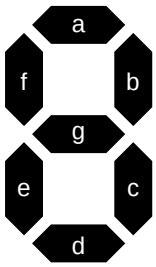
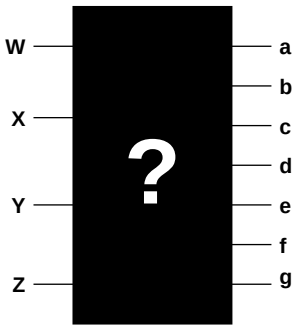
Given is the seven segment display below. Each segment represents one logic function (a,b,c,...). If the logic function is true(1) the segment lights up.



Given is the black box below. This black box has four inputs and the seven logic functions as outputs.



The functionality of this black box is given below:

W	X	Y	Z	
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5

W	X	Y	Z	
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	A

W	X	Y	Z	
1	0	1	1	b
1	1	0	0	C
1	1	0	1	d
1	1	1	0	E
1	1	1	1	F

Questions:

1. Draw the truth table for this black box.

W	X	Y	Z	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

2. Give the smallest representation for the functions using the method of the Karnaugh diagrams. Use for the functions **a**, **c**, **f**, and **g** the *max terms* and for the others the *min terms*.

$$a=\overline{\overline{W}\cdot\overline{X}\cdot\overline{Y}\cdot Z+\overline{W}\cdot X\cdot\overline{Y}\cdot\overline{Z}+W\cdot\overline{X}\cdot Y\cdot Z+W\cdot X\cdot\overline{Y}\cdot Z}$$

$$b=\overline{W}\cdot\overline{X}+\overline{X}\cdot\overline{Z}+\overline{W}\cdot\overline{Y}\cdot\overline{Z}+\overline{W}\cdot Y\cdot Z+W\cdot\overline{Y}\cdot Z$$

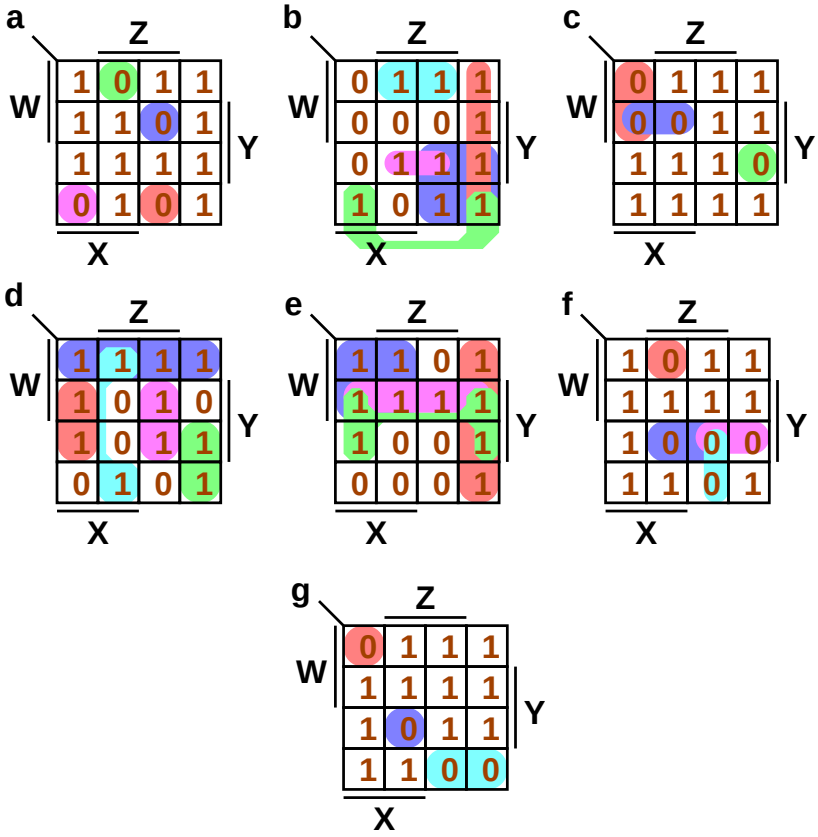
$$c=\overline{W\cdot X\cdot\overline{Z}+W\cdot X\cdot Y+\overline{W}\cdot\overline{X}\cdot Y\cdot\overline{Z}}$$

