

# Nodal Singularity Classifier — Summary

Goal: Learn to predict whether the projective point  $(1:0:0)$  is a nodal singularity for a random homogeneous cubic in three variables with complex coefficients.

Labeling (exact algebra): Singular at  $(1:0:0)$  iff  $a_0=a_1=a_2=0$ . Nodal iff  $\Delta=a_4^2-4 a_3 a_5 \neq 0$  in chart  $X=1$ .

Model: Pure NumPy MLP ( $20 \rightarrow 512 \rightarrow 1$ ), BCE-with-logits, Adam, L2. Optional explicit features  $\text{Re}(\Delta)$ ,  $\text{Im}(\Delta)$ .

Data: 10k samples with fixed composition; inputs are 20 real features (Re/Im of 10 complex coeffs); becomes 22 when  $\Delta$  is included.

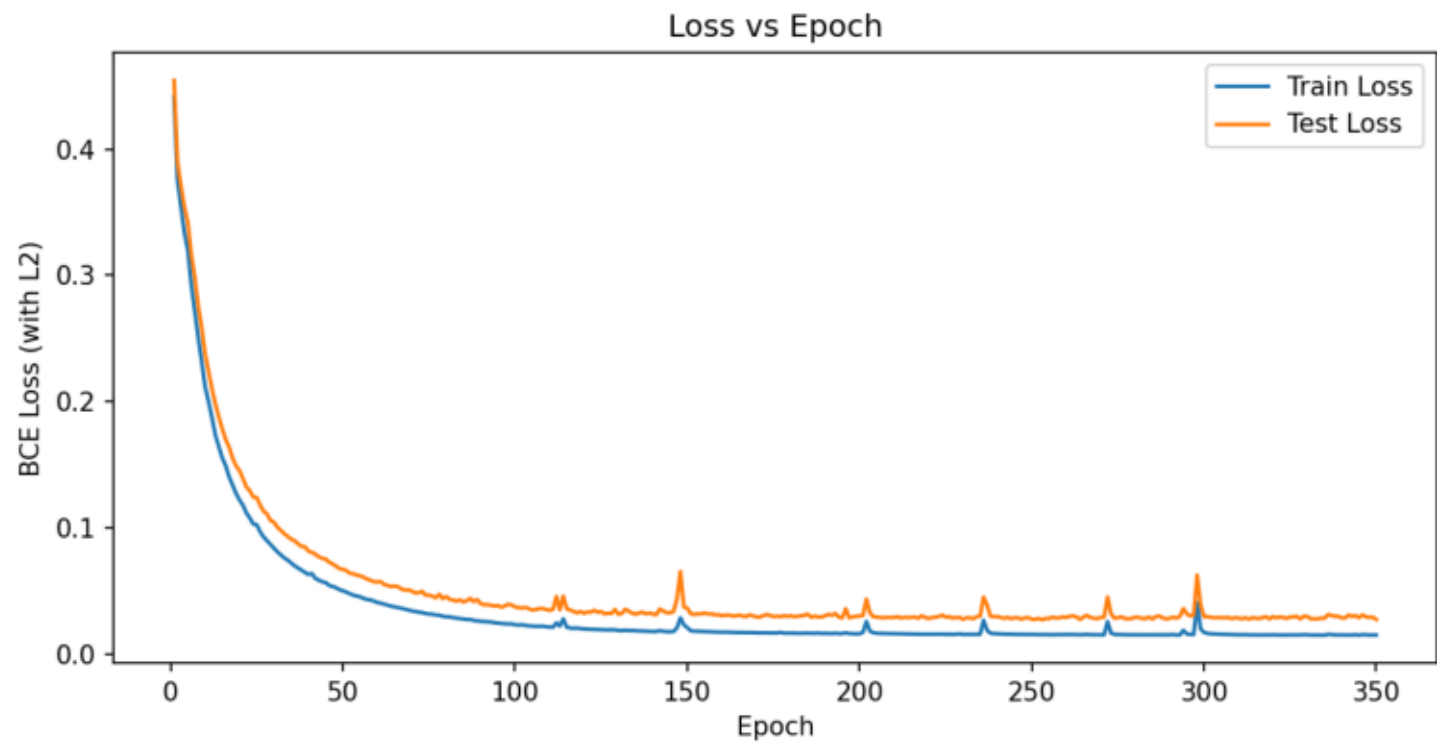
Outputs: Training curves, composition, sensitivity analyses,  $P(\text{nodal})$  vs  $|\Delta|$ , and  $\Delta$  ablation figures.

