

QF101 Introduction to Quantitative Finance

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Preliminaries

The Premise

- A digital playground for people interested in learning financial mathematics.
- Learn, discuss, build together

This course involves

Finance

- Markets
- Instruments

Mathematics

- Statistics
- Stochastic calculus

Programming

- Modeling
- Execution

Requirements

- Maths skills
 - Linear algebra
 - Differential calculus
 - Statistics/probability
- MATLAB
- Programming
 - Your favorite IDE & programming language
 - Consult a good programming course

Materials

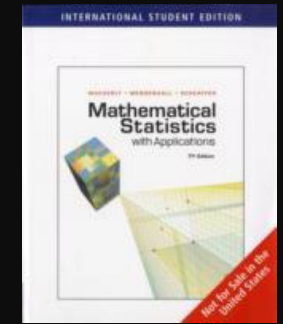
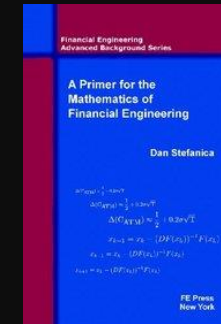
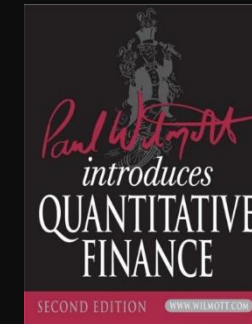
- Textbooks
 - 1 lecture = 4 screencasts (theory, tools, math, programming)
 - Source code
 - Articles
 - Exercise answers
 - Wiki/online demos
 - Anything *you* make :)
- } available online at
<http://github.com/hexlet/qf101>

Tests

- Questions from course textbooks
 - Groupwork
 - Create answer sets
- Individual and group analysis/programming exercises
 - Use the forums
 - Use/learn LaTeX
- If in trouble, ask

Course Texts

- Main text
 - *Paul Wilmott Introduces Quantitative Finance 2^e*
Paul Wilmott
- Mathematics
 - *A Primer for the Mathematics of Financial Engineering 2^e*
Dan Stefanica
 - *Mathematical Statistics with Applications 7^e*
(International Edition)
Wackerly, Mendenhall & Scheaffer
- Additional course materials announced later



1 Products and Markets

PWIQF Chapter 1

Value Measurement

- In commerce, people exchange goods and services
- Exchange only possible if value is measured correctly
- Barter – direct exchange
 - 1 apple = 1 orange = 4 potatoes = 1hr of hard labour
 - Alive and well today (taxed, too!)
 - Problems
 - Only works if both parties want each other's goods (supply matches demand)
 - Material goods are subject to supply issues, storage, decay, etc.
- Represent values of all things using one
 - 1 apple = 4 potatoes = 10 silver coins

Currency

- Uniform representation of value
- Previously tied to gold
 - Bretton Woods system
 - See *Battle of Bretton Woods*
- Printed (*minted*) and distributed by government
- Main currencies are
 - US Dollar (USD)
 - Euro (EUR)
 - Yen (JPY)
 - Pound (GBP)
- The value of one currency relative to another is called the *exchange rate*
- Main currency pairs
 - GBPUSD “cable”
 - USDJPY
 - EURUSD
- Cross pairs (e.g., GBPJPY)
- Minor pairs (e.g., USDSEK)
- Exotic pairs

Currency Markets

- Foreign Exchange (a.k.a. FX or ForEx)
- Global currency markets where pairs of currencies can be bought or sold
 - As well as CFDs and derivatives. More on them later.
- Values of currencies controlled entirely by bid/ask interest
 - Governments can intervene
- Currencies (and other instruments) can be used for
 - Hedging (fixing the conditions of an export deal)
 - Speculation (betting on market movement)
- Are offered by many institutions
 - E.g., LMAX
 - Part of a very dishonest market segment

Currency Risk

- Your operations have a *numeraire*
 - The unit all your profits/losses get converted to
- Currency risk affects
 - Value of home or foreign currency you're holding
 - Value of your positions which are expressed through foreign currency
- Transaction costs
 - Banks make a good profit on retail currency conversions

Time Value of Money

- \$1 is worth more tomorrow than today
- Inflation
 - Gradual loss of value
- Money is not 'working'
- Can trade the time value
 - Give money to bank
 - Bank uses the money for investment/trading
 - Bank rewards you with interest

Interest

- You have Q_0 in a bank account
- Bank pays an interest rate r
 - 5% $\rightarrow r = 0.05$
- After 1 year you have
 - $Q_1 = Q_0 + Q_0r = Q_0(1 + r)$
- After t years you have
 - $Q_t = Q_0(1 + r)^t$
- Bank decides to pay interest n times a year
 - $Q_t = Q_0(1 + r/n)^{tn}$
- If bank pays interest *continuously*
 - $\lim_{n \rightarrow \infty} (1 + r/n)^{tn} = e^{rt}$
- Future value needs to be *discounted* by e^{-rt}

Banking

- Current/savings account
 - Bank pays fixed interest rate
- Bank-managed investment portfolio
 - Bank invests the money for you
 - Choice of investment strategies (different risk & return)
 - Bank takes a part of the profit it makes
- Personal investment
 - Shares, options/warrants, ETFs, etc.
 - Provided by retail banks in addition to specialized institutions



Company Ownership

- Companies need to borrow to grow
- Can borrow money from bank
- Can issue bonds
 - Get \$ now, repay with interest
- Can give ownership of part of the company to the public
- IPO = Initial Public Offering
- Parts of the company are called *shares* or *stock*
- Stock is *listed* and traded on the *stock market*
- Price is defined by the supply and demand
- Anyone can buy and sell
- Selling is possible even if you don't own shares (short selling)

