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## Symmetric Cryptography

### Stream Ciphers (Stream Encryption)

#### Introduction to Stream Ciphers

#### Definition and Principle

Stream ciphers are a family of encryption systems characterized by:

- **Unit block size:** each encrypted block = 1 bit
- **Two-phase architecture:**

1. **Keystream generation:** production of the key sequence
2. **Substitution:** operation on plaintext bits based on the keystream

**Classic example:** the *one-time pad*

- Generation: (pseudo-)random generator
- Substitution: XOR operation ( $\oplus$ ) with the keystream

## General Characteristics

### Advantages:

- **Speed:** encryption at register level, ideal for real-time *streaming* (video)
- **Lightweight:** work on systems with limited CPU resources
- **Low memory:** little or no buffering needed
- **Non-propagated errors:** retransmission of defective packets is sufficient (suitable for wireless transmissions - WiFi)

### Disadvantages:

- **Dependency on keystream quality:** randomness determines robustness
- **Dangerous reuse:** keystream reuse allows easy cryptanalysis

## Original text

### Stream Ciphers

- **Stream ciphers** constitute a **family of encryption systems** where the **size of the encrypted block is equal to 1 bit**.
- Stream ciphers are generally composed of **two phases**:
  - A **generation phase** of the sequence of elements forming the key (the **keystream**).
  - A **substitution phase** where the *plaintext* bits undergo a specific operation dependent on the keystream.
- An obvious example of a stream cipher is the **one-time pad** with:
  - A keystream generation phase performed by a **(pseudo-)random generator**.
  - A substitution phase consisting of performing a **xor** ( $\oplus$ ) with the keystream.

### Stream Ciphers: Characteristics

- **Speed:** Encryption is done directly at the register level. Ideal for applications requiring “*on the fly*” encryption like **video streaming**.
- **Ease:** Operations can be performed by systems with **limited CPU resources**.

- No (or little...) need for memory/buffering.
- **Limited or absent error propagation:** retransmission of faulty packets is normally sufficient (suitable for applications where packet loss is frequent like wireless transmissions (WiFi)).
- **Disadvantages:**
  - The **quality in terms of randomness** of the generated keystream determines the **system's robustness**.
  - **Keystream reuse** allows **easy cryptanalysis** (cf. the one-time pad).



Quick revision

**Stream Ciphers** = encryption bit by bit in 2 phases (keystream generation + substitution).

**Advantages:** fast, lightweight, no error propagation.

**Disadvantages:** keystream quality critical, reuse = vulnerability.

## Synchronous Stream Ciphers

### Operating Principle

In a **synchronous stream cipher**, the keystream depends **only on the key**, independent of the plaintext and ciphertext.

**Process equations:**

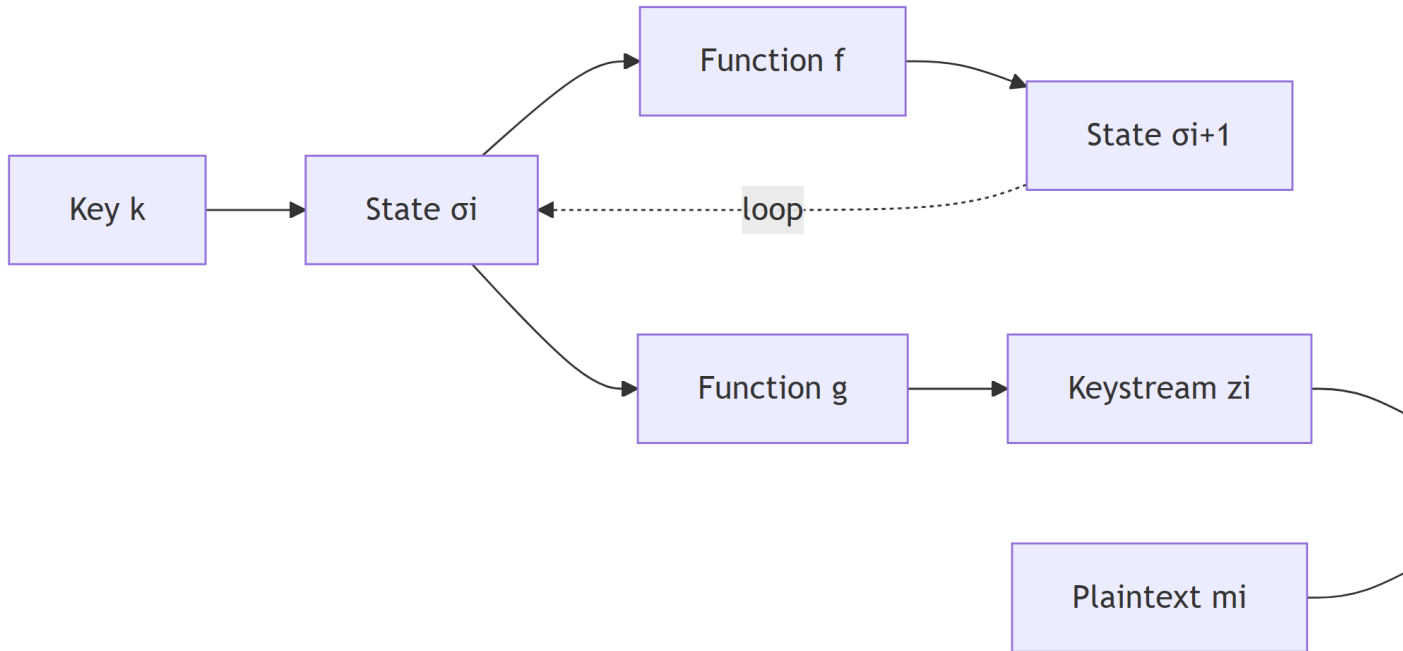
$$\sigma_{i+1} = f(\sigma_i, k)$$

$$z_i = g(\sigma_i, k)$$

$$c_i = h(z_i, m_i)$$

Where:

- $\sigma_i$ : state at time  $i$  (initial state  $\sigma_0$  may depend on  $k$ )
- $k$ : secret key
- $f$ : state transition function
- $g$ : keystream production function producing  $z_i$
- $h$ : output function producing ciphertext  $c_i$  from plaintext  $m_i$



### Characteristics

#### Synchronization requirement:

- Transmitter and receiver must share the same key  $k$  **AND** the same state  $\sigma_i$
- Loss of synchronization = need for external mechanisms (markers, redundancy analysis)

#### Properties:

- **No error propagation:** modification of ciphertext does not affect subsequent sequences
- **Attention:** deletion of a ciphertext = receiver desynchronization

#### Vulnerabilities to active attacks:

- **Detection:** insertion, elimination, replay of fragments
- **Bit modification:** adversary can modify bits and analyze impact on plaintext
- **Solution:** additional authentication mechanisms necessary

### Special case: Additive Stream Cipher

The most frequent case where:

- Functions  $f$  and  $g$  replaced by a random generator
- Function  $h$  = modulo 2 addition (XOR:  $\oplus$ )

**Formula:**  $c_i = z_i \oplus m_i$

 Original text

### Synchronous Stream Ciphers

- The **generated keystream depends only on the key** and not on the plaintext nor the ciphertext.
- The encryption process of a **synchronous stream cipher** is described by the following equations:

$$\sigma_{i+1} = f(\sigma_i, k)$$

$$z_i = g(\sigma_i, k)$$

$$c_i = h(z_i, m_i)$$

with  $\sigma_i$  the **initial state** which may depend on the key  $k$ ,  $f$  the **function determining the next state**,  $g$  the **function producing the keystream**  $z_i$  and  $h$  the **output function** producing the ciphertext  $c_i$  from the plaintext  $m_i$ .

### Synchronous Stream Ciphers: Characteristics

- **Require synchronization** of the transmitter and receiver: In addition to using the same key  $k$ , both must be in the **same state** for the process to work. If synchronization is lost, **external mechanisms** are needed to recover it (special markers, plaintext redundancy analysis, etc.)
- **No error propagation.** Modification of the ciphertext during transmission does not cause disturbances in subsequent ciphertext sequences (however, the **deletion** of a ciphertext would cause **desynchronization** of the receiver).
- **Active attacks:** Insertion, elimination or replay of parts of ciphertext are **detected** by the receiver. However, an adversary could **modify certain bits** of the ciphertext and analyze the impact on the corresponding plaintext. Additional **origin authentication mechanisms** are necessary to detect these attacks.
- **Most frequent case** of Synchronous Stream Ciphers: the **additive stream cipher** (cf. the one-time pad) where the functions  $f$  and  $g$  generating the keystream are replaced by a **random generator** and the function  $h$  is a **modulo 2 addition (xor)**.

 Quick revision

**Synchronous:** keystream =  $f(\text{key only})$ . Equations:  $\sigma_{i+1} = f(\sigma_i, k)$ ,  $z_i = g(\sigma_i, k)$ ,  $c_i = h(z_i, m_i)$ .

**Requires synchronization** transmitter/receiver. No error propagation but vulnerable to bit modifications.

**Frequent case:** additive cipher with XOR.

## Asynchronous Stream Ciphers

### Operating Principle

Also called **self-synchronizing ciphers**.

The keystream depends on the key **AND** a fixed number of previous ciphertexts.

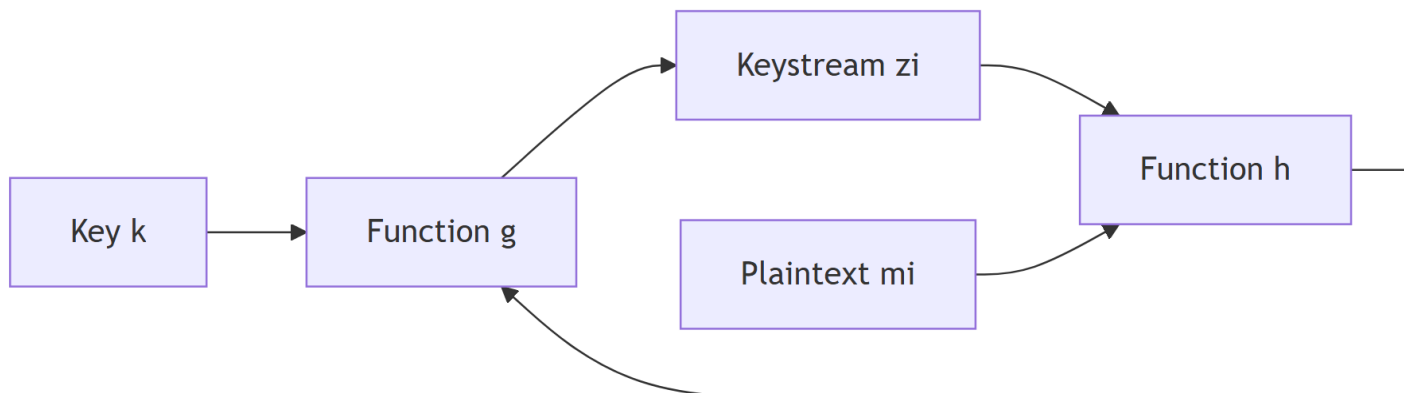
**Process equations:**

$$\sigma_i = (c_{i-t}, c_{i-t+1}, \dots, c_{i-1})$$

$$z_i = g(\sigma_i, k)$$

$$c_i = h(z_i, m_i)$$

Where  $\sigma_i$  represents a buffer of the last  $t$  ciphertexts.



### Characteristics

**Self-synchronization:**

- In case of insertion/elimination of ciphertexts, the receiver **automatically re-synchronizes**
- Mechanism: memorization (buffer) of the last ciphertexts

**Limited error propagation:**

- Error propagates only over the **buffer size** ( $t$  bits)
- After buffer exhaustion, correct decryption resumes

#### Security against active attacks:

- **Better detection:** modifications detected thanks to error propagation
- **Attention:** self-synchronization allows receiver to continue even after insertions/deletions
- **Solution:** verification of integrity and authenticity of entire stream necessary

#### Diffusion of plaintext statistics:

- Each plaintext bit influences **all subsequent ciphertexts**
- **Result:** better dispersion of statistics vs. synchronous case
- **Application:** use for low entropy or highly redundant plaintexts

#### Original text

##### Asynchronous Stream Ciphers

- Also called **self-synchronizing ciphers**.
- The **generated keystream depends on the key as well as a fixed number of previous ciphertexts**.
- The encryption process of an **asynchronous stream cipher** is described by the following equations:

$$\sigma_i = (c_{i-t}, c_{i-t+1}, \dots, c_{i-1})$$

$$z_i = g(\sigma_i, k)$$

$$c_i = h(z_i, m_i)$$

with  $\sigma_i$ ,  $g$  and  $h$  as for the synchronous case.

##### Asynchronous Stream Ciphers: Characteristics

- **Self-synchronization:** In case of elimination or insertion of ciphertexts along the way, the receiver is capable of **re-synchronizing with the transmitter** thanks to the **memorization (buffer)** of a number of previous ciphertexts.
- **Limited error propagation:** Error propagation extends only to the **number of ciphertext bits memorized** (buffer size). Afterwards, decryption proceeds correctly again.
- **Active attacks:** Modification of ciphertext fragments will be **more easily detected** than in the synchronous case because of error propagation. However, since the receiver is capable of self-synchronizing with the transmitter, even if ciphertexts are eliminated or inserted along the way, it is necessary to **verify the integrity and authenticity of the entire stream**.

