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OPTIMAL STRUCTURES FOR HIGH SPEED AND LOW ROUNDOFF NOISE DIGITAL FILTERS

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ABSTRACT

In this paper, we propose the optimization of the parallel and cascade connections of second-order sections, by selecting the best section for realizing each pair of complex-conjugate poles (and zeros). The optimization sense is, first, the minimization of the output roundoff noise power and, second, the minimization of the number of operations. The optimal second-order state-space section is the realization for those poles close to z=1 (or -1) and the second-order direct section for poles far from z=±1. These results are almost independent of the filter transfer function.

1. INTRODUCTION.

Digital implementation of discrete time systems implies finite precision in the representation of coefficients and operation results. The finite precision (quantization) effects have been studied during the last two decades and reported in many technical journals and conference proceedings. As is well known, the finite register length of digital realizations modifies the characteristics of the filters designed for infinite precision arithmetic. The transfer function degradation caused by quantization of filter coefficients, and the roundoff noise due to the operation result quantizations, are usually studied independently. However, some researchers concluded (by theoretical and experimental proofs) that structures of low-sensitivity with respect to the coefficients have also low roundoff noise power, and vice versa.

There is a tradeoff between minimum roundoff noise power and minimum number of multiplications in the implementation of a specific transfer function. Roughly speaking, for an nth-order filter, (n+1)(n+1) is the number of multiplications per output sample for the minimum roundoff noise state-space structure, and 2(n+1) for the direct structure (minimum number of operations), however direct structures have high roundoff noise power when poles are close to z=1 (or -1).

In [1] some results on roundoff noise power were given for several kinds of filters realized in parallel and cascade connections of second-order sections. For bandpass and bandstop filters, it was concluded that second-order direct sections have lower noise power than second-order optimal state-space sections, whenever both passband cutoff frequencies are in the interval $(0.2\pi,\ 0.8\pi)$. An interesting conclusion, not explicitly pointed out in [1], is that, for realizing Chebyshev and elliptic filters, the parallel form is better than the cascade form and, for Butterworth filters, cascade is better than parallel. We have observed differences in roundoff noise power up to 2, 7 and 20 dB for filter orders of 4, 8 and 12, respectively, when parallel and cascade forms are compared.

In this paper, we propose the optimization of the parallel and cascade connections of second-order sections, by selecting the best section for realizing each pair of complex-conjugate poles (and zeros). The optimization sense is, first, the minimization of the output roundoff noise power and, second, the minimization of the number of operations. The optimal second-order state-space section is the realization for those poles close to z=1 (or -1) and the second-order direct section for poles far from z=±1. These results are almost independent of the filter transfer function. The optimization criterion is explained an justified. Experimental results with Butterworth, Chebyshev and elliptic filters are given in curves in order to support the theory.

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2. OPTIMAL AND DIRECT SECOND-ORDER SECTIONS.

The signal flow graph of a second-order state-space section is shown in Fig. 1, and corresponds to the following state-space equations

$$\underline{x}(k+1) = \underline{A}\underline{x}(k) + \underline{b}u(k)$$

$$y(k) = \underline{c}\underline{x}(k) + du(k)$$

The state vector $\underline{x}(k)$ is 2×1, $y(k)$ and $u(k)$ are scalars, and the matrices $\underline{A}$, $\underline{b}$, $\underline{c}$ and d are of appropriate dimensions for compatibility:

$$\underline{A}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \underline{b}=\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad \underline{c}=[c_1 \; c_2],$$

d is a scalar

The transfer function $H(z)$ is expressed by

$$H(z)= \underline{c}(z\underline{I}-\underline{A})^{-1}\underline{b} + d$$

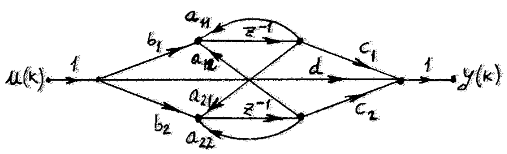

Fig. 1. Signal flow graph of a second order state-space section.