# Key Points

- 1) The network learns a two-stage rule: (a) gate on singularity via  $a_0, a_1, a_2$ ; (b) if singular, decide nodal via  $\Delta$ .
- 2) Providing  $\Delta$  explicitly improves optimization and raises training/test accuracy; the model relies on  $\Delta$  when relevant.
- 3) Global ablation (mask  $\Delta$  on whole test set) shows a modest gap because many samples are non-singular and  $\Delta$  can be reconstructed from  $(a_3, a_4, a_5)$ .
- 4) Singular-only ablation magnifies the gap: when conditioned on  $a_0=a_1=a_2=0$ ,  $\Delta$  becomes the pivotal signal.

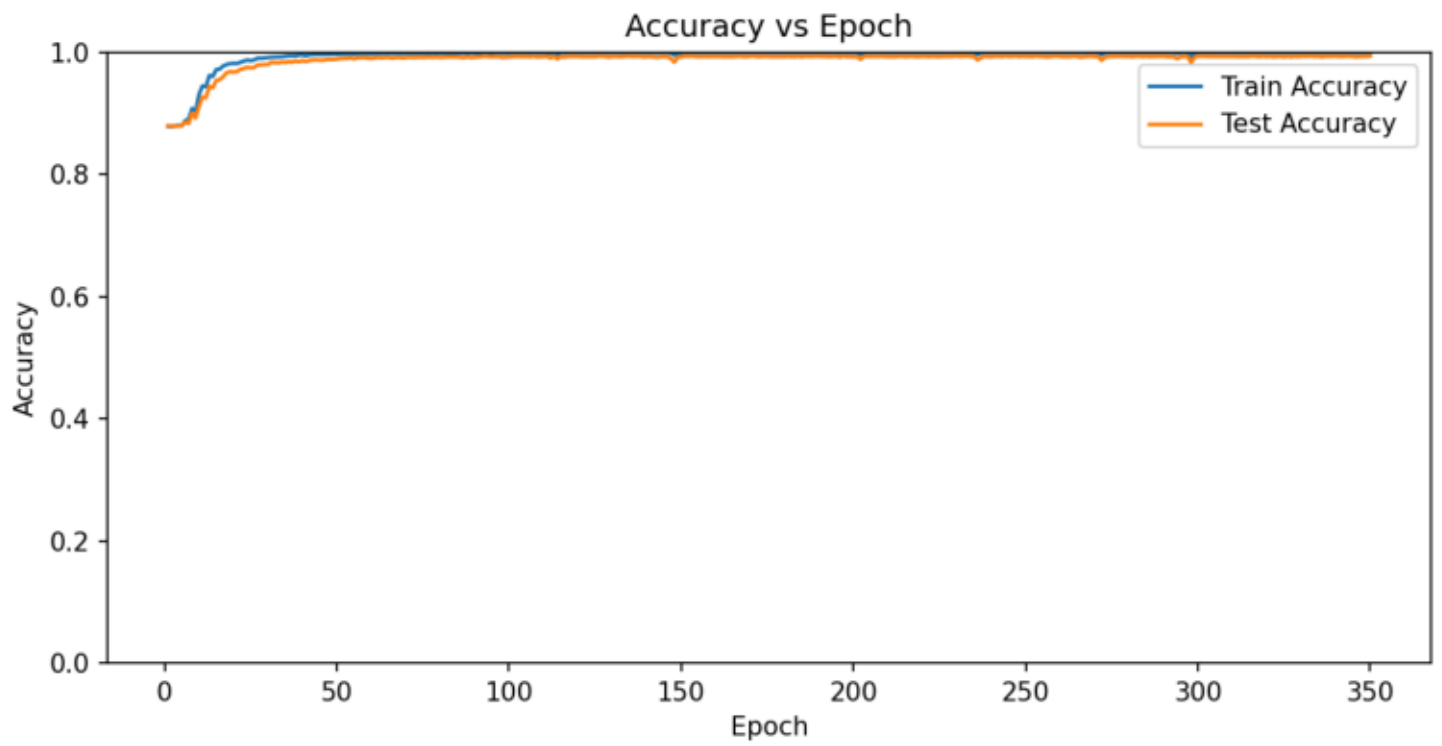
Loss vs Epoch (train/test)

