Transistor Hybrid Model for AC Analysis

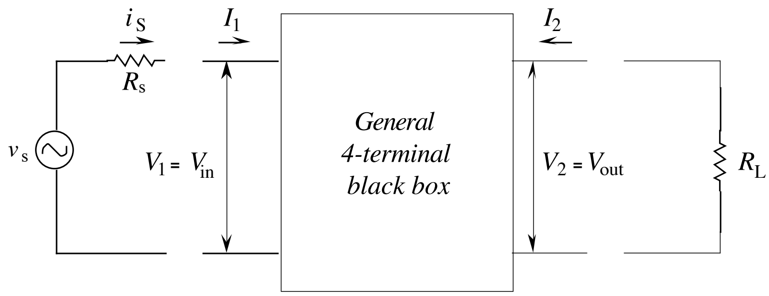


Figure 1

There are four variables: I_1, V_1, I_2, and V_2; they represent the instantaneous values of the currents and voltages. We can write two Kirchhoff voltage equations.

One for the input loop $f(I_1, V_1, I_2, V_2) = 0$

and

one for the output loop $g(I_1, V_1, I_2, V_2) = 0$.

The exact form of the function *f* and *g* will depend on the contents of the black box. We will pick two of the four variables to be independent (I_1, V_2) and two to be dependent (I_2, V_1). Starting with the function *f* we solve for V_1, then substitute V_1 into function *g*. Finally, we solve for I_2 in terms of I_1, and V_2. In a similar manner we can eliminate I_2 and solve for V_1 in terms of I_1, and V_2. We obtain the following general functions

$$I_2 = I_2(I_1, V_2) \qquad and \qquad V_1 = V_1(I_1, V_2).$$

In general we are interested in the response of the transistor to the application of small, low frequency AC input signals (differential changes). To obtain the expressions for the change in I_2 and V_1 as a function of the changes in I_1 and V_2 we take the differentials of the above functions.

Model Equations: $V_1 = V_1(I_1, V_2)$ and $I_2 = I_2(I_1, V_2)$

$$dI_2 = \left(\frac{\partial I_2}{\partial I_1}\right)_{V_2} dI_1 + \left(\frac{\partial I_2}{\partial V_2}\right)_{I_1} dV_2 \qquad \text{and} \qquad dV_1 = \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2} dI_1 + \left(\frac{\partial V_1}{\partial V_2}\right)_{I_1} dV_2$$

Letting lower case letters represent small AC voltages and currents

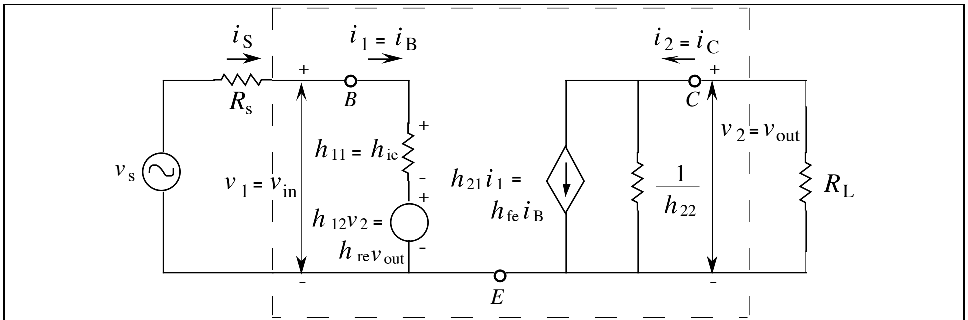
$$i_1 \equiv dI_1 \qquad i_2 \equiv dI_2 \qquad v_1 \equiv dV_1 \qquad and \qquad v_2 \equiv dV_2$$

and defining hybrid parameters h_{xy} yields

$$i_2 = \left(\frac{\partial I_2}{\partial I_1}\right)_{V_2} i_1 + \left(\frac{\partial I_2}{\partial V_2}\right)_{I_1} v_2 \qquad = \qquad h_{21}\, i_1 + h_{22}\, v_2$$

and

$$v_1 = \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2} i_1 + \left(\frac{\partial V_1}{\partial V_2}\right)_{I_1} v_2 \qquad = \qquad h_{11}\, i_1 + h_{12}\, v_2 .$$



