3 Blue 1 Brown

Essence of Calculus

#3B1B

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Chapter 1:

$$\boxed{A = \pi R^2}$$

$dR \to 0$

$$d(A) = \overbrace{(2\pi r)\, dR}^{dR}$$

$$A = \text{Ar } \Delta = \frac{1}{2}(2\pi R)(\underbrace{dR}_{\Sigma dR}) = \frac{1}{2} bh$$

$$= \pi R^2$$

Chapter 2: Paradox of derivatives

Instantaneous $\Longleftrightarrow$ Change (2 instance)

$\Delta S = S_2 - S_1$

Velocity in one instance
↓
moment

$$\left(\frac{\Delta s}{\Delta t}\right) \longleftrightarrow \frac{ds}{dt} \; \} \; ?$$

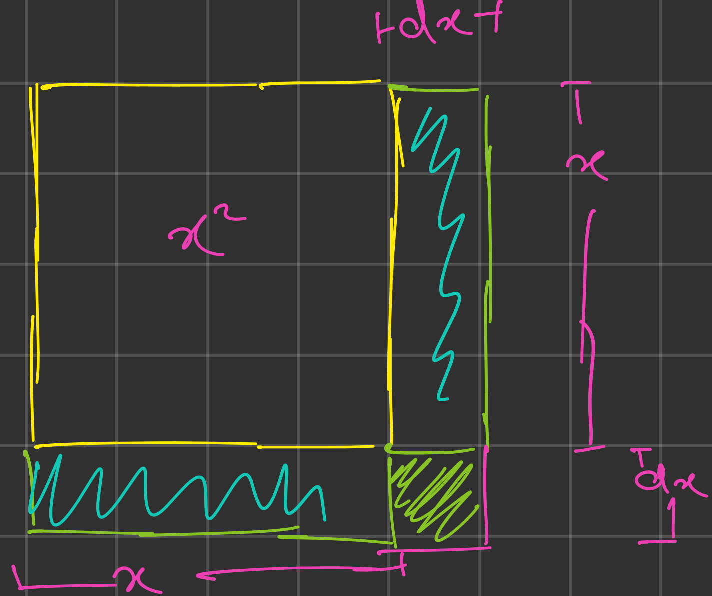
slope at tang. at point on graph

$$\frac{ds(t)}{dt} = \lim_{dt \to 0} \frac{s(dt+t) - s(t)}{dt} = \lim_{t \to 0} \frac{\Delta s}{\Delta t}$$

Compute sample: $dt \approx \varepsilon \Leftrightarrow 0.001s$

$$\tan\theta = \frac{ds}{dt} \qquad \theta = \tan^{-1}\left(\frac{ds}{dt}\right)$$

