

Convolution VS correlation

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Convolution: formula

$$(f * w)(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x - s, y - t)$$

Correlation: formula

$$(f \circ w)(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

Differences between convolution and correlation

	Convolution	Correlation
Associativity	$f * (h_1 * h_2) = (f * h_1) * h_2$	No
Commutativity	$f * h = h * f$	No
Distributivity	$f * (h_1 + h_2) = f * h_1 + f * h_2$	$f \circ (h_1 + h_2) = f \circ h_1 + f \circ h_2$

Differences between convolution and correlation

To make convolution, we have to flip (180) the filter and then to apply correlation

Convolution	Correlation
filtering operations	pattern matching

Distributivity of convolution

$$\begin{aligned}& (f * (h_1 + h_2))(x, y) \\&= \sum_s \sum_t (h_1 + h_2)(s, t) f(x - s, y - t) \\&= \sum_s \sum_t (h_1(s, t) + h_2(s, t)) f(x - s, y - t) \\&= \sum_s \sum_t (h_1(s, t) f(x - s, y - t) + h_2(s, t) f(x - s, y - t)) \\&= \sum_s \sum_t h_1(s, t) f(x - s, y - t) + \sum_s \sum_t h_2(s, t) f(x - s, y - t) \\&= (f * h_1)(x, y) + (f * h_2)(x, y)\end{aligned}$$

Distributivity of correlation

$$\begin{aligned} & (f \circ (h_1 + h_2))(x, y) \\ &= \sum_s \sum_t (h_1 + h_2)(s, t) f(x + s, y + t) \\ &= \sum_s \sum_t (h_1(s, t) + h_2(s, t)) f(x + s, y + t) \\ &= \sum_s \sum_t (h_1(s, t) f(x + s, y + t) + h_2(s, t) f(x + s, y + t)) \\ &= \sum_s \sum_t h_1(s, t) f(x + s, y + t) + \sum_s \sum_t h_2(s, t) f(x + s, y + t) \\ &= (f \circ h_1)(x, y) + (f \circ h_2)(x, y) \end{aligned}$$