QF101 Introduction to Quantitative Finance

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Preliminaries

The Premise

- A digital playground for people interested in learning financial mathematics.
- Learn, discuss, build together

This course involves

Finance

- Markets
- Instruments

Mathematics

- Statistics
- Stochastic calculus

Programming

- Modeling
- Execution

Requirements

- Maths skills
  - Linear algebra
  - Differential calculus
  - Statistics/probability
- MATLAB
- Programming
  - Your favorite IDE & programming language
  - Consult a good programming course

Materials

- Textbooks
- 1 lecture = 4 screencasts (theory, tools, math, programming)
- Source code
- Articles
- Exercise answers
- Wiki/online demos

available online at http://github.com/hexlet/qf101

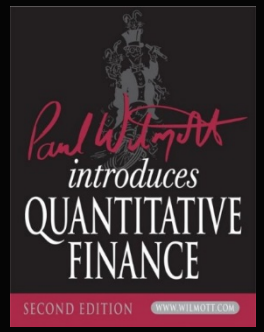
- Anything *you* make :)

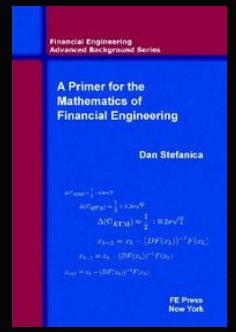
Tests

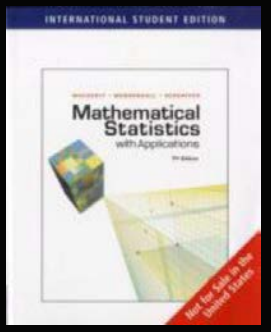
- Questions from course textbooks
  - Groupwork
  - Create answer sets
- Individual and group analysis/programming exercises
  - Use the forums
  - Use/learn LaTeX
- If in trouble, ask

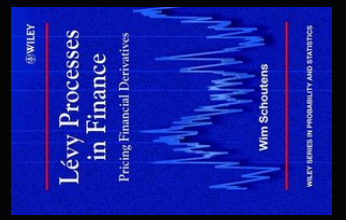
Course Texts

- Main text
  - *Paul Wilmott Introduces Quantitative Finance 2e*
    Paul Wilmott
- Mathematics
  - *A Primer for the Mathematics of Financial Engineering 2e*
    Dan Stefanica
  - *Mathematical Statistics with Applications 7e (International Edition)*
    Wackerly, Mendehall & Scheaffer
- Additional course materials announced later









1 Products and Markets

PWIQF Chapter 1

Value Measurement

- In commerce, people exchange goods and services
- Exchange only possible if value is measured correctly
- Barter – direct exchange
  - 1 apple = 1 orange = 4 potatoes = 1hr of hard labour
  - Alive and well today (taxed, too!)
  - Problems
    - Only works if both parties want each other's goods (supply matches demand)
    - Material goods are subject to supply issues, storage, decay, etc.
- Represent values of all things using one
  - 1 apple = 4 potatoes = 10 silver coins

Currency

- Uniform representation of value
- Previously tied to gold
  - Bretton Woods system
  - See *Battle of Bretton Woods*
- Printed (*minted*) and distributed by government
- Main currencies are
  - US Dollar (USD)
  - Euro (EUR)
  - Yen (JPY)
  - Pound (GBP)
- The value of one currency relative to another is called the *exchange rate*
- Main currency pairs
  - GBPUSD "cable"
  - USDJPY
  - EURUSD
- Cross pairs (e.g., GBPJPY)
- Minor pairs (e.g., USDSEK)
- Exotic pairs

Currency Markets

- Foreign Exchange (a.k.a. FX or ForEx)
- Global currency markets where pairs of currencies can be bought or sold
  - As well as CFDs and derivatives. More on them later.
- Values of currencies controlled entirely by bid/ask interest
  - Governments can intervene
- Currencies (and other instruments) can be used for
  - Hedging (fixing the conditions of an export deal)
  - Speculation (betting on market movement)
- Are offered by many institutions
  - E.g., LMAX
  - Part of a very dishonest market segment

