Machine Learning

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University of Illinois at Urbana-Champaign, 2018

L11: Structured Prediction (exhaustive search, dynamic programming)

Goals of this lecture

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- Getting to know structured prediction

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- Getting to know structured prediction
- Understanding basic structured inference algorithms

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Reading material:

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- Getting to know structured prediction
- Understanding basic structured inference algorithms

Reading material:

- D. Koller and N. Friedman; Probabilistic Graphical Models: Principles and Techniques;

Recap: General framework for learning:

$$\min_{\boldsymbol{w}} \frac{C}{2}\|\boldsymbol{w}\|_2^2 + \sum_{i\in\mathcal{D}} \epsilon \ln \sum_{\hat{y}} \exp \frac{L(y^{(i)}, \hat{y}) + F(\boldsymbol{w}, x^{(i)}, \hat{y})}{\epsilon} - F(\boldsymbol{w}, x^{(i)}, y^{(i)})$$

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- Scoring function

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How to get to

- Logistic regression
- Binary SVM
- Multiclass regression
- Multiclass SVM
- Deep Learning

Recap: Inference (how to find the highest scoring configuration):

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How to solve it?

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How to solve it?

Exhaustive search over all classes (easy for binary/few classes)

Example: why is structure/are correlations useful

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V

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I

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Z

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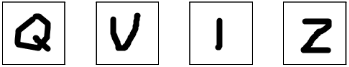
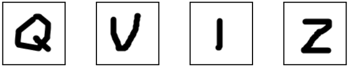


Q V I Z

Example: why is structure/are correlations useful



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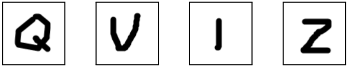

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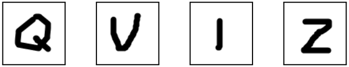

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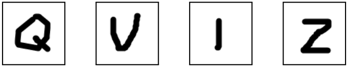

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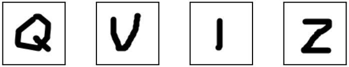

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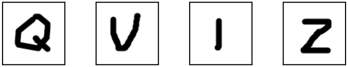

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Correlations not taken into account

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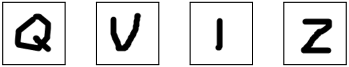

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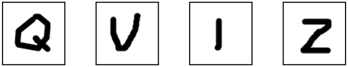

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Correlations taken into account

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- Problem:

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- Formulate it as prediction of all four letter words (multiclass prediction):

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- Problem: Really large **output space**

Example: Disparity map estimation

How big is the output space?

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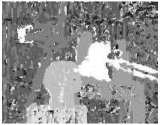
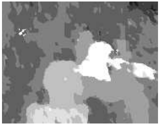
How big is the output space?

Image



Independent Prediction



Structured Prediction

Example: De-noising

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$\rightarrow$

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$\rightarrow$

Predictions from neighboring pixel are useful.

Structured Prediction estimates a complex object

$$x^{(i)} \quad \rightarrow \quad \boldsymbol{y}^{(i)} = (y_1^{(i)}, \ldots, y_D^{(i)})$$

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- Image segmentation (estimate a labeling)

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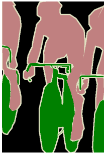

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- Image segmentation (estimate a labeling)
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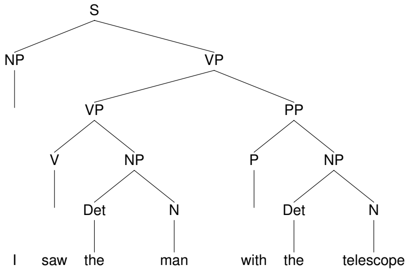
I saw the man with the telescope.