$v_1 = h_{11}\, i_1 + h_{12}\, v_2$	$i_2 = h_{21}\, i_1 + h_{22} v_2$

Figure 2

$h_{21} \equiv \left(\dfrac{\partial I_2}{\partial I_1}\right)_{V_2}$	a pure number *forward current transfer ratio*	represents how much of the input current i_1 is transferred to the output current i_2; The higher the value of h_{21}, the larger the change in output current for a given input current change.
$h_{22} \equiv \left(\dfrac{\partial I_2}{\partial V_2}\right)_{I_1}$	a conductance *output admittance*	an admittance or conductance directly across the output terminals
$h_{11} \equiv \left(\dfrac{\partial V_1}{\partial I_1}\right)_{V_2}$	a resistance *input resistance*	a resistance in the input circuit; it is usually called the input resistance
$h_{12} \equiv \left(\dfrac{\partial V_1}{\partial V_2}\right)_{I_1}$	a pure number *reverse voltage transfer ratio*	the amplitude of a voltage generator in the input; it represents how much of the output voltage v_2 is fed back into the input

Table 1

This mathematical development is exact; that is, no approximations have been made except that v and i must refer to small signals because the starting equations hold exactly only for infinitesimal changes in v's and i's. To show that this mathematical model can represent a *real* transistor a comparison of experimental *input* and *output* curves for a transistor must be looked at and h_{xy} values based on these curves must be chosen. The model will represent the real transistor if for a chosen set of h_{xy} parameters they represent the experimental curves over a wide range of currents and voltages.

The mathematical model developed above is a four terminal device while a transistor is a three terminal device. To apply this model to a transistor two of the terminals must be common between the input and output. Three configurations are possible: common emitter, common collector, and common base. The h_{xy} parameters will be different for each configuration. When a configuration is chosen letter subscripts are usually used instead of the numerical subscripts. The second letter denotes which terminal is common. In general the values of a particular h_{xy} are different for each configuration, e.g., $h_{fe} \neq h_{fb}$.

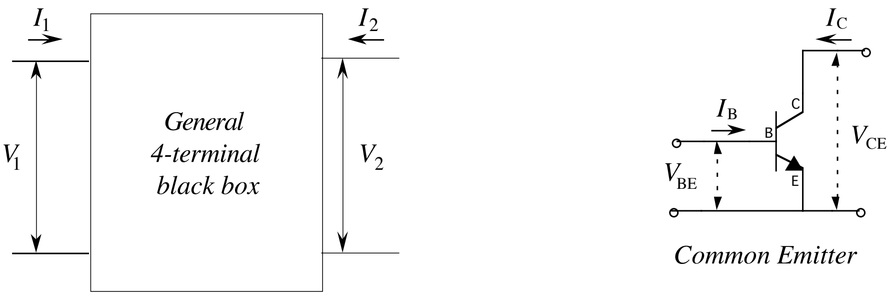
***h* Parameter Notation**			
***h* parameter**	**CE**	**CC**	**CB**
h_{11}	h_{ie}	h_{ic}	h_{ib}
h_{12}	h_{re}	h_{re}	h_{rb}
h_{21}	h_{fe}	h_{fc}	h_{fb}
h_{22}	h_{oe}	h_{oc}	h_{ob}
	Common Emitter	*Common Collector*	*Common Base*

Table 2

Equations for transistor alone (no biasing, collector, emitter, or load resistors	
Voltage Gain	$A_v = \dfrac{-h_{21}R_L}{(h_{11}h_{22} - h_{12}h_{21})R_L + h_{11}}$
Current Gain	$A_i = \dfrac{h_{21}}{1 + h_{22}R_L}$
Input Impedance	$Z_{T_{in}} = \dfrac{(h_{11}h_{22} - h_{12}h_{21})R_L + h_{11}}{1 + h_{22}R_L}$
Output Impedance	$Z_{T_{out}} = \dfrac{h_{11} + R_S}{(h_{11}h_{22} - h_{12}h_{21}) + h_{22}R_S}$

