

Nodal Singularity Classifier — Summary

Goal: Learn to predict whether the projective point (1:0:0) is a nodal singularity for a random homogeneous cubic in three variables with complex coefficients.

Labeling (exact algebra): Singular at (1:0:0) iff $a_0=a_1=a_2=0$. Nodal iff $\Delta=a_4^2-4 a_3 a_5 \neq 0$ in chart $X=1$.

Model: Pure NumPy MLP ($20 \rightarrow 512 \rightarrow 1$), BCE-with-logits, Adam, L2. Optional explicit features $\text{Re}(\Delta)$, $\text{Im}(\Delta)$.

Data: 10k samples with fixed composition; inputs are 20 real features (Re/Im of 10 complex coeffs); becomes 22 when Δ is included.

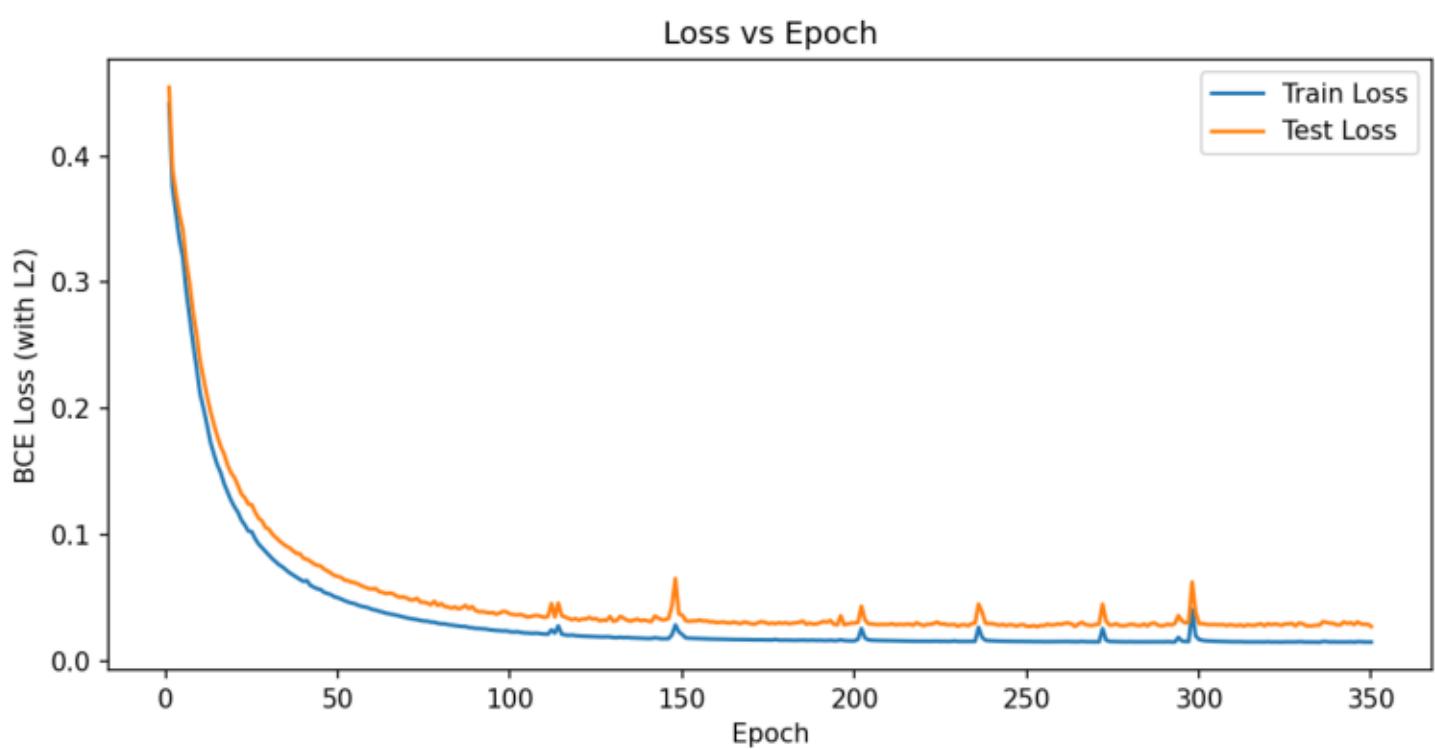
Outputs: Training curves, composition, sensitivity analyses, $P(\text{nodal})$ vs $|\Delta|$, and Δ ablation figures.