Structured Prediction estimates a complex object

- Image segmentation (estimate a labeling)
- Sentence parsing (estimate a parse tree)
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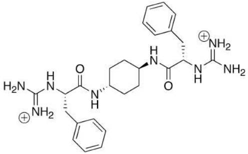

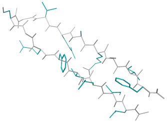

Structured Prediction estimates a complex object

- Image segmentation (estimate a labeling)
- Sentence parsing (estimate a parse tree)
- Protein folding (estimate a protein structure)
- Stereo vision (estimate a disparity map)

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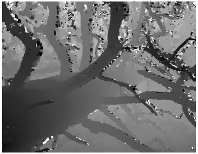

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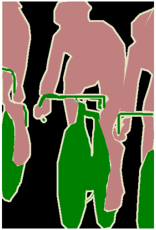

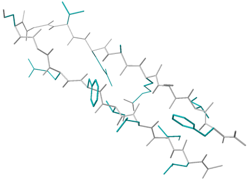

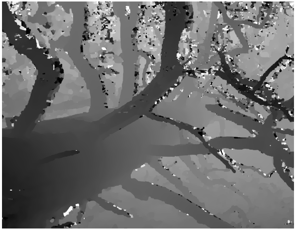

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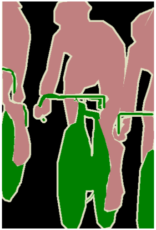

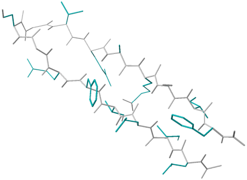

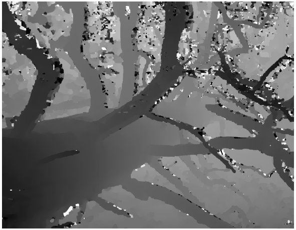

We can transition between both formulations. K possibly really large.

Formally:

$$\boldsymbol{y} = (y_1, \ldots, y_D) \qquad y_d \in \{1, \ldots, K\}$$

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How many possibilities do we have to store and explore?

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How many possibilities do we have to store and explore?

$$K^D$$

That's a problem. What can we do?

Separate prediction:

If

$$F(\boldsymbol{w}, x, \hat{y}_1, \ldots, \hat{y}_D) = \sum_{d=1}^{D} f_d(\boldsymbol{w}, x, \hat{y}_d)$$

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Why not predict every variable y_d from $\boldsymbol{y} = (y_1, \ldots, y_D)$ separately?

Correlations not explicitly taken into account

Score function decomposes less trivially:

$$F(\boldsymbol{w}, x, y_1, \ldots, y_D) = \sum_{r \in \mathcal{R}} f_r(\boldsymbol{w}, x, \boldsymbol{y}_r)$$

Restriction set $r \subseteq \{1, \ldots, D\}$
Set of all restrictions: $\mathcal{R}$

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Example: $r = \{1, 2\}$

$$f_{\{1,2\}}(\boldsymbol{y}_{\{1,2\}}) = f_{\{1,2\}}(y_1, y_2) = \left[f_{\{1,2\}}(1,1), f_{\{1,2\}}(1,2), \ldots\right]$$

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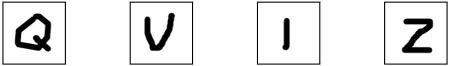
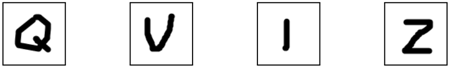
	Q	U	I	Z	V
Q	0	0.8	0.2	0.1	0.1
U		$\ddots$	$\cdots$		
I	$\vdots$				
Z					
V					



Q U I Z

Example:

$$F(\mathbf{w}, x, y_1, \ldots, y_4) = f_1(\mathbf{w}, x, y_1) + f_2(\mathbf{w}, x, y_2) + f_3(\mathbf{w}, x, y_3) + f_4(\mathbf{w}, x, y_4)$$
$$+ f_{1,2}(\mathbf{w}, x, y_1, y_2) + f_{2,3}(\mathbf{w}, x, y_2, y_3) + f_{3,4}(\mathbf{w}, x, y_3, y_4)$$



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Example:

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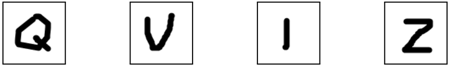
How many function values need to be stored if $y_d \in \{1, \ldots, 26\}\ \forall d$?



Q U I Z

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$$F(\boldsymbol{w}, x, y_1, \ldots, y_4) = f_1(\boldsymbol{w}, x, y_1) + f_2(\boldsymbol{w}, x, y_2) + f_3(\boldsymbol{w}, x, y_3) + f_4(\boldsymbol{w}, x, y_4) + f_{1,2}(\boldsymbol{w}, x, y_1, y_2) + f_{2,3}(\boldsymbol{w}, x, y_2, y_3) + f_{3,4}(\boldsymbol{w}, x, y_3, y_4)$$

How many function values need to be stored if $y_d \in \{1, \ldots, 26\}\ \forall d$?