Approximate Gains & Impedances for Transistor Circuits			
	CE	**CC**	**CB**
A_v	$-\dfrac{h_{fe}(R_C \parallel R_L)}{h_{ie}} \approx -h_{fe} \approx 100$	1	$\dfrac{h_{fe}R_L}{h_{ie}} \approx h_{fe} \approx 100$
A_i	$h_{fe} \approx 100$	$-h_{fe} \approx -100$	1
Z_{in}	$h_{ie} \parallel R_1 \parallel R_2$	$\sim h_{fe}(R_E \parallel R_L \parallel R_1 \parallel R_2)$	$\dfrac{h_{ie}}{h_{fe}} \parallel R_1 \parallel R_2$
Z_{out}	$\dfrac{h_{ie}}{(h_{11}h_{22} - h_{12}h_{21})} \parallel R_C \cong 5k\Omega \parallel R_C$	$\sim \dfrac{h_{ie}}{h_{fe}} \parallel R_E$	$\dfrac{h_{fe}R_S}{h_{11}h_{22} - h_{12}h_{21}} \parallel R_C$

Common Emitter Configuration

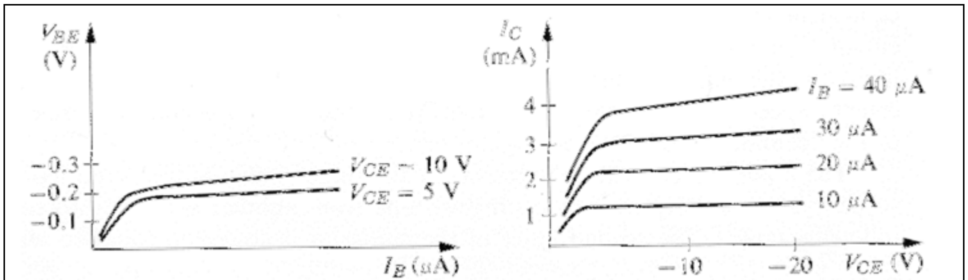


Common Emitter

$$I_1 \to I_B, \quad V_1 \to V_{BE}, \quad I_2 \to I_C, \quad V_2 \to V_{CE}$$

Figure 3

The experimental curves we look at are $I_C = I_C\left(I_B, V_{CE}\right)$ and $V_{BE} = \left(I_B, V_{CE}\right)$.



Experimental input and output graphs for low power germanium transistor in the common emitter configuration

