به نام خدا

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Question: You are given a network with n vertices, m edges, a source s and a sink t. Suppose your friend presents to you an s-t flow for the network by assigning for every edge e, a flow $f(e)$. Describe an $O(n + m)$ algorithm to test if your friend's flow assignment is a maximum s-t flow.

Answer: The answer is basically equivalent to a single iteration of the Ford-Fulkerson algorithm. You may also want to make sure the given flow is feasible too. The runtime is dominated by the DFS execution which is O(m+n).

```
IsMaxFlow(G, s, t, c, f) {
    // Check flow feasibility
    for each edge e in G.E {
        if c[e] < f[e] OR f[e] < 0
            return false // not a valid flow
    }
    for each node u in G.V {
        flow[u] = null
    }
    DFS(u, x, cf)
    if flow[t] = null {
        return true // max flow
    }
    return false // not a max flow
}

DFS(u, x, c, f) {
    flow[u] = x
    for each e = (u, v) as outgoing edges of u {
        if flow[v] = null and c[u, v] - f[u, v] > 0 {
            parent[v] = u
            DFS(v, min(x, c[u, v] - f[u, v]))
        }
    }
    for each e = (v, u) as incoming edges of u {
        if flow[v] = null and f[v, u] > 0 {
            parent[v] = u
            DFS(v, min(x, f[v, u]))
        }
    }
}
```