Данные задачи

```
clear; clc;
syms u v  e f g h;
F1 = e*u.*(1-u) - (  u.*v)/(1+g*u);
F2 = -f*v      + (h*u.*v)/(1+g*u);
F = [F1;F2]
```

F =

$$\begin{pmatrix} -e\,u\,(u-1)-\dfrac{u\,v}{g\,u+1} \\ \dfrac{h\,u\,v}{g\,u+1}-f\,v \end{pmatrix}$$

```
%determine parameters of interest
parameters = [e h];
F = subs(F,[f g], [1 1])
```

F =

$$\begin{pmatrix} -e\,u\,(u-1)-\dfrac{u\,v}{u+1} \\ \dfrac{h\,u\,v}{u+1}-v \end{pmatrix}$$

Поиск неподвижных точек

```
eqn = 0 == F;
vars = [u,v];
[solu, solv] = solve(eqn,vars);
solutions = [solu solv]
```

solutions =

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ \dfrac{1}{h-1} & -\dfrac{2\,e\,h-e\,h^2}{h^2-2\,h+1} \end{pmatrix}$$

Характер неподвижных точек

k-ая точка (k = 1:3)

```
k = 1;
static_point = solutions(k,:)
```

static_point = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

```
lim = 10;

[mu1, mu2] = get_static_point_eigenvalues(F,vars,static_point)
```

substitution = $\begin{pmatrix} x & y \end{pmatrix}$

F_new =

$$\begin{pmatrix} -e\,x\,(x-1) - \frac{x\,y}{x+1} \\ \frac{h\,x\,y}{x+1} - y \end{pmatrix}$$

Jac =

$$\begin{pmatrix} \frac{x\,y}{(x+1)^2} - e\,x - e\,(x-1) - \frac{y}{x+1} & -\frac{x}{x+1} \\ \frac{h\,y}{x+1} - \frac{h\,x\,y}{(x+1)^2} & \frac{h\,x}{x+1} - 1 \end{pmatrix}$$

Jac0 =

$$\begin{pmatrix} e & 0 \\ 0 & -1 \end{pmatrix}$$

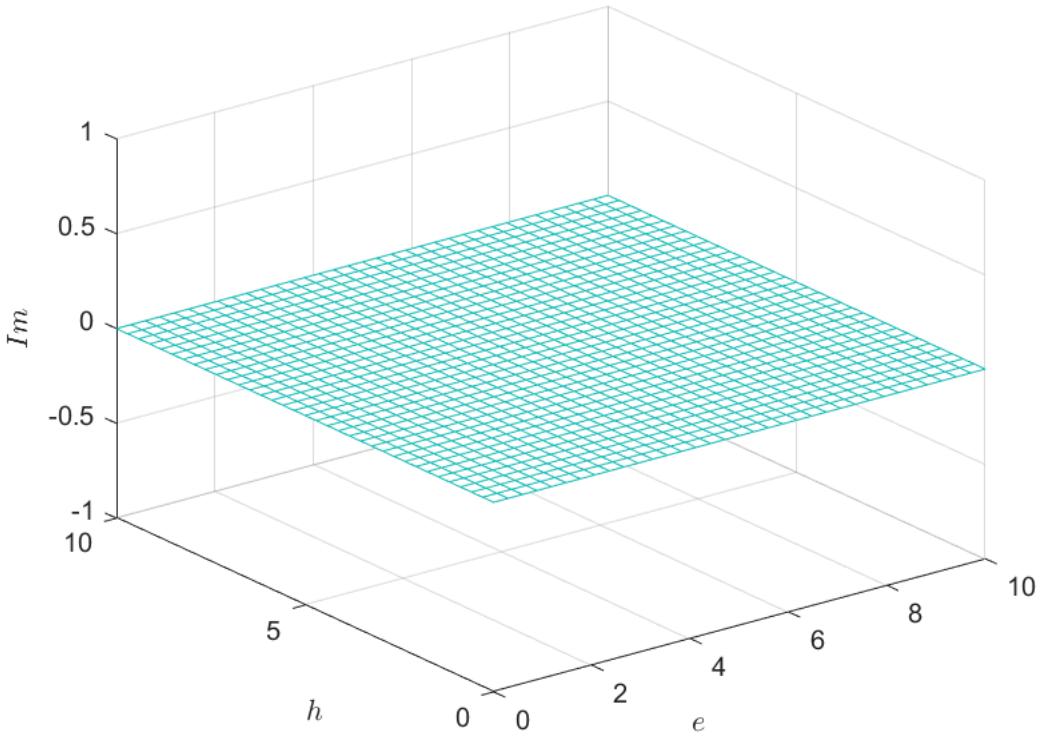
eigvecs =

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

mu1 = -1

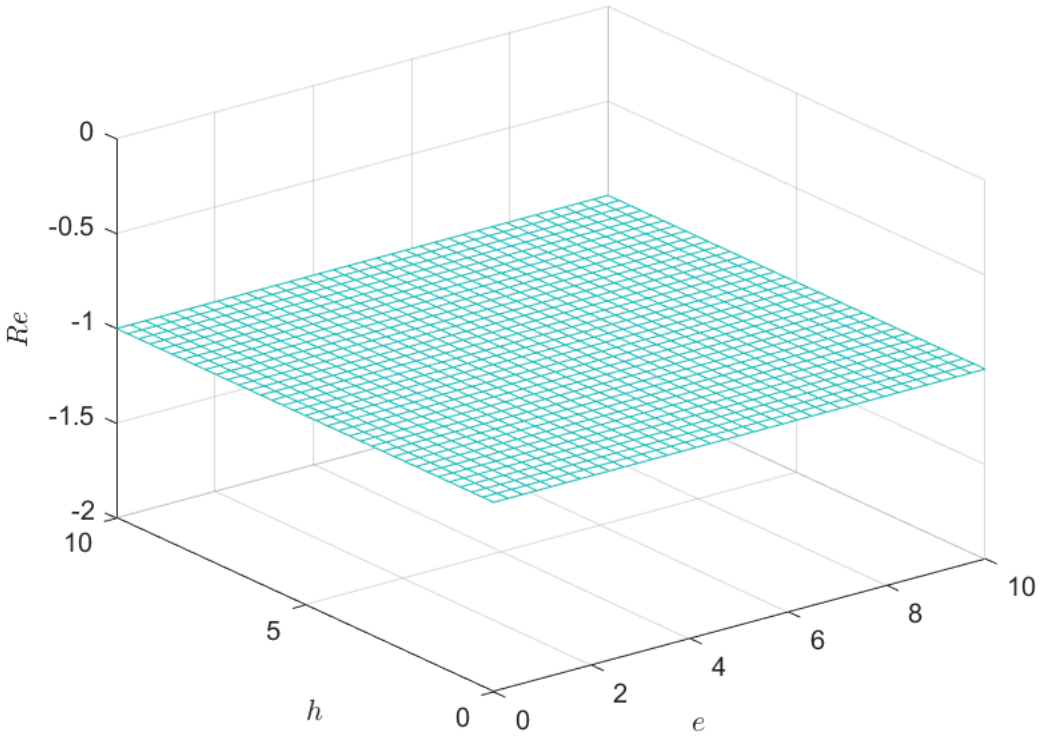
mu2 = e

```
fmesh(imag(mu1),[0 lim]); setplotstyleI('$Im$');
exportgraphics(gcf,'pictures\static_point_1\Im_mu1.pdf');
```

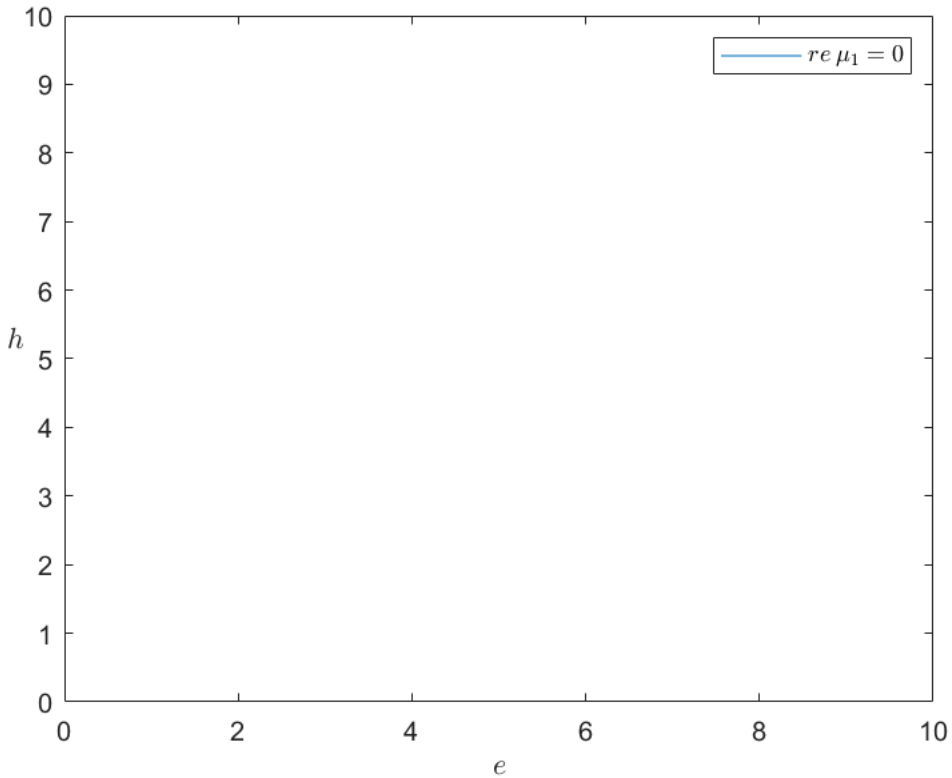



```
fmesh(real(mu1),[0 lim]); setplotstyleI('$Re$');
```