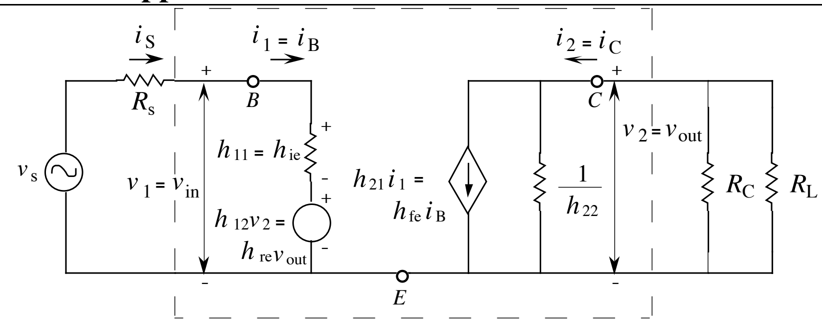
Figure 4

Common Emitter Configuration		
$h_{21} \equiv \left(\frac{\partial I_2}{\partial I_1}\right)_{V_2} \Rightarrow h_{fe} = \left(\frac{\partial I_C}{\partial I_B}\right)_{V_{CE}}$	*forward current transfer ratio* (a pure number)	$h_{fe} \approx 100$ AC current gain $h_{FE} = \beta = \frac{I_C}{I_B}$ DC current gain $h_{fe} \approx h_{FE}$ roughly constant for $\lvert V_{CE}\rvert \geq 1V$ although; it increases slightly as I_C increases. It decreases drastically when $\lvert V_{CE}\rvert$ is well below $1V$. Choose a quiescent point with V_{CE} greater than $1V$.
$h_{22} \equiv \left(\frac{\partial I_2}{\partial V_2}\right)_{I_1} \Rightarrow h_{oe} = \left(\frac{\partial I_C}{\partial V_{CE}}\right)_{I_B}$	*output admittance* (a conductance)	$h_{oe} \approx 2\times10^{-5}$ *siemens* For $V_{CE} > 1V$ it is essentially constant. It does increase with an increase in I_C
$h_{11} \equiv \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2} \Rightarrow h_{ie} = \left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}} = \left(\frac{\partial I_B}{\partial V_{BE}}\right)_{V_{CE}}^{-1}$ $h_{11} = \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2} = \left(\frac{\partial V_1}{\partial I_2}\frac{\partial I_2}{\partial I_1}\right)_{V_2} = h_{21}\left(\frac{\partial V_1}{\partial I_2}\right)_{V_2}$ $\frac{h_{11}}{h_{21}} = \left(\frac{\partial V_1}{\partial I_2}\right)_{V_2} \Rightarrow \frac{h_{ie}}{h_{fe}} = \left(\frac{\partial V_{BE}}{\partial I_C}\right)_{V_{CE}}$	*input resistance* (a resistance)	$h_{ie} = \frac{kT}{eI_B} = \frac{\beta kT}{eI_C}$ $h_{ie}(\beta = 100, T = 300°K) = \frac{2.59}{I_C}$ $v_{BE} = i_B h_{ie}$ where h_{ie} is the effective AC resistance of the forward biased base-emitter diode junction. h_{ie} is typically 2.6kΩ for I_C = 1 mA, and h_{ie} varies inversely with I_C (h_{ie} = 260Ω for I_C = 10 mA).
$h_{12} \equiv \left(\frac{\partial V_1}{\partial V_2}\right)_{I_1} \Rightarrow h_{re} = \left(\frac{\partial V_{BE}}{\partial V_{CE}}\right)_{I_B}$	*reverse voltage transfer ratio* (a pure number)	$h_{re} \approx 10^{-4}$ represents how much of the output voltage v_2 is fed back into the input; roughly constant for $I_B > 10\mu A$

Table 3

The h_{xy} parameters are essentially constant when $V_{CE} > 1V$, $I_B > 10\mu A$ ($I_C > 1$ mA).

Application of Kirchhoff's and Ohm's Laws



$(a)\ v_1 = h_{11}\, i_1 + h_{12}\, v_2$	$(c)\ i_2 = h_{21}\, i_1 + h_{22} v_2$
$(b)\ v_S = \left(R_S + h_{11}\right) i_1 + h_{12} v_2$	$(d)\ v_2 = -i_2\left(R_C \parallel R_L\right)$

Table 4

Voltage Gain

The voltage gain is defined as $A_v \equiv v_{out}/v_{in} = v_2/v_1$. Using equations (*a*) – (*d*) we obtain

$$i_2 = -\frac{v_2}{\left(R_C \parallel R_L\right)} = h_{21} i_1 + h_{22} v_2 \qquad \text{and} \qquad i_1 = \frac{v_1 - h_{12} v_2}{h_{11}} = \frac{v_S - h_{12} v_2}{\left(R_S + h_{11}\right)}$$

Subtituting for i_1

$$-\frac{v_2}{\left(R_C \parallel R_L\right)} = h_{21}\left(\frac{v_1 - h_{12} v_2}{h_{11}}\right) + h_{22} v_2$$

and solving for v_2/v_1

$$A_v = \frac{v_2}{v_1} = \frac{h_{21}}{h_{12} h_{21} - h_{11} h_{22} - \dfrac{h_{11}}{R_C \parallel R_L}} = \frac{h_{fe}}{h_{re} h_{fe} - h_{ie} h_{oe} - \dfrac{h_{ie}}{R_C \parallel R_L}}$$

For typical values for a low power transistor

$$A_v \approx \frac{-\,h_{21}}{h_{11}}\ R_C \parallel R_L = \frac{-\,h_{fe}}{h_{ie}}\ R_C \parallel R_L$$

Current Gain

The current gain is defined as $A_i \equiv i_2/i_1$. substituting $-i_2\left(R_C \parallel R_L\right)$ for v_2 yields

$$i_2 = h_{21} i_1 + \left[-i_2\left(R_C \parallel R_L\right)\right] h_{22}$$

Solving for i_2/i_1

$$A_i = \frac{i_2}{i_1} = \frac{h_{21}}{1 + h_{22}\left(R_C \parallel R_L\right)} = \frac{h_{fe}}{1 + h_{oe}\left(R_C \parallel R_L\right)}$$