Key Points

- 1) The network learns a two-stage rule: (a) gate on singularity via a_0, a_1, a_2 ; (b) if singular, decide nodal via Δ .
- 2) Providing Δ explicitly improves optimization and raises training/test accuracy; the model relies on Δ when relevant.
- 3) Global ablation (mask Δ on whole test set) shows a modest gap because many samples are non-singular and Δ can be reconstructed from (a_3, a_4, a_5) .
- 4) Singular-only ablation magnifies the gap: when conditioned on $a_0 = a_1 = a_2 = 0$, Δ becomes the pivotal signal.

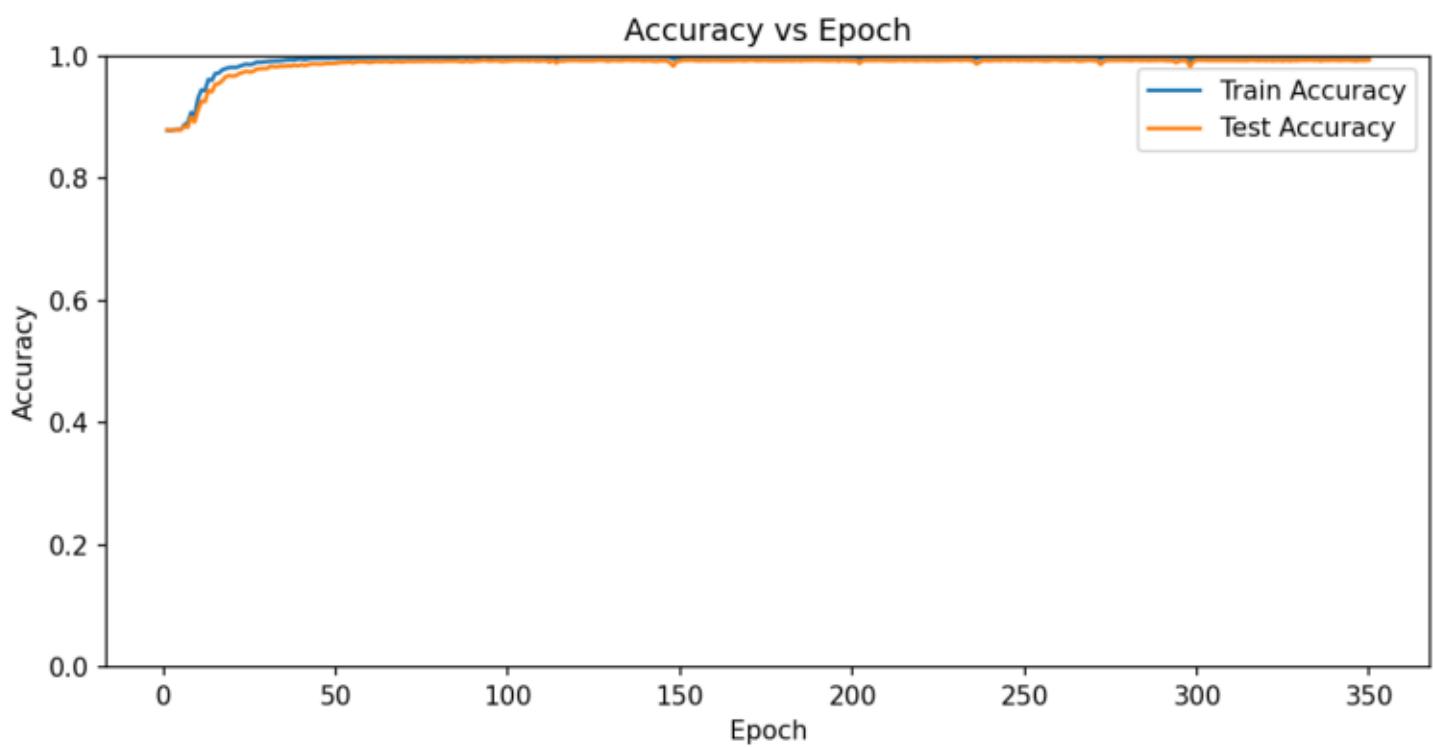
Loss vs Epoch (train/test)