- **Diffusion of plaintext statistics:** The fact that **each plaintext bit will influence all subsequent ciphertexts** results in a **greater dispersion of statistics** compared to the synchronous case...
- ... It is therefore advisable to use **asynchronous stream ciphers when the entropy of plaintexts is limited** and could allow targeted attacks on highly redundant plaintexts.



#### Quick revision

**Asynchronous** (self-synchronizing): keystream =  $f(\text{key} + \text{last ciphertexts})$ . State  $\sigma_i$  = buffer of  $t$  previous ciphertexts.

**Automatic self-synchronization.** Limited error propagation to buffer.

**Better diffusion** of statistics → ideal for redundant/low entropy plaintexts.

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## Keystream Generators: LSFR

### Context and Necessity

**Problem:** generate a keystream of length  $m$  from a secret key of length  $l$  with  $l \ll m$ .

**Solution:** Linear Feedback Shift Register (**LSFR** or **LFSR**)

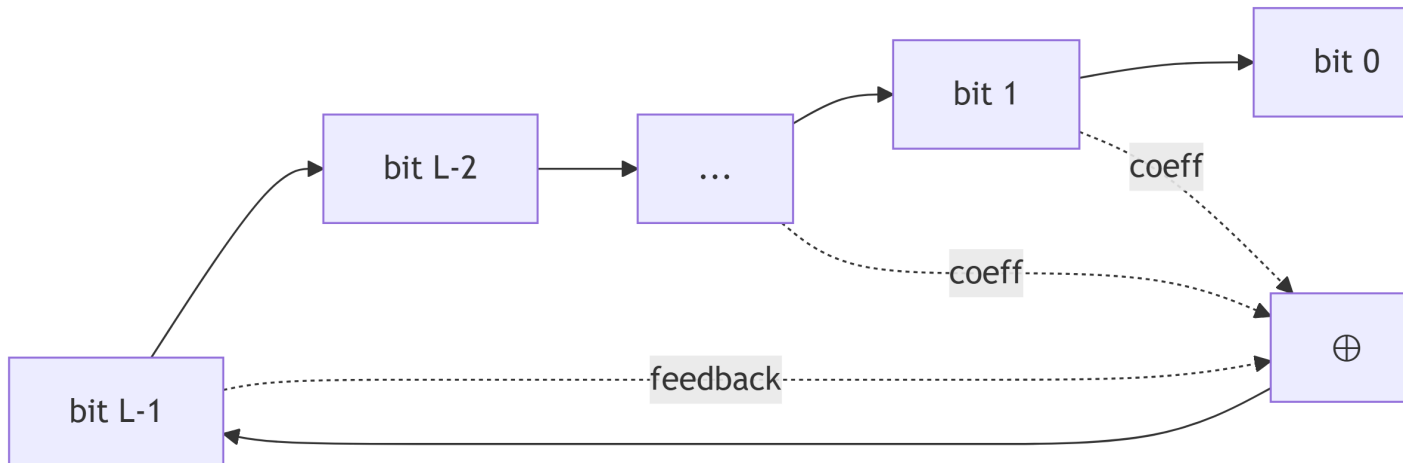
### LSFR Characteristics

#### Advantages:

- **Optimal hardware implementation:** very efficient circuits
- **Long periods:** sequences of great length
- **Good random quality:** notable randomness
- **Mathematical basis:** algebraic properties of linear combinations

**Generic structure:** LSFR of length  $L$





### Important Remarks on LSFR

#### History and Usage:

- Very widespread construction in cryptography and coding theory
- Many military stream ciphers based on LSFR

#### Security Limits:

- **Insufficient security level** compared to modern block ciphers
- **Vulnerability:** the Berlekamp-Massey algorithm allows to:
  - Determine the **linear complexity** of an LSFR
  - Calculate an arbitrary number of generated sequences

**Metric:** Linear complexity (*linear complexity*)

#### Improvement Solution:

Replace the linear combination with a **non-linear function**  $f$

→ **Non Linear Feedback Shift Registers** (NLFSR)

**i** Original text

#### Stream Ciphers: Keystream Generators

- When it is necessary to **generate a keystream of length**  $m$  from a **secret key of length**  $l$  with  $l \ll m$ , we call upon **keystream generators**.
- The most common of these generators is the **Linear Feedback Shift Register (LSFR)**.

- An LSFR has the following characteristics:
  - **Adapts very well to hardware implementations.**
  - Produces sequences of **long periods** and with **notable random quality** (quite strong randomness)
  - Based on the **algebraic properties of linear combinations.**

#### LSFRs: Some Remarks

- LSFRs are **very widespread constructions** in cryptography and coding theory.
- A **large number of stream ciphers** based on LSFRs (especially in the **military sphere**) were developed in the past.
- Unfortunately, the **security level offered by these systems is deemed insufficient** nowadays (compared to that of block ciphers...)
- The **metric** allowing analysis of an LSFR is its **linear complexity**. The **Berlekamp-Massey algorithm** allows determining the linear complexity of an LSFR and thus calculating an arbitrarily large number of sequences generated by an LSFR.
- A solution to **increase complexity** is to substitute the linear combination of ciphertext bits with a **non-linear function  $f$** . These are the **Non Linear Feedback Shift Registers**.

#### Quick revision

**LSFR:** long keystream generator ( $m$ ) from short key ( $l$ ). Base = linear combinations.

**Advantages:** efficient hardware, long periods.

**Problem:** insufficient security, vulnerable to Berlekamp-Massey (linear complexity calculation).

**Solution:** NLFSR (non-linear function).

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## RC4: Software Stream Cipher

### General Presentation

**RC4<sup>TM</sup>** (*Rivest Cipher 4*) developed in 1987 by Ron Rivest for RSA Security.

### Main characteristics:

- **Variable key:** flexible length
- **Extremely fast:** 10× faster than DES

- **Synchronous mode:** keystream independent of plaintext/ciphertext

#### History:

- 1987-1994: patented, details confidential (NDA contract required)
- 1994: unofficial publication in a newsgroup
- Since then: intensive analysis by cryptographic community

#### Architecture

##### Key components:

- **S-box:**  $8 \times 8$  substitution box (256 entries)
  - Content: permutation of numbers 0 to 255
  - Depends on the main key of variable length:  $0 < \text{len}(k) \leq 255$
- **Combinations:** linear and non-linear
- **Final encryption:** XOR between keystream and plaintext

#### Applications and Security

##### Commercial uses (numerous):

- Lotus Notes
- Oracle SQL
- Microsoft Windows
- SSL/TLS
- And many more...

##### Analyses and Vulnerabilities:

- Exhaustive work on key scheduling and PRGA
- **Major flaw:** implementation in WEP (WiFi Wired Equivalent Privacy)
  - WEP protocol completely compromised
  - Problem: faulty usage mode, not the RC4 algorithm itself

## Operation

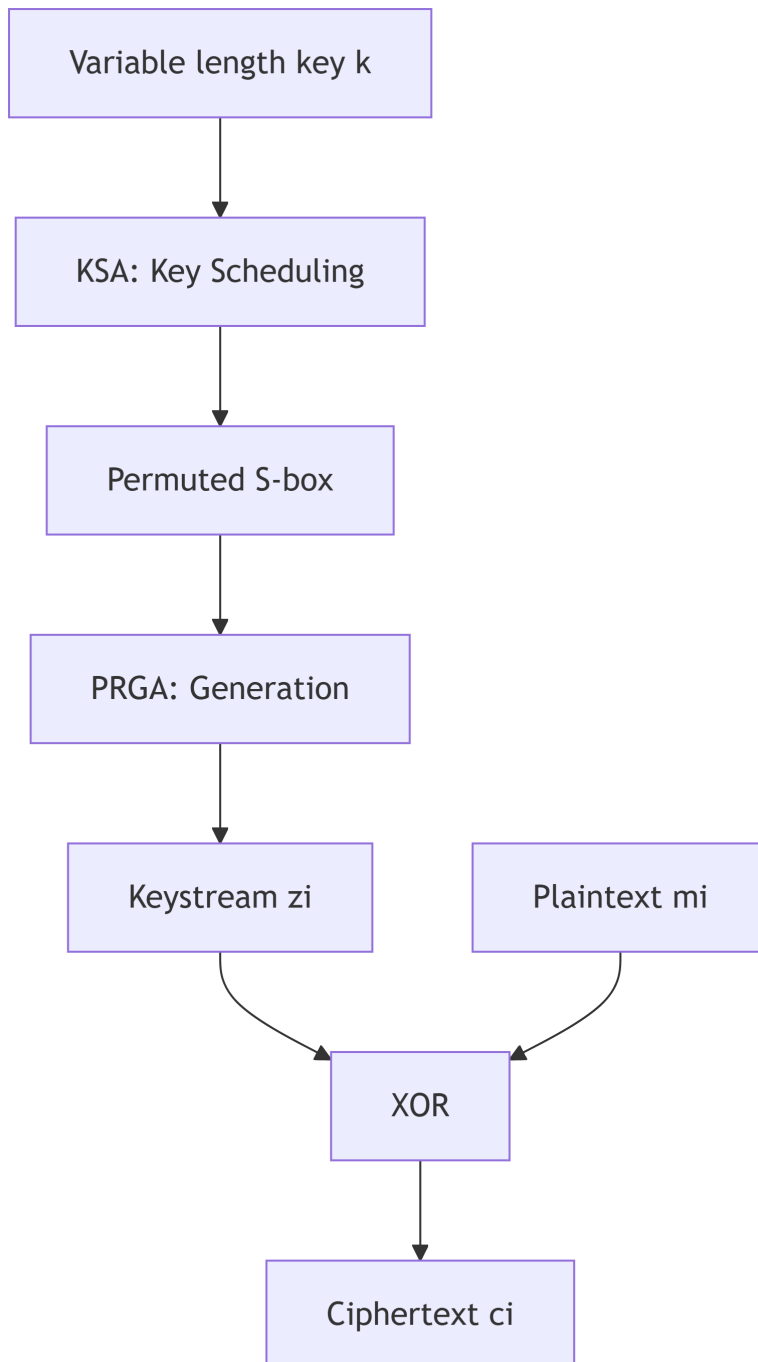
RC4 decomposes into **two steps**:

1. **Key Scheduling Algorithm (KSA)**

- Responsible for initial permutation of the S-box
- Function of the variable length key  $len(k) = l$

2. **Pseudo Random Generator Algorithm (PRGA)**

- Generates keystream of arbitrary size
- Relies on S-box permuted by KSA



## Original text

### Software Cipher Streams: RC4

- The major **disadvantage of stream ciphers based on registers** is that they are **very slow in programmed version** on a generic machine. **RC4™** is a **variable key stream cipher** developed in **1987 by Ron Rivest** for RSA security. It is **very fast (10 times faster than DES !)**
- For 7 years, this algorithm was **patented** and its internal operational details were disclosed only after **signing a confidentiality contract**. Since its **(unofficial) publication in a newsgroup in 1994**, it has been widely discussed and analyzed by the entire cryptographic community.
- The algorithm works in **synchronous mode** (the keystream is independent of the ciphertext and plaintext).
- It is composed of **linear and non-linear combinations**. The key element is an **8×8 substitution box (S-box)** whose entries are a **permutation of the numbers 0 to 255**. The permutation is a **function of the main key** of variable size with  $0 < \text{len}(k) \leq 255$ . The **final encryption is obtained by a xor** between the keystream and the plaintext.
- RC4 is used in a **large number of commercial applications**: Lotus Notes, Oracle SQL, MS Windows, SSL, etc. It is the subject of a **large number of analytical and exhaustive works** that have managed to **compromise the security** of the key scheduling and the PRGA.
- In particular the application of RC4 to the **Wired Equivalent Privacy (WiFi WEP) protocol** has been “**broken**” due to a **flaw in the protocol’s usage mode**.

### RC4: Operation

- The algorithm consists of **two steps**:
  - The **Key Scheduling Algorithm (KSA)**: Responsible for the **initial permutation** that will fill the S-box depending on the variable length key  $\text{len}(k) = l$ .
  - The **Pseudo Random Generator Algorithm (PRGA)**: Generates the **keystream of arbitrary size** relying on the S-box.

