۸۱۰۱۰۱۴۹۲ علیرضا کرمی

1)

$$f(x)=\begin{cases}\sin x & 0<x<\pi\\ -\sin x & -\pi<x<0\end{cases}\qquad f(x)=a_0+\sum_{n=1}^{\infty}\left[a_n\cos nx+b_n\sin nx\right]$$

تابع $f(x)=|\sin x|$ یک تابع زوج است.

$$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)\,dx \xrightarrow[\text{زوج بودن تابع}]{f(x)=f(-x)} a_0=\frac{1}{\pi}\int_0^{\pi}\sin x\,dx=\frac{-\cos x}{\pi}\Bigg|_0^{\pi}=\frac{2}{\pi}$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\underbrace{f(x)}^{\text{زوج}}\ \underbrace{\cos nx}^{\text{زوج}}\,dx \xrightarrow[\text{عبارت داخل انتگرال}]{\text{زوج بودن}} a_n=\frac{2}{\pi}\int_0^{\pi}\sin x\cos nx\,dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\left(\sin((1+n)x)+\sin((1-n)x)\right)dx=\frac{1}{\pi}\left(\frac{-\cos((1+n)x)}{1+n}-\frac{\cos((1-n)x)}{1-n}\right)\Bigg|_0^{\pi}$$

$$=\frac{1}{\pi}\left(\frac{(-1)^n}{1+n}+\frac{(-1)^n}{1-n}-\left(\frac{-1}{1+n}-\frac{1}{1-n}\right)\right)=\frac{1}{\pi}\left(\frac{2(-1)^n+2}{1-n^2}\right)$$

$$\Longrightarrow a_n=\begin{cases}\dfrac{4}{\pi(1-n^2)} & n=2k\\[2mm] 0 & n=2k+1\end{cases}$$

از آنجایی که $n=1$ مخرج را صفر می‌کند، آن را باید جداگانه بررسی کرد.

$$a_1=\frac{1}{\pi}\int_{-\pi}^{\pi}|\sin x|\cos x\,dx=\frac{2}{\pi}\int_0^{\pi}\sin x\cos x\,dx=\frac{1}{\pi}\int_0^{\pi}\sin 2x\,dx$$

$$=\frac{-1}{2\pi}\cos 2x\Big|_0^{\pi}=0\ \checkmark$$

چون فقط برای nهای زوج وجود دارد

$$\Longrightarrow f(x)=\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{4}{\pi(1-4n^2)}\cos 2nx$$

$$x=\frac{\pi}{2}\Longrightarrow f\left(\frac{\pi}{2}\right)=\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{4}{\pi(1-4n^2)}\underbrace{\cos n\pi}^{(-1)^n}$$

$$\Longrightarrow 1=\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{4}{\pi}\frac{(-1)^{n+1}}{4n^2-1}\longrightarrow 1-\frac{2}{\pi}=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2-1}$$

$$\Longrightarrow \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2-1}=\frac{\pi-2}{4}$$

2) $$f(x) = \overbrace{\frac{1}{3}}^{a_0} + \sum_{n=1}^{\infty}\left[\overbrace{\frac{1}{2(n^2+1)}}^{a_n}\cos nx + \overbrace{\frac{1}{2n^3}}^{b_n}\sin nx\right]$$

$$I = \int_{-\pi}^{\pi} f(x)\left[1+\cos 2x + \sin 3x\right]dx = \int_{-\pi}^{\pi} f(x)\,dx + \int_{-\pi}^{\pi} f(x)\cos(2x)\,dx + \int_{-\pi}^{\pi} f(x)\sin(3x)\,dx$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = a_0 \Rightarrow \int_{-\pi}^{\pi} f(x)\,dx = 2\pi a_0 = \frac{2\pi}{3}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = a_n \Rightarrow \int_{-\pi}^{\pi} f(x)\cos(2x)\,dx = \pi a_2 = \frac{\pi}{2(2^2+1)} = \frac{\pi}{10}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = b_n \Rightarrow \int_{-\pi}^{\pi} f(x)\sin(3x)\,dx = \pi b_3 = \frac{\pi}{2.3^3} = \frac{\pi}{54}$$

$$\Rightarrow I = \frac{2\pi}{3} + \frac{\pi}{10} + \frac{\pi}{54} = \pi\,\frac{180+27+5}{270} = \underline{\frac{212}{270}\pi}$$

