# Package 'sna'

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add.isolates

Add Isolates to a Graph

# **Description**

Adds n isolates to the graph (or graphs) in dat.

## Usage

```
add.isolates(dat, n, return.as.edgelist = FALSE)
```

# **Arguments**

dat one or more input graphs.

n the number of isolates to add.

return.as.edgelist

logical; should the input graph be returned as an edgelist (rather than an adja-

cency matrix)?

#### **Details**

If dat contains more than one graph, the n isolates are added to each member of dat.

## Value

The updated graph(s).

#### Note

Isolate addition is particularly useful when computing structural distances between graphs of different orders; see the above reference for details.

# Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Inter-Structural Analysis." CASOS Working Paper, Carnegie Mellon University.

#### See Also

isolates

## **Examples**

```
g<-rgraph(10,5) #Produce some random graphs
dim(g) #Get the dimensions of g
g<-add.isolates(g,2) #Add 2 isolates to each graph in g
dim(g) #Now examine g
g</pre>
```

bbnam

Butts' (Hierarchical) Bayesian Network Accuracy Model

# **Description**

Takes posterior draws from Butts' bayesian network accuracy/estimation model for multiple participant/observers (conditional on observed data and priors), using a Gibbs sampler.

## Usage

```
bbnam(dat, model="actor", ...)
bbnam.fixed(dat, nprior=0.5, em=0.25, ep=0.25, diag=FALSE,
    mode="digraph", draws=1500, outmode="draws", anames=NULL,
    onames=NULL)
bbnam.pooled(dat, nprior=0.5, emprior=c(1,11), epprior=c(1,11),
    diag=FALSE, mode="digraph", reps=5, draws=1500, burntime=500,
    quiet=TRUE, anames=NULL, onames=NULL, compute.sqrtrhat=TRUE)
bbnam.actor(dat, nprior=0.5, emprior=c(1,11), epprior=c(1,11),
    diag=FALSE, mode="digraph", reps=5, draws=1500, burntime=500,
    quiet=TRUE, anames=NULL, onames=NULL, compute.sqrtrhat=TRUE)
```

#### **Arguments**

dat

Input networks to be analyzed. This may be supplied in any reasonable form, but must be reducible to an array of dimension  $m \times n \times n$ , where n is |V(G)|, the first dimension indexes the observer (or information source), the second indexes the sender of the relation, and the third dimension indexes the recipient of the relation. (E.g., dat[i,j,k]==1 implies that i observed j sending the relation in question to k.) Note that only dichotomous data is supported at present, and missing values are permitted; the data collection pattern, however, is assumed to be ignorable, and hence the posterior draws are implicitly conditional on the observation pattern.

model

String containing the error model to use; options are "actor", "pooled", and "fixed".

. . .

Arguments to be passed by bbnam to the particular model method.

nprior Network prior matrix. This must be a matrix of dimension  $n \times n$ , containing

the arc/edge priors for the criterion network. (E.g., nprior[i,j] gives the prior probability of i sending the relation to j in the criterion graph.) Non-matrix values will be coerced/expanded to matrix form as appropriate. If no network prior is provided, an uninformative prior on the space of networks will be assumed

(i.e.,  $Pr(i \rightarrow j) = 0.5$ ). Missing values are not allowed.

em Probability of a false negative; this may be in the form of a single number, one

number per observation slice, one number per (directed) dyad, or one number

per dyadic observation (fixed model only).

ep Probability of a false positive; this may be in the form of a single number, one

number per observation slice, one number per (directed) dyad, or one number

per dyadic observation (fixed model only).

emprior Parameters for the (Beta) false negative prior; these should be in the form of an

 $(\alpha,\beta)$  pair for the pooled model, and of an  $n\times 2$  matrix of  $(\alpha,\beta)$  pairs for the actor model (or something which can be coerced to this form). If no emprior is given, a weakly informative prior (1,11) will be assumed; note that this may be

inappropriate, as described below. Missing values are not allowed.

epprior Parameters for the (Beta) false positive prior; these should be in the form of an

 $(\alpha,\beta)$  pair for the pooled model, and of an  $n\times 2$  matrix of  $(\alpha,\beta)$  pairs for the actor model (or something which can be coerced to this form). If no epprior is given, a weakly informative prior (1,11) will be assumed; note that this may be

inappropriate, as described below. Missing values are not allowed.

diag Boolean indicating whether loops (matrix diagonals) should be counted as data.

mode A string indicating whether the data in question forms a "graph" or a "digraph"

Number of replicate chains for the Gibbs sampler (pooled and actor models

only).

draws Integer indicating the total number of draws to take from the posterior distribu-

tion. Draws are taken evenly from each replication (thus, the number of draws

from a given chain is draws/reps).

burntime Integer indicating the burn-in time for the Markov Chain. Each replication is

iterated burntime times before taking draws (with these initial iterations being discarded); hence, one should realize that each increment to burntime increases execution time by a quantity proportional to reps. (pooled and actor models

only)

quiet Boolean indicating whether MCMC diagnostics should be displayed (pooled

and actor models only).

outmode posterior indicates that the exact posterior probability matrix for the criterion

graph should be returned; otherwise draws from the joint posterior are returned

instead (fixed model only).

anames A vector of names for the actors (vertices) in the graph.

onames A vector of names for the observers (possibly the actors themselves) whose

reports are contained in the input data.

compute.sqrtrhat

reps

A boolean indicating whether or not Gelman et al.'s potential scale reduction measure (an MCMC convergence diagnostic) should be computed (pooled and

actor models only).

#### **Details**

The bbnam models a set of network data as reflecting a series of (noisy) observations by a set of participants/observers regarding an uncertain criterion structure. Each observer is assumed to send false positives (i.e., reporting a tie when none exists in the criterion structure) with probability  $e^+$ , and false negatives (i.e., reporting that no tie exists when one does in fact exist in the criterion structure) with probability  $e^-$ . The criterion network itself is taken to be a Bernoulli (di)graph. Note that the present model includes three variants:

- Fixed error probabilities: Each edge is associated with a known pair of false negative/false
  positive error probabilities (provided by the researcher). In this case, the posterior for the
  criterion graph takes the form of a matrix of Bernoulli parameters, with each edge being
  independent conditional on the parameter matrix.
- 2. Pooled error probabilities: One pair of (uncertain) false negative/false positive error probabilities is assumed to hold for all observations. Here, we assume that the researcher's prior information regarding these parameters can be expressed as a pair of Beta distributions, with the additional assumption of independence in the prior distribution. Note that error rates and edge probabilities are *not* independent in the joint posterior, but the posterior marginals take the form of Beta mixtures and Bernoulli parameters, respectively.
- 3. Per observer ("actor") error probabilities: One pair of (uncertain) false negative/false positive error probabilities is assumed to hold for each observation slice. Again, we assume that prior knowledge can be expressed in terms of independent Beta distributions (along with the Bernoulli prior for the criterion graph) and the resulting posterior marginals are Beta mixtures and a Bernoulli graph. (Again, it should be noted that independence in the priors does *not* imply independence in the joint posterior!)

By default, the bbnam routine returns (approximately) independent draws from the joint posterior distribution, each draw yielding one realization of the criterion network and one collection of accuracy parameters (i.e., probabilities of false positives/negatives). This is accomplished via a Gibbs sampler in the case of the pooled/actor model, and by direct sampling for the fixed probability model. In the special case of the fixed probability model, it is also possible to obtain directly the posterior for the criterion graph (expressed as a matrix of Bernoulli parameters); this can be controlled by the outmode parameter.

As noted, the taking of posterior draws in the nontrivial case is accomplished via a Markov Chain Monte Carlo method, in particular the Gibbs sampler; the high dimensionality of the problem  $(O(n^2+2n))$  tends to preclude more direct approaches. At present, chain burn-in is determined ex ante on a more or less arbitrary basis by specification of the burntime parameter. Eventually, a more systematic approach will be utilized. Note that insufficient burn-in will result in inaccurate posterior sampling, so it's not wise to skimp on burn time where otherwise possible. Similarly, it is wise to employ more than one Markov Chain (set by reps), since it is possible for trajectories to become "trapped" in metastable regions of the state space. Number of draws per chain being equal, more replications are usually better than few; consult Gelman et al. for details. A useful measure of chain convergence, Gelman and Rubin's potential scale reduction  $(\sqrt{\hat{R}})$ , can be computed using the compute. sqrtrhat parameter. The potential scale reduction measure is an ANOVA-like comparison of within-chain versus between-chain variance; it approaches 1 (from above) as the chain converges, and longer burn-in times are strongly recommended for chains with scale reductions in excess of 1.2 or thereabouts.

Finally, a cautionary concerning prior distributions: it is important that the specified priors actually reflect the prior knowledge of the researcher; otherwise, the posterior will be inadequately informed. In particular, note that an uninformative prior on the accuracy probabilities implies that it is a priori equally probable that any given actor's observations will be informative or negatively informative (i.e., that i observing j sending a tie to k reduces  $Pr(j \to k)$ ). This is a highly unrealistic assumption, and it will tend to produce posteriors which are bimodal (one mode being related to the "informative" solution, the other to the "negatively informative" solution). Currently, the default error parameter prior is Beta(1,11), which is both diffuse and which renders negatively informative observers extremely improbable (i.e., on the order of 1e-6). Another plausible but still fairly diffuse prior would be Beta(3,5), which reduces the prior probability of an actor's being negatively informative to 0.16, and the prior probability of any given actor's being more than 50% likely to make a particular error (on average) to around 0.22. (This prior also puts substantial mass near the 0.5 point, which would seem consonant with the BKS studies.) For network priors, a reasonable starting point can often be derived by considering the expected mean degree of the criterion graph: if d represents the user's prior expectation for the mean degree, then d/(N-1) is a natural starting point for the cell values of nprior. Butts (2003) discusses a number of issues related to choice of priors for the bbnam model, and users should consult this reference if matters are unclear before defaulting to the uninformative solution.

#### Value

An object of class bbnam, containing the posterior draws. The components of the output are as follows:

anames A vector of actor names.

draws An integer containing the number of draws.

em A matrix containing the posterior draws for probability of producing false neg-

atives, by actor.

ep A matrix containing the posterior draws for probability of producing false posi-

tives, by actor.

nactors An integer containing the number of actors.

net An array containing the posterior draws for the criterion network.

reps An integer indicating the number of replicate chains used by the Gibbs sampler.

## Note

As indicated, the posterior draws are conditional on the observed data, and hence on the data collection mechanism if the collection design is non-ignorable. Complete data (e.g., a CSS) and random tie samples are examples of ignorable designs; see Gelman et al. for more information concerning ignorability.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

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## References

Butts, C. T. (2003). "Network Inference, Error, and Informant (In)Accuracy: A Bayesian Approach." *Social Networks*, 25(2), 103-140.

Gelman, A.; Carlin, J.B.; Stern, H.S.; and Rubin, D.B. (1995). *Bayesian Data Analysis*. London: Chapman and Hall.

Gelman, A., and Rubin, D.B. (1992). "Inference from Iterative Simulation Using Multiple Sequences." *Statistical Science*, 7, 457-511.

Krackhardt, D. (1987). "Cognitive Social Structures." Social Networks, 9, 109-134.

#### See Also

```
npostpred, event2dichot, bbnam.bf
```

# **Examples**

```
#Create some random data
g<-rgraph(5)
g.p < -0.8 * g + 0.2 * (1 - g)
dat<-rgraph(5,5,tprob=g.p)</pre>
#Define a network prior
pnet<-matrix(ncol=5,nrow=5)</pre>
pnet[,]<-0.5
#Define em and ep priors
pem<-matrix(nrow=5,ncol=2)</pre>
pem[,1]<-3
pem[,2]<-5
pep<-matrix(nrow=5,ncol=2)</pre>
pep[,1]<-3
pep[,2]<-5
#Draw from the posterior
b<-bbnam(dat,model="actor",nprior=pnet,emprior=pem,epprior=pep,
    burntime=100,draws=100)
#Print a summary of the posterior draws
summary(b)
```

bbnam.bf

Estimate Bayes Factors for the bbnam

# Description

This function uses monte carlo integration to estimate the BFs, and tests the fixed probability, pooled, and pooled by actor models. (See bbnam for details.)

bbnam.bf

#### Usage

```
bbnam.bf(dat, nprior=0.5, em.fp=0.5, ep.fp=0.5, emprior.pooled=c(1, 11),
    epprior.pooled=c(1, 11), emprior.actor=c(1, 11), epprior.actor=c(1, 11),
    diag=FALSE, mode="digraph", reps=1000)
```

#### **Arguments**

dat Input networks to be analyzed. This may be supplied in any reasonable form,

but must be reducible to an array of dimension  $m \times n \times n$ , where n is |V(G)|, the first dimension indexes the observer (or information source), the second indexes the sender of the relation, and the third dimension indexes the recipient of the relation. (E.g., dat[i,j,k]==1 implies that i observed j sending the relation in question to k.) Note that only dichotomous data is supported at present, and missing values are permitted; the data collection pattern, however, is assumed to be ignorable, and hence the posterior draws are implicitly conditional on the

observation pattern.

nprior Network prior matrix. This must be a matrix of dimension  $n \times n$ , containing

the arc/edge priors for the criterion network. (E.g., nprior[i,j] gives the prior probability of i sending the relation to j in the criterion graph.) Non-matrix values will be coerced/expanded to matrix form as appropriate. If no network prior is provided, an uninformative prior on the space of networks will be assumed

(i.e.,  $Pr(i \rightarrow j) = 0.5$ ). Missing values are not allowed.

em. fp Probability of false negatives for the fixed probability model

ep.fp Probability of false positives for the fixed probability model

emprior.pooled  $(\alpha, \beta)$  pairs for the (beta) false negative prior under the pooled model

epprior.pooled  $(\alpha, \beta)$  pairs for the (beta) false positive prior under the pooled model

emprior.actor Matrix of per observer  $(\alpha, \beta)$  pairs for the (beta) false negative prior under the

per observer/actor model, or something that can be coerced to this form

epprior.actor Matrix of per observer ( $(\alpha, \beta)$  pairs for the (beta) false positive prior under the

per observer/actor model, or something that can be coerced to this form

diag Boolean indicating whether or not the diagonal should be treated as valid data.

Set this true if and only if the criterion graph can contain loops. Diag is false by

default.

mode String indicating the type of graph being evaluated. "digraph" indicates that

edges should be interpreted as directed; "graph" indicates that edges are undi-

rected. Mode is set to "digraph" by default.

reps Number of Monte Carlo draws to take

# **Details**

The bbnam model (detailed in the bbnam function help) is a fairly simple model for integrating informant reports regarding social network data. bbnam.bf computes log Bayes Factors (integrated likelihood ratios) for the three error submodels of the bbnam: fixed error probabilities, pooled error probabilities, and per observer/actor error probabilities.

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By default, bbnam. bf uses weakly informative Beta(1,11) priors for false positive and false negative rates, which may not be appropriate for all cases. (Likewise, the initial network prior is uniformative.) Users are advised to consider adjusting the error rate priors when using this function in a practical context; for instance, it is often reasonable to expect higher false negative rates (on average) than false positive rates, and to expect the criterion graph density to be substantially less than 0.5. See the reference below for a discussion of this issue.

## Value

An object of class bayes. factor.

#### Note

It is important to be aware that the model parameter priors are essential components of the models to be compared; inappropriate parameter priors will result in misleading Bayes Factors.

# Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C. T. (2003). "Network Inference, Error, and Informant (In)Accuracy: A Bayesian Approach." *Social Networks*, 25(2), 103-140.

Robert, C. (1994). The Bayesian Choice: A Decision-Theoretic Motivation. Springer.

#### See Also

bbnam

# **Examples**

betweenness 13

betweenness	Compute the Betweenness Centrality Scores of Network Positions
betweenness	Compute the Betweenness Centrality Scores of Network Positions

# Description

betweenness takes one or more graphs (dat) and returns the betweenness centralities of positions (selected by nodes) within the graphs indicated by g. Depending on the specified mode, betweenness on directed or undirected geodesics will be returned; this function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

# Usage

# **Arguments**

dat	one or more input graphs.
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g=1.
nodes	vector indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. gmode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
cmode	string indicating the type of betweenness centrality being computed (directed or undirected geodesics, or a variant form – see below).
geodist.precom	р
	A geodist object precomputed for the graph to be analyzed (optional)
rescale	if true, centrality scores are rescaled such that they sum to 1.
ignore.eval	logical; ignore edge values when computing shortest paths?

# **Details**

The shortest-path betweenness of a vertex, v, is given by

$$C_B(v) = \sum_{i,j: i \neq j, i \neq v, j \neq v} \frac{g_{ivj}}{g_{ij}}$$

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where  $g_{ijk}$  is the number of geodesics from i to k through j. Conceptually, high-betweenness vertices lie on a large number of non-redundant shortest paths between other vertices; they can thus be thought of as "bridges" or "boundary spanners."

Several variant forms of shortest-path betweenness exist, and can be selected using the cmode argument. Supported options are as follows:

directed Standard betweenness (see above), calculated on directed pairs. (This is the default option.)

undirected Standard betweenness (as above), calculated on undirected pairs (undirected graphs only).

endpoints Standard betweenness, with direct connections counted towards ego's score. This expresses the intuition that individuals' control over their own direct contacts should be considered in their total score (e.g., when betweenness is interpreted as a measure of information control).

proximalsrc Borgatti's proximal source betweenness, given by

$$C_B(v) = \sum_{i,j: i \neq v, i \neq j, j \to v} \frac{g_{ivj}}{g_{ij}}.$$

This variant allows betweenness to accumulate only for the last intermediating vertex in each incoming geodesic; this expresses the notion that, by serving as the "proximal source" for the target, this particular intermediary will in some settings have greater influence or control than other intervening parties.

proximaltar Borgatti's proximal target betweenness, given by

$$C_B(v) = \sum_{i,j: i \neq v, i \to v, i \neq j} \frac{g_{ivj}}{g_{ij}}.$$

This counterpart to proximal source betweenness (above) allows betweenness to accumulate only for the first intermediating vertex in each outgoing geodesic; this expresses the notion that, by serving as the "proximal target" for the source, this particular intermediary will in some settings have greater influence or control than other intervening parties.

proximalsum The sum of Borgatti's proximal source and proximal target betweenness scores (above); this may be used when either role is regarded as relevant to the betweenness calculation.

lengthscaled Borgetti and Everett's length-scaled betweenness, given by

$$C_B(v) = \sum_{i,j:i \neq j, i \neq v, j \neq v} \frac{1}{d_{ij}} \frac{g_{ivj}}{g_{ij}},$$

where  $d_{ij}$  is the geodesic distance from i to j. This measure adjusts the standard betweenness score by downweighting long paths (e.g., as appropriate in circumstances for which such paths are less-often used).

linearscaled Geisberger et al.'s linearly-scaled betweenness:

$$C_B(v) = \sum_{i,j: i \neq j, i \neq v, j \neq v} \frac{1}{d_{ij}} \frac{g_{ivj}}{g_{ij}}.$$

This variant modifies the standard betweenness score by giving more weight to intermediaries which are closer to their targets (much like proximal source betweenness, above). This may be of use when those near the end of a path have greater direct control over the flow of influence or resources than those near its source.

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See Brandes (2008) for details and additional references. Geodesics for all of the above can be calculated using valued edges by setting ignore.eval=TRUE. Edge values are interpreted as distances for this purpose; proximity data should be transformed accordingly before invoking this routine.

#### Value

A vector, matrix, or list containing the betweenness scores (depending on the number and size of the input graphs).

## Warning

Rescale may cause unexpected results if all actors have zero betweenness.

#### Note

Judicious use of geodist.precomp can save a great deal of time when computing multiple path-based indices on the same network.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Borgatti, S.P. and Everett, M.G. (2006). "A Graph-Theoretic Perspective on Centrality." *Social Networks*, 28, 466-484.

Brandes, U. (2008). "On Variants of Shortest-Path Betweenness Centrality and their Generic Computation." *Social Networks*, 30, 136–145.

Freeman, L.C. (1979). "Centrality in Social Networks I: Conceptual Clarification." *Social Networks*, 1, 215-239.

Geisberger, R., Sanders, P., and Schultes, D. (2008). "Better Approximation of Betweenness Centrality." In *Proceedings of the 10th Workshop on Algorithm Engineering and Experimentation (ALENEX'08)*, 90-100. SIAM.

#### See Also

centralization, stresscent, geodist

#### **Examples**

16 bicomponent.dist

bicomponent.dist	Calculate the Bicomponents of a Graph
------------------	---------------------------------------

# Description

bicomponent.dist returns the bicomponents of an input graph, along with size distribution and membership information.

# Usage

```
bicomponent.dist(dat, symmetrize = c("strong", "weak"))
```

## **Arguments**

dat a graph or graph stack.

symmetrize symmetrization rule to apply when pre-processing the input (see symmetrize).

## **Details**

The bicomponents of undirected graph G are its maximal 2-connected vertex sets. bicomponent.dist calculates the bicomponents of G, after first coercing to undirected form using the symmetrization rule in symmetrize. In addition to bicomponent memberships, various summary statistics regarding the bicomponent distribution are returned; see below.

#### Value

# A list containing

members A list, with one entry per bicomponent, containing component members.

memberships A vector of component memberships, by vertex. (Note: memberships may not

be unique.) Vertices not belonging to any bicomponent have membership values

of NA.

csize A vector of component sizes, by bicomponent.

cdist A vector of length |V(G)| with the (unnormalized) empirical distribution func-

tion of bicomponent sizes.

#### Note

Remember that bicomponents can intersect; when this occurs, the relevant vertices' entries in the membership vector are assigned to one of the overlapping bicomponents on an arbitrary basis. The members element of the return list is the safe way to recover membership information.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

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## References

Brandes, U. and Erlebach, T. (2005). *Network Analysis: Methodological Foundations*. Berlin: Springer.

## See Also

```
component.dist, cutpoints, symmetrize
```

# **Examples**

```
#Draw a moderately sparse graph
g<-rgraph(25,tp=2/24,mode="graph")
#Compute the bicomponents
bicomponent.dist(g)</pre>
```

blockmodel

Generate Blockmodels Based on Partitions of Network Positions

# Description

Given a set of equivalence classes (in the form of an equiv.clust object, hclust object, or membership vector) and one or more graphs, blockmodel will form a blockmodel of the input graph(s) based on the classes in question, using the specified block content type.

