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16     def getShortestPath(self, destIndex):
17         self.dest = destIndex
18         path_edges = []
19         total_length = 0
20         src_node = self.network.nodes[self.source]
21         dest_node = self.network.nodes[self.dest]
22         prev_node = dest_node
23         while prev_node.prev is not None:
24             edge = prev_node.prev
25             path_edges.append((edge.src.loc, edge.dest.loc, '{:.0f}'.format(edge.length)))
26             total_length += edge.length
27             prev_node = edge.src
28         if prev_node != src_node:
29             return {'cost': float("inf"), 'path': []}
30
31         return {'cost': total_length, 'path': path_edges}
32
33     def computeShortestPaths(self, srcIndex, use_heap=False):
34         self.source = srcIndex
35         t1 = time.time()
36
37         if (use_heap):
38             self.heap_dijkstra(srcIndex)
39         else:
40             self.array_dijkstra(srcIndex)
41
42         t2 = time.time()
43         return (t2-t1)
44
45     def heap_dijkstra(self, srcIndex):
46         node_indices = {}
47         queue = []
48         self.network.nodes[srcIndex].dist = 0
49         self.make_heap_queue(queue, node_indices)
50         while len(queue) > 0:
51             node = self.heap_delete_min(queue, node_indices)
52             for edge in node.neighbors:
53                 if edge.dest.dist > edge.src.dist + edge.length:
54                     edge.dest.dist = edge.src.dist + edge.length
55                     edge.dest.prev = edge
56                     self.decreaseKey(queue, node_indices, edge.dest)
57

```

```

def array_dijkstra(self, srcIndex):
    queue = self.make_array_queue()
    queue[srcIndex].dist = 0
    while len(queue) > 0:
        node = self.array_delete_min(queue)
        for edge in node.neighbors:
            if edge.dest.dist > edge.src.dist + edge.length:
                edge.dest.dist = edge.src.dist + edge.length
                edge.dest.prev = edge

```

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def make_heap_queue(self, queue, node_indices):
    for node in self.network.nodes:
        queue.append(node)
        self.bubble_up(queue, len(queue) - 1, node_indices)

```

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def make_array_queue(self):
    return self.network.nodes.copy()

```

```

def heap_delete_min(self, queue, node_indices):
    if len(queue) > 1:
        min_node = queue[0]
        queue[0] = queue.pop()
    else:
        return queue.pop()
    self.sift_down(queue, 0, node_indices)
    return min_node

```

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def array_delete_min(self, queue):
    min_node = None
    min_index = -1
    for i in range(len(queue)):
        if min_node is None or min_node.dist > queue[i].dist:
            min_node = queue[i]
            min_index = i
    queue.pop(min_index)
    return min_node

```

```

def decreaseKey(self, queue, node_indices, node):
    self.bubble_up(queue, node_indices[node], node_indices)

```

```

def bubble_up(self, queue, index, node_indices):
    parent_index = (index - 1) // 2
    node_indices[queue[index]] = index
    while index != 0 and queue[index].dist < queue[parent_index].dist:
        node_indices[queue[index]], node_indices[queue[parent_index]] = (parent_index, index)
        queue[index], queue[parent_index] = (queue[parent_index], queue[index])
        index = parent_index
        parent_index = (index - 1) // 2

def sift_down(self, queue, index, node_indices):
    min_child_index = self.min_child(queue, index)
    node_indices[queue[index]] = index
    while min_child_index != 0 and queue[index].dist > queue[min_child_index].dist:
        node_indices[queue[index]], node_indices[queue[min_child_index]] = (min_child_index, index)
        queue[index], queue[min_child_index] = (queue[min_child_index], queue[index])
        index = min_child_index
        min_child_index = self.min_child(queue, index)

def min_child(self, queue, i):
    if (i + 1) * 2 > len(queue):
        return 0
    elif (i + 1) * 2 == len(queue):
        return ((i + 1) * 2) - 1
    else:
        a = ((i + 1) * 2) - 1
        b = (i + 1) * 2
        return a if queue[a].dist < queue[b].dist else b

```

For a heap, insertion is $O(\log(v))$, since it is simply appending to the queue(which is $O(1)$) and then bubbling up, which is $\log(v)$ time.

