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## Cryptographic Hash Functions and MACs

### One-Way Functions (OWF)

A function  $f$  is one-way if  $f(x) = y$  is easy to compute, but finding  $x$  from  $y$  is computationally impossible for the majority of values.

#### Examples:

- Squares modulo composite:  $f(x) = x^2 \bmod n$  with  $n = pq$
- DES construction:  $y = E_k(x) \oplus x$  with  $k$  fixed and known

**Note:** OWF  $\neq$  OWHF (hash functions impose compression and 2nd-preimage resistance).

#### Original Text

A function **f** is called **one-way** (one-way function or **OWF**) if for  $x \in X$  we can easily compute  $f(x) = y$  but for the vast majority of  $y \in Y$  it is **computationally impossible** to find an  $x$  such that  $f(x) = y$ .

#### Examples:

- computing squares modulo a composite:  $f(x) = x^2 \bmod n$  with  $n = pq$  (p and q

unknown) is a **one-way function** because the inverse is difficult (see the basic problem **SQROOTP**).

- we can construct a one-way function based on DES or any other block encryption system  $E$  as follows:  $y = f(x) = E_k(x) \oplus x$ ,  $\forall x$ , with  $k$  a fixed and known key. We can consider that  $E_k(x) \oplus x$  has (pseudo)random behavior by construction of  $E$ . Computing the inverse amounts to finding an  $x$  such that:  $x = E_k^{-1}(x \oplus y)$ , which is considered difficult given the properties of  $E$ . Note that  $f(x) = E_k(x)$  would not be sufficient to make an OWF because, with the key known, DES is reversible.

**OWF   OWHF:** Note that an OWHF as a hash function imposes additional restrictions on the source and image domains as well as on 2nd-preimage resistance that are not necessarily satisfied by OWFs.

**Example:**  $f(x) = x^2 \bmod n$  with  $n = pq$  ( $p$  and  $q$  unknown) is not an OWHF because given  $x$ ,  $-x$  is a trivial collision.

### Quick Revision

**OWF:** easy in one direction ( $f(x) \rightarrow y$ ), impossible in the other ( $y \rightarrow x$ ).

Examples: modular squares,  $E_k(x) \oplus x$ .

OWF   OWHF (hash functions = more constraints).

## Hash Functions: Definitions

A hash function  $h$  has two essential properties:

- **Compression:** transforms data of arbitrary length into fixed-length output
- **Ease of computation:**  $h(x)$  is fast to compute

**Classification:**

- **Unkeyed** (no key): MDC (Manipulation Detection Code)
- **Keyed** (with key): MAC (Message Authentication Code)

**Security properties:**

1. **Preimage resistance:** given  $y$ , impossible to find  $x$  such that  $h(x) = y$
2. **2nd-preimage resistance** (weak collision): given  $x$ , impossible to find  $x' \neq x$  such that  $h(x) = h(x')$
3. **Collision resistance** (strong collision): impossible to find any  $x \neq x'$  with  $h(x) = h(x')$

**Terminology:**

- **OWHF** (weak one-way): satisfies (1) and (2)

- **CRHF** (strong one-way): satisfies (2) and (3)

#### **i** Original Text

A **hash function** is a function  $h$  having the following properties:

- **compression**: the function  $h$  maps a set  $X$  composed of bit strings of finite but arbitrary length to a set  $Y$  composed of bit strings of finite and fixed length (and normally smaller than the size of  $X$ ) with  $h(x) = y$ , and  $x \in X$ ,  $y \in Y$ .
- **easy to compute**: given  $h$  and  $x \in X$ ,  $h(x)$  is easy to compute.

A hash function is called “**keyed**” (keyed hash function) if a key is involved in the computation of the function ( $h_k(x) = y$ ); otherwise it is called “**unkeyed**” (unkeyed hash function).

Hash functions have many computer applications including structured archiving facilitating search. On the security side we will study two main categories:

- **manipulation detection codes (MDC)** or message integrity codes (**MIC**): these are unkeyed functions allowing to provide an integrity service under certain conditions. The result of such a function is called **MDC-value** or simply **digest**.
- **message authentication codes (MAC)** which are keyed functions allowing to authenticate the source of the message and ensure its integrity without using additional (encryption) mechanisms.

