Homework 2

1. In any ordered sequence of elements of two kinds, each maximal subsequence of elements of like kind is called a run. For example the sequence $\alpha\alpha\alpha\beta\alpha\alpha\beta\beta\beta\alpha$ starts with a run of length 3, followed by runs of length 1,2,3,1, respectively. Given a indistinguishable α's and b indistinguishable β's

   (a) How many distinguishable orderings are there?

   (b) If there are n_1 runs of α, what are the possible number of runs of β?

   (c) Given that all the distinguishable orderings are equally likely, what is the probability that there are n_1 runs of α and $n_1 + 1$ runs of β?

2. If n persons, among whom are A and B, stand in a row, what is the probability that there will be exactly r persons between A and B? If they stand in a ring instead of in a row, show that the probability is independent of r and hence $1/(n-1)$. (In the circular arrangement consider only the arc leading from A to B in the positive direction)

3. Each of n sticks is broken into a long part and a short part, the parts jumbled up and recombined pairwise to form n new sticks. Find the probability

   (a) that the parts will be joined in the original order,

   (b) that the long parts are paired with short parts (If sticks represent chromosomes broken by, say X-ray irradiation, then a recombination of two long parts or two short parts causes cell death)

4. In a small town of n people a person passes a titbit of information to another person. A rumour is now launched with each recipient of the information passing it on to a randomly chosen individual. What is the probability that the rumour is told r times without

   (a) returning to the originator

   (b) being repeated to anyone