$$d=W\cdot\overline{Y}+X\cdot Y\cdot\overline{Z}+\overline{X}\cdot Y\cdot Z+\overline{W}\cdot\overline{X}\cdot\overline{Z}+X\cdot\overline{Y}\cdot Z$$

$$e=W\cdot X+W\cdot Y+\overline{X}\cdot\overline{Z}+Y\cdot\overline{Z}$$

$$f=\overline{W \cdot X \cdot \overline{Y} \cdot Z + \overline{W} \cdot Y \cdot Z + \overline{W} \cdot \overline{X} \cdot Y + \overline{W} \cdot \overline{X} \cdot Z}$$

$$g=\overline{\overline{W} \cdot \overline{X} \cdot \overline{Y} + W \cdot X \cdot \overline{Y} \cdot \overline{Z} + \overline{W} \cdot X \cdot Y \cdot Z}$$



3. Draw the gate implementation of these functions using only `AND`, `OR`, and `NOT` gates. Verify the correct functionality using logisim. ***Look in the logisim file.***

4. Draw and verify the gate implementation of function **a** using only `NOR`-gates (use logisim).

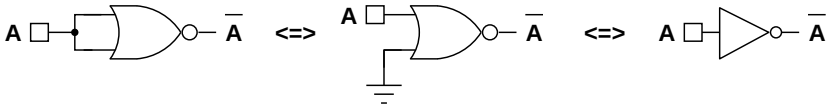
$$a=\overline{\overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot Z + \overline{W} \cdot X \cdot \overline{Y} \cdot \overline{Z} + W \cdot \overline{X} \cdot Y \cdot Z + W \cdot X \cdot \overline{Y} \cdot Z}$$

$$=\overline{\overline{\overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot Z} + \overline{\overline{W} \cdot X \cdot \overline{Y} \cdot \overline{Z}} + \overline{W \cdot \overline{X} \cdot Y \cdot Z} + \overline{W \cdot X \cdot \overline{Y} \cdot Z}}$$

$$=\overline{\overline{W + X + Y + \overline{Z}} + \overline{W + \overline{X} + Y + Z} + \overline{\overline{W} + X + \overline{Y} + \overline{Z}} + \overline{\overline{W} + \overline{X} + Y + \overline{Z}}}$$

and:

$$\overline{\mathbf{A}+\mathbf{A}} \Leftrightarrow \overline{\mathbf{A}+\mathbf{0}} \Leftrightarrow \overline{\mathbf{A}}$$



5. Draw and verify the gate implementation of function **b** using only `NAND`-gates (use logisim).

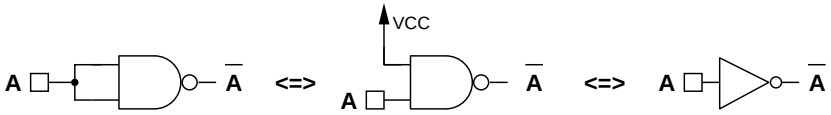
$$\mathbf{b}=\overline{\mathbf{W}}\cdot\overline{\mathbf{X}}+\overline{\mathbf{X}}\cdot\overline{\mathbf{Z}}+\overline{\mathbf{W}}\cdot\overline{\mathbf{Y}}\cdot\overline{\mathbf{Z}}+\overline{\mathbf{W}}\cdot\mathbf{Y}\cdot\mathbf{Z}+\mathbf{W}\cdot\overline{\mathbf{Y}}\cdot\mathbf{Z}$$