Figure 1: Training and Test Loss Curves



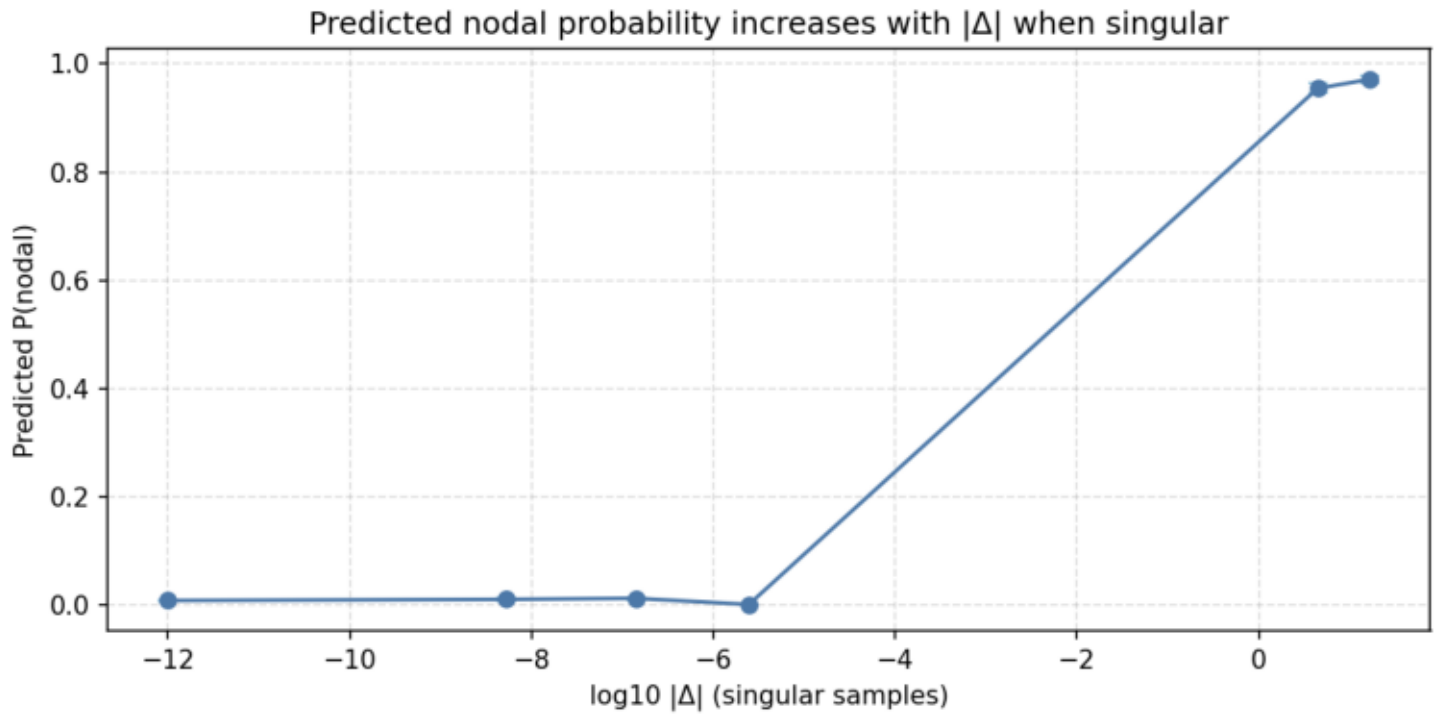
## Accuracy vs Epoch (train/test)

