四川大学期末考试试题(闭卷)　A

(2012-2013学年第一学期)

课程号: 201097050 课序号: 0,1 课程名称: 高等代数-1 任课教师: 彭国华,谭友军,洪剑勇 成绩:

适用专业年级: 2012级数学学院各专业 学生人数: 250 印题份数: 280 学号:　　　　姓名:

考试须知

四川大学学生参加由学校组织或由学校承办的各级各类考试, 必须严格执行《四川大学考试工作管理办法》和《四川大学考场规则》. 有考试违纪作弊行为的, 一律按照《四川大学学生考试违纪作弊处罚条例》进行处理.

四川大学各级各类考试的监考人员, 必须严格执行《四川大学考试工作管理办法》、《四川大学考场规则》和《四川大学监考人员职责》. 有违反学校有关规定的, 严格按照《四川大学教学事故认定及处理办法》进行处理.

注意: 试卷满分100分, 解答写在答题纸上; 可以用中文答题.

记号: $\mathbb{Q}$表示有理数域, $\mathbb{F}$表示一个数域.

$x^2-2ax+a^2$ 　 $x^3-3px+2q$

$x^2-2ax^2+a^2$

1. (40 points, 8 points for each) Complete the following questions.

   $p=a^2$, $q=a^3$

   (1) Find conditions on p, q, a so that $x^3-3px+2q$ is divisible by $x^2-2ax+a^2$.

   (2) Let k be an integer. Is $x^4-4kx+1$ irreducible over $\mathbb{Q}$? Prove your claim.

   (3) Let 　$y=f(x+1)$ 　 $\det A\neq 0$ 可逆

   $12+6+3-6$

   $$A=\begin{pmatrix}4 & 2 & 3\\ 1 & 1 & 0\\ -1 & 2 & 3\end{pmatrix}$$

   $2B-AB=A$

   and B is a 3×3 matrix such that $AB=2B-A$. Compute B.

   (4) Find the rank and a maximal linearly independent subset for the vector collection $\alpha_1,\alpha_2,\alpha_3,\alpha_4$, where 　$r=3$ 　$\alpha_1, \alpha_2, \alpha_4$

   $$\alpha_1=(1,-2,3,-1), \alpha_2=(-1,2,1,1), \alpha_3=(1,-2,15,-1), \alpha_4=(0,1,2,3).$$

   (5) Let A be an $n\times n$ matrix over $\mathbb{F}$ and let $\alpha,\beta\in\mathbb{F}^n$ be two column vectors satisfying that $\alpha+\beta\neq 0$ and $A\alpha=3\beta$, $A\beta=4\alpha$. Prove the rank $r(A^2-12E_n)<n$, where E_n denotes the $n\times n$ unit matrix.

   $\alpha+\beta\neq 0$

$\frac{A\alpha}{3}=\beta$

$(A^2-12E_n)\alpha=0$

同理 $(A^2-12E_n)\beta=0$

$(A^2-12E_n)(\alpha-\beta)=0$

$A=3\alpha\beta^{-1}$

$A=4\alpha\beta^{-1}$

$A\alpha=3\beta$ 　 $A\beta=4\alpha$ 　 $\alpha=\frac{A\beta}{4}$ 　 $\frac{A^2\beta}{4}=3\beta$ 　 $A^2=12$

(3, 0, 0) → (3, 0, 0)

(2, 1, 0) → (1, 1, 0)

(1, 1, 1) → (0, 0, 1)

$a_3=1$ $a_2=a$ $a_1=-2$ $a_0=1$

2. (10 points) Let $a \in \mathbb{Q}$ and let $\alpha_1, \alpha_2, \alpha_3$ be the roots of $x^3 + ax^2 - 2x + 1 = 0$ such that $\alpha_1^3 + \alpha_2^3 + \alpha_3^3 = -158$. Compute a.

5

$\sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3$

$\sigma_1 = (-1)\dfrac{a_{n-1}}{a_n} = -\dfrac{a_2}{a_3}$

$\sigma_2 = (-1)^2 \dfrac{a_{n-2}}{a_n}$

$\sigma_3 = (-1)^3 \dfrac{a_{n-3}}{a_n} = -\dfrac{a_1}{a_4}$

3. (12 points) Compute the determinant

$$\begin{vmatrix} 1 & 3 & 3 & \cdots & 3 \\ 2 & -1 & 0 & \cdots & 0 \\ 2 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 2 & 0 & 0 & \cdots & n-3 \end{vmatrix} \qquad D_n = \begin{vmatrix} 1 & 3 & 3 & \cdots & 3 \\ 3 & 2 & 3 & \cdots & 3 \\ 3 & 3 & 3 & \cdots & 3 \\ \vdots & \vdots & \vdots & & \vdots \\ 3 & 3 & 3 & \cdots & n \end{vmatrix}.$$

展开两次

4. (12 points) Let A, B be $n \times n$ matrices and $D = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$.

(1) Show that $|D| = |A + B| \cdot |A - B|$. $\begin{pmatrix} E_n & E_n \\ 0 & E_n \end{pmatrix}\begin{pmatrix} A & B \\ B & A \end{pmatrix} = \begin{pmatrix} A+B & A+B \\ B & A \end{pmatrix}\begin{pmatrix} E_n & -E_n \\ 0 & E_n \end{pmatrix} = \begin{pmatrix} A+B & 0 \\ B & A-B \end{pmatrix}$

(2) Is it true that $|D| = |A^2 - B^2|$? If it's true, give a proof. Otherwise, give a counterexample. 不对

5. (14 points) Determine the values of k so that the following system of linear equations is solvable and write out all the solutions when it is solvable:

$$\begin{cases} 2x_1 - x_2 - 2x_3 = 0, \\ 4x_1 - 3x_2 + 2kx_3 = 2, \\ 3x_1 - 2x_2 + (k-1)x_3 = 1, \\ 2x_1 - kx_2 + 6x_3 = 2. \end{cases}$$

6. (12 points) Let m, n be positive integers.

(1) Assume $m \mid n$. Prove $x^m - 1 \mid x^n - 1$.

(2) Let

$$f(x) = x^{m-1} + x^{m-2} + \cdots + x + 1, \; g(x) = x^{n-1} + x^{n-2} + \cdots + x + 1.$$

Show that $(f(x), g(x)) = 1$ if and only if $(m, n) = 1$.

教务处试题编号:

26+12

2 −1 −2

4 −3 −6

3 6