220A Discussion 1

1. Let
$$\Omega = \{1, 2, 3\},$$
and define
$$\mathcal{F}_1 = \{\emptyset, \Omega, \{1\}, \{2, 3\}\}, \qquad \mathcal{F}_2 = \{\emptyset, \Omega, \{1, 2\}, \{3\}\}.$$
   (a) Verify that $\mathcal{F}_1$ and $\mathcal{F}_2$ are σ-fields on Ω.
   (b) Show that $\mathcal{F}_1 \cup \mathcal{F}_2$ is not a field.
   (c) Compute the σ-field generated by $\mathcal{F}_1 \cup \mathcal{F}_2$, that is,
$$\sigma(\mathcal{F}_1 \cup \mathcal{F}_2).$$

2. Let $\Omega = \{1, 2, 3\}$, and define
$$\mathcal{F}_1 = \{\emptyset, \Omega, \{1\}, \{2, 3\}\}, \qquad \mathcal{F}_2 = \{\emptyset, \Omega, \{1, 2\}, \{3\}\}.$$
   (a) Describe all probability measures on $(\Omega, \mathcal{F}_1)$.
   (b) Describe all probability measures on $(\Omega, \mathcal{F}_2)$.
   (c) Let $a, b \in [0, 1]$. Suppose μ is a probability measure on $(\Omega, \mathcal{F}_1)$ such that
$$\mu(\{1\}) = a,$$
and ν is a probability measure on $(\Omega, \mathcal{F}_2)$ such that
$$\nu(\{3\}) = b.$$
Does there exist a probability measure P on $(\Omega, \mathcal{P}(\Omega))$ such that
$$P(A) = \mu(A) \text{ for all } A \in \mathcal{F}_1, \quad \text{and} \quad P(B) = \nu(B) \text{ for all } B \in \mathcal{F}_2?$$