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1

NMOS AND PMOS TRANSISTORS CHARACTERIZATION

1.1 I_{DS} function of V_{GS}

Plot, for the two types of transistors, the graph $I_{DS}(V_{DS}, V_{GS})$ for V_{DS} constant $= V_{DD}$, and for V_{GS} varying between between V_{SS} and V_{DD} . Define graphically the threshold voltages V_{tn} and V_{tp} of the two types of transistors.

1.1.1 • NMOS TRANSISTOR

We observe that $V_{DS} = V_{DD} \implies V_{DS} \ge V_{GS} - V_{tn} \implies$ the NMOS transistor is either cutoff or in saturation depending on the value of V_{GS} , thus

$$I_{DS}(V_{DS} = V_{DD}, V_{GS}) = \begin{cases} 0, & \text{if } V_{GS} < V_{tn} \text{ (cutoff)}, \\ \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_{tn})^2, & \text{if } V_{GS} \ge V_{tn} \text{ (saturation)}. \end{cases}$$
(1)

```
.include cmosws.mod

Vds 2 0 dc 3.3V

Vgs 1 0 dc 1V

M1 2 1 0 0 MODN L=0.6U W=3.0U

.dc Vgs 0 3.3 50mV

.end
```



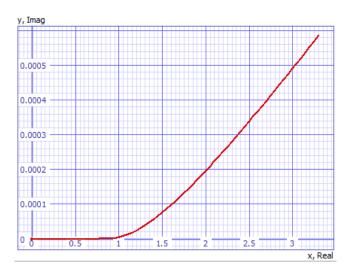


Figure 1: I_{DS} function of V_{GS} for NMOS transistor

We can define V_{tn} graphically as the limit value where I_{DS} becomes greater than 0. We get

$$V_{tn} \approx 0.87$$



Figure 2: Determining V_{tn}

1.1.2 • PMOS TRANSISTOR

In the PMOS transistor, to account for the reversed polarity we impose $V_{SD} = V_{DD} \implies V_{SD} \ge V_{SG} - V_{tp}$ where V_{tp} is negative.

Now, we expect

$$I_{DS}(V_{SD} = V_{DD}, V_{SG}) = \begin{cases} 0, & \text{if } V_{SG} < |V_{tp}| \text{ (cutoff)}, \\ -\frac{1}{2} k_p \frac{W}{L} (V_{SG} - |V_{tp}|)^2, & \text{if } V_{SG} \ge |V_{tp}| \text{ (saturation)}. \end{cases}$$
(2)

```
.include cmosws.mod

Vsd 2 0 dc 3.3V

Vsg 2 1 dc 1V

M1 0 1 2 2 MODP L=0.6U W=6.0U

dc Vsg 0 3.3 50mV

end
```





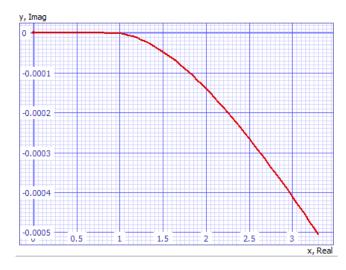


Figure 3: I_{DS} function of V_{SG} for PMOS transistor

We can identify $|V_{tp}|$ graphically as the limit value where I_{DS} becomes smaller than 0. We get

$$V_{tp} \approx -0.82$$

1.2 I_{DS} function of V_{DS}

Plot, for the two types of transistors, the graph $I_{DS}(V_{DS}, V_{GS})$ for V_{GS} constant, and for V_{DS} varying between between V_{SS} and V_{DD} . These plots will be done for several values of V_{GS} . Identify the linear and saturation domains for both types of transistors.

1.2.1 • NMOS TRANSISTOR

We will use V_{GS} equal to 1V, 2V and 3V (so we are never in the cutoff region because $V_{GS} > V_{tn}$).

