Indian Statistical Institute, Bangalore
B. Math.
First Year, First Semester

Analysis I — Instructor: B V Rajarama Bhat
Home Assignment I — Due Date : August 19, 2025

(1) Let C, D be sets with 4 and 5 elements respectively. Find the number of functions from C to D which are: (i) injective; (ii) surjective.
Similarly, find the number of functions from D to C which are: (iii) injective; (iv) surjective.

(2) Suppose X is a non-empty set and $f : X \to X$ is a function. Prove or disprove the following claims:
(i) f is injective if and only if $f \circ f$ is injective; (ii) f is surjective if and only if $f \circ f$ is surjective; (iii) f is bijective if and only if $f \circ f$ is bijective.

(3) Find three functions u, v, w from $\mathbb{N}$ to $\mathbb{N}$, which are injective and have disjoint ranges.

(4) Let R, S be two non-empty sets. Suppose there exists an injective function $g : R \to S$. Show that there exists a surjective function $h : S \to R$.

(5) Suppose A and B are countable sets. Show that $A \cup B$ is countable.

(6) Suppose $A_1, A_2, \ldots$ is a sequence of countable sets. Show that
$$\cup_{n=1}^{\infty} A_n = \{a : a \in A_n, \text{ for some } n \in \mathbb{N}\},$$
is countable. (In other words, a countable union of countable sets is countable.)

(7) Let X be a non-empty set. Show that the set of all functions from X to $\{0, 1\}$ is in bijective correspondence with the power set of X. (Here X need not be a finite set).

(8) Let Y be a non-empty set. What is the maximum possible number of distinct sets we can form using n- subsets $A_1, A_2, \ldots, A_n$ of Y, using set theoretic operations of union, intersection, complement in Y? For instance, when $n = 1$, the answer is 4. They are $A_1, A_1^c, \emptyset = A_1 \cap A_1^c, Y = A_1 \cup A_1^c$. For $n = 2$, the answer is 16, where the list goes on something like, $A_1, A_2, A_1 \cap A_2, A_1 \cup A_2, A_1 \cap A_2^c, A_1^c \cap A_2, A_1^c \cup A_2^c, \ldots.$ Guess the answer for general n and prove it. (Hint: Think of the Venn diagram).

(9) Let $K = \{0, 1\}$ and $L = \{0, 1, 2, 3\}$. Consider cartesian products of countably many copies of K and L:
$$M = K \times K \times \cdots$$
$$N = L \times L \times \cdots$$
Show that M and N are equipotent.

(10) A real number x is said to be a rational number if $x = \frac{p}{q}$, for some integers p, q, with $q \neq 0$. Let $\mathbb{Q}$ be the set of rational numbers. Show that $\mathbb{Q}$ is countable.

(11) Read about *'Proof by infinite descent'* and write down one such proof.

(12) Write down a mathematical puzzle or problem you *really* like with its solution.
(Last two questions are optional).