810101492 علیرضا کریمی

6)

$$v_y = u_x = 3ax^2 - 58x - 3ay^2 + 30$$

$$\xrightarrow[\text{گرفتیم}]{\text{انتگرال}} v(x,y) = 3ax^2y - 58xy - ay^3 + 30y + g(x)$$

$$v_x = -u_y \Longrightarrow 6axy - 58y + g'(x) = 6axy - 58y \Rightarrow g'(x) = 0$$

$$\Longrightarrow g(x) = c \longrightarrow v(x,y) = 3ax^2y - 58xy - a^3y + 30y + c$$

$$f(x,y) = ax^3 - 29x^2 + 30x - 3axy^2 + 29y^2 - 10 + i(3ax^2y - 58xy + 30y - a^3y + c)$$

حال باید c را پیدا کنیم.

$$\xRightarrow{\text{جایگذاری}} f(0) = -10 = f(0,0) = -10 + i(c) \longrightarrow c = 0$$

$$\xRightarrow[y=0]{x=z} f(z) = az^3 - 29z^2 + 30z - 10 \Longrightarrow f''(z) = 6az - 58$$

$$\Longrightarrow f''(i) = 6ai - 58$$

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$$u(x,y) = \frac{x + xy^2 + x^3}{x^2+y^2} = \frac{x}{x^2+y^2} + x$$

$$v(x,y) = \frac{x^2 y + y^3 - y}{x^2+y^2} = y - \frac{y}{x^2+y^2}$$

حال برقراری معادلات کوشی ریمان را بررسی می‌کنیم

$$u_x = v_y \Longrightarrow 1 + \frac{x^2+y^2 - x(2x)}{(x^2+y^2)^2} = 1 - \frac{x^2+y^2 - y(2y)}{(x^2+y^2)^2}$$

$$\Longrightarrow \frac{-x^2+y^2}{(x^2+y^2)^2} = \frac{-(x^2-y^2)}{(x^2+y^2)^2} \quad \checkmark$$

$$u_y = -v_x \Longrightarrow \frac{2xy}{(x^2+y^2)^2} = -\frac{-2xy}{(x^2+y^2)^2} \quad \checkmark$$

$f(z)$ تنها در نقطه $(x,y)=(0,0)$ یعنی (مبدأ) است چون تعریف نشده است، و در بقیه نقاط تحلیلی می‌باشد.

$$\xrightarrow[y=0]{x=z} : f(z) = \frac{z^3+z}{z^2} = z + \frac{1}{z} \longrightarrow f'''(z) = \frac{-6}{z^4}$$

$$\Longrightarrow f^{(3)}(i) = \frac{-6}{i^4} = -6$$

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الف)

$$u_{xx} + u_{yy} = 0 \Longrightarrow 6y + 4 - 6y - 4 = 0 \longrightarrow \text{همساز}$$

$$v_y = u_x = 6xy + 4x \xrightarrow[\text{می‌گیریم}]{\text{انتگرال}} v(x,y) = 3xy^2 + 4xy + g(x)$$

$$u_y = -v_x \Longrightarrow 3x^2 - 3y^2 - 4y = -3y^2 - 4y - g'(x)$$

$$\Longrightarrow g'(x) = -3x^2 \Longrightarrow g(x) = -x^3 + c$$

$$\longrightarrow v(x,y) = 3xy^2 + 4xy - x^3 + c$$

ادامه ـ 8)

ب)

$$u_{xx}+u_{yy}=0 \longrightarrow \frac{\partial}{\partial x}\left(\frac{2x}{x^2+y^2}\right)+\frac{\partial}{\partial y}\left(\frac{2y}{x^2+y^2}\right)=$$

$$\frac{2(x^2+y^2)-2x(2x)+2(x^2+y^2)-2y(2y)}{(x^2+y^2)^2}=0 \Longrightarrow$$ همساز

$$v_y=u_x=\frac{2x}{x^2+y^2} \xRightarrow{\text{انتگرال}} v(x,y)=2\,\mathrm{Arctg}\left(\frac{y}{x}\right)+g(x)$$

$$u_y=-v_x \Longrightarrow \frac{2y}{x^2+y^2}=\frac{2y}{x^2+y^2}-g'(x) \Longrightarrow g'(x)=0$$

$$\Longrightarrow g(x)=c \Longrightarrow v(x,y)=2\,\mathrm{Arctg}\left(\frac{y}{x}\right)+c$$

9)

$$v_\theta=r u_r=r\cos\theta\ln r+r\cos\theta-r\theta\sin\theta$$

$$\xRightarrow[\text{گیری}]{\text{انتگرال}} v(r,\theta)=\int(r\cos\theta\ln r+r\cos\theta-r\theta\sin\theta)\,d\theta=$$

$$r\ln r\sin\theta+r\sin\theta-r\sin\theta+r\theta\cos\theta+g(r)$$

$$\Longrightarrow v(r,\theta)=r\ln r\sin\theta+r\theta\cos\theta+g(r)$$

$$u_\theta=-r v_r \Longrightarrow -r\ln r\sin\theta-r\sin\theta-r\theta\cos\theta=$$

$$-r(\sin\theta\ln r+\sin\theta+\theta\cos\theta+g'(r)) \Longrightarrow g'(r)=0 \Longrightarrow g(r)=C$$

$$\Longrightarrow v(r,\theta)=r\ln r\sin\theta+r\theta\cos\theta+C$$

$$f(r,\theta)=u(r,\theta)+i\,v(r,\theta)=r\cos\theta\ln r-r\theta\sin\theta+i(r\sin\theta\ln r+$$

$$r\theta\cos\theta+C) \xRightarrow[\theta=0]{r=z} f(z)=z\ln z+iC \Longrightarrow f''(i)=\frac{1}{i}=-i$$

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$$f\{u_t\} - f\{u_{xx}\} = f\{e^{-5|x|}\}$$

$$\Longrightarrow \hat{u}_t - (-w^2\hat{u}) = \frac{10}{25+w^2} \Longrightarrow \hat{u}(w,t) = Ce^{-w^2t} + \frac{10}{w^2(25+w^2)}$$

$$f(x) = u(x,0) \Longrightarrow f\{f(x)\} = f\{u(x,0)\} \Longrightarrow F(w) = \hat{u}(w,0)$$

$$\Longrightarrow \hat{u}(w,0) = C + \frac{10}{w^2(25+w^2)} = F(w) \Longrightarrow C(w) = F(w) - \frac{10}{w^2(25+w^2)}$$

$$\Longrightarrow \hat{u}(w,t) = \left(F(w) - \frac{10}{w^2(25+w^2)}\right)e^{-w^2t} + \frac{10}{w^2(25+w^2)}$$

تبدیل فوریه معکوس

$$\Longrightarrow u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{u}(w,t)e^{iwx}\,dw =$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[\left(F(w) - \frac{10}{w^2(25+w^2)}\right)e^{-w^2t} + \frac{10}{w^2(25+w^2)}\right]e^{iwx}\,dw$$