## Usage

```
blockmodel(dat, ec, k=NULL, h=NULL, block.content="density",
    plabels=NULL, glabels=NULL, rlabels=NULL, mode="digraph",
    diag=FALSE)
```

# Arguments

dat	one or more input graphs.
ec	equivalence classes, in the form of an object of class equiv.clust or hclust, or a membership vector.
k	the number of classes to form (using cutree).
h	the height at which to split classes (using cutree).
block.content	string indicating block content type (see below).
plabels	a vector of labels to be applied to the individual nodes.
glabels	a vector of labels to be applied to the graphs being modeled.
rlabels	a vector of labels to be applied to the (reduced) roles.
mode	a string indicating whether we are dealing with graphs or digraphs.
diag	a boolean indicating whether loops are permitted.

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#### **Details**

Unless a vector of classes is specified, blockmodel forms its eponymous models by using cutree to cut an equivalence clustering in the fashion specified by k and h. After forming clusters (roles), the input graphs are reordered and blockmodel reduction is applied. Currently supported reductions are:

1. density: block density, computed as the mean value of the block

2. meanrowsum: mean row sums for the block

3. meancolsum: mean column sums for the block

4. sum: total block sum

5. median: median block value

6. min: minimum block value

7. max: maximum block value

8. types: semi-intelligent coding of blocks by "type." Currently recognized types are (in order of precedence) "NA" (i.e., blocks with no valid data), "null" (i.e., all values equal to zero), "complete" (i.e., all values equal to 1), "1 covered" (i.e., all rows/cols contain a 1), "1 row-covered" (i.e., all rows contain a 1), "1 col-covered" (i.e., all cols contain a 1), and "other" (i.e., none of the above).

Density or median-based reductions are probably the most interpretable for most conventional analyses, though type-based reduction can be useful in examining certain equivalence class hypotheses (e.g., 1 covered and null blocks can be used to infer regular equivalence classes). Once a given reduction is performed, the model can be analyzed and/or expansion can be used to generate new graphs based on the inferred role structure.

#### Value

An object of class blockmodel.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Doreian, P.; Batagelj, V.; and Ferligoj, A. (2005). *Generalized Blockmodeling*. Cambridge University Press.

White, H.C.; Boorman, S.A.; and Breiger, R.L. (1976). "Social Structure from Multiple Networks I: Blockmodels of Roles and Positions." *American Journal of Sociology*, 81, 730-779.

## See Also

equiv.clust, blockmodel.expand

blockmodel.expand 19

# **Examples**

blockmodel.expand

Generate a Graph (or Stack) from a Given Blockmodel Using Particular Expansion Rules

# **Description**

blockmodel.expand takes a blockmodel and an expansion vector, and expands the former by making copies of the vertices.

# Usage

```
blockmodel.expand(b, ev, mode="digraph", diag=FALSE)
```

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## **Arguments**

D	biockinodel object.
ev	a vector indicating the number of copies to make of each class (respectively).
mode	a string indicating whether the result should be a "graph" or "digraph".
diag	a boolean indicating whether or not loops should be permitted.

# **Details**

The primary use of blockmodel expansion is in generating test data from a blockmodeling hypothesis. Expansion is performed depending on the content type of the blockmodel; at present, only density is supported. For the density content type, expansion is performed by interpreting the interclass density as an edge probability, and by drawing random graphs from the Bernoulli parameter matrix formed by expanding the density model. Thus, repeated calls to blockmodel.expand can be used to generate a sample for monte carlo null hypothesis tests under a Bernoulli graph model.

## Value

An adjacency matrix, or stack thereof.

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#### Note

Eventually, other content types will be supported.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Doreian, P.; Batagelj, V.; and Ferligoj, A. (2005). *Generalized Blockmodeling*. Cambridge: Cambridge University Press.

White, H.C.; Boorman, S.A.; and Breiger, R.L. (1976). "Social Structure from Multiple Networks I: Blockmodels of Roles and Positions." *American Journal of Sociology*, 81, 730-779.

#### See Also

blockmodel

## **Examples**

bn

Fit a Biased Net Model

## **Description**

Fits a biased net model to an input graph, using moment-based or maximum pseudolikelihood techniques.

# Usage

```
bn(dat, method = c("mple.triad", "mple.dyad", "mple.edge",
    "mtle"), param.seed = NULL, param.fixed = NULL,
    optim.method = "BFGS", optim.control = list(),
    epsilon = 1e-05)
```

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## **Arguments**

dat a single input graph.

method the fit method to use (see below).

param. seed seed values for the parameter estimates.

param.fixed parameter values to fix, if any.
optim.method method to be used by optim.
optim.control control parameter for optim.

epsilon tolerance for convergence to extreme parameter values (i.e., 0 or 1).

#### **Details**

The biased net model stems from early work by Rapoport, who attempted to model networks via a hypothetical "tracing" process. This process may be described loosely as follows. One begins with a small "seed" set of vertices, each member of which is assumed to nominate (generate ties to) other members of the population with some fixed probability. These members, in turn, may nominate new members of the population, as well as members who have already been reached. Such nominations may be "biased" in one fashion or another, leading to a non-uniform growth process. Specifically, let  $e_{ij}$  be the random event that vertex i nominates vertex j when reached. Then the conditional probability of  $e_{ij}$  is given by

$$\Pr(e_{ij}|T) = 1 - (1 - \Pr(B_e)) \prod_k (1 - \Pr(B_k|T))$$

where T is the current state of the trace,  $B_e$  is the a Bernoulli event corresponding to the baseline probability of  $e_{ij}$ , and the  $B_k$  are "bias events." Bias events are taken to be independent Bernoulli trials, given T, such that  $e_{ij}$  is observed with certainty if any bias event occurs. The specification of a biased net model, then, involves defining the various bias events (which, in turn, influence the structure of the network).

Although other events have been proposed, the primary bias events employed in current biased net models are the "parent bias" (a tendency to return nominations); the "sibling bias" (a tendency to nominate alters who were nominated by the same third party); and the "double role bias" (a tendency to nominate alters who are both siblings and parents). These bias events, together with the baseline edge events, are used to form the standard biased net model. It is standard to assume homogeneity within bias class, leading to the four parameters  $\pi$  (probability of a parent bias event),  $\sigma$  (probability of a sibling bias event),  $\rho$  (probability of a double role bias event), and d (probability of a baseline event).

Unfortunately, there is no simple expression for the likelihood of a graph given these parameters (and hence, no basis for likelihood based inference). However, Skvoretz et al. have derived a class of maximum pseudo-likelihood estimators for the the biased net model, based on local approximations to the likelihood at the edge, dyad, or triad level. These estimators may be employed within bn by selecting the appropriate MPLE for the *method* argument. Alternately, it is also possible to derive expected triad census rates for the biased net model, allowing an estimator which maximizes the likelihood of the observed triad census (essentially, a method of moments procedure). This last may be selected via the argument mode="mtle". In addition to estimating model parameters, bn generates predicted edge, dyad, and triad census statistics, as well as structure statistics (using the Fararo-Sunshine recurrence). These can be used to evaluate goodness-of-fit.

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print, summary, and plot methods are available for bn objects. See rgbn for simulation from biased net models.

#### Value

An object of class bn.

#### Note

Asymptotic properties of the MPLE are not known for this model. Caution is strongly advised.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Fararo, T.J. and Sunshine, M.H. (1964). "A study of a biased friendship net." Syracuse, NY: Youth Development Center.

Rapoport, A. (1957). "A contribution to the theory of random and biased nets." *Bulletin of Mathematical Biophysics*, 15, 523-533.

Skvoretz, J.; Fararo, T.J.; and Agneessens, F. (2004). "Advances in biased net theory: definitions, derivations, and estimations." *Social Networks*, 26, 113-139.

# See Also

```
rgbn, structure.statistics
```

# **Examples**

```
#Generate a random graph
g<-rgraph(25)

#Fit a biased net model, using the triadic MPLE
gbn<-bn(g)

#Examine the results
summary(gbn)
plot(gbn)

#Now, fit a model containing only a density parameter
gbn<-bn(g,param.fixed=list(pi=0,sigma=0,rho=0))
summary(gbn)
plot(gbn)</pre>
```

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ı	bonpow	Find Bonacich Power Centrality Scores of Network Positions

## **Description**

bonpow takes one or more graphs (dat) and returns the Boncich power centralities of positions (selected by nodes) within the graphs indicated by g. The decay rate for power contributions is specified by exponent (1 by default). This function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

## Usage

```
bonpow(dat, g=1, nodes=NULL, gmode="digraph", diag=FALSE,
    tmaxdev=FALSE, exponent=1, rescale=FALSE, tol=1e-07)
```

## **Arguments**

dat	one or more input graphs.
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g=1.
nodes	vector indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. This is currently ignored.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. Diag is FALSE by default.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev=FALSE.
exponent	exponent (decay rate) for the Bonacich power centrality score; can be negative
rescale	if true, centrality scores are rescaled such that they sum to 1.
tol	tolerance for near-singularities during matrix inversion (see solve)

# Details

Bonacich's power centrality measure is defined by  $C_{BP}(\alpha,\beta) = \alpha (\mathbf{I} - \beta \mathbf{A})^{-1} \mathbf{A}\mathbf{1}$ , where  $\beta$  is an attenuation parameter (set here by exponent) and  $\mathbf{A}$  is the graph adjacency matrix. (The coefficient  $\alpha$  acts as a scaling parameter, and is set here (following Bonacich (1987)) such that the sum of squared scores is equal to the number of vertices. This allows 1 to be used as a reference value for the "middle" of the centrality range.) When  $\beta \to 1/\lambda_{\mathbf{A}1}$  (the reciprocal of the largest eigenvalue of  $\mathbf{A}$ ), this is to within a constant multiple of the familiar eigenvector centrality score; for other values of  $\beta$ , the behavior of the measure is quite different. In particular,  $\beta$  gives positive and negative weight to even and odd walks, respectively, as can be seen from the series expansion  $C_{BP}(\alpha,\beta) = \alpha \sum_{k=0}^{\infty} \beta^k \mathbf{A}^{k+1} \mathbf{1}$  which converges so long as  $|\beta| < 1/\lambda_{\mathbf{A}1}$ . The magnitude of  $\beta$ 

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controls the influence of distant actors on ego's centrality score, with larger magnitudes indicating slower rates of decay. (High rates, hence, imply a greater sensitivity to edge effects.)

Interpretively, the Bonacich power measure corresponds to the notion that the power of a vertex is recursively defined by the sum of the power of its alters. The nature of the recursion involved is then controlled by the power exponent: positive values imply that vertices become more powerful as their alters become more powerful (as occurs in cooperative relations), while negative values imply that vertices become more powerful only as their alters become *weaker* (as occurs in competitive or antagonistic relations). The magnitude of the exponent indicates the tendency of the effect to decay across long walks; higher magnitudes imply slower decay. One interesting feature of this measure is its relative instability to changes in exponent magnitude (particularly in the negative case). If your theory motivates use of this measure, you should be very careful to choose a decay parameter on a non-ad hoc basis.

#### Value

A vector, matrix, or list containing the centrality scores (depending on the number and size of the input graphs).

#### Warning

Singular adjacency matrices cause no end of headaches for this algorithm; thus, the routine may fail in certain cases. This will be fixed when I get a better algorithm. bonpow will not symmetrize your data before extracting eigenvectors; don't send this routine asymmetric matrices unless you really mean to do so.

#### Note

The theoretical maximum deviation used here is not obtained with the star network, in general. For positive exponents, at least, the symmetric maximum occurs for an empty graph with one complete dyad (the asymmetric maximum is generated by the outstar). UCINET V seems not to adjust for this fact, which can cause some oddities in their centralization scores (thus, don't expect to get the same numbers with both packages).

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Bonacich, P. (1972). "Factoring and Weighting Approaches to Status Scores and Clique Identification." *Journal of Mathematical Sociology*, 2, 113-120.

Bonacich, P. (1987). "Power and Centrality: A Family of Measures." *American Journal of Sociology*, 92, 1170-1182.

#### See Also

centralization, evcent

brokerage 25

## **Examples**

```
#Generate some test data
dat<-rgraph(10,mode="graph")
#Compute Bonpow scores
bonpow(dat,exponent=1,tol=1e-20)
bonpow(dat,exponent=-1,tol=1e-20)</pre>
```

brokerage

Perform a Gould-Fernandez Brokerage Analysis

## **Description**

Performs the brokerage analysis of Gould and Fernandez on one or more input graphs, given a class membership vector.

#### Usage

```
brokerage(g, cl)
```

# Arguments

g one or more input graphs.

cl a vector of class memberships.

#### **Details**

Gould and Fernandez (following Marsden and others) describe brokerage as the role played by a social actor who mediates contact between two alters. More formally, vertex v is a broker for distinct vertices a and b iff  $a \to v \to b$  and  $a \not\to b$ . Where actors belong to a priori distinct groups, group membership may be used to segment brokerage roles into particular types. Let  $A \to B \to C$  denote the two-path associated with a brokerage structure, such that some vertex from group B brokers the connection from some vertex from group A to a vertex in group C. The types of brokerage roles defined by Gould and Fernandez (and their accompanying two-path structures) are then defined in terms of group membership as follows:

- $w_I$ : Coordinator role; the broker mediates contact between two individuals from his or her own group. Two-path structure:  $A \to A \to A$
- $w_O$ : Itinerant broker role; the broker mediates contact between two individuals from a single group to which he or she does not belong. Two-path structure:  $A \to B \to A$
- $b_{IO}$ : Representative role; the broker mediates an incoming contact from an out-group member to an in-group member. Two-path structure:  $A \to B \to B$
- $b_{OI}$ : Gatekeeper role; the broker mediates an outgoing contact from an in-group member to an out-group member. Two-path structure:  $A \to A \to B$
- $b_O$ : Liaison role; the broker mediates contact between two individuals from different groups, neither of which is the group to which he or she belongs. Two-path structure:  $A \to B \to C$
- t: Total (cumulative) brokerage role occupancy. (Any of the above two-paths.)

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The *brokerage score* for a given vertex with respect to a given role is the number of ordered pairs having the appropriate group membership(s) brokered by said vertex. brokerage computes the brokerage scores for each vertex, given an input graph and vector of class memberships. Aggregate scores are also computed at the graph level, which correspond to the total frequency of each role type within the network structure. Expectations and variances of the brokerage scores conditional on size and density are computed, along with approximate *z*-tests for incidence of brokerage. (Note that the accuracy of the normality assumption is not known in the general case; see Gould and Fernandez (1989) for details. Simulation-based tests may be desirable as an alternative.)

## Value

An object of class brokerage, containing the following elements:

raw.nli	The matrix of observed brokerage scores, by vertex
exp.nli	The matrix of expected brokerage scores, by vertex
sd.nli	The matrix of predicted brokerage score standard deviations, by vertex
z.nli	The matrix of standardized brokerage scores, by vertex
raw.gli	The vector of observed aggregate brokerage scores
exp.gli	The vector of expected aggregate brokerage scores
sd.gli	The vector of predicted aggregate brokerage score standard deviations
z.gli	The vector of standardized aggregate brokerage scores
exp.grp	The matrix of expected brokerage scores, by group
sd.grp	The matrix of predicted brokerage score standard deviations, by group
cl	The vector of class memberships
clid	The original class names
n	The input class sizes
N	The order of the input network

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Gould, R.V. and Fernandez, R.M. 1989. "Structures of Mediation: A Formal Approach to Brokerage in Transaction Networks." *Sociological Methodology*, 19: 89-126.

## See Also

triad.census, gtrans

centralgraph 27

## **Examples**

```
#Draw a random network with 3 groups
g<-rgraph(15)
cl<-rep(1:3,5)

#Compute a brokerage object
b<-brokerage(g,cl)
summary(b)</pre>
```

centralgraph

Find the Central Graph of a Labeled Graph Stack

# **Description**

Returns the central graph of a set of labeled graphs, i.e. that graph in which i->j iff i->j in >=50% of the graphs within the set. If normalize==TRUE, then the value of the i,jth edge is given as the proportion of graphs in which i->j.

## Usage

```
centralgraph(dat, normalize=FALSE)
```

# **Arguments**

dat one or more input graphs.

normalize boolean indicating whether the results should be normalized. The result of this

is the "mean matrix". By default, normalize==FALSE.

#### **Details**

The central graph of a set of graphs S is that graph C which minimizes the sum of Hamming distances between C and G in S. As such, it turns out (for the dichotomous case, at least), to be analogous to both the mean and median for sets of graphs. The central graph is useful in a variety of contexts; see the references below for more details.

# Value

A matrix containing the central graph (or mean matrix)

## Note

0.5 is used as the cutoff value regardless of whether or not the data is dichotomous (as is tacitly assumed). The routine is unaffected by data type when normalize==TRUE.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

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## References

Banks, D.L., and Carley, K.M. (1994). "Metric Inference for Social Networks." *Journal of Classification*, 11(1), 121-49.

## See Also

hdist

## **Examples**

```
#Generate some random graphs
dat<-rgraph(10,5)
#Find the central graph
cg<-centralgraph(dat)
#Plot the central graph
gplot(cg)
#Now, look at the mean matrix
cg<-centralgraph(dat,normalize=TRUE)
print(cg)</pre>
```

centralization

Find the Centralization of a Given Network, for Some Measure of Centrality

# **Description**

Centralization returns the centralization GLI (graph-level index) for a given graph in dat, given a (node) centrality measure FUN. Centralization follows Freeman's (1979) generalized definition of network centralization, and can be used with any properly defined centrality measure. This measure must be implemented separately; see the references below for examples.

## Usage

```
centralization(dat, FUN, g=NULL, mode="digraph", diag=FALSE,
    normalize=TRUE, ...)
```

## **Arguments**

dat	one or more input graphs.
FUN	Function to return nodal centrality scores.
g	Integer indicating the index of the graph for which centralization should be computed. By default, all graphs are employed.
mode	String indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.
diag	Boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops, diag is FALSE by default.

centralization 29

normalize Boolean indicating whether or not the centralization score should be normalized

to the theoretical maximum. (Note that this function relies on FUN to return this value when called with tmaxdev==TRUE.) By default, tmaxdev==TRUE.

value when called with that dev—Troe.) by default, that dev—Troe

... Additional arguments to FUN.

#### **Details**

The centralization of a graph G for centrality measure C(v) is defined (as per Freeman (1979)) to be:

$$C^*(G) = \sum_{i \in V(G)} \left| \max_{v \in V(G)} (C(v)) - C(i) \right|$$

Or, equivalently, the absolute deviation from the maximum of C on G. Generally, this value is normalized by the theoretical maximum centralization score, conditional on |V(G)|. (Here, this functionality is activated by normalize.) Centralization depends on the function specified by FUN to return the vector of nodal centralities when called with dat and g, and to return the theoretical maximum value when called with the above and tmaxdev==TRUE. For an example of such a centrality routine, see degree.

#### Value

The centralization of the specified graph.

#### Note

See cugtest for null hypothesis tests involving centralization scores.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Freeman, L.C. (1979). "Centrality in Social Networks I: Conceptual Clarification." *Social Networks*, 1, 215-239.

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

# See Also

cugtest

## **Examples**

```
#Generate some random graphs
dat<-rgraph(5,10)
#How centralized is the third one on indegree?
centralization(dat,g=3,degree,cmode="indegree")
#How about on total (Freeman) degree?
centralization(dat,g=3,degree)
```

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clique.census

Compute Cycle Census Information

## **Description**

clique.census computes clique census statistics on one or more input graphs. In addition to aggregate counts of maximal cliques, results may be disaggregated by vertex and co-membership information may be computed.

#### Usage

```
clique.census(dat, mode = "digraph", tabulate.by.vertex = TRUE,
    clique.comembership = c("none", "sum", "bysize"), enumerate = TRUE,
    na.omit = TRUE)
```

#### **Arguments**

dat one or more input graphs.

mode "digraph" for directed graphs, or "graph" for undirected graphs.

tabulate.by.vertex

logical; should maximal clique counts be tabulated by vertex?

clique.comembership

the type of clique co-membership information to be tabulated, if any. "sum" returns a vertex by vertex matrix of clique co-membership counts; these are disaggregated by clique size if "bysize" is used. If "none" is given, no co-

membership information is computed.

enumerate logical; should an enumeration of all maximal cliques be returned?

na.omit logical; should missing edges be omitted?

#### **Details**

A (maximal) *clique* is a maximal set of mutually adjacenct vertices. Cliques are important for their role as cohesive subgroups, but show up in many other contexts as well.

A *subgraph census statistic* is a function which, for any given graph and subgraph, gives the number of copies of the latter contained in the former. A collection of subgraph census statistics is referred to as a *subgraph census*; widely used examples include the dyad and triad censuses, implemented in sna by the dyad.census and triad.census functions (respectively). Likewise, kpath.census and kcycle.census compute a range of census statistics related to k-paths and k-cycles. clique.census provides similar functionality for the census of maximal cliques, including:

- Aggregate counts of maximal cliques by size.
- Counts of cliques to which each vertex belongs (when tabulate.byvertex==TRUE).
- Counts of clique co-memberships, potentially disaggregated by size (when the appropriate co-membership argument is set to bylength).

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These calculations are intrinsically expensive (clique enumeration is NP hard in the general case), and users should be aware that computing the census can be impractical on large graphs (unless they are very sparse). On the other hand, the algorithm employed here (a variant of Makino and Uno (2004)) is generally fast enough to suport enumeration for even dense graphs of several hundred vertices on a typical desktop computer.

Calling this function with mode=="digraph", forces and initial symmetrization step, which can be avoided with mode=="graph". While incorrectly employing the default is harmless (except for the relatively small cost of verifying symmetry), setting mode=="graph" incorrectly may result in problematic behavior. When in doubt, stick with the default.

#### Value

A list with the following elements:

clique.count If tabulate.byvertex==FALSE, a vector of aggregate counts by clique size.

Otherwise, a matrix whose first column is a vector of aggregate clique counts, and whose succeeding columns contain vectors of clique counts for each vertex.

clique.comemb If clique.comembership!="none", a matrix or array containing co-membership

in cliques by vertex pairs. If clique.comembership="sum", only a matrix of co-memberships is returned; if bysize is used, however, co-memberships are returned in a maxsize by n by n array whose i, j, kth cell is the number of cliques of size i containing j and k (with maxsize being the size of the largest

maximal clique).

cliques If enumerate=TRUE, a list of length equal to the maximum clique size, each

element of which is in turn a list of all cliques of corresponding size (given as

vectors of vertices).

## Warning

The computational cost of calculating cliques grows very sharply in size and network density. It is possible that the expected completion time for your calculation may exceed your life expectancy (and those of subsequent generations).