Input Impedance

Looking into the transistor

$$Z_{T_{in}} \equiv \frac{v_1}{i_1} = h_{11} - \frac{h_{12}h_{21}}{h_{22} + \dfrac{1}{R_C \parallel R_L}} \approx h_{11} = h_{ie}$$ (for typical low power transistor values)

What the source v_S sees

The source drives both the transistor and the bias network. The impedance the source sees is

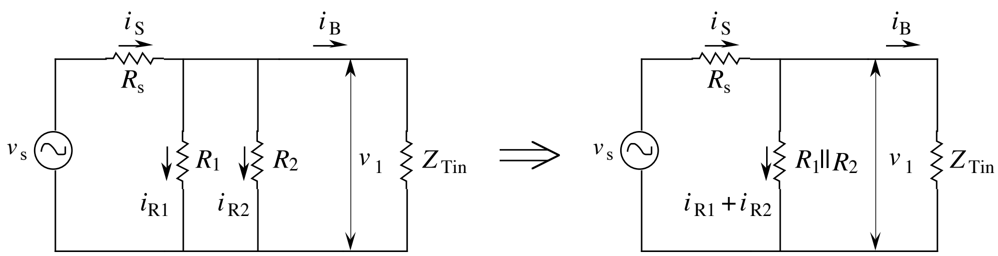


Figure 5

$$\frac{1}{Z_S} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{Z_{T_{in}}} \quad \Rightarrow \quad Z_{S_{in}} \approx Z_{T_{in}} \quad \text{if} \quad R_1 \parallel R_2 \gg Z_{T_{in}}$$

If $R_1 \parallel R_2$ is small compared to $Z_{T_{in}}$ then most of the current supplied by the signal source will flow through $R_1 \parallel R_2$, thus lowering the i_B. This results in lowering the transistor output (i.e, lowering the transistor gain).

Output Impedance

The output impedance looking back into the two output terminals is important for two reasons:

1. Determines the high frequency response of the circuit (particularly when driving a capacitive load where it represents the resistance in a low-pass filter)
2. Determines how much power is transferred to the load attached to the two output terminals.

Another thing to note is that for a given fixed voltage source and a fixed transistor output impedance:

1. The output voltage across the load is maximum if $R_L \gg Z_{T_{out}}$
2. The output current through the load will be a maximum when $R_L \ll Z_{T_{out}}$
3. For purely a resistive load and output impedance of the transistor the output power through the load will be maximum when $R_L = R_{out}$. If the load is fixed, maximum power is delivered to the load when $Z_{T_{out}} = 0$.

To calculate the transistor output impedance in terms of the h_{xy} parameters we again start with the equations in Table 4.

$$v_S - i_1 R_S = v_1 = h_{11} i_1 + h_{12} v_2$$
$$i_2 = h_{21} i_1 + h_{22} v_2$$

If we let v_S be set to zero to avoid letting the output depend on the applied input signal (v_S=0) then the transistor output impedance $Z_{T_{out}} = v_2/i_2$ can be obtained by eliminating i_1.

$$-i_1 R_S = v_1 = h_{11} i_1 + h_{12} v_2$$

$$i_1 = -\frac{h_{12} v_2}{h_{11} + R_S}$$

$$i_2 = h_{21} i_1 + h_{22} v_2$$

$$i_2 = h_{21}\left(-\frac{h_{12} v_2}{h_{11} + R_S}\right) + h_{22} v_2$$

$$i_2 = \left\{\frac{(h_{11} + R_S) h_{22} - h_{12} h_{21}}{h_{11} + R_S}\right\} v_2$$

$$Z_{T_{out}} \equiv \frac{v_2}{i_2} = \frac{h_{11} + R_S}{(h_{11} h_{22} - h_{12} h_{21}) + h_{22} R_S} = \frac{h_{ie} + R_S}{(h_{ie} h_{oe} - h_{re} h_{fe}) + h_{oe} R_S}$$

Note:

1. If the reverse voltage generator is not present ($h_{12} = h_{re} = 0$) then $Z_{T_{out}} = 1/h_{22} = 1/h_{oe}$ which is reasonable since h_{22} (h_{oe}) is an admittance.
2. When $h_{12} = h_{re} \neq 0$ then the transistor output impedance is a function of h_{11} (h_{ie}) and R_S due to the coupling provided by h_{12} (h_{re}).