Theoretically we expect

$$I_{DS}(V_{DS}, V_{GS} = \text{constant}) = \begin{cases} k_n \frac{W}{L} \left[(V_{GS} - V_{tn}) V_{DS} - \frac{V_{DS}^2}{2} \right], & \text{if } V_{DS} < V_{GS} - V_{tn} \text{ (triode or linear)}, \\ \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_{tn})^2, & \text{if } V_{GS} \ge V_{tn} \text{ (saturation)}. \end{cases}$$

$$(3)$$

.include cmosws.mod

Vds 2 0 dc 3.3V



```
Vgs 1 0 dc 1V

M1 2 1 0 0 MODN L=0.6U W=3.0U

dc Vds 0 3.3 50mV Vgs 1 3 1V

end
```

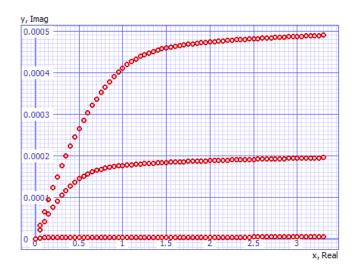


Figure 4: I_{DS} function of V_{DS} for NMOS transistor

1.2.2 • PMOS TRANSISTOR

Once again for the PMOS, we make adaptions to account for reversed polarity.

We will use V_{SG} equal to 1V, 2V and 3V (so we are never in the cutoff region because $V_{SG} > |V_{tp}|$).

Theoretically we expect

$$I_{DS}(V_{SD}, V_{SG} = \text{constant}) = \begin{cases} -k_p \frac{W}{L} \left[(V_{SG} - |V_{tp}|) V_{SD} - \frac{V_{SD}^2}{2} \right], & \text{if } V_{SD} < V_{SG} - |V_{tp}| \text{ (triode or linear)}, \\ -\frac{1}{2} k_p \frac{W}{L} (V_{SG} - |V_{tp}|)^2, & \text{if } V_{SG} \ge |V_{tp}| \text{ (saturation)}. \end{cases}$$
(4)

```
.include cmosws.mod

Vsd 2 0 dc 3.3V

Vsg 2 1 dc 1V

M1 0 1 2 2 MODP L=0.6U W=6.0U

.dc Vsd 0 3.3 50mV Vsg 1 3 1V

.end
```



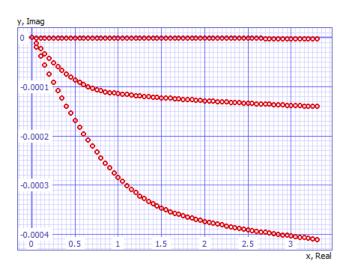
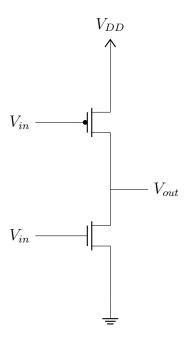


Figure 5: I_{DS} function of V_{SD} for PMOS transistor

2 CMOS INVERTER STATIC CHARACTERIZATION





2.1 Centered Inverter

Plot the static transfer function for a centered inverter. Deduce the noise margin on the input low state and high state.

```
.include cmosws.mod

Vin 1 0 dc 1V

Vdd 2 0 dc 3.3V

M1 3 1 0 0 MODN L=0.6U W=3.0U

M2 2 1 3 2 MODP L=0.6U W=6.0U

.dc Vin 0 3.3 50mV

end
```



Figure 6: Transfer function for centered inverter

From the graphic, we have

- $V_{IL} = 0,92V$ (maximum V_{in} for which V_{out} is high)
- $V_{IH} = 2,3V$ (minimum V_{in} for which V_{out} is low)
- and we consider $V_{OL} = 0V$ and $V_{OH} = 3, 3V$.

We conclude that

- $NML = V_{IL} V_{OL} = 0,92V$
- $NMH = V_{OH} V_{IH} = 3, 3 2, 3 = 1V$

We observe that $W_p = 2W_n$ did not give us a perfectly centered inverter, we expected NML = NMH.



2.2 Decentered Inverter

Plot the transfer function for a decentered inverter with

- $W_n = W_p = 3,0\mu m$
- $W_p = 12,0 \mu m \text{ and } W_n = 3,0 \mu m$

Deduce the noise margins on the input low state and high state.