## Quick revision

**RC4**: software stream cipher, variable key, 10× faster than DES.

**Architecture**: 8×8 S-box (permutation 0-255) + XOR.

**2 steps**: KSA (S-box permutation) + PRGA (keystream generation). Synchronous mode.

**Vulnerability**: WEP broken (usage flaw). Used in SSL, Windows, Oracle...

## Block Ciphers (Block Encryption)

### 1. Introduction to Block Ciphers

#### Definition and Principle

A **block cipher** is a cryptographic function that:

- **Transforms fixed-size blocks:** maps a block of  $n$  bits to another block of the same size
- **Is parameterized by a key:** the key  $K$  of  $k$  bits defines the transformation
- **Must be bijective:** to allow unique decryption
- **Each key = different bijection:** guarantees variability

**Nominal size:** input block size on which encryption is applied

#### Quality Criteria

##### 1. Key size/Entropy

- Keys ideally **equiprobable** with entropy =  $k$  bits
- Strong entropy protects against **brute-force attacks**
- **Minimum required:** 128 bits for modern block ciphers

##### 2. Performance

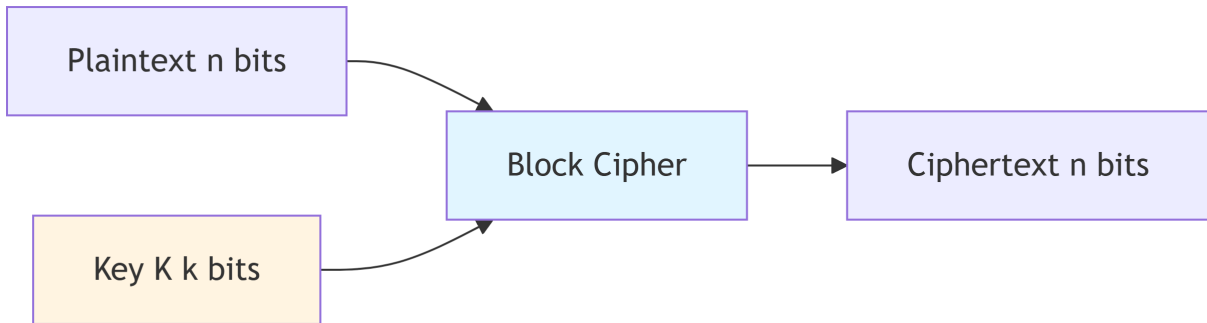
- Execution speed
- Software/hardware efficiency

##### 3. Block size

- Too small block = vulnerability to **plaintext/ciphertext dictionaries**
- **Modern standard:** blocks 128 bits

##### 4. Cryptographic resistance

- Resistance to known techniques:
  - Linear cryptanalysis
  - Differential cryptanalysis
  - Meet in the middle
- **Cryptanalysis effort** equivalent to brute force



### **i** Original text

#### **Block Ciphers**

- **Symmetric block ciphers** constitute the **cornerstone of cryptography**. Their main functionality is **confidentiality** but they are also the basis for **authentication**, **hashing functions**, **random generation**, etc.
- **Definition:** A block cipher is a **function** that maps a **block of  $n$  bits** to another block of **the same size**. The function is **parameterized by a key  $K$  of  $k$  bits**. To allow **unique decryption**, the function must be **bijective**. **Each key defines a different bijection**. The **input block size** on which encryption is applied is also called **nominal algorithm size**.
- **Criteria to evaluate the quality** of a block cipher:
  - **Key size/Entropy:** Ideally, keys are **equiprobable** and the key space has an **entropy equal to  $k$** . A **strong key entropy** protects against **brute-force attacks** from chosen/known plaintexts. Modern block ciphers must have **keys of at least 128 bits**.
  - **Performance**
  - **Block size:** A too small block would allow attacks where **plaintext/ciphertext “dictionaries”** could be built. Nowadays, **blocks of size 128 bits** are becoming common.
  - **Cryptographic resistance:** The block cipher must show **resistance** to known cryptanalysis techniques: **linear or differential cryptanalysis**, **meet in the middle**, etc. The **inherent effort** of these attacks (complexity, storage, parallelization, etc.) must be **equivalent to that of a brute force attack**.

### **💡** Quick revision

**Block cipher:** bijective function transforming blocks of  $n$  bits with key  $K$  of  $k$  bits.  
**Criteria:** key entropy 128 bits, block size 128 bits, cryptanalysis resistance = brute



force effort. **Usage:** confidentiality, authentication, hashing, random generation.

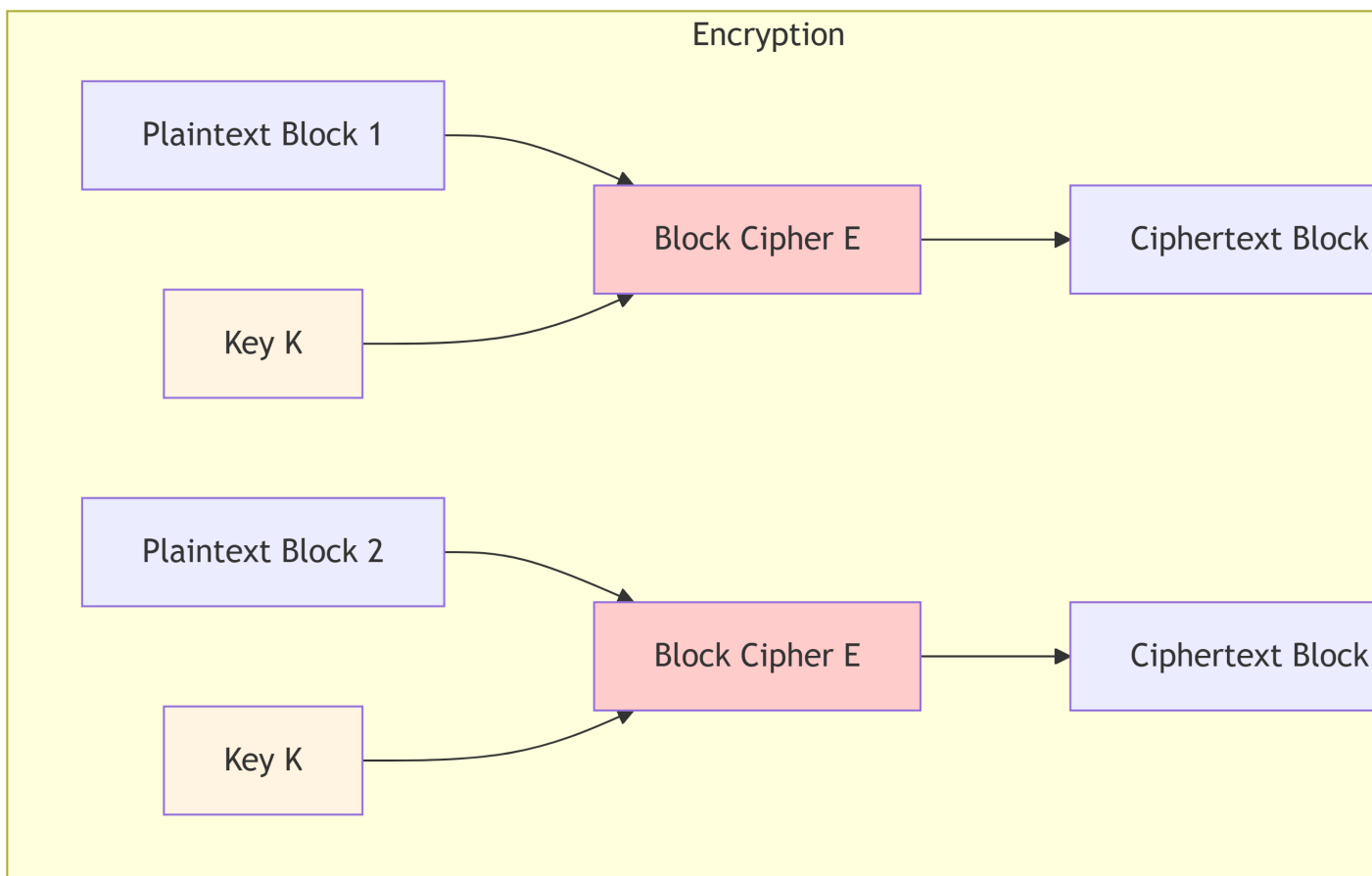
## 2. Block Cipher Modes of Operation

### 2.1 Electronic Codebook (ECB)

**Principle:** each plaintext block is encrypted **independently** with the same key.

$$c_i = E_K(m_i)$$

$$m_i = D_K(c_i)$$



**Characteristics:**

- **Identical plaintexts** → identical ciphertexts (predictable)
- **No error propagation:** error on  $c_j$  affects only  $m_j$
- **Visible patterns:** plaintext structure transparent in ciphertext
- **Parallelizable:** each block processed independently

**Major vulnerability:** Should NOT be used for redundant data

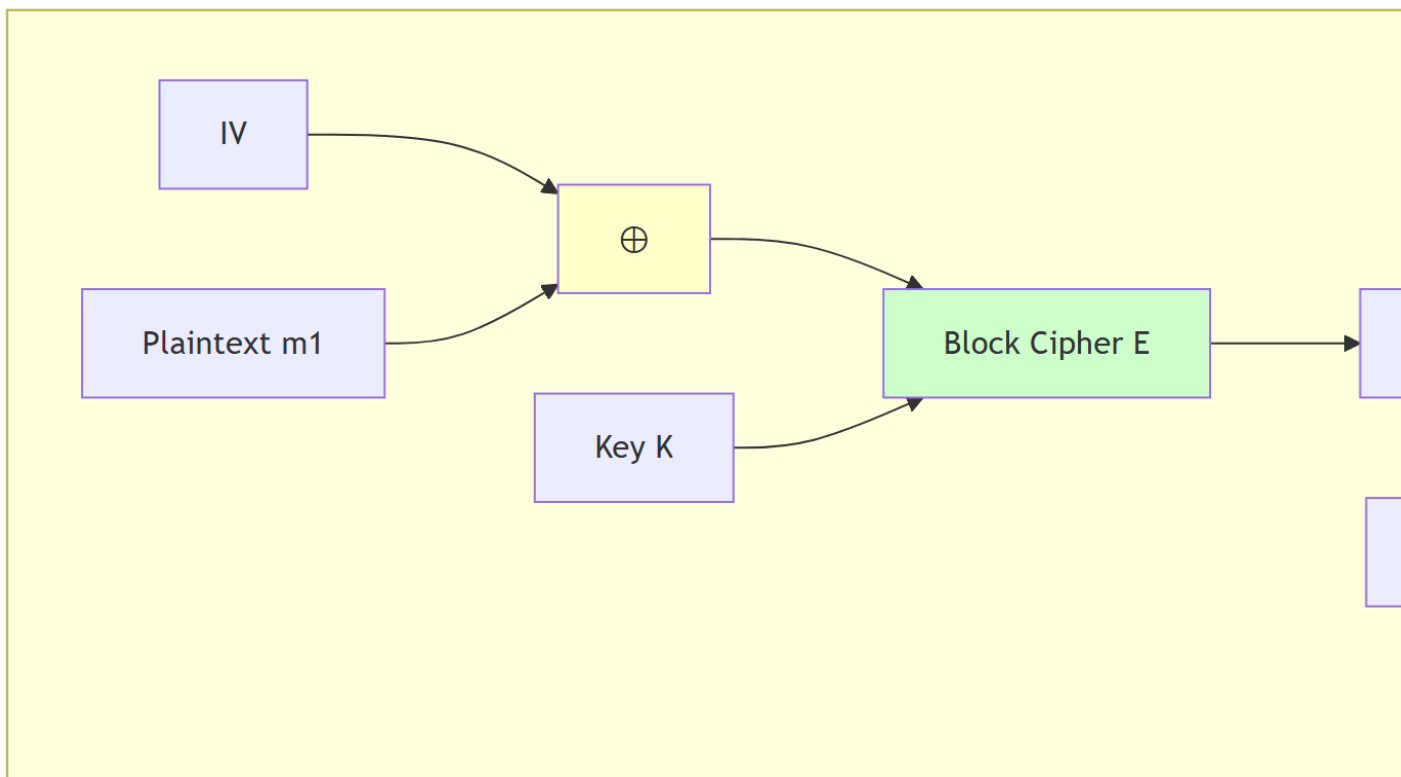
## 2.2 Cipher Block Chaining (CBC)

**Principle:** each plaintext block is **XORed with the previous ciphertext** before encryption.

$$c_i = E_K(m_i \oplus c_{i-1})$$

$$m_i = D_K(c_i) \oplus c_{i-1}$$

With  $c_0 = IV$  (Initialization Vector)



**Characteristics:**

- **Identical plaintexts** → different ciphertexts (if IV changes)
- **Patterns erased**: chaining masks the structure
- **Limited error propagation**: error on  $c_j$  affects  $m_j$  and  $m_{j+1}$  only
- **Not parallelizable** in encryption (sequential)
- **Parallelizable** in decryption

#### IV (Initialization Vector):

- Must be **random** or **pseudo-random**
- Can be transmitted **in clear**
- Must be **different** for each message with the same key

