Note: Code to support this assignment hand-in can be found by reference to [1]

1.

$$A^2 = \begin{pmatrix} 2 & 7 & 13 \\ 4 & 11 & 9 \\ 1 & 6 & 10 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ -0.375 & 0.375 & 0.25 \\ 0.125 & -0.125 & 0.25 \end{pmatrix}, \quad \det(A) = 8, \tag{1}$$

$$\lambda_1 = 4.5115 + 0.0000i, \quad \lambda_2 = 1.2442 + 0.4745i, \quad \lambda_3 = 1.2442 - 0.4745i. \tag{2}$$

$$\lambda_1 \lambda_2 \lambda_3 = 8 = \det(A) \tag{3}$$

2.

$$e^A \approx \texttt{expm}(A) = \begin{pmatrix} 11.3532 & 35.2345 & 49.1500 \\ 14.0269 & 46.4762 & 49.3730 \\ 7.0692 & 28.1653 & 39.4070 \end{pmatrix} \tag{4}$$

This matrix is approached, in every entry, by the taylor series $I + A + A^2/2! + ...$ The difference in the 2-Norm of the solution matrices as a function of the number of terms in this series is shown in the Figure 1.

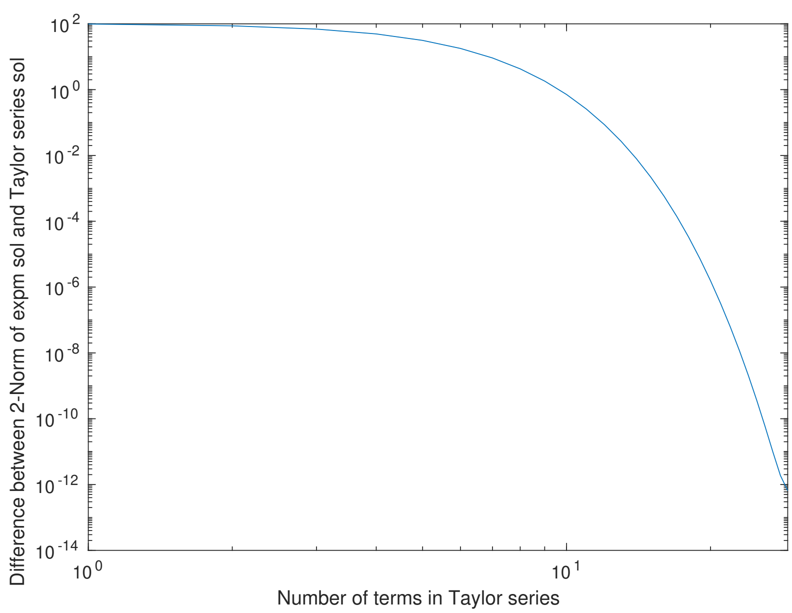

Figure 1: Difference in the 2-Norm of the matrices given by `expm` and the series $I + A + A^2/2!...$ as a function of the number of terms in this series.

3. The size of $A(N)$, which is defined in Question 3, is (N^3, N^3). We also have

$$A(2) = \begin{pmatrix} 6 & -1 & -1 & 0 & -1 & 0 & 0 & 0 \\ -1 & 6 & 0 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 6 & -1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 6 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 6 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 6 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & 0 & 6 & -1 \\ 0 & 0 & 0 & -1 & 0 & -1 & -1 & 6 \end{pmatrix}. \tag{5}$$

The spy plot of $A(4)$ is shown in Figure 2.

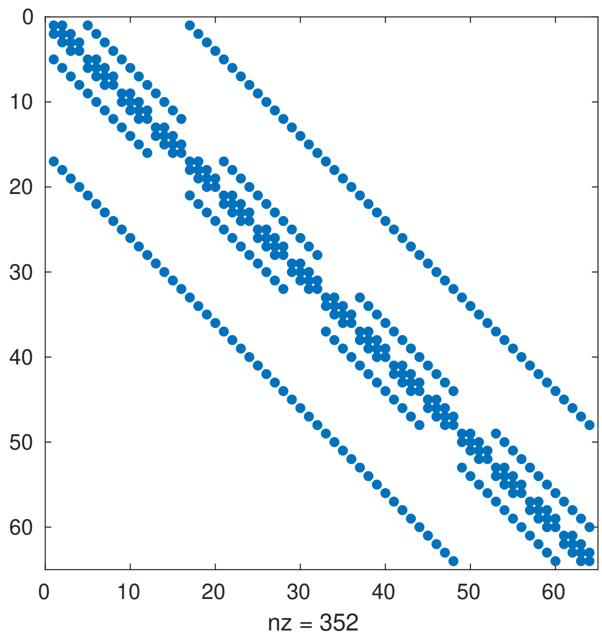

Figure 2: Spy plot of $A(4)$.

The solution times for $Ax = b$ using `backslash`, with b defined in Question 3, for different N, are shown in Table 1, and in the `loglog` plot in Figure 3. The line N^2 is also shown in the `loglog` plot. Because the matrix given by the function $A(N)$ is sparse, the solution time scales with N^2 rather than with N^3, which is the solution time scaling that we would expect for a random matrix.

N	Time
10	0.0064
20	0.0440
30	0.2588
40	0.8721
50	2.9758
60	8.7434
70	25.1357

Table 1: Solution times of $x = A\backslash b$ for different N.

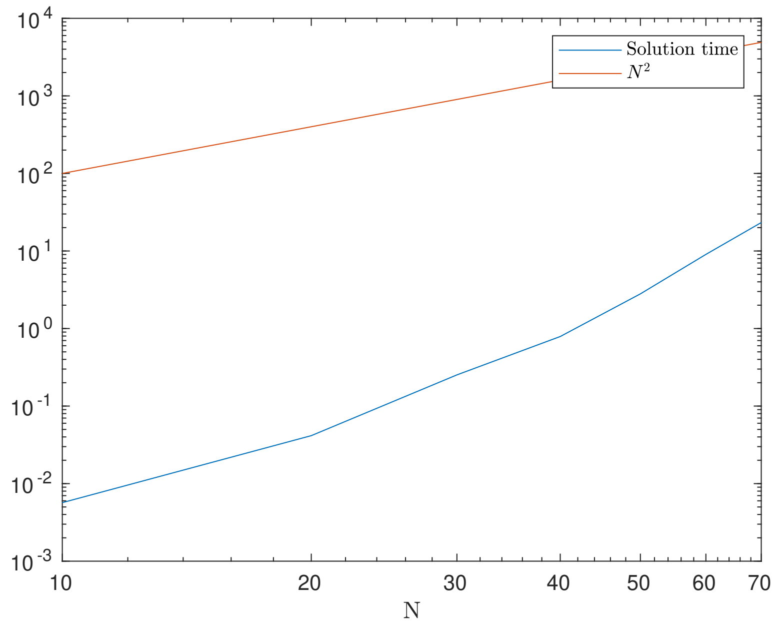

Figure 3: The solution time of $x = A\backslash b$ as a function of N, and the line N^2, in a `loglog` plot.

References

[1] A. Wey. Numerical methods assignments. https://github.com/ArkadyWey/NumericalMethodsAssignments. Accessed: 2010-09-30.