Steps:

1. Load the dataset.
2. Load the autoencoder.
3. Load the trained GAN.
4. Apply the trained GAN to the latent code.

In [1]:

```python
import torch
from torch import nn

import numpy as np

from torch.utils.data import Dataset, DataLoader
from torch.nn import functional as F

from matplotlib import pyplot as plt


from torchvision.datasets import MNIST
from torchvision import transforms as T
from torchvision import models
```

```
/home/harkhymadhe/miniforge3/lib/python3.11/site-packages/torchvision/io/image.py:13: UserWarning: Failed to load image Python extension: '/home/harkhymadhe/miniforge3/lib/python3.11/site-packages/torchvision/image.so: undefined symbol: _ZN3c106detail23torchInternalAssertFailEPKcS2_jS2_RKSs'If you don't plan on using image functionality from `torchvision.io`, you can ignore this warning. Otherwise, there might be something wrong with your environment. Did you have `libjpeg` or `libpng` installed before building `torchvision` from source?
  warn(
```

In [2]:

```python
# Load data files
train_dataset = MNIST(root = ".", download = True, train = True)
test_dataset = MNIST(root = ".", download = True, train = False)
```

In [3]:

```python
class MNISTDataset(Dataset):
    def __init__(self, dataset, return_tensor = False):
        super().__init__()

        self.return_tensor = return_tensor

        self.base_rotator = T.Compose(
            [
                T.ToTensor(),
                T.Normalize([0.5,], [0.5,])
            ]
        )

        self.rotators = [
            T.Compose(
                [
                    T.ToTensor(),
```

```python
                    T.Normalize([0.5,], [0.5,]),
                    T.RandomRotation((30 * factor, 30 * factor)),
                ]
            )

            for factor in range(1, 12)
        ]

        self.dataset = dataset

    def __getitem__(self, index):
        mapped_index = int(index / 12) # Index to base image to be rotated
        rotation_index = index % 12    # Index for angular rotation to be ap

        image, label = self.dataset.__getitem__(mapped_index)
        image = self.rotators[rotation_index - 1](image) if rotation_index >

        return (image, label) if self.return_tensor else (T.ToPILImage()(ima


    def __len__(self):
        return self.dataset.__len__() * 12
```

In [4]:
```python
train_dataset = MNISTDataset(dataset = train_dataset, return_tensor = True)
test_dataset = MNISTDataset(dataset = test_dataset, return_tensor = True)
```

In [5]:
```python
BATCH_SIZE = 128

train_dl = DataLoader(train_dataset, batch_size = BATCH_SIZE, shuffle = True
test_dl = DataLoader(test_dataset, batch_size = BATCH_SIZE, shuffle = False,
```

In [ ]:

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In [6]:
```python
class ParticleEncoder(nn.Module):
    def __init__(self, latent_dim, p = .4):
        super(ParticleEncoder, self).__init__()

        self.latent_dim = latent_dim
        self.p = p

        self.l1 = nn.Conv2d(in_channels = 1, out_channels = 8, stride = 1, p
        self.b1 = nn.BatchNorm2d(8)
        self.d1 = nn.Dropout(self.p)

        self.l2 = nn.Conv2d(in_channels = 8, out_channels = 16, stride = 1,
        self.b2 = nn.BatchNorm2d(16)
        self.d2 = nn.Dropout(self.p)

        self.l3 = nn.Conv2d(in_channels = 16, out_channels = 32, stride = 3,
        self.b3 = nn.BatchNorm2d(32)
```

```python
        self.d3 = nn.Dropout(self.p)

        self.l4 = nn.Conv2d(in_channels = 32, out_channels = 64, stride = 2,
        self.b4 = nn.BatchNorm2d(64)

        self.linear = nn.Linear(64*6*6, self.latent_dim)

        self.flatten = nn.Flatten()

    def forward(self, x):
        y = self.d1(F.relu(self.b1(self.l1(x))))
        y = self.d2(F.relu(self.b2(self.l2(y))))
        y = self.d3(F.relu(self.b3(self.l3(y))))

        y = F.relu(self.b4(self.l4(y)))

        y = self.flatten(y)
        y = self.linear(y)

        return y
```