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Wasserman, S. and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

Makino, K. and Uno, T. (2004.) "New Algorithms for Enumerating All Maximal Cliques." In T. Hagerup and J. Katajainen (eds.), *SWAT 2004*, LNCS 3111, 260-272. Berlin: Springer-Verlag.

#### See Also

dyad.census, triad.census, kcycle.census, kpath.census

32 closeness

## **Examples**

closeness

Compute the Closeness Centrality Scores of Network Positions

# **Description**

closeness takes one or more graphs (dat) and returns the closeness centralities of positions (selected by nodes) within the graphs indicated by g. Depending on the specified mode, closeness on directed or undirected geodesics will be returned; this function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

# Usage

```
closeness(dat, g=1, nodes=NULL, gmode="digraph", diag=FALSE,
    tmaxdev=FALSE, cmode="directed", geodist.precomp=NULL,
    rescale=FALSE, ignore.eval=TRUE)
```

# **Arguments**

dat	one or more input graphs.
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g=1.
nodes	list indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. gmode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
cmode	string indicating the type of closeness centrality being computed (distances on directed or undirected pairs, or an alternate measure).

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geodist.precomp

a geodist object precomputed for the graph to be analyzed (optional)

rescale if true, centrality scores are rescaled such that they sum to 1.

ignore.eval logical; should edge values be ignored when calculating geodesics?

## **Details**

The closeness of a vertex v is defined as

$$C_C(v) = \frac{|V(G)| - 1}{\sum_{i:i \neq v} d(v, i)}$$

where d(i, j) is the geodesic distance between i and j (where defined). Closeness is ill-defined on disconnected graphs; in such cases, this routine substitutes Inf. It should be understood that this modification is not canonical (though it is common), but can be avoided by not attempting to measure closeness on disconnected graphs in the first place! Intuitively, closeness provides an index of the extent to which a given vertex has short paths to all other vertices in the graph; this is one reasonable measure of the extent to which a vertex is in the "middle" of a given structure.

An alternate form of closeness (apparently due to Gil and Schmidt (1996)) is obtained by taking the sum of the inverse distances to each vertex, i.e.

$$C_C(v) = \frac{\sum_{i:i\neq v} \frac{1}{d(v,i)}}{|V(G)| - 1}.$$

This measure correlates well with the standard form of closeness where both are well-defined, but lacks the latter's pathological behavior on disconnected graphs. Computation of alternate closeness may be performed via the argument cmode="suminvdir" (directed case) and cmode="suminvundir" (undirected case). The corresponding arguments cmode="directed" and cmode="undirected" return the standard closeness scores in the directed or undirected cases (respectively). Although treated here as a measure of closeness, this index was originally intended to capture power or efficacy; in its original form, the Gil-Schmidt power index is a renormalized version of the above. Specifically, let R(v,G) be the set of vertices reachable by v in  $V\setminus v$ . Then the Gil-Schmidt power index is defined as

$$C_{GS}(v) = \frac{\sum_{i \in R(v,G)} \frac{1}{d(v,i)}}{|R(v,G)|}.$$

with  $C_{GS}$  defined to be 0 for vertices with no outneighbors. This may be obtained via the argument cmode="gil-schmidt".

#### Value

A vector, matrix, or list containing the closeness scores (depending on the number and size of the input graphs).

# Note

Judicious use of geodist.precomp can save a great deal of time when computing multiple path-based indices on the same network.

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#### Author(s)

Carter T. Butts, <buttsc@uci.edu>

#### References

Freeman, L.C. (1979). "Centrality in Social Networks I: Conceptual Clarification." *Social Networks*, 1, 215-239.

Gil, J. and Schmidt, S. (1996). "The Origin of the Mexican Network of Power". Proceedings of the International Social Network Conference, Charleston, SC, 22-25.

Sinclair, P.A. (2009). "Network Centralization with the Gil Schmidt Power Centrality Index" *Social Networks*, 29, 81-92.

#### See Also

centralization

# **Examples**

```
g<-rgraph(10) #Draw a random graph with 10 members closeness(g) #Compute closeness scores
```

coleman

Coleman's High School Friendship Data

## **Description**

James Coleman (1964) reports research on self-reported friendship ties among 73 boys in a small high school in Illinois over the 1957-1958 academic year. Networks of reported ties for all 73 informants are provided for two time points (fall and spring).

# Usage

```
data(coleman)
```

#### **Format**

An adjacency array containing two directed, unvalued 73-node networks:

```
[1,,] Fall binary matrix Friendship for Fall, 1957
[2,,] Spring binary matrix Friendship for Spring, 1958
```

#### **Details**

Both networks reflect answers to the question, "What fellows here in school do you go around with most often?" with the presence of an (i, j, k) edge indicating that j nominated k in time

component.dist 35

period i. The data are unvalued and directed; although the self-reported ties are highly reciprocal, unreciprocated nominations are possible.

It should be noted that, although this data is usually described as "friendship," the sociometric item employed might be more accurately characterized as eliciting "frequent elective interaction." This should be borne in mind when interpreting this data.

#### References

Coleman, J. S. (1964). Introduction to Mathermatical Sociology. New York: Free Press.

## **Examples**

```
data(coleman)
#Plot showing edges by time point
gplot(coleman[1,,]|coleman[2,,],edge.col=2*coleman[1,,]+3*coleman[2,,])
```

component.dist

Calculate the Component Size Distribution of a Graph

## **Description**

component.dist returns a list containing a vector of length n such that the ith element contains the number of components of graph G having size i, and a vector of length n giving component membership (where n is the graph order). Component strength is determined by the connected parameter; see below for details.

component.largest identifies the component(s) of maximum order within graph G. It returns either a logical vector indicating membership in a maximum component or the adjacency matrix of the subgraph of G induced by the maximum component(s), as determined by result. Component strength is determined as per component.dist.

#### Usage

#### **Arguments**

dat one or more input graphs.

connected a string selecting strong, weak, unilateral or recursively connected components;

by default, "strong" components are used.

result a string indicating whether a vector of membership indicators or the induced

subgraph of the component should be returned.

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```
return.as.edgelist
```

logical; if result=="graph", should the resulting structure be returned in edgelist form?

#### **Details**

Components are maximal sets of mutually connected vertices; depending on the definition of "connected" one employs, one can arrive at several types of components. Those supported here are as follows (in increasing order of restrictiveness):

- 1. weak:  $v_1$  is connected to  $v_2$  iff there exists a semi-path from  $v_1$  to  $v_2$  (i.e., a path in the weakly symmetrized graph)
- 2. unilateral:  $v_1$  is connected to  $v_2$  iff there exists a directed path from  $v_1$  to  $v_2$  or a directed path from  $v_2$  to  $v_1$
- 3. strong:  $v_1$  is connected to  $v_2$  iff there exists a directed path from  $v_1$  to  $v_2$  and a directed path from  $v_2$  to  $v_1$
- 4. recursive:  $v_1$  is connected to  $v_2$  iff there exists a vertex sequence  $v_a, \ldots, v_z$  such that  $v_1, v_a, \ldots, v_z, v_2$  and  $v_2, v_z, \ldots, v_a, v_1$  are directed paths

Note that the above definitions are distinct for directed graphs only; if dat is symmetric, then the connected parameter has no effect.

#### Value

For component.dist, a list containing:

membership A vector of component memberships, by vertex csize A vector of component sizes, by component

cdist A vector of length |V(G)| with the (unnormalized) empirical distribution func-

tion of component sizes

If multiple input graphs are given, the return value is a list of lists.

For component.largest, either a logical vector of component membership indicators or the adjacency matrix/edgelist of the subgraph induced by the largest component(s) is returned. If multiple graphs were given as input, a list of results is returned.

#### Note

Unilaterally connected component partitions may not be well-defined, since it is possible for a given vertex to be unilaterally connected to two vertices that are not unilaterally connected with one another. Consider, for instance, the graph  $a \to b \leftarrow c \to d$ . In this case, the maximal unilateral components are ab and bcd, with vertex b properly belonging to both components. For such graphs, a unique partition of vertices by component does not exist, and we "solve" the problem by allocating each "problem vertex" to one of its components on an essentially arbitrary basis. (component.dist generates a warning when this occurs.) It is recommended that the unilateral option be avoided where possible.

Do not make the mistake of assuming that the subgraphs returned by component.largest are necessarily connected. This is *usually* the case, but depends upon the uniqueness of the largest component.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

### References

West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, N.J.: Prentice Hall.

### See Also

```
components, symmetrize, reachability geodist
```

### **Examples**

```
g<-rgraph(20,tprob=0.06)
                           #Generate a sparse random graph
#Find weak components
cd<-component.dist(g,connected="weak")</pre>
cd$membership
                           #Who's in what component?
cd$csize
                           #What are the component sizes?
                           #Plot the size distribution
plot(1:length(cd$cdist),cd$cdist/sum(cd$cdist),ylim=c(0,1),type="h")
lgc<-component.largest(g,connected="weak") #Get largest component</pre>
gplot(g,vertex.col=2+lgc) #Plot g, with component membership
                           #Plot largest component itself
gplot(component.largest(g,connected="weak",result="graph"))
#Find strong components
cd<-component.dist(g,connected="strong")</pre>
cd$membership
                           #Who's in what component?
cd$csize
                           #What are the component sizes?
                           #Plot the size distribution
plot(1:length(cd$cdist),cd$cdist/sum(cd$cdist),ylim=c(0,1),type="h")
lgc<-component.largest(g,connected="strong") #Get largest component</pre>
gplot(g,vertex.col=2+lgc) #Plot g, with component membership
                           #Plot largest component itself
gplot(component.largest(g,connected="strong",result="graph"))
```

```
component.size.byvertex
```

Get Component Sizes, by Vertex

### **Description**

This function computes the component structure of the input network, and returns a vector whose ith entry is the size of the component to which i belongs. This is useful e.g. for studies of diffusion or similar applications.

#### Usage

#### **Arguments**

dat one or more input graphs (for best performance, sna edgelists or network objects

are suggested).

connected a string selecting the connectedness definition to use; by default, "strong" com-

ponents are used.

#### **Details**

Component sizes are here computed using component.dist; see this function for additional information.

In an undirected graph, the size of v's component represents the maximum number of nodes that can be reached by a diffusion process along the edges of the graph originating with node v; the expectation of component sizes by vertex (rather than the mean component size) is thus one measure of the maximum average diffusion potential of a graph. Because this quantity is monotone with respect to edge addition, it can be bounded using Bernoulli graphs (see Butts (2011)). In the directed case, multiple types of components are possible; see component.dist for details.

#### Value

A vector of length equal to the number of vertices in dat, whose *i*th element is the number of vertices in the component to which the *i*th vertex belongs.

### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

## References

```
West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, N.J.: Prentice Hall. Butts, C.T. (2011). "Bernoulli Bounds for General Random Graphs." Sociological Methodology, 41, 299-345.
```

#### See Also

```
component.dist
```

```
#Generate a random undirected graph
g<-rgraph(100,tprob=1.5/99,mode="graph",return.as.edgelist=TRUE)
#Get the component sizes for each vertex
cs<-component.size.byvertex(g)
cs</pre>
```

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components

Find the Number of (Maximal) Components Within a Given Graph

## **Description**

Returns the number of components within dat, using the connectedness rule given in connected.

# Usage

```
components(dat, connected="strong", comp.dist.precomp=NULL)
```

# **Arguments**

dat one or more input graphs.

connected the the component definition to be used by component.dist during component

extraction.

comp.dist.precomp

a component size distribution object from component.dist (optional).

#### **Details**

The connected parameter corresponds to the rule parameter of component.dist. By default, components returns the number of strong components, but other component types can be returned if so desired. (See component.dist for details.) For symmetric matrices, this is obviously a moot point.

#### Value

A vector containing the number of components for each graph in dat

## Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

### References

West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, NJ: Prentice Hall.

### See Also

```
component.dist, symmetrize
```

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### **Examples**

```
g<-rgraph(20,tprob=0.05) #Generate a sparse random graph
#Find weak components
components(g,connected="weak")
#Find strong components
components(g,connected="strong")</pre>
```

connectedness

Compute Graph Connectedness Scores

## **Description**

connectedness takes one or more graphs (dat) and returns the Krackhardt connectedness scores for the graphs selected by g.

### Usage

```
connectedness(dat, g=NULL)
```

# **Arguments**

dat one or more graphs.

g index values for the graphs to be utilized; by default, all graphs are selected.

### Details

Krackhardt's connectedness for a digraph G is equal to the fraction of all dyads,  $\{i, j\}$ , such that there exists an undirected path from i to j in G. (This, in turn, is just the density of the weak reachability graph of G.) Obviously, the connectedness score ranges from 0 (for the null graph) to 1 (for weakly connected graphs).

Connectedness is one of four measures (connectedness, efficiency, hierarchy, and lubness) suggested by Krackhardt for summarizing hierarchical structures. Each corresponds to one of four axioms which are necessary and sufficient for the structure in question to be an outtree; thus, the measures will be equal to 1 for a given graph iff that graph is an outtree. Deviations from unity can be interpreted in terms of failure to satisfy one or more of the outtree conditions, information which may be useful in classifying its structural properties.

## Value

A vector containing the connectedness scores

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#### Note

The four Krackhardt indices are, in general, nondegenerate for a relatively narrow band of size/density combinations (efficiency being the sole exception). This is primarily due to their dependence on the reachability graph, which tends to become complete rapidly as size/density increase. See Krackhardt (1994) for a useful simulation study.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Krackhardt, David. (1994). "Graph Theoretical Dimensions of Informal Organizations." In K. M. Carley and M. J. Prietula (Eds.), *Computational Organization Theory*, 89-111. Hillsdale, NJ: Lawrence Erlbaum and Associates.

#### See Also

connectedness, efficiency, hierarchy, lubness, reachability

### **Examples**

```
#Get connectedness scores for graphs of varying densities connectedness(rgraph(10,5,tprob=c(0.1,0.25,0.5,0.75,0.9)))
```

consensus

Estimate a Consensus Structure from Multiple Observations

## **Description**

consensus estimates a central or consensus structure given multiple observations, using one of several algorithms.

## Usage

```
consensus(dat, mode="digraph", diag=FALSE, method="central.graph",
     tol=1e-06, maxiter=1e3, verbose=TRUE, no.bias=FALSE)
```

## **Arguments**

dat	a set of input graphs (must have same order).
mode	"digraph" for directed data, else "graph".
diag	logical; should diagonals (loops) be treated as data?
method	one of "central.graph", "single.reweight", "iterative.reweight", "romney.batchelder", "PCA.reweight", "LAS.intersection", "LAS.union", "OR.row", or "OR.col".
tol	convergence tolerance for the iterative reweighting and B-R algorithms.

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maxiter	maximum number of iterations to take (regardless of convergence) for the iterative reweighting and B-R algorithms.
verbose	logical; should bias and competency parameters be reported (where computed)?
no.bias	logical; should responses be assumed to be unbiased?

#### **Details**

The term "consensus structure" is used by a number of authors to reflect a notion of shared or common perceptions of social structure among a set of observers. As there are many interpretations of what is meant by "consensus" (and as to how best to estimate it), several algorithms are employed here:

- 1. central.graph: Estimate the consensus structure using the central graph. This corresponds to a "median response" notion of consensus.
- single.reweight: Estimate the consensus structure using subject responses, reweighted by mean graph correlation. This corresponds to an "expertise-weighted vote" notion of consensus
- 3. iterative.reweight: Similar to single.reweight, but the consensus structure and accuracy parameters are estimated via an iterated proportional fitting scheme. The implementation employed here uses both bias and competency parameters.
- 4. romney.batchelder: Fits a Romney-Batchelder informant accuracy model using IPF. This is very similar to iterative.reweight, but can be interpreted as the result of a process in which each informant report is correct with a probability equal to the informant's competency score, and otherwise equal to a Bernoulli trial with parameter equal to the informant's bias score.
- 5. PCA. reweight: Estimate the consensus using the (scores on the) first component of a network PCA. This corresponds to a "shared theme" or "common element" notion of consensus.
- 6. LAS.intersection: Estimate the consensus structure using the locally aggregated structure (intersection rule). In this model, an i->j edge exists iff i *and* j agree that it exists.
- 7. LAS.union: Estimate the consensus structure using the locally aggregated structure (union rule). In this model, an i->j edge exists iff i *or* j agree that it exists.
- 8. OR. row: Estimate the consensus structure using own report. Here, we take each informant's outgoing tie reports to be correct.
- 9. OR.col: Estimate the consensus structure using own report. Here, we take each informant's incoming tie reports to be correct.

Note that the results returned by the single weighting algorithms are not dichotomized by default; since some algorithms thus return valued graphs, dichotomization may be desirable prior to use.

It should be noted that a model for estimating an underlying criterion structure from multiple informant reports is provided in bbnam; if your goal is to reconstruct an "objective" network from informant reports, this (or the R-B model) may prove more useful than the ad-hoc solutions.

### Value

An adjacency matrix representing the consensus structure

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#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Banks, D.L., and Carley, K.M. (1994). "Metric Inference for Social Networks." *Journal of Classification*, 11(1), 121-49.

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Inter-Structural Analysis." CASOS Working Paper, Carnegie Mellon University.

Krackhardt, D. (1987). "Cognitive Social Structures." Social Networks, 9, 109-134.

Romney, A.K.; Weller, S.C.; and Batchelder, W.H. (1986). "Culture as Consensus: A Theory of Culture and Informant Accuracy." *American Anthropologist*, 88(2), 313-38.

#### See Also

bbnam, centralgraph

### **Examples**

```
#Generate some test data
g<-rgraph(5)
g.pobs<-g*0.9+(1-g)*0.5
g.obs<-rgraph(5,5,tprob=g.pobs)

#Find some consensus structures
consensus(g.obs) #Central graph
consensus(g.obs,method="single.reweight") #Single reweighting
consensus(g.obs,method="PCA.reweight") #1st component in network PCA</pre>
```

cug.test

Univariate Conditional Uniform Graph Tests

## **Description**

cug. test takes an input network and conducts a conditional uniform graph (CUG) test of the statistic in FUN, using the conditioning statistics in cmode. The resulting test object has custom print and plot methods.

## Usage

```
cug.test(dat, FUN, mode = c("digraph", "graph"), cmode = c("size",
    "edges", "dyad.census"), diag = FALSE, reps = 1000,
    ignore.eval = TRUE, FUN.args = list())
```

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## **Arguments**

dat	one or more input	graphs.

FUN the function generating the test statistic; note that this must take a graph as its

first argument, and return a single numerical value.

mode graph if dat is an undirected graph, else digraph.

cmode string indicating the type of conditioning to be applied.

diag logical; should self-ties be treated as valid data?
reps number of Monte Carlo replications to use.

ignore.eval logical; should edge values be ignored? (Note: TRUE is usually more efficient.)

FUN. args a list containing any additional arguments to FUN.

### **Details**

cug.test is an improved version of cugtest, for use only with univariate CUG hypotheses. Depending on cmode, conditioning on the realized size, edge count (or exact edge value distribution), or dyad census (or dyad value distribution) can be selected. Edges are treated as unvalued unless ignore.eval=FALSE; since the latter setting is less efficient for sparse graphs, it should be used only when necessary.

A brief summary of the theory and goals of conditional uniform graph testing can be found in the reference below. See also cugtest for a somewhat informal description.

#### Value

An object of class cug. test.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, Carter T. (2008). "Social Networks: A Methodological Introduction." *Asian Journal of Social Psychology*, 11(1), 13–41.

# See Also

cugtest

```
#Draw a highly reciprocal network
g<-rguman(1,15,mut=0.25,asym=0.05,null=0.7)

#Test transitivity against size, density, and the dyad census
cug.test(g,gtrans,cmode="size")
cug.test(g,gtrans,cmode="edges")
cug.test(g,gtrans,cmode="dyad.census")</pre>
```

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cugtest	Perform Conditional Uniform Graph (CUG) Hypothesis Tests for Graph-Level Indices

## **Description**

cugtest tests an arbitrary GLI (computed on dat by FUN) against a conditional uniform graph null hypothesis, via Monte Carlo simulation. Some variation in the nature of the conditioning is available; currently, conditioning only on size, conditioning jointly on size and estimated tie probability (via expected density), and conditioning jointly on size and (bootstrapped) edge value distributions are implemented. Note that fair amount of flexibility is possible regarding CUG tests on functions of GLIs (Anderson et al., 1999). See below for more details.

## Usage

```
cugtest(dat, FUN, reps=1000, gmode="digraph", cmode="density",
    diag=FALSE, g1=1, g2=2, ...)
```

## Arguments

dat	graph(s) to be analyzed.
FUN	function to compute GLIs, or functions thereof. FUN must accept dat and the specified g arguments, and should return a real number.
reps	integer indicating the number of draws to use for quantile estimation. Note that, as for all Monte Carlo procedures, convergence is slower for more extreme quantiles. By default, reps==1000.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. gmode is set to "digraph" by default.
cmode	string indicating the type of conditioning assumed by the null hypothesis. If cmode is set to "density", then the density of the graph in question is used to determine the tie probabilities of the Bernoulli graph draws (which are also conditioned on $ V(G) $ ). If $cmode="ties"$ , then draws are bootstrapped from the distribution of edge values within the data matrices. If $cmode="order"$ , then draws are uniform over all graphs of the same order (size) as the graphs within the input stack. By default, $cmode$ is set to "density".
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
g1	integer indicating the index of the first graph input to the GLI. By default, $g1==1$ .
g2	integer indicating the index of the second graph input to the GLI. (FUN can ignore this, if one wishes to test the GLI value of a single graph, but it should recognize the argument.) By default, $g2==2$ .
	additional arguments to FUN.

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#### **Details**

The null hypothesis of the CUG test is that the observed GLI (or function thereof) was drawn from a distribution equivalent to that of said GLI evaluated (uniformly) on the space of all graphs conditional on one or more features. The most common "features" used for conditioning purposes are order (size) and density, both of which are known to have strong and nontrivial effects on other GLIs (Anderson et al., 1999) and which are, in many cases, exogenously determined. (Note that maximum entropy distributions conditional on expected statistics are not in general correctly referred to as "conditional uniform graphs", but have been described as such for independent-dyad models; this is indeed the case for this function, although such terminology is not really proper. See cug. test for CUG tests with exact conditioning.) Since theoretical results regarding functions of arbitrary GLIs on the space of graphs are not available, the standard approach to CUG testing is to approximate the quantiles of the observed statistic associated with the null hypothesis using Monte Carlo methods. This is the technique utilized by cugtest, which takes appropriately conditioned draws from the set of graphs and computes on them the GLI specified in FUN, thereby accumulating an approximation to the true quantiles.

The cugtest procedure returns a cugtest object containing the estimated distribution of the test GLI under the null hypothesis, the observed GLI value of the data, and the one-tailed p-values (estimated quantiles) associated with said observation. As usual, the (upper tail) null hypothesis is rejected for significance level alpha if p>=observation is less than alpha (or p<=observation, for the lower tail). Standard caveats regarding the use of null hypothesis testing procedures are relevant here: in particular, bear in mind that a significant result does not necessarily imply that the likelihood ratio of the null model and the alternative hypothesis favors the latter.

Informative and aesthetically pleasing portrayals of cugtest objects are available via the print.cugtest and summary.cugtest methods. The plot.cugtest method displays the estimated distribution, with a reference line signifying the observed value.

### Value

An object of class cugtest, containing

testval The observed GLI value.

dist A vector containing the Monte Carlo draws.

pgreq The proportion of draws which were greater than or equal to the observed GLI

value.

pleeq The proportion of draws which were less than or equal to the observed GLI

value.

#### Note

This function currently conditions only on expected statistics, and is somewhat cumbersome. cug.test is now recommended for univariate CUG tests (and will eventually supplant this function).

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

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### References

Anderson, B.S.; Butts, C.T.; and Carley, K.M. (1999). "The Interaction of Size and Density with Graph-Level Indices." *Social Networks*, 21(3), 239-267.

#### See Also

