220A Discussion 3

1. **Same distribution, different random variables.** Let $(\Omega, \mathcal{F}, P)$ be a probability space.

   (a) Construct random variables $X, Y : \Omega \to \mathbb{R}$ such that $X \neq Y$ almost surely, but X and Y have the same distribution.

   (b) Show that for every Borel set $A \subset \mathbb{R}$,
   $$P(X \in A) = P(Y \in A).$$

   (c) Explain briefly why knowledge of the distribution of a random variable does not determine the random variable itself.

2. **Stability under continuous transformations.** Show that if X is a random variable and $f : \mathbb{R} \to \mathbb{R}$ is continuous, then $f(X)$ is a random variable.