For a lower value of W_p , the PMOS transistor has a smaller $g_m = \frac{\partial I_D}{\partial V_{SG}} = k_p \frac{W}{L} (V_{SG} - |V_{tp}|)$. Thus, the PMOS is less sensitive to variations in V_{SG} which makes the high-low transition slower. We expect NML < NMH.

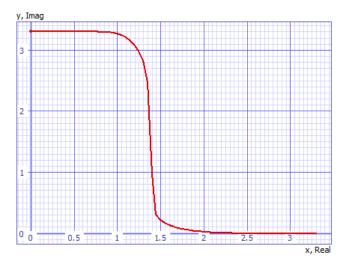


Figure 7: Transfer function for decentered inverter with $W_n = W_p = 3,0 \mu m$

We get from the graphic

- $V_{IL} = 0.85 \text{V}$ (maximum V_{in} for which V_{out} is high)
- $V_{IH} = 2,22V$ (minimum V_{in} for which V_{out} is low)
- again we consider $V_{OL} = 0V$ and $V_{OH} = 3, 3V$.

We conclude that

- $NML = V_{IL} V_{OL} = 0.85V$
- $NMH = V_{OH} V_{IH} = 3, 3 2, 22 = 1,08V$



Now for a higher value of W_p , the PMOS transistor has a higher $g_m = \frac{\partial I_D}{\partial V_{SG}} = k_p \frac{W}{L} (V_{SG} - |V_{tp}|)$. Thus, the PMOS is more sensitive to variations in V_{SG} which makes the high-low transition faster. We expect NML > NMH.



Figure 8: Transfer function for decentered inverter with $W_p=12,0\mu m$ and $W_n=3,0\mu m$

We get from the graphic

- $V_{IL} = 0,99V$ (maximum V_{in} for which V_{out} is high)
- $V_{IH} = 2,35 \text{V}$ (minimum V_{in} for which V_{out} is low)
- again we consider $V_{OL}=0{\rm V}$ and $V_{OH}=3,3{\rm V}.$

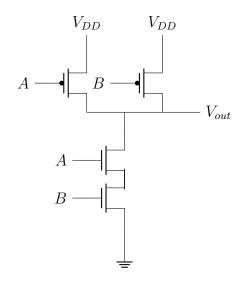
We conclude that

•
$$NML = V_{IL} - V_{OL} = 0,99V$$

•
$$NMH = V_{OH} - V_{IH} = 3, 3 - 2, 35 = 0,95V$$



2.3 NAND2 TRANSFER FUNCTION



Plot the static transfer function of the NAND2 gate in the following cases:

- The entries A and B of this NAND gate are connected to the same input signal which varies from V_{SS} to V_{DD} . In this scenario the two inputs switch at the same time.
- The A input is at high state (voltage V_{DD}), and the B voltage varies from V_{SS} to V_{DD} . This corresponds to commutations which do not occur at the same time.

Deduce the noise margin on the input low state and high state.

2.3.1 • Simultaneous switch

We observe that $A = B = 0 \implies A \cdot B = 1$ and $A = B = 1 \implies A \cdot B = 0$. Thus, we expect the output graphic to look like an inverter and we can compute the margins like we did in the previous question.

```
. include cmosws.mod

Vdd 1 0 dc 3.3V

Vin 2 0 dc 1.0V

M1 1 2 4 1 MODP L=0.6U W=6.0U

M2 1 2 4 1 MODP L=0.6U W=6.0U

M3 4 2 5 0 MODN L=0.6U W=6.0U

M4 5 2 0 0 MODN L=0.6U W=6.0U

. dc Vin 0 3.3 50mV
```



11

.end



Figure 9: In red, we have inputs A and B. In green the output of the NAND2 gate.

We get from the graphic

- $V_{IL} = 0,97$ V (maximum A,B for which V_{out} is high)
- $V_{IH} = 2,32V$ (minimum A,B for which V_{out} is low)
- again we consider $V_{OL} = 0$ V and $V_{OH} = 3, 3$ V.