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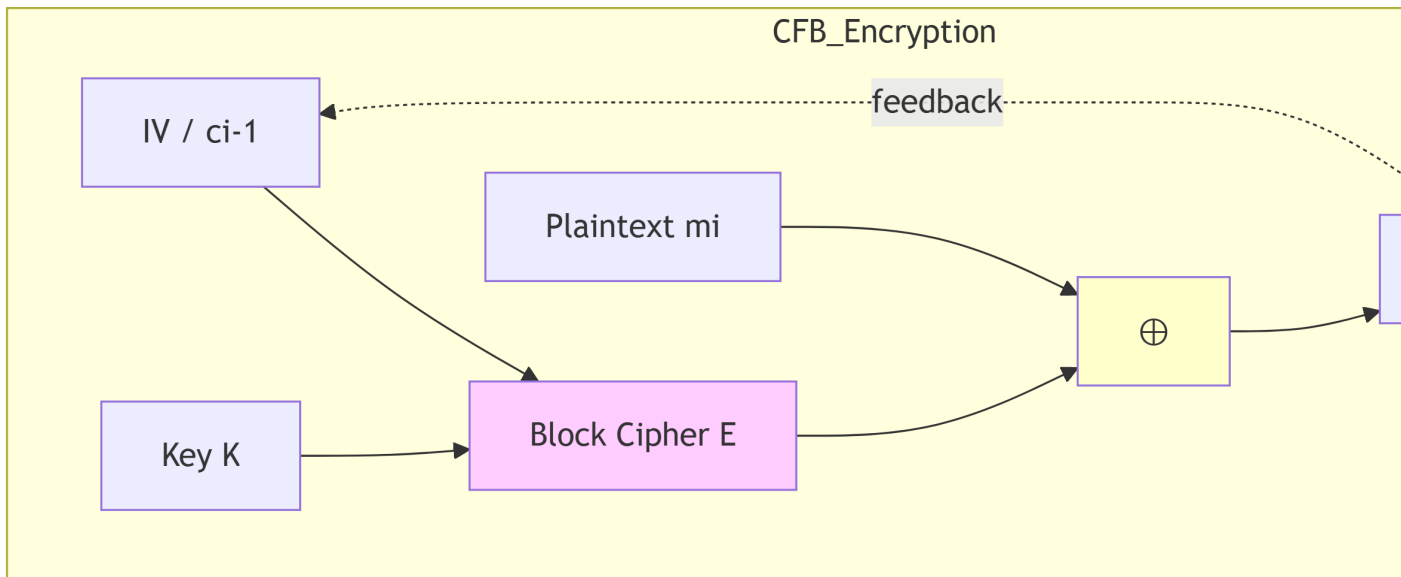
### 2.3 Cipher Feedback Mode (CFB)

**Principle**: works like a **stream cipher** where the keystream is generated by the block cipher. The keystream depends on **previous ciphertexts** (**asynchronous** mode).

$$c_i = m_i \oplus E_K(c_{i-1})$$

$$m_i = c_i \oplus E_K(c_{i-1})$$

With  $c_0 = IV$



**Characteristics:**

- **Identical plaintexts** → different ciphertexts (if IV changes)
- **Chaining**: dependencies between ciphertexts
- **Error propagation**: error on  $c_j$  affects  $\frac{n}{r}$  following blocks
  - $n$  = nominal size of block cipher
  - $r$  = size of plaintexts
- **Not parallelizable**
- **IV non-confidential** but must be transmitted

**Usage**: suitable for transmissions with frequent packet loss

## 2.4 Output Feedback Mode (OFB)

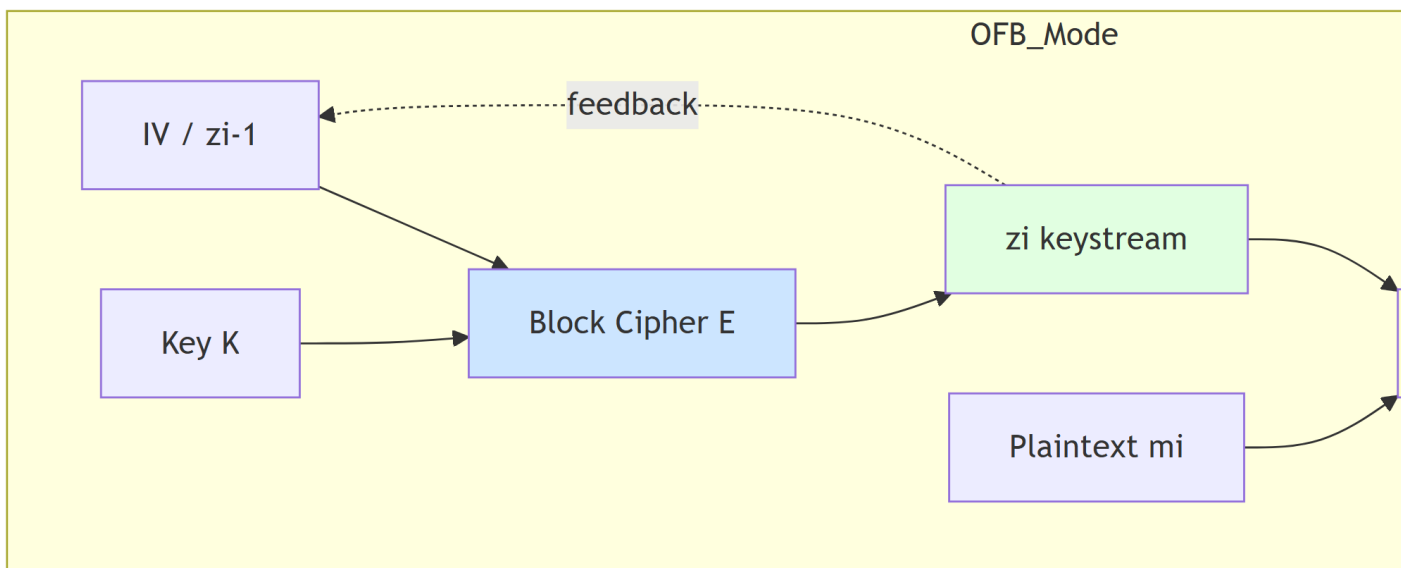
**Principle**: works like a **synchronous stream cipher**. The keystream is **entirely determined** by the key and IV, **independent** of plaintext and ciphertext.

$$z_i = E_K(z_{i-1})$$

$$c_i = m_i \oplus z_i$$

$$m_i = c_i \oplus z_i$$

With  $z_0 = IV$



**Characteristics**:

- **Identical plaintexts** → different ciphertexts (if IV changes)
- **No error propagation:** error on  $c_j$  affects only  $m_j$
- **Pre-computable keystream:** efficient
- **CRITICAL:** NEVER reuse the same IV with the same key (otherwise identical keystream)
- **Parallelizable** if keystream pre-computed

**Reuse warning:** Change IV for each new message!

### Original text (CFB and OFB Modes)

#### **CFB and OFB Modes: Characteristics**

The **CFB** and **OFB** modes work as a **stream cipher** with a **keystream** generated by the encryption block. In **CFB**, the keystream depends on **previous ciphertexts** (asynchronous) whereas in **OFB**, the keystream is **entirely determined by the key and the IV** (synchronous).

#### **Particularities of CFB:**

- As in CBC mode, **identical plaintexts** are translated into **different ciphertexts** if the **IV changes**. The **IV is not necessarily confidential** and can be exchanged in clear between parties.
- **Chaining** also introduces **dependencies** between current ciphertexts and previous ciphertexts. In particular, if  $n$  is the **nominal algorithm size** and  $r$  is the **plaintext size**, the current ciphertext will depend on the  $\frac{n}{r}$  **previous ciphertexts** (each iteration will shift the faulty input by  $r$  positions, after  $\frac{n}{r}$  iterations the faulty ciphertext will be completely “expelled”).
- **Error propagation** follows the same principle: an error in a ciphertext will result in incorrect decryption of the  $\frac{n}{r}$  following ciphertexts.

#### **Particularities of OFB:**

- OFB has **identical behavior** to CBC and CFB modes for **encryption of identical plaintexts**.
- **No error propagation** on adjacent ciphertexts.
- **Modify the IV** if the key does not change to **avoid keystream reuse !!!**

### Quick revision (CFB/OFB)

**CFB** (asynchronous): keystream =  $f(\text{previous ciphertexts})$ . Limited error propagation ( $\frac{n}{r}$  blocks).

**OFB** (synchronous): keystream =  $f(\text{key} + \text{IV only})$ . No error propagation.

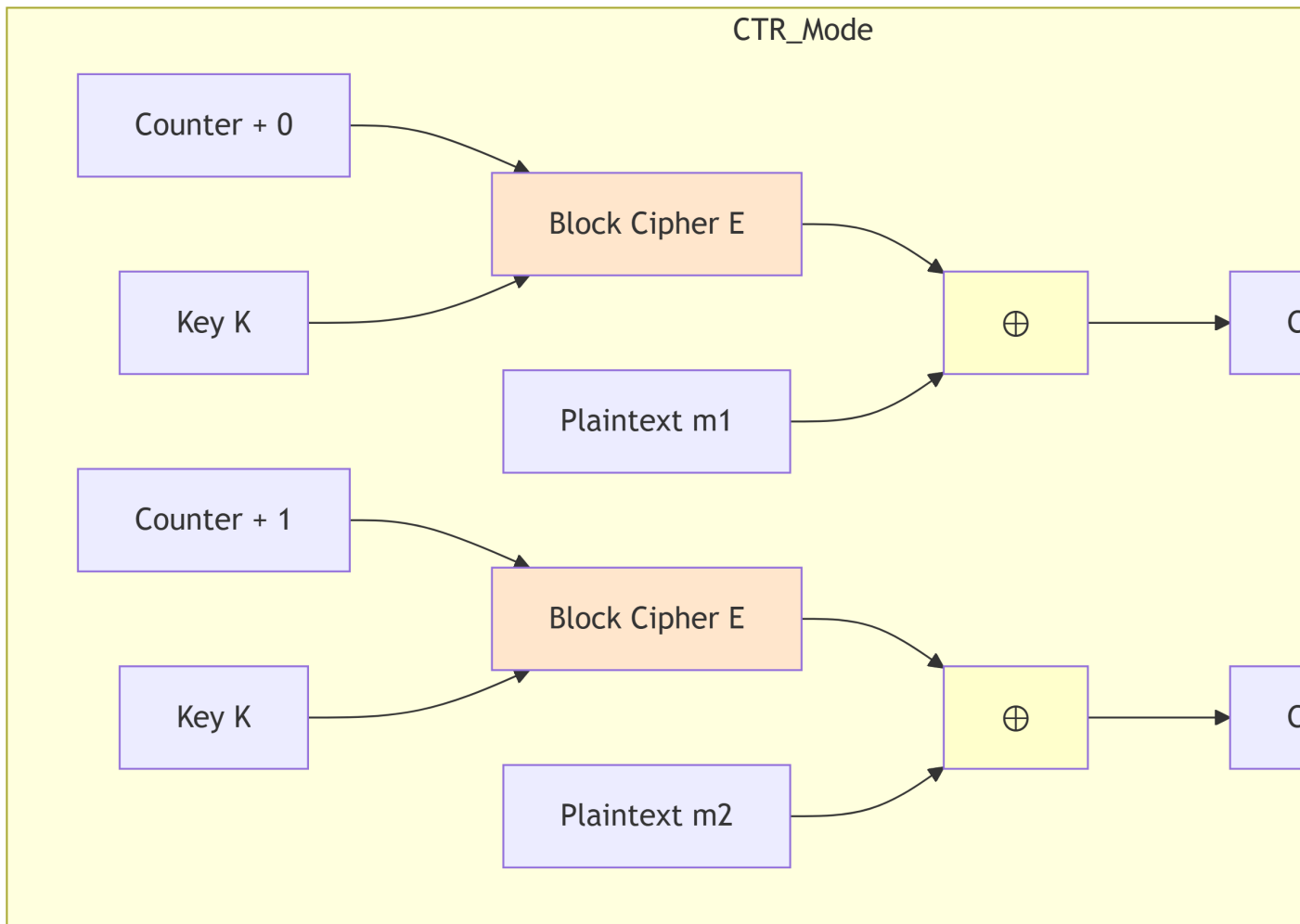
**CRITICAL:** NEVER reuse same IV with same key. IV transmissible in clear.

## 2.5 Counter Mode (CTR)

**Principle:** the keystream is generated by **encryption of a counter** incremented at each block.

$$c_i = m_i \oplus E_K(\text{counter} + i)$$

$$m_i = c_i \oplus E_K(\text{counter} + i)$$



**Characteristics:**

- **Synchronous mode:**  $\text{keystream} = f(\text{counter})$
- **Parallelizable:** keystream pre-computable for encryption AND decryption
- **Random access:** each block decryptable independently
- **No error propagation**

- **Benefits from SIMD architectures:** no dependencies between blocks
- **Counter:** must be of size  $2^b$  ( $b$  = block size)
- **CRITICAL:** NEVER reuse the same counter with the same key

#### Counter management:

- **Increment modulo  $2^b$**  after each iteration
- **Solution:** always increment for each encrypted stream
- First block of stream  $i + 1 >$  last block of stream  $i$

#### Applications:

- **ATM** (Asynchronous Transfer Mode)
- **IPsec** (IP security)
- **High-speed lines:** selective transmission of blocks
- **Large volume transfers:** video

#### **i** Original text (Counter Mode)

##### **Counter Mode (CTR Mode)**

Frequently used as encryption support in data transfer protocols like **ATM** (Asynchronous Transfer Mode) and **IPsec** (IP security).