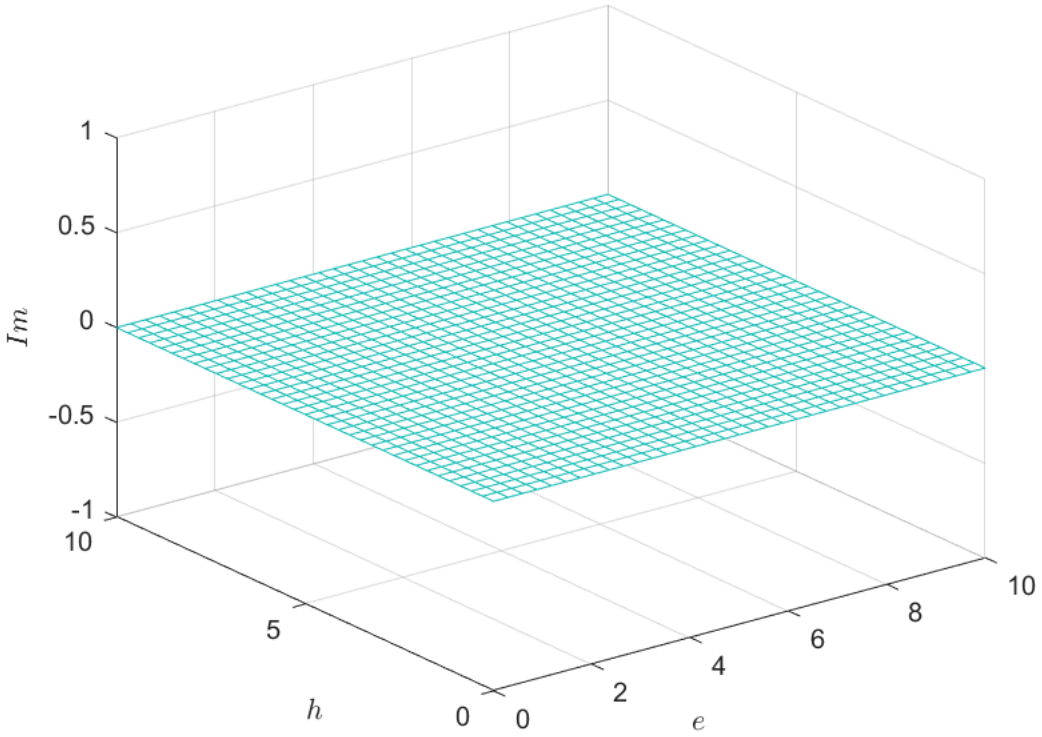
```
exportgraphics(gcf,'pictures\static_point_1\Re_mu1.pdf');
```



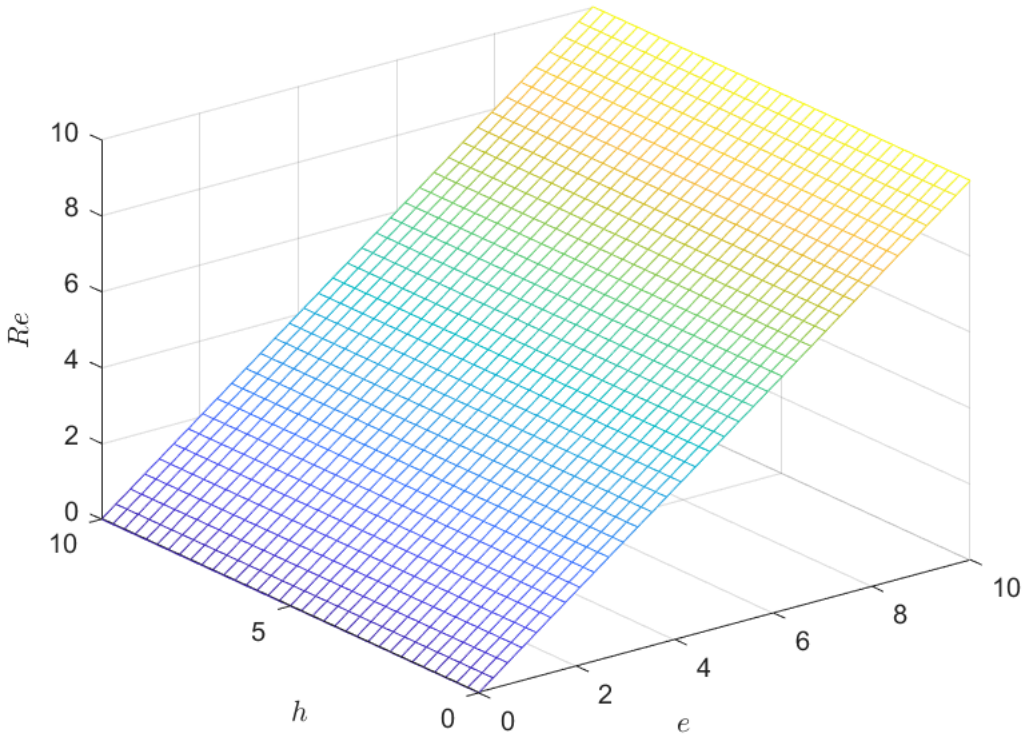

```
fimplicit(real(mu1) == 0, [0 lim]); setplotstyleII('$re\:\mu_1 = 0$');
```



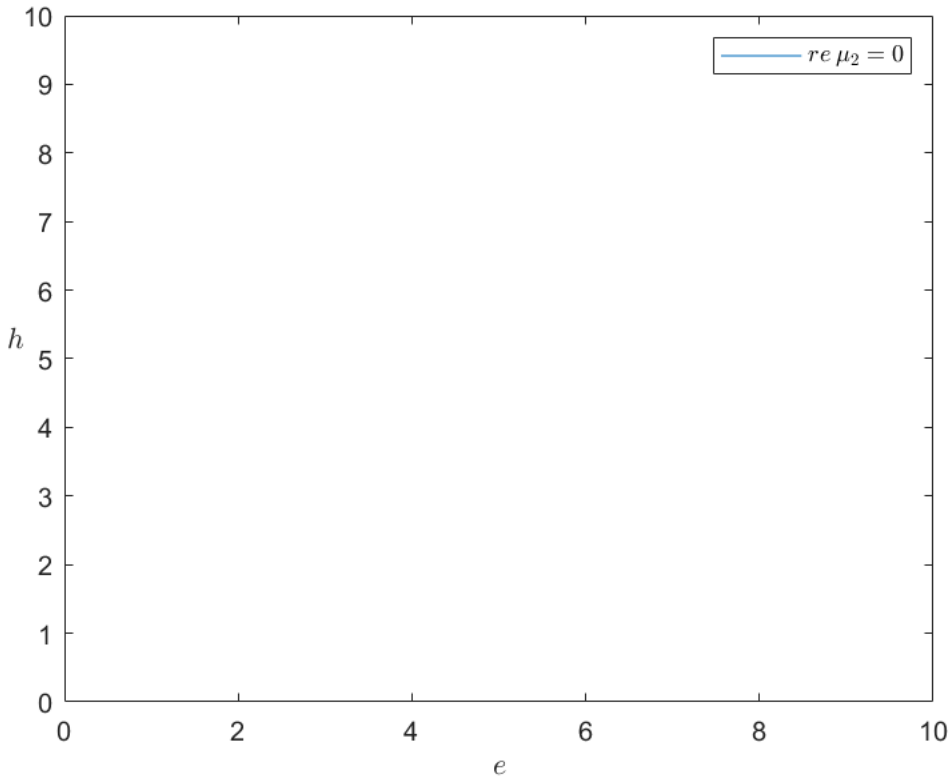
```
fmesh(imag(mu2),[0 lim]); setplotstyleI('$Im$');
exportgraphics(gcf,'pictures\static_point_1\Im_mu2.pdf');
```




```
fmesh(real(mu2),[0 lim]); setplotstyleI('$Re$');
exportgraphics(gcf,'pictures\static_point_1\Re_mu2.pdf');
```




```
fimplicit(real(mu2) == 0, [0 lim]); setplotstyleII('$re\:\mu_2 = 0$');
```



```
k = 2
```

k = 2

```
static_point = solutions(k,:)
```

static_point = $\begin{pmatrix} 1 & 0 \end{pmatrix}$

```
lim = 10;

[mu1, mu2] = get_static_point_eigenvalues(F,vars,static_point)
```

substitution = $\begin{pmatrix} x+1 & y \end{pmatrix}$

F_new =

$$\begin{pmatrix} -e\,x\,(x+1) - \frac{y\,(x+1)}{x+2} \\ \frac{h\,y\,(x+1)}{x+2} - y \end{pmatrix}$$