We conclude that

- $NML = V_{IL} V_{OL} = 0,97V$
- $NMH = V_{OH} V_{IH} = 3, 3 2, 32 = 0,98V$

2.3.2 • Non-simultaneous switch

Now we fix A = 1. For B = 0, we have $A \cdot B = 1$ and for B = 1, we have $A \cdot B = 0$. Thus, if we consider $V_{in} = B$ we have once again an inverter and we can compute the margins like we did previously.

```
1 .include cmosws.mod

2 
3     Vdd 1 0 dc 3.3V

4     Va 2 0 dc 3.3V

5     Vb 3 0 dc 1.0V

6     M1 1 2 4 1 MODP L=0.6U W=6.0U

7     M2 1 3 4 1 MODP L=0.6U W=6.0U

8     M3 4 2 5 0 MODN L=0.6U W=6.0U
```



```
9 M4 5 3 0 0 MODN L=0.6U W=6.0U

10 .dc Vb 0 3.3 50mV

12 .end
```



Figure 10: In red, we have input B. In green the output of the NAND2 gate.

We get from the graphic

- $V_{IL} = 0,87$ V (maximum A, B for which V_{out} is high)
- $V_{IH} = 2,31 \text{V}$ (minimum A,B for which V_{out} is low)
- again we consider $V_{OL} = 0$ V and $V_{OH} = 3, 3$ V.

We conclude that

- $NML = V_{IL} V_{OL} = 0.87V$
- $NMH = V_{OH} V_{IH} = 3, 3 2, 31 = 0,99V$

Comparing the margins calculated, we conclude that the simultaneous switch gives a more centered curve.

DYNAMICAL CHARACTERIZATION OF THE CMOS INVERTER



3.1 Output capacitance influence

We consider a centered CMOS inverter for which the output S is connected to a charge capacitance C. This capacitance C modeled the sum of the capacitance of the input gates addressed by the signal S, plus the interconnection capacitance. We now use

- $L_n = 0,6\mu m \text{ and } W_n = 3,0\mu m;$
- $L_p = 0.6 \mu m$ and $W_p = 6.0 \mu m$;

Measure the propagation time $T_p(\text{up} \to \text{down})$ and $T_p(\text{down} \to \text{up})$ for C equal to 0.0pF, 0.1pF, 0.2pF, 0.5pF and 1.0pF. Plot $T_p(C)$.

We modify the code for the centered inverter to introduce the capacitance C. I could not find support for *parameter sweep* in Spice Opus, so we must run the script with a different value of C to measure each propagation time.

```
.include cmosws.mod
        Vin 1 0 pulse (0.0V 3.3V 5ns 1ns 1ns 3ns 8ns)
3
        Vdd 2 0 dc 3.3V
            3 1 0 0 MODN L=0.6U W=3.0U
            2 1 3 2 MODP L=0.6U W=6.0U
6
           3 0 0.0pF
            3 0 0.1pF
        *C1
8
        *C1
            3 0 0.2pF
9
        *C1
            3 0 0.5pF
            3 0 1.0pF
11
        .tran 0.05ns 13ns
        .end
14
```

SPICE ELECTRICAL SIMULATION



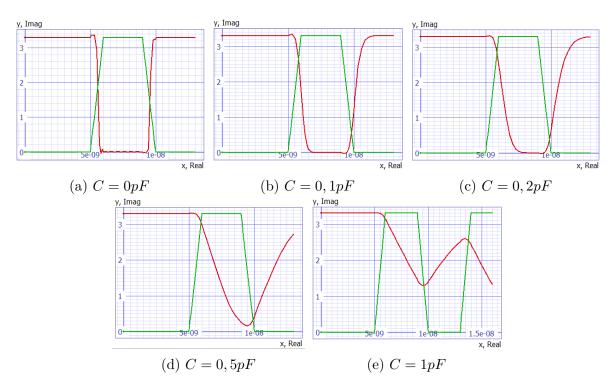


Figure 11: Output V_{out} in red and input V_{in} in green for different values of C