3) $f(x) = e^{-\left|\frac{x}{2}\right|} \sin 5\pi x \qquad \sin nx = \frac{1}{2i}\left(e^{inx} - e^{-inx}\right) \Rightarrow \sin 5\pi x = \frac{1}{2i}\left(e^{i5\pi x} - e^{-i5\pi x}\right)$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi}{2}x} \quad , \quad C_n = \frac{1}{8i}\int_{-2}^{2} e^{-\left|\frac{x}{2}\right|}\left(e^{i5\pi x} - e^{-i5\pi x}\right) e^{-\frac{in\pi}{2}x}\, dx$$

$$= \frac{1}{8i}\int_{-2}^{0} e^{\frac{x}{2}}\left(e^{i5\pi x} - e^{-i5\pi x}\right) e^{-\frac{in\pi}{2}x}\, dx + \frac{1}{8i}\int_{0}^{2} e^{-\frac{x}{2}}\left(e^{i5\pi x} - e^{-i5\pi x}\right) e^{-\frac{in\pi}{2}x}\, dx$$

$$= \frac{1}{8i}\int_{-2}^{0}\left[e^{\left(\frac{1-in\pi+i10\pi}{2}\right)x} - e^{\left(\frac{1-in\pi-i10\pi}{2}\right)x}\right] dx + \frac{1}{8i}\int_{0}^{2}\left[e^{\left(\frac{-1-in\pi+i10\pi}{2}\right)x} - e^{\left(\frac{-1-in\pi-i10\pi}{2}\right)x}\right] dx$$

$$= \frac{1}{8i}\left[\frac{2}{1-in\pi+i10\pi} e^{\left(\frac{1-in\pi+i10\pi}{2}\right)x} - \frac{2}{1-in\pi-i10\pi} e^{\left(\frac{1-in\pi-i10\pi}{2}\right)x}\right]_{-2}^{0} +$$

$$\frac{1}{8i}\left[\frac{2}{-1-in\pi+i10\pi} e^{\left(\frac{-1-in\pi+i10\pi}{2}\right)x} - \frac{2}{-1-in\pi-i10\pi} e^{\left(\frac{-1-in\pi-i10\pi}{2}\right)x}\right]_{0}^{2}$$

$$e^{i10\pi} = \cos(10\pi) + i\sin(10\pi) = \cos(10\pi) - i\sin(10\pi) = e^{-i10\pi} \quad (*)$$

$$e^{in\pi} = \cos n\pi + i\sin n\pi = \cos n\pi - i\sin n\pi = e^{-in\pi} \quad (**)$$

$$(*),(**) \Longrightarrow e^{i10\pi + in\pi} = e^{i10\pi - in\pi} = e^{-i10\pi + in\pi} = e^{-i10\pi - in\pi} = \cos(10\pi)\cos n\pi = (-1)^n$$

$$\Longrightarrow C_n = \frac{40\, in\pi}{e} \cdot \frac{(-1)^n - e}{(100\pi^2 - n^2\pi^2 + 1)^2 + 4\pi^2 n^2} = \frac{40\, in\pi}{e} \cdot \frac{(-1)^n - e}{\pi^4(n^2-100)^2 + 2\pi^2(n^2+100) + 1}$$

$$\Longrightarrow f(x) = \sum_{n=-\infty}^{+\infty} C_n e^{\frac{in\pi}{2}x} \Rightarrow y = \sum_{n=-\infty}^{+\infty} D_n e^{\frac{in\pi}{2}x}$$

$$\Rightarrow y' = \sum_{n=-\infty}^{+\infty} D_n \left(\frac{in\pi}{2}\right) e^{\frac{in\pi}{2}x} \Longrightarrow y'' = \sum_{n=-\infty}^{+\infty} D_n \underbrace{\left(\frac{in\pi}{2}\right)^2}_{-\frac{n^2\pi^2}{4}} e^{\frac{in\pi}{2}x}$$

3-ب)

$$\Longrightarrow \quad D_n\left(\frac{-n^2\pi^2}{4} + \alpha\,\frac{in\pi}{2} - 1\right) = C_n =$$

$$\frac{40\,in\pi}{e} \cdot \frac{(-1)^n - e}{\pi^4(n^2-100)^2 + 2\pi^2(n^2+100) + 1}$$

$$\Longrightarrow D_n = \frac{4}{-n^2\pi^2 + 2\alpha\, in\pi - 4} \times \frac{40\, in\pi}{e} \times \frac{(-1)^n - e}{\pi^2(n^2-100)^2 + 2\pi^2(n^2+100) + 1}$$

$$\Longrightarrow \quad y = \sum_{n=-\infty}^{+\infty} D_n\, e^{\frac{in\pi}{2}x}$$

