PARETO OPTIMAL DESIGNS OF LOW SENSITIVITY DIGITAL FILTERS: PARALLEL AND CASCADE FORM STRUCTURES

Tarek A. Tutunji and Victor E. DeBrunner
School of Electrical Engineering
The University of Oklahoma
202 West Boyd Street, Room 219
Norman, Ok 73019-0631 USA

ABSTRACT

We use the Pareto optimal multi-criterion optimization method to design low sensitivity state-space digital filters for the parallel form and cascade form structures. Our designs balance the desire to match a desired filter response with the desire to produce a low sensitivity filter. Some work on direct form II designs which solidifies some previous work is shown. We find that pole/zero cancellation pairs added to a desired direct form II filter are Pareto optimal.

1. INTRODUCTION

Low sensitivity digital filters are desirable because they produce low output coefficient noise power. Much attention has recently been concentrated on the development of such filters [2,3,4,5]. Beex and DeBrunner [6,7] have used the L_2 norm as a coefficient sensitivity measure for state-space recursive, finite wordlength (FWL) digital filters. Later, DeBrunner [8] has used Pareto optimal multi-criterion to design low sensitivity, direct form II, digital filters.

In this paper, a Pareto optimal multi-criterion method is used to design low sensitivity state-space digital filters. In traditional filter design, recursive methods can be used to match a desired impulse response with minimum error [8]. The sensitivity of the filter is ignored. In this paper, a ***concurrent*** reduction in the filter sensitivity and impulse response error is implemented using recursive methods. The Pareto optimization is used to guarantee low sensitivity and small error: weighting coefficients on the sensitivity and the error are used to balance our desires of matching the impulse response and producing a low sensitivity design for a given implementation structure.

Three filter forms have been examined: direct form II, parallel form, and cascade form. In the case of direct form II filters, the filter parameters converge to apparent global minima. Furthermore, an increase in the relative sensitivity weighting produces a decrease in the sensitivity of the designed filter and a corresponding increase in its impulse response error. In general, designing a filter with higher order than the objective response results in pole/zero cancellations that produce low sensitivity filters, reinforcing work done in [7]. Furthermore, we see the presence of local minima in the objective function and a corresponding increase in the time to convergence of the design algorithm. However, low sensitivity filters with small errors are designed using simulated annealing concepts and judicious initial filter parameter guesses.

2. OPTIMIZATION METHOD

Here, we develop a Pareto multi-criterion optimization algorithm using the Recursive Prediction Error Method (RPEM) to design digital filters. First, we define our objectives.

2.1 The Sensitivity and Impulse Response Measures

The sensitivity measure $S(\theta)$ is a scalar function of the NP system parameters and is defined as [6,7,8]

$$S(\theta) = \sum_{i=1}^{NP} \frac{1}{2\pi j} \oint \frac{\partial H(z)}{\partial \theta_i} \frac{\partial H(z^{-1})}{\partial \theta_i} \frac{dz}{z} = \sum_{i=1}^{NP} \sum_{n=0}^{\infty} g_n^{(i)2} \quad (1)$$

where H(z) is the filter transfer function, the θ_i are the NP parameters of the filter, and the $g_n^{(i)}$ are the impulse responses of the systems with transfer functions $\frac{\partial H(z)}{\partial \theta_i}$. The contour of integration is counter-clockwise around the unit circle.

In our design, the impulse response of a high sensitivity filter is available; our objective is to design a low sensitivity filter with a similar impulse response. We define the impulse response error

$$e_n \equiv y_n - \hat{h}_n(\theta) \quad (2)$$

where y_n is the desired impulse response and $\hat{h}_n$ is the designed impulse response at time n.

2.2 Pareto Optimization Multi-Criterion

In our case, the multi-criterion optimization problem is most fundamental. We can use objective weighting in order to minimize the total weight between the impulse response error the implementation sensitivity. Our cost function is

$$J(\theta) = \gamma_1 S(\theta) + \gamma_2 T(\theta) \quad (3)$$

where $\gamma_1 \equiv \frac{w_1}{S^o(\theta)}$ and $\gamma_2 \equiv \frac{w_2}{T^o(\theta)}$. $S(\theta)$ is the sensitivity, $T(\theta)$ is the 2-norm of the impulse response error, $T^o(\theta)$ and $S^o(\theta)$ are individual minima used to equalize magnitudes, and the weights are chosen such that $w_1+w_2=1$ [1]. To guarantee convergence to a global minima, both the sensitivity and the error must be convex functions [9].

