STAT 220A: Probability & Measure Theory

Arkajyoti Saha
arkajyos@uci.edu
University of California, Irvine
Department of Statistics

Introduction

Roadmap

Part 1: Foundations (Before Midterm)

- Probability spaces, measures, and events
- Random variables and distributions
- Integrals, expectation, and conditioning
- Joint structure and independence

Part 2: Limits and Asymptotics (After Midterm)

- Modes of convergence
- Laws of large numbers
- Central limit theorem

Logistics

- **Lecture:** Monday & Wednesday; 11 - 12:20 PM; ICS 180

- **Discussion:** Thursday; 1 - 1:50 PM; RH 188 (Rowland Hall).

  Each week, 2 designated students will present a selected problem and help lead group discussion.

  I'll moderate, step in when needed, and provide feedback.

- **Office hours:**

  Instructor: 30 minutes per week (time to be announced)

  TA (Lynn Gao, lmgao@uci.edu): 1 hour per week (time to be announced)

Evaluations

- **Homework:** 4 HWs.
- **Exams:** Midterm 1 (Foundations), Midterm 2 (Limits & Asymptotics); Both exams will be held in class.
- **Grades:** HW: 30%, Discussion presentation & engagement: 10%, Midterm 1: 30%, Midterm 2: 30%.

Feedback

- Student feedback is important for improving the course. A **mid-quarter evaluation** will be conducted to gather instructional feedback and help guide adjustments during the quarter.

- To encourage participation in the **official end-of-quarter course evaluation**, the lowest-scoring completed homework will be dropped for all students who submit the evaluation.

References

Measure Theory & Measure-Theoretic Probability

- **Probability: Theory and Examples — Rick Durrett**
- A First Look at Rigorous Probability Theory — Jeffrey S. Rosenthal
- Probability and Measure — Patrick Billingsley
- Probability for Statisticians — Galen R. Shorack

Real Analysis Background

- Introduction to Real Analysis — Bartle & Sherbert
- Mathematical Analysis — Tom M. Apostol
- Principles of Mathematical Analysis — Walter Rudin

What is probability?

Probability

Probability theory analyzes experiments that depend on chance.

Examples:

- Coin tosses and dice rolls
- Card games and lotteries
- Weather forecasting
- Insurance and risk pricing
- Reliability and lifetimes of systems
- Data-driven decision making

Randomness has structure

- At first glance, this seems contradictory:
  if outcomes are random, what can we analyze?

- **The key idea:**
  randomness exhibits structure and regularity.

- Individual outcomes are unpredictable.
  Collections of outcomes behave regularly.

- If a deck of cards is shuffled and 13 cards are dealt, how likely is it to get all four aces?

Understanding a chance experiment

- To analyze a chance experiment, we must first understand the experiment.
- That means:
  - What are the possible outcomes?
  - What questions are we allowed to ask about them?
- Before assigning probabilities, we must agree on what objects probabilities apply to.

When probability works smoothly

In **discrete settings**, probability behaves well.

- We can list all possible outcomes
- We assign probabilities to individual outcomes
- Probabilities of events are obtained by adding them up

Continuous outcomes change the rules

- Many real experiments are **not discrete**.
- Example: Pick a number uniformly from the interval (0, 1].
- What happens?
  - Every individual point must have probability zero
  - Yet intervals clearly have positive probability
- This is fundamentally different from the discrete case.

A necessary shift in perspective

- Probability cannot live on individual outcomes.

- Instead: Probability must be assigned to **sets of outcomes**

- This works well for intervals and many familiar collections.

- But there is a problem.

We cannot assign probability to all sets

- If we try to assign probabilities to **every subset:**
  - fundamental inconsistencies appear
  - The theory breaks down

- The resolution:
- Carefully decide which subsets count as events
- Enough to answer meaningful questions
- Restricted enough to remain mathematically consistent

- This is where measure theory enters.