Advantages of separable filter

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Non separable filter

Size of f: (M, N)

Size of w: (m, n)

$$(f * w)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t)$$

$\Rightarrow MNmn$ operations !

Separable filter

Size of f: (M, N)

Size of w: (m, n)

$$w(x, y) = w_1(x)w_2(y)$$

$$(f * w)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)f(x - s, y - t)$$

$$= \sum_{s=-a}^{a} \sum_{t=-b}^{b} w_1(s)w_2(t)f(x - s, y - t)$$

$$= \sum_{s=-a}^{a} w_1(s) \left(\sum_{t=-b}^{b} w_2(t)f(x - s, y - t) \right)$$

$$= mMN + MNn \text{ operations}$$

Gain

$$\frac{mnMN}{MN(m+n)} = \frac{mn}{m+n}$$

(Very large if m and n are large !)