**Some basic properties of hash functions:**

- **1) preimage resistance**: given a  $y \in Y$ , it is computationally impossible to find a preimage  $x \in X$  satisfying  $h(x) = y$ .
- **2) 2nd-preimage resistance**: given an  $x \in X$  and its image  $y \in Y$ , with  $h(x) = y$ , it is computationally impossible to find an  $x' \neq x$  such that  $h(x) = h(x')$ . Also called **weak collision resistance**.
- **3) collision resistance**: it is computationally impossible to find two distinct preimages  $x, x' \in X$  for which  $h(x) = h(x')$  (no restriction on the choice of values). Also called **strong collision resistance**.

A **one-way hash function (OWHF)** is an MDC satisfying 1) and 2). Also called: **weak one-way hash function**.

A **collision resistant hash function (CRHF)** is an MDC satisfying properties 2) and 3). (Note that 3)  $\Rightarrow$  2)). Also called: **strong one-way hash function**.

**OWF** **OWHF**: Note that an OWHF as a hash function imposes additional restrictions on the source and image domains as well as on 2nd-preimage resistance that are not necessarily satisfied by OWFs.

**Example**:  $f(x) = x^2 \bmod n$  with  $n = pq$  ( $p$  and  $q$  unknown) is not an OWHF because given  $x$ ,  $-x$  is a trivial collision.

### 💡 Quick Revision

**Hash function:** compression + easy computation

**MDC** (unkeyed) for integrity

**MAC** (keyed) for authentication

**Properties**

1. preimage resistance
2. 2nd-preimage resistance
3. collision resistance

**OWHF** = (1)+(2)

**CRHF** = (2)+(3).

## Message Authentication Codes (MACs)

A MAC is a family of functions  $h_k$  parameterized by a secret key  $k$ :

**Properties:**

1. **Compression:** arbitrary input  $\rightarrow$  fixed output
2. **Easy to compute:** with known  $k$ ,  $h_k(x)$  is fast
3. **Computation-resistance:** without  $k$ , impossible to compute valid pairs  $(x, h_k(x))$

**Implications:**

- Key non-recovery: impossible to recover  $k$  from pairs  $(x_i, h_k(x_i))$
- Preimage and collision resistance for anyone not possessing  $k$

**Usage:** Source authentication + message integrity without directly revealing secrets.

### i Original Text

A **Message Authentication Code (MAC)** is a family of functions  $h_k$  parameterized by a secret key  $k$  having the following properties:

- **1) compression:** as for generic hash functions but applied to  $h_k$ .
- **2) easy to compute:** from a function  $h_k$ , and a known key  $k$ , we can easily compute  $h_k(x)$ . The result is called a **MAC-value** or simply a **MAC**.
- **3) computational resistance** (computation-resistance): without knowledge of the symmetric key  $k$ , it is (computationally) impossible to compute pairs  $(x, h_k(x))$  from 0 or several known pairs  $(x_i, h_k(x_i))$  for any  $x \neq x_i$ .

Property 3) implies that the pairs  $(x_i, h_k(x_i))$  cannot be used to compute the key  $k$  (**key**

**non-recovery**). However the key non-recovery property does not imply computation-resistance because chosen/known-plaintext attacks could lead to forged pairs  $(x, h_k(x))$ . The impossibility of computing pairs  $(x, h_k(x))$  also translates to preimage and collision resistance (cf. previous slide) for any entity not possessing the key  $k$ .

### 💡 Quick Revision

**MAC** = hash with key  $k$

Without  $k$ : impossible to forge  $(x, h_k(x))$  or recover  $k$

Guarantees source authentication + integrity.

## Attacks on MDCs

### 2nd-Preimage Resistance Attack

**Problem:** Given  $h(x) = y$ , find  $x'$  such that  $h(x') = h(x)$ .

**Probabilistic analysis:**

For an  $m$ -bit digest ( $n = 2^m$  possible outputs), the probability of having at least one collision after  $k$  attempts is:

$$P(\text{collision}) \approx 1 - (1 - 1/n)^k \approx k/n$$

For  $P = 0.5$ :  $k = n/2 = 2^{m-1}$

**Conclusion:** For an  $m$ -bit digest, approximately  $2^{m-1}$  attempts are needed to find a 2nd-preimage with probability 0.5.