Currency Risk

- Your operations have a *numeraire*
  - The unit all your profits/losses get converted to
- Currency risk affects
  - Value of home or foreign currency you're holding
  - Value of your positions which are expressed through foreign currency
- Transaction costs
  - Banks make a good profit on retail currency conversions

Time Value of Money

- $1 is worth more tomorrow than today
- Inflation
  - Gradual loss of value
- Money is not ‘working’
- Can trade the time value
  - Give money to bank
  - Bank uses the money for investment/trading
  - Bank rewards you with interest

Interest

- You have Q_0 in a bank account
- Bank pays an interest rate r
  - $5\% \rightarrow r = 0.05$
- After 1 year you have
  - $Q_1 = Q_0 + Q_0 r = Q_0(1 + r)$
- After t years you have
  - $Q_t = Q_0(1 + r)^t$

- Bank decides to pay interest n times a year
  - $Q_t = Q_0(1 + {}^{r}/_{n})^{tn}$
- If bank pays interest *continuously*
  - $\lim_{n \to \infty} (1 + {}^{r}/_{n})^{tn} = e^{rt}$
- Future value needs to be *discounted* by e^{-rt}

Banking

- Current/savings account
  - Bank pays fixed interest rate
- Bank-managed investment portfolio
  - Bank invests the money for you
  - Choice of investment strategies (different risk & return)
  - Bank takes a part of the profit it makes
- Personal investment
  - Shares, options/warrants, ETFs, etc.
  - Provided by retail banks in addition to specialized institutions

Company Ownership

- Companies need to borrow to grow
- Can borrow money from bank
- Can issue bonds
  - Get $ now, repay with interest
- Can give ownership of part of the company to the public
- IPO = Initial Public Offering

- Parts of the company are called *shares* or *stock*
- Stock is *listed* and traded on the *stock market*
- Price is defined by the supply and demand
- Anyone can buy and sell
- Selling is possible even if you don’t own shares (short selling)

Shares

- Company value divided into shares
  - Shares represent ownership
  - Having >50% means you control the company
  - Different *classes* of shares give different powers (e.g., voting rights)
- A single share has a price listed on an exchange
- Every company is (naturally) interested in its share price going up

Yahoo Finance

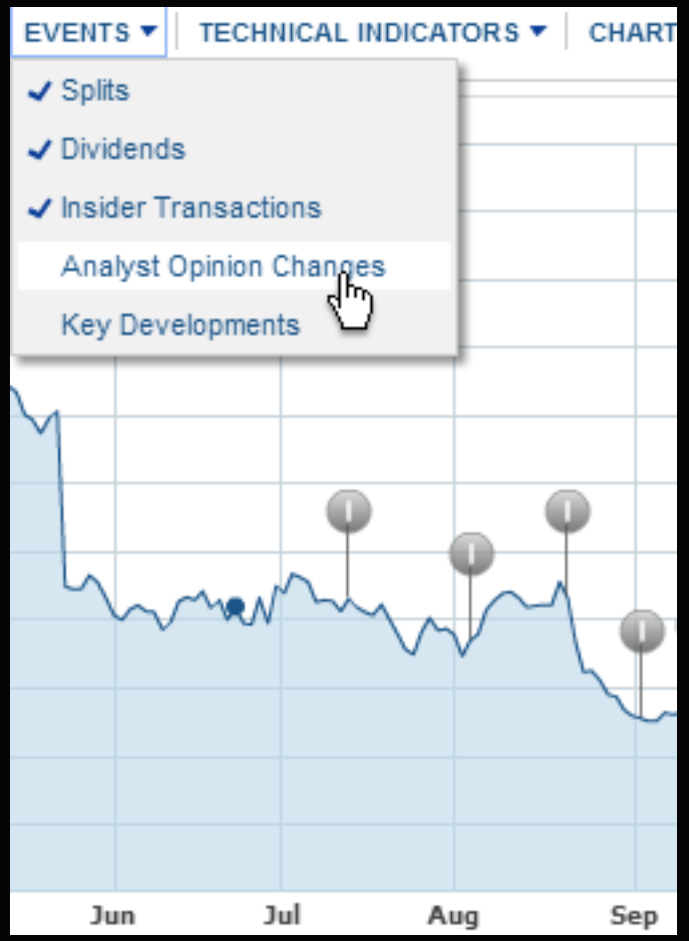
- http://finance.yahoo.com

Prices						
Date	Open	High	Low	Close	Volume	Adj Close*
Dec 21, 2012	10.35	10.43	10.19	10.43	22,982,600	10.43
Dec 20, 2012	10.49	10.55	10.41	10.50	11,979,200	10.50
Dec 19, 2012	10.56	10.57	10.47	10.49	11,885,900	10.49
Dec 18, 2012	10.30	10.62	10.29	10.56	19,559,300	10.56