The output impedance of a common emitter amplifier is $Z_{out} = Z_{T_{out}} \parallel R_C \approx R_C$ when R_C is in the range of 2 to 20kΩ

P-N Junction	
The current-voltage equation for a *p-n* junction can be modeled as	
$I(V_B, T) = I_0\left(e^{eV_B/kT} - 1\right)$	e electronic charge, 1.60x10^{-19} C k Boltzmann's constant, 1.38x10^{-23} J/°K T absolute temperature °K V_B is positive for forward bias, negative for reverse bias I_0 the reverse current for large junction reverse bias
$I(V_B, T = 300°K) = I_0\left(e^{V_B/(25.875 mV)} - 1\right)$	$\left(\frac{kT}{e}\right)_{T=300°K} = \frac{300 \times 1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 0.025875V$

Table 5

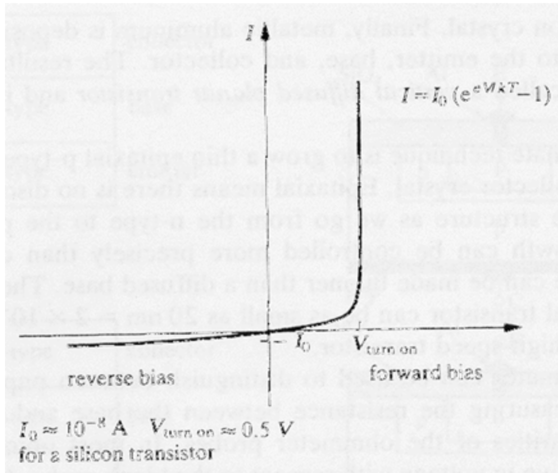


Figure 6

For the *base-emitter p-n* junction of the transistor the base-emitter current is

$$I_E\left(V_{BE},T\right) = I_0\left(e^{eV_{BE}/kT}-1\right)$$

$$I_E\left(V_{BE},T\right) = \left(\beta+1\right)I_B$$

$$I_B\left(V_{BE},T,\beta\right) = \frac{I_0}{\left(\beta+1\right)}\left(e^{eV_{BE}/kT}-1\right)$$

$$\frac{\partial I_B}{\partial V_{BE}} = \frac{e\,I_0}{\left(\beta+1\right)kT}\,e^{eV_{BE}/kT}.$$

$$V_{BE} = \frac{kT}{e}\ln\left(\frac{\left(\beta+1\right)I_B}{I_0}+1\right)$$

$$h_{11} = \left(\frac{\partial V_1}{\partial I_1}\right)_{V_2} \quad\Rightarrow\quad h_{ie} = \left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}} = \left(\frac{\partial I_B}{\partial V_{BE}}\right)_{V_{CE}}^{-1} = \frac{\left(\beta+1\right)kT}{e\,I_0}\,e^{-eV_{BE}/kT}$$

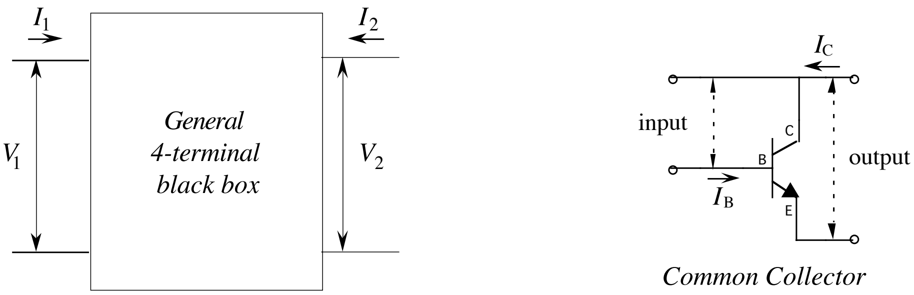
If $V_{BE} > 0.6V$ or equivalently if $I_B > 0.1\mu A$ then $I_C > 10\mu A$. The transistor is turned on and $e^{eV_{BE}/kT} \gg 1$. We can neglect the minus one term in the base current.

$$I_B\left(V_{BE},T,\beta\right) \approx \frac{I_0}{\left(\beta+1\right)}\,e^{eV_{BE}/kT}$$