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2.3 Recursive Prediction Error Method (RPEM)

Since $J(\theta)$ is a scalar function, recursive methods can be used directly with proper linear combinations. The following RPEM is used in our filter design [10]

$$
\begin{aligned}
\theta_n &= \theta_{n-1} + K_n\varepsilon_n \\
K_n &= P_n\Psi_n \qquad\qquad (4) \\
P_n &= P_{n-1} - P_{n-1}\Psi_n[1 + \Psi_n^T P_{n-1}\Psi_n]^{-1} \; \Psi_n^T P_{n-1}
\end{aligned}
$$

θ_n is a vector of the predicted NP parameters of the system at iteration n, P_n is an NPxNP matrix, and ε_n and Ψ_n are given:

$$
\begin{aligned}
\Psi_n &= -\frac{\partial\varepsilon_n}{\partial\theta} \qquad\qquad (5) \\
\varepsilon_n &= \pm\sqrt{2\gamma_1 g_n^{(1)2} + \ldots + 2\gamma_1 g_n^{(NP)2} + \gamma_2 e_n^2}
\end{aligned}
$$

3. EXAMPLES

In this section, several designs of low sensitivity parallel and cascade form filters are examined. In each case, our desired response is the 4^{th}-order causal filter with transfer function

$$H(z) = \frac{0.294z^{-2} - 0.346z^{-3} + 0.1z^{-4}}{1 + 1.8z^{-1} + 1.74z^{-2} + 1.152z^{-3} + 0.34z^{-4}} \qquad (6)$$

Section 3.3 presents a direct form II example of the use of the technique in a low-pass filter design. This example directly supports the work from [8]. Furthermore, we have noted that designed direct form II filters with excess order have pole/zero cancellation pairs [7] which are Pareto Optimal.

3.1 Parallel Form

3.1.1 Weighting Effects

Different 4^{th}-order parallel form designs are realized by changing the weights (see Table 1). In general, the larger w_1, the smaller the sensitivity and the higher the error. However, this is not always true since the parallel form contains local minima which affects the parameter convergence. To minimize the effect of local minima, the direct form II designed systems were converted to parallel form and these parameters were used as initial parameters for the parallel form systems. Also, the parallel form filters need more iterations than the direct form II in order for the parameters to converge [11]. The use of simulated annealing methods should help.

3.1.2 Filter Design

Three different order filters are designed. The pole/zero plots are shown in Figure 1. Figure 1a shows the original system while Figure 3b is obtained from the 4^{th}-order designed system: the location of the poles are changed and a zero is pushed outside the unit circle. Figure 3c shows the 3^{rd}-order designed system; this figure is similar to the direct form II 3^{rd}-order designed filter [11]. Figure 1d shows the pole/zero plot of the 5^{th}-order designed system; this system contains double poles on the negative real axis. In fact, those double poles are located in the same sub-filter which results in a high sensitivity -- the sub-filter has a sensitivity of 365 while the whole system has a sensitivity of 374. We cannot explain this at present.

3.2 Cascade Form

3.2.1 Weighting Effects

We find that the convergence of the parameters depends on the initial values. Table 3 summarizes the results of using the direct form II output as initial parameter values in a 4^{th}-order design. In general, we can design systems with different sensitivity and error by changing the weights. During some designs, the change in the weights did not produce the expected change in sensitivity and impulse response error due to the presence of local minima in the multi-objective criterion. Incorporating a simulated annealing procedure improved performance.

3.2.2 Filter Design

Figure 2 shows the pole/zero plots. In Figure 2b, two of the poles are pushed a little further from each other to lie on the real axis while the other two poles are pushed away from the unit circle. Figure 4c shows the 3^{rd}-order system which is similar to the 3^{rd}-order parallel design. In Figure 1d, we notice a double pole at the left of the real axis. Those two poles exist in two different sub-filters, and so are somewhat isolated from each other. The results are summarized in Table 4.