Figure 1: Training and Test Loss Curves



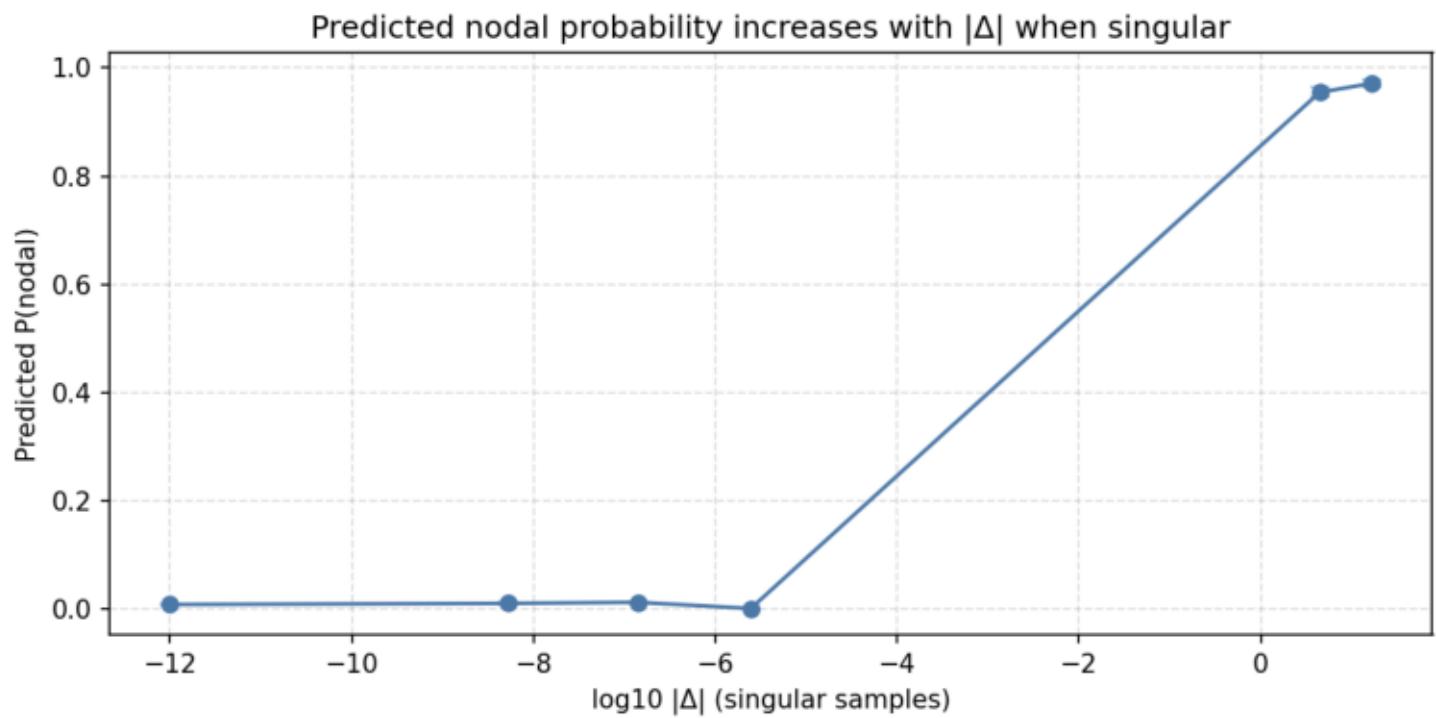
Accuracy vs Epoch (train/test)