4)

الف) $f(x) = \frac{A}{T}x \;,\; f(x) = \sum_{n=-\infty}^{+\infty} C_n e^{\frac{i2n\pi}{T}x} \;,\; C_n = \frac{1}{T}\int_0^T \frac{A}{T}x\, e^{\overbrace{-\frac{i2n\pi}{T}x}^{-i\omega_n x}} dx$

$$= \frac{A}{T^2}\left(\frac{i}{\omega_n} x e^{-i\omega_n x}\Big|_0^T - \frac{i}{\omega_n}\int_0^T e^{-i\omega_n x} dx \right) = \frac{A}{T^2}\left(\left(\frac{iT^2}{2\pi n} e^{-i2\pi n}\right) + \frac{1}{\omega_n^2} e^{-i\omega_n x}\Big|_0^T\right)$$

$$= \frac{A}{T^2}\left(\frac{iT}{2\pi n} T e^{-i2\pi n}\right) + \frac{A}{T^2}\frac{T^2}{4\pi^2 n^2} e^{-i2\pi n} - \frac{A}{T^2}\frac{T^2}{4\pi^2 n^2}$$

$$e^{-i2\pi n} = \cos 2\pi n - i \sin 2\pi n = 1 \Longrightarrow C_n = \frac{iA}{2\pi n}$$

$$C_0 = \frac{1}{T}\int_0^T \frac{A}{T}(x) e^0 dx = \frac{A}{T^2}\left(\frac{x^2}{2}\right)_0^T = \frac{A}{2}$$

$$\Longrightarrow f(x) = \frac{A}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{iA}{2\pi n} e^{\frac{i2n\pi}{T}x}$$

ب) $\sum_{n=-\infty}^{+\infty} |C_n|^2 = \frac{1}{T}\int_0^T \underbrace{\frac{A^2}{T^2}x^2}_{f^2(x)} dx = \frac{A^2}{T^3}\left.\frac{x^3}{3}\right)_0^T = \frac{A^2}{3}$

$$f(x) = \sum_{n=-\infty}^{+\infty} C_n e^{inx}$$

5) $$\sin^3 x \cos 2x = \left(\frac{e^{ix} - e^{-ix}}{2i} \right)^3 \left(\frac{e^{2ix} + e^{-2ix}}{2} \right)$$

$$= \frac{e^{3ix} - 3e^{ix} + 3e^{-ix} - e^{-3ix}}{-8i} \times \frac{e^{2ix} + e^{-2ix}}{2} = \frac{e^{5ix} - 3e^{3ix} + 4e^{ix} - 4e^{-ix} + 3e^{-3ix} - e^{-5ix}}{-16i}$$

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx = \frac{1}{-32i\pi} \int_{-\pi}^{\pi} \left[e^{i(5-n)x} - 3e^{i(3-n)x} + 4e^{i(1-n)x} - 4e^{i(-1-n)x} + 3e^{i(-3-n)x} - e^{i(-5-n)x} \right] dx$$

$$= \frac{1}{-32i\pi} \int_{-\pi}^{\pi} \Big[\cos(5x-nx) + i\sin(5x-nx) - 3\big(\cos(3x-nx) + i\sin(3x-nx)\big) + 4\big(\cos(x-nx) + i\sin(x-nx)\big)$$

$$- 4\big(\cos(-x-nx) + i\sin(-x-nx)\big) + 3\big(\cos(-3x-nx) + i\sin(-3x-nx)\big) - \big(\cos(-5x-nx) + i\sin(-5x-nx)\big) \Big] dx$$

در انتگرال گیری از عبارت فوق، از آنجا که $\sin$ های ایجاد شده به صورت $\sin k\pi$ هستند، مقدار صفر خواهند گرفت و از آنجا که $\cos$ های ایجاد شده به صورت $\cos(2k+1-n)\pi$ هستند، با هم خط می‌خورند. درنتیجه خواهیم داشت:

$$C_n = 0$$

حال باید حالت‌های خاص را پیدا کنیم. حالت‌هایی که توان‌ها را صفر می‌کنند (یا توان e ها را صفر می‌کنند) و آنها را جداگانه حساب کنیم.

