Данные задачи

```
clear; clc;
syms u v  e f g h;
F1 = e*u.*(1-u) - (  u.*v)/(1+g*u);
F2 = -f*v      + (h*u.*v)/(1+g*u);
F = [F1;F2]
```

F =

$$\begin{pmatrix} -e\,u\,(u-1)-\dfrac{u\,v}{g\,u+1} \\ \dfrac{h\,u\,v}{g\,u+1}-f\,v \end{pmatrix}$$

```
%determine parameters of interest
parameters = [e h];
F = subs(F,[f g], [1 1])
```

F =

$$\begin{pmatrix} -e\,u\,(u-1)-\dfrac{u\,v}{u+1} \\ \dfrac{h\,u\,v}{u+1}-v \end{pmatrix}$$

Поиск неподвижных точек

```
eqn = 0 == F;
vars = [u,v];
[solu, solv] = solve(eqn,vars);
solutions = [solu solv]
```

solutions =

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ \dfrac{1}{h-1} & -\dfrac{2\,e\,h-e\,h^2}{h^2-2\,h+1} \end{pmatrix}$$

Характер неподвижных точек

Первая точка

```
k = 1;
static_point = solutions(k,:)
```

static_point = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
[tr, dt] = get_trace_n_determinant(F,vars,static_point);
```

new_vars = $\begin{pmatrix} x & y \end{pmatrix}$

substitution = $\begin{pmatrix} x & y \end{pmatrix}$

F_new =

$$\begin{pmatrix} -e\,x\,(x-1) - \frac{x\,y}{x+1} \\ \frac{h\,x\,y}{x+1} - y \end{pmatrix}$$

Jac =

$$\begin{pmatrix} \frac{x\,y}{(x+1)^2} - e\,x - e\,(x-1) - \frac{y}{x+1} & -\frac{x}{x+1} \\ \frac{h\,y}{x+1} - \frac{h\,x\,y}{(x+1)^2} & \frac{h\,x}{x+1} - 1 \end{pmatrix}$$

Jac0 =

$$\begin{pmatrix} e & 0 \\ 0 & -1 \end{pmatrix}$$

tr = $e - 1$

dt = $-e$

```
sign_map(tr,   [0 1],0,'$\mathop{\mathrm{tr}}J > 0$','$\mathop{\mathrm{tr}}J < 0$');
exportgraphics(gcf,'pictures/stp1/trJ.png');
```

```
sign_map(dt,       0,0,'$\det J > 0$','$\det J < 0$');
exportgraphics(gcf,'pictures/stp1/detJ.pdf');
```

```
sign_map(4*dt-tr^2,0,0,'$\det J > \frac{1}{4}\mathop{\mathrm{tr}}^2 J$','$4\det J < \mathop
exportgraphics(gcf,'pictures/stp1/trJdetJ.pdf');
```

```
lim = 5;
stp_parametric_portrait(tr,dt,lim, [0 1], 0);
    hold on;
    text(0.075*lim, 0.5*lim, '$VI$','Interpreter','latex');
    text(0.4  *lim, 0.5*lim, '$V$' ,'Interpreter','latex');
    hold off;
exportgraphics(gcf,'pictures/stp1/stability_portrait.png');
```

Вторая точка

```
k = 2
```

k = 2

```
static_point = solutions(k,:)
```

static_point = $\begin{pmatrix} 1 & 0 \end{pmatrix}$

```
[tr, dt] = get_trace_n_determinant(F,vars,static_point);
```

new_vars = $\begin{pmatrix} x & y \end{pmatrix}$

substitution = $\begin{pmatrix} x+1 & y \end{pmatrix}$

F_new =

$$\begin{pmatrix} -e\,x\,(x+1) - \frac{y\,(x+1)}{x+2} \\ \frac{h\,y\,(x+1)}{x+2} - y \end{pmatrix}$$

Jac =

$$\begin{pmatrix} \frac{y\,(x+1)}{(x+2)^2} - e\,x - e\,(x+1) - \frac{y}{x+2} & -\frac{x+1}{x+2} \\ \frac{h\,y}{x+2} - \frac{h\,y\,(x+1)}{(x+2)^2} & \frac{h\,(x+1)}{x+2} - 1 \end{pmatrix}$$

Jac0 =

$$\begin{pmatrix} -e & -\frac{1}{2} \\ 0 & \frac{h}{2} - 1 \end{pmatrix}$$

tr =

$$\frac{h}{2} - e - 1$$

dt =

$$-\frac{e\,(h-2)}{2}$$