- Prices: high, low, open, close
- Different timeframes (1d, 3m, etc.)
- Adjusted close (close adjusted for stock splits)

Events

- Stock split (adjusted close)
- Dividend payouts
- Insider transactions
- Change in analyst ratings
- Key company events



Commodities

- Raw materials
  - Gold, oil, corn, etc.
- Same uses as any other instrument
  - Farmer can hedge against the price of corn
  - Speculator can bet on rising price of oil
- Often traded in futures
  - No need to supply product

Indices

- An index is a composite calculation based on several stocks
- Stocks chosen can represent
  - A segment of the economy
  - The whole economy
- Well-known indices: S&P 500, FTSE 100, RTSI

Forward

- An obligation to buy or sell something at some time in the future
- You: “I agree to buy 100 apples for $100 1 year from now”
- Counterparty: “I agree to sell 100 apples for $100 1 year from now”
- Determined by
  - F – price of a forward contract
  - $S(t)$ – current price of asset
  - $(T - t)$ – time remaining until the contract ends
    - T is when the contract ends
    - Often expressed as either t or τ
    - Example: 3 months from now $\rightarrow t = 0.25$

Futures

- A forward contract can specify *any* price and expiration
- To ensure *liquidity* (possibility of trading) for forward contracts, they are standardized into *futures contracts* or *futures*
- A futures contract specifies settlement dates
  - E.g., RTS-9.13 (coded as RIU3) is for 16.09.2013
- Futures can involve stock, stock indices, commodities, currencies
- Typically, subjects of future contract are not supplied

No Arbitrage

- Arbitrage
  - Ability to make money due to market mispricing
- No arbitrage principle
  - The idea that there is *no* arbitrage opportunity in the market
- *Statistical arbitrage* (stat arb) is an investment strategy that looks for arbitrage opportunities
- For example, with forward contracts we have *spot-future parity*

$$F = S(t)e^{rt}$$

- Violation of the above creates an *arbitrage opportunity*

2 Derivatives

PWIQF Chapter 2, Stefanica pp.16-26

What's an Option?

- A futures contract is an *obligation* to buy or sell something
- An *option* is a right, but not an obligation
  - Will only be *exercised* (used) if it brings profit
- Direction
  - A *call option* is the right to buy something
  - A *put option* is the right to sell something
- Anyone can buy and sell calls and puts

Option Types

- European
  - Can only be exercised at expiry
- American
  - Can be exercised at any moment
- Exotic
  - Has additional exercise rules
  - Can be path-dependent

European Non-Dividend Pricing

- K – strike
- S – price of underlying
- t – time to expiration in days
  - Often written as $(T - t)$ where t is current time, T expiry time
  - Sometimes written as τ
- r – risk-free rate
- σ – volatility

Direction to Market

- With stocks/futures/commodities
  - If you bought a stock, you are *long* or have a *long position*
  - If you sold a stock, you are *short*
- With options
  - You are long if you bought a call or sold a put
  - You are short otherwise

Moneyness

- Option is *In The Money*
  - Brings profit at expiry
  - $K < S$ for calls, $K > S$ for puts
- Option *At The Money* (ATM)
  - K is close or equal to S
- Option *Out of The Money*

Option Pricing

- Premium – price of the option (market price of risk)
- Intrinsic value
  - How much money you will get if option expires *now*
  - Only nonzero for options in the money
- Time value
  - How much the premium exceeds intrinsic value
  - “Payment for uncertainty”
  - Time value goes to zero towards expiry
- Premium = Intrinsic Value + Time Value