The optimal section is the state-space section with minimum "roundoff noise power" (σ^2), and satisfies the equivalent conditions [2,3]:

$$a_{11}=a_{22} \quad \text{and} \quad b_1c_1=b_2c_2$$

The second-order direct section satisfies:

$a_{11}=0$, $a_{12}=1$ and $b_1=0$, and the signal flow graph is shown in Fig. 2a. The transfer function $H(z)$ is

$$H(z) = d + \frac{b_2c_2z^{-1}+b_2c_1z^{-2}}{1 - a_{22}z^{-1}-a_{21}z^{-2}}$$

The second-order direct form II, corresponding to the transfer function

$$H(z) = \frac{\beta_0+\beta_1z^{-1}+\beta_2z^{-2}}{1 - \alpha_1z^{-1}-\alpha_2z^{-2}}$$

is shown in Fig. 2b, and the parameter relations are

$\alpha_1=a_{22}$, $\alpha_2=a_{21}$, $\beta_0=d$, $\beta_1=b_2c_2-da_{22}$,

$\beta_2=b_2c_1-da_{21}$

The direct sections have the minimum number of multiplications.

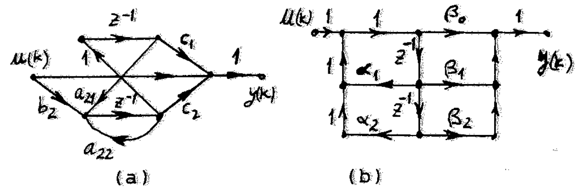

Fig. 2. Signal flow graph of second-order (a) direct section, (b) direct form II

3. COMPUTER RESULTS.

The roundoff noise power (σ^2) was computed for Butterworth, Chebyshev and elliptic low-pass and bandpass filters, realized in parallel and cascade connections of second-order sections. The fundamental parameters are the order n of the filter and the cutoff frequencies (Wc, Wc1 and Wc2). The maximum magnitude variation of the transfer function in passbands are 0.5 dB (roundoff noise power is strongly insensitive to this variation).

We have considered optimal (Fig.1) and direct (Fig.2a) sections. The optimal section is better than direct section when poles are close to $z=\pm1$, and direct is better than optimal for poles far from $z=\pm1$. The hybrid structure is composed of the best sections for realizing each pair of complex-conjugate poles, and has lesser both roundoff noise power and number of operations than the structure of optimal sections.

Considering the hybrid structure and representing in the z-plane the poles only realized by the direct sections, we have the diagram shown in the Fig.5. The shadow zones inside the unit circle and delimited by the discontinuous lines are the

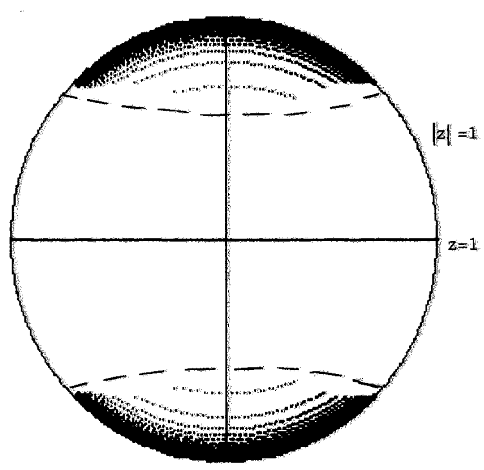

Fig.5. Shadow zones are the locations of complex-conjugate poles realized by direct sections in hybrid structures.

locations of complex-conjugate poles whose direct second-order realizations are better than optimal ones.

Figs.3 show output-roundoff noise power of some kinds of filters realized in parallel. For each figure and n fixed, we can see two curves: the continuous line corresponds to a structure of only optimal sections, the discontinuous line corresponds to the hybrid structure composed of the best combination of direct and optimal sections.

Figs.4 show the cascade case, but Chebyshev filters are not considered because results are similar to the elliptic ones (compare Fig.3b and Fig.3d). On the other hand, no optimization was done by ordering the sections.