Figure 2: Training and Test Accuracy over Epochs



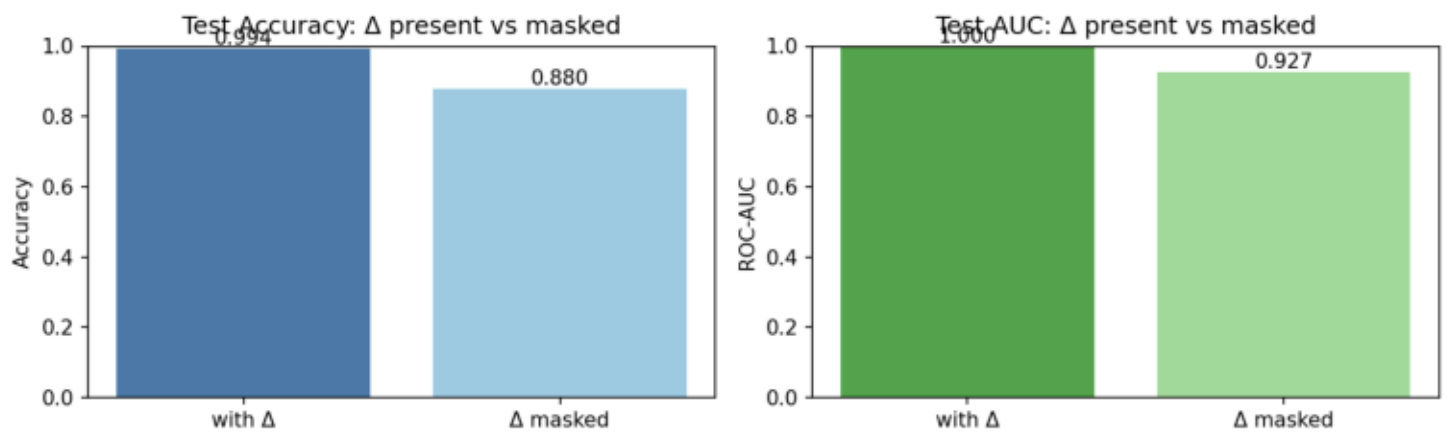
## Predicted $P(\text{nodal})$ vs $\log_{10} |\Delta|$ (singular)

Figure 6: Evidence of Discriminant Usage by the Network



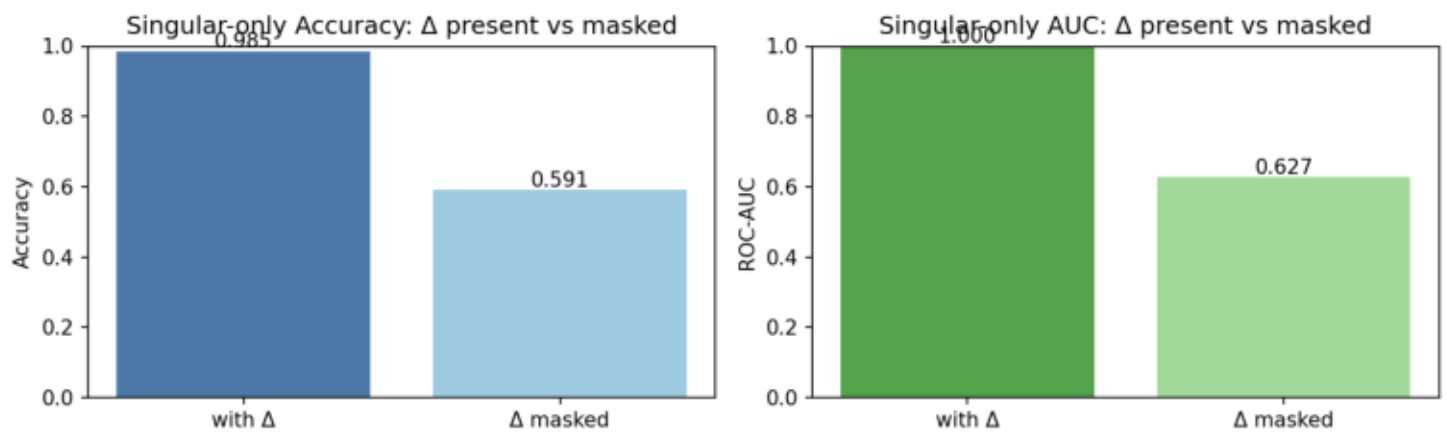
## $\Delta$ Feature Ablation (global test set)

Figure 7: Impact of Explicit  $\Delta$  Feature ( $\Delta$  sensitivity  $\sim 2.142\text{e}+00$ )