Jac =

$$\begin{pmatrix} \frac{y\,(x+1)}{(x+2)^2} - e\,x - e\,(x+1) - \frac{y}{x+2} & -\frac{x+1}{x+2} \\ \frac{h\,y}{x+2} - \frac{h\,y\,(x+1)}{(x+2)^2} & \frac{h\,(x+1)}{x+2} - 1 \end{pmatrix}$$

Jac0 =

$$\begin{pmatrix} -e & -\frac{1}{2} \\ 0 & \frac{h}{2} - 1 \end{pmatrix}$$

eigvecs =

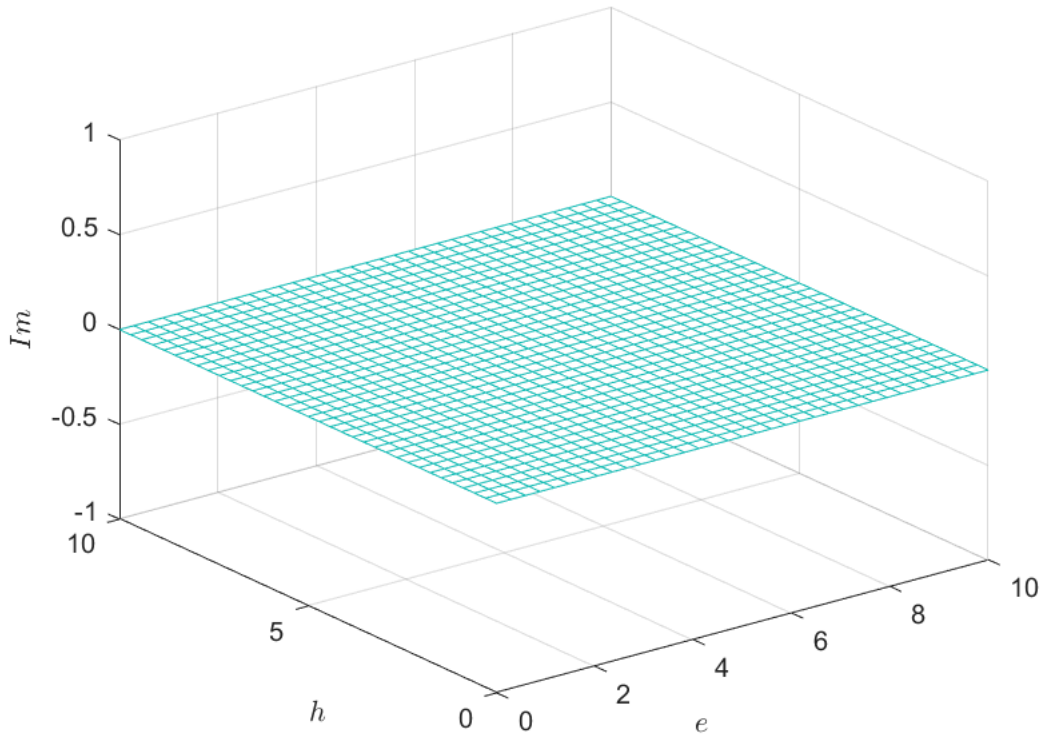
$$\begin{pmatrix} -\frac{1}{2\,e+h-2} & 1 \\ 1 & 0 \end{pmatrix}$$

mu1 =

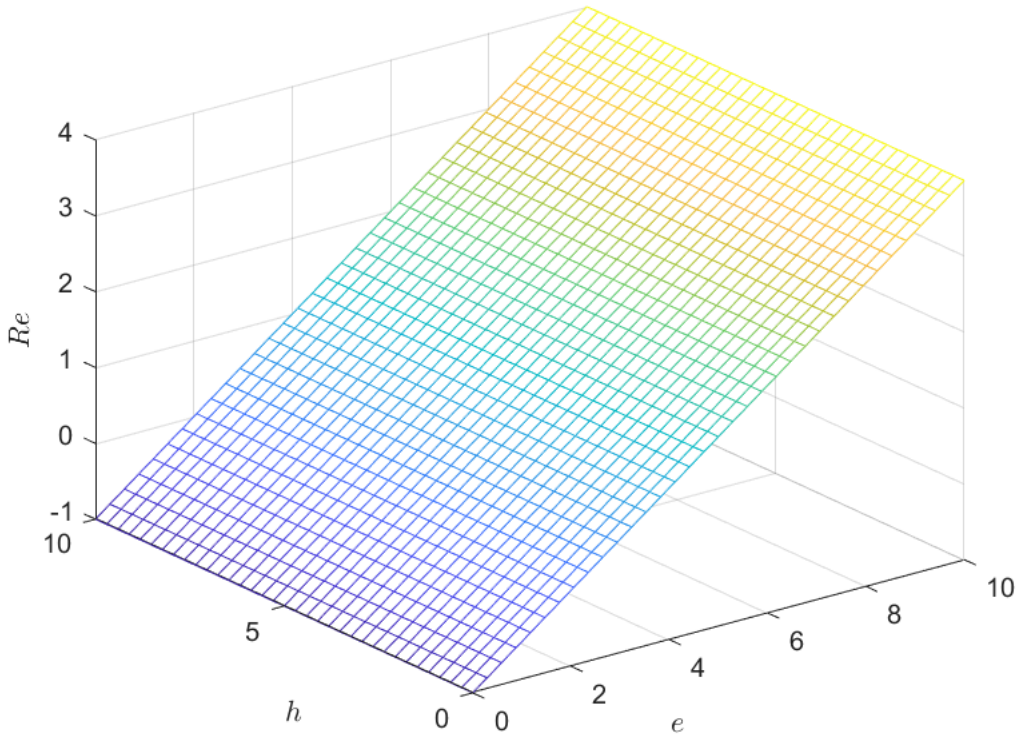
$$\frac{h}{2} - 1$$

mu2 = $-e$

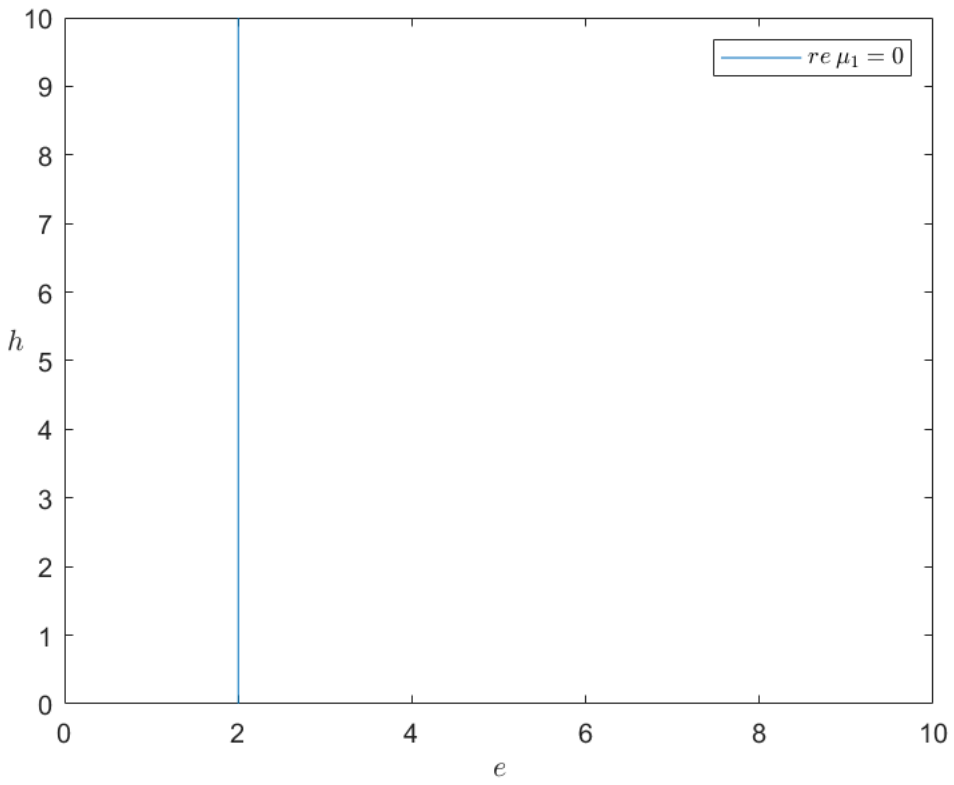
```
fmesh(imag(mu1),[0 lim]); setplotstyleI('$Im$');
exportgraphics(gcf,'pictures\static_point_2\Im_mu1.pdf');
```