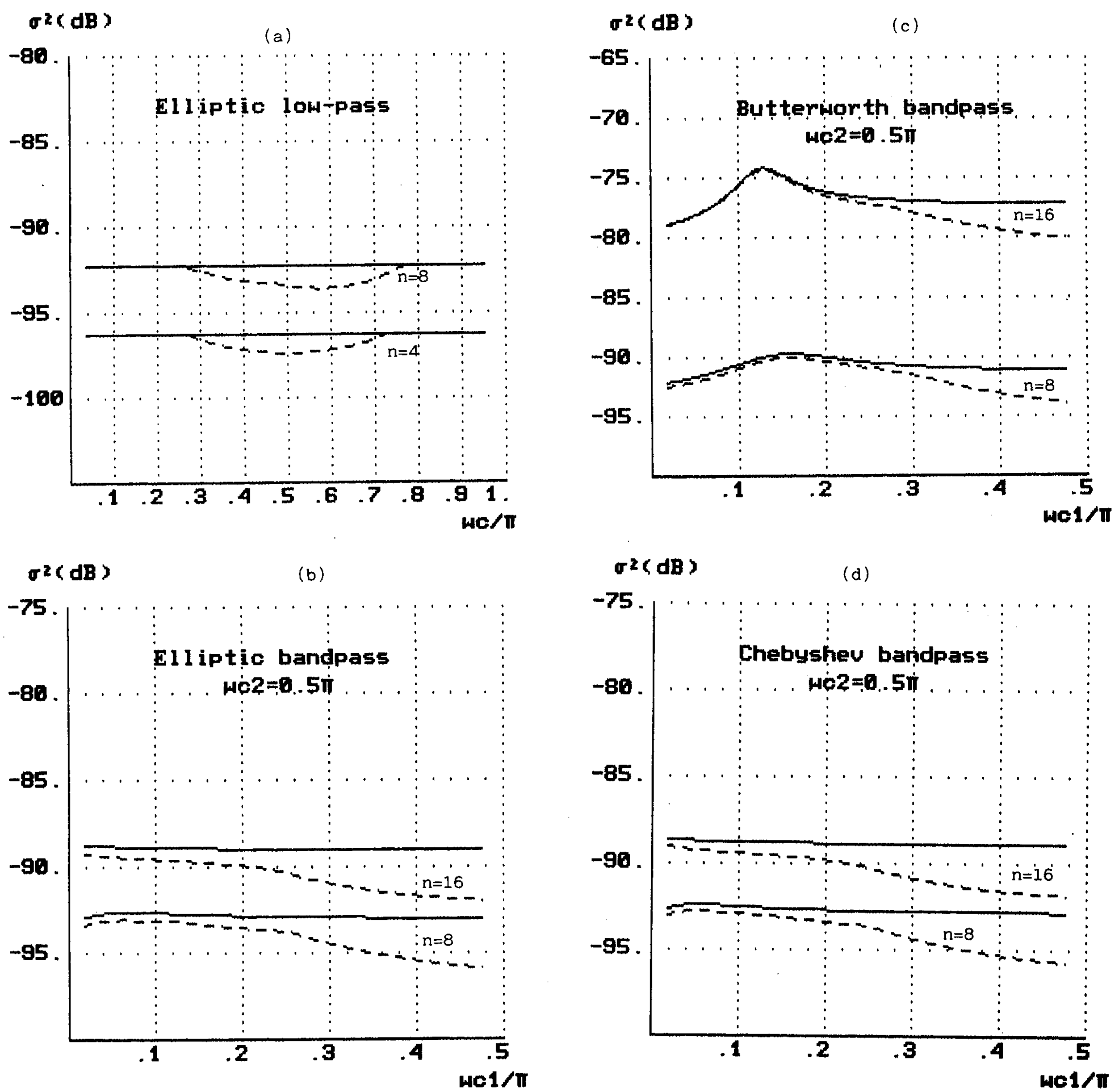

Fig.3. Output-noise power versus cutoff frequencies for nth-order filters realized in PARALLEL connections of second-order sections and fixed-point 16-bit numbers. (a) Low-pass filters, Wc is the cutoff frequency. (b), (c) and (d) Bandpass filters, Wc1 and Wc2 are the lower and the upper cutoff frequencies, respectively.