Chapter 5: $\left(\frac{d}{dt}\right)$ through Geometry



$f(x) = x^2$

$df = 2[x\,dx] + (dx)^2$

Ignore
too small

$$\boxed{\frac{df}{dx} = 2x}$$

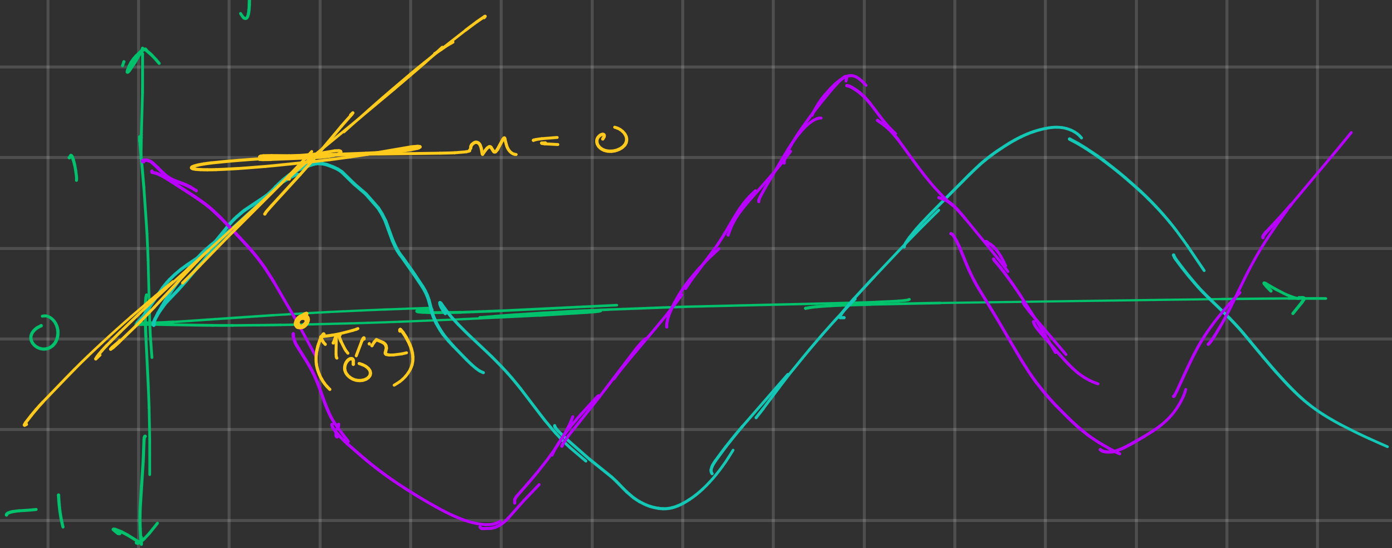
Think of $x^3 \rightarrow$ cube

Power Rule

$f(x) = x^n$ $\quad \frac{df}{dx} = n x^{n-1}$

Trig.

$f(\theta) = \sin\theta \xrightarrow{f'} \cos\theta$



Chapter 4 : Chain & Product Rule

peel onions!

Function	Add	Multiply	Composing
$f(x), g(x)$	$(f+g)(x)$	$[f \cdot g](x)$	$f(g(x))$

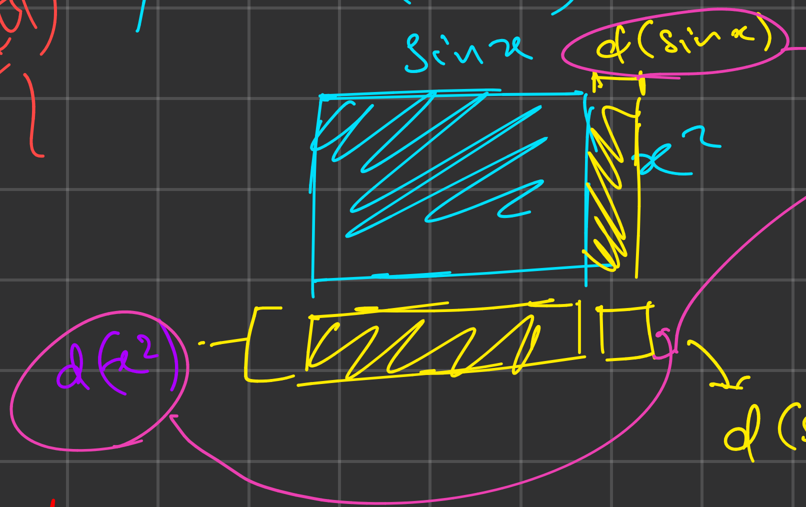
Ex. $f(x) = \sin x + x^2 \Rightarrow df = d(\sin x) + d(x^2)$

Add

$$\frac{df}{dx} = \frac{\cos x \, \cancel{dx} + 2x \, \cancel{dx}}{\cancel{dx}}$$