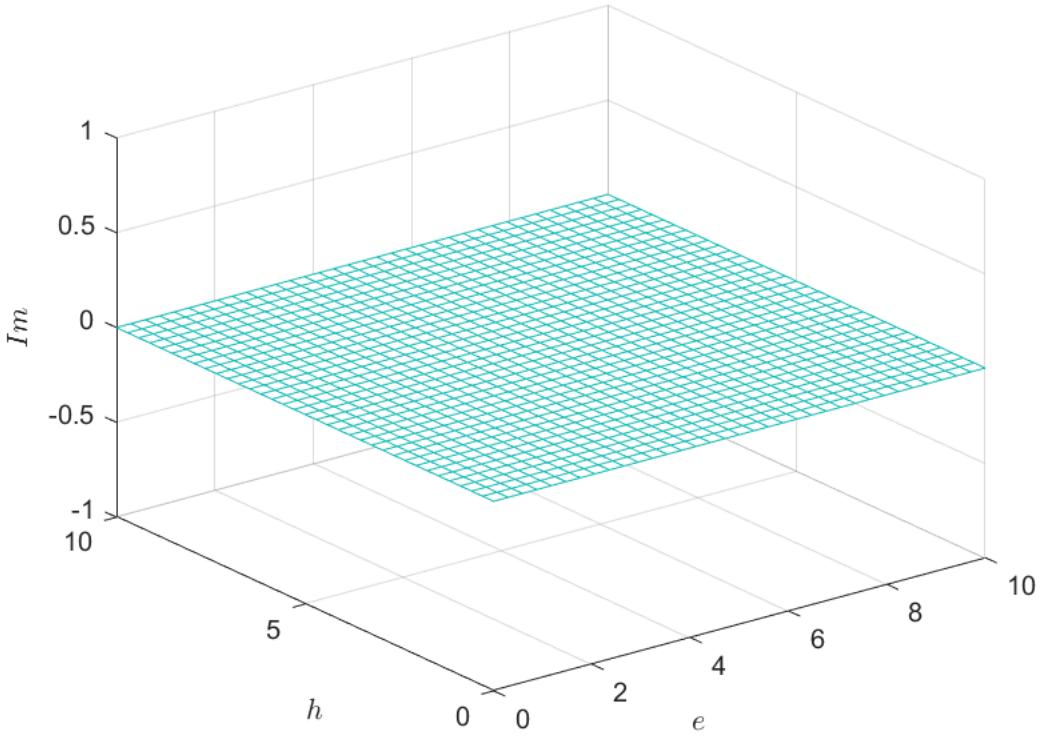

```
fmesh(real(mu1),[0 lim]); setplotstyleI('$Re$');
exportgraphics(gcf,'pictures\static_point_2\Re_mu1.pdf');
```



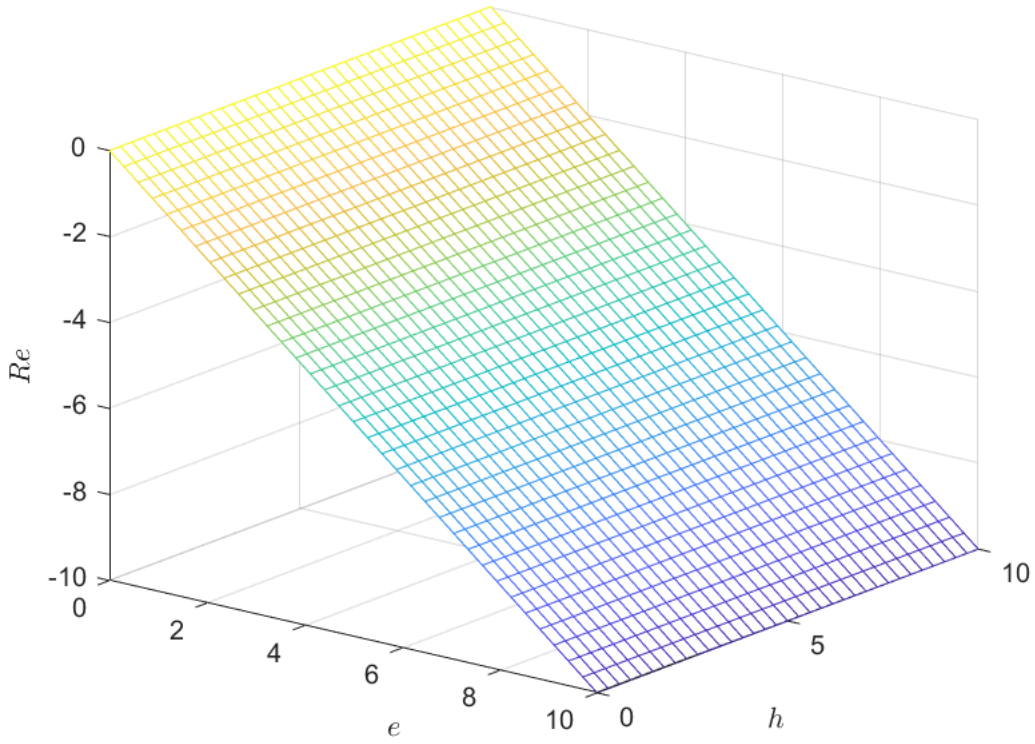
```
fimplicit(real(mu1) == 0, [0 lim]); setplotstyleII('$re\:\mu_1 = 0$');
```



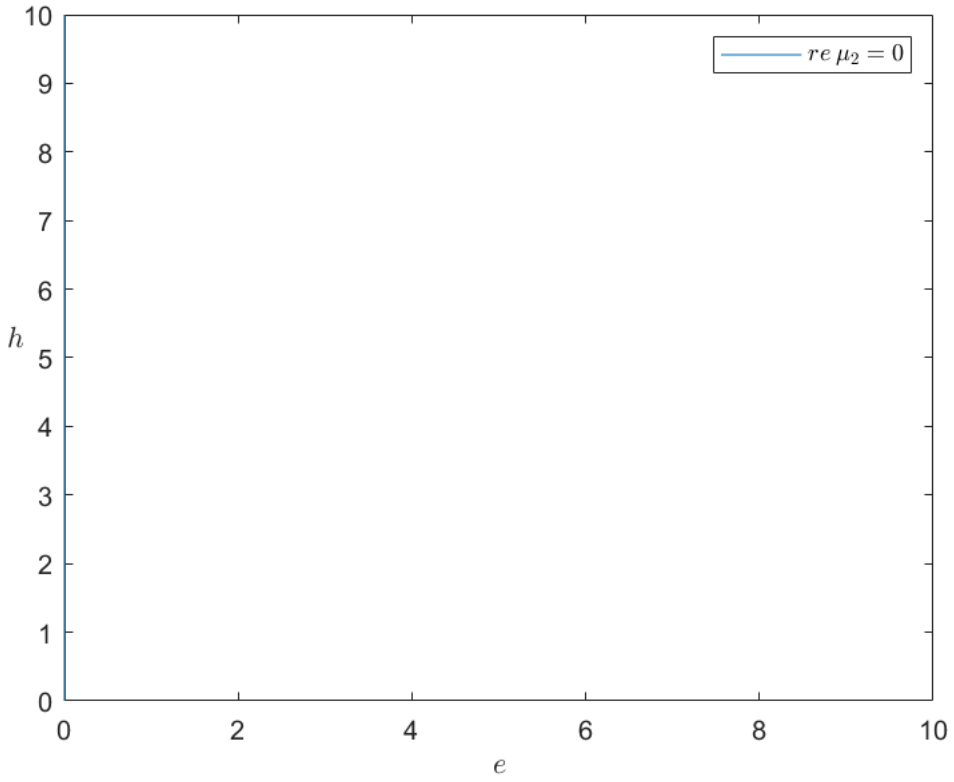

```
fmesh(imag(mu2),[0 lim]); setplotstyleI('$Im$');
exportgraphics(gcf,'pictures\static_point_2\Im_mu2.pdf');
```




```
fmesh(real(mu2),[0 lim]); setplotstyleI('$Re$'); view([38 23]);
exportgraphics(gcf,'pictures\static_point_2\Re_mu2.pdf');
```



```
fimplicit(real(mu2) == 0, [0 lim]); setplotstyleII('$re\:\mu_2 = 0$');
```




```
k = 3
```

k = 3

```
static_point = solutions(k,:)
```

static_point =

$$\left(\frac{1}{h-1} \quad -\frac{2\,e\,h-e\,h^2}{h^2-2\,h+1}\right)$$

```
lim = 10;
```

Неподвижная точка $\left(\frac{1}{h-1} \quad -\frac{2\,e\,h-e\,h^2}{h^2-2\,h+1}\right)$, очевидно, не принаделижит фазовому пространству, при $h < 2$.

```
elb = 0; erb = lim; hlb = 2; hrb = lim;
bnds = [elb erb hlb hrb];

[mu1, mu2] = get_static_point_eigenvalues(F,vars,static_point)
```

substitution =

$$\left(x+\frac{1}{h-1}\quad y-\frac{2\,e\,h-e\,h^2}{h^2-2\,h+1}\right)$$

F_new =

$$\begin{pmatrix}-\dfrac{(y-\sigma_1)\,\left(x+\dfrac{1}{h-1}\right)}{x+\dfrac{1}{h-1}+1}-e\,\left(x+\dfrac{1}{h-1}\right)\,\left(x+\dfrac{1}{h-1}-1\right)\\ \sigma_1-y+\dfrac{h\,(y-\sigma_1)\,\left(x+\dfrac{1}{h-1}\right)}{x+\dfrac{1}{h-1}+1}\end{pmatrix}$$