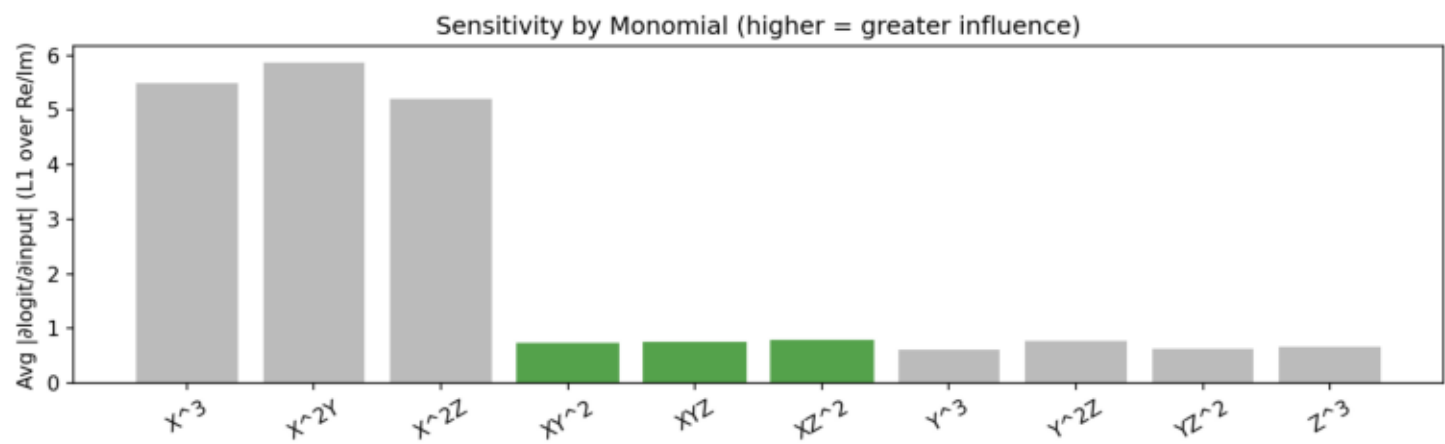
## $\Delta$ Feature Ablation (singular-only)

