Convolution VS correlation

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January 16, 2024

Convolution: formula

$$(f * w)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t)$$

Correlation: formula

$$(f \circ w)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

Associativity

$$f * (h_1 * h_2) = (f * h_1) * h_2?$$

$$f * (h_1 * h_2)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} (h_1 * h_2)(s, t) f(x - s, y - t)$$

$$= \sum_{s=-a}^{a} \sum_{t=-b}^{b} (\sum_{u=-c}^{c} \sum_{v=-d}^{d} h_2(u, v) h_1(s - u, t - v)) f(x - s, y - t)$$

$$= \sum_{s=-a}^{a} \sum_{t=-b}^{b} \sum_{u=-c}^{c} \sum_{v=-d}^{d} h_2(u, v) h_1(s - u, t - v) f(x - s, y - t)$$

$$f * (h_1 * h_2)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t)$$

and

Commutativity

Distributivity