Theory

To simulate mode-lock pulsing in that laser cavity Shrödinger equation in spectral domain is used [1, 2]:

$$\frac{dU(z,\omega)}{dz} = i\sum_{n=2}^{3}\beta_n\omega^n U(z,\omega) + i\gamma G(z,\omega) + \left(1-\frac{\omega^2}{\Omega^2}\right) g(z)U(z,\omega), \quad (1)$$

where $U(\omega,z) = \int_{-\infty}^{\infty}\widetilde{U}(z,t)\,e^{-i\omega t}dt$ is the pulse spectrum, $\widetilde{U}(z,t)$ is the pulse electric field profile, ω is the pulse spectrum angular frequency deviation from central carrying frequency ω_0, z is the coordinate along the fiber, β_2 and β_3 are the dispersion coefficients at pulse spectrum central wavelenght λ_0, γ is the nonlinear coefficient, $G(\omega,z) = \int_{-\infty}^{\infty}\left|\widetilde{U}(z,t)\right|^2\widetilde{U}(z,t)\,e^{-i\omega t}dt$ is the Fourier transform of pulse phase self modulation term, Ω is the parameter that defines active fiber parabolic amplification spectrum width, $g(z)$ is the gain along the active fiber.

Active fiber amplification spectrum is represented by formula:

$$A(\omega) = 1-\frac{\omega^2}{\Omega^2}.$$

As a result of simultaneous action of pulse group velocity dispersion and self phase modulation pulse gets frequency chirp:

$$\omega(z,t)_{ch} = \frac{\partial}{\partial t}\left(\tan^{-1}\frac{Im\left(\widetilde{U}(z,t)\right)}{Re\left(\widetilde{U}(z,t)\right)}\right).$$

Pulse frequency chirp can be represented as wavelength chirp:

$$\lambda(z,t)_{ch} = \frac{2\pi v_g}{\omega(z,t)_{ch}+\omega_0},$$

where v_g is the pulse group velocity.

Active fiber gain saturation is determine by equation:

$$g(z) = \frac{g_0}{1+\frac{E_p}{P_g T_R}}, \qquad (2)$$

where g_0 is the small signal gain, P_g is the gain saturation power, T_R is the resonator round trip time, $E_p = \int\left|\widetilde{U}(z,t)\right|^2 dt$ is the pulse energy.

Pulse profile and spectral intensity is determined consequently:

$$P_A(z,t) = \left|\widetilde{U}(z,t)\right|^2,$$

$$P_\omega(z,\omega) = |U(\omega,z)|^2.$$

Saturable absorber intensity depended loss dynamics is governed by the rate equation:

$$\frac{\partial q(t)}{\partial t} = -\frac{q(t)-q_0}{\tau_a} - \frac{q(t)|\widetilde{U}(z,t)|^2}{\tau_a P_a}, \qquad (3)$$

where $q(t)$ is the pulse intensity depended loss, q_0 is the unsaturable losses, τ_a is the absorber recovery time, P_a is the absorber saturation power.

To find pulse time-bandwidth product its necessary to find the FWHM of pulse intensity profile $P_A(z,t)$ and spectral intensity $P_\omega(z,\omega)$. For Gauss and sech like pulses time-bandwidth product is equal 0.441 and 0.315 consequently.

Filter induced losses determined by super-Gaussian function:

$$F(\lambda) = \exp(-((\lambda-\lambda_0+\lambda_F)^2/2\Lambda^2)^p),$$

where λ_F is the filter center wavelength position deviation from active fiber amplification bandwidth center at λ_0, Λ and p is the parameters that defines filter bandwidth and steepness.

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Summary This chapter contains sections titled: Overview of Short Pulse Fiber Lasers Modeling of Ultrafast Fiber Lasers Implementation and Control of Advanced Components Conclusions and Future Outlook References

Simulation model

The simulation purpose is to find $\widetilde{U}(z,t)$ at some point in cavity.
To do this in the equation (1) the $\widetilde{U}(z,t)$ is substituted with $U(\omega,z)$ by use of Fourier transform $\widetilde{U}(z,t) = \int_{-\infty}^{\infty} U(\omega,z)\, e^{i\omega t} dt.$
After that we have the eq.(1) in frequency domain.
Now we have N point in frequency space set of z dependent equations.
And this is solved by Scipy odeint.
To find Sesam action over pulse the pulse is turned to time domain and is transformed by (3)

How the program works

Simulation starts:

```
anim = FuncAnimation(fig, cavityshow, fargs=(frames0-1,),
init_func=initgraph, frames=frames0, interval=1000, blit=True, repeat=False)
```

FuncAnimation used to show state of pulse at some iteration.
cavityshow return lines to show pulse state, makes figures when simulation is finished and saved data when iteration number is equal to *multiple2savedata*
cavityshow calls ***cavityrun*** with iteration number and *y0w* – pulse spectrum that is used to go through cavity elements. ***cavityrun*** returns pulse spectrum *yout* as *y0w* pass all cavity elements.

In model used to cavity types:
def lzrcrcl(y0w): **-** circular cavity
def lzrlnr(y0w): - linear cavity

Cavity type can be chose: cavitytype = {1: lzrcrcl, 2: lzrlnr}
Cavity structure and elements parameters is determined in **def cavitystruct():**
For example:
cavstruct = [activeA, passiveA, nalmA, couplerA, passiveB, filterB]
makes cavity when first goes element **activeA**, later **passiveA**, …

Cavity element is represented as a python list. For example, some active fiber element is made as follows:
activeA = [active, 'activefA', beta, gama1, OOm2, g01, Ez, Psg, Tr, direction, La1].
Here is:
active – function name, that simulates element action
'activefA' – element name used for indicating lines in figures
Later goes some element intrinsic parameters: beta – fiber dispersion coefficients, gama1 – fiber nonlinear coeff, OOm2 – param determines amplification bandwidth, g01 – small signal amplification gain, Ez – array of pulse energy through cavity length, Psg - active fiber saturation power, Tr - round trip time, direction – sign, that determines direction of pulse propagation, La1 – fiber length.

Active fiber medium amplification is saturated (decrease amplification g) by pulse with energy Egz (2).
In linear cavity in (2) Ep = Egz0 + Egz1
In circular cavity Ep = Egz0

Egz0 = pulse energy calculated at point z
Egz1 = pulse energy from previous iteration.

```
Egz0 = np.sum(np.abs(Ax) ** 2) * dt * 1e-12 / Psg / Tr
Egz1 = tvl(Xt, z, Ez[0], La[0]) / Psg / Tr
Egz = Egz0 + Egz1
gt = g0 / (1 + Egz) # saturated gain
```

tcf is function to get Taylor coefficients of pulse energy through fiber length
tvl is opposite function: gets pulse energy at some point in fiber.

Its possible to make scripts that defines some element parameter to change with iteration.
All scripts have to be defined placed in **def changecvtstruct(ij):**
The step in iteration with which parameter is changed is *multiple2savedata*