```
    sign_map(tr,       0,[0 2],'$\mathop{\mathrm{tr}}J > 0$','$\mathop{\mathrm{tr}}J < 0$');
    exportgraphics(gcf,'pictures/stp2/trJ.png');
```

```
    sign_map(dt,       0,[0 2],'$\det J > 0$','$\det J < 0$');
    exportgraphics(gcf,'pictures/stp2/detJ.png');
```

```
    sign_map(4*dt-tr^2,0,0,'$\det J > \frac{1}{4}\mathop{\mathrm{tr}}^2 J$','$4\det J < \mathop
    exportgraphics(gcf,'pictures/stp2/trJdetJ.png');
```

```
    line2 = isolate(tr == 0, h)
```

line2 = $h = 2e + 2$

```
    lim = 5;
    stp_parametric_portrait(tr,dt,lim, 0, [0 2]);
        hold on;
        text(0.5 *lim, 1,    '$IV$','Interpreter','latex');
        text(0.6*lim, 3.75, '$VI$','Interpreter','latex');
        text(0.1*lim, 4.25,'$V$' ,'Interpreter','latex');
        hold off;
    exportgraphics(gcf,'pictures/stp2/stability_portrait.png');
```



Третья точка

```
k = 3
```

k = 3

```
static_point = solutions(k,:)
```

static_point =

$$\left(\frac{1}{h-1} \quad -\frac{2eh - eh^2}{h^2 - 2h + 1}\right)$$

NB! Неподвижная точка $\left(\frac{1}{h-1} \quad -\frac{2eh - eh^2}{h^2 - 2h + 1}\right)$ не принаделижит фазовому пространству, при $h < 2$.

```
subs(static_point,h,2)
```

ans = $(1 \quad 0)$

При h = 2 н.т. (3) сливается с н.т. (2)

```
[tr, dt] = get_trace_n_determinant(F,vars,static_point);
```

new_vars = $(x \quad y)$
substitution =

$$\left(x + \frac{1}{h-1} \quad y - \frac{2eh - eh^2}{h^2 - 2h + 1}\right)$$

F_new =

$$\begin{pmatrix} -\dfrac{(y-\sigma_1)\left(x+\dfrac{1}{h-1}\right)}{x+\dfrac{1}{h-1}+1} - e\left(x+\dfrac{1}{h-1}\right)\left(x+\dfrac{1}{h-1}-1\right) \\ \sigma_1 - y + \dfrac{h\,(y-\sigma_1)\left(x+\dfrac{1}{h-1}\right)}{x+\dfrac{1}{h-1}+1} \end{pmatrix}$$

where

$$\sigma_1 = \frac{2\,e\,h - e\,h^2}{h^2 - 2\,h + 1}$$

```
Jac =
```

$$\begin{pmatrix} \dfrac{\sigma_1\,\sigma_3}{{\sigma_2}^2} - e\,\sigma_3 - e\left(x+\dfrac{1}{h-1}-1\right) - \dfrac{\sigma_1}{\sigma_2} & -\dfrac{\sigma_3}{\sigma_2} \\ \dfrac{h\,\sigma_1}{\sigma_2} - \dfrac{h\,\sigma_1\,\sigma_3}{{\sigma_2}^2} & \dfrac{h\,\sigma_3}{\sigma_2} - 1 \end{pmatrix}$$

where

$$\sigma_1 = y - \frac{2\,e\,h - e\,h^2}{h^2 - 2\,h + 1}$$

$$\sigma_2 = x + \frac{1}{h-1} + 1$$

$$\sigma_3 = x + \frac{1}{h-1}$$

```
Jac0 =
```

$$\begin{pmatrix} \frac{\sigma_1}{\sigma_3} - \frac{e}{h-1} - e\left(\frac{1}{h-1} - 1\right) - \frac{\sigma_1}{\sigma_2} & -\frac{1}{\sigma_4\,(h-1)} \\ \frac{h\,\sigma_1}{\sigma_2} - \frac{h\,\sigma_1}{\sigma_3} & \frac{h}{\sigma_4\,(h-1)} - 1 \end{pmatrix}$$

where

$$\sigma_1 = 2\,e\,h - e\,h^2$$

$$\sigma_2 = {\sigma_4}^2\,(h-1)\,(h^2 - 2\,h + 1)$$

$$\sigma_3 = \sigma_4\,(h^2 - 2\,h + 1)$$

$$\sigma_4 = \frac{1}{h-1} + 1$$