$$=\overline{\overline{\overline{\mathbf{W}}\cdot\overline{\mathbf{X}}+\overline{\mathbf{X}}\cdot\overline{\mathbf{Z}}+\overline{\mathbf{W}}\cdot\overline{\mathbf{Y}}\cdot\overline{\mathbf{Z}}+\overline{\mathbf{W}}\cdot\mathbf{Y}\cdot\mathbf{Z}+\mathbf{W}\cdot\overline{\mathbf{Y}}\cdot\mathbf{Z}}}$$

$$=\overline{\overline{\overline{\mathbf{W}}\cdot\overline{\mathbf{X}}}\cdot\overline{\overline{\mathbf{X}}\cdot\overline{\mathbf{Z}}}\cdot\overline{\overline{\mathbf{W}}\cdot\overline{\mathbf{Y}}\cdot\overline{\mathbf{Z}}}\cdot\overline{\overline{\mathbf{W}}\cdot\mathbf{Y}\cdot\mathbf{Z}}\cdot\overline{\mathbf{W}\cdot\overline{\mathbf{Y}}\cdot\mathbf{Z}}}$$

and:

$$\overline{\mathbf{A}\cdot\mathbf{A}} \Leftrightarrow \overline{\mathbf{A}\cdot\mathbf{1}} \Leftrightarrow \overline{\mathbf{A}}$$



6. Draw and verify the gate implementation of function **c** using only `OR` and `XOR`-gates (use logisim).

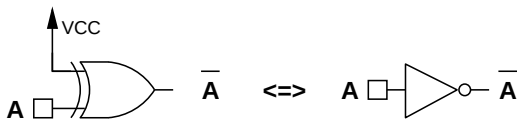
$$\mathbf{c}=\overline{\mathbf{W}\cdot\mathbf{X}\cdot\overline{\mathbf{Z}}+\mathbf{W}\cdot\mathbf{X}\cdot\mathbf{Y}+\overline{\mathbf{W}}\cdot\overline{\mathbf{X}}\cdot\mathbf{Y}\cdot\overline{\mathbf{Z}}}$$

$$=\overline{\overline{\overline{\mathbf{W}\cdot\mathbf{X}\cdot\overline{\mathbf{Z}}}}+\overline{\overline{\mathbf{W}\cdot\mathbf{X}\cdot\mathbf{Y}}}+\overline{\overline{\overline{\mathbf{W}}\cdot\overline{\mathbf{X}}\cdot\mathbf{Y}\cdot\overline{\mathbf{Z}}}}}$$

$$=\overline{\overline{\overline{\mathbf{W}}+\overline{\mathbf{X}}+\mathbf{Z}}+\overline{\overline{\mathbf{W}}+\overline{\mathbf{X}}+\overline{\mathbf{Y}}}+\overline{\mathbf{W}+\mathbf{X}+\overline{\mathbf{Y}}+\mathbf{Z}}}$$

and:

$$\mathbf{A}\oplus\mathbf{1} \Leftrightarrow \overline{\mathbf{A}}$$



7. Draw and verify the gate implementation of function **e** using the conjunctive canonical form [*product of sums*] (use logisim).

$$\begin{aligned}\mathbf{e} &= \mathbf{W}\cdot\mathbf{X} + \mathbf{W}\cdot\mathbf{Y} + \overline{\mathbf{X}}\cdot\overline{\mathbf{Z}} + \mathbf{Y}\cdot\overline{\mathbf{Z}} \\ &= \overline{\overline{\mathbf{W}\cdot\mathbf{X} + \mathbf{W}\cdot\mathbf{Y} + \overline{\mathbf{X}}\cdot\overline{\mathbf{Z}} + \mathbf{Y}\cdot\overline{\mathbf{Z}}}} \\ &= \overline{(\overline{\mathbf{W}} + \overline{\mathbf{X}})\cdot(\overline{\mathbf{W}} + \overline{\mathbf{Y}})\cdot(\mathbf{X} + \mathbf{Z})\cdot(\overline{\mathbf{Y}} + \mathbf{Z})}\end{aligned}$$