Figure 2: Training and Test Accuracy over Epochs



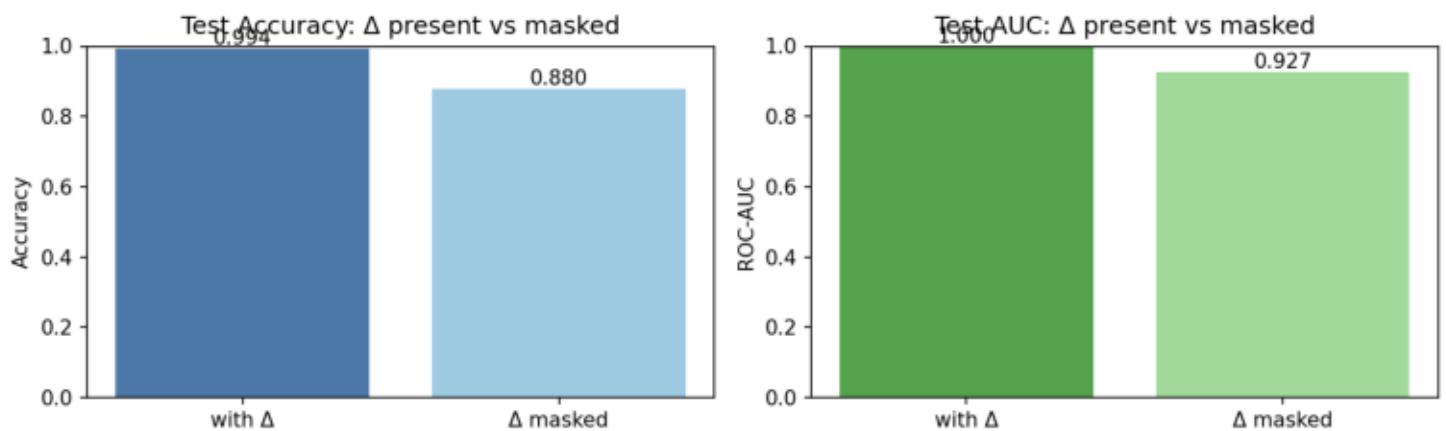
Predicted P(nodal) vs $\log_{10} |\Delta|$ (singular)

Figure 6: Evidence of Discriminant Usage by the Network



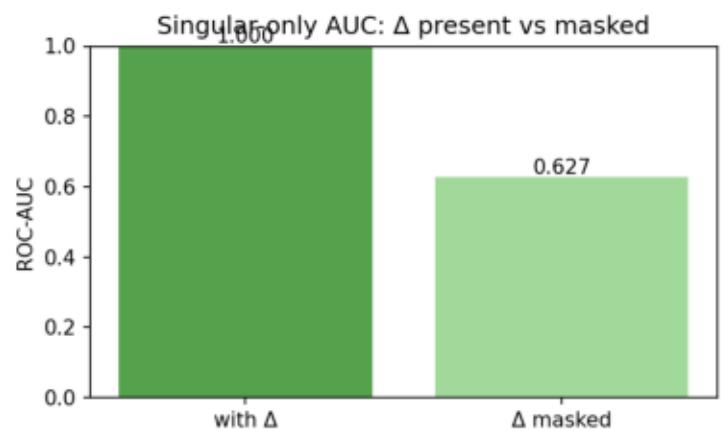
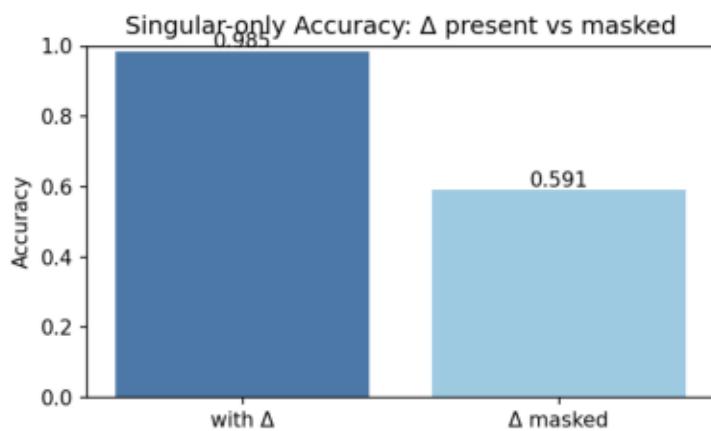
Δ Feature Ablation (global test set)

Figure 7: Impact of Explicit Δ Feature (Δ sensitivity $\sim 2.142\text{e+}00$)