$$n=5 \longrightarrow C_5 = \frac{1}{-32i\pi} \int_{-\pi}^{\pi} 1\, dx = \frac{1}{-16i}$$

$$n=3 \longrightarrow C_3 = \frac{1}{-32i\pi} \int_{-\pi}^{\pi} (-3)\, dx = \frac{+3}{16i}$$

$$n=1 \longrightarrow C_1 = \frac{1}{-32i\pi} \int_{-\pi}^{\pi} 4\, dx = \frac{1}{-4i}$$

$$n=-1 \longrightarrow C_{-1} = \frac{1}{-32i\pi} \int_{-\pi}^{\pi} (-4)\, dx = \frac{1}{4i}$$

$$n=-3 \longrightarrow C_{-3} = \frac{1}{-32i\pi} \int_{-\pi}^{\pi} 3\, dx = \frac{-3}{16i}$$

$$n=-5 \longrightarrow C_{-5} = \frac{1}{-32i\pi} \int_{-\pi}^{\pi} (-1)\, dx = \frac{1}{16i}$$

$$\Longrightarrow f(t) = \frac{1}{-16i} e^{i\omega_5 t} + \frac{3}{16i} e^{i\omega_3 t} - \frac{4}{16i} e^{i\omega_1 t} + \frac{4}{16i} e^{i\omega_{-1} t} - \frac{3}{16i} e^{i\omega_{-3} t} + \frac{1}{16i} e^{i\omega_{-5} t}$$

ب)

$$\Longrightarrow \sum_{-\infty}^{+\infty} |C_n|^2 = \frac{1+9+16+16+9+1}{256} = \frac{52}{256} = \frac{13}{64} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f^2(x)\, dx$$

$$\Longrightarrow \frac{13}{64} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{\overset{\text{زوج}}{\sin^4(x)}\ \overset{\text{زوج}}{\cos^2(2x)}}_{\text{زوج}}\, dx = \frac{1}{\pi} \int_{0}^{\pi} \sin^4(x) \cos^2(2x)\, dx$$

$$\Longrightarrow \int_0^{\pi} \sin^4(x) \cos^2(2x)\, dx = \frac{13}{64}\pi$$

4) $(\sin^2 x + \cos 5x)^2 \sin 3x = \left(\frac{1-\cos 2x}{2} + \cos 5x\right)^2 \sin 3x =$

$$\left(\frac{1-2\cos 2x+\cos^2 2x}{4} + \cos^2 5x + \cos 5x - \cos 2x \cos 5x\right)\sin 3x =$$

$$\left(\frac{1}{4} - \frac{2\cos 2x}{4} + \frac{1+\cos 4x}{8} + \frac{1+\cos 10x}{2} + \cos 5x - \frac{\cos 7x}{2} - \frac{\cos 3x}{2}\right)\sin 3x =$$

$$\frac{1}{4}\sin 3x - \frac{1}{4}\sin 5x - \frac{1}{4}\sin x + \frac{1}{8}\sin 3x + \frac{1}{16}\sin 7x - \frac{1}{16}\sin x + \frac{1}{2}\sin 3x + \frac{1}{4}\sin 13x - \frac{1}{4}\sin 7x$$

$$+ \frac{1}{2}\sin 8x - \frac{1}{2}\sin 2x - \frac{1}{4}\sin 10x + \frac{1}{4}\sin 4x - \frac{1}{4}\sin 6x$$

$$\Rightarrow I = \pi\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{2}\right)b_3 + \pi\left(-\frac{1}{4}\right)b_5 + \pi\left(-\frac{1}{4}-\frac{1}{16}\right)b_1 + \pi\left(\frac{1}{16}-\frac{1}{4}\right)b_7 +$$

$$\frac{\pi}{4}b_{13} + \frac{\pi}{2}b_8 - \frac{\pi}{2}b_2 - \frac{\pi}{4}b_{10} + \frac{\pi}{4}b_4 - \frac{\pi}{4}b_6$$

