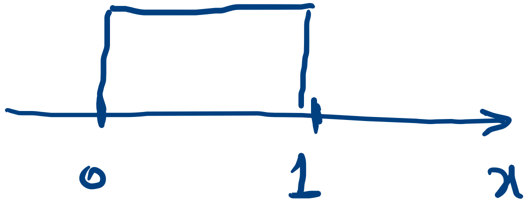
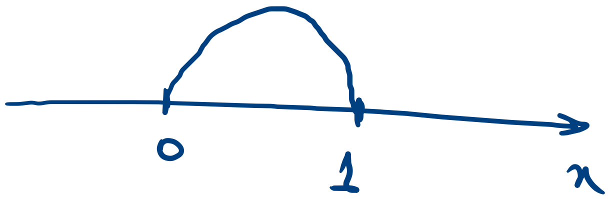
Beta Distribution

$0 \le x \le 1$

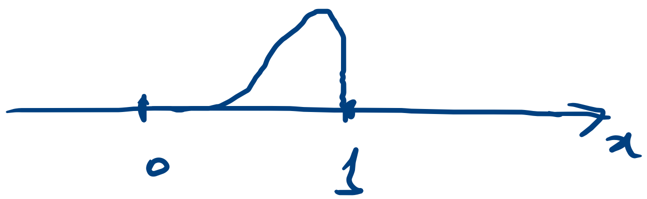
frequentist $x = 0.2$



Baysian

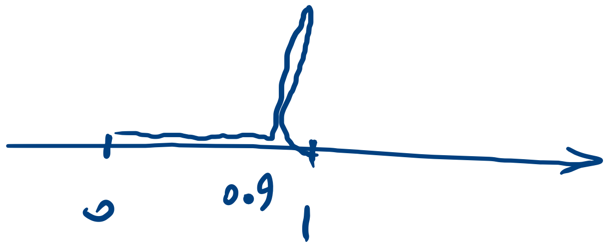


$f_X(x) \longrightarrow$ prior احتمال پیشین



$\downarrow$

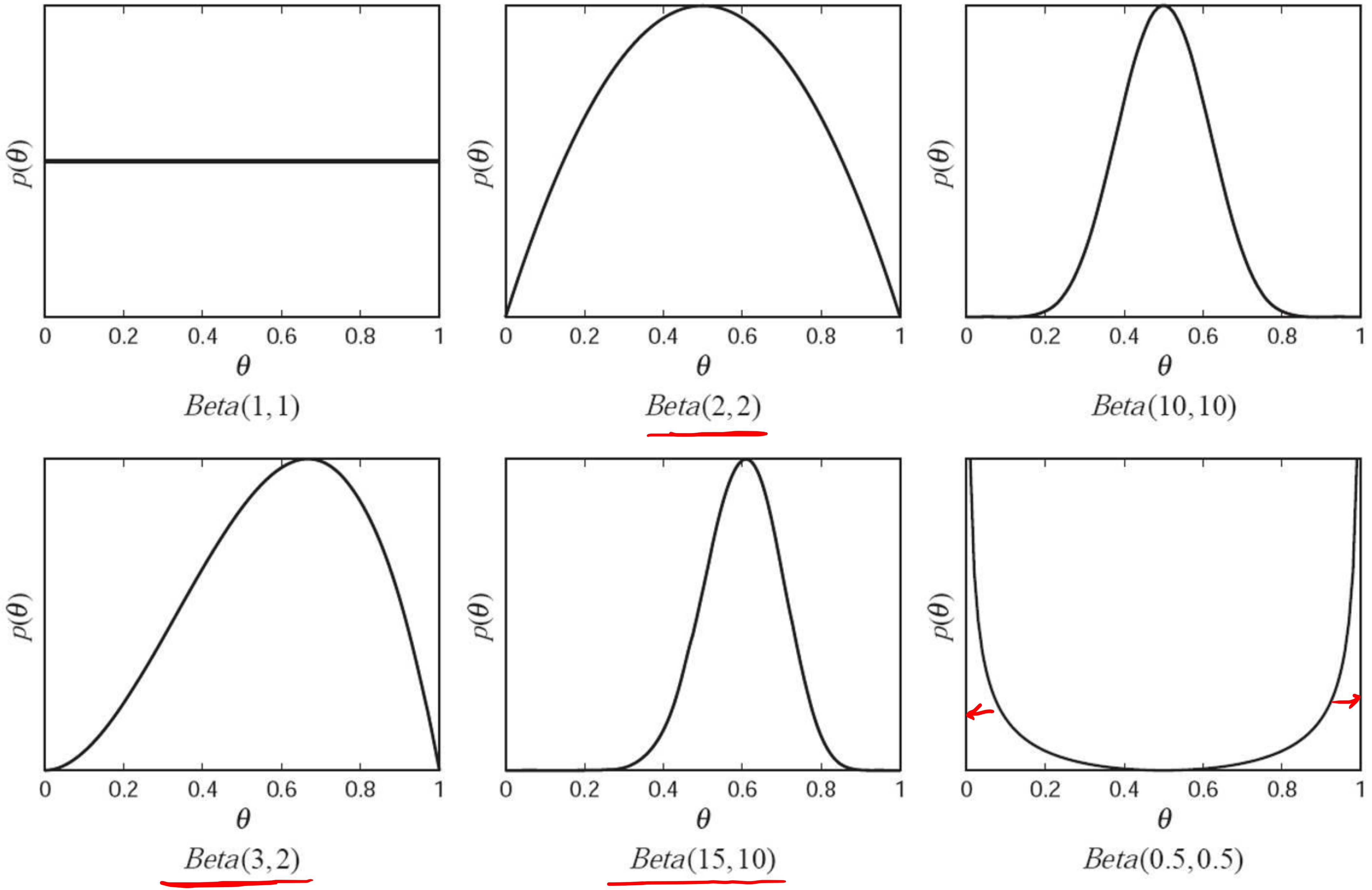
$f_{X|N}(x|n) \longrightarrow$ Posterior احتمال پسین