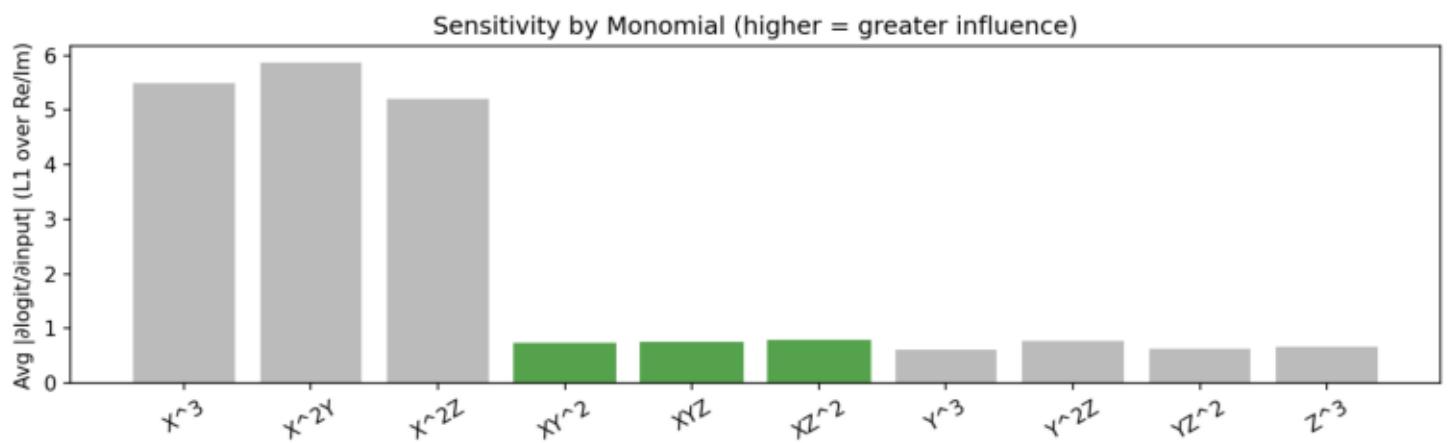
Δ Feature Ablation (singular-only)

