به نام او

Quiz 5

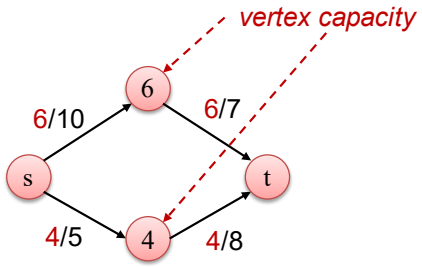
Question:

Suppose that, in addition to edge capacities, a flow network has *vertex capacities*, i.e., each vertex v (except *s* and *t*) has a limit $l(v)$ on how much flow can pass through v.

a) Show how to transform a flow network $G = (V, E)$ with vertex capacities into an equivalent flow network $G' = (V', E')$ without vertex capacities, such that a maximum flow in G' has the same value as a maximum flow in G.

b) How many vertices and edges does G' have?

Example:



Answer (100pts): Part a (80 pts), part b (20 pts)

a) For each vertex v (except *s* and t vertices), transform it to an edge (v, v') with capacity $l(v)$. The resultant graph G' ensures that the total flow passing through *v* in G' does not exceed $l(v)$. The original edges in *G* do not change.

Equivalence of flows:

- Any feasible flow in *G* can be transformed into a feasible flow in G′ by routing the flow through the split vertices while respecting edge capacities and vertex capacities.
- Conversely, any feasible flow in G′ corresponds to a feasible flow in *G* by collapsing the split vertices.
- The maximum flow values in G and G′ are thus the same.

b) G' has $2|V| - 2$ vertices and $|E| + |V| - 2$ edges.