##### **Counter Mode (II)**

- The **keystream** is generated by the **encryption of a random counter** of size  $2^b$  (with  $b$  the block size) and necessary for decryption. This counter is **incremented modulo  $2^b$**  after each iteration.
- Works in **synchronous mode**. **Reuse of the same counter** results in an **identical keystream** !
- **Solution:** Always **increment the counter** for each encrypted stream such that the counter of the first block of a stream is **larger than the last block** of the previous stream.
- **Easily parallelizable:** The keystream can be **pre-calculated** both for encryption and decryption. Fully benefits from **SIMD architectures** because unlike other chaining modes there are no **dependencies between operations** of different blocks.
- **Random access** to encryption/decryption of each block: Unlike other chaining modes where the  $i$ -th operation depends on the  $(i - 1)$ -th operation.
- If we add **absence of error propagation**, the counter mode facilitates **selective (re)transmission** of ciphertext blocks, making it very attractive for **securing high-speed lines** as well as for **encrypted transfers of large volumes** of information (e.g. video).

💡 Quick revision (Counter Mode)

**CTR:** keystream =  $E_K(\text{counter} + i)$ .

**Advantages:** parallelizable (encryption + decryption), random access, no error propagation, SIMD-friendly.

**CRITICAL:** never reuse counter.

**Usage:** ATM, IPsec, high speed, video.

---

### 3. Product Ciphers and Feistel Ciphers

#### Product Ciphers

**Definition:** encryption scheme combining a **series of successive transformations** to strengthen resistance to cryptanalysis.

**Common transformations:**

- Transpositions (permutations)
- Substitutions (S-boxes)
- XORs
- Linear combinations
- Modular multiplications

#### Feistel Ciphers

**Definition:** iterative product cipher with specific structure.

**Operating principle:**

- **Input:** plaintext of  $2t$  bits =  $(L_0, R_0)$  (two sub-blocks of  $t$  bits)
- **Output:** ciphertext of  $2t$  bits =  $(R_r, L_r)$  after  $r$  steps (rounds)
- **Each step:** invertible bijection (for unique decryption)

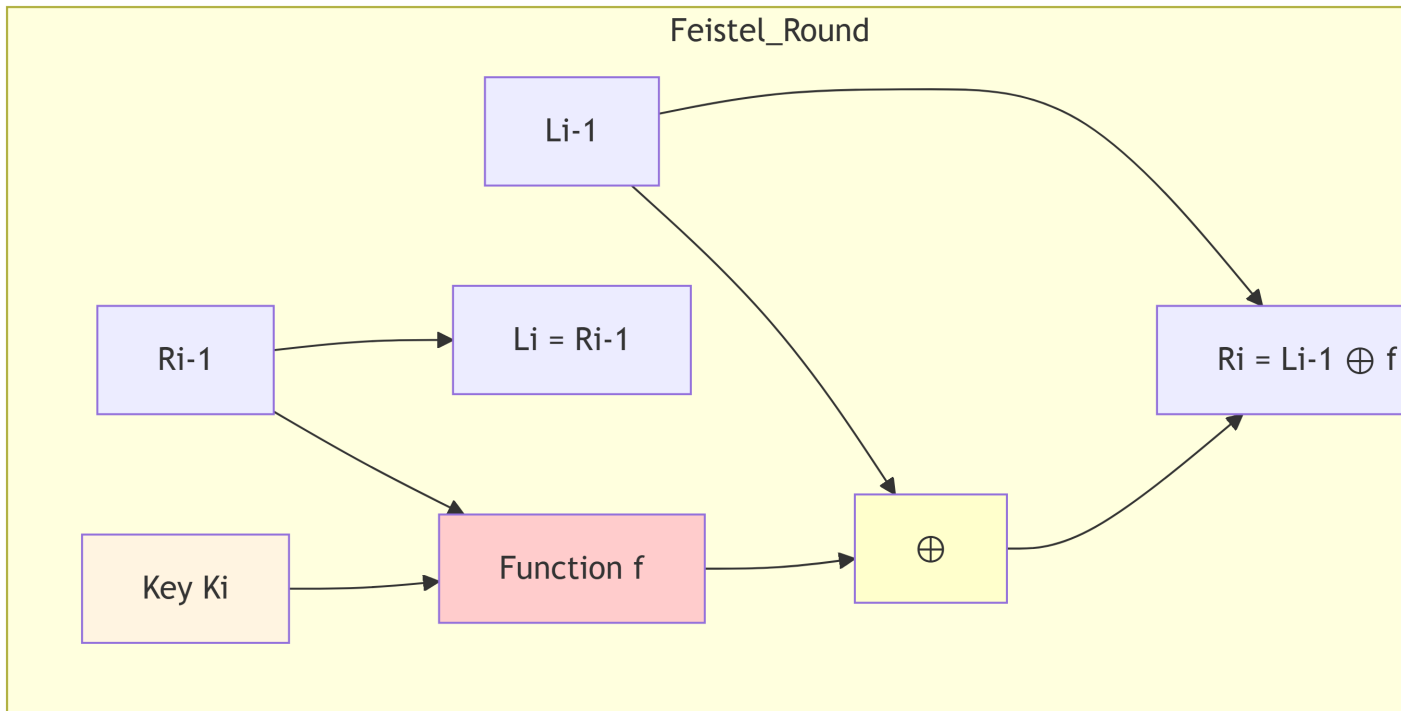
**Equations of step  $i$  ( $1 \leq i \leq r$ ):**

$$(L_{i-1}, R_{i-1}) \xrightarrow{K_i} (L_i, R_i)$$

With:

- $L_i = R_{i-1}$
- $R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$





#### Characteristics:

- $K_i$ : subkeys generated from the main key  $K$
- Number of steps  $r$ : generally **even** and  $\geq 3$ 
  - Example: DES has 16 steps
- **Final permutation**:  $(L_r, R_r) \rightarrow (R_r, L_r)$
- **Decryption**: identical to encryption but subkeys applied in **reverse order** (from  $K_r$  to  $K_1$ )

#### Frequent operations:

- Permutations
- Substitutions (S-boxes)

**i** Original text

#### Product Ciphers and Feistel Ciphers

- A **product cipher** is an **encryption scheme** combining a **series of successive transformations** to **strengthen resistance to cryptanalysis**. Common transformations for a product cipher are: **transpositions**, **substitutions**, **XORs**, **linear combinations**, **modular multiplications**, etc.

- A **Feistel cipher** is an **iterative product cipher** capable of transforming a **plaintext of  $2t$  bits** of the form  $(L_0, R_0)$  composed of two **sub-blocks**  $L_0$  and  $R_0$  of  $t$  bits into a **ciphertext of size  $2t$**  of the form  $(R_r, L_r)$  after  $r$  **successive steps (rounds)** with  $r \geq 1$ . **Each step** defines a **bijection (invertible !)** to allow unique decryption.
- **Permutations** and **substitutions** are the most frequent operations.
- The steps  $1 \leq i \leq r$  are written:  $(L_{i-1}, R_{i-1}) \xrightarrow{K_i} (L_i, R_i)$  with  $L_i = R_{i-1}$  and  $R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$ . The  $K_i$  are **sub-keys, different for each step**, generated from the **main key**  $K$  of the encryption scheme.
- The **number of steps** proper to a Feistel cipher is normally **even** and  $\geq 3$  (e.g. **DES has 16 steps**)
- After execution of all steps, a Feistel cipher performs a **permutation** of the two parts  $(L_r, R_r)$  into  $(R_r, L_r)$ .
- The **decryption** of a Feistel Cipher is **identical to encryption** except that the sub-keys  $K_i$  are applied in **reverse order** (From  $K_r$  to  $K_1$ ).



#### Quick revision

**Product cipher:** combination of successive transformations (transpositions, substitutions, XOR).

**Feistel cipher:**

- iterative product cipher
- plaintext  $2t$  bits =  $(L_0, R_0)$
- $r$  rounds with  $L_i = R_{i-1}$  and  $R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$ .
- Decryption = encryption with reversed sub-keys.
- Example: DES (16 rounds).

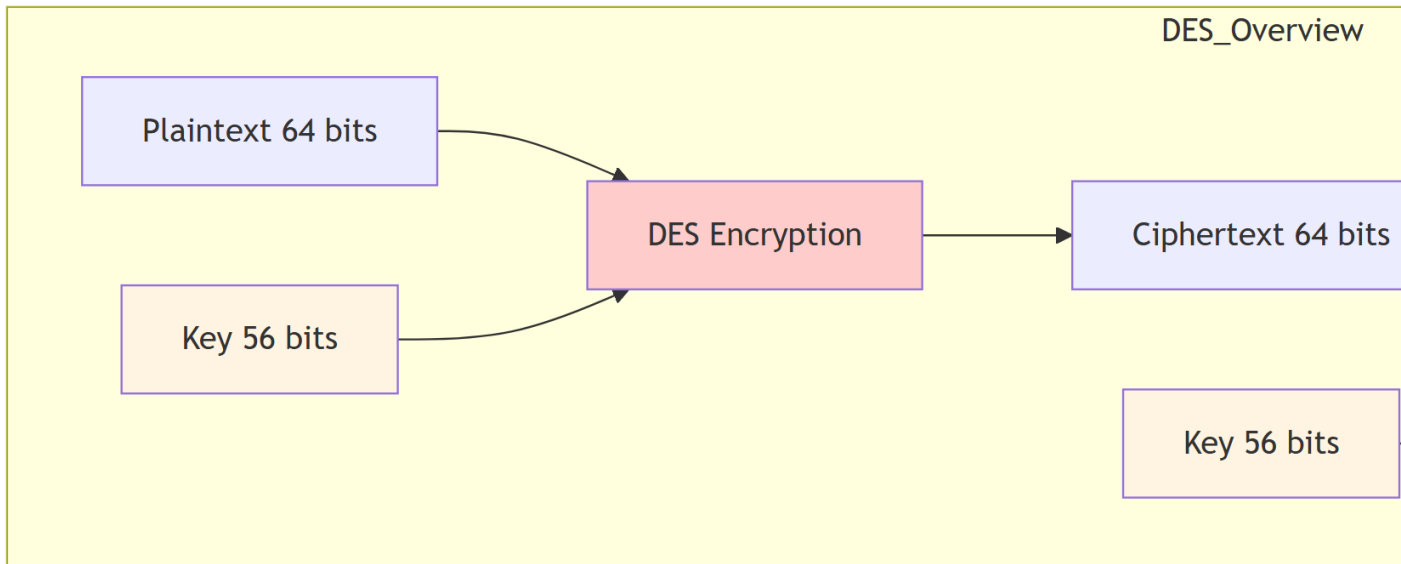
## 4. Data Encryption Standard (DES)

### General Presentation

**DES** (Data Encryption Standard): most important cryptographic algorithm until the advent of AES in 2001.

**Main characteristics:**

- **Type:** Feistel Cipher
- **Block size:** 64 bits (nominal size)
- **Key size:** 56 effective bits (64 total bits with 8 parity bits)
- **Number of steps:** 16 rounds
- **Subkeys:** 16 subkeys of 48 bits (one per step)
- **Usage modes:** ECB, CBC, CFB, OFB



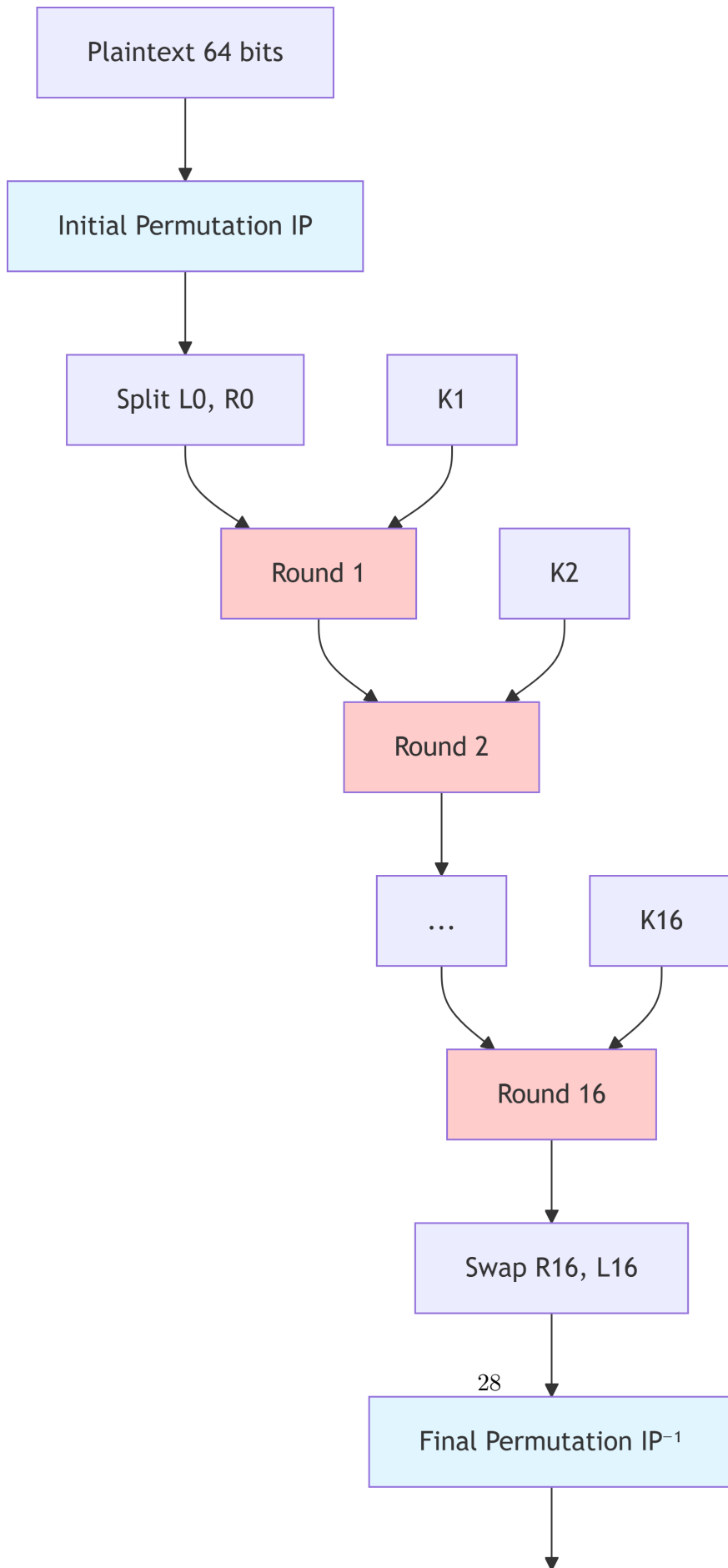
## DES Structure

### Main components:

1. **Initial permutation (IP):** permutation of the 64 input bits
2. **16 Feistel rounds:** iterative transformation
3. **Final permutation (IP<sup>-1</sup>):** inverse of IP