For the unsorted array, insertion is $O(1)$, as it would simply append to the queue; in this case I just returned a copy of the array in `make_array`, which is $O(v)$, but there would be v insertions anyway, so it is the same time complexity.

Bubble_up is $\log(v)$ because it will at worst iterate a number of times equal to the height of the tree.

Heap_delete_min is $\log(v)$ because `pop()` is an $O(1)$ operation and `sift_down` is $\log(v)$ (for the same reason `bubble_up` is $\log(v)$)

Array_delete_min is $O(n)$ because it has to iterate though the list to find the correct index, then `pop(index)` is $O(n)$

The array implementation does not have a `decreaseKey` function, as it would be pointless. Since the array is unsorted, there is no need to notify anything but the node itself when a distance is updated.

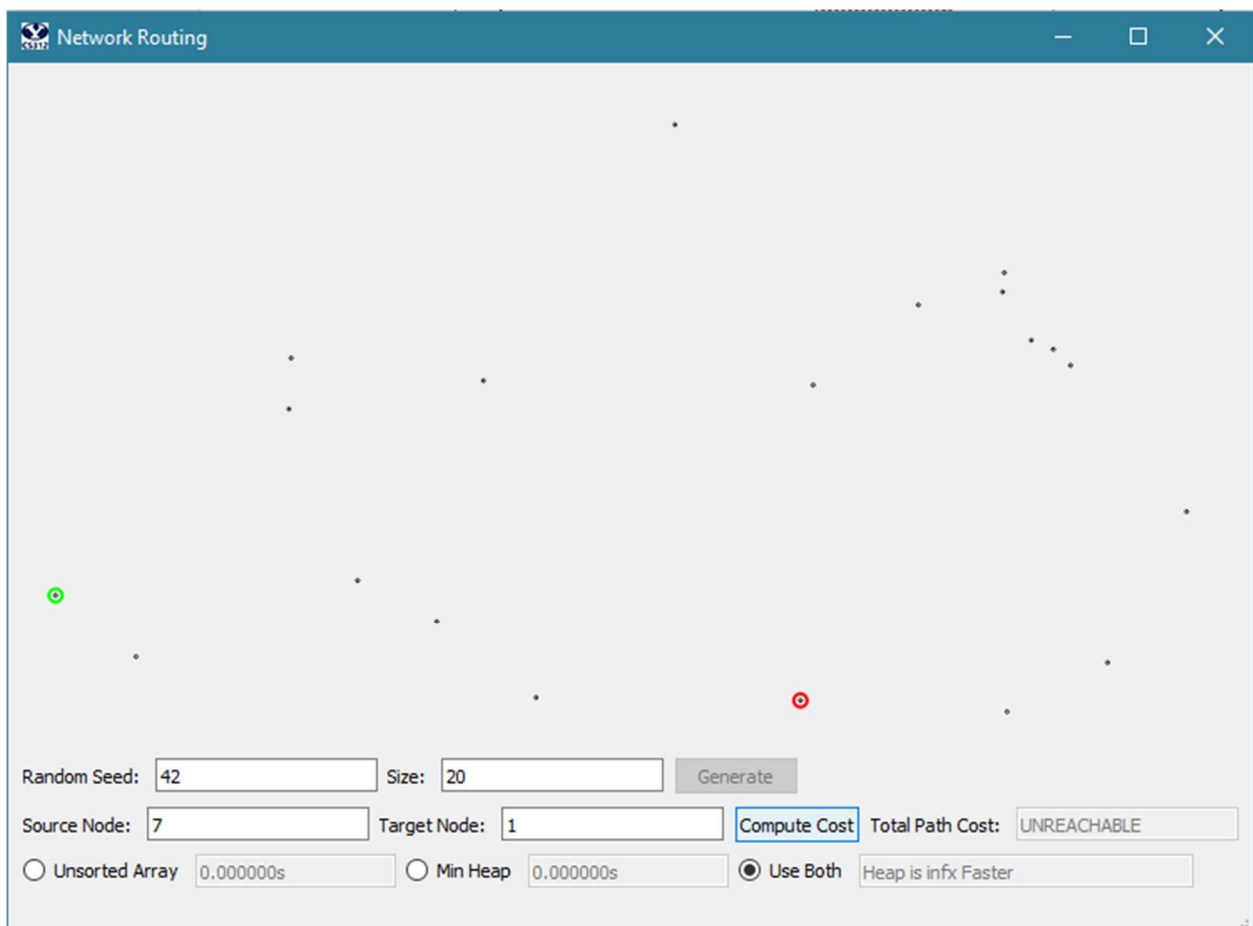
Decrease_key is also $\log(v)$ since all it does is call bubble_up; the node value in the queue has already been updated on the line previous to the call to decrease_key