where

$$\sigma_1=\frac{2\,e\,h-e\,h^2}{h^2-2\,h+1}$$

Jac =

$$\begin{pmatrix}\dfrac{\sigma_1\,\sigma_3}{{\sigma_2}^2}-e\,\sigma_3-e\,\left(x+\dfrac{1}{h-1}-1\right)-\dfrac{\sigma_1}{\sigma_2} & -\dfrac{\sigma_3}{\sigma_2}\\ \dfrac{h\,\sigma_1}{\sigma_2}-\dfrac{h\,\sigma_1\,\sigma_3}{{\sigma_2}^2} & \dfrac{h\,\sigma_3}{\sigma_2}-1\end{pmatrix}$$

where

$$\sigma_1=y-\frac{2\,e\,h-e\,h^2}{h^2-2\,h+1}$$

$$\sigma_2=x+\frac{1}{h-1}+1$$

$$\sigma_3=x+\frac{1}{h-1}$$

Jac0 =

$$\begin{pmatrix} \frac{\sigma_1}{\sigma_3} - \frac{e}{h-1} - e\left(\frac{1}{h-1} - 1\right) - \frac{\sigma_1}{\sigma_2} & -\frac{1}{\sigma_4\,(h-1)} \\ \frac{h\,\sigma_1}{\sigma_2} - \frac{h\,\sigma_1}{\sigma_3} & \frac{h}{\sigma_4\,(h-1)} - 1 \end{pmatrix}$$

where

$$\sigma_1 = 2\,e\,h - e\,h^2$$

$$\sigma_2 = {\sigma_4}^2\,(h-1)\,(h^2 - 2\,h + 1)$$

$$\sigma_3 = \sigma_4\,(h^2 - 2\,h + 1)$$

$$\sigma_4 = \frac{1}{h-1} + 1$$

eigvecs =

$$\begin{pmatrix} -\frac{e + \sigma_1}{(2\,e - e\,h)\,(h - h^2)} & -\frac{e - \sigma_1}{(2\,e - e\,h)\,(h - h^2)} \\ 1 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \sqrt{e\,(-h^4 + 4\,h^3 - 5\,h^2 + 2\,h + e)}$$

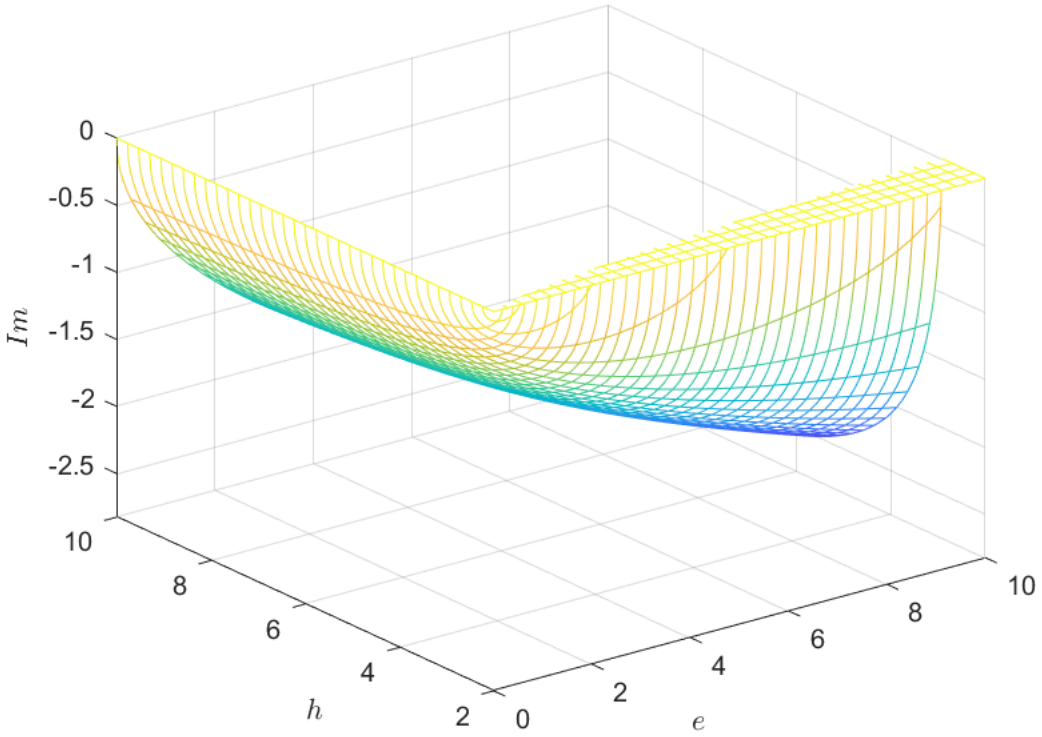
mu1 =

$$\frac{e + \sqrt{e\,(-h^4 + 4\,h^3 - 5\,h^2 + 2\,h + e)}}{h - h^2}$$

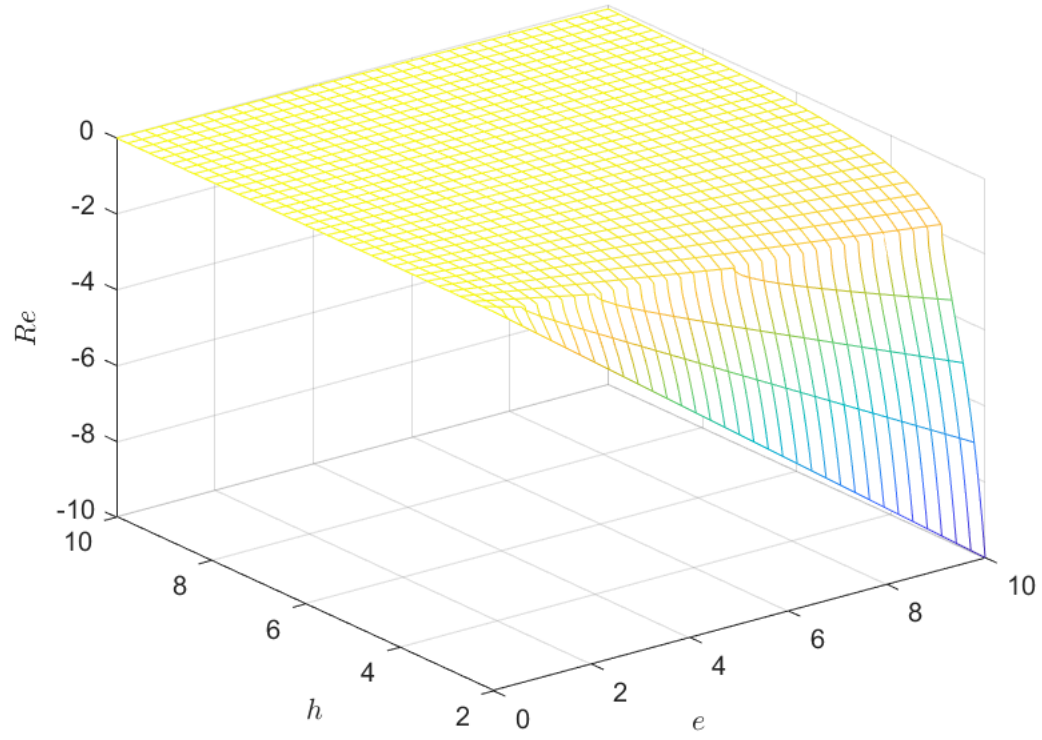
mu2 =

$$\frac{e - \sqrt{e\,(-h^4 + 4\,h^3 - 5\,h^2 + 2\,h + e)}}{h - h^2}$$

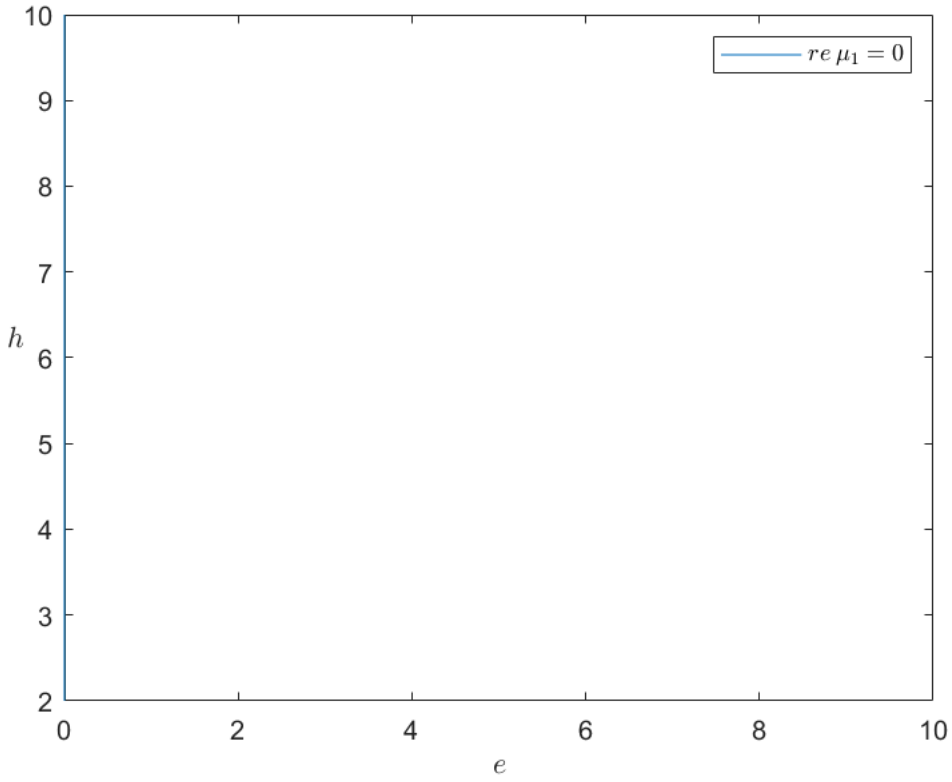
```
fmesh(imag(mu1),bnds); setplotstyleI('$Im$');
exportgraphics(gcf,'pictures\static_point_3\Im_mu1.pdf');
```