Earlier: 26^4 v.s. now: $3 \cdot 26^2 (+4 \cdot 26)$

Visualization of the decomposition:

$$\begin{aligned} F(\boldsymbol{w}, x, y_1, \ldots, y_4) = & f_1(\boldsymbol{w}, x, y_1) + f_2(\boldsymbol{w}, x, y_2) + f_3(\boldsymbol{w}, x, y_3) + f_4(\boldsymbol{w}, x, y_4) \\ & + f_{1,2}(\boldsymbol{w}, x, y_1, y_2) + f_{2,3}(\boldsymbol{w}, x, y_2, y_3) + f_{3,4}(\boldsymbol{w}, x, y_3, y_4) \end{aligned}$$

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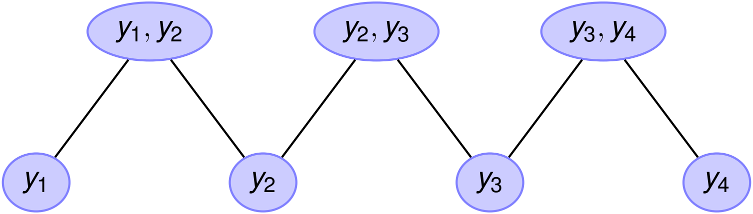

Edges denote subset relationship

Special cases

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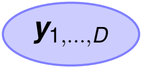
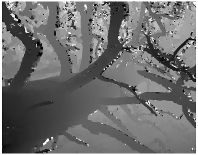


Example: stereo vision



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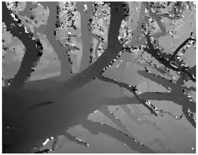

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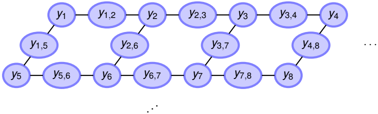

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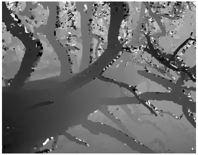

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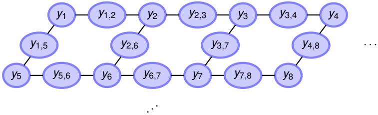

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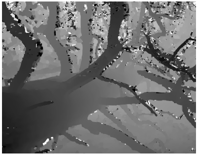

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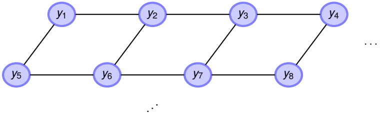

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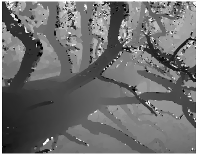

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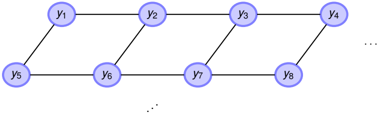

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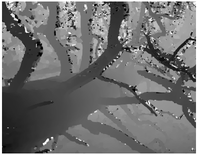

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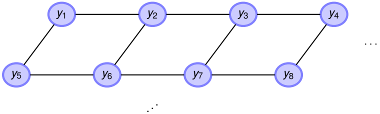

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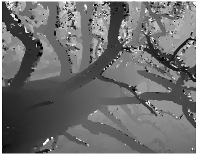

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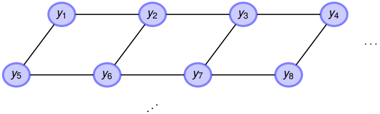

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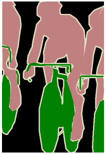

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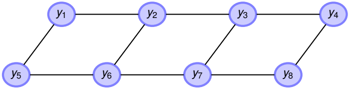

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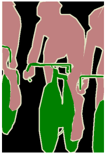

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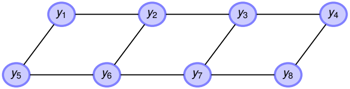

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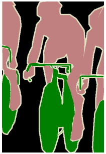

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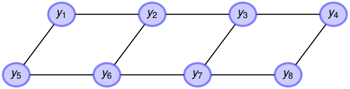