```
tr =
```

$$\frac{2\,e\,h - e\,h^2}{\sigma_1\,(h^2 - 2\,h + 1)} - \frac{e}{h-1} - e\left(\frac{1}{h-1} - 1\right) + \frac{h}{\sigma_1\,(h-1)} - \frac{2\,e\,h - e\,h^2}{{\sigma_1}^2\,(h-1)\,(h^2 - 2\,h + 1)} - 1$$

where

$$\sigma_1 = \frac{1}{h-1} + 1$$

```
dt =
```

$$-\frac{2\,e - e\,h}{h}$$

```
line3 = isolate(tr^2 == 4*dt,h)
```

```
line3 =
```

$$h = \frac{\sqrt{2}\,\sqrt{\sqrt{4\,e+1}+1}}{2} + 1$$

```
    sign_map(tr,       0,[0 1],'$\mathop{\mathrm{tr}}J > 0$','$\mathop{\mathrm{tr}}J < 0$');
    exportgraphics(gcf,'pictures/stp3/trJ.png');
```

```
    sign_map(dt,       0,[0 2],'$\det J > 0$','$\det J < 0$');
    exportgraphics(gcf,'pictures/stp3/detJ.png');
```

```
    sign_map(4*dt-tr^2,0,[0 2],'$\det J > \frac{1}{4}\mathop{\mathrm{tr}}^2 J$','$4\det J < \ma
```

```
    exportgraphics(gcf,'pictures/stp3/trJdetJ.png');
```

```
    lim = 5;
    stp_parametric_portrait(tr,dt,lim, 0, [0 2]);
        hold on;
        text(0.5 *lim, 1,  '\textbf{---}'  ,'Interpreter','latex');
        text(0.6*lim, 2.25,  '$IV$'  ,'Interpreter','latex');
        text(0.5 *lim, 3.5,'$I$'  ,'Interpreter','latex');
        text(0.75 *lim, 2.9,'$h = \sqrt{\frac{\sqrt{4e+1}+1}{2}} + 1$' ,'Interpreter','latex');
        hold off;
    exportgraphics(gcf,'pictures/stp3/stability_portrait.png');
```

Параметрический портрет системы

```
    lim = 5;
    figure;
    ax = axes;
    hold on;
        ax.XLim  = [0 lim]; ax.YLim = [0 lim];
        X = ax.XLim; Y = ax.YLim;
        ax.XTick = [0 1.5]; ax.YTick = [0 2];
        ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
        ax.XLabel.String = '$e$'; ax.YLabel.String = '$h$';
        ax.XLabel.Position = [0.95*X(2) -0.025*Y(2)]; ax.YLabel.Position = [-0.05*X(2) 0.9*Y(2)
        ax.YLabel.Rotation = 0;

        xline(0,'Color','#A2142F'); yline(0,'Color','#A2142F');
        yline(2,'Color','#A2142F');
        fimplicit(line3,'Color','#A2142F');

        text(0.5 *lim, 1,  '$iii$'  ,'Interpreter','latex','FontSize',12);
        text(0.6*lim, 2.25,  '$ii$'  ,'Interpreter','latex','FontSize',12);
        text(0.5 *lim, 3.5,'$i$'  ,'Interpreter','latex','FontSize',12);

        xline(1.5,'Color','#77AC30','LineStyle','--');
        plot(1.5, 3,   'Marker','.','MarkerSize',10);
        plot(1.5, 2.1, 'Marker','.','MarkerSize',10);
        plot(1.5, 1.5, 'Marker','.','MarkerSize',10);

        text(1.6, 3.05,  '$b_1$'  ,'Interpreter','latex','FontSize',9);
        text(1.6, 2.15,'$b_2$'  ,'Interpreter','latex','FontSize',9);
        text(1.6, 1.55,'$b_3$'  ,'Interpreter','latex','FontSize',9);

   hold off;

exportgraphics(gcf,'pictures\phase_portraits\parametric_portrait.png');
```

Фазовый портрет i(k=3 -- устойчивый фокус)

```
fix_params = [1.5 3]; grd = 0.01:0.08:2.01;

stable_point = subs(solutions(3,:), parameters, fix_params)
```

```
stable_point =
```

$$\left(\frac{1}{2} \quad \frac{9}{8}\right)$$

```
Ffix = subs(F,parameters, fix_params);