C	$T_p(\text{up} \to \text{down})$	$T_p(\text{down} \to \text{up})$
0pF	0,11ns	0,12ns
0,1pF	0,52ns	0,62ns
0,2pF	0,81ns	1,02ns
0,5pF	1,69ns	1,82ns
1pF	3,11ns	1,08ns

Table 1: Propagation times measured in each plot

When we plot the propagation times, we observe a clear linear relation between T_p and C. We can use linear regression to plot $T_p(\text{up} \to \text{down})$ and $T_p(\text{down} \to \text{up})$. For high values of C however, it is importance to notice that because the capacitor never manages to fully decharge, $T_p(\text{down} \to \text{up})$ starts to decrease and we lose this linear behavior. The plot was made using Python.



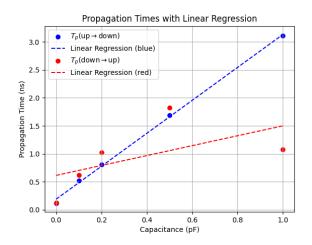


Figure 12: Propagation time T_p as function of capacitance C

3.2 W AND L SIZING INFLUENCE

We select a charge capacitance of 0, 5pF.

- Measure the variation of the propagation times when the resistance of the N transistor is decreased by increasing W_n . Plot the graph $T_p^{\text{up}\to\text{down}}(W_n)$.
- Measure the variation of the propagation times when the resistance of the N transistor is decreased by increasing W_n . Plot the graph $T_p^{\text{down}\to\text{up}}(W_p)$.

3.2.1 • $T_p(\text{UP} \to \text{DOWN})$ function of W_n

The values of W_n chosen for simulation and measurement were: $2\mu m$, $3\mu m$, $6\mu m$, $9\mu m$, $12\mu m$. I tried $W_n = 1\mu m$ but for low values of W_n , the output does not even reach $\frac{V_{DD}}{2}$.

W_n	$T_p(\mathrm{up} \to \mathrm{down})$
$2\mu m$	2,48ns
$3\mu m$	1,69ns
$6\mu m$	0,91ns
$9\mu m$	0,69ns
$12\mu m$	0,56ns

Table 2: Propagation time measured for each value of W_n

SPICE ELECTRICAL SIMULATION



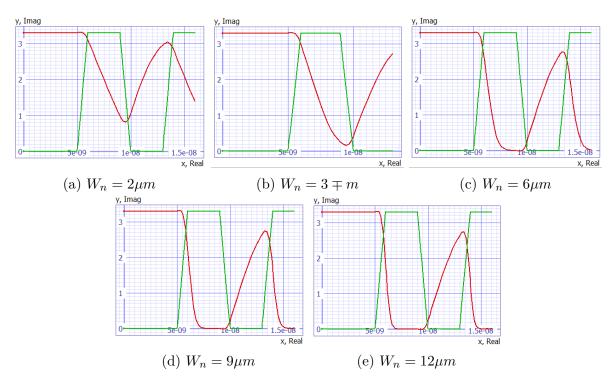


Figure 13: Output V_{out} in red and input V_{in} in green for different values of W_n

When we plot $T_p(\text{up} \to \text{down})$ as a function of W_n , we observe that $T_p(\text{up} \to \text{down})$ seems to decrease exponentially and for large values of W_n it approaches some constant. Therefore, the appropriate fit is

$$T_p^{\text{up}\to\text{down}}(W_n) = a \cdot e^{-bW_n} + c$$

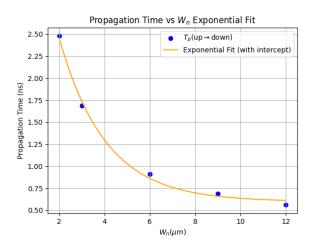


Figure 14: Propagation time $T_p(\text{up} \to \text{down})$ as function of W_n



3.2.2 • $T_p(\text{DOWN} \rightarrow \text{UP})$ function of W_p

The values of W_p chosen for simulation and measurement were: $4\mu m$, $5\mu m$, $6\mu m$, $9\mu m$, $12\mu m$. I tried $W_p = 3\mu m$ but similarly to what we verified for W_n , for low values of W_p , the output does not even reach $\frac{V_{DD}}{2}$.