Figure 7s: Δ Feature Ablation on Singular-only Test Subset



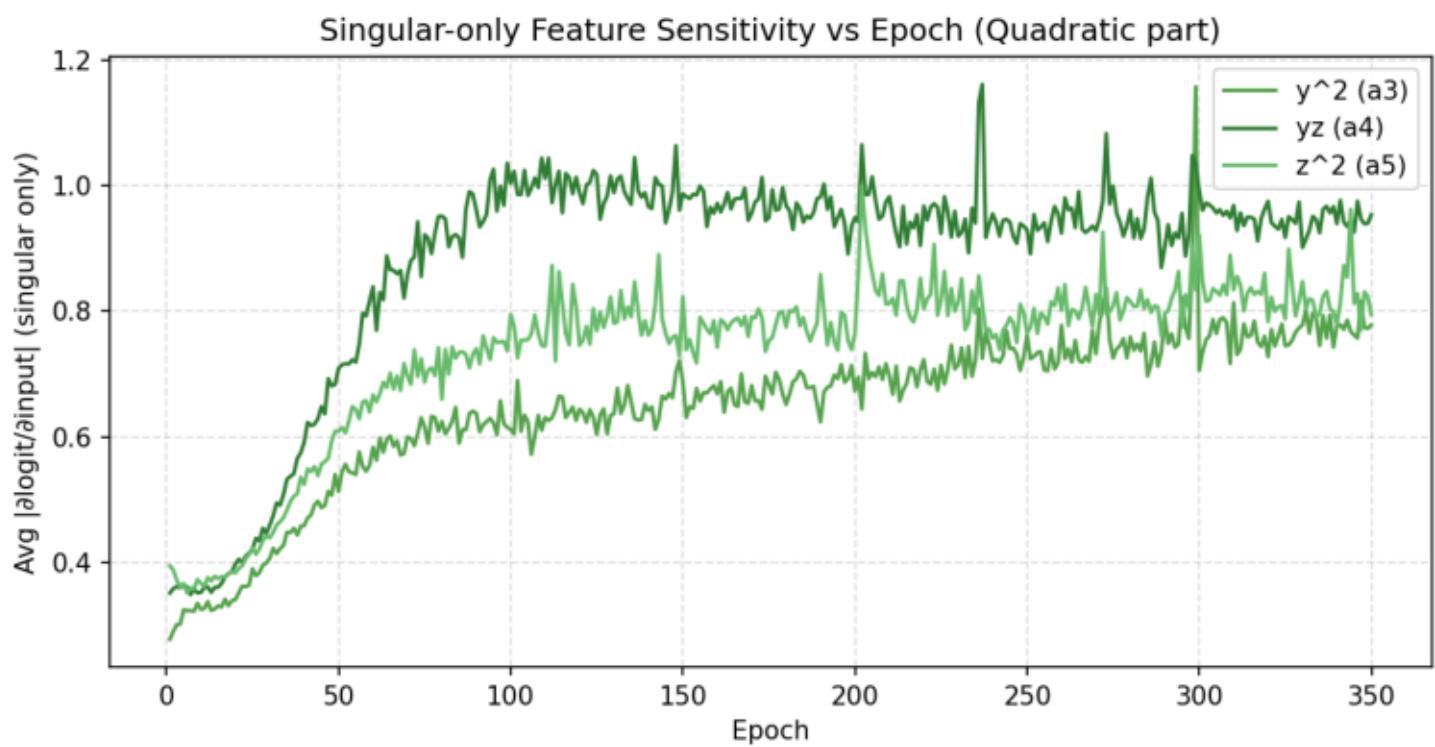
Sensitivity by Monomial

Figure 5: Feature Sensitivity — Quadratic part (y^2 , yz , z^2) highlighted



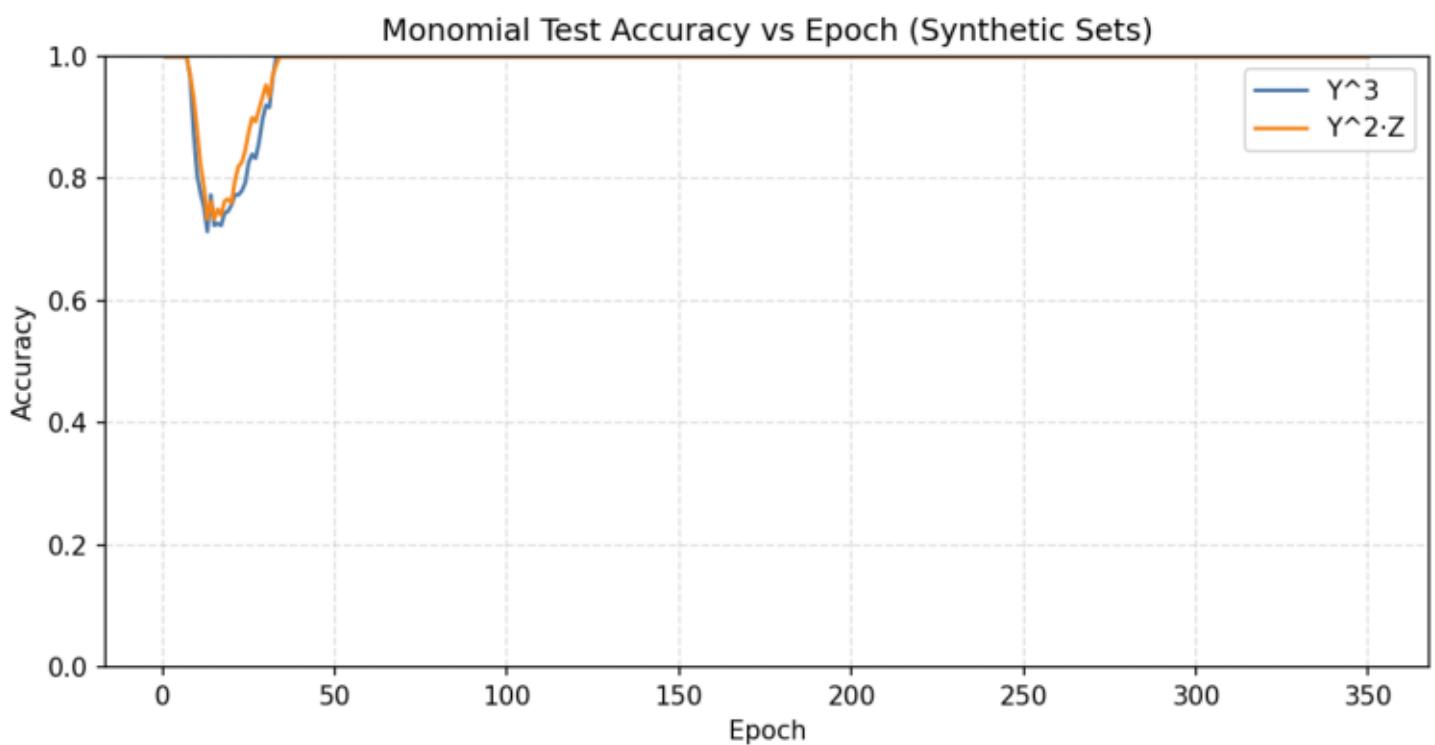
Singular-only Sensitivity vs Epoch (y^2 , yz , z^2)