```
cug.test, qaptest, gliop
```

### **Examples**

```
#Draw two random graphs, with different tie probabilities
dat<-rgraph(20,2,tprob=c(0.2,0.8))
#Is their correlation higher than would be expected, conditioning
#only on size?
cug<-cugtest(dat,gcor,cmode="order")
summary(cug)
#Now, let's try conditioning on density as well.
cug<-cugtest(dat,gcor)
summary(cug)</pre>
```

cutpoints

Identify the Cutpoints of a Graph or Digraph

## **Description**

cutpoints identifies the cutpoints of an input graph. Depending on mode, either a directed or undirected notion of "cutpoint" can be used.

#### Usage

```
cutpoints(dat, mode = "digraph", connected = c("strong","weak","recursive"),
    return.indicator = FALSE)
```

# Arguments

dat one or more input graphs.

mode "digraph" for directed graphs, or "graph" for undirected graphs.

connected string indicating the type of connectedness rule to apply (only relevant where

mode=="digraph").

return.indicator

logical; should the results be returned as a logical (TRUE/FALSE) vector of indi-

cators, rather than as a vector of vertex IDs?

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#### **Details**

A cutpoint (also known as an articulation point or cut-vertex) of an undirected graph, G is a vertex whose removal increases the number of components of G. Several generalizations to the directed case exist. Here, we define a strong cutpoint of directed graph G to be a vertex whose removal increases the number of strongly connected components of G (see component.dist). Likewise, weak and recursive cutpoints of G are those vertices whose removal increases the number of weak or recursive cutpoints (respectively). By default, strong cutpoints are used; alternatives may be selected via the connected argument.

Cutpoints are of particular interest when seeking to identify critical positions in flow networks, since their removal by definition alters the connectivity properties of the graph. In this context, cutpoint status can be thought of as a primitive form of centrality (with some similarities to betweenness).

Cutpoint computation is significantly faster for the undirected case (and for the weak/recursive cases) than for the strong directed case. While calling cutpoints with mode="digraph" on an undirected graph will give the same answer as mode="graph", it is thus to one's advantage to use the latter form. Do not, however, employ mode="graph" with directed data, unless you enjoy unpredictable behavior.

#### Value

A vector of cutpoints (if return.indicator==FALSE), or else a logical vector indicating cutpoint status for each vertex.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Berge, Claude. (1966). The Theory of Graphs. New York: John Wiley and Sons.

#### See Also

component.dist, bicomponent.dist, betweenness

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```
cpdr<-cutpoints(gd,connected="recursive",return.indicator=TRUE)
#Visualize the difference
gplot(gd,vertex.col=2+cpdw)
gplot(gd,vertex.col=2+cpdr)</pre>
```

degree

Compute the Degree Centrality Scores of Network Positions

## Description

Degree takes one or more graphs (dat) and returns the degree centralities of positions (selected by nodes) within the graphs indicated by g. Depending on the specified mode, indegree, outdegree, or total (Freeman) degree will be returned; this function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

## Usage

## **Arguments**

dat	one or more input graphs.
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g=1.
nodes	vector indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. gmode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
cmode	string indicating the type of degree centrality being computed. "indegree", "outdegree", and "freeman" refer to the indegree, outdegree, and total (Freeman) degree measures, respectively. The default for cmode is "freeman".
rescale	if true, centrality scores are rescaled such that they sum to 1.
ignore.eval	logical; should edge values be ignored when computing degree scores?

50 degree

#### **Details**

Degree centrality is the social networker's term for various permutations of the graph theoretic notion of vertex degree: for unvalued graphs, indegree of a vertex, v, corresponds to the cardinality of the vertex set  $N^+(v) = \{i \in V(G) : (i,v) \in E(G)\}$ ; outdegree corresponds to the cardinality of the vertex set  $N^-(v) = \{i \in V(G) : (v,i) \in E(G)\}$ ; and total (or "Freeman") degree corresponds to  $|N^+(v)| + |N^-(v)|$ . (Note that, for simple graphs, indegree=outdegree=total degree/2.) Obviously, degree centrality can be interpreted in terms of the sizes of actors' neighborhoods within the larger structure. See the references below for more details.

When ignore.eval==FALSE, degree weights edges by their values where supplied.ignore.eval==TRUE ensures an unweighted degree score (independent of input). Setting gmode=="graph" forces behavior equivalent to cmode=="indegree" (i.e., each edge is counted only once); to obtain a total degree score for an undirected graph in which both in- and out-neighborhoods are counted separately, simply use gmode=="digraph".

## Value

A vector, matrix, or list containing the degree scores (depending on the number and size of the input graphs).

### Author(s)

Carter T. Butts <buttsc@uci.edu>

### References

Freeman, L.C. (1979). "Centrality in Social Networks I: Conceptual Clarification." *Social Networks*, 1, 215-239.

#### See Also

centralization

```
#Create a random directed graph
dat<-rgraph(10)
#Find the indegrees, outdegrees, and total degrees
degree(dat,cmode="indegree")
degree(dat,cmode="outdegree")
degree(dat)</pre>
```

diag.remove 51

diag.remove

Remove the Diagonals of Adjacency Matrices in a Graph Stack

# Description

Returns the input graphs, with the diagonal entries removed/replaced as indicated.

# Usage

```
diag.remove(dat, remove.val=NA)
```

## Arguments

dat one or more graphs.

remove.val the value with which to replace the existing diagonals

### **Details**

diag.remove is simply a convenient way to apply diag to an entire collection of adjacency matrices/network objects at once.

### Value

The updated graphs.

### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

### See Also

```
diag, upper.tri.remove, lower.tri.remove
```

```
#Generate a random graph stack
g<-rgraph(3,5)
#Remove the diagonals
g<-diag.remove(g)</pre>
```

52 dyad.census

dyad.census

Compute a Holland and Leinhardt MAN Dyad Census

### **Description**

dyad.census computes a Holland and Leinhardt dyad census on the graphs of dat selected by g.

## Usage

```
dyad.census(dat, g=NULL)
```

### **Arguments**

dat one or more graphs.

g the elements of dat to be included; by default, all graphs are processed.

#### **Details**

Each dyad in a directed graph may be in one of four states: the null state  $(a \nleftrightarrow b)$ , the complete or mutual state  $(a \leftrightarrow b)$ , and either of two asymmetric states  $(a \leftarrow b \text{ or } a \rightarrow b)$ . Holland and Leinhardt's dyad census classifies each dyad into the mutual, asymmetric, or null categories, counting the number of each within the digraph. These counts can be used as the basis for null hypothesis tests (since their distributions are known under assumptions such as constant edge probability), or for the generation of random graphs (e.g., via the UlMAN distribution, which conditions on the numbers of mutual, asymmetric, and null dyads in each graph).

#### Value

A matrix whose three columns contain the counts of mutual, asymmetric, and null dyads (respectively) for each graph

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Holland, P.W. and Leinhardt, S. (1970). "A Method for Detecting Structure in Sociometric Data." *American Journal of Sociology*, 76, 492-513.

Wasserman, S., and Faust, K. (1994). "Social Network Analysis: Methods and Applications." Cambridge: Cambridge University Press.

#### See Also

mutuality, grecip, rguman triad.census, kcycle.census, kpath.census

efficiency 53

### **Examples**

#Generate a dyad census of random data with varying densities dyad.census(rgraph(15,5,tprob=c(0.1,0.25,0.5,0.75,0.9)))

efficiency

Compute Graph Efficiency Scores

## Description

efficiency takes one or more graphs (dat) and returns the Krackhardt efficiency scores for the graphs selected by g.

### Usage

```
efficiency(dat, g=NULL, diag=FALSE)
```

## **Arguments**

dat one or more graphs.

g index values for the graphs to be utilized; by default, all graphs are selected.

diag TRUE if the diagonal contains valid data; by default, diag==FALSE.

#### **Details**

Let  $G = \bigcup_{i=1}^n G_i$  be a digraph with weak components  $G_1, G_2, \ldots, G_n$ . For convenience, we denote the cardinalities of these components' vertex sets by |V(G)| = N and  $|V(G_i)| = N_i$ ,  $\forall i \in 1, \ldots, n$ . Then the Krackhardt efficiency of G is given by

$$1 - \frac{|E(G)| - \sum_{i=1}^{n} (N_i - 1)}{\sum_{i=1}^{n} (N_i (N_i - 1) - (N_i - 1))}$$

which can be interpreted as 1 minus the proportion of possible "extra" edges (above those needed to weakly connect the existing components) actually present in the graph. A graph which an efficiency of 1 has precisely as many edges as are needed to connect its components; as additional edges are added, efficiency gradually falls towards 0.

Efficiency is one of four measures (connectedness, efficiency, hierarchy, and lubness) suggested by Krackhardt for summarizing hierarchical structures. Each corresponds to one of four axioms which are necessary and sufficient for the structure in question to be an outtree; thus, the measures will be equal to 1 for a given graph iff that graph is an outtree. Deviations from unity can be interpreted in terms of failure to satisfy one or more of the outtree conditions, information which may be useful in classifying its structural properties.

### Value

A vector of efficiency scores

54 ego.extract

#### Note

The four Krackhardt indices are, in general, nondegenerate for a relatively narrow band of size/density combinations (efficiency being the sole exception). This is primarily due to their dependence on the reachability graph, which tends to become complete rapidly as size/density increase. See Krackhardt (1994) for a useful simulation study.

The violation normalization used before version 0.51 was  $N(N-1)\sum_{i=1}^{n}(N_i-1)$ , based on a somewhat different interpretation of the definition in Krackhardt (1994). The former version gave results which more closely matched those of the cited simulation study, but was less consistent with the textual definition.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Krackhardt, David. (1994). "Graph Theoretical Dimensions of Informal Organizations." In K. M. Carley and M. J. Prietula (Eds.), *Computational Organization Theory*, 89-111. Hillsdale, NJ: Lawrence Erlbaum and Associates.

#### See Also

```
connectedness, efficiency, hierarchy, lubness, gden
```

### **Examples**

```
#Get efficiency scores for graphs of varying densities efficiency(rgraph(10,5,tprob=c(0.1,0.25,0.5,0.75,0.9)))
```

ego.extract

Extract Egocentric Networks from Complete Network Data

## **Description**

ego.extract takes one or more input graphs (dat) and returns a list containing the egocentric networks centered on vertices named in ego, using adjacency rule *neighborhood* to define inclusion.

# Usage

#### **Arguments**

dat one or more graphs.

ego a vector of vertex IDs, or NULL if all are to be selected.

neighborhood the neighborhood to use.

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### **Details**

The egocentric network (or "ego net") of vertex v in graph G is defined as  $G[v \cup N(v)]$  (i.e., the subgraph of G induced by v and its neighborhood). The neighborhood employed by ego.extract is selected by the eponymous argument: "in" selects in-neighbors, "out" selects out-neighbors, and "combined" selects all neighbors. In the event that one of the vertices selected by ego has no qualifying neighbors, ego.extract will return a degenerate (1 by 1) adjacency matrix containing that individual's diagonal entry.

Vertices within the returned matrices are maintained in their original order, save for ego (who is always listed first). The ego nets themselves are returned in the order specified in the ego parameter (or their vertex order, if no value was specified).

ego.extract is useful for finding local properties associated with particular vertices. To compute functions of neighbors' covariates, see gapply.

#### Value

A list containing the adjacency matrices for the ego nets of each vertex in ego.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Wasserman, S. and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

#### See Also

gapply

```
#Generate a sample network
g<-rgraph(10,tp=1.5/9)

#Extract some ego nets
g.in<-ego.extract(g,neighborhood="in")
g.out<-ego.extract(g,neighborhood="out")
g.comb<-ego.extract(g,neighborhood="in")

#View some networks
g.comb

#Compare ego net size with degree
all(sapply(g.in,NROW)==degree(g,cmode="indegree")+1)  #TRUE
all(sapply(g.out,NROW)==degree(g,cmode="outdegree")+1)  #TRUE
all(sapply(g.comb,NROW)==degree(g)/2+1)  #Usually FALSE!

#Calculate egocentric network density
ego.size<-sapply(g.comb,NROW)</pre>
```

56 equiv.clust

```
if(any(ego.size>2))
  sapply(g.comb[ego.size>2],function(x){gden(x[-1,-1])})
```

equiv.clust

Find Clusters of Positions Based on an Equivalence Relation

### **Description**

equiv.clust uses a definition of approximate equivalence (equiv.fun) to form a hierarchical clustering of network positions. Where dat consists of multiple relations, all specified relations are considered jointly in forming the equivalence clustering.

## Usage

```
equiv.clust(dat, g=NULL, equiv.dist=NULL, equiv.fun="sedist",
    method="hamming", mode="digraph", diag=FALSE,
    cluster.method="complete", glabels=NULL, plabels=NULL, ...)
```

## **Arguments**

dat	one or more graphs.
g	the elements of dat to use in clustering the vertices; by default, all structures are used.
equiv.dist	a matrix of distances, by which vertices should be clustered. (Overrides equiv. fun.)
equiv.fun	the distance function to use in clustering vertices (defaults to sedist).
method	method parameter to be passed to equiv.fun.
mode	"graph" or "digraph," as appropriate.
diag	a boolean indicating whether or not matrix diagonals (loops) should be interpreted as useful data.
${\tt cluster.method}$	the hierarchical clustering method to use (see hclust).
glabels	labels for the various graphs in dat.
plabels	labels for the vertices of dat.
	additional arguments to equiv.dist.

#### **Details**

This routine is essentially a joint front-end to hclust and various positional distance functions, though it defaults to structural equivalence in particular. Taking the specified graphs as input, equiv.clust computes the distances between all pairs of positions using equiv.fun (unless distances are supplied in equiv.dist), and then performs a cluster analysis of the result. The return value is an object of class equiv.clust, for which various secondary analysis methods exist.

#### Value

An object of class equiv.clust

eval.edgeperturbation 57

### Note

See sedist for an example of a distance function compatible with equiv.clust.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Breiger, R.L.; Boorman, S.A.; and Arabie, P. (1975). "An Algorithm for Clustering Relational Data with Applications to Social Network Analysis and Comparison with Multidimensional Scaling." *Journal of Mathematical Psychology*, 12, 328-383.

Burt, R.S. (1976). "Positions in Networks." Social Forces, 55, 93-122.

Wasserman, S., and Faust, K. Social Network Analysis: Methods and Applications. Cambridge: Cambridge University Press.

#### See Also

```
sedist, blockmodel
```

## **Examples**

 $\begin{array}{ll} \hbox{\it eval.edgeperturbation} & \textit{\it Compute the Effects of Single-Edge Perturbations on Structural Indices} \\ \end{array}$ 

## **Description**

Evaluates a given function on an input graph with and without a specified edge, returning the difference between the results in each case.

## Usage

```
eval.edgeperturbation(dat, i, j, FUN, ...)
```

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## Arguments

dat	A single adjacency matrix
i	The row(s) of the edge(s) to be perturbed
j	The column(s) of the edge(s) to be perturbed
FUN	The function to be computed
	Additional arguments to FUN

### **Details**

Although primarily a back-end utility for pstar, eval.edgeperturbation may be useful in any circumstance in which one wishes to assess the stability of a given structural index with respect to single edge perturbations. The function to be evaluated is calculated first on the input graph with all marked edges set to present, and then on the same graph with said edges absent. (Obviously, this is sensible only for dichotomous data.) The difference is then returned.

In pstar, calls to eval.edgeperturbation are used to construct a perturbation effect matrix for the GLM.

### Value

The difference in the values of FUN as computed on the perturbed graphs.

#### Note

length(i) and length(j) must be equal; where multiple edges are specified, the row and column listings are interpreted as pairs.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Anderson, C.; Wasserman, S.; and Crouch, B. (1999). "A p\* Primer: Logit Models for Social Networks. *Social Networks*, 21,37-66.

### See Also

pstar

```
#Create a random graph
g<-rgraph(5)

#How much does a one-edge change affect reciprocity?
eval.edgeperturbation(g,1,2,grecip)</pre>
```

evcent 59

evcent	Find Eigenvector Centrality Scores of Network	Positions

### **Description**

evcent takes one or more graphs (dat) and returns the eigenvector centralities of positions (selected by nodes) within the graphs indicated by g. This function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

### Usage

```
evcent(dat, g=1, nodes=NULL, gmode="digraph", diag=FALSE,
    tmaxdev=FALSE, rescale=FALSE, ignore.eval=FALSE, tol=1e-10,
    maxiter=1e5, use.eigen=FALSE)
```

## **Arguments**

dat	one or more input graphs.
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g=1.
nodes	vector indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. This is currently ignored.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
rescale	if true, centrality scores are rescaled such that they sum to 1.
ignore.eval	logical; should edge values be ignored?
tol	convergence tolerance for the eigenvector computation.
maxiter	maximum iterations for eigenvector calculation.
use.eigen	logical; should we use R's eigen routine instead of the (faster but less robust) internal method?

### **Details**

Eigenvector centrality scores correspond to the values of the first eigenvector of the graph adjacency matrix; these scores may, in turn, be interpreted as arising from a reciprocal process in which the centrality of each actor is proportional to the sum of the centralities of those actors to whom he or she is connected. In general, vertices with high eigenvector centralities are those which are connected to many other vertices which are, in turn, connected to many others (and so on). (The perceptive

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may realize that this implies that the largest values will be obtained by individuals in large cliques (or high-density substructures). This is also intelligible from an algebraic point of view, with the first eigenvector being closely related to the best rank-1 approximation of the adjacency matrix (a relationship which is easy to see in the special case of a diagonalizable symmetric real matrix via the  $S\Lambda S^{-1}$  decomposition).)

By default, a sparse-graph power method is used to obtain the principal eigenvector. This procedure scales well, but may not converge in some cases. In the event that the convergence objective set by tol is not obtained, evcent will return a warning message. Correctives in this case include increasing maxiter, or setting use.eigen to TRUE. The latter will cause evcent to use R's standard eigen method to calculate the principal eigenvector; this is far slower for sparse graphs, but is also more robust.

The simple eigenvector centrality is generalized by the Bonacich power centrality measure; see bonpow for more details.

#### Value

A vector, matrix, or list containing the centrality scores (depending on the number and size of the input graphs).

#### WARNING

evcent will not symmetrize your data before extracting eigenvectors; don't send this routine asymmetric matrices unless you really mean to do so.

#### Note

The theoretical maximum deviation used here is not obtained with the star network, in general. For symmetric data, the maximum occurs for an empty graph with one complete dyad; the maximum deviation for asymmetric data is generated by the outstar. UCINET V seems not to adjust for this fact, which can cause some oddities in their centralization scores (and results in a discrepancy in centralizations between the two packages).

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Bonacich, P. (1987). "Power and Centrality: A Family of Measures." *American Journal of Sociology*, 92, 1170-1182.

Katz, L. (1953). "A New Status Index Derived from Sociometric Analysis." *Psychometrika*, 18, 39-43.

## See Also

centralization, bonpow

event2dichot 61

### **Examples**

```
#Generate some test data
dat<-rgraph(10,mode="graph")
#Compute eigenvector centrality scores
evcent(dat)</pre>
```

event2dichot

Convert an Observed Event Matrix to a Dichotomous matrix

### **Description**

Given one or more valued adjacency matrices (possibly derived from observed interaction "events"), event2dichot returns dichotomized equivalents.

### Usage

```
event2dichot(m, method="quantile", thresh=0.5, leq=FALSE)
```

#### **Arguments**

m one or more (valued) input graphs.

method one of "quantile," "rquantile," "cquantile," "mean," "rmean," "cmean," "abso-

lute," "rank," "rrank," or "crank".

thresh dichotomization thresholds for ranks or quantiles.

leq boolean indicating whether values less than or equal to the threshold should be

taken as existing edges; the alternative is to use values strictly greater than the

threshold.

### **Details**

The methods used for choosing dichotomization thresholds are as follows:

1. quantile: specified quantile over the distribution of all edge values

2. rquantile: specified quantile by row

3. cquantile: specified quantile by column

4. mean: grand mean5. rmean: row mean6. cmean: column mean

7. absolute: the value of thresh itself

8. rank: specified rank over the distribution of all edge values

9. rrank: specified rank by row

10. crank: specified rank by column

Note that when a quantile, rank, or value is said to be "specified," this refers to the value of thresh.

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#### Value

The dichotomized data matrix (or matrices)

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Wasserman, S. and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

### **Examples**

```
#Draw a matrix of normal values
n<-matrix(rnorm(25),nrow=5,ncol=5)
#Dichotomize by the mean value
event2dichot(n,"mean")
#Dichotomize by the 0.95 quantile
event2dichot(n,"quantile",0.95)</pre>
```

flowbet

Calculate Flow Betweenness Scores of Network Positions

## **Description**

flowbet takes one or more graphs (dat) and returns the flow betweenness scores of positions (selected by nodes) within the graphs indicated by g. Depending on the specified mode, flow betweenness on directed or undirected geodesics will be returned; this function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

### Usage

```
flowbet(dat, g = 1, nodes = NULL, gmode = "digraph", diag = FALSE,
    tmaxdev = FALSE, cmode = "rawflow", rescale = FALSE,
    ignore.eval = FALSE)
```

### Arguments

dat one or more input graphs.

g integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g=1.

