Explain what is the affine matrix. Explain the shift, scaling, flipping and change of aspect ratio.

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Notation

The coordinate of a point of the image is given by: $\begin{bmatrix} w \\ z \end{bmatrix}$

and the new coordinate is given by: $\begin{bmatrix} x \\ y \end{bmatrix}$

Shift

$$
\begin{aligned}
x = \quad & w + \alpha = \quad 1 \times w + 0 \times z + 1 \times \alpha \\
y = \quad & z + \beta = \quad 0 \times w + 1 \times z + 1 \times \beta
\end{aligned}
$$

We can write it as a matrix product:

$$
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 1 & \beta \end{bmatrix} \begin{bmatrix} w \\ z \\ 1 \end{bmatrix} = A \begin{bmatrix} w \\ z \\ 1 \end{bmatrix}
$$

Scaling

$$
\begin{aligned}
x = & \quad \alpha \times w \\
y = & \quad \beta \times z
\end{aligned}
$$

For the scaling:

$$\alpha = \beta \geq 0$$

We can write it as a matrix product:

$$
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} w \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \end{bmatrix} \begin{bmatrix} w \\ z \\ 1 \end{bmatrix} = A \begin{bmatrix} w \\ z \\ 1 \end{bmatrix}
$$

Note

To flip the image, we have:

$$\alpha = -\beta$$

To change aspect ratio, we have:

$$\alpha \neq \beta$$

To have the same length of input and output, we write the affix transform matrix like this:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} & A & \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ z \\ 1 \end{bmatrix}$$