### i Original Text

**Problem:** given  $h(x) = y$ , find  $x'$  such that  $h(x') = h(x)$ .

**Practical example:** we have a text with an associated digest bearing a digital signature; we want to create a fake text bearing the same signature (without control over the original text). What are our chances from a probabilistic point of view?

Let a hash function  $h$  with  $n$  possible outputs and a given value  $h(x)$ . If  $h$  is applied to  $k$  random values, what must be the value of  $k$  so that the probability of having at least one  $y$  such that  $h(x) = h(y)$  is 0.5?

For the first value of  $y$ , the probability that  $h(x) = h(y)$  is  $1/n$ . Conversely, the probability that  $h(x) \neq h(y)$  is  $1 - 1/n$ . For  $k$  values, the probability of having no collision is:  $(1 - 1/n)^k$ , i.e.:

$$\left(1 - \frac{1}{n}\right)^k = 1 - \frac{k}{n} + \frac{1}{2!} \left(\frac{k}{n}\right)^2 - \frac{1}{3!} \left(\frac{k}{n}\right)^3 + \dots$$

which for very large  $n$  can be approximated by  $1 - k/n$ . Therefore, the complementary probability of having at least one collision is about  $k/n$ ; which gives us  $k = n/2$  for a probability of 0.5.

**Conclusion:** for an  $m$ -bit digest, the number of attempts needed to find a  $y$  such that  $h(x) = h(y)$  with a probability of 0.5 is  $2^{m-1}$ .

### 💡 Quick Revision

To break 2nd-preimage resistance with  $m$ -bit digest:  $2^{m-1}$  attempts (prob 0.5).

## Collision Resistance Attack: Birthday Paradox

**Problem:** Find two distinct values  $x, x'$  such that  $h(x) = h(x')$ .

**Birthday paradox:** In a group of 23 people, probability  $> 0.5$  of having two people with the same birthday.

**Mathematical result:**

For  $n$  possible outputs, the probability of collision after  $k$  computations:

$$P(\text{at least 1 collision}) = 1 - e^{-k(k-1)/(2n)}$$

For  $P \geq 0.5$ :  $k \approx 1.17\sqrt{n}$

**Cryptographic consequence:** For an  $m$ -bit digest ( $n = 2^m$  outputs), approximately  $2^{m/2}$  computations are needed to find a collision with probability  $> 0.5$ .

**Practical example:** Modification of a contract into 237 variations to find a fraudulent version having the same digest as the legitimate version.

### i Original Text

**Problem:** find two values  $x, x'$  distinct such that  $h(x) = h(x')$ .

**Practical example:** We have to have someone sign a text and we want to apply this signature to a falsified text (we control the original text). What are our chances of finding two original texts satisfying this criterion?

The **birthday paradox** is a classic probabilistic problem that shows that in a gathering of only 23 people, there is already a 50% chance of having two people with the same birthday.

Let  $y_1, y_2, \dots, y_n$  all the possible outputs of a hash function. How many  $h(x_i)$ :  $h(x_1), h(x_2), \dots, h(x_k)$  must we compute to have a probability of collision equal to or greater than 0.5?

The first choice for  $h(x_1)$  is arbitrary (prob = 1), the second  $h(x_2) \neq h(x_1)$  has a probability of  $1 - 1/n$ , the third of  $1 - 2/n$ , etc. This gives us a probability of having no collisions equal to:

$$P_{\text{no collision}} = \prod_{i=1}^{k-1} \left(1 - \frac{i}{n}\right)$$

We easily prove (series expansion of  $e^{-x}$ ) that for  $0 \leq x \leq 1$ :  $1 - x \leq e^{-x}$  and therefore:

$$P_{\text{no coll}} \leq \prod_{i=1}^{k-1} \left(1 - \frac{i}{n}\right) \leq \prod_{i=1}^{k-1} e^{-i/n} = e^{-k(k-1)/(2n)}$$

The probability of having at least one collision is  $P_{\text{at least 1}} = 1 - P_{\text{no-coll}}$ . To know the value of  $k$  for which  $P_{\text{at least 1}}$  is greater than 0.5, it suffices to calculate:

$$\frac{1}{2} \leq 1 - e^{-k(k-1)/(2n)}$$

If  $k$  is large, we replace  $k(k-1)$  by  $k^2$  and we obtain after simple calculations:

$$k \geq \sqrt{2 \ln(2) \cdot n} \approx 1.17\sqrt{n}$$

Taking  $n = 365$  for the birthday, we get  $k = 22.3$ , which confirms the statement of the problem.