Margins

- Options are risky
- Exchange needs guarantees that you can cover your position
- Initial margin
  - Takes into account your portfolio
- Variation margin
  - Premium not paid
  - No premium → no excess cash you can invest short term
  - $r = 0 \therefore e^{\pm rt} = 1$

Early Exercise

- Applicable to American put options
- Only makes sense if
  - $r > 0$
  - Option is in the money
- Thus, for options with margin requirements
  - Early exercise doesn't make sense
  - American option can be treated as European

Order Book

- Market players post orders to the market
- An order might be a
  - Bid
  - Ask
  - Remove
- The market matches orders
- A listing of orders is called an *order book*
- At certain times the market clears all orders

10000
200
130
125
120

Bid = 100, Ask = 120, Spread = 20, Price = 110

100
95
93
4

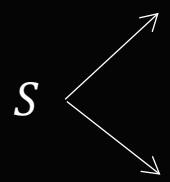
3 Binomial Model

PWIQF Chapter 3

Option Valuation

- Need to price options
- *Option Pricing: A Simplified Approach*
  Cox, Ross, Rubinstein
  Journal of Financial Economics 7 (1979), pp.229-263
- Discrete model

Underlying Price



uS with probability q,

dS with probability $1 - q$

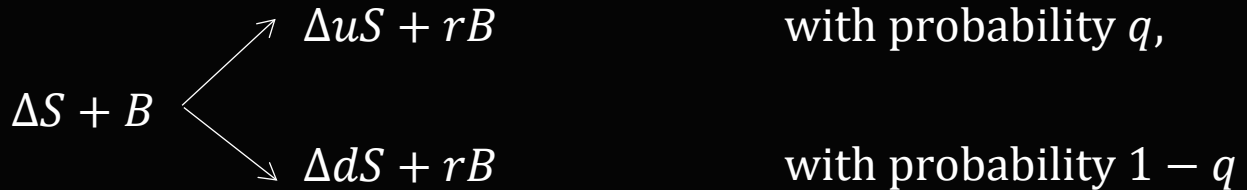
Call Option

$Z^+ = \max[Z, 0]$

C → $C_u = (uS - K)^+$ with probability q,

C → $C_d = (dS - K)^+$ with probability $1 - q$

Portfolio: Stock + Riskless Bonds



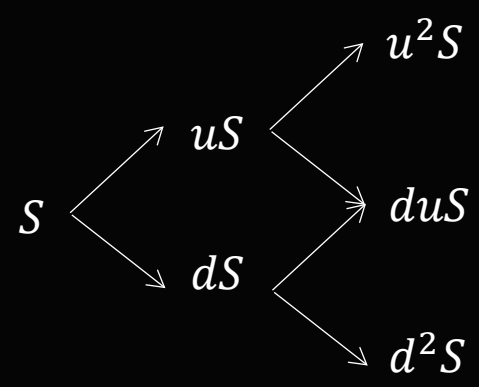
Equate Positions

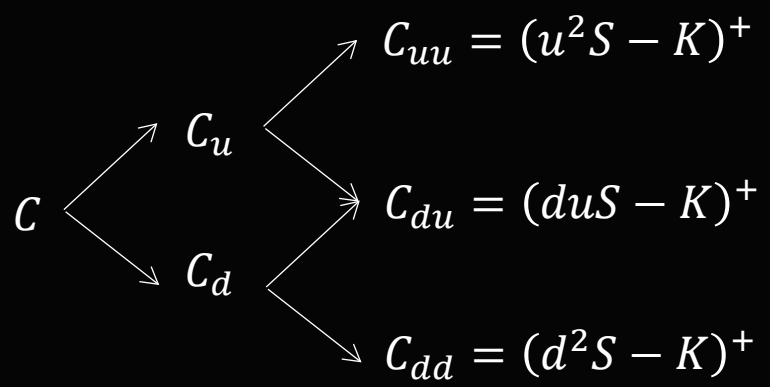
- $\Delta uS + rB = C_u$
- $\Delta dS + rB = C_d$
- $\Delta = \frac{C_u - C_d}{(u-d)S}$
- $B = \frac{uC_d - dC_u}{(u-d)r}$
- Hedging portfolio