The effective *AC* signal resistance of base-emitter junction is

$$h_{ie} \equiv \left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}} = \left(\frac{\partial I_B}{\partial V_{BE}}\right)_{V_{CE}}^{-1} = \frac{\left(\beta+1\right)kT}{e\,I_0}\,e^{-eV_{BE}/kT} = \frac{\left(\beta+1\right)kT}{e\,I_0\,e^{eV_{BE}/kT}} = \frac{kT}{e\,I_B} = \frac{\beta\,kT}{e\,I_C}.$$

Common Collector Configuration (Emitter Follower)



Common Collector

$$I_1 \to I_B, \quad V_1 \to V_{BE}, \quad I_2 \to I_C, \quad V_2 \to V_{CE}$$

Figure 7

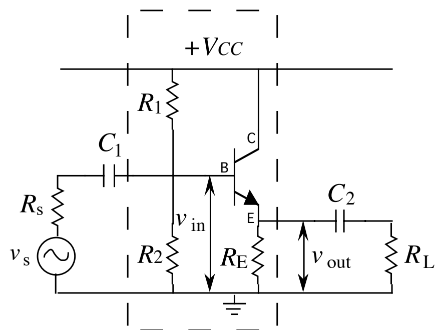
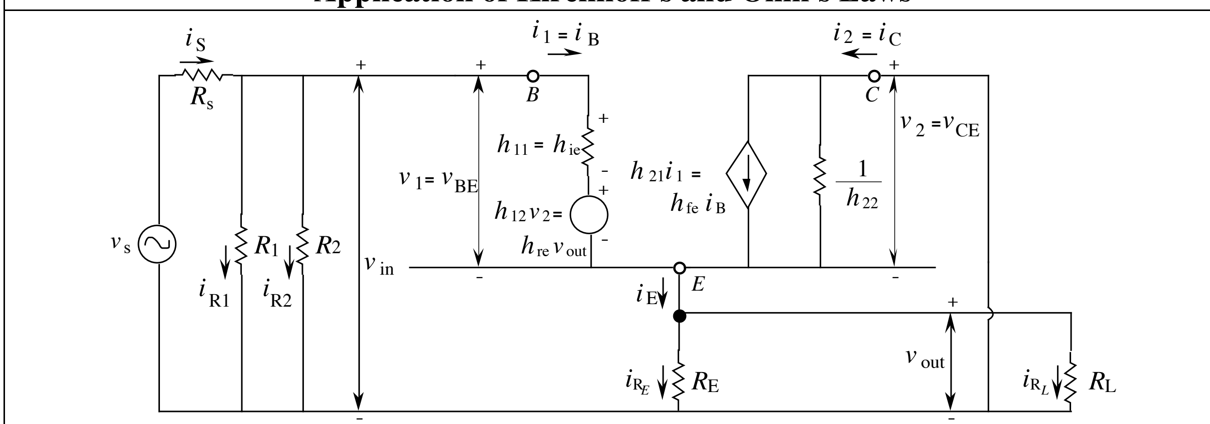


Figure 8

Application of Kirchhoff's and Ohm's Laws



$(a)\ v_1 = h_{11}i_1 + h_{12}v_2$ $(b)\ v_{in} = v_1 + v_{out}$	$(c)\ i_2 = h_{21}i_1 + h_{22}v_2$ $(d)\ v_{out} = i_{R_E}R_E = i_{R_L}R_L = -v_2$
$(e)\ i_1 + i_2 = i_E = i_{R_E} + i_{R_L}$ (*from current conservation*)	

Table 6

Current Gain

$$A_i = \frac{i_{out}}{i_1} = \frac{i_{R_L}}{i_1}$$

1) $i_1 + i_2 = i_{R_E} + i_{R_L}$

2) $i_2 = h_{21} i_1 + h_{22} v_2$

$$\left(1+h_{21}\right) i_1 + \left(h_{22} + \frac{1}{R_E} + \frac{1}{R_L}\right) v_2 = 0$$

3) $v_2 = i_{R_L} R_L$

$$\left(1+h_{21}\right) i_1 + \left(h_{22} + \frac{1}{R_E} + \frac{1}{R_L}\right)\left(-i_{R_L} R_L\right) = 0$$

$$A_i = \frac{\left(1+h_{21}\right)}{1 + \frac{R_L}{R_E} + h_{22} R_L}$$

$$= \frac{\left(1+h_{fe}\right)}{1 + \frac{R_L}{R_E} + h_{oe} R_L} \approx h_{21} = h_{fe}$$