In [7]:

```python
class ParticleDecoder(nn.Module):
    def __init__(self, latent_dim):
        super(ParticleDecoder, self).__init__()

        self.latent_dim = latent_dim

        self.linear = nn.Linear(self.latent_dim, 64*6*6)

        self.l1 = nn.ConvTranspose2d(in_channels = 64, out_channels = 32, ke
        self.b1 = nn.BatchNorm2d(32)

        self.l2 = nn.ConvTranspose2d(in_channels = 32, out_channels = 16, ke
        self.b2 = nn.BatchNorm2d(16)

        self.l3 = nn.ConvTranspose2d(in_channels = 16, out_channels = 8, ker
        self.b3 = nn.BatchNorm2d(8)

        self.l4 = nn.Conv2d(in_channels = 8, out_channels = 1, kernel_size =

    def forward(self, x):
        x = F.relu(self.linear(x))
        y = F.relu(self.b1(self.l1(x.view(-1, 64, 6, 6))))
        y = F.relu(self.b2(self.l2(y)))
        y = F.relu(self.b3(self.l3(y)))

        y = torch.tanh(self.l4(y).view(-1, 1, 28, 28))

        return y
```

In [8]:

```python
class ParticleAutoEncoder(nn.Module):
    def __init__(self, in_dim = 128, out_dim = 128, encoder = None, decoder
        super(ParticleAutoEncoder, self).__init__()
```

```python
        self.in_dim, self.out_dim = in_dim, out_dim

        self.encoder = encoder
        self.decoder = decoder

    def forward(self, x):
        x_encoded = self.encoder(x)
        return self.decoder(x_encoded)

    @torch.no_grad()
    def generate_images(self, x_, code_type = 'code'):
        self.eval()
        if code_type == 'code':
            img = self.decoder(x_)
        elif code_type == 'image':
            img = self(x_)
        else:
            raise Exception('Provide a random code [-1, 1] or an image [-1,

        return img

    @torch.no_grad()
    def generate_code(self, image):
        self.eval()
        code = self.encoder(image)
        return code
```

In [ ]:

In [ ]:

In [9]:
```python
class LieGenerator(nn.Module):
    def __init__(self, latent_dim):
        super().__init__()

        self.latent_dim = latent_dim

        self.layer_norm = nn.LayerNorm(self.latent_dim)

        self.linear1 = nn.Linear(in_features = self.latent_dim, out_features
        self.bn1 = nn.BatchNorm1d(self.latent_dim * 2)

        self.linear2 = nn.Linear(in_features = self.latent_dim * 2, out_feat
        self.bn2 = nn.BatchNorm1d(self.latent_dim * 2)

        self.linear3 = nn.Linear(in_features = self.latent_dim * 2, out_feat

    def forward(self, x):
        x = self.layer_norm(x)

        x = self.bn1(self.linear1(x))
        x = F.relu(x)

        x = self.bn2(self.linear2(x))
        x = F.relu(x)
```

```python
        x = self.linear3(x)

        return F.tanh(x)
```

In [10]:
```python
class LieDiscriminator(nn.Module):
    def __init__(self, latent_dim):
        super().__init__()

        self.latent_dim = latent_dim

        self.layer_norm = nn.LayerNorm(self.latent_dim)

        self.linear1 = nn.Linear(in_features = self.latent_dim, out_features
        self.bn1 = nn.BatchNorm1d(self.latent_dim * 2)

        self.linear2 = nn.Linear(in_features = self.latent_dim * 2, out_feat
        self.bn2 = nn.BatchNorm1d(self.latent_dim * 2)

        self.linear3 = nn.Linear(in_features = self.latent_dim * 2, out_feat

    def forward(self, x):
        x = self.layer_norm(x)

        x = self.bn1(self.linear1(x))
        x = F.relu(x)

        x = self.bn2(self.linear2(x))
        x = F.relu(x)

        x = self.linear3(x)

        return F.softmax(x, dim = -1)
```

In [11]:
```python
class LieGAN(nn.Module):
    def __init__(self, generator = None, discriminator = None):
        super().__init__()

        self.discriminator = discriminator
        self.generator = generator

    def forward(self, x, noise):
        lie_group_element = self.generator(noise)
        transformed_x = x * lie_group_element

        return self.discriminator(transformed_x)
```

In [ ]:
```python

```

In [ ]:
```python

```

In [12]:
```python
latent_dim = 32
DEVICE = "cuda"
```

In [13]:
```
encoder = ParticleEncoder(latent_dim = latent_dim)
decoder = ParticleDecoder(latent_dim = latent_dim)
```