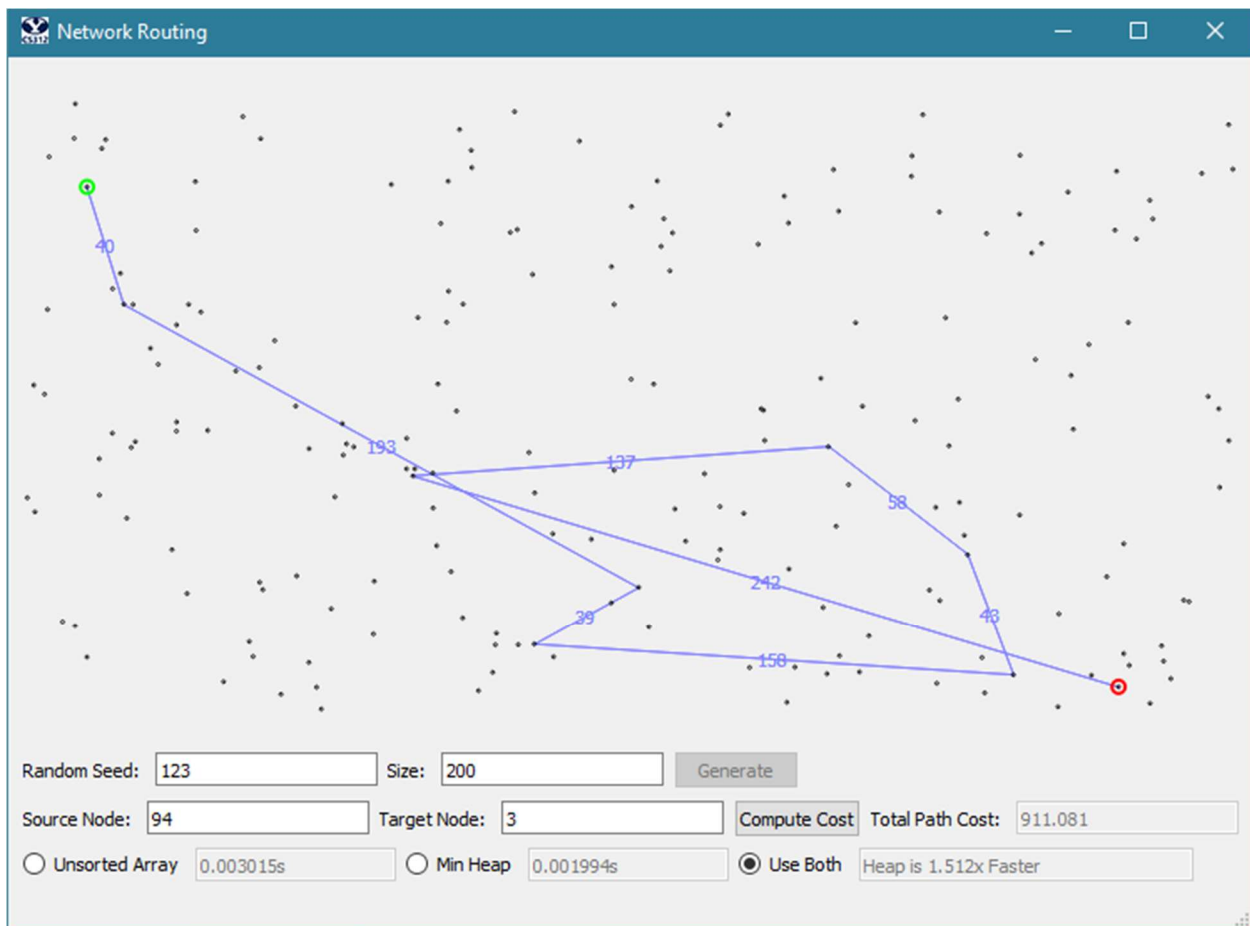
The space complexity of the unsorted array implementation is simply $O(v)$, as it only ever keeps track of the single array