Figure 7s:  $\Delta$  Feature Ablation on Singular-only Test Subset



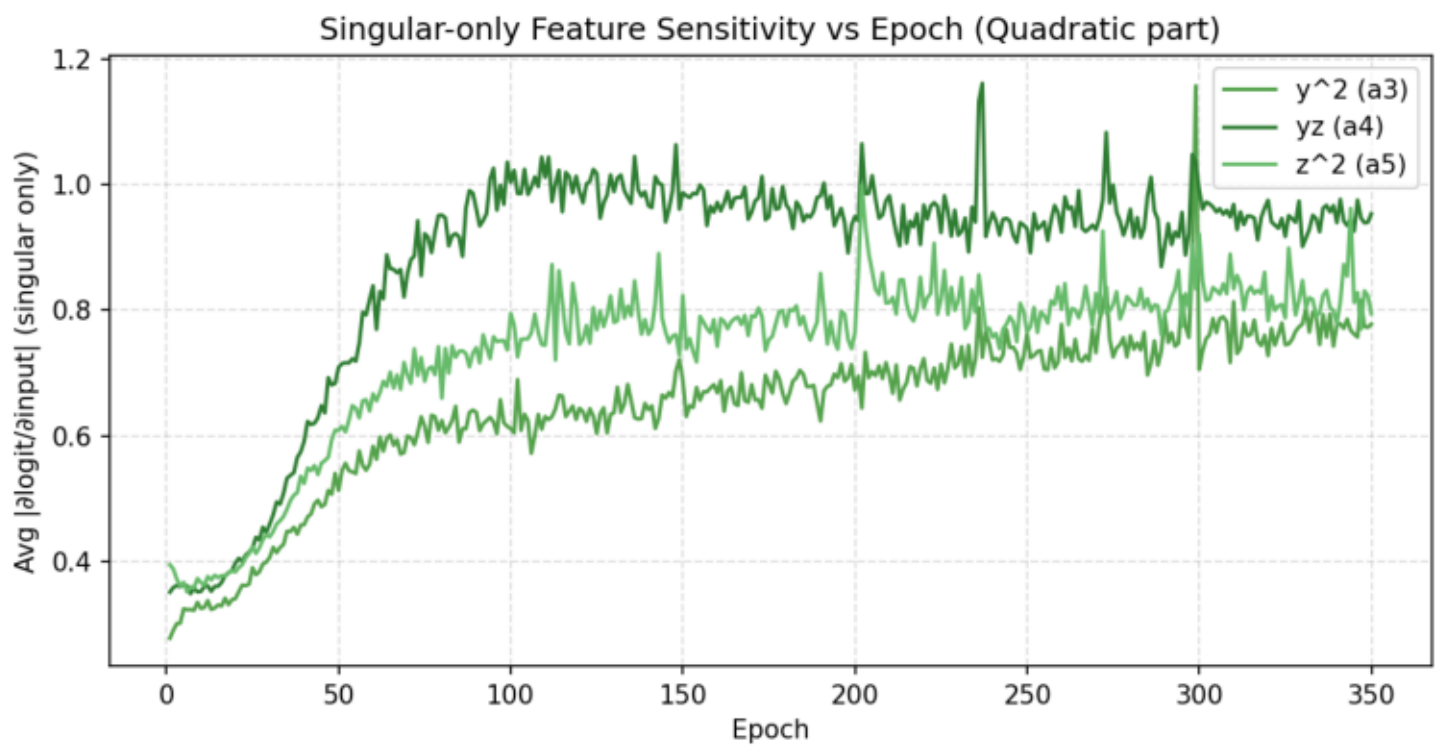
# Sensitivity by Monomial

Figure 5: Feature Sensitivity — Quadratic part ( $y^2$ ,  $yz$ ,  $z^2$ ) highlighted



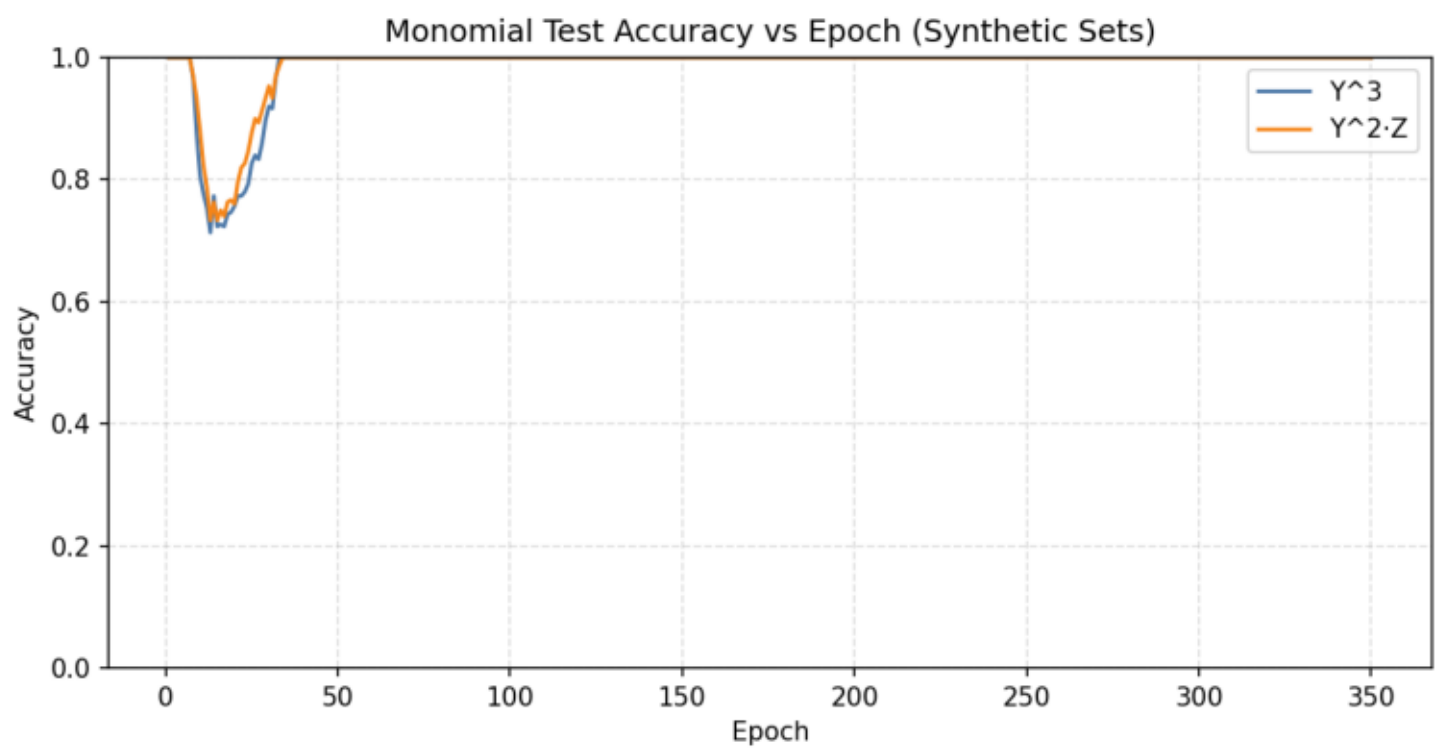
## Singular-only Sensitivity vs Epoch ( $y^2$ , $yz$ , $z^2$ )

