به نام او

# Quiz 4

## Question:

Suppose that we wish to maintain the transitive closure of a directed graph $G= (V, E)$ as we insert edges into $E$. After each edge has been inserted, we want to update the transitive closure of the edges inserted so far. Assume that the graph $G$ has no edges initially and that we represent the transitive closure as a boolean matrix. Show how to update the transitive closure $G^*=(V, E^*)$ of a graph $G=(V, E)$ in $\mathcal{O}(V^2)$ time when a new edge is added to $G$.

## Answer (100pts): Pseudocode (60pts) + Explanation (40pts)

Let $T$ be the transitive closure matrix and is initialized as follows:

$$T[i,j] = \begin{cases} 1 & \text{when } i = j, \\ 0 & otherwise \end{cases}$$

$T$ is updated when an edge $(u, v)$ is added to $G$ as follows:

```
UpdateTransitiveClosure(T, u, v) {
    for i = 1 to |V|
        for j = 1 to |V|
            if T[i, u] == 1 and T[v, j] == 1 {
                T[i, j] = 1
            }
}
```

With this procedure, the effect of adding edge $(u, v)$ is to create a path (via the new edge) from every vertex that could already reach $u$ to every vertex that could already be reached from $v$. Note that the procedure sets $T$[u, v]=1 , because both $T$[u, u] and $T$[v, v] are initialized to 1. This procedure takes $\Theta(V^2)$ time because of the two nested loops.