In [14]:
```
auto_encoder = ParticleAutoEncoder(
    encoder = encoder,
    decoder = decoder
).to(DEVICE)
```

In [15]:
```
auto_encoder.load_state_dict(torch.load("new_autoencoder.pt"))
```

Out[15]:
```
<All keys matched successfully>
```

In [ ]:

In [16]:
```
generator = LieGenerator(latent_dim = latent_dim)
discriminator = LieDiscriminator(latent_dim = latent_dim)
```

In [ ]:

In [17]:
```
# Instantiate GAN model
gan_model = LieGAN(generator = generator, discriminator = discriminator).to(
```

In [18]:
```
gan_model.load_state_dict(torch.load("latent_gan.pt"))
```

Out[18]:
```
<All keys matched successfully>
```

In [ ]:

In [19]:
```
for p in auto_encoder.parameters():
    p.requires_grad_(False)

for p in gan_model.parameters():
    p.requires_grad_(False)

auto_encoder.eval()
gan_model.eval()
```

Out[19]:
```
LieGAN(
  (discriminator): LieDiscriminator(
    (layer_norm): LayerNorm((32,), eps=1e-05, elementwise_affine=True)
    (linear1): Linear(in_features=32, out_features=64, bias=True)
    (bn1): BatchNorm1d(64, eps=1e-05, momentum=0.1, affine=True, track_runn
ing_stats=True)
    (linear2): Linear(in_features=64, out_features=64, bias=True)
    (bn2): BatchNorm1d(64, eps=1e-05, momentum=0.1, affine=True, track_runn
ing_stats=True)
    (linear3): Linear(in_features=64, out_features=2, bias=True)
  )
  (generator): LieGenerator(
    (layer_norm): LayerNorm((32,), eps=1e-05, elementwise_affine=True)
    (linear1): Linear(in_features=32, out_features=64, bias=True)
    (bn1): BatchNorm1d(64, eps=1e-05, momentum=0.1, affine=True, track_runn
ing_stats=True)
    (linear2): Linear(in_features=64, out_features=64, bias=True)
    (bn2): BatchNorm1d(64, eps=1e-05, momentum=0.1, affine=True, track_runn
ing_stats=True)
    (linear3): Linear(in_features=64, out_features=32, bias=True)
  )
)
```

In [ ]:
```

```

In [20]:
```
sample_images, sample_labels = next(iter(train_dl))
```

In [21]:
```
sample_images.shape
```

Out[21]:
```
torch.Size([128, 1, 28, 28])
```

In [22]:
```
sample_labels
```

Out[22]:
```
tensor([0, 3, 6, 9, 0, 5, 5, 1, 8, 2, 7, 4, 0, 1, 9, 3, 1, 8, 5, 9, 6, 4,
8, 5,
        6, 8, 9, 9, 0, 0, 5, 4, 2, 9, 9, 1, 4, 6, 6, 8, 0, 9, 5, 0, 4, 1,
0, 7,
        8, 0, 0, 7, 6, 0, 4, 2, 6, 8, 2, 0, 9, 2, 7, 6, 0, 4, 2, 8, 6, 8,
0, 7,
        5, 4, 1, 0, 2, 7, 4, 4, 9, 9, 6, 0, 8, 3, 2, 2, 3, 6, 9, 2, 1, 5,
2, 8,
        0, 6, 2, 1, 3, 0, 3, 9, 3, 9, 4, 7, 9, 9, 9, 1, 6, 2, 7, 7, 2, 1,
7, 1,
        4, 2, 5, 1, 7, 1, 5, 6])
```

In [23]:
```
# Generate code for image batch
sample_code = auto_encoder.encoder(sample_images.to(DEVICE))
```

In [24]:
```
sample_code.shape
```

Out[24]:
```
torch.Size([128, 32])
```

In [25]:
```
### Code mean (determined via pretraining)
MEAN = .02
```

```
STD = .233
```

In [26]:
```
noise = torch.distributions.Normal(loc = MEAN, scale = STD).sample(sample_co
```