Multiply

$f(x) = (\sin x) \cdot x^2 \Rightarrow$ Area



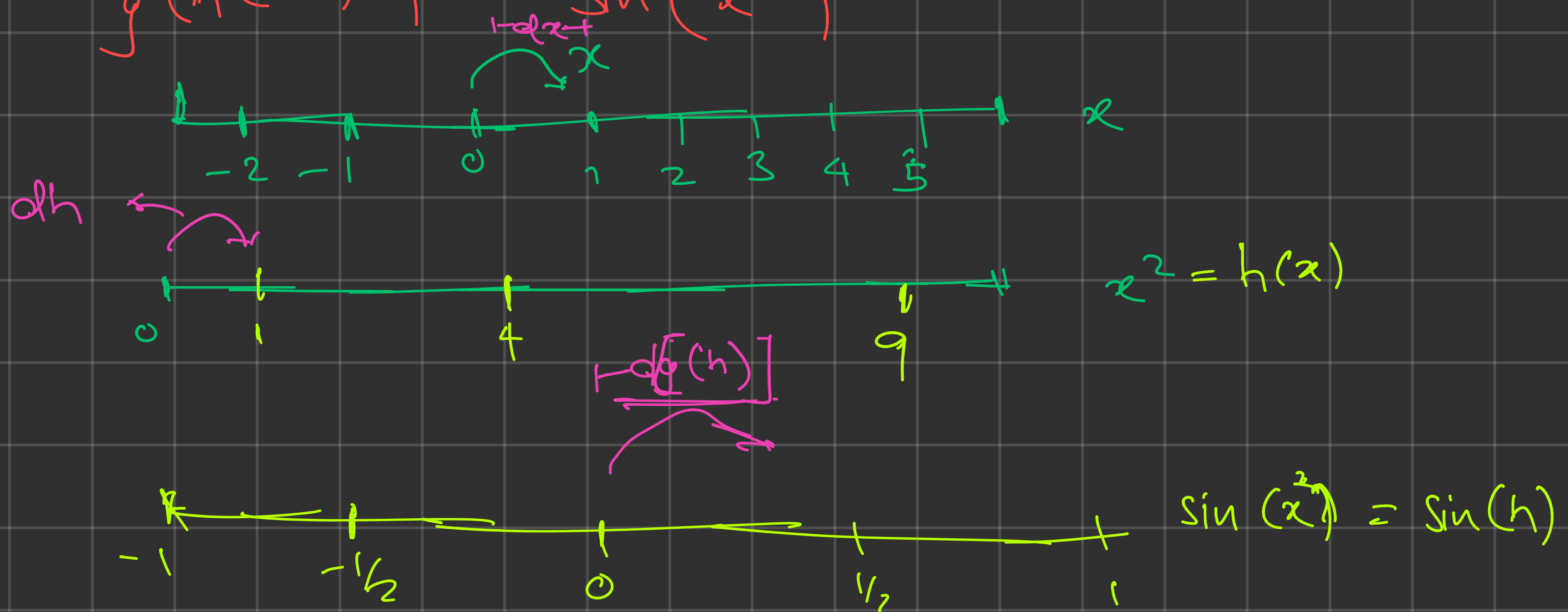
$$\frac{df}{dx} = g(x)\left(\frac{dh}{dx}\right) + h(x)\left(\frac{dg}{dx}\right)$$

$* \; df = g \cdot dh + h \cdot dg$

change

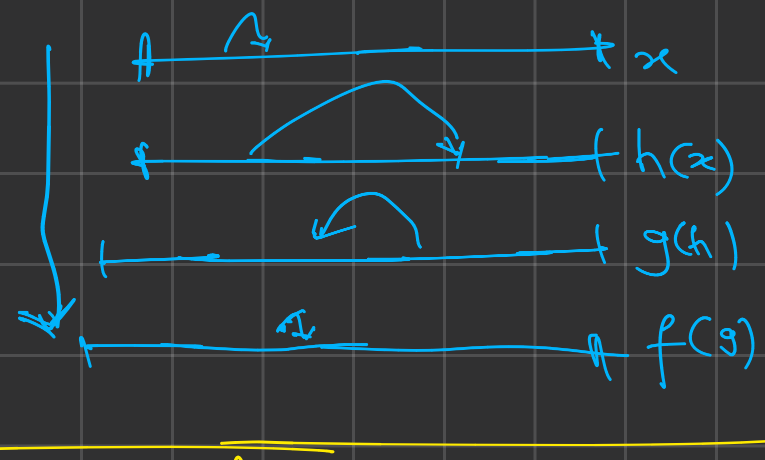
$d(\sin x) \cdot d(x^2) \rightarrow 0$ (negligible)

$g(h(x)) = \sin(x^2)$



nudge (change)

$$\left\{\begin{array}{lcl} d\sin(x^2) & \longrightarrow & d\sin(h) \\ dx^2 & \longrightarrow & dh \\ dx & \longleftrightarrow & \end{array}\right.$$



Chain Rule

$h = x^2$

$$d(h) = d(x^2) = 2x\,dx$$

$$d\sin(h) = \cos(h)\,dh = \cos(x^2)\cdot 2x\,dx$$

$$\frac{d(\sin(h))}{dx} = \cos(x^2)2x$$

Chapter: 5

$y = e^x$

Not sure?