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nodes	vector indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. digraph indicates that edges should be interpreted as directed (with flows summed over directed dyads); graph indicates that edges are undirected (with only undirected pairs considered). gmode is set to digraph by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
cmode	one of rawflow, normflow, or fracflow (see below).
rescale	if true, centrality scores are rescaled such that they sum to 1.
ignore.eval	logical; ignore edge values when computing maximum flow (alternately, edge values will be assumed to carry capacity information)?

### **Details**

The ("raw," or unnormalized) flow betweenness of a vertex,  $v \in V(G)$ , is defined by Freeman et al. (1991) as

$$C_F(v) = \sum_{i,j: i \neq j, i \neq v, j \neq v} (f(i,j,G) - f(i,j,G \setminus v)),$$

where f(i, j, G) is the maximum flow from i to j within G (under the assumption of infinite vertex capacities, finite edge capacities, and non-simultaneity of pairwise flows). Intuitively, unnormalized flow betweenness is simply the total maximum flow (aggregated across all pairs of third parties) mediated by v.

The above flow betweenness measure is computed by flowbet when cmode=="rawflow". In some cases, it may be desirable to normalize the raw flow betweenness by the total maximum flow among third parties (including v); this leads to the following normalized flow betweenness measure:

$$C_F'(v) = \frac{\sum_{i,j: i \neq j, i \neq v, j \neq v} \left( f(i,j,G) - f(i,j,G \setminus v) \right)}{\sum_{i,j: i \neq j, i \neq v, j \neq v} f(i,j,G)}.$$

This variant can be selected by setting cmode=="normflow"

Finally, it may be noted that the above normalization (from Freeman et al. (1991)) is rather different from that used in the definition of shortest-path betweenness, which normalizes within (rather than across) third-party dyads. A third flow betweenness variant has been suggested by Koschutzki et al. (2005) based on a normalization of this type:

$$C_F''(v) = \sum_{i,j:i \neq j, i \neq v, j \neq v} \frac{(f(i,j,G) - f(i,j,G \setminus v))}{f(i,j,G)}$$

where 0/0 flow ratios are treated as 0 (as in shortest-path betweenness). Setting cmode=="fracflow" selects this variant.

#### Value

A vector of centrality scores.

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### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Freeman, L.C.; Borgatti, S.P.; and White, D.R. (1991). "Centrality in Valued Graphs: A Measure of Betweenness Based on Network Flow." *Social Networks*, 13(2), 141-154.

Koschutzki, D.; Lehmann, K.A.; Peeters, L.; Richter, S.; Tenfelde-Podehl, D.; Zlotowski, O. (2005). "Centrality Indices." In U. Brandes and T. Erlebach (eds.), *Network Analysis: Methodological Foundations*. Berlin: Springer.

### See Also

betweenness, maxflow

### **Examples**

```
g<-rgraph(10)  #Draw a random graph
flowbet(g)  #Raw flow betweenness
flowbet(g,cmode="normflow")  #Normalized flow betweenness
g<-g*matrix(rpois(100,4),10,10)  #Add capacity constraints
flowbet(g)  #Note the difference!</pre>
```

gapply

Apply Functions Over Vertex Neighborhoods

### **Description**

Returns a vector or array or list of values obtained by applying a function to vertex neighborhoods of a given order.

## Usage

```
gapply(X, MARGIN, STATS, FUN, ..., mode = "digraph", diag = FALSE,
    distance = 1, thresh = 0, simplify = TRUE)
```

# **Arguments**

Χ	one or more input graphs.
MARGIN	a vector giving the "margin" of X to be used in calculating neighborhoods. 1 indicates rows (out-neighbors), 2 indicates columns (in-neighbors), and $c(1,2)$ indicates rows and columns (total neighborhood).
STATS	the vector or matrix of vertex statistics to be used.
FUN	the function to be applied. In the case of operators, the function name must be quoted.

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	additional arguments to FUN.
mode	"graph" if X is a simple graph, else "digraph".
diag	boolean; are the diagonals of X meaningful?
distance	the maximum geodesic distance at which neighborhoods are to be taken. 1 signifies first-order neighborhoods, 2 signifies second-order neighborhoods, etc.
thresh	the threshold to be used in dichotomizing X.
simplify	boolean; should we attempt to coerce output to a vector if possible?

### **Details**

For each vertex in X, gapply first identifies all members of the relevant neighborhood (as determined by MARGIN and distance) and pulls the rows of STATS associated with each. FUN is then applied to this collection of values. This provides a very quick and easy way to answer questions like:

- How many persons are in each ego's 3rd-order neighborhood?
- What fraction of each ego's alters are female?
- What is the mean income for each ego's trading partners?
- etc.

With clever use of FUN and STATS, a wide range of functionality can be obtained.

### Value

The result of the iterated application of FUN to each vertex neighborhood's STATS.

#### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

### See Also

```
apply, sapply
```

```
#Generate a random graph
g<-rgraph(6)

#Calculate the degree of g using gapply
all(gapply(g,1,rep(1,6),sum)==degree(g,cmode="outdegree"))
all(gapply(g,2,rep(1,6),sum)==degree(g,cmode="indegree"))
all(gapply(g,c(1,2),rep(1,6),sum)==degree(symmetrize(g),cmode="freeman")/2)

#Find first and second order neighborhood means on some variable
gapply(g,c(1,2),1:6,mean)
gapply(g,c(1,2),1:6,mean,distance=2)</pre>
```

66 gelust.boxstats

gclust.boxstats

Plot Statistics Associated with Graph Clusters

#### **Description**

gclust.boxstats creates side-by-side boxplots of graph statistics based on a hierarchical clustering of networks (cut into k sets).

## Usage

```
gclust.boxstats(h, k, meas, ...)
```

## Arguments

h an hclust object, presumably formed by clustering a set of structural distances.

k the number of groups to evaluate.

meas a vector of length equal to the number of graphs in h, containing a GLI to be

evaluated.

... additional parameters to boxplot.

#### **Details**

gclust.boxstats simply takes the hclust object in h, applies cutree to form k groups, and then uses boxplot on the distribution of meas by group. This can be quite handy for assessing graph clusters.

### Value

None

#### Note

Actually, this function will work with any hclust object and measure matrix; the data need not originate with social networks. For this reason, the clever may also employ this function in conjunction with sedist or equiv.clust to plot NLIs against clusters of positions within a graph.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS working paper, Carnegie Mellon University.

#### See Also

```
gclust.centralgraph, gdist.plotdiff, gdist.plotstats
```

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### **Examples**

```
#Create some random graphs
g<-rgraph(10,20,tprob=c(rbeta(10,15,2),rbeta(10,2,15)))

#Find the Hamming distances between them
g.h<-hdist(g)

#Cluster the graphs via their Hamming distances
g.c<-hclust(as.dist(g.h))

#Now display boxplots of density by cluster for a two cluster solution
gclust.boxstats(g.c,2,gden(g))</pre>
```

gclust.centralgraph

Get Central Graphs Associated with Graph Clusters

### **Description**

Calculates central graphs associated with particular graph clusters (as indicated by the k partition of h).

### Usage

```
gclust.centralgraph(h, k, dat, ...)
```

### **Arguments**

h an hclust object, based on a graph stack in dat.
 k the number of groups to evaluate.
 dat one or more graphs (on which the clustering was performed).
 additional arguments to centralgraph.

### Details

gclust.centralgraph uses cutree to cut the hierarchical clustering in h into k groups. centralgraph is then called on each cluster, and the results are returned as a graph stack. This is a useful tool for interpreting clusters of (labeled) graphs, with the resulting central graphs being subsequently analyzed using standard SNA methods.

#### Value

An array containing the stack of central graph adjacency matrices

### Author(s)

Carter T. Butts <buttsc@uci.edu>

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### References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS working paper, Carnegie Mellon University.

### See Also

```
hclust, centralgraph, gclust.boxstats, gdist.plotdiff, gdist.plotstats
```

## **Examples**

```
#Create some random graphs
g<-rgraph(10,20,tprob=c(rbeta(10,15,2),rbeta(10,2,15)))
#Find the Hamming distances between them
g.h<-hdist(g)

#Cluster the graphs via their Hamming distances
g.c<-hclust(as.dist(g.h))

#Now find central graphs by cluster for a two cluster solution
g.cg<-gclust.centralgraph(g.c,2,g)

#Plot the central graphs
gplot(g.cg[1,,])
gplot(g.cg[2,,])</pre>
```

gcor

Find the (Product-Moment) Correlation Between Two or More Labeled Graphs

## **Description**

gcor finds the product-moment correlation between the adjacency matrices of graphs indicated by g1 and g2 in stack dat (or possibly dat2). Missing values are permitted.

# Usage

```
gcor(dat, dat2=NULL, g1=NULL, g2=NULL, diag=FALSE, mode="digraph")
```

## Arguments

dat	one or more input graphs.
dat2	optionally, a second stack of graphs.
g1	the indices of dat reflecting the first set of graphs to be compared; by default, all members of dat are included.
g2	the indices or dat (or dat2, if applicable) reflecting the second set of graphs to be compared; by default, all members of dat are included.

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diag boolean indicating whether or not the diagonal should be treated as valid data.

Set this true if and only if the data can contain loops. diag is FALSE by default.

string indicating the type of graph being evaluated. "Digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.

### **Details**

mode

The (product moment) graph correlation between labeled graphs G and H is given by

$$cor(G, H) = \frac{cov(G, H)}{\sqrt{cov(G, G)cov(H, H)}}$$

where the graph covariance is defined as

$$cov(G, H) = \frac{1}{\binom{|V|}{2}} \sum_{\{i,j\}} \left( A_{ij}^G - \mu_G \right) \left( A_{ij}^H - \mu_H \right)$$

(with  $A^G$  being the adjacency matrix of G). The graph correlation/covariance is at the center of a number of graph comparison methods, including network variants of regression analysis, PCA, CCA, and the like.

Note that gcor computes only the correlation between *uniquely labeled* graphs. For the more general case, gscor is recommended.

#### Value

A graph correlation matrix

### Note

The gcor routine is really just a front-end to the standard cor method; the primary value-added is the transparent vectorization of the input graphs (with intelligent handling of simple versus directed graphs, diagonals, etc.). As noted, the correlation coefficient returned is a standard Pearson's product-moment coefficient, and output should be interpreted accordingly. Classical null hypothesis testing procedures are not recommended for use with graph correlations; for nonparametric null hypothesis testing regarding graph correlations, see cugtest and qaptest. For multivariate correlations among graph sets, try netcancor.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS Working Paper, Carnegie Mellon University.

Krackhardt, D. (1987). "QAP Partialling as a Test of Spuriousness." Social Networks, 9, 171-86

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### See Also

```
gscor, gcov, gscov
```

### **Examples**

```
#Generate two random graphs each of low, medium, and high density g < -rgraph(10,6,tprob = c(0.2,0.2,0.5,0.5,0.8,0.8))
#Examine the correlation matrix gcor(g)
```

gcov

Find the Covariance(s) Between Two or More Labeled Graphs

### **Description**

gcov finds the covariances between the adjacency matrices of graphs indicated by g1 and g2 in stack dat (or possibly dat2). Missing values are permitted.

### Usage

```
gcov(dat, dat2=NULL, g1=NULL, g2=NULL, diag=FALSE, mode="digraph")
```

## Arguments

dat	one or more input graphs.
dat2	optionally, a second graph stack.
g1	the indices of dat reflecting the first set of graphs to be compared; by default, all members of dat are included.
g2	the indices or dat (or dat2, if applicable) reflecting the second set of graphs to be compared; by default, all members of dat are included.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.

## **Details**

The graph covariance between two labeled graphs is defined as

$$cov(G, H) = \frac{1}{\binom{|V|}{2}} \sum_{\{i, j\}} \left( A_{ij}^G - \mu_G \right) \left( A_{ij}^H - \mu_H \right)$$

(with  $A^G$  being the adjacency matrix of G). The graph correlation/covariance is at the center of a number of graph comparison methods, including network variants of regression analysis, PCA, CCA, and the like.

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Note that gcov computes only the covariance between *uniquely labeled* graphs. For the more general case, gscov is recommended.

#### Value

A graph covariance matrix

#### Note

The gcov routine is really just a front-end to the standard cov method; the primary value-added is the transparent vectorization of the input graphs (with intelligent handling of simple versus directed graphs, diagonals, etc.). Classical null hypothesis testing procedures are not recommended for use with graph covariance; for nonparametric null hypothesis testing regarding graph covariance, see cugtest and qaptest.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS Working Paper, Carnegie Mellon University.

#### See Also

```
gscov, gcor, gscor
```

### **Examples**

```
#Generate two random graphs each of low, medium, and high density g<-rgraph(10,6,tprob=c(0.2,0.2,0.5,0.5,0.8,0.8))

#Examine the covariance matrix gcov(g)
```

gden

Find the Density of a Graph

### **Description**

gden computes the density of the graphs indicated by g in collection dat, adjusting for the type of graph in question.

#### Usage

```
gden(dat, g=NULL, diag=FALSE, mode="digraph", ignore.eval=FALSE)
```

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### **Arguments**

dat	one or more input graphs.
g	integer indicating the index of the graphs for which the density is to be calculated (or a vector thereof). If g==NULL (the default), density is calculated for all graphs in dat.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.
ignore.eval	logical; should edge values be ignored when calculating density?

### **Details**

The density of a graph is here taken to be the sum of tie values divided by the number of possible ties (i.e., an unbiased estimator of the graph mean); hence, the result is interpretable for valued graphs as the mean tie value when ignore.eval==FALSE. The number of possible ties is determined by the graph type (and by diag) in the usual fashion.

Where missing data is present, it is removed prior to calculation. The density/graph mean is thus taken relative to the observed portion of the graph.

#### Value

The graph density

## Author(s)

Carter T. Butts <buttsc@uci.edu>

### References

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

```
#Draw three random graphs
dat<-rgraph(10,3)
#Find their densities
gden(dat)</pre>
```

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gdist.plotdiff	Plot Differences in Graph-level Statistics Against Inter-graph Dis-
	tances

# **Description**

For a given graph set, gdist.plotdiff plots the distances between graphs against their distances (or differences) on a set of graph-level measures.

# Usage

# **Arguments**

d	A matrix containing the inter-graph distances
meas	An n x m matrix containing the graph-level indices; rows of this matrix must correspond to graphs, and columns to indices
method	The distance method used by dist to establish differences/distances between graph GLI values. By default, absolute ("manhattan") differences are used.
jitter	Should values be jittered prior to display?
xlab	A label for the X axis
ylab	A label for the Y axis
lm.line	Include a least-squares line?
	Additional arguments to plot

## **Details**

gdist.plotdiff works by taking the distances between all graphs on meas and then plotting these distances against d for all pairs of graphs (with, optionally, an added least-squares line for reference value). This can be a useful exploratory tool for relating inter-graph distances (e.g., Hamming distances) to differences on other attributes.

# Value

None

# Note

This function is actually quite generic, and can be used with node-level – or even non-network – data as well.

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## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS working paper, Carnegie Mellon University.

#### See Also

```
gdist.plotstats, gclust.boxstats, gclust.centralgraph
```

# **Examples**

```
#Generate some random graphs with varying densities
g<-rgraph(10,20,tprob=runif(20,0,1))

#Find the Hamming distances between graphs
g.h<-hdist(g)

#Plot the relationship between distance and differences in density
gdist.plotdiff(g.h,gden(g),lm.line=TRUE)</pre>
```

gdist.plotstats

Plot Various Graph Statistics Over a Network MDS

## **Description**

Plots a two-dimensional metric MDS of d, with the corresponding values of meas indicated at each point. Various options are available for controlling how meas is to be displayed.

# Usage

```
gdist.plotstats(d, meas, siz.lim=c(0, 0.15), rescale="quantile",
    display.scale="radius", display.type="circleray", cex=0.5, pch=1,
    labels=NULL, pos=1, labels.cex=1, legend=NULL, legend.xy=NULL,
    legend.cex=1, ...)
```

# **Arguments**

d	A matrix containing the inter-graph distances
meas	An nxm matrix containing the graph-level measures; each row must correspond to a graph, and each column must correspond to an index
siz.lim	The minimum and maximum sizes (respectively) of the plotted symbols, given as fractions of the total plotting range
rescale	One of "quantile" for ordinal scaling, "affine" for max-min scaling, and "normalize" for rescaling by maximum value; these determine the scaling rule to be used in sizing the plotting symbols

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display.scale One of "area" or "radius"; this controls the attribute of the plotting symbol which

is rescaled by the value of meas

display.type One of "circle", "ray", "circleray", "poly", or "polyray"; this determines the

type of plotting symbol used (circles, rays, polygons, or come combination of

these)

cex Character expansion coefficient

pch Point types for the base plotting symbol (not the expanded symbols which are

used to indicate meas values)

labels Point labels, if desired

pos Relative position of labels (see par)
labels.cex Character expansion factor for labels

legend Add a legend?

legend.xy x,y coordinates for legend

legend.cex Character expansion factor for legend

... Additional arguments to plot

#### **Details**

gdist.plotstats works by performing an MDS (using cmdscale) on d, and then using the values in meas to determine the shape of the points at each MDS coordinate. Typically, these shapes involve rays of varying color and length indicating meas magnitude, with circles and polygons of the appropriate radius and/or error being options as well. Various options are available (described above) to govern the details of the data display; some tinkering may be needed in order to produce an aesthetically pleasing visualization.

The primary use of gdist.plotstats is to explore broad relationships between graph properties and inter-graph distances. This routine complements others in the gdist and gclust family of interstructural visualization tools.

# Value

None

# Note

This routine does not actually depend on the data's being graphic in origin, and can be used with any distance matrix/measure matrix combination.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS working paper, Carnegie Mellon University.

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## See Also

```
gdist.plotdiff, gclust.boxstats, gclust.centralgraph
```

## **Examples**

```
#Generate random graphs with varying density
g<-rgraph(10,20,tprob=runif(20,0,1))

#Get Hamming distances between graphs
g.h<-hdist(g)

#Plot the association of distance, density, and reciprocity
gdist.plotstats(g.h,cbind(gden(g),grecip(g)))</pre>
```

geodist

Fund the Numbers and Lengths of Geodesics Among Nodes in a Graph

# **Description**

geodist uses a BFS to find the number and lengths of geodesics between all nodes of dat. Where geodesics do not exist, the value in inf.replace is substituted for the distance in question.

## Usage

```
geodist(dat, inf.replace=Inf, count.paths=TRUE, predecessors=FALSE,
    ignore.eval=TRUE, na.omit=TRUE)
```

### **Arguments**

dat	one or more input graphs.
inf.replace	the value to use for geodesic distances between disconnected nodes; by default, this is equal Inf.
count.paths	logical; should a count of geodesics be included in the returned object?
predecessors	logical; should a predecessor list be included in the returned object?
ignore.eval	logical; should edge values be ignored when computing geodesics?
na.omit	logical; should NA-valued edges be removed?

#### **Details**

This routine is used by a variety of other functions; many of these will allow the user to provide manually precomputed geodist output so as to prevent expensive recomputation. Note that the choice of infinite path length for disconnected vertex pairs is non-canonical (albeit common), and some may prefer to simply treat these as missing values. geodist (without loss of generality) treats all paths as directed, a fact which should be kept in mind when interpreting geodist output.

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By default, geodist ignores edge values (except for NAed edges, which are dropped when na.omit==TRUE). Setting ignore.eval=FALSE will change this behavior, with edge values being interpreted as distances; where edge values reflect proximity or tie strength, transformation may be necessary. Edge values should also be non-negative. Because the valued-case algorithm is significantly slower than the unvalued-case algorithm, ignore.eval should be set to TRUE wherever possible.

## Value

#### A list containing:

counts If count.paths==TRUE, a matrix containing the number of geodesics between

each pair of vertices

gdist A matrix containing the geodesic distances between each pair of vertices

predecessors If predecessors, a list whose ith element is a list of vectors, the jth of which

contains the intervening vertices on all shortest paths from i to j

# Author(s)

Carter T. Butts <buttsc@uci.edu>

### References

Brandes, U. (2000). "Faster Evaluation of Shortest-Path Based Centrality Indices." *Konstanzer Schriften in Mathematik und Informatik*, 120.

West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, N.J.: Prentice Hall.

# See Also

```
component.dist, components
```

## **Examples**

```
#Find geodesics on a random graph
gd<-geodist(rgraph(15))</pre>
```

#Examine the number of geodesics
gd\$counts

#Examine the geodesic distances
gd\$gdist

78 gilschmidt

gilschmidt	Compute the Gil-Schmidt Power Index
0	· · · · · · · · · · · · · · · · · · ·

# **Description**

gilschmidt computes the Gil-Schmidt Power Index for all nodes in dat, with or without normalization.

#### **Usage**

```
gilschmidt(dat, g = 1, nodes = NULL, gmode = "digraph", diag = FALSE,
    tmaxdev = FALSE, normalize = TRUE)
```

## **Arguments**

dat	one or more input graphs (for best performance, sna edgelists or network objects are suggested).
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g=1.
nodes	list indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. gmode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. (This has no effect on this index, but is included for compatibility with centralization.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
normalize	logical; should the index scores be normalized?

## **Details**

For graph G=(V,E), let R(v,G) be the set of vertices reachable by v in  $V\setminus v$ . Then the Gil-Schmidt power index is defined as

$$C_{GS}(v) = \frac{\sum_{i \in R(v,G)} \frac{1}{d(v,i)}}{|R(v,G)|}.$$

where d(v,i) is the geodesic distance from v to i in G; the index is taken to be 0 for isolates. The measure takes a value of 1 when v is adjacent to all reachable vertices, and approaches 0 as the distance from v to each vertex approaches infinity. (For finite N=|V|, the minimum value is 0 if v is an isolate, and otherwise 1/(N-1).)

If normalize=FALSE is selected, then normalization by |R(v,G)| is not performed. This measure has been proposed as a better-behaved alternative to closeness (to which it is closely related).

The closeness function in the sna library can also be used to compute this index.

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## Value

A vector of centrality scores.

#### Author(s)

```
Carter T. Butts, <buttsc@uci.edu>
```

### References

Gil, J. and Schmidt, S. (1996). "The Origin of the Mexican Network of Power". Proceedings of the International Social Network Conference, Charleston, SC, 22-25.