W_p	$T_p(\text{down} \to \text{up})$
$4\mu m$	2,82ns
$5\mu m$	2,22
$6\mu m$	1,81ns
$9\mu m$	1,31ns
$12\mu m$	1,01ns

Table 3: Propagation time measured for each value of W_p

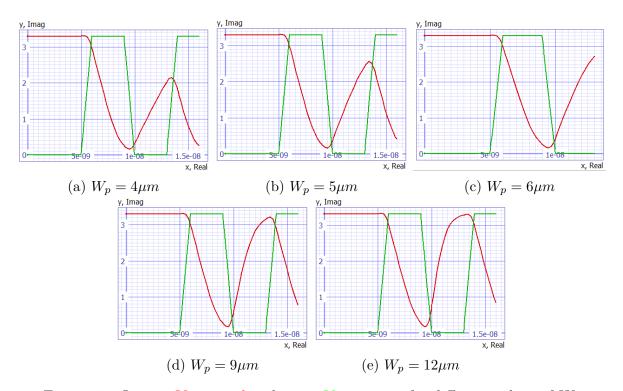


Figure 15: Output V_{out} in red and input V_{in} in green for different values of W_p

Now, when we plot $T_p(\text{down} \to \text{up})$ as a function of W_p , we observe that $T_p(\text{down} \to \text{up})$ seems to decrease exponentially and for large values of W_p it approaches some constant. Therefore, the appropriate fit is

$$T_p^{\text{down}\to \text{up}}(W_p) = a \cdot e^{-bW_p} + c$$



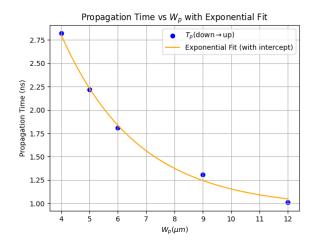


Figure 16: Propagation time $T_p(\text{down} \to \text{up})$ as function of W_p

3.3 Temperature and threshold voltage influence

Measure the variation of the propagation time for the reference inverter $(W_n = 3\mu m)$ and $W_p = 6\mu m$ for an output capacitance C of 0, 5pF in the four following cases:

- -40°C, worst case
- +125°C, worst case
- -40°C, typical
- +125°C, typical



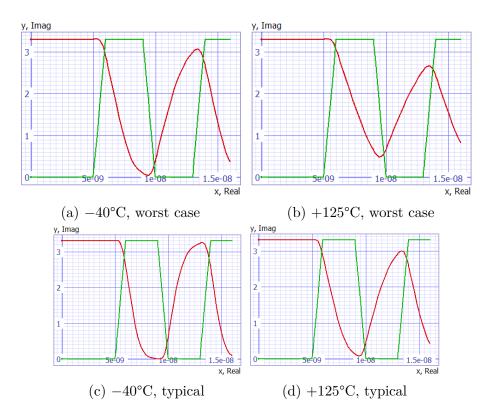


Figure 17: Output V_{out} in red and input V_{in} in green in all 4 scenarios

	-40°C	+125°C
worst case	1,43ns	2,07ns
typical	0,99ns	1,43ns

Table 4: $T_p(\text{down} \to \text{up})$ measured in all 4 scenarios

	-40°C	+125°C
worst case	1,61ns	1,91ns
typical	1,11ns	1,61ns

Table 5: $T_p(\text{up} \to \text{down})$ measured in all 4 scenarios

We observe that $T_p(\text{down} \to \text{up})$ and $T_p(\text{up} \to \text{down})$ are bigger for the worst case parameters as expected. And they are also bigger at higher temperatures, which we also expected because heating causes the transistor's performance to deteriorate.