**Consequence for hash functions:** Let a hash function with  $2^m$  possible outputs. If  $h$  is applied to  $k = 2^{m/2}$  inputs we have a probability greater than 0.5 of obtaining  $h(x_i) = h(x_j)$ .

### Quick Revision

**Birthday paradox:** to break collision resistance with  $m$ -bit digest:  $2^{m/2}$  attempts (prob  $> 0.5$ ).

Example: 23 people suffice for identical birthdays.

## Computational Resistance: Recap

For a hash function with  $n$ -bit digest and MAC key of  $t$  bits:

Type	Property	Difficulty	Recommended Size
<b>OWHF</b>	Preimage	$2^n$	$n \geq 128$ bits
	2nd-preimage	$2^{n-1}$	
<b>CRHF</b>	Collision	$2^{n/2}$	$n \geq 256$ bits
<b>MAC</b>	Key recovery	$2^t$	$t \geq 256$ bits
	Computation	$\min(2^t, 2^n)$	$n \geq 128$ bits

### Practical implications:

- For integrity only (OWHF): 128 bits sufficient
- For collision resistance (CRHF): minimum 256 bits
- MACs: 256-bit key, 128-bit digest minimum

#### Original Text

$n$ : size of the MDC-value or MAC-value resulting from the application of the hash function  
 $t$ : size of the MAC key

Hash Fct. Type	Characteristic	Computational Difficulty	Attack Goal	Recommended digest/key size
<b>OWHF</b>	preimage resistance	$2^n$	find a preimage	$n \geq 128$ bits
	2nd-preimage resistance	$2^{n-1}$	find $x'$ with $h(x') = h(x)$	
<b>CRHF</b>	collision resistance	$2^{n/2}$	find a collision	$n \geq 256$ bits
<b>MAC</b>	key non-recovery	$2^t$	find the key	$n \geq 128$
	computation resistance	$\min(2^t, 2^n)$	produce a $(x, h_k(x))$	$t \geq 256$

#### Quick Revision

**Efforts:** preimage  $2^n$ , 2nd-preimage  $2^{n-1}$ , collision  $2^{n/2}$ .

**Sizes:** OWHF 128 bits, CRHF 256 bits, MAC key 256 bits.



## MDCs Based on Encryption Systems

**Principle:** Use a symmetric encryption algorithm (DES, AES) to construct an MDC.

**Challenges to solve:**

- Break the reversibility of symmetric algorithms
- Increase the nominal width (DES = 64 bits insufficient for CRHF)

**Operation:**

- Sequential processing of blocks
- Chaining operations with XOR
- Combination of  $n$  boxes for digests of size  $n \times$  nominal width

**Classical models:**

1. **Matyas-Meyer-Oseas:**  $H_i = E_{m_i}(H_{i-1}) \oplus H_{i-1}$
2. **Davies-Meyer:**  $H_i = E_{m_i}(H_{i-1}) \oplus m_i$
3. **Miyaguchi-Preneel:**  $H_i = E_{m_i}(H_{i-1}) \oplus H_{i-1} \oplus m_i$

**Practical examples:**

- **MDC-2:** uses 2 DES boxes  $\rightarrow$  128-bit digest
- **MDC-4:** uses 4 DES boxes  $\rightarrow$  128-bit digest

**Limitation:** Security strongly dependent on the underlying algorithm.

### Original Text

**Idea:** use a known symmetric encryption system to construct an MDC.

**Problems to solve:**

- we must “break” the reversibility of symmetric algorithms to make them OWHF or CRHF.
- The “nominal width” of some encryption systems (eg. DES) is 64 bits, which is not sufficient to build CRHF.

**Operating principle:**

- the text blocks are sequentially processed by the encryption “box”.
- compression is based on chaining operations with the blocks resulting from previous iterations and logical functions (fundamentally XOR). This also makes the process irreversible.
- If necessary,  $n$  encryption boxes will be combined to obtain digest lengths  $n$  times greater than the nominal width of the boxes used.