Sinclair, P.A. (2009). "Network Centralization with the Gil Schmidt Power Centrality Index" *Social Networks*, 29, 81-92.

## See Also

```
closeness, centralization
```

# **Examples**

```
data(coleman) #Load Coleman friendship network
gs<-gilschmidt(coleman,g=1:2) #Compute the Gil-Schmidt index
#Plot G-S values in the fall, versus spring
plot(gs,xlab="Fall",ylab="Spring",main="G-S Index")
abline(0,1)</pre>
```

gliop

Return a Binary Operation on GLI Values Computed on Two Graphs

# Description

gliop is a wrapper which allows for an arbitrary binary operation on GLIs to be treated as a single call. This is particularly useful for test routines such as cugtest and qaptest.

## Usage

```
gliop(dat, GFUN, OP="-", g1=1, g2=2, ...)
```

# Arguments

dat	a collection of graphs.
GFUN	a function taking single graphs as input.
OP	the operator to use on the output of GFUN.
g1	the index of the first input graph.
g2	the index of the second input graph.
	Additional arguments to GFUN

## **Details**

gliop operates by evaluating GFUN on the graphs indexed by g1 and g2 and returning the result of OP as applied to the GFUN output.

# Value

```
OP(GFUN(dat[g1, , ],...),GFUN(dat[g2, , ],...))
```

#### Note

If the output of GFUN is not sufficiently well-behaved, undefined behavior may occur. Common sense is advised.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

# References

Anderson, B.S.; Butts, C.T.; and Carley, K.M. (1999). "The Interaction of Size and Density with Graph-Level Indices." *Social Networks*, 21(3), 239-267.

# See Also

```
cugtest, qaptest
```

# Examples

```
#Draw two random graphs
g<-rgraph(10,2,tprob=c(0.2,0.5))

#What is their difference in density?
gliop(g,gden,"-",1,2)</pre>
```

gplot

Two-Dimensional Visualization of Graphs

# Description

gplot produces a two-dimensional plot of graph g in collection dat. A variety of options are available to control vertex placement, display details, color, etc.

## Usage

```
gplot(dat, g = 1, gmode = "digraph", diag = FALSE,
    label = NULL, coord = NULL, jitter = TRUE, thresh = 0,
    thresh.absval=TRUE, usearrows = TRUE, mode = "fruchtermanreingold",
    displayisolates = TRUE, interactive = FALSE, interact.bycomp = FALSE,
    xlab = NULL, ylab = NULL, xlim = NULL, ylim = NULL, pad = 0.2,
    label.pad = 0.5, displaylabels = !is.null(label), boxed.labels = FALSE,
    label.pos = 0, label.bg = "white", vertex.enclose = FALSE,
    vertex.sides = NULL, vertex.rot = 0, arrowhead.cex = 1, label.cex = 1,
    loop.cex = 1, vertex.cex = 1, edge.col = 1, label.col = 1,
    vertex.col = NULL, label.border = 1, vertex.border = 1, edge.lty = NULL,
    edge.lty.neg=2, label.lty = NULL, vertex.lty = 1, edge.lwd = 0,
    label.lwd = par("lwd"), edge.len = 0.5, edge.curve = 0.1,
    edge.steps = 50, loop.steps = 20, object.scale = 0.01, uselen = FALSE,
    usecurve = FALSE, suppress.axes = TRUE, vertices.last = TRUE,
    new = TRUE, layout.par = NULL, ...)
```

# Arguments

dat	a graph or set thereof. This data may be valued.
g	integer indicating the index of the graph which is to be plotted. By default, g==1.
gmode	String indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected; "twomode" indicates that data should be interpreted as two-mode (i.e., rows and columns are distinct vertex sets). gmode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
label	a vector of vertex labels, if desired; defaults to the vertex index number.
coord	user-specified vertex coordinates, in an NCOL( $dat$ )x2 matrix. Where this is specified, it will override the mode setting.
jitter	boolean; should the output be jittered?
thresh	real number indicating the lower threshold for tie values. Only ties of value >thresh (by default in absolute value - see thresh.absval)are displayed. By default, thresh=0.
thresh.absval	boolean; should the absolute value of edge weights be used when thresholding? (Defaults to TRUE; setting to FALSE leads to thresholding by signed weights.)
usearrows	boolean; should arrows (rather than line segments) be used to indicate edges?
mode	the vertex placement algorithm; this must correspond to a <code>gplot.layout</code> function.
displayisolates	S
	boolean; should isolates be displayed?
interactive	boolean; should interactive adjustment of vertex placement be attempted?
interact.bycom	p

boolean; if interactive==TRUE, should all vertices in the component be moved?

xlab	x axis label.
ylab	y axis label.
xlim	the x limits (min, max) of the plot.
ylim	the y limits of the plot.
pad	amount to pad the plotting range; useful if labels are being clipped.
label.pad	amount to pad label boxes (if boxed.labels==TRUE), in character size units.
displaylabels	boolean; should vertex labels be displayed?
boxed.labels	boolean; place vertex labels within boxes?
label.pos	position at which labels should be placed, relative to vertices. @ results in labels which are placed away from the center of the plotting region; 1, 2, 3, and 4 result in labels being placed below, to the left of, above, and to the right of vertices (respectively); and label.pos>=5 results in labels which are plotted with no offset (i.e., at the vertex positions).
label.bg	background color for label boxes (if boxed.labels==TRUE); may be a vector, if boxes are to be of different colors.
vertex.enclose	boolean; should vertices be enclosed within circles? (Can increase legibility for polygonal vertices.)
vertex.sides	number of polygon sides for vertices; may be given as a vector, if vertices are to be of different types. By default, 50 sides are used (or 50 and 4, for two-mode data).
vertex.rot	angle of rotation for vertices (in degrees); may be given as a vector, if vertices are to be rotated differently.
arrowhead.cex	expansion factor for edge arrowheads.
label.cex	character expansion factor for label text.
loop.cex	expansion factor for loops; may be given as a vector, if loops are to be of different sizes.
vertex.cex	expansion factor for vertices; may be given as a vector, if vertices are to be of different sizes.
edge.col	color for edges; may be given as a vector or adjacency matrix, if edges are to be of different colors.
label.col	color for vertex labels; may be given as a vector, if labels are to be of different colors.
vertex.col	color for vertices; may be given as a vector, if vertices are to be of different colors. By default, red is used (or red and blue, for two-mode data).
label.border	label border colors (if boxed.labels==TRUE); may be given as a vector, if label boxes are to have different colors.
vertex.border	border color for vertices; may be given as a vector, if vertex borders are to be of different colors.
edge.lty	line type for (positive weight) edges; may be given as a vector or adjacency matrix, if edges are to have different line types.
edge.lty.neg	line type for negative weight edges, if any; may be given as per edge.lty.

label.lty	line type for label boxes (if boxed.labels==TRUE); may be given as a vector, if label boxes are to have different line types.
vertex.lty	line type for vertex borders; may be given as a vector or adjacency matrix, if vertex borders are to have different line types.
edge.lwd	line width scale for edges; if set greater than 0, edge widths are scaled by edge.lwd*dat. May be given as a vector or adjacency matrix, if edges are to have different line widths.
label.lwd	line width for label boxes (if boxed.labels==TRUE); may be given as a vector, if label boxes are to have different line widths.
edge.len	if uselen==TRUE, curved edge lengths are scaled by edge.len.
edge.curve	if usecurve==TRUE, the extent of edge curvature is controlled by edge.curv. May be given as a fixed value, vector, or adjacency matrix, if edges are to have different levels of curvature.
edge.steps	for curved edges (excluding loops), the number of line segments to use for the curve approximation.
loop.steps	for loops, the number of line segments to use for the curve approximation.
object.scale	base length for plotting objects, as a fraction of the linear scale of the plotting region. Defaults to 0.01.
uselen	boolean; should we use edge.len to rescale edge lengths?
usecurve	boolean; should we use edge.curve?
suppress.axes	boolean; suppress plotting of axes?
vertices.last	boolean; plot vertices after plotting edges?
new	boolean; create a new plot? If new==FALSE, vertices and edges will be added to the existing plot.
layout.par	parameters to the gplot.layout function specified in mode.
	additional arguments to plot.

## **Details**

gplot is the standard network visualization tool within the sna library. By means of clever selection of display parameters, a fair amount of display flexibility can be obtained. Graph layout – if not specified directly using coord – is determined via one of the various available algorithms. These should be specified via the mode argument; see gplot.layout for a full list. User-supplied layout functions are also possible – see the aforementioned man page for details.

Note that where <code>gmode=="twomode"</code>, the supplied two-mode network is converted to bipartite form prior to computing coordinates (if not in that form already). <code>vertex.col</code> or other settings may be used to differentiate row and column vertices — by default, row vertices are drawn as red circles, and column vertices are rendered as blue squares. If <code>interactive==TRUE</code>, then the user may modify the initial graph layout by selecting an individual vertex and then clicking on the location to which this vertex is to be moved; this process may be repeated until the layout is satisfactory. If <code>interact.bycomp==TRUE</code> as well, the vertex and all other vertices in the same component as that vertex are moved together.

gplot.arrow

## Value

A two-column matrix containing the vertex positions as x,y coordinates.

### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
Alex Montgomery <ahm@reed.edu>
```

#### References

Wasserman, S. and Faust, K. (1994) *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

# See Also

```
plot, gplot.layout
```

## **Examples**

```
gplot(rgraph(5))  #Plot a random graph
gplot(rgraph(5),usecurv=TRUE) #This time, use curved edges
gplot(rgraph(5),mode="mds") #Try an alternative layout scheme

#A colorful demonstration...
gplot(rgraph(5,diag=TRUE),diag=TRUE,vertex.cex=1:5,vertex.sides=3:8,
    vertex.col=1:5,vertex.border=2:6,vertex.rot=(0:4)*72,
    displaylabels=TRUE,label.bg="gray90")
```

gplot.arrow

Add Arrows or Segments to a Plot

# Description

gplot.arrow draws a segment or arrow between two pairs of points; unlike arrows or segments, the new plot element is drawn as a polygon.

#### Usage

```
gplot.arrow(x0, y0, x1, y1, length = 0.1, angle = 20, width = 0.01,
    col = 1, border = 1, lty = 1, offset.head = 0, offset.tail = 0,
    arrowhead = TRUE, curve = 0, edge.steps = 50, ...)
```

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### Arguments

x0	A vector of x coordinates for points of origin
y0	A vector of y coordinates for points of origin
x1	A vector of x coordinates for destination points
y1	A vector of y coordinates for destination points
length	Arrowhead length, in current plotting units

angle Arrowhead angle (in degrees)

width Width for arrow body, in current plotting units (can be a vector)

col Arrow body color (can be a vector)
border Arrow border color (can be a vector)
lty Arrow border line type (can be a vector)
offset.head Offset for destination point (can be a vector)
offset.tail Offset for origin point (can be a vector)

arrowhead Boolean; should arrowheads be used? (Can be a vector))

curve Degree of edge curvature (if any), in current plotting units (can be a vector)
edge.steps For curved edges, the number of steps to use in approximating the curve (can be

a vector)

.. Additional arguments to polygon

## **Details**

gplot.arrow provides a useful extension of segments and arrows when fine control is needed over the resulting display. (The results also look better.) Note that edge curvature is quadratic, with curve providing the maximum horizontal deviation of the edge (left-handed). Head/tail offsets are used to adjust the end/start points of an edge, relative to the baseline coordinates; these are useful for functions like gplot, which need to draw edges incident to vertices of varying radii.

# Value

None.

# Author(s)

Carter T. Butts <buttsc@uci.edu>

# See Also

```
gplot, gplot.loop, polygon
```

# Examples

```
#Plot two points
plot(1:2,1:2)

#Add an edge
gplot.arrow(1,1,2,2,width=0.01,col="red",border="black")
```

gplot.layout

Vertex Layout Functions for gplot

# Description

Various functions which generate vertex layouts for the gplot visualization routine.

## Usage

```
gplot.layout.adj(d, layout.par)
gplot.layout.circle(d, layout.par)
gplot.layout.circrand(d, layout.par)
gplot.layout.eigen(d, layout.par)
gplot.layout.fruchtermanreingold(d, layout.par)
gplot.layout.geodist(d, layout.par)
gplot.layout.hall(d, layout.par)
gplot.layout.kamadakawai(d, layout.par)
gplot.layout.mds(d, layout.par)
gplot.layout.princoord(d, layout.par)
gplot.layout.random(d, layout.par)
gplot.layout.rmds(d, layout.par)
gplot.layout.segeo(d, layout.par)
gplot.layout.seham(d, layout.par)
gplot.layout.spring(d, layout.par)
gplot.layout.springrepulse(d, layout.par)
gplot.layout.target(d, layout.par)
```

#### **Arguments**

```
d an adjacency matrix, as passed by gplot. layout.par a list of parameters.
```

### Details

Vertex layouts for network visualization pose a difficult problem – there is no single, "good" layout algorithm, and many different approaches may be valuable under different circumstances. With this in mind, gplot allows for the use of arbitrary vertex layout algorithms via the gplot.layout.\* family of routines. When called, gplot searches for a gplot.layout function whose third name matches its mode argument (see gplot help for more information); this function is then used to generate the layout for the resulting plot. In addition to the routines documented here, users may add their own layout functions as needed. The requirements for a gplot.layout function are as follows:

- 1. the first argument, d, must be the (dichotomous) graph adjacency matrix;
- 2. the second argument, layout.par, must be a list of parameters (or NULL, if no parameters are specified); and

3. the return value must be a real matrix of dimension c(2,NROW(d)), whose rows contain the vertex coordinates.

Other than this, anything goes. (In particular, note that layout.par could be used to pass additional matrices, if needed.)

The graph.layout functions currently supplied by default are as follows:

**circle** This function places vertices uniformly in a circle; it takes no arguments.

**eigen** This function places vertices based on the eigenstructure of the adjacency matrix. It takes the following arguments:

layout.par\\$var This argument controls the matrix to be used for the eigenanalysis. "symupper", "symlower", "symstrong", "symweak" invoke symmetrize on d with the respective symmetrizing rule. "user" indicates a user-supplied matrix (see below), while "raw" indicates that d should be used as-is. (Defaults to "raw".)

layout.par\\$evsel If "first", the first two eigenvectors are used; if "size", the two eigenvectors whose eigenvalues have the largest magnitude are used instead. Note that only the real portion of the associated eigenvectors is used. (Defaults to "first".)

**layout.par\\$mat** If layout.par\$var=="user", this matrix is used for the eigenanalysis. (No default.)

**fruchtermanreingold** This function generates a layout using a variant of Fruchterman and Reingold's force-directed placement algorithm. It takes the following arguments:

**layout.par\\$niter** This argument controls the number of iterations to be employed. Larger values take longer, but will provide a more refined layout. (Defaults to 500.)

layout.par\\$max.delta Sets the maximum change in position for any given iteration. (Defaults to n.)

layout.par\\$area Sets the "area" parameter for the F-R algorithm. (Defaults to n^2.)

**layout.par\\$cool.exp** Sets the cooling exponent for the annealer. (Defaults to 3.)

**layout.par\\$repulse.rad** Determines the radius at which vertex-vertex repulsion cancels out attraction of adjacent vertices. (Defaults to area\*log(n).)

layout.par\\$ncell To speed calculations on large graphs, the plot region is divided at each iteration into ncell by ncell "cells", which are used to define neighborhoods for force calculation. Moderate numbers of cells result in fastest performance; too few cells (down to 1, which produces "pure" F-R results) can yield odd layouts, while too many will result in long layout times. (Defaults to n^0.4.)

**layout.par\\$cell.jitter** Jitter factor (in units of cell width) used in assigning vertices to cells. Small values may generate "grid-like" anomalies for graphs with many isolates. (Defaults to 0.5.)

layout.par\\$cell.pointpointrad Squared "radius" (in units of cells) such that exact point interaction calculations are used for all vertices belonging to any two cells less than or equal to this distance apart. Higher values approximate the true F-R solution, but increase computational cost. (Defaults to 0.)

layout.par\\$cell.pointcellrad Squared "radius" (in units of cells) such that approximate point/cell interaction calculations are used for all vertices belonging to any two cells less than or equal to this distance apart (and not within the point/point radius). Higher values provide somewhat better approximations to the true F-R solution at slightly increased computational cost. (Defaults to 18.)

layout.par\\$cell.cellcellrad Squared "radius" (in units of cells) such that approximate cell/cell interaction calculations are used for all vertices belonging to any two cells less than or equal to this distance apart (and not within the point/point or point/cell radii). Higher values provide somewhat better approximations to the true F-R solution at slightly increased computational cost. Note that cells beyond this radius (if any) do not interact, save through edge attraction. (Defaults to ncel1^2.)

- **layout.par\\$seed.coord** A two-column matrix of initial vertex coordinates. (Defaults to a random circular layout.)
- hall This function places vertices based on the last two eigenvectors of the Laplacian of the input matrix (Hall's algorithm). It takes no arguments.
- **kamadakawai** This function generates a vertex layout using a version of the Kamada-Kawai forcedirected placement algorithm. It takes the following arguments:
  - **layout.par\\$niter** This argument controls the number of iterations to be employed. (Defaults to 1000.)
  - **layout.par\\$sigma** Sets the base standard deviation of position change proposals. (Defaults to NROW(d)/4.)
  - **layout.par\\$initemp** Sets the initial "temperature" for the annealing algorithm. (Defaults to 10.)
  - **layout.par\\$cool.exp** Sets the cooling exponent for the annealer. (Defaults to 0.99.)
  - layout.par\\$kkconst Sets the Kamada-Kawai vertex attraction constant. (Defaults to NROW(d)^2.)
  - **layout.par\\$elen** Provides the matrix of interpoint distances to be approximated. (Defaults to the geodesic distances of d after symmetrizing, capped at sqrt(NROW(d)).)
  - **layout.par\\$seed.coord** A two-column matrix of initial vertex coordinates. (Defaults to a gaussian layout.)
- **mds** This function places vertices based on a metric multidimensional scaling of a specified distance matrix. It takes the following arguments:
  - layout.par\\$var This argument controls the raw variable matrix to be used for the subsequent distance calculation and scaling. "rowcol", "row", and "col" indicate that the rows and columns (concatenated), rows, or columns (respectively) of d should be used. "rcsum" and "rcdiff" result in the sum or difference of d and its transpose being employed. "invadj" indicates that max{d}-d should be used, while "geodist" uses geodist to generate a matrix of geodesic distances from d. Alternately, an arbitrary matrix can be provided using "user". (Defaults to "rowcol".)
  - layout.par\\$dist The distance function to be calculated on the rows of the variable matrix. This must be one of the method parameters to dist ("euclidean", "maximum", "manhattan", or "canberra"), or else "none". In the latter case, no distance function is calculated, and the matrix in question must be square (with dimension dim(d)) for the routine to work properly. (Defaults to "euclidean".)
  - **layout.par\\$exp** The power to which distances should be raised prior to scaling. (Defaults to 2.)
  - **layout.par\\$vm** If layout.par\$var=="user", this matrix is used for the distance calculation. (No default.)
  - Note: the following layout functions are based on mds:
  - adj scaling of the raw adjacency matrix, treated as similarities (using "invadj").
  - **geodist** scaling of the matrix of geodesic distances.

**rmds** euclidean scaling of the rows of d.

**segeo** scaling of the squared euclidean distances between row-wise geodesic distances (i.e., approximate structural equivalence).

**seham** scaling of the Hamming distance between rows/columns of d (i.e., another approximate structural equivalence scaling).

**princoord** This function places vertices based on the eigenstructure of a given correlation/covariance matrix. It takes the following arguments:

layout.par\\$var The matrix of variables to be used for the correlation/covariance calculation.
 "rowcol", "col", and "row" indicate that the rows/cols, columns, or rows (respectively)
 of d should be employed. "rcsum" "rcdiff" result in the sum or difference of d and
 t(d) being used. "user" allows for an arbitrary variable matrix to be supplied. (Defaults
 to "rowcol".)

**layout.par\\$cor** Should the correlation matrix (rather than the covariance matrix) be used? (Defaults to TRUE.)

**layout.par\\$vm** If layout.par\$var=="user", this matrix is used for the correlation/covariance calculation. (No default.)

**random** This function places vertices randomly. It takes the following argument:

layout.par\\$dist The distribution to be used for vertex placement. Currently, the options are "unif" (for uniform distribution on the square), "uniang" (for a "gaussian donut" configuration), and "normal" (for a straight Gaussian distribution). (Defaults to "unif".)

Note: circrand, which is a frontend to the "uniang" option, is based on this function.

spring This function places vertices using a spring embedder. It takes the following arguments:

**layout.par\\$mass** The vertex mass (in "quasi-kilograms"). (Defaults to 0.1.)

layout.par\\$equil The equilibrium spring extension (in "quasi-meters"). (Defaults to 1.)

**layout.par\\$k** The spring coefficient (in "quasi-Newtons per quasi-meter"). (Defaults to 0.001.)

**layout.par\\$repeqdis** The point at which repulsion (if employed) balances out the spring extension force (in "quasi-meters"). (Defaults to 0.1.)

**layout.par\\$kfr** The base coefficient of kinetic friction (in "quasi-Newton quasi-kilograms"). (Defaults to 0.01.)

**layout.par\\$repulse** Should repulsion be used? (Defaults to FALSE.)

Note: springrepulse is a frontend to spring, with repulsion turned on.

**target** This function produces a "target diagram" or "bullseye" layout, using a Brandes et al.'s force-directed placement algorithm. (See also gplot.target.) It takes the following arguments:

**layout.par\\$niter** This argument controls the number of iterations to be employed. (Defaults to 1000.)

**layout.par\\$radii** This argument should be a vector of length NROW(d) containing vertex radii. Ideally, these should lie in the [0,1] interval (and odd behavior may otherwise result). (Defaults to the affine-transformed Freeman degree centrality scores of d.)

**layout.par\\$minlen** Sets the minimum edge length, below which edge lengths are to be adjusted upwards. (Defaults to 0.05.)

**layout.par\\$area** Sets the initial "temperature" for the annealing algorithm. (Defaults to 10.) **layout.par\\$cool.exp** Sets the cooling exponent for the annealer. (Defaults to 0.99.)

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**layout.par\\$maxdelta** Sets the maximum angular distance for vertex moves. (Defaults to pi.)

**layout.par\\$periph.outside** Boolean; should "peripheral" vertices (in the Brandes et al. sense) be placed together outside the main target area? (Defaults to FALSE.)

**layout.par\\$periph.outside.offset** Radius at which to place "peripheral" vertices if periph.outside==TRUE. (Defaults to 1.2.)

**layout.par\\$disconst** Multiplier for the Kamada-Kawai-style distance potential. (Defaults to 1.)

layout.par\\$crossconst Multiplier for the edge crossing potential. (Defaults to 1.)

layout.par\\$repconst Multiplier for the vertex-edge repulsion potential. (Defaults to 1.)

**layout.par\\$minpdis** Sets the "minimum distance" parameter for vertex repulsion. (Defaults to 0.05.)

#### Value

A matrix whose rows contain the x,y coordinates of the vertices of d.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Brandes, U.; Kenis, P.; and Wagner, D. (2003). "Communicating Centrality in Policy Network Drawings." *IEEE Transactions on Visualization and Computer Graphics*, 9(2):241-253.

Fruchterman, T.M.J. and Reingold, E.M. (1991). "Graph Drawing by Force-directed Placement." *Software - Practice and Experience*, 21(11):1129-1164.

Kamada, T. and Kawai, S. (1989). "An Algorithm for Drawing General Undirected Graphs." *Information Processing Letters*, 31(1):7-15.

# See Also