$y = e^x$

$$\boxed{e = \lim_{h\to 0}\left(1 - \frac{1}{h}\right)^h ?}$$

$M(t) = N^t$

$$\lim_{\varepsilon\to 0}\frac{N^{t+\varepsilon} - N^t}{\varepsilon} = \lim_{dt\to 0}\frac{N^t\left[N^{dt} - 1\right]}{dt}$$

$$N^t\left[\lim_{dt\to 0}\frac{N^{dt} - 1}{dt}\right]$$

$N = e^{\ln(N)}$

this is e

2.7.

defining $\boxed{e}$ $\left\{\lim_{dt\to 0}\frac{N^{dt} - 1}{dt} = 1\right\}$

$$\boxed{N^t = e^{\ln(N)\cdot t}}$$

Money

$$\frac{dM(t)}{dt} = (1+r)M$$

Population

$$\frac{dP(t)}{dt} = kP(t)$$

Temp

$$\frac{d\Delta T}{dt} = -k\,\Delta T$$

Chapter 6: Implicit Differentiation

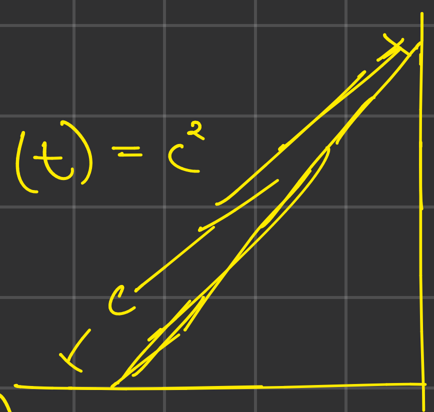
$$ax^2 + ay^2 = c^2 \quad \text{(const radius)} \qquad a \geq 0$$

$$\left(\frac{df}{dx}\right) \Rightarrow \text{Tangent at a point}$$

Example

Related Rates

Ladder



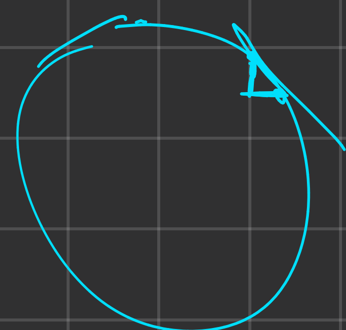
$$x^2(t) + y^2(t) = c^2$$

$$\frac{d(x^2(t))}{dt} + \frac{d(y^2(t))}{dt} = 0$$

$$2x\frac{dx}{\cancel{dt}} + 2y\frac{dy}{\cancel{dt}} = \frac{0}{\cancel{dt}} \Rightarrow \left\{\frac{dy}{dx} = -\frac{y}{x}\right\}$$

$$s(x,y) = x^2 + y^2 \qquad (c>0)$$

For $s(x,y) = (c^2)$, now for what ds still keeps it on curve $s(x,y)$



$$ds = 2x\,dx + 2y\,dy = 0$$

$\lim_{\to 0} dx$, dy ← tangential for circle.

Chapter I: lim, (ε, δ), L'Hopital

→ Continuous*

$\delta \to dx$

$\varepsilon \to dy$ → stupid!

$\frac{0}{0}$, $\frac{\infty}{\infty}$, 1^{∞}, ...