In [27]:
```
noise.shape
```

Out[27]:
```
torch.Size([128, 32])
```

In [28]:
```
generated_symmetries = gan_model.generator(noise)
```

In [29]:
```
generated_symmetries.shape
```

Out[29]:
```
torch.Size([128, 32])
```

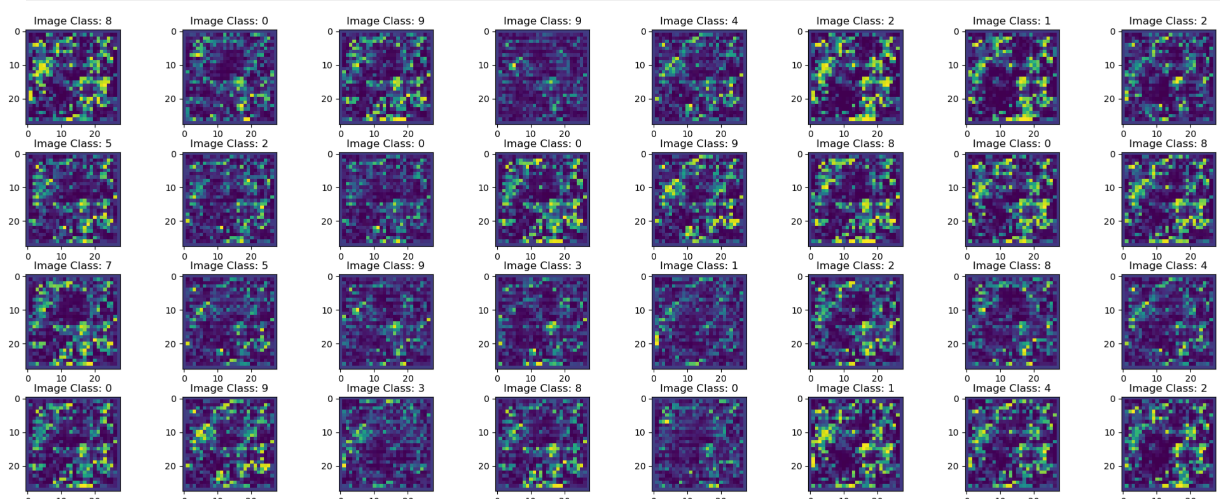
In [30]:
```
sample_code_orbit = generated_symmetries * sample_code.to(DEVICE)
```

In [31]:
```
decoded_sample_orbit = auto_encoder.decoder(sample_code_orbit)
decoded_sample_code = auto_encoder.decoder(sample_code)
```

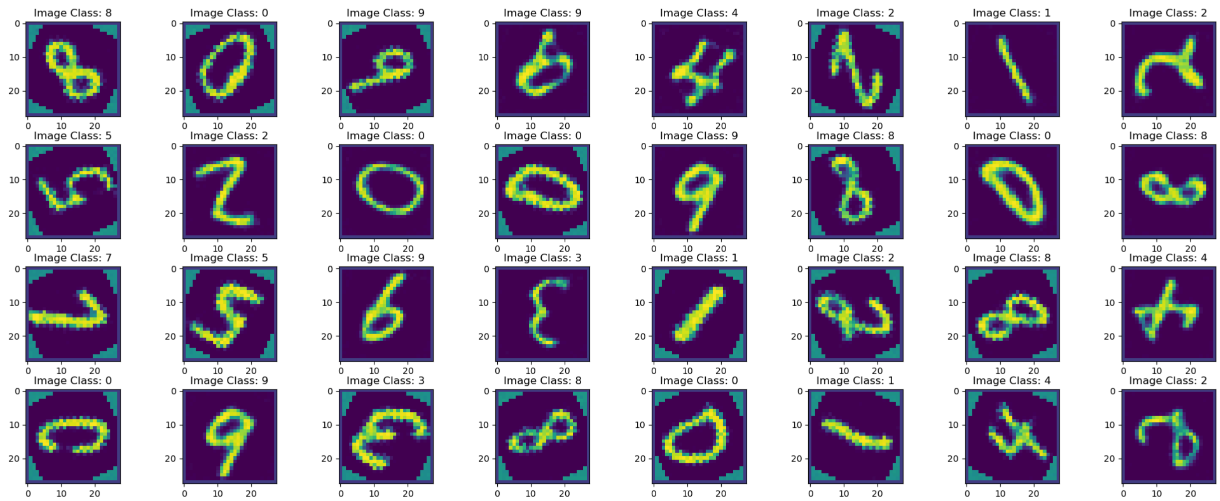
In [32]:
```
decoded_sample_orbit.shape
```