X0 = [2.00 2.00 2.00 2.00 2.00 2.00 2.00 0.10 0.01 ;
      0.05 0.30 0.55 0.80 1.05 1.35 1.80 2.00 2.00];

figure;
add_phase_portrait_plot(Ffix,vars,grd,fix_params(1),fix_params(2),X0,stable_point);

exportgraphics(gcf,'pictures\phase_portraits\i.pdf');
```

Фазовый портерт ii (k=3 -- устойчивый узел)

```
fix_params = [1.5 2.1]; grd = 0.01:0.08:2.01;

stable_point = subs(solutions(3,:), parameters, fix_params)
```

```
stable_point =
```

$$\left(\frac{10}{11} \quad \frac{63}{242}\right)$$

```
Ffix = subs(F,parameters, fix_params);

X0 = [2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 0.10 0.03 0.003;
      0.05 0.30 0.55 0.80 1.05 1.30 1.55 1.90 2.50 3.50 2.00 2.00 2.000];

figure;
add_phase_portrait_plot(Ffix,vars,grd,fix_params(1),fix_params(2),X0, stable_point);

exportgraphics(gcf,'pictures\phase_portraits\ii.pdf');
```

Фазовый портет iii (k=2 -- устойчивый узел)

```
fix_params = [1.5 1.5]; grd = 0.01:0.08:2.01;

stable_point = solutions(2,:)
```

stable_point = $(1 \quad 0)$

```
Ffix = subs(F,parameters, fix_params);

X0 = [2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00;
      0.05 0.30 0.55 0.80 1.05 1.30 1.55 1.90 2.50 3.50 6.00 50.0];

figure;
add_phase_portrait_plot(Ffix,vars,grd,fix_params(1),fix_params(2),X0, stable_point);

exportgraphics(gcf,'pictures\phase_portraits\iii.pdf');
```

Дивергенция векторного поля (частный случай)

```
divF = divergence(F,vars);
[N D] = numden(divF)
```

N = $e - 2\,u - v + h\,u - 3\,e\,u^2 - 2\,e\,u^3 + h\,u^2 - u^2 - 1$

D = $(u+1)^2$

```
divF = N/D
```

divF =

$$-\frac{2\,u - e + v - h\,u + 3\,e\,u^2 + 2\,e\,u^3 - h\,u^2 + u^2 + 1}{(u+1)^2}$$

```
fmesh(subs(divF,parameters,[1 1])); ax = gca; ax.XLim = [0 Inf]; ax.YLim = [0 Inf];
```

Дополнительные рисунки

```
fplot(@(x) 0.25*x.^2,'Color','#000000');
ax = gca;
    t = 5;
    ax.XLim = [-t t]; ax.YLim = [-t t];
    ax.XTick = []; ax.YTick = [];
    ax.XAxisLocation = 'origin'; ax.YAxisLocation = 'origin'; ax.Box = "off";
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
    ax.XLabel.Position = [0.95*t -0.15*t]; ax.YLabel.Position = [-0.2*t 0.95*t];
```

```
    ax.YLabel.Rotation = 0;
    ax.XLabel.String = '$\mathop{\mathrm{tr}}J$'; ax.YLabel.String = '$\det J$';
text(-0.4*t, 0.7*t,'$I$'  ,'Interpreter','latex');
text( 0.4*t, 0.7*t,'$II$' ,'Interpreter','latex');
text( 0.8*t, 0.2*t,'$III$','Interpreter','latex');
text(-0.8*t, 0.2*t,'$IV$' ,'Interpreter','latex');
text( 0.5*t,-0.5*t,'$V$'  ,'Interpreter','latex');
text(-0.5*t,-0.5*t,'$VI$' ,'Interpreter','latex');

exportgraphics(gcf,'pictures\stabilities.pdf');
```

Вспомогательные функции

```
function res = odefun(t,x,e,f,g,h)
    res =  [e*x(1).*(1-x(1)) - (  x(1).*x(2))./(1+g*x(1));
            -f*x(2)          + (h*x(1).*x(2))./(1+g*x(1))];
end

function [tr, dt] = get_trace_n_determinant(F,vars,static_point)
    syms x y; %substitution
    new_vars = [x y]
        %x = u - u* => u = x + u*; \dot u = f(x+u*); v~--- analogously
    substitution = new_vars + static_point
    F_new = subs(F,vars, substitution)
    Jac = jacobian(F_new,new_vars)
    Jac0 = subs(Jac,[x y], [0 0])