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{df(a)/dx}{dg(a)/dx}$$

when $f(a) = g(a) = 0$

$$f(x) \to \frac{df}{dx} = \frac{\text{Rise}}{\text{Run}}$$

$$\frac{df}{dx}(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

Explain "Approach"

→ (ε, δ)

Example

dN/dx $\qquad$ $d(\quad)/dx$

$$\lim_{x \to 1} \frac{\sin(\pi x)}{x^2 - 1} = \frac{0}{0} \text{ At } 1$$

$\frac{d(D)}{dx}$ $\qquad$ $d(\quad)/dx$

$$\lim_{x \to a} \frac{N(a)}{D(a)} = \frac{0}{0}, \frac{\infty}{\infty}, 0^0, \infty^0, \infty^\infty$$

Chapter 8: Integration & fundamental theorem

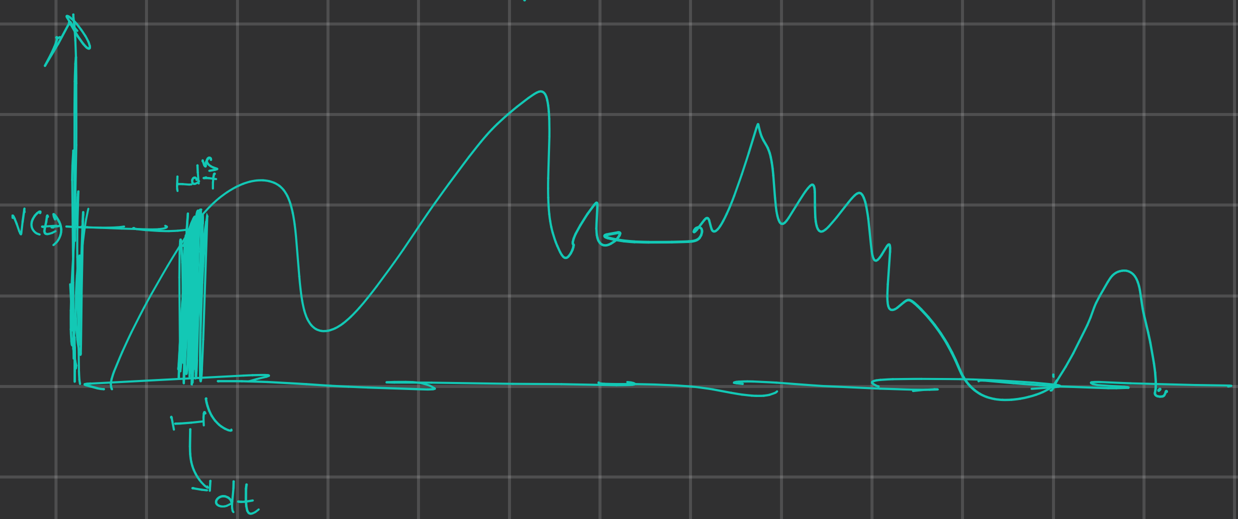
$\int \cdot dt \rightarrow$ Area

displacement $\rightleftarrows$ Velocity

$\frac{d}{dt} \rightarrow$ slope.

Σ Ar [trapezium] $\approx$

Σ Ar [rectangular] $\approx$



$$v(t) = \frac{ds}{dt} \longrightarrow \int_{t_1}^{t_2} v\,dt = s(t_2) - s(t_1)$$

Chapter 9: Slope --- Area

Average of function b/w (a, b)

$$\text{Avg.}\left(f(t)\right) = \frac{1}{(b-a)}\int_a^b f(t)\,dt = \frac{1}{b-a}\left[F(b) - F(a)\right]$$

$$\boxed{\begin{matrix}\text{Avg} \\ \text{Height}\end{matrix}} = \frac{\text{Area}}{\text{Width}}$$

Chapter 10: Higher Order Derivatives

$\int v \cdot dt = s$ = displacement

$\int a \cdot dt = v = \frac{ds}{dt}$ = Velocity

$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$ = Acceleration

$\frac{da}{dt} = \frac{d^2 v}{dt^2} = \frac{d^3 s}{dt^3}$ = Jerk

Chapter 10: Taylor Series

Approx. functions ⟷ Application

Calculations!

Ex.

$$\cos\theta = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

polynomial is easier to deal w/

$\{(c_n)\}$ degrees of freedom

For IRL, use an Equation sheet!

Curve fit: on computer