TD1: Wireless Propagation

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# 1 Path-loss models

For our calculations, let's convert the received power from `dBm` to `mW`.

| Distance $d$ [m] | Received power [dBm] | Received power [mW] |
|---|---|---|
| 50 | 1.2 | 1.32 |
| 100 | $-10.4$ | $9.12 \cdot 10^{-2}$ |
| 250 | $-19.8$ | $1.05 \cdot 10^{-2}$ |
| 500 | $-30.4$ | $9.12 \cdot 10^{-4}$ |
| 1000 | $-50.4$ | $9.12 \cdot 10^{-6}$ |

We also convert the antenna gains $g_t = 39.81$ and $g_r = 1$, and we convert the transmitted power $p_t = 39.81$W.

### Question 1

In the simplified path loss model we have

$$p_r(d) = p_t K \left(\frac{d_0}{d}\right)^\alpha \tag{1}$$

we compare this expression with the free-space path-loss model

$$\frac{p_t}{p_r} = \left(\frac{4\pi d}{\lambda\sqrt{g_t g_r}}\right)^2 \tag{2}$$

with $\lambda = \frac{c}{f_c}$. At $d = d_0$, we conclude that

$$K = \left(\frac{\lambda\sqrt{g_t g_r}}{4\pi d_0}\right)^2 \tag{3}$$

which gives $K = 2.8 \cdot 10^{-2}$ or $K = -15.5$dB.

### Question 2

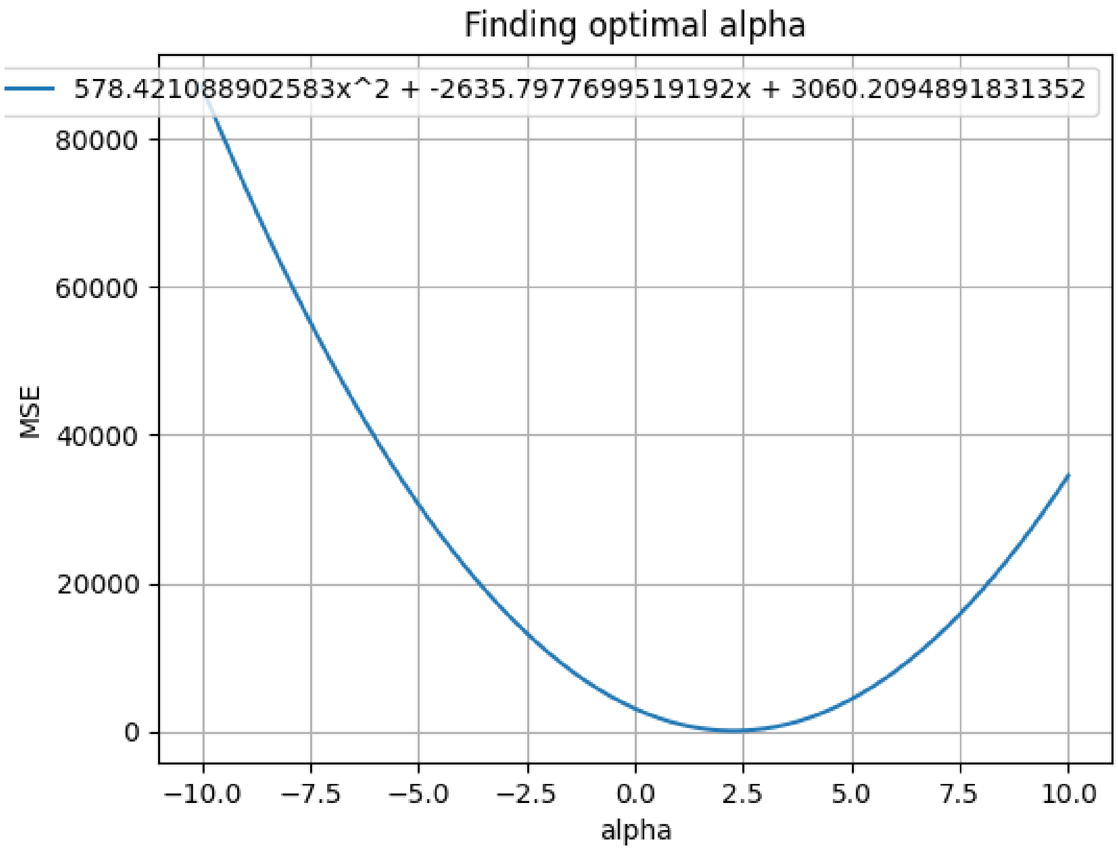
$$p_r[dBm] = 10\log_{10}(K \cdot p_t[mW]) + 10\alpha\log_{10}\left(\frac{d_0}{d}\right) \tag{4}$$