**Attention:** the security of these algorithms is strongly dependent on the properties of the underlying encryption boxes.

**Examples:**

- The models of **Matyas-Meyer-Oseas**, **Davies-Meyer** and **Miyaguchi-Preneel**.
- **MDC-2** and **MDC-4** using respectively 2 and 4 DES boxes. Digest = 128 bits.

#### 💡 Quick Revision

MDCs from symmetric crypto: break reversibility + chaining XOR.

Models: Matyas-Meyer-Oseas, Davies-Meyer, Miyaguchi-Preneel.

MDC-2/4 with DES → 128 bits.

## Customized MDCs

Functions specifically designed for digest generation, optimized for speed and security.

**Construction elements:**

- Padding + adding the message length
- Predefined constants to increase dispersion
- Successive rounds with logical operations and rotations
- Chaining of outputs between rounds
- Every bit of the digest depends on every input bit

**Main algorithms:**

Algorithm	Year	Digest	Status
<b>MD5</b>	1992	128 bits	Broken
<b>SHA-0</b>	1993	160 bits	Collisions in $2^{39}$
<b>SHA-1</b>	1995	160 bits	Collisions in $2^{63}$
<b>SHA-2</b>	-	224-512 bits	Currently secure
<b>SHA-3</b> (Keccak)	2012	224-512 bits	Current standard

**Attack evolution:**

- 2004: Full collisions on MD5 (X. Wang)
- 2005: SHA-1 theoretically broken ( $2^{63}$  operations)
- 2008: Creation of fraudulent CA certificates via MD5
- 2012: SHA-3 adopted as new standard

## Original Text

These are functions designed exclusively to generate integrity codes (digests) with a main concern for speed and security.

Their operation is based on the following elements:

- initialization operations (**padding** + adding the length).
- a set of **predefined constants** chosen specifically to increase dispersion.
- a set of “steps” (**rounds**) that will sequentially apply to all the original data blocks. These rounds will perform a combination of logical operations and rotations on the data and constants.
- **chaining** operations involving the outputs of previous rounds.

In these functions, every bit of the digest is a function of every bit of the inputs.

The most famous are:

- **MD5**: R. Rivest, 1992; RFC 1321. Digest = 128 bits. **Broken!**
- **SHA-0**: NIST, 1993. Digest = 160 bits. Collisions in  $2^{39}$  operations instead of  $2^{80}$
- **SHA-1**: NIST, 1995. Digest = 160 bits. Revision of SHA-0 with additional bit rotation. Collisions in  $2^{63}$  operations (instead of  $2^{80}$ ).
- **SHA-2**: NIST (FIPS 190-3). Includes: SHA-224, SHA-256, SHA-384 and SHA-512. Digest sizes range from 224 to 512 bits.
- **SHA-3**: Keccak Algorithm (digest size variable from 224 to 512 bits)

### Latest Developments:

- X.Wang et al. culminated in 2004 a long work aiming to find collisions in the MD5 algorithm. They publish two pairs of collisions for 1024-bit messages.
- In 2005, X.Wang et al. prove at the CRYPTO'05 conference that the number of operations needed to find collisions on SHA-1 (current standard for secure hash functions) is only  $2^{63}$ .
- These attacks target the search for arbitrary collisions but during CRYPTO'06 researchers from the University of Graz in Austria propose a method to partially control the content of collisions.
- In December 2008 it is shown that controlled collisions on MD5 can be generated and thus create an illicit Certification Authority allowing to forge certificates accepted by any browser.
- These results rely on **analytical** approaches (as opposed to brute force!)
- The selection process for SHA-1's successor is similar to the one that designated AES as a block encryption standard. NIST decided (October 2012) that **Keccak** would be the base algorithm for **SHA-3**.

### Quick Revision

#### Customized MDCs

- MD5 (broken)
- SHA-0 (broken)
- SHA-1 (weak)
- SHA-2 (secure)
- SHA-3/Keccak (current standard).

Construction: padding + constants + rounds + chaining.