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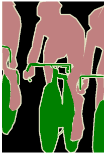

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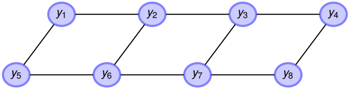

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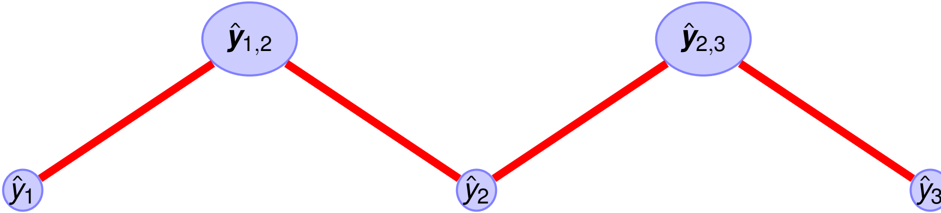
Dynamic Programming



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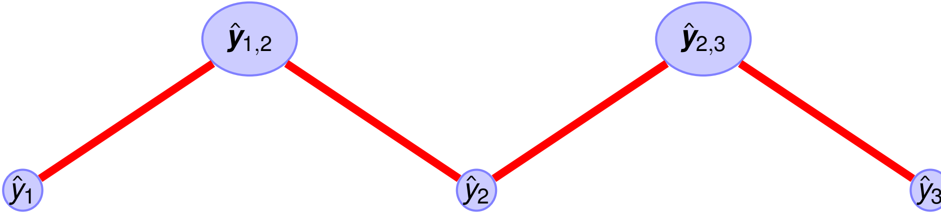

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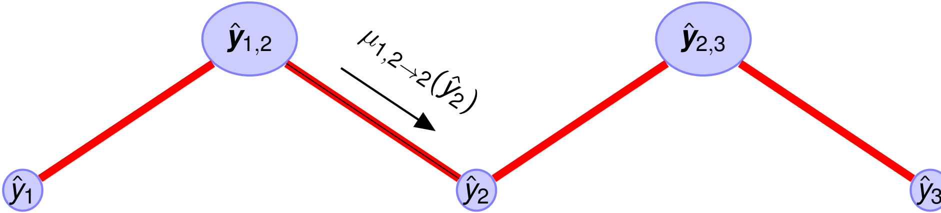

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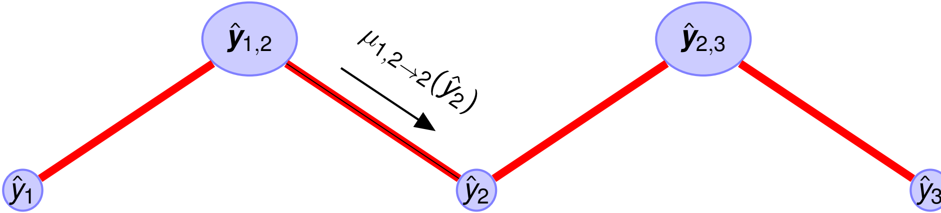

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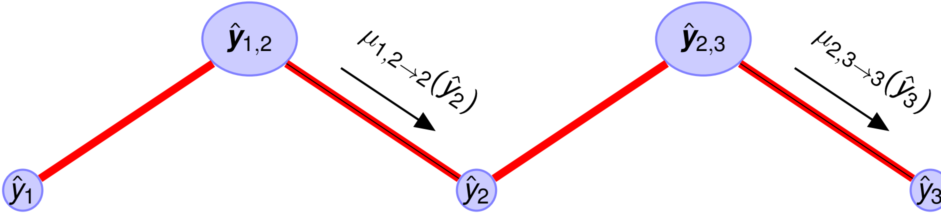

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Dynamic Programming

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Dynamic Programming

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What to do for general loopy graphs?

- Integer Linear Programs
- Linear Programming relaxations
- Dynamic programming extensions (message passing)
- Graph cut algorithms

Quiz:

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- Why structured output spaces?

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- Why structured output spaces?
- What makes computation with structured spaces hard?
- Inference algorithms for structured output spaces?

Important topics of this lecture

- Getting to know structured prediction
- Understood some inference algorithms

Up next:

- More inference algorithms for structured output spaces