Low-pass filtering

Michel Donnet

January 16, 2024

Box filter

Pros:

- Remove additive gaussian noise

Cons:

- Image is blurred

Box filter: example

Box filter: structure

Filter size: (N, N)

$$\begin{bmatrix} \frac{1}{N^2} & \cdots & \frac{1}{N^2} \\ \vdots & \ddots & \vdots \\ \frac{1}{N^2} & \cdots & \frac{1}{N^2} \end{bmatrix} = \frac{1}{N^2} \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$$

So we have

$$\sum^{N^2} \frac{1}{N^2} = \frac{N^2}{N^2} = 1$$

Gaussian filter

Pros:

- Reduce noise in images
- Can be usefull to sharp images: image + k(image - image gaussian filtered)

Cons:

- Image is blurred

Gaussian filter: structure

$$w(s,t) = Ke^{-\frac{s^2+t^2}{2\sigma^2}}$$

with

$$K = \frac{1}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} e^{-\frac{s^2+t^2}{2\sigma^2}}}$$

So we have

$$\sum_{x=-a}^{a}\sum_{y=-b}^{b} Ke^{-\frac{x^2+y^2}{2\sigma^2}} = \frac{\sum_{x=-a}^{a}\sum_{y=-b}^{b} e^{-\frac{x^2+y^2}{2\sigma^2}}}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} e^{-\frac{s^2+t^2}{2\sigma^2}}} = 1$$