Shares

- Company value divided into shares
 - Shares represent ownership
 - Having >50% means you control the company
 - Different *classes* of shares give different powers (e.g., voting rights)
- A single share has a price listed on an exchange
- Every company is (naturally) interested in its share price going up

Yahoo Finance

- <http://finance.yahoo.com>

| Prices | | | | | | |
|--------------|-------|-------|-------|-------|------------|------------|
| Date | Open | High | Low | Close | Volume | Adj Close* |
| Dec 21, 2012 | 10.35 | 10.43 | 10.19 | 10.43 | 22,982,600 | 10.43 |
| Dec 20, 2012 | 10.49 | 10.55 | 10.41 | 10.50 | 11,979,200 | 10.50 |
| Dec 19, 2012 | 10.56 | 10.57 | 10.47 | 10.49 | 11,885,900 | 10.49 |
| Dec 18, 2012 | 10.30 | 10.62 | 10.29 | 10.56 | 19,559,300 | 10.56 |

- Prices: high, low, open, close
- Different timeframes (1d, 3m, etc.)
- Adjusted close (close adjusted for stock splits)

Events

- Stock split (adjusted close)
- Dividend payouts
- Insider transactions
- Change in analyst ratings
- Key company events



Commodities

- Raw materials
 - Gold, oil, corn, etc.
- Same uses as any other instrument
 - Farmer can hedge against the price of corn
 - Speculator can bet on rising price of oil
- Often traded in futures
 - No need to supply product

Indices

- An index is a composite calculation based on several stocks
- Stocks chosen can represent
 - A segment of the economy
 - The whole economy
- Well-known indices: S&P 500, FTSE 100, RTSI

Forward

- An obligation to buy or sell something at some time in the future
- You: “I agree to buy 100 apples for \$100 1 year from now”
- Counterparty: “I agree to sell 100 apples for \$100 1 year from now”
- Determined by
 - F – price of a forward contract
 - $S(t)$ – current price of asset
 - $(T - t)$ – time remaining until the contract ends
 - T is when the contract ends
 - Often expressed as either t or τ
 - Example: 3 months from now $\rightarrow t = 0.25$

Futures

- A forward contract can specify *any* price and expiration
- To ensure *liquidity* (possibility of trading) for forward contracts, they are standardized into *futures contracts* or *futures*
- A futures contract specifies settlement dates
 - E.g., RTS-9.13 (coded as RIU3) is for 16.09.2013
- Futures can involve stock, stock indices, commodities, currencies
- Typically, subjects of future contract are not supplied

No Arbitrage

- Arbitrage
 - Ability to make money due to market mispricing
- No arbitrage principle
 - The idea that there is *no* arbitrage opportunity in the market
- *Statistical arbitrage* (stat arb) is an investment strategy that looks for arbitrage opportunities
- For example, with forward contracts we have *spot-future parity*
$$F = S(t)e^{rt}$$
- Violation of the above creates an *arbitrage opportunity*

2 Derivatives

PWIQF Chapter 2, Stefanica pp.16-26

What's an Option?

- A futures contract is an *obligation* to buy or sell something
- An *option* is a right, but not an obligation
 - Will only be *exercised* (used) if it brings profit
- Direction
 - A *call option* is the right to buy something
 - A *put option* is the right to sell something
- Anyone can buy and sell calls and puts

Option Types

- European
 - Can only be exercised at expiry
- American
 - Can be exercised at any moment
- Exotic
 - Has additional exercise rules
 - Can be path-dependent

European Non-Dividend Pricing

- K – strike
- S – price of underlying
- t – time to expiration in days
 - Often written as $(T - t)$ where t is current time, T expiry time
 - Sometimes written as τ
- r – risk-free rate
- σ – volatility

Direction to Market

- With stocks/futures/commodities
 - If you bought a stock, you are *long* or have a *long position*
 - If you sold a stock, you are *short*
- With options
 - You are long if you bought a call or sold a put
 - You are short otherwise

Moneyness

- Option is *In The Money*
 - Brings profit at expiry
 - $K < S$ for calls, $K > S$ for puts
- Option *At The Money* (ATM)
 - K is close or equal to S
- Option *Out of The Money*

Option Pricing

- Premium – price of the option (market price of risk)
- Intrinsic value
 - How much money you will get if option expires *now*
 - Only nonzero for options in the money
- Time value
 - How much the premium exceeds intrinsic value
 - “Payment for uncertainty”
 - Time value goes to zero towards expiry
- $\text{Premium} = \text{Intrinsic Value} + \text{Time Value}$

Margins

- Options are risky
- Exchange needs guarantees that you can cover your position
- Initial margin
 - Takes into account your portfolio
- Variation margin
 - Premium not paid
 - No premium \rightarrow no excess cash you can invest short term
 - $r = 0 \therefore e^{\pm rt} = 1$

Early Exercise

- Applicable to American put options
- Only makes sense if
 - $r > 0$
 - Option is in the money
- Thus, for options with margin requirements
 - Early exercise doesn't make sense
 - American option can be treated as European

Order Book

- Market players post orders to the market
- An order might be a
 - Bid
 - Ask
 - Remove
- The market matches orders
- A listing of orders is called an *order book*
- At certain times the market clears all orders

| |
|-------|
| 10000 |
| 200 |
| 130 |
| 125 |
| 120 |

Bid = 100, Ask = 120, Spread = 20, Price = 110

| |
|-----|
| 100 |
| 95 |
| 93 |
| 4 |
| |