## MACs Based on Encryption Systems

**CBC-MAC:** Uses a block cipher algorithm in CBC mode.

### Operation:

- CBC mode with  $IV = 0$
- Elimination of intermediate ciphertexts
- Only the last encrypted block is kept as MAC

### With DES:

- Key length: 56 bits (112 in optional Triple-DES)
- MAC length: 64 bits

### Advantages:

- Reuse of existing encryption infrastructure
- Acceptable performance

### Limitations:

- Security limited by block size (64 bits for DES)
- Vulnerable if used incorrectly (ex: without variable IV)

### Original Text

**CBC-MAC algorithm based on DES-CBC with  $IV = 0$  and elimination of intermediate ciphertexts**

- key length = 56 bits (112 in case of using the optional part)
- MAC-value length = 64 bits

The diagram shows the sequential processing of message blocks  $M_1, M_2, M_3$  with the encryption algorithm  $E$  and the key  $k$ . The intermediate ciphertexts  $C_1, C_2$  are eliminated. Only the last block  $C_3$  constitutes the MAC.

#### Quick Revision

**CBC-MAC:** CBC mode + IV=0, only last block kept. DES: key 56/112 bits, MAC 64 bits.

## Nested MACs and HMACs

**Nested MAC (NMAC):** Composition of two MAC families  $G$  and  $H$ :

$$\text{NMAC}_{k,l}(x) = g_k(h_l(x))$$

**Security:** Depends on two criteria:

- $G$  resistant to collisions
- $H$  resistant to specific MAC attacks

**HMAC (FIPS 198 standard, 2002):** Nested MAC using unkeyed MDCs (SHA-1, SHA-256).

**Construction:**

- Constants:  $\text{ipad} = 0x363636 \dots 36$  and  $\text{opad} = 0x5C5C5C \dots 5C$  (512 bits)
- Key  $k$  of 512 bits

$$\text{HMAC-256}_k(x) = \text{SHA-256}((k \oplus \text{opad}) \parallel \text{SHA-256}((k \oplus \text{ipad}) \parallel x))$$

**Advantages:**

- Most widely used MACs in practice
- Attacks on SHA more difficult with secret key
- Excellent performance
- Standardized and widely supported

#### Original Text

A **Nested MAC** or **NMAC** is a composition of 2 families of MAC functions  $G$  and  $H$  parameterized by keys  $k$  and  $l$  such that:

$$G \circ H = \{g \circ h \text{ with } g \in G \text{ and } h \in H\} \text{ with } g \circ h_{(k,l)}(x) = g_k(h_l(x))$$

The security of an NMAC depends on two criteria:

- The family of functions  $G$  is collision resistant.
- The family of functions  $H$  is resistant to specific attacks for MACs, i.e.: It is impossible to find a pair  $(x, y)$  and a fixed but unknown key  $m$ , such that:  $\text{MAC}_m(x) = y$ .

An **HMAC** (FIPS 198, 2002) is a Nested MAC using at its base dedicated unkeyed MDCs like SHA-1 or SHA-256.

An HMAC uses two 512-bit constants called **ipad** and **opad** such that:

- $\text{opad} := 363636 \dots 36$
- $\text{ipad} := 5C5C5C \dots 5C$

and a key  $k$  of 512 bits.

The operating scheme of HMAC-256 (based on SHA-256) is as follows:

$$\text{HMAC-256}_k(x) := \text{SHA-256}((k \oplus \text{opad}) \parallel \text{SHA-256}((k \oplus \text{ipad}) \parallel x))$$

**HMACs** are the most used MACs. The attacks mentioned on the functions of the SHA family are more difficult to carry out on an HMAC because of the key  $k$ .

### Quick Revision

**HMAC**: double hash with derived keys (**ipad/opad**).  $\text{HMAC}_k(x) = H((k \oplus \text{opad}) \parallel H((k \oplus \text{ipad}) \parallel x))$ . Standard, secure, performant.

## Hash Functions Applications

### Data Integrity

Three main approaches:

#### 1. MAC only:

- $A \rightarrow B : X, \text{MAC}_k(X)$
- Authentication + integrity guaranteed
- Requires shared key

#### 2. MDC + Encryption:

- $A \rightarrow B : E_k(X, \text{MDC}(X))$

- Confidentiality + integrity
- Shared symmetric key

### 3. MDC + Authentic channel:

- $A \rightarrow B : X$  (normal channel)
- $A \rightarrow B : \text{MDC}(X)$  (authentic channel)
- Channel separation

**Limitations:** These simple protocols offer no protection against replay attacks.

**Solution:** add timestamps or sequence numbers.

#### Original Text

##### MAC Only:

$$A \rightarrow B : X, \text{MAC}_k(X)$$

If  $B$  computes on its side  $\text{MAC}_k(X)$  and obtains the same value the message comes from  $A$ .