No Arbitrage

- $C = \Delta S + B$
- $p = \frac{r-d}{u-d}$
- $C = \frac{pC_u+(1-p)C_d}{r}$

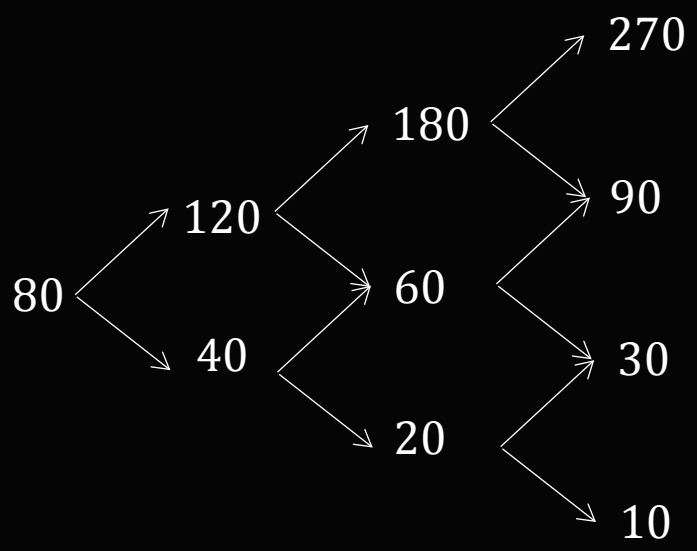
Multi-Period

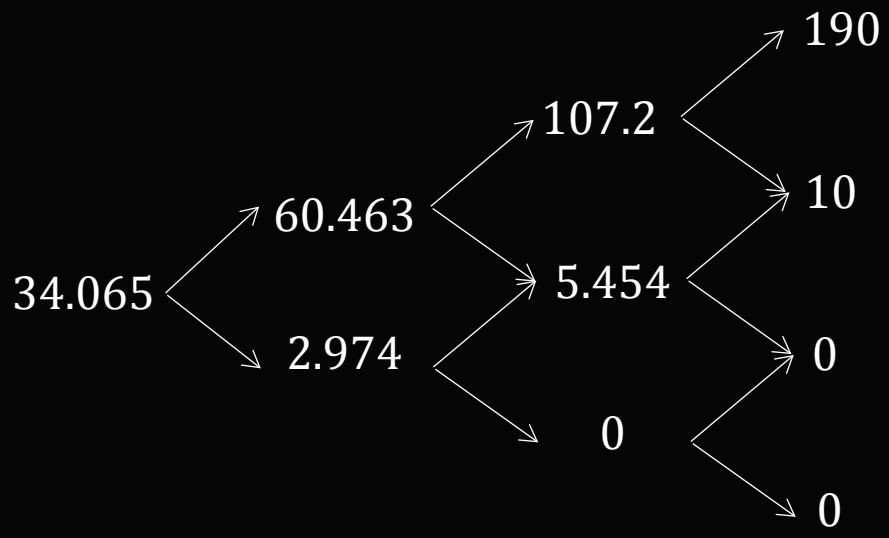




- $C_u = \frac{pC_{uu}+(1-p)C_{ud}}{r}$
- $C_d = \frac{pC_{du}+(1-p)C_{dd}}{r}$
- where $p = \frac{r-d}{u-d}$

Example





- $S = 80, n = 3, K = 80, u = 1.5, d = 0.5, r = 1.1$
- $p = \frac{1.1-0.5}{1.5-0.5} = 0.6$
- $C = \frac{pC_u+(1-p)C_d}{r}$

General Formula

- Given n periods of the model,

$$C = \frac{1}{r^n}\left[\sum_{j=0}^{n}\left(\frac{n!}{j!\,(n-j)!}\right)p^j(1-p)^{n-j}\left(u^j d^{n-j}S - K\right)^+\right]$$

4 Random Behavior of Assets

PWIQF Chapter 4

Financial Analysis

- Fundamental Analysis
  - Analysis of cheques and ledgers
- Technical analysis
  - Search for patterns in the chaos
  - No scientific basis
- Quantitative analysis
  - That’s what we do ☺

Assets Prices Are Random