### Each round applies:

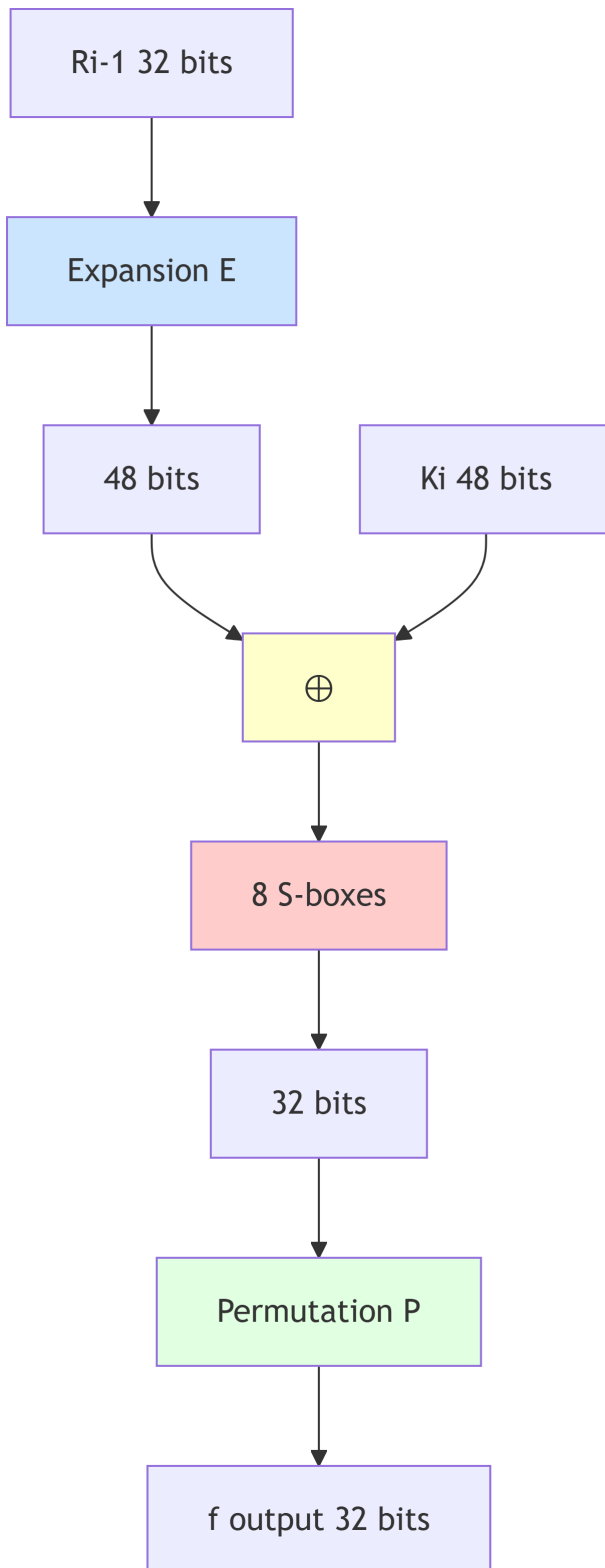
- Division into two halves:  $L_{i-1}$  and  $R_{i-1}$  (32 bits each)
- Function  $f$  on  $R_{i-1}$  with subkey  $K_i$
- XOR with  $L_{i-1}$
- Exchange of halves



## DES Cipher Function

The **function**  $f$  for each round:

1. **Expansion E**: 32 bits  $\rightarrow$  48 bits (table E)
2. **Key Addition**: XOR with subkey  $K_i$  (48 bits)
3. **S-boxes**: 8 S-boxes transform 48 bits  $\rightarrow$  32 bits
  - Each S-box: 6 bits input  $\rightarrow$  4 bits output
4. **Permutation P**: permutation of the resulting 32 bits



### S-box operation:

Input:  $a_1a_2a_3a_4a_5a_6$  (6 bits)

- **Row:**  $a_1 + 2a_6$  (external bits)
- **Column:**  $a_2 + 2a_3 + 4a_4 + 8a_5$  (internal bits)
- **Output:** value of the corresponding cell (4 bits)

### Subkey Generation

Process:

1. Main key: 64 bits (56 effective + 8 parity)
2. **Permuted Choice 1 (PC-1):** selection of 56 bits
3. Division into two halves:  $C_0$  and  $D_0$  (28 bits each)
4. For each round  $i$ :
  - Left circular rotation of  $C_{i-1}$  and  $D_{i-1}$
  - **Permuted Choice 2 (PC-2):** selection of 48 bits for  $K_i$

Rotations:

- Rounds 1, 2, 9, 16: 1 position
- Other rounds: 2 positions

**i** Original text (DES Operation)

#### DES: Operation

##### Cipher Function

- **Expansion E:** The **32 bits of the input** are transformed into a vector of **48 bits** using the **table E**. The first line of this table indicates how the first sub-block of 6 bits will be generated: first take the 32nd bit then bits 1,2,3,4,5. The second sub-block starts with the 4th bit then bits 5,6,7,8,9 and so on...
- **Key addition:** **XOR of the 48-bit vector** with the key.
- **S-boxes:** Apply **8 S-boxes** on the resulting 48-bit vector. Each of these S-boxes takes a **6-bit sub-block** and transforms it into a **4-bit sub-block**. The operation is performed as follows: If we denote the 6 input bits of the S-box as:  $a_1a_2a_3a_4a_5a_6$ . The output is given by the content of the cell located in the **row**  $a_1 + 2a_6$  and the **column**  $a_2 + 2a_3 + 4a_4 + 8a_5$ .
- **Permutation P:** Permutation P works as follows: The first bit is sent to the 16th position, the second to the 7th position and so on.

#### Permutations IP and IP<sup>-1</sup>

- Act respectively at the **beginning** and at the **end** of the block processing and on the **entirety of the 64 bits**.

#### 💡 Quick revision (DES)

**DES:** Feistel cipher, 64-bit blocks, 56-bit effective key, 16 rounds.

**Function  $f$ :** Expansion E (32→48 bits) → XOR  $K_i$  → 8 S-boxes (48→32 bits) → Permutation P.

**S-box:** 6 bits input → 4 bits output via table (row = external bits, column = internal bits).

**Permutations:** IP (initial) and  $IP^{-1}$  (final) on 64 bits.

## 5. Triple-DES and DES Security

### DES Vulnerabilities

**Main problem:** key space size  $\{0,1\}^{56}$  insufficient.

**Brute force attack:**

- **1999:** key found in **24 hours**
- Technique: massively parallel brute force (100,000 PCs on Internet)
- Known plaintext attack

### Triple-DES (3DES)

**Solution:** increase key space to  $\{0,1\}^{112}$ .

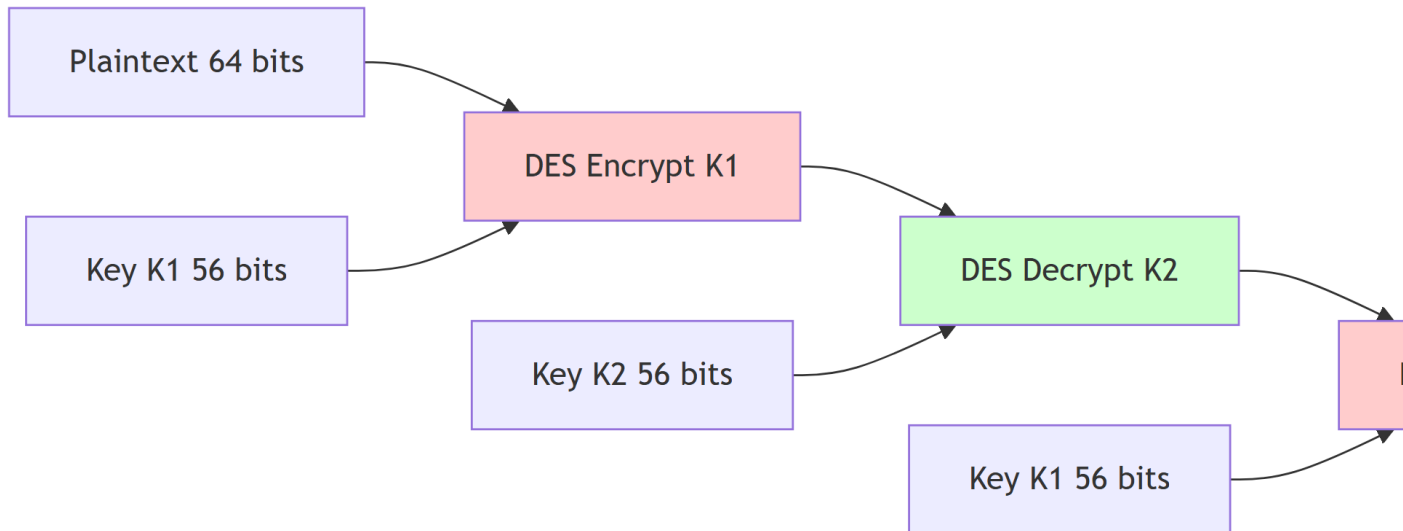
**Scheme:**

$$C = E_{K_1}(D_{K_2}(E_{K_1}(P)))$$

With:

- $E$ : DES encryption
- $D$ : DES decryption
- $K_1, K_2$ : two 56-bit keys





#### Advantages:

- **Satisfactory security:** key space  $2^{112}$
- **Compatibility:** reuse of existing DES hardware/software
- **Gradual migration:** while waiting for AES

#### Disadvantage:

- **Performance:**  $3\times$  slower (3 successive DES executions)

### DES Properties

#### 1. DES is not a group

DES is NOT a group under composition:

$$\nexists K_3 \text{ such that } E_{K_3}(E_{K_2}(E_{K_1}(x))) = E_{K_3}(x)$$

**Consequence:** composite encryption (Triple-DES) considerably increases security.

**If DES were a group:** exhaustive search on  $\{0,1\}^{56}$  would break the algorithm regardless of the number of consecutive executions.

#### 2. Weak and semi-weak keys

- **Weak key:**  $E_K(E_K(x)) = x$
- **Pair of semi-weak keys:**  $E_{K_1}(E_{K_2}(x)) = x$

**Characteristic:** weak keys generate identical subkeys in pairs:

- $k_1 = k_{16}, k_2 = k_{15}, \dots, k_8 = k_9$
- Facilitates cryptanalysis

**DES has 4 weak keys:**

Weak key (hexadecimal)
0101 0101 0101 0101
0101 0101 FEFE FEFE
FEFE FEFE FEFE FEFE
FEFE FEFE 0101 0101

**And 6 pairs of semi-weak keys**

**i** Original text (DES and 3DES)

### DES and Triple-DES

- The **size of the key set** ( $\{0, 1\}^{56}$ ) constitutes the **greatest threat** weighing on DES with current computing resources. In **1999** it took only **24 hours** to find the key from a **known plaintext** using a **massively parallel brute force technique** (100,000 PCs connected to the Internet).
- **Triple DES** protects us from these **brute force attacks** by increasing the **possible key space** to  $\{0, 1\}^{112}$ .
- This alternative allows continuing to use **DES “boxes”** (hardware and software) while waiting for migration to AES.
- The **security level** obtained by this solution is **very satisfactory**.
- The **performance impact** of three successive DES executions remains a **disadvantage** for some applications.

### DES: properties

- **DES is not a group** (in the algebraic sense) under composition: In other words, DES being a permutation:  $\{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$ , if DES were a group under composition, this would mean that:  $\exists K_3$  such that  $E_{K_3}(E_{K_2}(x)) = E_{K_3}(x)$

This property ensures that **composite encryption** (like Triple-DES) **considerably increases the security** of DES. If DES were a group, exhaustive search on the possible key set ( $\{0, 1\}^{56}$ ) would allow “breaking” the algorithm **regardless of the number of consecutive executions** of DES.