Figure 5b: Per-epoch Sensitivity on Singular Subset —  $y^2$ ,  $yz$ ,  $z^2$



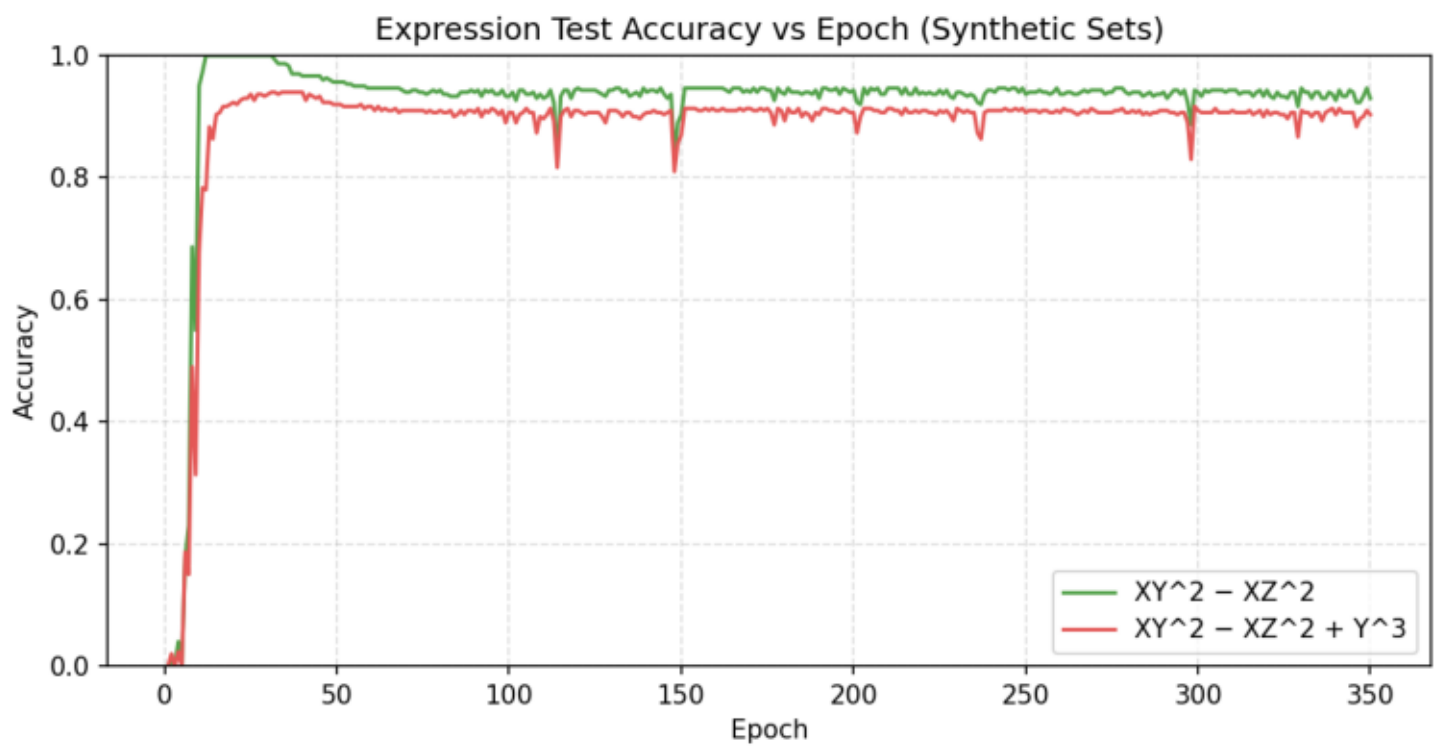
# Monomial Accuracy Curves ( $Y^3$ , $Y^2 \cdot Z$ )