—— optimal sections, - - - hybrid sections

4. REFERENCES.

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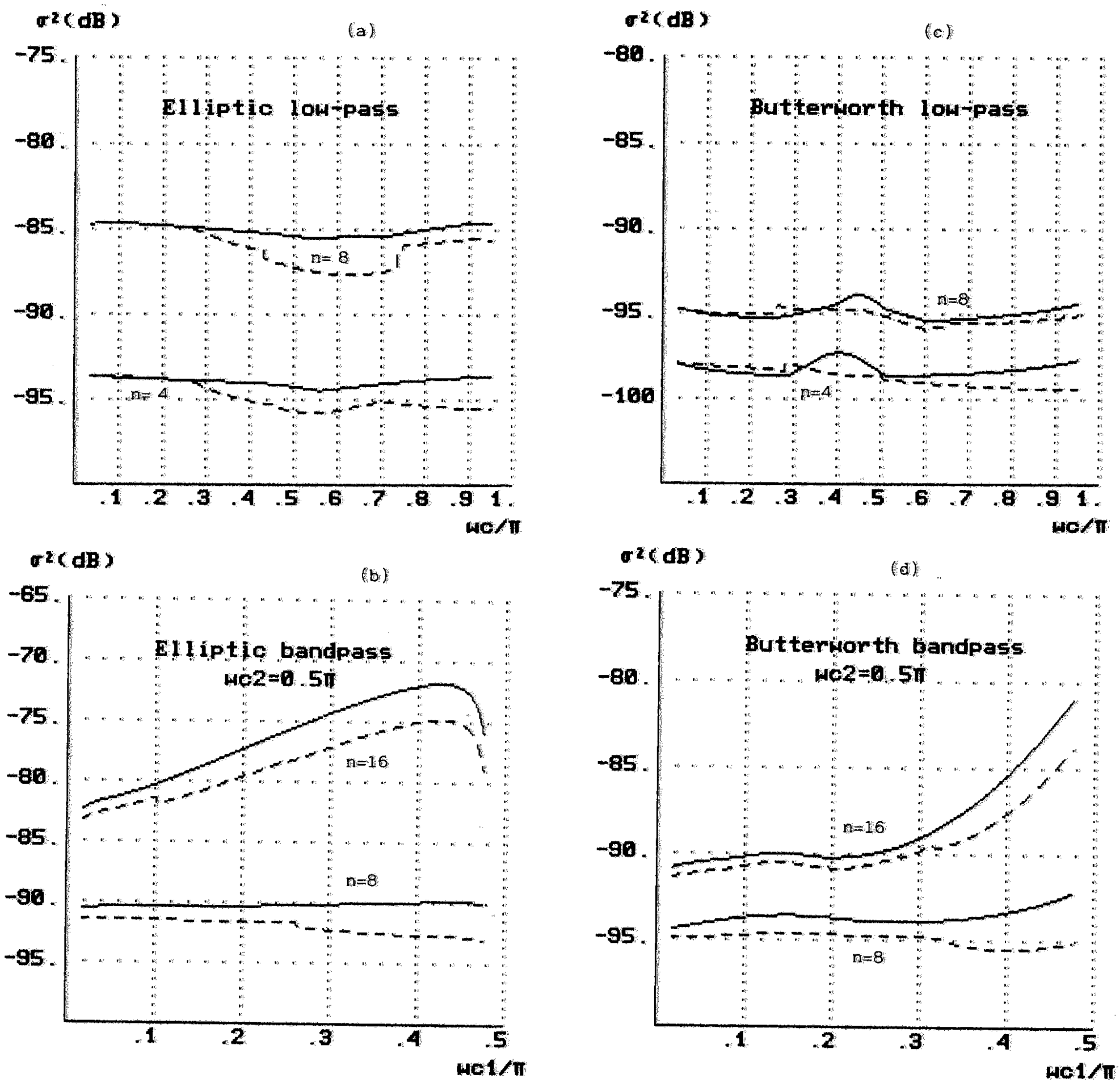

Fig.4. Output-noise power versus cutoff frequencies for nth-order filters realized in CASCADE connections of second-order sections and fixed-point 16-bit numbers. (a) and (c) Low-pass filters, Wc is the cutoff frequency. (b) and (d) Bandpass filters, Wc1 and Wc2 are the lower and the upper cutoff frequencies, respectively.

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