The mean squared error is MSE $= \frac{1}{5}\sum_{i=1}^{5}\left(p_r^{\text{model}}(d_i) - p_r^{\text{measured}}(d_i)\right)^2$.
Let's define $C_i(d_i) = 10\log_{10}(K \cdot p_t[mW]) - p_r^{\text{measured}}(d_i)[dBm]$ to ease the notation.
We observe that

$$\begin{aligned} MSE &= \frac{1}{5}\sum_{i=1}^{5}\left(\alpha 10\log_{10}\left(\frac{d_0}{d_i}\right) + C_i(d_i)\right)^2 && (5)\\ &= \frac{1}{5}\sum_{i=1}^{5}\alpha^2 100\log_{10}^2\left(\frac{d_0}{d_i}\right) + 2\alpha 10\log_{10}\left(\frac{d_0}{d_i}\right)C_i(d_i) + C_i(d_i)^2 && (6)\\ &= \alpha^2\sum_{i=1}^{5}20\log_{10}^2\left(\frac{d_0}{d_i}\right) + \alpha\sum_{i=1}^{5}4\log_{10}\left(\frac{d_0}{d_i}\right)C_i(d_i) + \frac{1}{5}\sum_{i=1}^{5}C_i(d_i)^2 && (7)\end{aligned}$$

is a second order polynomial in $\alpha$. We conclude that

$$\begin{aligned} \alpha_{\text{opt}} &= -\frac{\sum_{i=1}^{5}4\log_{10}\left(\frac{d_0}{d_i}\right)C_i(d_i)}{2\sum_{i=1}^{5}20\log_{10}^2\left(\frac{d_0}{d_i}\right)} && (8)\\ \implies \alpha_{\text{opt}} &= 2.28 && (9)\end{aligned}$$




Now we can compute the received power $p_r$ at $d = 750$m.

$$p_r(d = 750\text{m}) = -35\text{dBm} \tag{10}$$

## Question 3

We compute the variance relative to the simplified model (with powers in dBm),

$$\text{MSE} = \frac{1}{5}\sum_{i=1}^{5}\left(p_r^{\text{model}}(d_i) - p_r^{\text{measured}}(d_i)\right)^2 = 57.45 \tag{11}$$

this gives us a shadowing standard deviation

$$\sigma = \sqrt{\text{MSE}} = 7.58 \tag{12}$$

In dB, we have $\sigma = 8.8$dB

## Question 4

We can reuse the expression we got at question 1 to compute the new $K$,

$$K = \left(\frac{\lambda\sqrt{g_t g_r}}{4\pi d_0}\right)^2 \tag{13}$$

which gives $K = 2.0 \cdot 10^{-3}$ or $K = -26.98$dB.
For a configuration with $N$ antennas, with the simplified model, we have

$$p_r[dBm] = 10\log_{10}\left(Kp_t\left(\frac{d_0}{d}\right)^{\alpha}\right) + 10\log_{10} N \tag{14}$$

We want $p_r(d = 750\text{m}) = -35$dBm, which implies

$$N = 10^{\frac{p_r - 10\log_{10}\left(Kp_t\left(\frac{d_0}{d}\right)^{\alpha}\right)}{10}} = 60.64 \tag{15}$$

We round up to $N = 61$ antennas.

## Question 5

We have

$$10\log_{10} N_d = 16\text{dBi} \implies N_d = 39.81 \tag{16}$$

which we round up to $N_d = 40$ $\lambda/2-$dipoles.