```
gplot, gplot.target, gplot3d.layout, cmdscale, eigen
```

gplot.loop

Add Loops to a Plot

# **Description**

gplot.loop draws a "loop" at a specified location; this is used to designate self-ties in gplot.

# Usage

```
gplot.loop(x0, y0, length = 0.1, angle = 10, width = 0.01, col = 1,
border = 1, lty = 1, offset = 0, edge.steps = 10, radius = 1,
arrowhead = TRUE, xctr=0, yctr=0, ...)
```

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# Arguments

a vector of x coordinates for points of origin.
 a vector of y coordinates for points of origin.
 length arrowhead length, in current plotting units.

angle arrowhead angle (in degrees).

width width for loop body, in current plotting units (can be a vector).

col loop body color (can be a vector).
border loop border color (can be a vector).
lty loop border line type (can be a vector).
offset offset for origin point (can be a vector).

edge.steps number of steps to use in approximating curves.

radius loop radius (can be a vector).

arrowhead boolean; should arrowheads be used? (Can be a vector.)

xctr x coordinate for the central location away from which loops should be oriented.
yctr y coordinate for the central location away from which loops should be oriented.

... additional arguments to polygon.

#### **Details**

gplot.loop is the companion to gplot.arrow; like the latter, plot elements produced by gplot.loop are drawn using polygon, and as such are scaled based on the current plotting device. By default, loops are drawn so as to encompass a circular region of radius radius, whose center is offset units from x0,y0 and at maximum distance from xctr,yctr. This is useful for functions like gplot, which need to draw loops incident to vertices of varying radii.

# Value

None.

# Author(s)

Carter T. Butts <buttsc@uci.edu>

### See Also

```
gplot.arrow, gplot, polygon
```

## **Examples**

```
#Plot a few polygons with loops
plot(0,0,type="n",xlim=c(-2,2),ylim=c(-2,2),asp=1)
gplot.loop(c(0,0),c(1,-1),col=c(3,2),width=0.05,length=0.4,
    offset=sqrt(2)/4,angle=20,radius=0.5,edge.steps=50,arrowhead=TRUE)
polygon(c(0.25,-0.25,-0.25,0.25,NA,0.25,-0.25,-0.25,0.25),
    c(1.25,1.25,0.75,0.75,NA,-1.25,-1.25,-0.75,-0.75),col=c(2,3))
```

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gplot.target Display a Graph in Target Diagram Form

# **Description**

Displays an input graph (and associated vector) as a "target diagram," with vertices restricted to lie at fixed radii from the origin. Such displays are useful ways of representing vertex characteristics and/or local structural properties for graphs of small to medium size.

# Usage

```
gplot.target(dat, x, circ.rad = (1:10)/10, circ.col = "blue",
    circ.lwd = 1, circ.lty = 3, circ.lab = TRUE, circ.lab.cex = 0.75,
    circ.lab.theta = pi, circ.lab.col = 1, circ.lab.digits = 1,
    circ.lab.offset = 0.025, periph.outside = FALSE,
    periph.outside.offset = 1.2, ...)
```

# **Arguments**

```
dat
                  an input graph.
                  a vector of vertex properties to be plotted (must match the dimensions of dat).
circ.rad
                  radii at which to draw reference circles.
circ.col
                  reference circle color.
circ.lwd
                  reference circle line width.
circ.lty
                  reference circle line type.
circ.lab
                  boolean; should circle labels be displayed?
circ.lab.cex
                  expansion factor for circle labels.
circ.lab.theta angle at which to draw circle labels.
                  color for circle labels.
circ.lab.col
circ.lab.digits
                  digits to display for circle labels.
circ.lab.offset
                   offset for circle labels.
periph.outside boolean; should "peripheral" vertices be drawn together beyond the normal ver-
                  tex radius?
periph.outside.offset
                  radius at which "peripheral" vertices should be drawn if periph.outside==TRUE.
                   additional arguments to gplot.
```

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#### **Details**

gplot.target is a front-end to gplot which implements the target diagram layout of Brandes et al. (2003). This layout seeks to optimize various aesthetic criteria, given the constraint that all vertices lie at fixed radii from the origin (set by x). One important feature of this algorithm is that vertices which belong to mutual dyads (described by Brandes et al. as "core" vertices) are treated differently from vertices which do not ("peripheral" vertices). Layout is optimized for core vertices prior to placing peripheral vertices; thus, the result may be misleading if mutuality is not a salient characteristic of the data.

The layout for gplot.target is handled by gplot.layout.target; additional parameters are specied on the associated manual page. Standard arguments may be passed to gplot, as well.

#### Value

A two-column matrix of vertex positions (generated by gplot.layout.target)

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Brandes, U.; Kenis, P.; and Wagner, D. (2003). "Communicating Centrality in Policy Network Drawings." *IEEE Transactions on Visualization and Computer Graphics*, 9(2):241-253.

## See Also

```
gplot.layout.target, gplot
```

# **Examples**

```
#Generate a random graph
g<-rgraph(15)

#Produce a target diagram, centering by betweenness
gplot.target(g,betweenness(g))</pre>
```

gplot.vertex

Add Vertices to a Plot

## **Description**

gplot.vertex adds one or more vertices (drawn using polygon) to a plot.

# Usage

```
gplot.vertex(x, y, radius = 1, sides = 4, border = 1, col = 2,
    lty = NULL, rot = 0, ...)
```

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# **Arguments**

a vector of x coordinates. Х a vector of y coordinates. a vector of vertex radii. radius sides a vector containing the number of sides to draw for each vertex. border a vector of vertex border colors. col a vector of vertex interior colors. a vector of vertex border line types. lty a vector of vertex rotation angles (in degrees). rot Additional arguments to polygon

# **Details**

gplot.vertex draws regular polygons of specified radius and number of sides, at the given coordinates. This is useful for routines such as gplot, which use such shapes to depict vertices.

## Value

None

# Author(s)

Carter T. Butts <buttsc@uci.edu>

# See Also

```
gplot, polygon
```

# **Examples**

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gplot3d Three-Dimensional Visualization of Graphs
---

# **Description**

gplot3d produces a three-dimensional plot of graph g in set dat. A variety of options are available to control vertex placement, display details, color, etc.

## Usage

```
gplot3d(dat, g = 1, gmode = "digraph", diag = FALSE,
    label = NULL, coord = NULL, jitter = TRUE, thresh = 0,
    mode = "fruchtermanreingold", displayisolates = TRUE,
    displaylabels = !missing(label), xlab = NULL, ylab = NULL,
    zlab = NULL, vertex.radius = NULL, absolute.radius = FALSE,
    label.col = "gray50", edge.col = "black", vertex.col = NULL,
    edge.alpha = 1, vertex.alpha = 1, edge.lwd = NULL, suppress.axes = TRUE,
    new = TRUE, bg.col = "white", layout.par = NULL)
```

## **Arguments**

	dat	a graph or set thereof. This data may be valued.
	g	integer indicating the index of the graph (from dat) which is to be displayed.
	gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected; "twomode" indicates that data should be interpreted as two-mode (i.e., rows and columns are distinct vertex sets).
	diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops.
	label	a vector of vertex labels; setting this to a zero-length string (e.g., "") omits
	coord	user-specified vertex coordinates, in an NCOL(dat)x3 matrix. Where this is specified, it will override the mode setting.
	jitter	boolean; should vertex positions be jittered?
	thresh	real number indicating the lower threshold for tie values. Only ties of value >thresh are displayed.
	mode	the vertex placement algorithm; this must correspond to a ${\tt gplot3d.layout}$ function.
displayisolates		
		boolean; should isolates be displayed?
	displaylabels	boolean; should vertex labels be displayed?
	xlab	X axis label.
	ylab	Y axis label.
	zlab	Z axis label.

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vertex.radius	vertex radius, relative to the baseline (which is set based on layout features); may be given as a vector, if radii vary across vertices.
absolute.radius	· ·
	vertex radius, specified in absolute terms; this may be given as a vector.
label.col	color for vertex labels; may be given as a vector, if labels are to be of different colors.
edge.col	color for edges; may be given as a vector or adjacency matrix, if edges are to be of different colors.
vertex.col	color for vertices; may be given as a vector, if vertices are to be of different colors. By default, red is used (or red and blue, if gmode=="twomode").
edge.alpha	alpha (transparency) values for edges; may be given as a vector or adjacency matrix, if edge transparency is to vary.
vertex.alpha	alpha (transparency) values for vertices; may be given as a vector, if vertex transparency is to vary.
edge.lwd	line width scale for edges; if set greater than 0, edge widths are rescaled by edge.lwd*dat. May be given as a vector or adjacency matrix, if edges are to have different line widths.
suppress.axes	boolean; suppress plotting of axes?
new	boolean; create a new plot? If new==FALSE, the RGL device will not be cleared prior to adding vertices and edges.
bg.col	background color for display.

## **Details**

layout.par

gplot3d is the three-dimensional companion to gplot. As with the latter, clever manipulation of parameters can allow for a great deal of flexibility in the resulting display. (Displays produced by gplot3d are also interactive, to the extent supported by rgl.) If vertex positions are not specified directly using coord, vertex layout is determined via one of the various available algorithms. These should be specified via the mode argument; see gplot3d.layout for a full list. User-supplied layout functions are also possible - see the aforementioned man page for details.

list of parameters to the gplot.layout function specified in mode.

Note that where gmode="twomode", the supplied two-mode graph is converted to bipartite form prior to computing coordinates (assuming it is not in this form already). It may be desirable to use parameters such as vertex.col to differentiate row and column vertices; by default, row vertices are colored red, and column vertices blue.

# Value

A three-column matrix containing vertex coordinates

# Requires

rgl

## Author(s)

Carter T. Butts <buttsc@uci.edu>

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## References

Wasserman, S. and Faust, K. (1994) *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

## See Also

```
gplot, gplot3d.layout, rgl
```

# **Examples**

```
## Not run:
#A three-dimensional grid...
gplot3d(rgws(1,5,3,1,0))

#...rewired...
gplot3d(rgws(1,5,3,1,0.05))

#...some more!
gplot3d(rgws(1,5,3,1,0.2))

## End(Not run)
```

gplot3d.arrow

Add Arrows a Three-Dimensional Plot

# Description

gplot3d.arrow draws an arrow between two pairs of points.

# Usage

```
gplot3d.arrow(a, b, radius, color = "white", alpha = 1)
```

# Arguments

a	a vector or three-column matrix containing origin X,Y,Z coordinates.
b	a vector or three-column matrix containing origin X,Y,Z coordinates.
radius	the arrow radius, in current plotting units. May be a vector, if multiple arrows are to be drawn.
color	the arrow color. May be a vector, if multiple arrows are being drawn.
alpha	alpha (transparency) value(s) for arrows. (May be a vector.)

# **Details**

gplot3d.arrow draws one or more three-dimensional "arrows" from the points given in a to those given in b. Note that the "arrows" are really cones, narrowing in the direction of the destination point.

#### Value

None.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### See Also

```
gplot3d, gplot3d.loop, rgl.primitive
```

gplot3d.layout

Vertex Layout Functions for gplot3d

## **Description**

Various functions which generate vertex layouts for the gplot3d visualization routine.

# Usage

```
gplot3d.layout.adj(d, layout.par)
gplot3d.layout.eigen(d, layout.par)
gplot3d.layout.fruchtermanreingold(d, layout.par)
gplot3d.layout.geodist(d, layout.par)
gplot3d.layout.hall(d, layout.par)
gplot3d.layout.kamadakawai(d, layout.par)
gplot3d.layout.mds(d, layout.par)
gplot3d.layout.princoord(d, layout.par)
gplot3d.layout.random(d, layout.par)
gplot3d.layout.rmds(d, layout.par)
gplot3d.layout.segeo(d, layout.par)
gplot3d.layout.segeo(d, layout.par)
gplot3d.layout.seham(d, layout.par)
```

#### **Arguments**

d an adjacency matrix, as passed by gplot3d.

layout.par a list of parameters.

#### **Details**

Like gplot, gplot3d allows for the use of arbitrary vertex layout algorithms via the gplot3d.layout.\* family of routines. When called, gplot3d searches for a gplot3d.layout function whose third name matches its mode argument (see gplot3d help for more information); this function is then used to generate the layout for the resulting plot. In addition to the routines documented here, users may add their own layout functions as needed. The requirements for a gplot3d.layout function are as follows:

- 1. the first argument, d, must be the (dichotomous) graph adjacency matrix;
- 2. the second argument, layout.par, must be a list of parameters (or NULL, if no parameters are specified); and
- 3. the return value must be a real matrix of dimension c(3,NROW(d)), whose rows contain the vertex coordinates.

Other than this, anything goes. (In particular, note that layout.par could be used to pass additional matrices, if needed.)

The gplot3d.layout functions currently supplied by default are as follows:

**eigen** This function places vertices based on the eigenstructure of the adjacency matrix. It takes the following arguments:

layout.par\\$var This argument controls the matrix to be used for the eigenanalysis. "symupper", "symlower", "symstrong", "symweak" invoke symmetrize on d with the respective symmetrizing rule. "user" indicates a user-supplied matrix (see below), while "raw" indicates that d should be used as-is. (Defaults to "raw".)

**layout.par\\$evsel** If "first", the first three eigenvectors are used; if "size", the three eigenvectors whose eigenvalues have the largest magnitude are used instead. Note that only the real portion of the associated eigenvectors is used. (Defaults to "first".)

**layout.par\\$mat** If layout.par\$var=="user", this matrix is used for the eigenanalysis. (No default.)

**fruchtermanreingold** This function generates a layout using a variant of Fruchterman and Reingold's force-directed placement algorithm. It takes the following arguments:

**layout.par\\$niter** This argument controls the number of iterations to be employed. (Defaults to 300.)

**layout.par\\$max.delta** Sets the maximum change in position for any given iteration. (Defaults to NROW(d).)

**layout.par\\$volume** Sets the "volume" parameter for the F-R algorithm. (Defaults to NROW(d) ^3.)

layout.par\\$cool.exp Sets the cooling exponent for the annealer. (Defaults to 3.)

**layout.par\\$repulse.rad** Determines the radius at which vertex-vertex repulsion cancels out attraction of adjacent vertices. (Defaults to volume\*NROW(d).)

**layout.par\\$seed.coord** A three-column matrix of initial vertex coordinates. (Defaults to a random spherical layout.)

hall This function places vertices based on the last three eigenvectors of the Laplacian of the input matrix (Hall's algorithm). It takes no arguments.

**kamadakawai** This function generates a vertex layout using a version of the Kamada-Kawai force-directed placement algorithm. It takes the following arguments:

**layout.par\\$niter** This argument controls the number of iterations to be employed. (Defaults to 1000.)

**layout.par\\$sigma** Sets the base standard deviation of position change proposals. (Defaults to NROW(d)/4.)

**layout.par\\$initemp** Sets the initial "temperature" for the annealing algorithm. (Defaults to 10.)

**layout.par\\$cool.exp** Sets the cooling exponent for the annealer. (Defaults to 0.99.)

layout.par\\$kkconst Sets the Kamada-Kawai vertex attraction constant. (Defaults to NROW(d)^3.)

**layout.par\\$elen** Provides the matrix of interpoint distances to be approximated. (Defaults to the geodesic distances of d after symmetrizing, capped at sqrt(NROW(d)).)

**layout.par\\$seed.coord** A three-column matrix of initial vertex coordinates. (Defaults to a gaussian layout.)

**mds** This function places vertices based on a metric multidimensional scaling of a specified distance matrix. It takes the following arguments:

layout.par\\$var This argument controls the raw variable matrix to be used for the subsequent distance calculation and scaling. "rowcol", "row", and "col" indicate that the rows and columns (concatenated), rows, or columns (respectively) of d should be used. "rcsum" and "rcdiff" result in the sum or difference of d and its transpose being employed. "invadj" indicates that max{d}-d should be used, while "geodist" uses geodist to generate a matrix of geodesic distances from d. Alternately, an arbitrary matrix can be provided using "user". (Defaults to "rowcol".)

layout.par\\$dist The distance function to be calculated on the rows of the variable matrix. This must be one of the method parameters to dist ("euclidean", "maximum", "manhattan", or "canberra"), or else "none". In the latter case, no distance function is calculated, and the matrix in question must be square (with dimension dim(d)) for the routine to work properly. (Defaults to "euclidean".)

**layout.par\\$exp** The power to which distances should be raised prior to scaling. (Defaults to 2.)

**layout.par\\$vm** If layout.par\$var=="user", this matrix is used for the distance calculation. (No default.)

Note: the following layout functions are based on mds:

adj scaling of the raw adjacency matrix, treated as similarities (using "invadj").

**geodist** scaling of the matrix of geodesic distances.

rmds euclidean scaling of the rows of d.

**segeo** scaling of the squared euclidean distances between row-wise geodesic distances (i.e., approximate structural equivalence).

**seham** scaling of the Hamming distance between rows/columns of d (i.e., another approximate structural equivalence scaling).

**princoord** This function places vertices based on the eigenstructure of a given correlation/covariance matrix. It takes the following arguments:

layout.par\\$var The matrix of variables to be used for the correlation/covariance calculation. "rowcol", "col", and "row" indicate that the rows/cols, columns, or rows (respectively) of d should be employed. "rcsum" "rcdiff" result in the sum or difference of d and t(d) being used. "user" allows for an arbitrary variable matrix to be supplied. (Defaults to "rowcol".)

**layout.par\\$cor** Should the correlation matrix (rather than the covariance matrix) be used? (Defaults to TRUE.)

**layout.par\\$vm** If layout.par\$var=="user", this matrix is used for the correlation/covariance calculation. (No default.)

**random** This function places vertices randomly. It takes the following argument:

**layout.par\\$dist** The distribution to be used for vertex placement. Currently, the options are "unif" (for uniform distribution on the unit cube), "uniang" (for a "gaussian sphere" configuration), and "normal" (for a straight Gaussian distribution). (Defaults to "unif".)

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## Value

A matrix whose rows contain the x,y,z coordinates of the vertices of d.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Fruchterman, T.M.J. and Reingold, E.M. (1991). "Graph Drawing by Force-directed Placement." *Software - Practice and Experience*, 21(11):1129-1164.

Kamada, T. and Kawai, S. (1989). "An Algorithm for Drawing General Undirected Graphs." *Information Processing Letters*, 31(1):7-15.

#### See Also

```
gplot3d, gplot, gplot.layout, cmdscale, eigen
```

gplot3d.loop	Add Loops to a Three-Dimensional Plot
--------------	---------------------------------------

# **Description**

gplot3d.loop draws a "loop" at a specified location; this is used to designate self-ties in gplot3d.

#### Usage

```
gplot3d.loop(a, radius, color = "white", alpha = 1)
```

### **Arguments**

a	a vector or three-column matrix containing origin X,Y,Z coordinates.
radius	the loop radius, in current plotting units. May be a vector, if multiple loops are to be drawn.
color	the loop color. May be a vector, if multiple loops are being drawn.

alpha alpha (transparency) value(s) for loops. (May be a vector.)

## **Details**

gplot3d.loop is the companion to gplot3d.arrow. The "loops" produced by this routine currently look less like loops than like "hats" – they are noticable as spike-like structures which protrude from vertices. Eventually, something more attractice will be produced by this routine.

## Value

None.

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# Author(s)

Carter T. Butts <buttsc@uci.edu>

#### See Also

```
gplot3d.arrow, gplot3d, rgl-package
```

graphcent

Compute the (Harary) Graph Centrality Scores of Network Positions

# Description

graphcent takes one or more graphs (dat) and returns the Harary graph centralities of positions (selected by nodes) within the graphs indicated by g. Depending on the specified mode, graph centrality on directed or undirected geodesics will be returned; this function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

# Usage

## **Arguments**

ignore.eval

dat	one or more input graphs.
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g==1.
nodes	list indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. gmode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
cmode	string indicating the type of graph centrality being computed (directed or undirected geodesics).
geodist.precom	р
	a geodist object precomputed for the graph to be analyzed (optional)
rescale	if true, centrality scores are rescaled such that they sum to 1.

logical; should edge values be ignored when calculating geodesics?

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## **Details**

The Harary graph centrality of a vertex v is equal to  $\frac{1}{\max_u d(v,u)}$ , where d(v,u) is the geodesic distance from v to u. Vertices with low graph centrality scores are likely to be near the "edge" of a graph, while those with high scores are likely to be near the "middle." Compare this with closeness, which is based on the reciprocal of the sum of distances to all other vertices (rather than simply the maximum).

#### Value

A vector, matrix, or list containing the centrality scores (depending on the number and size of the input graphs).

#### Note

Judicious use of geodist.precomp can save a great deal of time when computing multiple path-based indices on the same network.

# Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

#### References

Hage, P. and Harary, F. (1995). "Eccentricity and Centrality in Networks." *Social Networks*, 17:57-63.

### See Also

