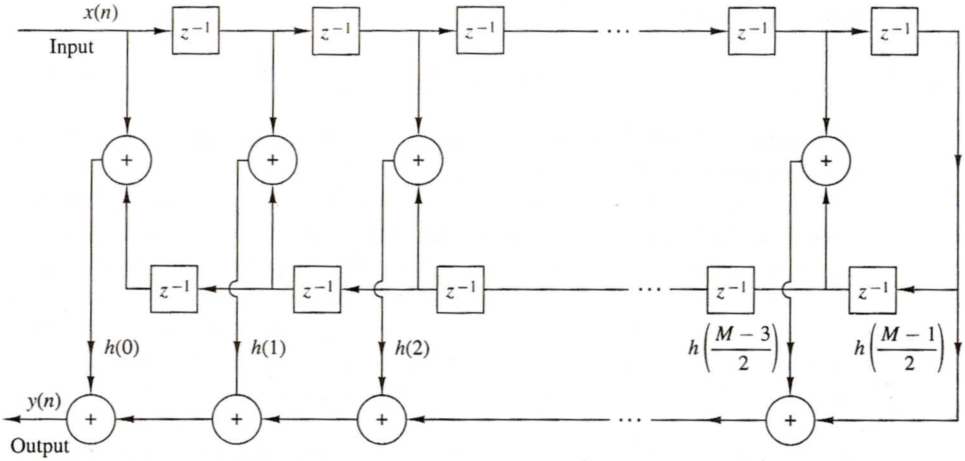

Figure 7.2 Direct-form realization of linear-phase FIR system (M odd).

system. Consequently, the direct-form realization is often called a transversal or tapped-delay-line filter.

When the FIR system has linear phase, as described in Section 8.2, the unit sample response of the system satisfies either the symmetry or asymmetry condition

$$h(n) = \pm h(M - 1 - n) \tag{7.2.5}$$

For such a system the number of multiplications is reduced from M to $M/2$ for M even and to $(M-1)/2$ for M odd. For example, the structure that takes advantage of this symmetry is illustrated in Fig. 7.2 for the case in which M is odd.

7.2.2 Cascade-Form Structures

The cascade realization follows naturally from the system function given by (7.2.2). It is a simple matter to factor $H(z)$ into second-order FIR systems so that

$$H(z) = \prod_{k=1}^{K} H_k(z) \tag{7.2.6}$$

where

$$H_k(z) = b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2} \qquad k = 1, 2, \ldots, K \tag{7.2.7}$$

and K is the integer part of $(M+1)/2$. The filter parameter b_0 may be equally distributed among the K filter sections, such that $b_0 = b_{10}b_{20}\cdots b_{K0}$ or it may be assigned to a single filter section. The zeros of $H(z)$ are grouped in pairs to produce the second-order FIR systems of the form (7.2.7). It is always desirable to form pairs of complex-conjugate roots so that the coefficients $\{b_{ki}\}$ in (7.2.7) are real valued. On the other hand, real-valued roots can be paired in any arbitrary manner. The cascade-form realization along with the basic second-order section are shown in Fig. 7.3.

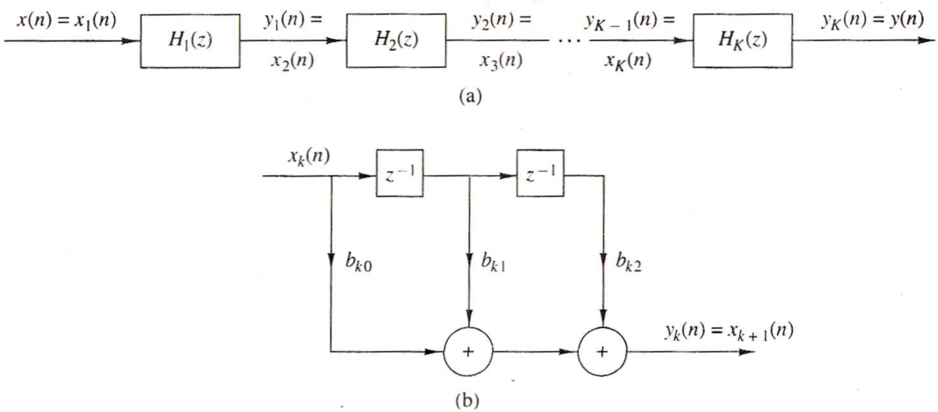

Figure 7.3 Cascade realization of an FIR system.

In the case of linear-phase FIR filters, the symmetry in $h(n)$ implies that the zeros of $H(z)$ also exhibit a form of symmetry. In particular, if z_k and z_k^* are a pair of complex-conjugate zeros then $1/z_k$ and $1/z_k^*$ are also a pair of complex-conjugate zeros (see Sec. 8.2). Consequently, we gain some simplification by forming fourth-order sections of the FIR system as follows

$$\begin{aligned} H_k(z) &= c_{k0}(1 - z_k z^{-1})(1 - z_k^* z^{-1})(1 - z^{-1}/z_k)(1 - z^{-1}/z_k^*) \\ &= c_{k0} + c_{k1} z^{-1} + c_{k2} z^{-2} + c_{k1} z^{-3} + z^{-4} \end{aligned} \tag{7.2.8}$$

where the coefficients $\{c_{k1}\}$ and $\{c_{k2}\}$ are functions of z_k. Thus, by combining the two pairs of poles to form a fourth-order filter section, we have reduced the number of multiplications from six to three (i.e., by a factor of 50%). Figure 7.4 illustrates the basic fourth-order FIR filter structure.

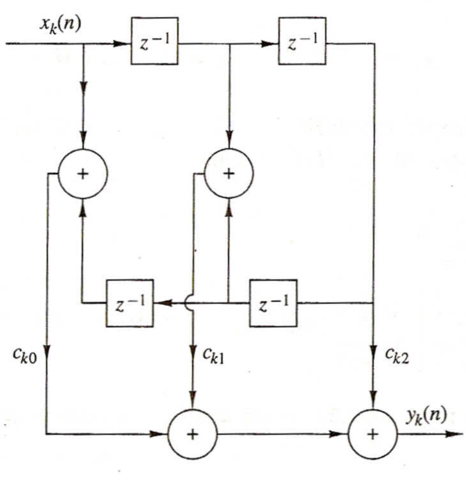

Figure 7.4 Fourth-order section in a cascade realization of an FIR system.