Out[32]:
```
torch.Size([128, 1, 28, 28])
```

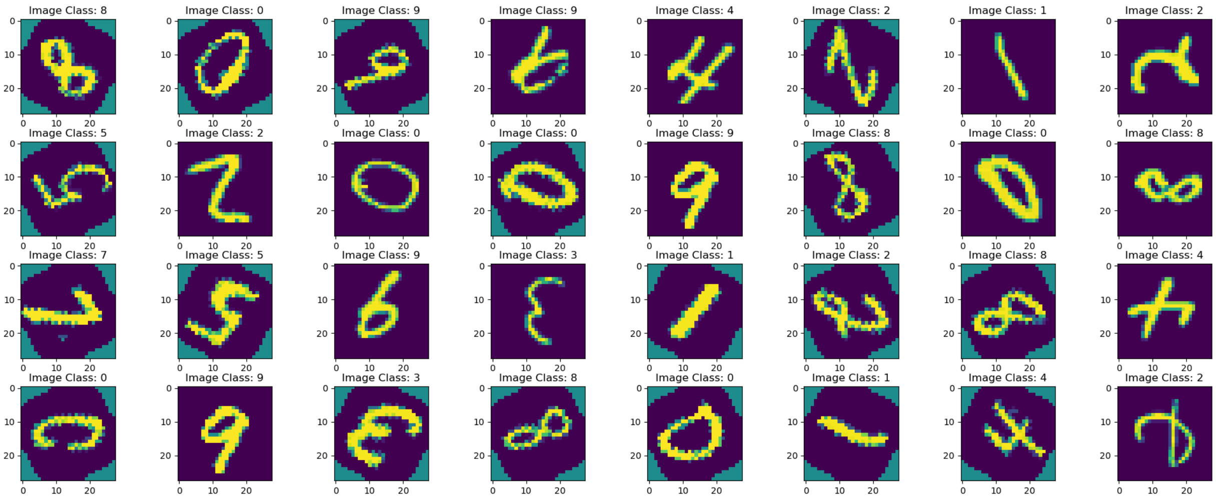
In [33]:
```
# Visualization of generated images (symmetries + code)
fig, ax = plt.subplots(nrows = 4, ncols = 8, figsize = (20, 8))

ix = 0
ixs = np.random.randint(low = 0, high = len(decoded_sample_orbit), size = (3
for i in range(4):
    for j in range(8):
        index = ixs[ix]
        ax[i, j].imshow(decoded_sample_orbit[index].detach().cpu().squeeze()
        ax[i, j].set_title(f"Image Class: {sample_labels[index].item()}")
        ix += 1

plt.tight_layout(h_pad = 0.01)
plt.show(); plt.close('all')
```



In [34]:
```
# Visualization of generated images (code)
fig, ax = plt.subplots(nrows = 4, ncols = 8, figsize = (20, 8))

ix = 0
for i in range(4):
    for j in range(8):
        index = ixs[ix]
        ax[i, j].imshow(decoded_sample_code[index].cpu().squeeze(),)
        ax[i, j].set_title(f"Image Class: {sample_labels[index].item()}")
        ix += 1

plt.tight_layout(h_pad = 0.01)
plt.show(); plt.close('all')
```



In [35]:
```
# Visualization of original images
fig, ax = plt.subplots(nrows = 4, ncols = 8, figsize = (20, 8))

ix = 0
for i in range(4):
    for j in range(8):
        index = ixs[ix]
        ax[i, j].imshow(sample_images[index].cpu().squeeze(),)
        ax[i, j].set_title(f"Image Class: {sample_labels[index].item()}")
        ix += 1

plt.tight_layout(h_pad = 0.01)
plt.show(); plt.close('all')
```



Observation

As can be observed from the images visualized above, although the autoencoder does it job almost perfectly, the symmetry generator is strongly lacking. Rather than generate actual meaningful symmetries, it only generates noise. Possible reasons for this may include:

1. **GAN Issues**: GANs are notoriously difficult to train.

2. **Symmetry-Code Combination**: The image code and the generated symmetry were combined via element-wise multiplication. This is not necessarily the best way to go, seeing as we are dealing with a latent space. More seasoned combination may be required.

3. **Training Time**: The GAN was only trained for 20 epochs. More time might be needed for the GAN to learn any meaningful symmetries.

4. **Data Distilation Techniques**: In this task, I was supposed to implement a data distillation technique to eliminate outliers from the data orbit. I thought on the problem quite a lot, but I was unable to come up with a meaningful approach due to time constraints. If I had more time, I'd look into this.

In [ ]:
```
```