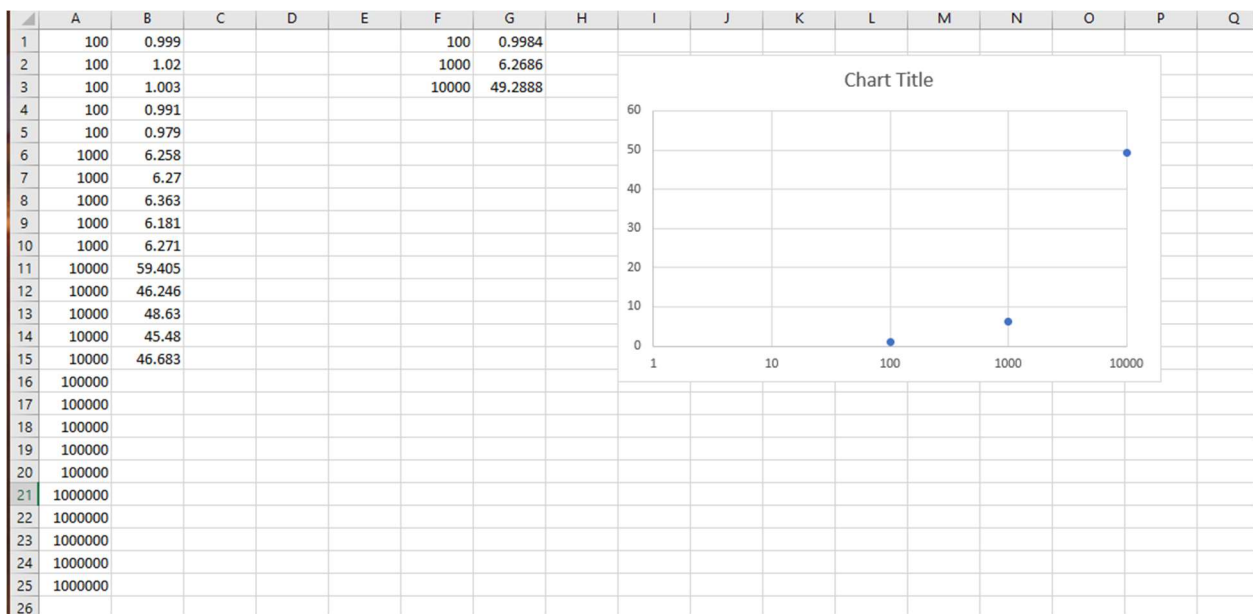
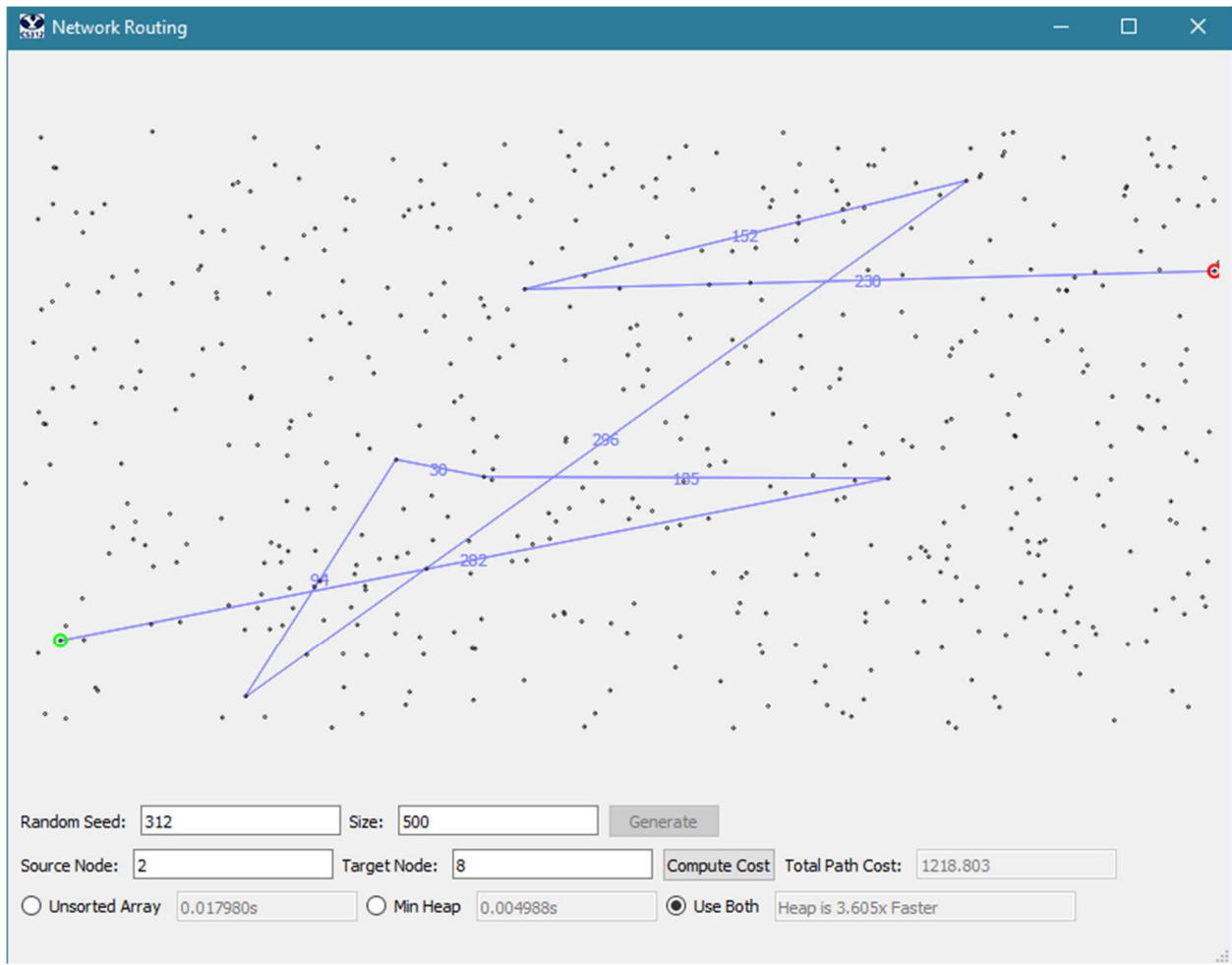
The space complexity of the heap implementation is $O(2v) = O(v)$, as it keeps track of the array as well as a dictionary of node_indices which is size v .

The overall time complexity for the unsorted array implementation is $O(v^2)$, as it will go through each node ($O(v)$) and for each node, call delete_min, which adds another factor of v .

The overall time complexity for the heap implementation is $O(v \log(v))$, as it will go through each node (again, $O(v)$) and for each node, call each of insert, delete_min, and decrease_key, adding up to $O(\log(V))$ asymptotically, resulting in $O(v \log(v))$ overall.







Unfortunately I ran out of time to compare array to heap implementation on the 100000 tests, but my heap seemed to get about another 10x faster the more nodes it had to go through, so I would

estimate that my heap would be about 5000x faster than the array implementation for 1000000 nodes.