$$\underbrace{f_{X|N}(x|n)}_{\text{posterior}} = \frac{\overbrace{p_{N|X}(n|x)}^{\text{Likelihood}}\ \overbrace{f_X(x)}^{\text{prior}}}{p_N(n)}$$

توزیع بتا

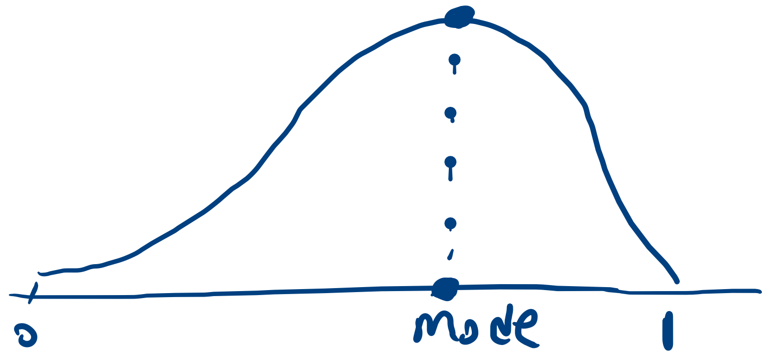
$$\underbrace{f_X(x)}_{\text{prior}} = \begin{cases} \frac{1}{c} x^{a-1}(1-x)^{b-1} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

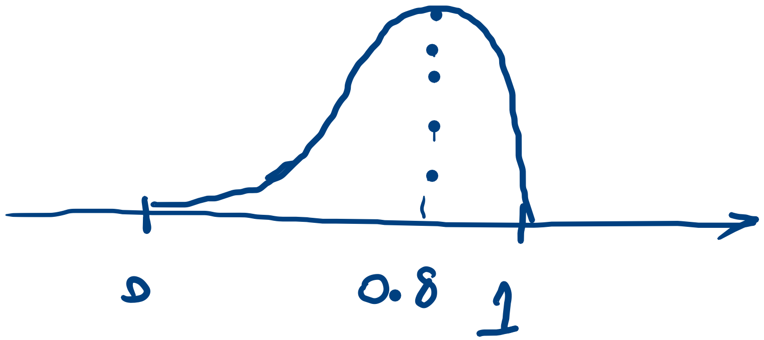

$p(\theta)$
0 0.2 0.4 0.6 0.8 1
θ
$Beta(1,1)$
$Beta(2,2)$
$Beta(10,10)$
$Beta(3,2)$
$Beta(15,10)$
$Beta(0.5,0.5)$

میانگین و مد توزیع بتا

$$E[X] = \frac{a}{a+b}$$

$$x = \frac{a-1}{a+b-2}$$





$$\theta = E[X] = \frac{a}{a+b}$$

$$\theta = \frac{a-1}{a+b-2} = \frac{a-1}{a-1+b-1}$$

مد توزیع بتا

$$x = \frac{a-1}{a+b-2}$$

$$x = \frac{a-1}{a-1+b-1}$$

$$E[x] = \frac{a}{a+b}$$

$$X \sim U(0,1) = Beta(1,1)$$

$$m+n \qquad N|X \sim Bin(n+m, x)$$

$m \downarrow$ خط $\qquad n \downarrow$ شیر

$$\boxed{f_X(x) = Beta(1,1)}$$

$$f_{N|X}(n|x) = Bin(n+m, x)$$

Conjugate prior

$$f_{X|N}(x|n) = ?$$

$$\underbrace{f_{X|N}(x|n)} = \frac{f_X(x)\, P_{N|X}(n|x)}{P_N(n)} = \frac{1 \times \binom{n+m}{n} x^n (1-x)^m}{P_N(n)}$$

$$= \frac{1}{c} x^n (1-x)^m = \boxed{Beta(n+1, m+1)}$$

10 بار $\xrightarrow{n}$ 2 شیر

10 بار $\xrightarrow{m}$ 8 خط

$x = 0.2 \leftarrow$ Frequentist

$$\underline{Beta(1,1)} \longrightarrow Beta(n+1, m+1) = Beta(3,9)$$

$$\boxed{x_{mode} = \frac{a-1}{a+b-2} = \frac{2}{12-2} = \frac{2}{10}}$$

$$x_{Exp} = \frac{a}{a+b} = \frac{3}{12}$$

prior

posterior

$\mathrm{Beta}(1,1) \longrightarrow \mathrm{Beta}(1+n, 1+m)$

$\boxed{\mathrm{Beta}(a,b)} \longrightarrow \mathrm{Beta}(a+n, b+m)$

$\downarrow$

$$\text{mode} = \frac{a+n-1}{a+b+m+n-2} \xrightarrow[n,m\to\infty]{} = \frac{n}{m+n}$$

$$\frac{n}{m+n}$$