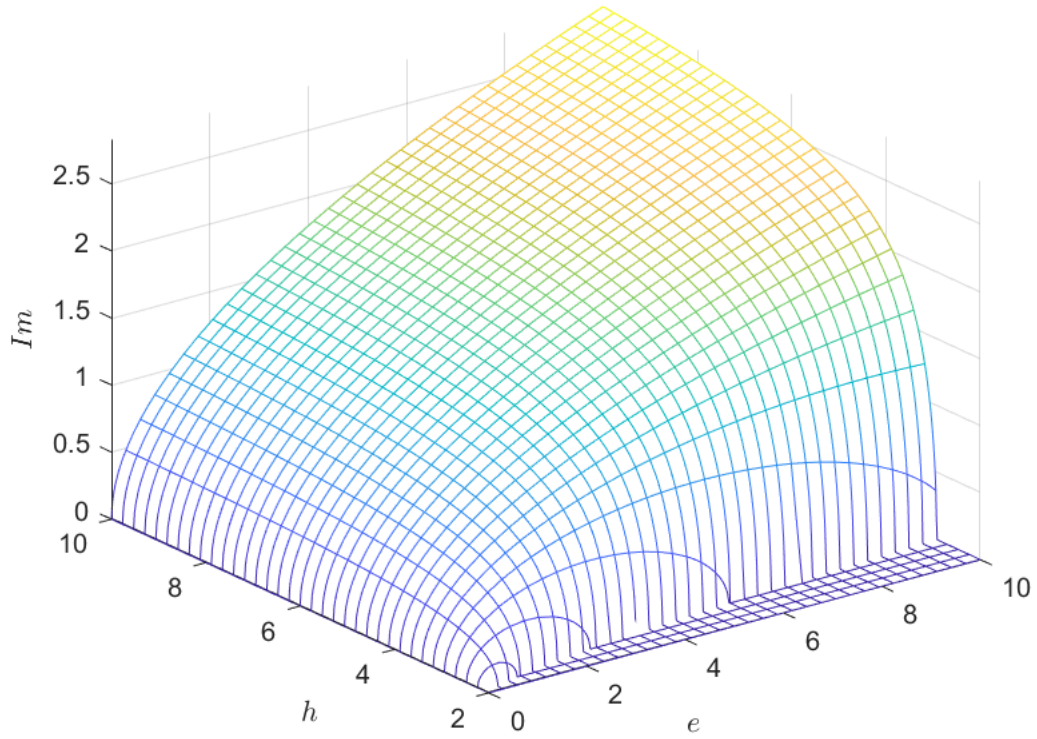

```
fmesh(real(mu1),bnds); setplotstyleI('$Re$');
exportgraphics(gcf,'pictures\static_point_3\Re_mu1.pdf');
```



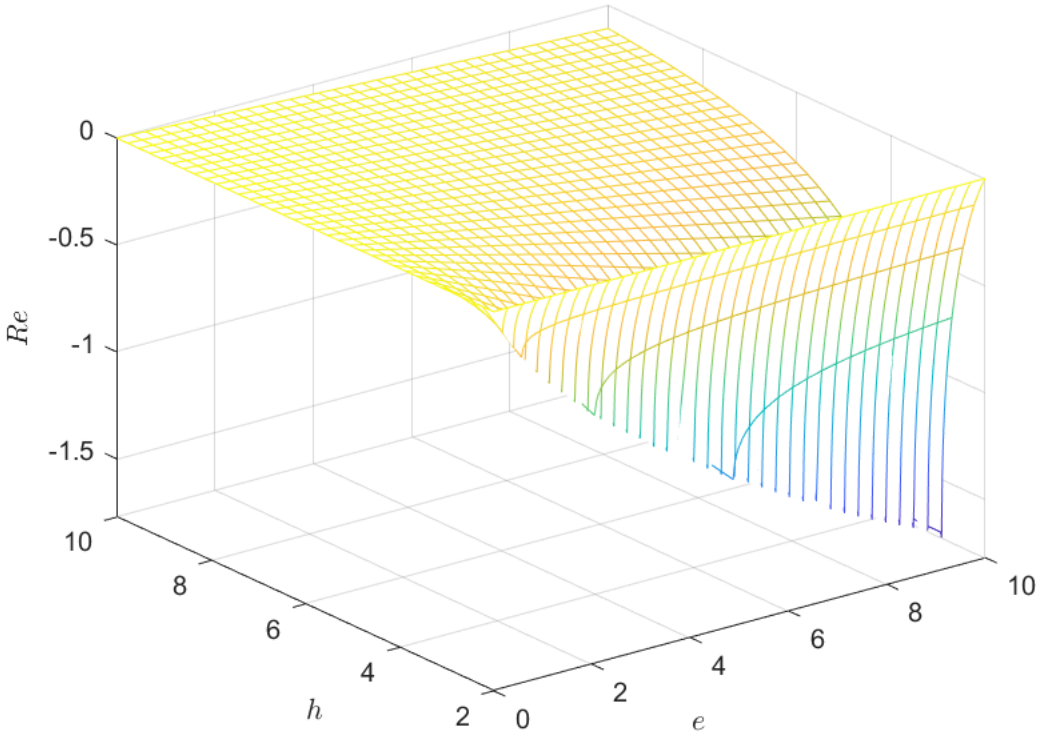
```
%fplot(real(subs(mu1,e,1)),[hlb hrb]); hold on; fplot(0,[0 lim]); hold off;
fimplicit(real(mu1) == 0, bnds); setplotstyleII('$re\:\mu_1 = 0$');
```



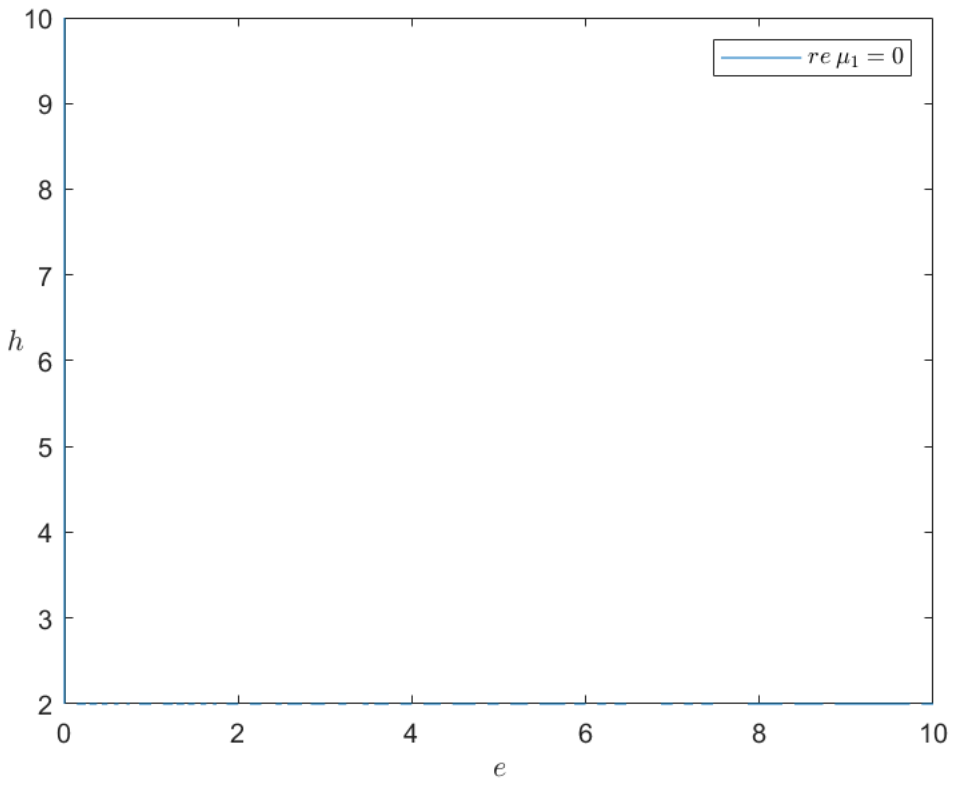

```
fmesh(imag(mu2),bnds); setplotstyleI('$Im$');
exportgraphics(gcf,'pictures\static_point_3\Im_mu2.pdf');
```




```
fmesh(real(mu2),bnds); setplotstyleI('$Re$');
exportgraphics(gcf,'pictures\static_point_3\Re_mu2.pdf');
```



```
fimplicit(real(mu2) == 0, bnds); setplotstyleII('$re\:\mu_1 = 0$');
```