**MDC + symmetric encryption** (key  $k$  known to  $A$  and  $B$ )

$$A \rightarrow B : X, E_k(\text{MDC}(X))$$

$B$  computes  $\text{MDC}(X)$  and then  $E_k(\text{MDC}(X))$ . If equal message comes from  $A$ .

**As 2) with confidentiality of  $X$  added:**

$$A \rightarrow B : E_k(X, \text{MDC}(X))$$

##### MDC + digital signature:

$$A \rightarrow B : X, \text{Sig}_{\text{priv-A}}(\text{MDC}(X))$$

$B$  computes  $\text{MDC}(X)$  and verifies  $\text{Sig}_{\text{priv-A}}(\text{MDC}(X))$  with an authentic copy of **pub-A**. If equality  $A$  is the origin of the message. This solution additionally offers **origin non-repudiation**.

These simple protocols offer no support for uniqueness nor for the timeliness of received messages and are exposed to **replay attacks!** They require mechanisms taking into account time or the transaction context (cf. entity authentication).

#### Quick Revision

**Integrity:** MAC only, MDC+crypto, MDC+signature.

Vulnerable to replay without timestamps/nonces.

## Blockchains and Proof of Work

**Bitcoin and blockchains:** Use of hash functions to chain transaction blocks.

### Characteristics:

- Public and visible transactions
- Blocks chained via cryptographic hash functions
- Mining = solving a cryptographic puzzle (proof of work)

### Proof of Work:

- Find a nonce such that  $\text{hash}(\text{block} \parallel \text{nonce}) < \text{target}$
- Computationally expensive puzzle, rapid validation
- First miner to solve receives bitcoin reward

### Security:

- Blockchain = public, decentralized, immutable ledger
- Falsification would require effort  $>$  all honest miners
- Protection based on CRHF properties

### Bitcoin statistics (October 2025):

- Difficulty: 150.84 T
- Target:  $\approx 2^{177}$  (pseudo-collision on 79 bits)
- Hashrate:  $\sim 1.1$  ZH/sec ( $1.1 \times 10^{21}$  hash/sec)
- Average block generation time: 10 minutes

#### Original Text

Bitcoin transactions are published and visible by all participants. They are encapsulated in blocks chained using cryptographic hash functions.

**Mining** consists of iteratively adding new blocks containing current transactions.

Generating a valid block requires solving a **cryptographic puzzle** (proof of work) very costly in computation time (finding pseudo-collisions in cryptographic hash functions). Validation remains very efficient.

The first miner able to generate a valid block will receive a monetary reward (in bitcoins).

The mining process is open to all miners but only the first is rewarded.

The resulting chain of blocks (**blockchain**) then becomes a public ledger, decentralized and **immutable** protecting all past transactions. Falsification/modification of data protected by the blockchain would require computational effort greater than that performed by all honest miners.

**Bitcoin Statistics 13/10/2025:**



- **Difficulty:** 150.84 T
- **Target:**  $2^{224}/\text{Difficulty} \approx 2^{177}$ . The valid digest to generate a block must be less than  $2^{177}$ , which means a pseudo-collision on the 79 most significant bits. The variation on the inputs depends on the **nonce**.
- **Hashrate:**  $\sim 1.1 \text{ ZH/sec}$  ( $1.1 \times 10^{21}$  hashes /sec)
- **Hash functions executed to obtain a block:**  $\sim 660 \times 10^{21}$
- **Average block generation time:** 10 min

### Quick Revision

**Blockchain:** chaining of blocks via hash.

**Proof of Work:** find nonce for hash  $<$  target.

Security = effort  $>$  all miners.

Bitcoin:  $\sim 10$  min/block,  $10^{21}$  hash/sec.