$$= \frac{-5\pi}{16}b_1 - \frac{\pi}{2}b_2 + \frac{7\pi}{8}b_3 + \frac{\pi}{4}b_4 - \frac{\pi}{4}b_5 - \frac{\pi}{4}b_6 - \frac{3}{16}b_7 + \frac{\pi}{2}b_8 - \frac{\pi}{4}b_{10} + \frac{\pi}{4}b_{13}$$

$$\xrightarrow{b_n = \frac{n}{n^3+1}} \frac{-5\pi}{32} - \frac{\pi}{9} + \frac{3\pi}{32} + \frac{\pi}{65} - \frac{5\pi}{504} - \frac{3\pi}{434} - \frac{21\pi}{16(344)} + \frac{4\pi}{513} - \frac{5\pi}{2002} + \frac{13\pi}{4\times(13^3+1)}$$

$$\simeq -0{,}1721\,\pi \simeq -0{,}5407$$

درواقع داشتیم : $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \Rightarrow \pi b_n = \int_{-\pi}^{\pi} f(x)\sin nx\,dx$

7) $\sin^7 x = (\sin^2 x)^3 \sin x = \left(\frac{1-\cos 2x}{2}\right)^3 \sin x = \left(\frac{1-3\cos 2x+3\cos^2 2x-\cos^3 2x}{8}\right)\sin x$

$$= \left(\frac{1}{8}-\frac{3}{8}\cos 2x+\frac{3}{16}+\frac{3}{16}\cos 4x-\frac{1}{16}\cos 2x-\frac{1}{16}\cos 2x\cos 4x\right)\sin x$$

$$= \left(\frac{1}{8}-\frac{3}{8}\cos 2x+\frac{3}{16}+\frac{3}{16}\cos 4x-\frac{1}{16}\cos 2x-\frac{1}{32}\cos 6x-\frac{1}{32}\cos 2x\right)\sin x$$

$$= \left(\frac{5}{16}-\frac{15}{32}\cos 2x+\frac{3}{16}\cos 4x-\frac{1}{32}\cos 6x\right)\sin x$$

$$= \frac{5}{16}\sin x-\frac{15}{64}\sin 3x+\frac{15}{64}\sin x+\frac{3}{32}\sin 5x-\frac{3}{32}\sin 3x-\frac{1}{64}\sin 7x+\frac{1}{64}\sin 5x$$

$$= \frac{35}{64}\sin x-\frac{21}{64}\sin 3x+\frac{7}{64}\sin 5x-\frac{1}{64}\sin 7x \longrightarrow$$

همان سری فوریه برای بازه $-\pi < x < \pi$ است

چون فقط ترم های $\sin$ دارد.

الف) از آنجا که تابع فرد است، و بازه متقارن است، هارمونیک های کسینوسی نداریم:

$a_0 = a_n = 0$

فرد: $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}\Big[\frac{35}{128}\big(\cos(1-n)x-\cos(1+n)x\big)-\frac{21}{128}\times$

$\big(\cos(3-n)x-\cos(3+n)x\big)+\frac{7}{128}\big(\cos(5-n)x-\cos(5+n)x\big)-\frac{1}{128}\big(\cos(7-n)x-\cos(7+n)x\big)\Big]dx$

از آنجا که این ترم های cos ها با انتگرال گرفتن به $\sin k\pi$ تبدیل می‌شوند، جواب همه آن ها صفر خواهد بود. حال باید حالات خاص (حالاتی که مخرج ها صفر می‌شود) را به طور جداگانه بررسی کنیم.

$n=1 \Rightarrow b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{35}{64}\sin^2 x = \frac{35}{128\pi}x\Big)_{-\pi}^{\pi} = \frac{35}{64}$

$n=3 \Rightarrow b_3 = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{-21}{64}\sin^2 3x = \frac{-21}{64}$

$n=5 \Rightarrow b_5 = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{7}{64}\sin^2 5x = \frac{7}{64}$, $n=7 \Rightarrow b_7 = \frac{-1}{64}$

$$\Longrightarrow f(x) = \frac{35}{64}\sin x-\frac{21}{64}\sin 3x+\frac{7}{64}\sin 5x-\frac{1}{64}\sin 7x$$