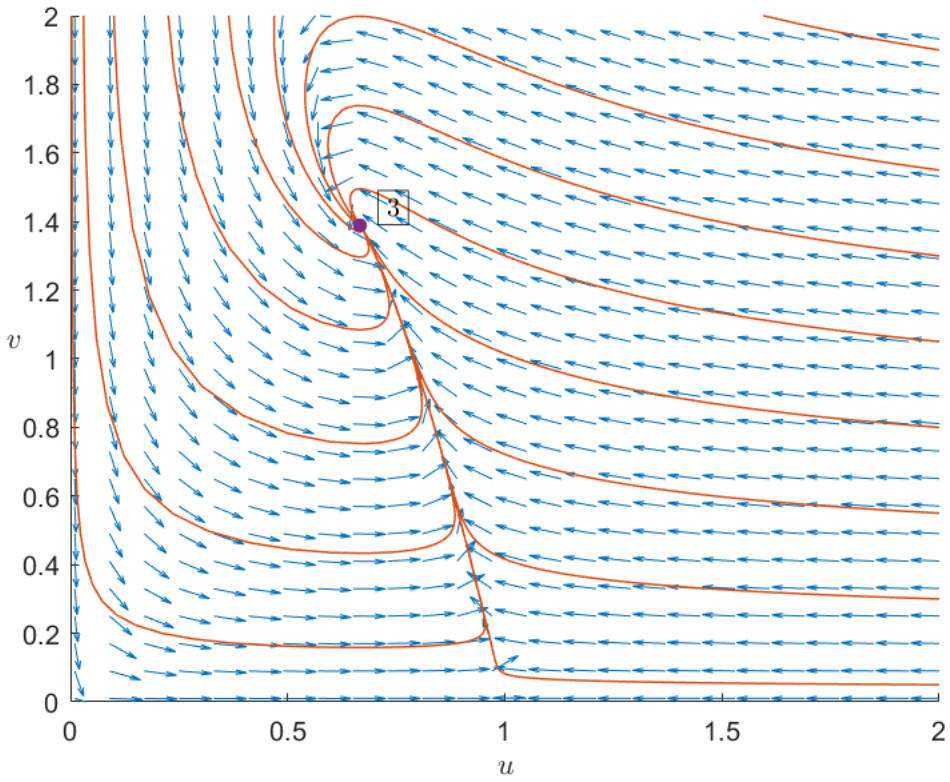
Фазовый портрет i

```
%                e   h
fix_params = [2.5 2.5]; grd = 0.01:0.08:2.01;

stable_point = subs(solutions(3,:), parameters, fix_params)
```

```
stable_point =
```

$$\left(\frac{2}{3} \quad \frac{25}{18}\right)$$

```
Ffix = subs(F,parameters, fix_params);

X0 = [2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 0.10 0.03 0.003;
      0.05 0.30 0.55 0.80 1.05 1.30 1.55 1.90 2.50 3.50 2.00 2.00 2.000];

figure;
add_phase_portrait_plot(Ffix,vars,grd,fix_params(1),fix_params(2),X0,stable_point,'\fbox{$3$}')

exportgraphics(gcf,'pictures\phase_portraits\i.pdf');
```



Фазовый портет ii

```
%                e   h
fix_params = [1.5 3]; grd = 0.01:0.08:2.01;

stable_point_1 = subs(solutions(3,:), parameters, fix_params)
```

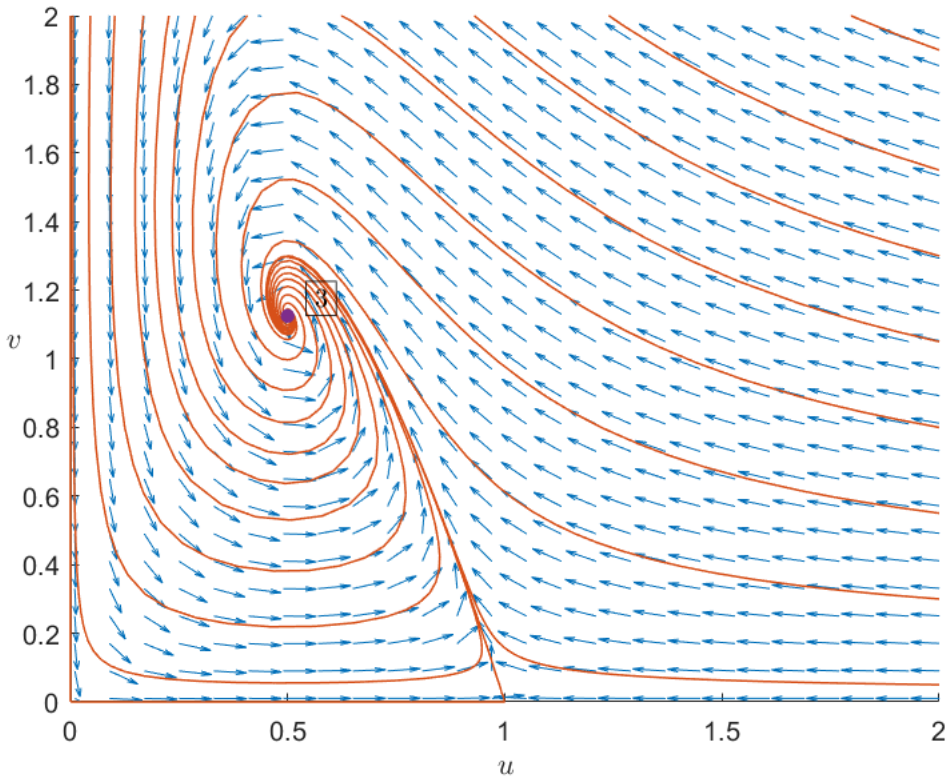
```
stable_point_1 =
```

$$\left(\frac{1}{2} \quad \frac{9}{8}\right)$$

```
stable_point_2 = solutions(2,:)
```

```
stable_point_2 = (1   0)
```

```

Ffix = subs(F,parameters, fix_params);

X0 = [2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00;
      0.05 0.30 0.55 0.80 1.05 1.30 1.55 1.90 2.50 3.50 6.00 50.0];

figure;
add_phase_portrait_plot(Ffix,vars,grd,fix_params(1),fix_params(2),X0,stable_point_1,'\fbox{$3$}

exportgraphics(gcf,'pictures\phase_portraits\ii.pdf');
```

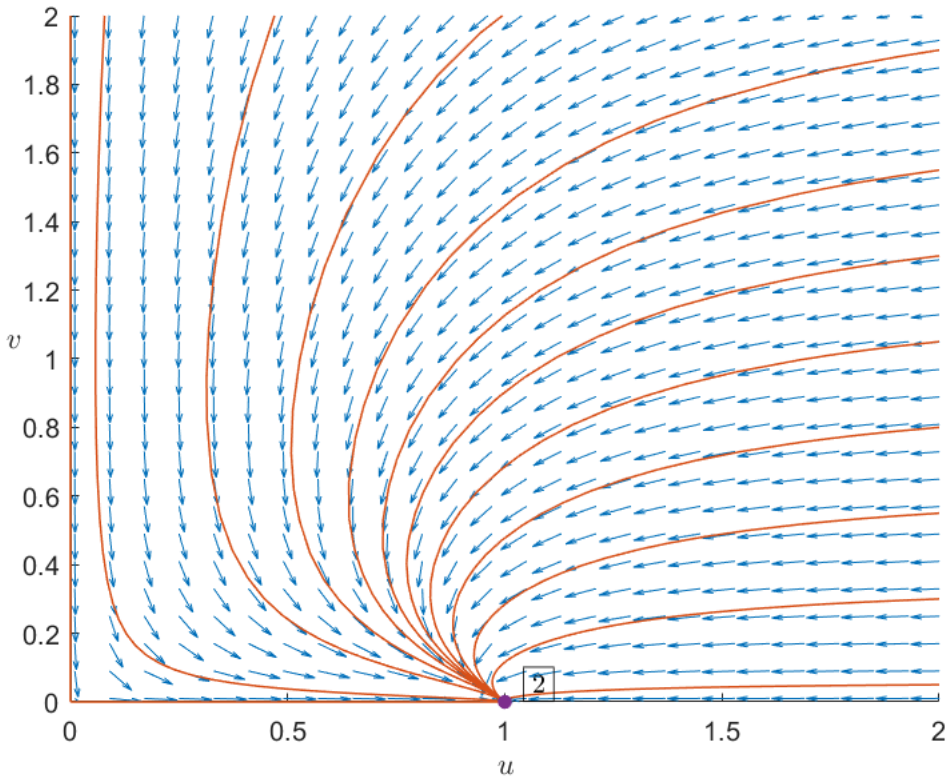


Фазовый портет iii

```
%              e h
fix_params = [1 1]; grd = 0.01:0.08:2.01;

stable_point = solutions(2,:)
```

```
stable_point = (1   0)
```

```


Ffix = subs(F,parameters, fix_params);

X0 = [2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00;
      0.05 0.30 0.55 0.80 1.05 1.30 1.55 1.90 2.50 3.50 6.00 50.0];

figure;
add_phase_portrait_plot(Ffix,vars,grd,fix_params(1),fix_params(2),X0,stable_point,'\fbox{$2$}')

exportgraphics(gcf,'pictures\phase_portraits\iii.pdf');
```