Figure 5b: Per-epoch Sensitivity on Singular Subset — y^2 , yz , z^2



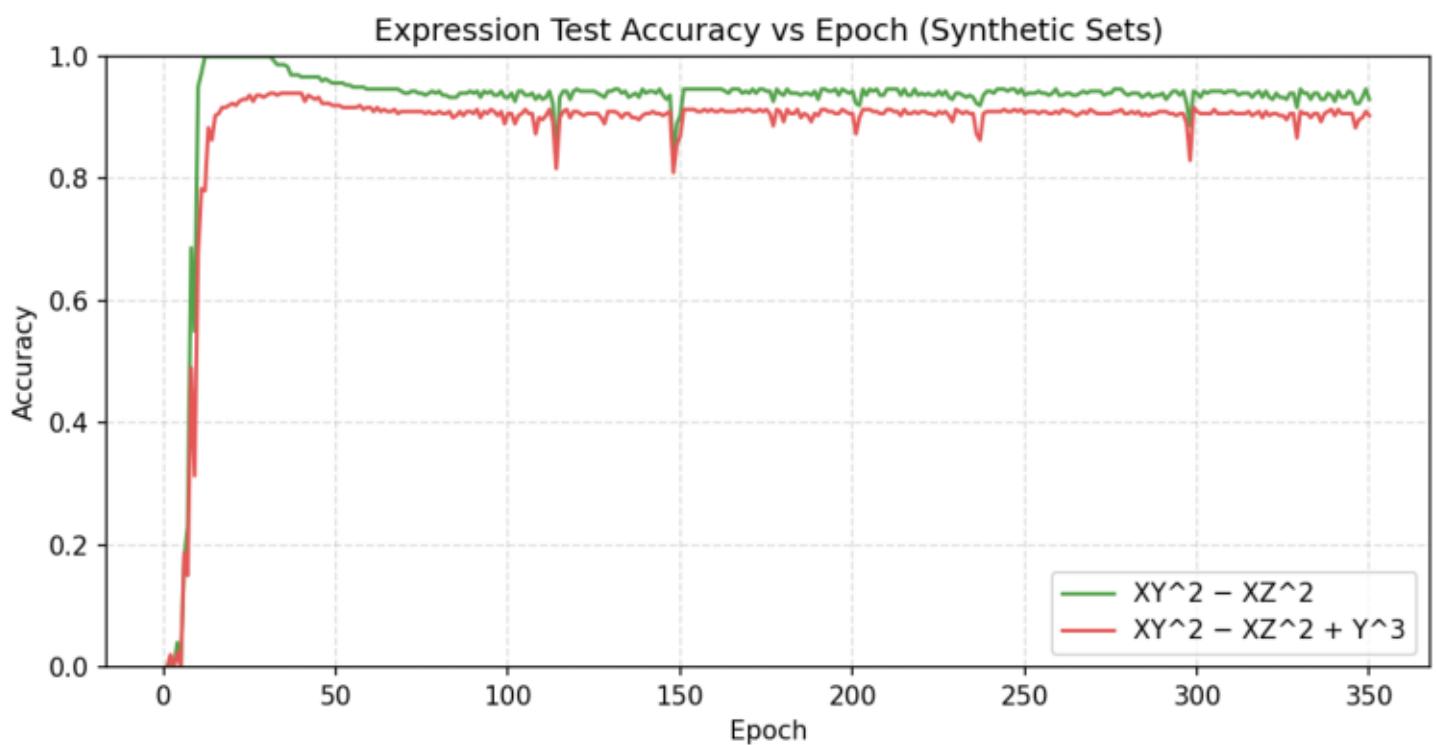
Monomial Accuracy Curves (Y^3 , $Y^2 \cdot Z$)