3.3 A Butterworth Filter Design Example

To show some further capabilities of the design method, we examine the design of a 4^{th}-order lowpass butterworth filter, which has a cutoff frequency at frequency $\omega = 0.32$ radians. This filter has a direct form sensitivity $S(\theta) = 17.9x10^3$. A 6^{th}-order direct form II design can be achieved with a sensitivity less than one third of the sensitivity without affecting the corner frequency of the filter. The cost of the lower sensitivity is an increase in the reject band gain. While other implementations based on this designed filter are possible, they do not necessarily have a lower sensitivity than the corresponding implementation of the original 4^{th}-order butterworth filter. For instance, the optimal form implementation [12] of the 4^{th}-order filter has a sensitivity of 17, while that of the 6^{th}-order design has a sensitivity of 122. It is this fact which requires us to have algorithms for each filter implementation structure.

4. CONCLUSIONS

Low sensitivity systems produce low output quantization noise power since small changes in the parameters do not change the system behavior. A Pareto optimization procedure was used to produce low sensitivity systems with small impulse response errors by applying different relative weighting between the sensitivity and the impulse response error criteria. In parallel and cascade form filters, the parameter convergence contains local minima and therefore some designed filters had higher sensitivity and/or higher error than expected. This required the use of different initial conditions and more iterations to convergence. In most cases we were able to design low sensitivity filters with small errors. We wish to do more work concerning the use of simulated annealing.

Future work includes the use of Lagrange multipliers in a constrained search to design scaled fixed-point filters and the extension to other implementation forms.

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Table 1. Weighting effects on a parallel form filter

w_1	w_2	Sensitivity	Error
0.2	0.8	516	2.40e-05
0.4	0.6	488	1.15e-04
0.45	0.55	479	2.19e-04
0.4575	0.5425	248	4.83e-02
0.46	0.54	216	3.82e-02
0.5	0.5	121	4.86e-02

Table 2. Design of parallel form filters

System	Sensitivity	Error
desired	536	---
4th-order designed	216	3.82e-02
3rd-order designed	188	4.66e-02
5th-order designed	374	1.1e-03

Table 3. Weighting effects on a cascade form filter

w_1	w_2	Sensitivity	Error
0.37	0.63	468	1.4859e-04
0.39	0.61	454	4.0392e-04
0.395	0.605	247	1.58e-02
0.4	0.6	188	3.14e-02
0.45	0.55	82	5.57e-02
0.5	0.5	76	3.17e-02

Table 4. Design of cascade form filters

System	Sensitivity	Error
desired	501	---
4th-order designed	247	1.58e-02
3rd-order designed	86	3.88e-02
5th-order designed	79	4.89e-02

Figure 1.a, b, c and d: Pole/zero plots of parallel form filters

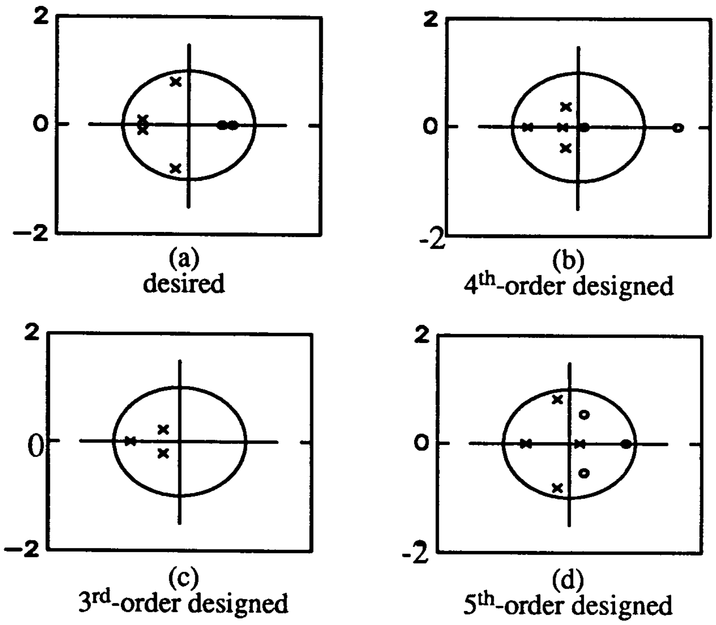

Figure 2.a, b, c and d: Pole/zero plots of cascade form filters