## Other Applications

### 1. Authentication:

- Data origin authentication (DOA)
- Transaction authentication (DOA + temporal parameters)

### 2. Virus checking:

- Creator publishes digest  $= h(\text{software})$  via secure channel
- Users verify integrity by recalculating the digest

### 3. Public key distribution:

- Publish  $h(\text{public key})$  instead of the complete key
- Simplified authenticity verification

### 4. Document timestamping:

- Timestamp applied to digest rather than complete document
- Reduction of data to sign

### 5. One-time password (S-Key):

- Hash chain:  $x_1 = h(x_0), x_2 = h(x_1), \dots, x_n = h(x_{n-1})$
- System stores  $x_n$ , user provides  $x_{n-1}$
- Verification:  $h(x_{n-1}) = x_n$
- After validation, system stores  $x_{n-1}$  for next time

## Original Text

### **Authentication:**

- data origin authentication (DOA)
- transaction authentication (= DOA + time-variant parameters)

### **Virus checking:**

- The creator of software creates a digest =  $h(x)$  with  $x$  being the original and distributes it via a secure channel (eg. CD-ROM).

### **Distribution of public keys:**

- Allows controlling the authenticity of a public key.

### **Timestamp on a document:**

- The document on which we want to perform the timestamp is first submitted to a hash function. The timestamp (with the signature of the corresponding entity) then applies only to the digest.

### **One-time password (S-Key) (identification mechanism):**

- From a secret seed  $x_0$ , we create a chain of hash-values:  $x_1 = h(x_0)$ ,  $x_2 = h(x_1)$ , ...  $x_n = h(x_{n-1})$ .
- The system stores  $x_n$  and the user enters  $x_{n-1}$ . If  $h(x_{n-1}) == x_n$  OK.
- The system then stores  $x_{n-1}$  and so on.

## Quick Revision

### **Applications**

- authentication
- virus checking
- public key distribution
- timestamping
- one-time passwords (hash chain)

## Randomized Hash Functions: UNIX Example

**Problem:** Deterministic hash functions always produce the same result for the same password.

### Risks:

- Detection of identical passwords
- Offline dictionary attacks (pre-computed codebooks)
- Rainbow tables

### UNIX solution: Salt

- Addition of a 12-bit pseudo-random element (salt) before hashing
- Different salt for each user
- 4096 possibilities ( $2^{12}$ ) for each password

### Advantages:

- Prevents detection of duplicates
- Pre-computed codebooks become ineffective
- Each password requires 4096 dictionary entries

### UNIX implementation:

- File `/etc/passwd` globally accessible
- Format: `username:hash(salt+password):uid:gid:...`
- Hash based on modified DES (25 iterations)
- Salt stored in clear (first 2 characters of hash)

### Example:

```
root:Jw87u9bebeb9i:0:1:Operator:/:/bin/csh
pp:1Qhw.oihEtHK6:359:355:PP:/net/spp_telecom/pp:/bin/cs
```

### Limitations:

- Effective protection against pre-computed dictionaries
- Online attacks limited by the system (number of attempts)
- Offline attacks possible if file compromised

#### Original Text

UNIX keeps its passwords in a globally accessible file (or possibly distributed by NIS). The stored information corresponds to the result produced by a hash function.

### Example (fictional):

```
root:Jw87u9bebeb9i:0:1:Operator:/:/bin/csh
pp:1Qhw.oihEtHK6:359:355:PP:/net/spp_telecom/pp:/bin/cs
```

### Problems:

- the hash function being deterministic, it produces the same result for identical passwords.
- one could create “books” (codebooks) containing the result of applying the hash function to given inputs (eg. a dictionary) and easily compare them (off-line) with the strings stored by UNIX (**brute force dictionary attack**).

### Solution:

- Add a (pseudo) random element of **12 bits** different for each password (called **salt**) before computing the hash function and during verification.
- This element allows adding a random factor of **4096 possibilities** for each password and prevents detection of duplicates.

The operating scheme uses DES with 25 iterations, the password as key, and the salt to modify the E-boxes. The final 64-bit result is converted to 11 ASCII characters.

User awareness (not visiting dubious sites) decreases the effectiveness of this technique in malware transmission.

Dictionary attacks are normally less effective **online** because operating systems limit the number of unsuccessful authentication attempts.

### Quick Revision

**UNIX salt:** 12 random bits added to password before hash.

4096 possible variations.

Prevents pre-computed codebooks and duplicate detection.