Figure 3: Accuracy Curves for Y^3 and $Y^2 \cdot Z$ over Training



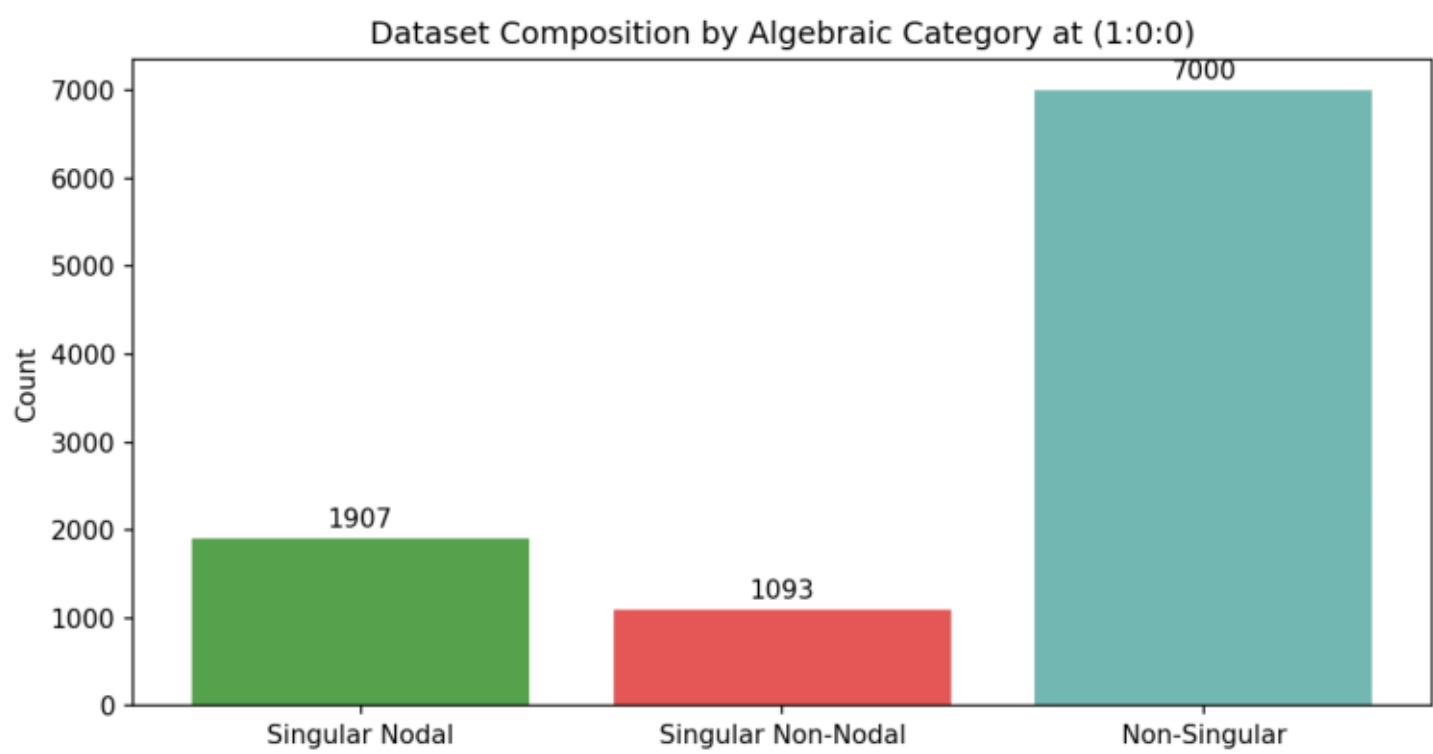
Expression Accuracy ($XY^2 - XZ^2$, $+Y^3$)

Figure 3b: Accuracy Curves for $XY^2 - XZ^2$ and $XY^2 - XZ^2 + Y^3$



Dataset Composition by Algebraic Category

Figure 4: Dataset Composition — Nodal, Singular Non-Nodal, and Non-Singular



Run Information

Outputs source timestamp: 20251106_005830

Report generated: 20251106_010230

Embedded figures: loss_curve, accuracy_curve, prob_vs_discriminant, delta_feature_ablation, delta_feature_ablation_singular_only, feature_sensitivity, feature_sensitivity_singular_epochs, monomial_accuracy_curves, expression_accuracy_curves, dataset_composition