(ب

$$T=\pi \longrightarrow L=\frac{\pi}{2} \longrightarrow a_0=\frac{1}{2L}\int_0^{2L} f(x)\,dx=\frac{1}{\pi}\int_0^{\pi} \sin^7 x\,dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\left[\frac{35}{64}\sin x-\frac{21}{64}\sin 3x+\frac{7}{64}\sin 5x-\frac{1}{64}\sin 7x\right]dx=\frac{1}{\pi}\left(\frac{35}{32}-\frac{21}{3\times 32}+\frac{7}{5\times 32}-\frac{1}{7\times 64}\right)$$

$$=\frac{1}{\pi}\left(\frac{35}{32}-\frac{7}{32}+\frac{7}{160}-\frac{1}{224}\right)=\frac{32}{35\pi} \quad ,$$

$$b_n=\frac{2}{\pi}\int_0^{\pi}\left[\frac{35}{64}\sin x\sin 2nx-\frac{21}{64}\sin 3x\sin 2nx+\frac{7}{64}\sin 5x\sin 2nx-\frac{1}{64}\sin 7x\sin 2nx\right]dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\left[\frac{35}{64}\left(\cos(2n-1)x-\cos(2n+1)x\right)-\frac{21}{64}\left(\cos(2n-3)x-\cos(2n+3)x\right)+\frac{7}{64}\left(\cos(2n-5)x-\cos(2n+5)x\right)-\frac{1}{64}\left(\cos(2n-7)x-\cos(2n+7)x\right)\right]dx$$

از آنجایی که همه‌ی این ترم‌ها به صورت $\cos kx$ هستند، با انتگرال‌گیری و قرار دادن $x=\pi,0$ ، ترم‌های $\sin kx$ ساخته می‌شوند که همه برابر صفر هستند. بنابراین:

$$b_n=0$$

$$a_n=\frac{2}{\pi}\int_0^{\pi}\left[\frac{35}{64}\sin x\cos 2nx-\frac{21}{64}\sin 3x\cos 2nx+\frac{7}{64}\sin 5x\cos 2nx-\frac{1}{64}\sin 7x\cos 2nx\right]dx$$

$$=\frac{1}{\pi}\int_0^{\pi}\frac{35}{64}\left(\sin(1+2n)x+\sin(1-2n)x\right)-\frac{21}{64}\left(\sin(3+2n)x+\sin(3-2n)x\right)+\frac{7}{64}\left(\sin(5+2n)x+\sin(5-2n)x\right)-\frac{1}{64}\left(\sin(7+2n)x+\sin(7-2n)x\right)dx$$

$$=\frac{1}{\pi}\left(\frac{-35}{64}\left(\frac{-2}{1+2n}+\frac{-2}{1-2n}\right)+\frac{21}{64}\left(\frac{-2}{3+2n}+\frac{-2}{3-2n}\right)-\frac{7}{64}\left(\frac{-2}{5+2n}+\frac{-2}{5-2n}\right)+\frac{1}{64}\left(\frac{-2}{7+2n}+\frac{-2}{7-2n}\right)\right)$$

$$=\frac{1}{\pi}\left(\frac{35}{16}\left(\frac{1}{1-4n^2}\right)-\frac{21}{16}\left(\frac{1}{9-4n^2}\right)+\frac{7}{16}\left(\frac{1}{25-4n^2}\right)-\frac{1}{16}\left(\frac{1}{49-4n^2}\right)\right)$$

$$\Longrightarrow f(x)=\frac{32}{35\pi}+\frac{1}{\pi}\sum_{n=1}^{\infty}\left[\frac{35}{16}\left(\frac{1}{1-4n^2}\right)-\frac{21}{16}\left(\frac{1}{9-4n^2}\right)+\frac{7}{16}\left(\frac{1}{25-4n^2}\right)-\frac{1}{16}\left(\frac{1}{49-4n^2}\right)\right]\cos 2nx$$

تابع DC ندارد

۸) $f(x) = \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$

$$= \sum_{n=1}^{\infty} \frac{1}{\pi} \cos nx \int_{-\pi}^{\pi} f(x') \cos nx' \, dx' + \frac{1}{\pi} \sin nx \int_{-\pi}^{\pi} f(x') \sin nx' \, dx'$$

$$= \frac{1}{\pi} \sum_{n=1}^{\infty} \int_{-\pi}^{\pi} f(x') \left[\cos nx \cos nx' + \sin nx \sin nx' \right] dx'$$

$$= \frac{1}{\pi} \sum_{n=1}^{\infty} \int_{-\pi}^{\pi} f(x') \underbrace{\cos(nx - nx')}_{= \cos(nx' - nx)} \, dx'$$