    tr = trace(Jac0)
    dt = det(Jac0)
end

function sign_map(func, x_ticks,y_ticks, label_lz,label_gz)
             fmesh(func > 0);
    hold on; fimplicit(func == 0); hold off;
    ax = gca;
        ax.XLim(1) = 0; ax.YLim(1) = 0;
        X = ax.XLim; Y = ax.YLim;
        ax.XTick = x_ticks; ax.YTick = y_ticks;
        ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
        ax.XLabel.String = '$e$'; ax.YLabel.String = '$h$';
        ax.XLabel.Position = [0.95*X(2) -0.025*Y(2)]; ax.YLabel.Position = [-0.05*X(2) 0.9*Y(2)
        ax.YLabel.Rotation = 0;
    colormap(lines(3));
    cb = colorbar('Ticks',[0 1],'TickLabels',{label_lz,label_gz},'TickLabelInterpreter',"latex"
    cb.Location = 'northoutside';
    cb.FontSize = 11;
    view([0 90]);
end

function stp_parametric_portrait(tr,dt,lim, x_ticks,y_ticks)
    figure;
```

```
    ax = axes;
    hold on;
        ax.XLim  = [0 lim]; ax.YLim = [0 lim];
        X = ax.XLim; Y = ax.YLim;
        ax.XTick = x_ticks; ax.YTick = y_ticks;
        ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
        ax.XLabel.String = '$e$'; ax.YLabel.String = '$h$';
        ax.XLabel.Position = [0.95*X(2) -0.025*Y(2)]; ax.YLabel.Position = [-0.05*X(2) 0.9*Y(2)
        ax.YLabel.Rotation = 0;

    fimplicit(tr == 0, [0 lim],'Color','#7E2F8E');
    fimplicit(dt == 0, [0 lim],'Color','#7E2F8E');
    fimplicit(tr^2 == 4*dt, [0 lim],'Color','#7E2F8E');
    hold off;
end

function add_phase_portrait_plot(Fexmp, vars, grd, e_fix, h_fix, X0, stp) %, stp_txt
    hold on;

    [X,Y] = meshgrid(grd, grd);
    F1exmp = symfun(Fexmp(1),vars); F2exmp = symfun(Fexmp(2),vars);
    NORM = symfun(sqrt(F1exmp.^2 + F2exmp.^2),vars);
    F1exmp = F1exmp./NORM; F2exmp = F2exmp./NORM;
    Udot = F1exmp(X,Y); Vdot = F2exmp(X,Y);

    q = quiver(X,Y,Udot,Vdot,'AutoScaleFactor',0.7);
    ax = gca;
        ax.XLim = [0 2]; ax.YLim = [0 2];
        X = ax.XLim; Y = ax.YLim;
        ax.XTick = [0 1]; ax.YTick = 0;
        ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
        ax.XLabel.String = '$u$'; ax.YLabel.String = '$v$';
        ax.XLabel.Position = [0.95*X(2) -0.025*Y(2)]; ax.YLabel.Position = [-0.05*X(2) 0.9*Y(2)
        ax.YLabel.Rotation = 0;

    line_col = '#D95319'; line_width = 0.75;
    for cnt = 1:length(X0)
        [t,y] = ode45(@(t,x) odefun(t,x, e_fix,1,1,h_fix),[0 100],X0(:,cnt));
        plot(y(:,1),y(:,2),'LineWidth',line_width,'Color', line_col);
    end

    plot(stp(1),stp(2),'Marker','.',"MarkerSize", 14,'Color','#A2142F');
    %text(stp(1)+0.04,stp(2)+0.05, stp_txt,'Interpreter',"latex");

    setplotstyleIII();

    hold off;
end

function setplotstyleI(z_label)
    hold on;
    ax = gca;
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex'; ax.ZLabel.Interpreter = '
    ax.XLabel.String = '$e$'; ax.YLabel.String = '$h$'; ax.ZLabel.String = z_label;
```

```
    hold off
end

function setplotstyleII(lgd_text)
    hold on;
    ax = gca;
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
    ax.XLabel.String = '$e$'; ax.YLabel.String = '$h$';
    ax.YLabel.Rotation = 0;
    legend(lgd_text,'Interpreter','latex','AutoUpdate','off');
    hold off;
end

function setplotstyleIII()
    hold on;
    ax = gca;
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
    ax.XLabel.String = '$u$'; ax.YLabel.String = '$v$';
    ax.YLabel.Rotation = 0;
    hold off;
end
```