- **Weak and semi-weak keys** (weak and semi-weak keys):
  - A key  $K$  is said to be **weak** if  $E_K(E_K(x)) = x$ .
  - A pair of keys  $(K_1, K_2)$  is said to be **semi-weak** if  $E_{K_1}(E_{K_2}(x)) = x$ .
- Weak keys have the particularity of generating **identical subkeys in pairs** ( $k_1 = k_{16}, k_2 = k_{15}, \dots, k_8 = k_9$ ), which **facilitates cryptanalysis**.
- **DES has 4 weak keys** (and 6 pairs of semi-weak keys).



Quick revision (3DES and security)

**DES vulnerability:** key space  $2^{56}$  breakable in 24h (1999). **Triple-DES:**  $E_{K_1}(D_{K_2}(E_{K_1}(P)))$ , space  $2^{112}$ , reuses DES hardware,  $3\times$  slower. **DES group**  $\rightarrow$  composite encryption strengthens security. **4 weak keys** generating identical subkeys in pairs  $\rightarrow$  facilitates cryptanalysis.

## 6. Advanced Encryption Standard (AES)

### General Presentation

**AES** (Advanced Encryption Standard): standard adopted in November 2001.

**Design:** Johan Daemen and Vincent Rijmen (original name: **Rijndael**)

**Main characteristics:**

- **Type:** iterative block cipher (but **NOT a Feistel Cipher**)
- **Block size:** 128 bits
- **Variable key size:** 128, 192 or 256 bits
- **Number of rounds:** depends on key size
  - 10 rounds for 128-bit key
  - 12 rounds for 192-bit key
  - 14 rounds for 256-bit key
- **Usage modes:** ECB, CBC, CFB, OFB, CTR

**Advantages over DES:**

- **Open process:** consultation and analysis by worldwide experts
- **$\sim 2\times$  more performant** in software
- **$\sim 10^{22}$  times more secure** (theoretically)

- **Scalable:** key size can be increased if necessary

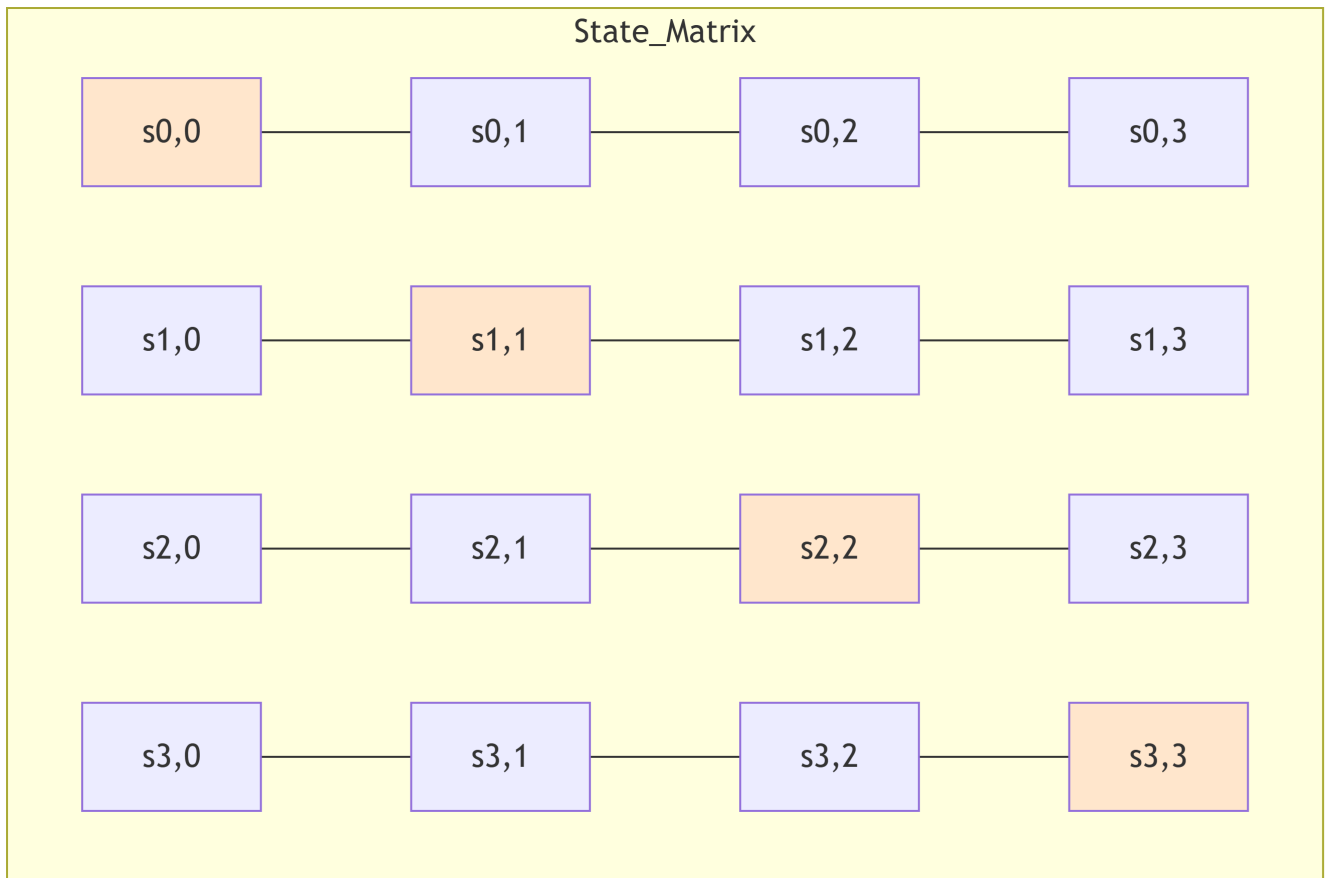
## AES Structure

**Basic unit:** State matrix of 4 rows  $\times$  4 columns (for 128-bit key)

- Each element = 1 byte
- **Total:** 16 bytes = 128 bits

**Operations on field  $GF(2^8)$ :**

- Byte = element of  $GF(2^8)$
- Finite field of polynomials of degree 7 with coefficients in  $GF(2)$
- Additions, multiplications defined in  $GF(2^8)$



## AES Round Detail

Four operations per round:

### 1. SubBytes (ByteSub)

- Non-linear substitution via **S-box**
- Each byte transformed independently
- Resistance to linear and differential cryptanalysis

### 2. ShiftRows

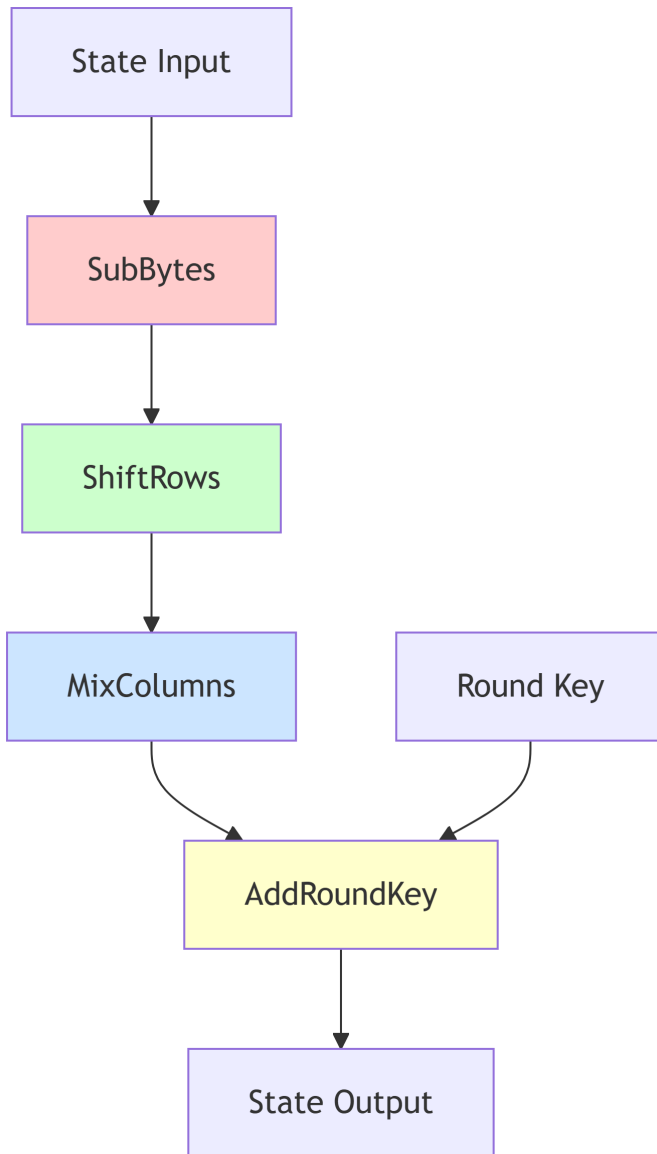
- **Permutation of bytes** with variable shifts per row
- Row 0: no shift
- Row 1: left shift 1 position
- Row 2: left shift 2 positions
- Row 3: left shift 3 positions

### 3. MixColumns

- Each column = linear combination of other columns
- **Matrix multiplication** in  $GF(2^8)$
- Maximum diffusion

### 4. AddRoundKey

- **XOR** of the State matrix with the round subkey
- Subkey = result of Key Schedule



**Final round:** identical EXCEPT **no MixColumns**

### Key Schedule (Subkey Generation)

**Process:**

1. **Key Expansion:** generation of an extended matrix
  - Key 128 bits  $\rightarrow$  matrix  $4 \times 4 \times (N_e + 1)$  bytes
  - $N_e$  = number of rounds

## 2. Key Selection: extraction of subkeys

- First subkey: first 4 columns
- Second subkey: next 4 columns
- Etc.

### Operations:

- Byte rotations
- Substitutions via S-box
- XOR with constants (Rcon)

### AES Pseudo-code

```
Rijndael(State, CipherKey) {  
    KeyExpansion(CipherKey, ExpandedKey); // Key Schedule  
  
    AddRoundKey(State, ExpandedKey[0..3]); // Initial XOR  
  
    for(i = 1; i < Ne; i++) {  
        Round(State, ExpandedKey[4*i...(4*i)+3]);  
    }  
  
    FinalRound(State, ExpandedKey[4*Ne...4*Ne+3]); // Without MixColumns  
}
```

### AES Decryption

**Principle:** apply the **inverse operations** in each round.

#### Inverse operations:

- **InvSubBytes:** inverse substitution via S-box <sup>1</sup>
- **InvShiftRows:** right shifts (instead of left)
- **InvMixColumns:** inverse matrix multiplication
- **AddRoundKey:** self-inverse (XOR)

**Order:** inverse of encryption with subkeys in reverse order

**i** Original text (AES)

#### Advanced Encryption Standard (AES)

- Adopted as **standard in November 2001**, designed by **Johan Daemen and Vincent Rijmen** (hence its original name **Rijndael**).

- It is also an **iterative block cipher** (like DES) but **not a Feistel Cipher**.
- **Plaintext/Ciphertext Blocks: 128 bits.**
- **Variable key length: 128, 192, or 256 bits.**
- Unlike DES, AES comes from an **open consultation and analysis process** involving worldwide experts.
- Techniques similar to DES (substitutions, permutations, XOR...) complemented by **simple and very performant algebraic operations**.
- All operations are performed in the **field  $GF(2^8)$** : the finite field of **polynomials of degree 7** with **coefficients in  $GF(2)$** .
- In particular, a **byte for AES is an element in  $GF(2^8)$**  and the **operations on bytes** (additions, multiplications,...) are **defined as in  $GF(2^8)$** .
- **~2 times more performant** (in software) and **~ $10^{22}$  times (in theory...) more secure** than DES...
- **Scalable:** The key size can be increased if necessary.

#### **Detail of an AES Step (round)**

The **basic unit** on which calculations are applied is a **matrix of 4 rows and 4 columns** (in the case of a 128-bit key) whose elements are **bytes**:

- **ByteSub: Non-linear operation (S-box)** designed to resist **linear and differential cryptanalysis**.
- **ShiftRow: Permutation of bytes** introducing **variable shifts** on the rows.
- **MixColumn:** Each column is replaced by **linear combinations** of the other columns (**matrix multiplication !**)
- **AddRoundKey: XOR** of the current matrix with the **subkey** corresponding to the current step.

#### **AES: Global Operation**

- The **number of steps** of AES varies depending on the **key size**. For a **128-bit key**, **10 steps** must be performed. Each increase of 32 bits in the key size entails an **additional step** (14 steps for 256-bit keys).
- **Decryption** consists of applying the **inverse operations** in each of the steps (**InvSubBytes, InvShiftRows, InvMixColumns**). **AddRoundKey** (because of XOR) is **its own inverse**.



- The **Key Schedule** consists of:
  - An operation of **key expansion** of the main key. If  $N_e$  is the number of steps (depending on the key), a **matrix of 4 rows and  $4 \times (N_e + 1)$  columns** is generated.
  - An operation of **step key selection**: The **first subkey** will be constituted by the **first 4 columns** of the matrix generated during expansion and so on.

#### Quick revision (AES)

**AES** (Rijndael 2001): iterative block cipher (NOT Feistel), 128-bit blocks, keys 128/192/256 bits  $\rightarrow$  10/12/14 rounds.