At $f_1 = 900$MHz, each dipole has length $\frac{\lambda_1}{2} = \frac{c}{2f_1} = 0.167$m.
If we organize our dipoles in 2 columns of $\frac{N_d}{2} = 20$ dipoles $\implies$ each column has $20 \cdot \frac{\lambda_1}{2} = 3.34$m.
At $f_2 = 30$GHz, each dipole has length $\frac{\lambda_2}{2} = \frac{c}{2f_2} = 5 \cdot 10^{-3}$m.
The height $N$ of the square panel (uniform linear array) is equal to the length of the dipole $\implies N = 5 \cdot 10^{-3}$m.

## 2 Cell radius

### Question 6

Assuming that there is no gain at the edge device ($g_r = 0$dBi), we repeat our procedure from Question 1 to get

$$K = \left(\frac{c\sqrt{g_t g_r}}{f_c \cdot 4\pi d_0}\right)^2 \tag{17}$$

which gives $K = 6.7 \cdot 10^{-3}$ or $K = -21.74$dB.
Assuming the Shannon formula is achievable, we can fix $C_{\text{Shannon}} = 10$Mbits/s and compute the SNR at cell edge,

$$C_{\text{Shannon}} = W \log_2(1 + \gamma_{\text{edge}}) \implies \gamma_{\text{edge}} = 2^{\frac{C_{\text{Shannon}}}{W}} - 1 \tag{18}$$

which gives $\gamma_{\text{edge}} = 0.414$ or $-3.83$dB.
**We observe that this SNR value is very low. The signal power is lower than the noise power.**
We can now calculate the received power at cell edge $P_{\text{r,edge}}$.
The noise power spectral density is $N_0[dBm] = 10\log_{10}(kT) = -203.83$dBm. We have

$$P_{\text{r,edge}} = \gamma_{\text{edge}} P_{\text{noise}} = \gamma_{\text{edge}} N_0 W \tag{19}$$

which gives $P_{\text{r,edge}} = 3.43 \cdot 10^{-14}$mW or $-134.65$dBm.
To compute the cell size $d_{\text{edge}}$ we use the outage probability formula.
We want $\mathbb{P}[P_r < P_{\text{r,edge}}] = P_{\text{out}} = 1 - 0.99$, so

$$1 - Q\left(\frac{P_{\text{r,edge}}[dBm] - \left[P_t[dBm] + K[dB] + 10\alpha\log_{10}\left(\frac{d_0}{d_{\text{edge}}}\right)\right]}{\sigma[dB]}\right) = 1 - 0.99 \tag{20}$$

in this expression, we used the simplified path loss model and assumed no co-channel interference.
We conclude

$$d_{\text{edge}} = 14.6\text{km} \tag{21}$$

### Question 7

We set

$$u = Q(k) \implies u' = Q'(k)k' \tag{22}$$

$$v' = r \implies v = \frac{r^2}{2} \tag{23}$$

Using integration by parts (and the fact that $Q'(k) = -\frac{e^{-\frac{k^2}{2}}}{\sqrt{2\pi}}$) we have

$$\begin{aligned}
C &= \frac{2}{R^2}\left(\frac{r^2}{2}Q(k)\Big|_0^R - \int_0^R \frac{rb}{2}Q'(k)dr\right) && (24)\\
&= Q(a) - \frac{2}{R^2}\int_{-\infty}^{a} \frac{b}{2}Re^{\left(\frac{k-a}{b}\right)}\left(-\frac{e^{-\frac{k^2}{2}}}{\sqrt{2\pi}}\right)\frac{R}{b}e^{\left(\frac{k-a}{b}\right)}dk \quad \text{we make the substitution } r \to k && (25)\\
&= Q(a) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{\frac{2(k-a)}{b} - \frac{k^2}{2}}dk && (26)\\
&= Q(a) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{\frac{-\frac{b^2k^2}{2}+2kb-2ab}{b^2}}dk \quad \text{now we can complete the square on the exponent} && (27)\\
&= Q(a) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{\frac{-\left(\frac{bk}{\sqrt{2}}-\sqrt{2}\right)^2+2-2ab}{b^2}}dk && (28)\\
&= Q(a) + e^{\frac{2-2ab}{b^2}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{\frac{-\left(k-\frac{2}{b}\right)^2}{2}}dk && (29)\\
&= Q(a) + e^{\frac{2-2ab}{b^2}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a-\frac{2}{b}} e^{-\frac{k^2}{2}}dk \quad \text{we make the translation } k \to k - \frac{2}{b} && (30)\\
&= Q(a) + e^{\frac{2-2ab}{b^2}}\frac{1}{\sqrt{2\pi}}\int_{-a+\frac{2}{b}}^{+\infty} e^{-\frac{k^2}{2}}dk \quad \text{we manipulate the integration limits using the fact that the function is even} && (31)\\
&= Q(a) + e^{\frac{2-2ab}{b^2}}Q\left(\frac{2}{b} - a\right) && (32)
\end{aligned}$$