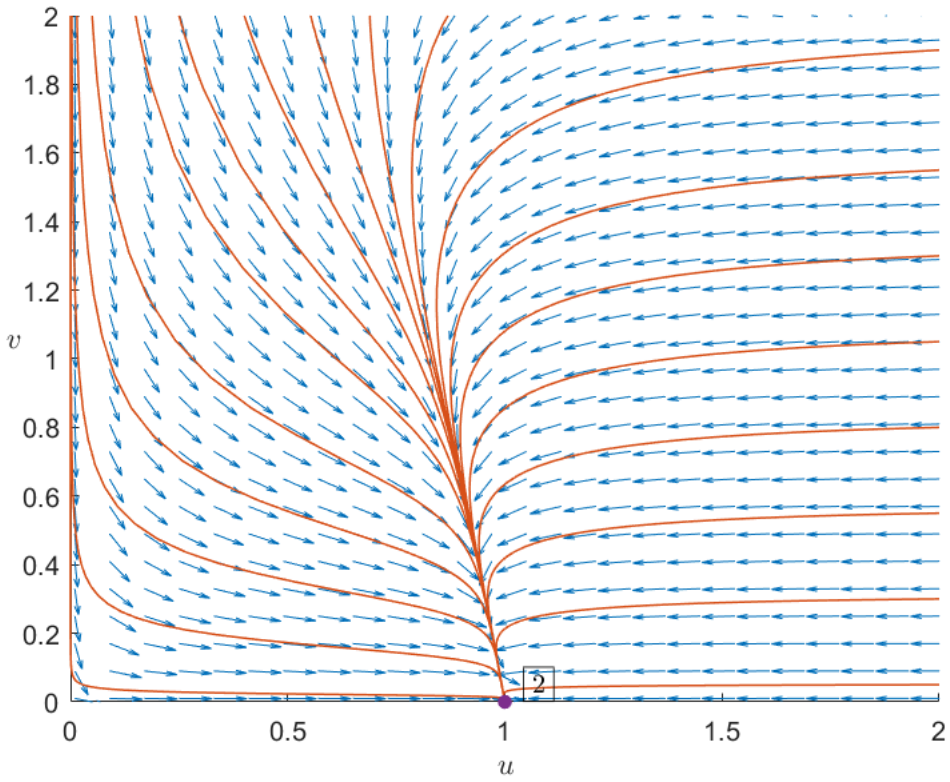


Фазовый портрет iv

```
%              e h
fix_params = [4 1]; grd = 0.01:0.08:2.01;

seddle_point = solutions(2,:)
```

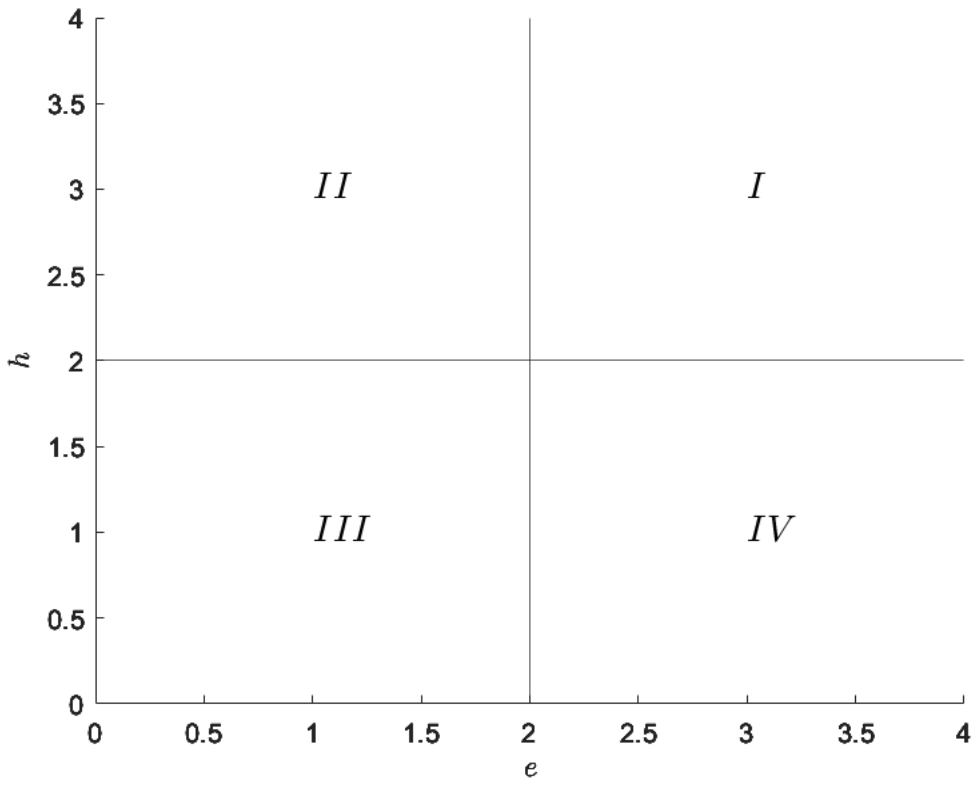
```
seddle_point = (1   0)
```

```
Ffix = subs(F,parameters, fix_params);

X0 = [2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.00 0.40 0.25 0.15 0.05 0.015 0.0
      0.05 0.30 0.55 0.80 1.05 1.30 1.55 1.90 2.50 3.50 6.00 50.0 2.00 2.00 2.00 2.00 2.000 2.0

figure;
add_phase_portrait_plot(Ffix,vars,grd,fix_params(1),fix_params(2),X0, seddle_point,'\fbox{$2$}'

exportgraphics(gcf,'pictures\phase_portraits\iv.pdf');
```



Параметрический портрет системы

```
ax = axes('XLim',[0 4],'YLim',[0 4]);
ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
ax.XLabel.String = '$e$'; ax.YLabel.String = '$h$';
xline(2); yline(2);
text(3,3, '$I$'  ,'Interpreter',"latex",'FontSize',14);
text(1,3, '$II$' ,'Interpreter',"latex",'FontSize',14);
text(1,1, '$III$','Interpreter',"latex",'FontSize',14);
text(3,1, '$IV$' ,'Interpreter',"latex",'FontSize',14);

exportgraphics(gcf,'pictures\phase_portraits\parameters.pdf');
```



Вспомогательные функции

```
function res = odefun(t,x,e,f,g,h)
    res =  [e*x(1).*(1-x(1)) - (  x(1).*x(2))./(1+g*x(1));
            -f*x(2)          + (h*x(1).*x(2))./(1+g*x(1))];
end

function [mu1, mu2] = get_static_point_eigenvalues(F,vars,static_point)
    syms x y; %substitution
    new_vars = [x y];
        %x = u - u* => u = x + u*; \dot u = f(x+u*); v~--- analogously
    substitution = new_vars + static_point
    F_new = subs(F,vars, substitution)
    Jac = jacobian(F_new,new_vars)
    Jac0 = subs(Jac,[x y], [0 0])
    [eigvecs,lambda] = eig(Jac0);
    eigvecs
    mu1 = lambda(1,1); mu2 = lambda(2,2);
end

function add_phase_portrait_plot(Fexmp, vars, grd, e_fix, h_fix, X0, stp, stp_txt)
    hold on;

    [X,Y] = meshgrid(grd, grd);
    F1exmp = symfun(Fexmp(1),vars); F2exmp = symfun(Fexmp(2),vars);
    NORM = symfun(sqrt(F1exmp.^2 + F2exmp.^2),vars);
    F1exmp = F1exmp./NORM; F2exmp = F2exmp./NORM;
```

```
    Udot = F1exmp(X,Y); Vdot = F2exmp(X,Y);

    q = quiver(X,Y,Udot,Vdot,'AutoScaleFactor',0.7);
    ax = gca; ax.XLim = [0 2]; ax.YLim = [0 2];


    line_col = '#D95319'; line_width = 0.75;
    for cnt = 1:length(X0)
        [t,y] = ode45(@(t,x) odefun(t,x, e_fix,1,1,h_fix),[0 100],X0(:,cnt));
        plot(y(:,1),y(:,2),'LineWidth',line_width,'Color', line_col);
    end

    plot(stp(1),stp(2),'Marker','.',"MarkerSize", 18,'Color','#7E2F8E');
    text(stp(1)+0.04,stp(2)+0.05, stp_txt,'Interpreter',"latex");

    setplotstyleIII();

    hold off;
end

function setplotstyleI(z_label)
    hold on;
    ax = gca;
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex'; ax.ZLabel.Interpreter = '
    ax.XLabel.String = '$e$'; ax.YLabel.String = '$h$'; ax.ZLabel.String = z_label;
    hold off
end

function setplotstyleII(lgd_text)
    hold on;
    ax = gca;
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
    ax.XLabel.String = '$e$'; ax.YLabel.String = '$h$';
    ax.YLabel.Rotation = 0;
    legend(lgd_text,'Interpreter','latex','AutoUpdate','off');
    hold off;
end

function setplotstyleIII()
    hold on;
    ax = gca;
    ax.XLabel.Interpreter = 'latex'; ax.YLabel.Interpreter = 'latex';
    ax.XLabel.String = '$u$'; ax.YLabel.String = '$v$';
    ax.YLabel.Rotation = 0;
    hold off;
end
```