Figure 3: Accuracy Curves for  $Y^3$  and  $Y^2 \cdot Z$  over Training



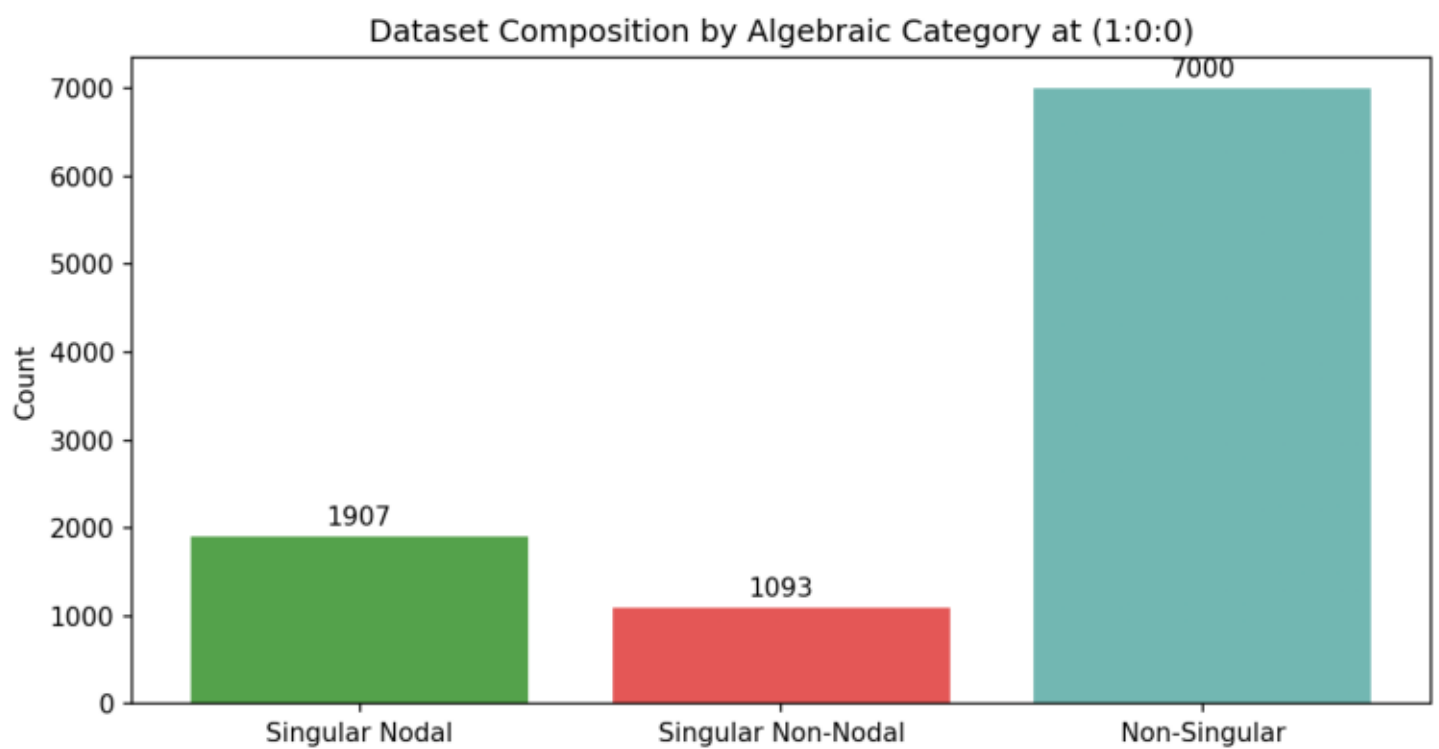
## Expression Accuracy ( $XY^2 - XZ^2$ , $+Y^3$ )

Figure 3b: Accuracy Curves for  $XY^2 - XZ^2$  and  $XY^2 - XZ^2 + Y^3$



# Dataset Composition by Algebraic Category

Figure 4: Dataset Composition — Nodal, Singular Non-Nodal, and Non-Singular



# Run Information

Outputs source timestamp: 20251106\_005830

Report generated: 20251106\_010230

Embedded figures: loss\_curve, accuracy\_curve, prob\_vs\_discriminant, delta\_feature\_ablation, delta\_feature\_ablation\_singular\_only, feature\_sensitivity, feature\_sensitivity\_singular\_epochs, monomial\_accuracy\_curves, expression\_accuracy\_curves, dataset\_composition