### Question 8

We describe the cell area in radial coordinates $(r, \theta)$ with $0 \leq r \leq R$ and $0 \leq \theta \leq 2\pi$.

We use $d_0 = R$ to compute $\bar{P}(r)$ using the simplified path loss model. We have

$$\bar{P}(r) = P_t + K_{\text{dB}} - 10\alpha \log_{10}\left(\frac{r}{R}\right) + P_{\text{shadowing}} \tag{33}$$

Because we changed the reference distance, we need to recompute $K$. We can use the outage probability at the edge, we have

$$1 - \mathbb{P}[P_{\text{shadowing}} \geq P_r^* - P_t - K_{\text{dB}}] = 0.1 \tag{34}$$

$$\iff Q\left(\frac{P_r^* - P_t - K_{\text{dB}}}{\sigma}\right) = 0.9 \tag{35}$$

which gives $K = 145.75$dB.

The proportion $p_{\text{covered}}$ of the covered cell area is given by

$$p_{\text{covered}} = \frac{\int_0^{2\pi}\int_0^R r\mathbb{I}_{\{\bar{P}(r,\theta) > P_r^*\}} dr d\theta}{\pi R^2} \quad \text{but we know that the received power depends only on } r \tag{36}$$

$$= \frac{2}{R^2}\int_0^R r\mathbb{I}_{\{\bar{P}(r) > P_r^*\}} dr \tag{37}$$

$$= \frac{2}{R^2}\int_0^R r\mathbb{I}_{\{P_{\text{shadowing}} > P_r^* - P_t - K_{\text{dB}} + 10\alpha\log_{10}\left(\frac{r}{R}\right)\}} dr \tag{38}$$

$$= \frac{2}{R^2}\int_0^R rQ\left(a + b\ln\left(\frac{r}{R}\right)\right) \quad \text{let } a = \frac{P_r^* - P_t - K_{\text{dB}}}{\sigma} \text{ and } b = \frac{10\alpha}{\sigma\ln 10} \tag{39}$$

$$= Q(a) + e^{\frac{2-2ab}{b^2}} Q\left(\frac{2}{b} - a\right) \quad \text{using question 7} \tag{40}$$

We get $p_{\text{covered}} \approx 1$.

## 3 SIR and SINR

### Question 9

On the downlink, we consider all antennas simultaneously transmitting to the receiver. We want to compute

$$\text{SIR} = \frac{p_{\text{signal}}}{p_{\text{interference}}} \tag{41}$$

We can use the formula for $g(r)$ to compute $p_{\text{signal}}$, we have

$$p_{\text{signal}} = p_t K r^{-\eta} \tag{42}$$

We compute $p_{\text{interference}}$ using the superposition principle.

$$p_{\text{interference}} = 2\sum_{j=0}^{+\infty} p_t K (r + jR)^{-\frac{\eta}{2}} \tag{43}$$

we conclude

$$\text{SIR} = \frac{r^{-\eta}}{2\sum_{j=0}^{+\infty}(r + jR)^{-\frac{\eta}{2}}} \tag{44}$$

$\implies$ the SIR does not depend on the transmit power $p_t$, only on the distance $r$.