**State**:  $4 \times 4$  byte matrix in  $GF(2^8)$ .

**4 operations/round**:

- SubBytes (non-linear S-box)
- ShiftRows (row shifts)
- MixColumns (linear combinations)
- AddRoundKey (XOR subkey).

$2\times$  faster than DES,  $10^{22}$  times more secure.

## 7. Attacks and AES Security

### AES Strengths

**Simplicity and performance:**

- Simple and efficient algorithm
- Works on limited platforms (8-bit smart cards)
- Hardware and software optimizations

### Published Attacks

#### 1. Algebraic attacks (2002)

**XSL technique** (N. Courtois and P. Pieprzyk):

- Represents AES as **system of 8000 quadratic equations** with 1600 binary unknowns
- **Estimated effort**:  $2^{100}$  operations (still a conjecture)

- **Characteristic:** requires few known plaintexts
- **Distinction:** different from linear/differential attacks

**Critique:** based on the “highly algebraic” character of AES (largely contested)

## 2. Related Key Attacks (2009-2011)

**Principle:** attacks based on **similar keys**

- Interesting results on **reduced versions** of AES
- Do not compromise full AES

## 3. Side Channel Attacks

**Principle:** attacks on **implementation** (not the algorithm)

**Techniques:**

- **Cache timing attacks:** cache access analysis
- **Power analysis:** power consumption
- **Electromagnetic analysis:** electromagnetic emissions

**Example** (2005): Osvik, Shamir, Tromer

- Extraction of 128-bit key with **6-7 plaintext/ciphertext pairs**
- Based on **cache access** analysis

## 4. Meet in the Middle on biclique structures (2011-2015)

**Result:**

- Reduces effort for AES-128 to  $2^{126}$  (factor 4 vs brute force)
- **Remains well above** current capabilities

## Practical Security

**Fundamental assumption:** key of **maximum entropy**

**Recent attacks** (WPA2, etc.):

- Exploit **weakness of passwords/passphrases**
- No flaw in AES itself
- Problem: key generation from weak passwords

**Critical reminder:** key quality = system security

### **AES: Final Remarks and Attacks (I)**

- The greatest **strength of AES** lies in its **simplicity** and its **performance**, including on **reduced computing capacity platforms** (e.g. **smart cards** with 8-bit processors).
- Since its official publication, **many cryptanalysis works** have been published with very interesting results. In particular, **N. Courtois and P. Pieprzyk** presented a technique called **XSL** allowing to represent AES as a **system of 8000 quadratic equations** with **1600 binary unknowns**. The **effort needed** to break this system is estimated (it is still a **conjecture...**) to be  $2^{100}$ .
- These attacks are based on the **highly algebraic character** (and largely contested...) of AES. Moreover, only **a few known plaintexts** are needed to set them up, which distinguishes them from linear and differential attacks.
- In recent years (2009-2011) **attacks based on similar keys** (related key attacks) have obtained interesting results on **reduced versions** of AES.
- Another family of attacks called **side channel attacks** acting directly on the **algorithm implementation** allows extracting cryptographically relevant information during encryption execution.

### **AES: Final remarks and Attacks (II)**

- In **2015** a **Meet in the Middle** type attack based on **biclique structures** showed that it was possible to reduce the **effort needed** to find an AES-128 key to  $2^{126}$ , i.e., a **factor of 4** compared to brute force. This nevertheless remains **well above** current computing capabilities.
- Another family of attacks called **side channel attacks** acting directly on the **algorithm implementation** allows extracting cryptographically relevant information during encryption execution. In particular, the authors manage to **extract the 128-bit key** with only **6-7 plaintext/ciphertext pairs** based on **cache accesses**.
- The **security of AES** (as for any other encryption algorithm) is always based on the assumption of a **key of maximum entropy**. The **attacks published recently** on protocols based on AES (like WPA2) exploit the **weakness of passwords/passphrases** that are the origin of the keys used.

💡 Quick revision (AES Security)

**Strengths:** simplicity, performance (even 8-bit cards). **Attacks:** XSL ( $2^{100}$ , algebraic), related keys (reduced versions), side channel (implementation, cache), Meet-in-Middle biclique ( $2^{126}$ ). **Security:** assumption of max entropy key. Practical attacks = weak passwords, not AES flaw.

---

## 8. Block Cipher Cryptanalysis Techniques

### 8.1 Differential Cryptanalysis

**Principle:** chosen plaintext attack analyzing the **propagation of differences** between two plaintexts through the rounds.

**Method:**

1. Choose two plaintexts with known difference:  $x_a$  and  $x_b$
2. Observe propagation:  $\Delta x = x_a \oplus x_b$
3. Analyze ciphertexts:  $\Delta y = y_a \oplus y_b$
4. **Assign probabilities to keys** according to observed changes
5. **Most probable key** = correct key (after many trials)

**Characteristics:**

- Requires  $2^{47}$  **chosen plaintext pairs** for DES
- **Probabilities:** depend on S-boxes and structure
- The more pairs increase, the more success probability increases

**Sensitivity:** very sensitive to **number of rounds**

- Chances of success increase **exponentially** when rounds decrease

### 8.2 Linear Cryptanalysis

**Principle:** known plaintext attack creating a **linear simulator** of the block cipher.

**Method:**

1. Create **linear approximations** of the algorithm
2. Analyze a large number of plaintext/ciphertext pairs
3. The bits of the simulator key **tend to coincide** with those of the real key (probabilistic calculation)

### Complexity for DES:

- $2^{38}$  **known plaintexts**  $\rightarrow$  10% probability of guessing correctly
- $2^{43}$  **known plaintexts**  $\rightarrow$  85% success probability

### Characteristics:

- **Most powerful analytical attack** to date on block ciphers
- Also **sensitive to number of rounds**

## 8.3 Differential vs Linear Comparison

### Common difficulties:

- **Parallelization**: less efficient than parallel brute force
- **Sensitivity to rounds**: efficiency decreases exponentially with number of rounds

### DES and these attacks:

- Widespread conjecture: DES designers **knew these attacks** (1970s, unpublished at the time)
- **S-box design**: very high resistance to both techniques

## 8.4 Meet-in-the-Middle Attack

**Principle**: exploits **composite constructions** of type  $y = E_{K_2}(E_{K_1}(x))$ .

### Method:

1. Build list  $L_1$ :  $L_1 = \{E_{K_1}(x) \mid K_1 \in \text{KeySpace}\}$
2. Build list  $L_2$ :  $L_2 = \{D_{K_2}(y) \mid K_2 \in \text{KeySpace}\}$
3. Identify **repeated elements** in  $L_1$  and  $L_2$
4. Verify hypothesis with **second known plaintext**
5. The associated keys  $K_1$  and  $K_2$  are probably the sought keys

### Example for DES:

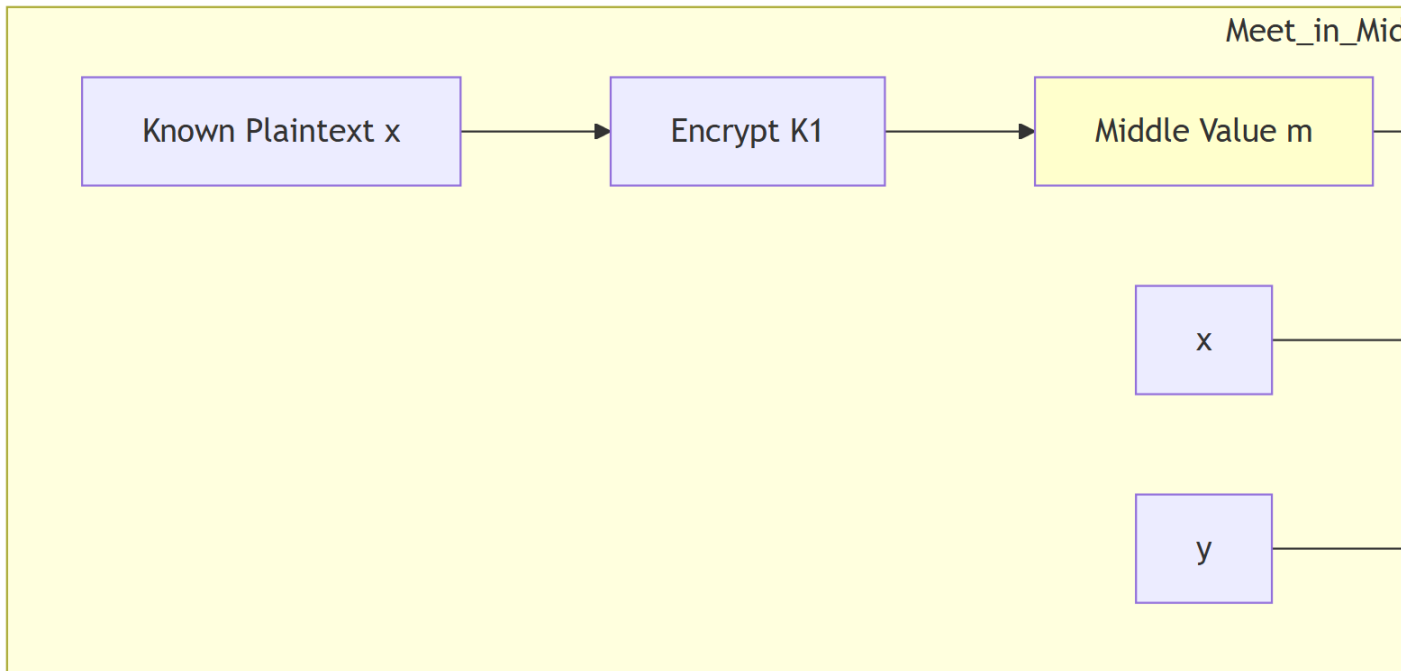
Intuitive key space for  $E_{K_2}(E_{K_1}(x))$ :  $\{0, 1\}^{112}$

### Actual effort:

- $2^{57}$  **operations** to establish the two lists
- $2^{56}$  **blocks** of 64 bits storage
- **Significantly lower** than the intuitive  $2^{112}$

### Applications:

- Attacks on **composite constructions**
- **Internal** cryptanalysis of block ciphers



**i** Original text (Cryptanalysis)

### Block Cipher Cryptanalysis Techniques

#### Differential Cryptanalysis

- This is a **chosen plaintext attack** that focuses on the **propagation of differences** in two plaintexts as they evolve through the different steps of the algorithm.
- It **assigns probabilities to keys** it “guesses” based on the **changes** they induce on the ciphertexts. The **most probable key** has a good chance of being the correct key after a **large number** of plaintext/ciphertext pairs.
- Requires  $2^{47}$  **chosen plaintext pairs** (for DES) to obtain correct results.

#### Linear Cryptanalysis

- This is a **known plaintext attack** that creates a **block simulator** from **linear approximations**. By analyzing a **large number** of plaintext/ciphertext pairs, the **bits of the simulator key** tend to **coincide** with those of the analyzed block cipher (**probabilistic calculation**)

- For DES an attack based on this technique requires  $2^{38}$  **known plaintexts** to obtain a probability of **10%** of guessing correctly and  $2^{43}$  **for 85%** !
- It is the **most powerful analytical attack** to date on block ciphers.

### Block Cipher Cryptanalysis Techniques (II)

- The practical implementation of **differential and linear attacks** presents **difficulties in parallelizing** calculations compared to an exhaustive key search.
- These two attacks are **very sensitive to the number of steps** of the block cipher: chances of success increase **exponentially** as the number of algorithm steps decreases.
- A widespread conjecture among cryptographers is that these attacks, at the time **unpublished**, were **known to the designers of DES**. In particular, the **design of the S-boxes** offers a **very high resistance** to both techniques.

### Meet-in-the-Middle Attack

- Applies to constructions of the type  $y := E_{K_2}(E_{K_1}(x))$ . For DES, the key space for this solution would be  $\{0, 1\}^{112}$ . First build **two lists**  $L_1$  and  $L_2$  of  $2^{56}$  messages of the form:  $L_1 = E_{K_1}(x)$  and  $L_2 = D_{K_2}(y)$  with  $E$  and  $D$  the encryption and decryption operations respectively. Then **identify elements that repeat** in both lists and **verify our hypothesis** with a second known plaintext. The  $K_1$  and  $K_2$  associated with this pair of known plaintexts will (in all likelihood) be **the sought keys** !
- **Effort required** to carry out the attacks (for DES):  $2^{57}$  **operations** to establish the two lists +  $2^{56}$  **blocks** of 64 bits of storage to memorize intermediate results... **significantly lower** than the intuitive  $2^{112}$ ...
- These meet-in-the-middle techniques are also applied to the **internal cryptanalysis** of block ciphers.

### Quick revision (Cryptanalysis)

**Differential:** chosen plaintext, difference propagation, probabilities on keys,  $2^{47}$  pairs (DES).

**Linear:** known plaintext, linear approximations,  $2^{38}$ - $2^{43}$  plaintexts (DES), most powerful attack.

**Meet-in-Middle:** composite constructions, 2 lists  $2^{56}$ , effort  $2^{57} \ll 2^{112}$ .

**Sensitivity:** very dependent on number of rounds.