ax.YLim;
    height = abs(Y(2)-Y(1)); width = abs(X(2)-X(1));
    ax.XAxisLocation = 'origin'; ax.YAxisLocation = 'origin'; ax.Box = "off";
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
    ax.XLabel.Position = [X(2)-width*0.02 Y(1)+height*0.05]; ax.YLabel.Position = [X(1)+width*0
    ax.YLabel.Rotation = 0;
    ax.XLabel.String = x_label; ax.YLabel.String = y_label;
end

function plotNS(r,u0,iters)
    f  = @(u) ds2(u,r);
    U = iterate(f,iters,u0);
    plot(U(1,:),U(2,:),'.','MarkerSize', 2,"Color",'#0072BD');
    ax = gca;
    ax.XLim = [0 1]; ax.YLim = [0 1];
end
```