Voltage Gain

$$A_v = \frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_1 + v_{out}}$$

1) $i_1 + i_2 = i_{R_E} + i_{R_L}$

2) $i_2 = h_{21} i_1 + h_{22} v_2$

$$\left(1+h_{21}\right) i_1 + \left(h_{22} + \frac{1}{R_E} + \frac{1}{R_L}\right) v_2 = 0$$

$$i_1 = -\frac{\left(h_{22} + \frac{1}{R_E} + \frac{1}{R_L}\right)}{\left(1+h_{21}\right)} v_2$$

3) $v_1 = h_{11} i_1 + h_{12} v_2$

$$v_1 = \left[h_{12} - \frac{h_{11}\left(h_{22} + \frac{1}{R_E} + \frac{1}{R_L}\right)}{\left(1+h_{21}\right)}\right] v_2 = \left[\frac{h_{11}\left(h_{22} + \frac{1}{R_E} + \frac{1}{R_L}\right)}{\left(1+h_{21}\right)} - h_{12}\right] v_{out}$$

$$
\begin{aligned}
A_v &= \frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_1 + v_{out}} \\
&= \frac{v_{out}}{\left\{\left[\dfrac{h_{11}\left(h_{22} + \dfrac{1}{R_E} + \dfrac{1}{R_L}\right)}{\left(1+h_{21}\right)} - h_{12}\right] + 1\right\} v_{out}} \\
&= \frac{1+h_{21}}{\left(1-h_{12}\right)\left(1+h_{21}\right) + h_{11}h_{22} + \dfrac{h_{11}}{R_E} + \dfrac{h_{11}}{R_L}} \\
&= \frac{1+h_{fe}}{\left(1-h_{re}\right)\left(1+h_{fe}\right) + h_{ie}h_{oe} + \dfrac{h_{ie}}{R_E} + \dfrac{h_{ie}}{R_L}} \approx 1
\end{aligned}
$$

The voltage gain can be seen to be almost exactly 1.0. If the base-emitter junction is forward biased, V_{BE} will remain essentially constant at 0.6V for a silicon transistor. If the input changes by ΔV, V_{B} changes by ΔV and V_{E} will change by approximately the same amount to keep V_{BE} constant. The output voltage change approximately equals the input voltage change $\Delta V_E = \left(0.98 \text{ or } 0.99\right) \times \Delta V_B$. The change in emitter voltage is in phase with the change in base voltage.

Input Impedance

$$Z_{in} = \frac{v_{in}}{i_{in}}$$

The input impedance looking into the base of the transistor is

$$Z_{B_{in}} = h_{11} + \left(1+h_{21}\right)\left(R_E \parallel R_L\right) = h_{ie} + \left(1+h_{fe}\right)\left(R_E \parallel R_L\right)$$

The input impedance of the circuit is the parallel combination of the base resistors R_1 and R_2 and the input impedance looking into the base of the transistor.

$$Z_{in} = \frac{1}{\dfrac{1}{R_1 \parallel R_2} + \dfrac{1}{h_{ie} + \left[\left(1+h_{fe}\right)\left(R_E \parallel R_L\right)\right]}} \approx \frac{1}{\dfrac{1}{R_1 \parallel R_2} + \dfrac{1}{h_{ie} + h_{fe}\left(R_E \parallel R_L\right)}}$$

Output Impedance

The output impedance looking back into the emitter of the transistor is

$$
\begin{aligned}
Z_{out} &= \frac{v_{out}}{i_{out}} \\
&= \frac{1}{\dfrac{1}{R_E} + h_{22} + \dfrac{1}{\dfrac{\left(R_1 \parallel R_2 \parallel R_S\right) + h_{11}}{\left(1 + h_{21}\right)}}} \\
&= \frac{1}{\dfrac{1}{R_E} + h_{oe} + \dfrac{1}{\dfrac{\left(R_1 \parallel R_2 \parallel R_S\right) + h_{ie}}{\left(1 + h_{fe}\right)}}} \approx \frac{1}{\dfrac{1}{R_E} + \dfrac{1}{\left[\dfrac{\left(R_1 \parallel R_2 \parallel R_S\right) + h_{ie}}{h_{fe}}\right]}}
\end{aligned}
$$