```
centralization
```

# **Examples**

```
g<-rgraph(10)  #Draw a random graph with 10 members
graphcent(g)  #Compute centrality scores</pre>
```

grecip

Compute the Reciprocity of an Input Graph or Graph Stack

## **Description**

grecip calculates the dyadic reciprocity of the elements of dat selected by g.

#### **Usage**

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### **Arguments**

dat one or more input graphs.

g a vector indicating which graphs to evaluate (optional).

measure one of "dyadic" (default), "dyadic.nonnull", "edgewise", "edgewise.lrr",

or "correlation".

#### **Details**

The dyadic reciprocity of a graph is the proportion of dyads which are symmetric; this is computed and returned by grecip for the graphs indicated. (dyadic.nonnull returns the ratio of mutuals to non-null dyads.) Note that the dyadic reciprocity is distinct from the *edgewise* or *tie reciprocity*, which is the proportion of *edges* which are reciprocated. This latter form may be obtained by setting measure="edgewise". Setting measure="edgewise.lrr" returns the log of the ratio of the edgewise reciprocity to the density; this is measure (called  $r_4$  by Butts (2008)) can be interpreted as the relative log-odds of an edge given a reciprocation, versus the baseline probability of an edge. Finally, measure="correlation" returns the correlation between within-dyad edge values, where this is defined by

 $\frac{2\sum_{\{i,j\}}(Y_{ij} - \mu_G)(Y_{ji} - \mu_G)}{(2N_d - 1)\sigma_G^2}$ 

with Y being the graph adjacency matrix,  $\mu_G$  being the mean non-loop edge value,  $\sigma_G^2$  being the variance of non-loop edge values, and  $N_d$  being the number of dyads. (Note that this quantity is unaffected by dyad orientation.) The correlation measure may be interpreted as the net tendency for edges of similar relative value (with respect to the mean edge value) to occur within the same dyads. For dichotomous data, adjacencies are interpreted as having values of 0 (no edge present) or 1 (edge present), but edge values are used where supplied. In cases where all edge values are identical (e.g., the complete or empty graph), the correlation reciprocity is taken to be 1 by definition.

Note that grecip calculates values based on non-missing data; dyads containing missing data are removed from consideration when calculating reciprocity scores (except for the correlation measure, which uses non-missing edges within missing dyads when calculating the graph mean and variance).

#### Value

The graph reciprocity value(s)

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

Butts, C.T. (2008). "Social Networks: A Methodological Introduction." *Asian Journal of Social Psychology*, 11(1), 13-41.

#### See Also

mutuality, symmetrize

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# **Examples**

```
\#Calculate the dyadic reciprocity scores for some random graphs grecip(rgraph(10,5))
```

gscor

Find the Structural Correlations Between Two or More Graphs

# **Description**

gscor finds the product-moment structural correlation between the adjacency matrices of graphs indicated by g1 and g2 in stack dat (or possibly dat2) given exchangeability list exchange.list. Missing values are permitted.

# Usage

```
gscor(dat, dat2=NULL, g1=NULL, g2=NULL, diag=FALSE,
    mode="digraph", method="anneal", reps=1000, prob.init=0.9,
    prob.decay=0.85, freeze.time=25, full.neighborhood=TRUE,
    exchange.list=0)
```

# **Arguments**

dat	a stack of input graphs.
dat2	optionally, a second graph stack.
g1	the indices of dat reflecting the first set of graphs to be compared; by default, all members of dat are included.
g2	the indices or dat (or dat2, if applicable) reflecting the second set of graphs to be compared; by default, all members of dat are included.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.
method	method to be used to search the space of accessible permutations; must be one of "none", "exhaustive", "anneal", "hillclimb", or "mc".
reps	number of iterations for Monte Carlo method.
prob.init	initial acceptance probability for the annealing routine.
prob.decay	cooling multiplier for the annealing routine.
freeze.time	freeze time for the annealing routine.
full.neighborhood	
	should the annealer evaluate the full neighborhood of pair exchanges at each iteration?
exchange.list	information on which vertices are exchangeable (see below); this must be a

single number, a vector of length n, or a nx2 matrix.

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#### **Details**

The structural correlation coefficient between two graphs G and H is defined as

$$scor(G, H | L_G, L_H) = \max_{L_G, L_H} cor(\ell(G), \ell(H))$$

where  $L_G$  is the set of accessible permutations/labelings of G,  $\ell(G)$  is a permutation/relabeling of G, and  $\ell(G) \in L_G$ . The set of accessible permutations on a given graph is determined by the *theoretical exchangeability* of its vertices; in a nutshell, two vertices are considered to be theoretically exchangeable for a given problem if all predictions under the conditioning theory are invariant to a relabeling of the vertices in question (see Butts and Carley (2001) for a more formal exposition). Where no vertices are exchangeable, the structural correlation becomes the simple graph correlation. Where *all* vertices are exchangeable, the structural correlation reflects the correlation between unlabeled graphs; other cases correspond to correlation under partial labeling.

The accessible permutation set is determined by the exchange.list argument, which is dealt with in the following manner. First, exchange.list is expanded to fill an nx2 matrix. If exchange.list is a single number, this is trivially accomplished by replication; if exchange.list is a vector of length n, the matrix is formed by cbinding two copies together. If exchange.list is already an nx2 matrix, it is left as-is. Once the nx2 exchangeability matrix has been formed, it is interpreted as follows: columns refer to graphs 1 and 2, respectively; rows refer to their corresponding vertices in the original adjacency matrices; and vertices are taken to be theoretically exchangeable iff their corresponding exchangeability matrix values are identical. To obtain an unlabeled graph correlation (the default), then, one could simply let exchange.list equal any single number. To obtain the standard graph correlation, one would use the vector 1:n.

Because the set of accessible permutations is, in general, very large (o(n!)), searching the set for the maximum correlation is a non-trivial affair. Currently supported methods for estimating the structural correlation are hill climbing, simulated annealing, blind monte carlo search, or exhaustive search (it is also possible to turn off searching entirely). Exhaustive search is not recommended for graphs larger than size 8 or so, and even this may take days; still, this is a valid alternative for small graphs. Blind monte carlo search and hill climbing tend to be suboptimal for this problem and are not, in general recommended, but they are available if desired. The preferred (and default) option for permutation search is simulated annealing, which seems to work well on this problem (though some tinkering with the annealing parameters may be needed in order to get optimal performance). See the help for lab.optimize for more information regarding these options.

Structural correlation matrices are p.s.d., and are p.d. so long as no graph within the set is a linear combination of any other under any accessible permutation. Their eigendecompositions are meaningful and they may be used in linear subspace analyses, so long as the researcher is careful to interpret the results in terms of the appropriate set of accessible labelings. Classical null hypothesis tests should not be employed with structural correlations, and QAP tests are almost never appropriate (save in the uniquely labeled case). See cugtest for a more reasonable alternative.

#### Value

An estimate of the structural correlation matrix

#### Warning

The search process can be *very slow*, particularly for large graphs. In particular, the *exhaustive* method is order factorial, and will take approximately forever for unlabeled graphs of size greater

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than about 7-9.

# Note

Consult Butts and Carley (2001) for advice and examples on theoretical exchangeability.

# Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

#### References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS Working Paper, Carnegie Mellon University.

# See Also

```
gscov, gcor, gcov
```

## **Examples**

```
#Generate two random graphs
g.1<-rgraph(5)
g.2<-rgraph(5)

#Copy one of the graphs and permute it
perm<-sample(1:5)
g.3<-g.2[perm,perm]

#What are the structural correlations between the labeled graphs?
gscor(g.1,g.2,exchange.list=1:5)
gscor(g.1,g.3,exchange.list=1:5)

#What are the structural correlations between the underlying
#unlabeled graphs?
gscor(g.1,g.2)
gscor(g.1,g.3)
gscor(g.2,g.3)</pre>
```

gscov

Find the Structural Covariance(s) Between Two or More Graphs

# Description

gscov finds the structural covariance between the adjacency matrices of graphs indicated by g1 and g2 in stack dat (or possibly dat2) given exchangeability list exchange.list. Missing values are permitted.

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## Usage

```
gscov(dat, dat2=NULL, g1=NULL, g2=NULL, diag=FALSE, mode="digraph",
    method="anneal", reps=1000, prob.init=0.9, prob.decay=0.85,
    freeze.time=25, full.neighborhood=TRUE, exchange.list=0)
```

# **Arguments**

dat	one or more input graphs.	
dat2	optionally, a second graph stack.	
g1	the indices of dat reflecting the first set of graphs to be compared; by default, all members of dat are included.	
g2	the indices or dat (or dat2, if applicable) reflecting the second set of graphs to be compared; by default, all members of dat are included.	
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.	
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.	
method	method to be used to search the space of accessible permutations; must be one of "none", "exhaustive", "anneal", "hillclimb", or "mc".	
reps	number of iterations for Monte Carlo method.	
prob.init	initial acceptance probability for the annealing routine.	
prob.decay	cooling multiplier for the annealing routine.	
freeze.time	freeze time for the annealing routine.	
full.neighborhood		
	dhould the annealer evaluate the full neighborhood of pair exchanges at each iteration?	
exchange.list	information on which vertices are exchangeable (see below); this must be a single number, a vector of length n, or a nx2 matrix.	

### **Details**

The structural covariance between two graphs G and H is defined as

$$scov(G, H | L_G, L_H) = \max_{L_G, L_H} cov(\ell(G), \ell(H))$$

where  $L_G$  is the set of accessible permutations/labelings of G,  $\ell(G)$  is a permutation/labeling of G, and  $\ell(G) \in L_G$ . The set of accessible permutations on a given graph is determined by the theoretical exchangeability of its vertices; in a nutshell, two vertices are considered to be theoretically exchangeable for a given problem if all predictions under the conditioning theory are invariant to a relabeling of the vertices in question (see Butts and Carley (2001) for a more formal exposition). Where no vertices are exchangeable, the structural covariance becomes the simple graph covariance. Where all vertices are exchangeable, the structural covariance reflects the covariance between unlabeled graphs; other cases correspond to covariance under partial labeling.

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The accessible permutation set is determined by the exchange.list argument, which is dealt with in the following manner. First, exchange.list is expanded to fill an nx2 matrix. If exchange.list is a single number, this is trivially accomplished by replication; if exchange.list is a vector of length n, the matrix is formed by chinding two copies together. If exchange.list is already an nx2 matrix, it is left as-is. Once the nx2 exchangeability matrix has been formed, it is interpreted as follows: columns refer to graphs 1 and 2, respectively; rows refer to their corresponding vertices in the original adjacency matrices; and vertices are taken to be theoretically exchangeable iff their corresponding exchangeability matrix values are identical. To obtain an unlabeled graph covariance (the default), then, one could simply let exchange.list equal any single number. To obtain the standard graph covariance, one would use the vector 1:n.

Because the set of accessible permutations is, in general, very large (o(n!)), searching the set for the maximum covariance is a non-trivial affair. Currently supported methods for estimating the structural covariance are hill climbing, simulated annealing, blind monte carlo search, or exhaustive search (it is also possible to turn off searching entirely). Exhaustive search is not recommended for graphs larger than size 8 or so, and even this may take days; still, this is a valid alternative for small graphs. Blind monte carlo search and hill climbing tend to be suboptimal for this problem and are not, in general recommended, but they are available if desired. The preferred (and default) option for permutation search is simulated annealing, which seems to work well on this problem (though some tinkering with the annealing parameters may be needed in order to get optimal performance). See the help for lab.optimize for more information regarding these options.

Structural covariance matrices are p.s.d., and are p.d. so long as no graph within the set is a linear combination of any other under any accessible permutation. Their eigendecompositions are meaningful and they may be used in linear subspace analyses, so long as the researcher is careful to interpret the results in terms of the appropriate set of accessible labelings. Classical null hypothesis tests should not be employed with structural covariances, and QAP tests are almost never appropriate (save in the uniquely labeled case). See cugtest for a more reasonable alternative.

### Value

An estimate of the structural covariance matrix

## Warning

The search process can be *very slow*, particularly for large graphs. In particular, the *exhaustive* method is order factorial, and will take approximately forever for unlabeled graphs of size greater than about 7-9.

## Note

Consult Butts and Carley (2001) for advice and examples on theoretical exchangeability.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS Working Paper, Carnegie Mellon University.

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### See Also

```
gscor, gcov, gcor
```

### **Examples**

```
#Generate two random graphs
g.1<-rgraph(5)
g.2<-rgraph(5)

#Copy one of the graphs and permute it
perm<-sample(1:5)
g.3<-g.2[perm,perm]

#What are the structural covariances between the labeled graphs?
gscov(g.1,g.2,exchange.list=1:5)
gscov(g.1,g.3,exchange.list=1:5)
gscov(g.2,g.3,exchange.list=1:5)

#What are the structural covariances between the underlying
#unlabeled graphs?
gscov(g.1,g.2)
gscov(g.1,g.3)
gscov(g.2,g.3)</pre>
```

gt

Transpose an Input Graph

### **Description**

gt returns the graph transpose of its input. For an adjacency matrix, this is the same as using t; however, this function is also applicable to sna edgelists (which cannot be transposed in the usual fashion). Code written using gt instead of t is thus guaranteed to be safe for either form of input.

### Usage

```
gt(x, return.as.edgelist = FALSE)
```

#### **Arguments**

## **Details**

The transpose of a (di)graph, G = (V, E), is the graph G = (V, E') where  $E' = \{(j, i) : (i, j) \in E\}$ . This is simply the graph formed by reversing the sense of the edges.

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## Value

The transposed graph(s).

### Author(s)

Carter T. Butts <buttsc@uci.edu>

### See Also

```
symmetrize, t
```

# **Examples**

```
#Create a graph....
g<-rgraph(5)
g

#Transpose it
gt(g)
gt(g)==t(g)  #For adjacency matrices, same as t(g)

#Now, see both versions in edgelist form
as.edgelist.sna(g)
gt(g,return.as.edgelist=TRUE)</pre>
```

gtrans

Compute the Transitivity of an Input Graph or Graph Stack

### **Description**

gtrans returns the transitivity of the elements of dat selected by g, using the definition of measure. Triads involving missing values are omitted from the analysis.

## Usage

```
gtrans(dat, g=NULL, diag=FALSE, mode="digraph", measure = c("weak",
    "strong", "weakcensus", "strongcensus", "rank", "correlation"),
    use.adjacency = TRUE)
```

# Arguments

dat	a collection of input graphs.
g	a vector indicating the graphs which are to be analyzed; by default, all graphs are analyzed.
diag	a boolean indicating whether or not diagonal entries (loops) are to be taken as valid data.
mode	"digraph" if directed triads are sought, or else "graph".

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measure one of "weak" (default), "strong", "weakcensus", "strongcensus", "rank",

or "correlation".

use.adjacency logical; should adjacency matrices (versus sparse graph methods) be used in the

transitivity computation?

#### **Details**

Transitivity is a triadic, algebraic structural constraint. In its weak form, the transitive constraint corresponds to  $a \to b \to c \Rightarrow a \to c$ . In the corresponding strong form, the constraint is  $a \to b \to c \Leftrightarrow a \to c$ . (Note that the weak form is that most commonly employed.) Where measure=="weak", the fraction of potentially intransitive triads obeying the weak condition is returned. With the measure=="weakcensus" setting, by contrast, the total *number* of transitive triads is computed. The strong versions of the measures are similar to the above, save in that the set of all triads is considered (since all are "at risk" for intransitivity).

Note that where missing values prevent the assessment of whether a triple is transitive, that triple is omitted.

Generalizations of transitivity to valued graphs are numerous. The above strong and weak forms ignore edge values, treating any non-zero edge as present. Two additional notions of transitivity are also supported valued data. The "rank" condition treads an (i,j,k) triple as transitive if the value of the (i,k) directed dyad is greater than or equal to the minimum of the values of the (i,j) and (j,k) dyads. The "correlation" option implements the correlation transitivity of David Dekker, which is defined as the matrix correlation of the valued adjacency matrix A with its second power (i.e.,  $A^2$ ), omitting diagonal entries where inapplicable.

Note that the base forms of transitivity can be calculated using either matrix multiplication or sparse graph methods. For very large, sparse graphs, the sparse graph method (which can be forced by use.adjacency=FALSE) may be preferred. The latter provides much better scaling, but is significantly slower for networks of typical size due to the overhead involved (and R's highly optimized matrix operations). Where use.adjacency is set to TRUE, gtrans will attempt some simple heuristics to determine if the edgelist method should be used instead (and will do so if indicated). These heuristics depend on recognition of the input data type, and hence may behave slightly differently depending on the form in which dat is given. Note that the rank measure can at present be calculated only via sparse graph methods, and the correlation measure only by adjacency matrices. For these measures, the use.adjacency argument is ignored.

#### Value

A vector of transitivity scores

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Holland, P.W., and Leinhardt, S. (1972). "Some Evidence on the Transitivity of Positive Interpersonal Sentiment." *American Journal of Sociology*, 72, 1205-1209.

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

gvectorize 113

### See Also

```
triad.classify, cugtest
```

### **Examples**

```
#Draw some random graphs
g<-rgraph(5,10)

#Find transitivity scores
gtrans(g)</pre>
```

gvectorize

Vectorization of Adjacency Matrices

## Description

gvectorize takes an input graph set and converts it into a corresponding number of vectors by row concatenation.

## Usage

```
gvectorize(mats, mode="digraph", diag=FALSE, censor.as.na=TRUE)
```

### **Arguments**

mats one or more input graphs.

mode "digraph" if data is taken to be directed, else "graph".

diag boolean indicating whether diagonal entries (loops) are taken to contain mean-

ingful data.

censor.as.na if TRUE, code unused parts of the adjacency matrix as NAs prior to vectorizing;

otherwise, unused parts are simply removed.

#### **Details**

The output of gvectorize is a matrix in which each column corresponds to an input graph, and each row corresponds to an edge. The columns of the output matrix are formed by simple row-concatenation of the original adjacency matrices, possibly after removing cells which are not meaningful (if censor.as.na==FALSE). This is useful when preprocessing edge sets for use with glm or the like.

#### Value

An nxk matrix, where n is the number of arcs and k is the number of graphs; if censor. as.na==FALSE, n will be reflect the relevant number of uncensored arcs.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

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## **Examples**

```
#Draw two random graphs
g<-rgraph(10,2)

#Examine the vectorized form of the adjacency structure
gvectorize(g)</pre>
```

hdist

Find the Hamming Distances Between Two or More Graphs

## Description

hdist returns the Hamming distance between the labeled graphs g1 and g2 in set dat for dichotomous data, or else the absolute (manhattan) distance. If normalize is true, this distance is divided by its dichotomous theoretical maximum (conditional on |V(G)|).

### Usage

## **Arguments**

dat	a stack of input graphs.
dat2	a second graph stack (optional).
g1	a vector indicating which graphs to compare (by default, all elements of dat).
g2	a vector indicating against which the graphs of g1 should be compared (by default, all graphs).
normalize	divide by the number of available dyads?
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.

## Details

The Hamming distance between two labeled graphs  $G_1$  and  $G_2$  is equal to  $|\{e:(e\in E(G_1),e\not\in E(G_2))\land (e\not\in E(G_1),e\in E(G_2))\}|$ . In more prosaic terms, this may be thought of as the number of addition/deletion operations required to turn the edge set of  $G_1$  into that of  $G_2$ . The Hamming distance is a highly general measure of structural similarity, and forms a metric on the space of graphs (simple or directed). Users should be reminded, however, that the Hamming distance is extremely sensitive to nodal labeling, and should not be employed directly when nodes are interchangeable. The structural distance (Butts and Carley (2001)), implemented in structdist, provides a natural generalization of the Hamming distance to the more general case of unlabeled graphs.

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Null hypothesis testing for Hamming distances is available via cugtest, and qaptest; graphs which minimize the Hamming distances to all members of a graph set can be found by centralgraph. For an alternative means of comparing the similarity of graphs, consider gcor.

## Value

A matrix of Hamming distances

#### Note

For non-dichotomous data, the distance which is returned is simply the sum of the absolute edgewise differences.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Banks, D., and Carley, K.M. (1994). "Metric Inference for Social Networks." *Journal of Classification*, 11(1), 121-49.

Butts, C.T. and Carley, K.M. (2005). "Some Simple Algorithms for Structural Comparison." *Computational and Mathematical Organization Theory*, 11(4), 291-305.

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS Working Paper, Carnegie Mellon University.

Hamming, R.W. (1950). "Error Detecting and Error Correcting Codes." *Bell System Technical Journal*, 29, 147-160.

### See Also

```
sdmat, structdist
```

### **Examples**

```
#Get some random graphs
g<-rgraph(5,5,tprob=runif(5,0,1))
#Find the Hamming distances
hdist(g)</pre>
```

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Compute Graph Hierarchy Scores

## **Description**

hierarchy takes a graph set (dat) and returns reciprocity or Krackhardt hierarchy scores for the graphs selected by g.

## Usage

```
hierarchy(dat, g=NULL, measure=c("reciprocity", "krackhardt"))
```

### **Arguments**

dat a stack of input graphs.

g index values for the graphs to be utilized; by default, all graphs are selected.

measure one of "reciprocity" or "krackhardt".

#### **Details**

Hierarchy measures quantify the extent of asymmetry in a structure; the greater the extent of asymmetry, the more hierarchical the structure is said to be. (This should not be confused with how *centralized* the structure is, i.e., the extent to which centralities of vertex positions are highly concentrated.) hierarchy provides two measures (selected by the measure argument) as follows:

- 1. reciprocity: This setting returns one minus the dyadic reciprocity for each input graph (see grecip)
- 2. krackhardt: This setting returns the Krackhardt hierarchy score for each input graph. The Krackhardt hierarchy is defined as the fraction of non-null dyads in the reachability graph which are asymmetric. Thus, when no directed paths are reciprocated (e.g., in an in/outtree), Krackhardt hierarchy is equal to 1; when all such paths are reciprocated, by contrast (e.g., in a cycle or clique), the measure falls to 0.

Hierarchy is one of four measures (connectedness, efficiency, hierarchy, and lubness) suggested by Krackhardt for summarizing hierarchical structures. Each corresponds to one of four axioms which are necessary and sufficient for the structure in question to be an outtree; thus, the measures will be equal to 1 for a given graph iff that graph is an outtree. Deviations from unity can be interpreted in terms of failure to satisfy one or more of the outtree conditions, information which may be useful in classifying its structural properties.

Note that hierarchy is inherently density-constrained: as densities climb above 0.5, the proportion of mutual dyads must (by the pigeonhole principle) increase rapidly, thereby reducing possibilities for asymmetry. Thus, the interpretation of hierarchy scores should take density into account, particularly if density is artifactual (e.g., due to a particular dichotomization procedure).

#### Value

A vector of hierarchy scores

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### Note

The four Krackhardt indices are, in general, nondegenerate for a relatively narrow band of size/density combinations (efficiency being the sole exception). This is primarily due to their dependence on the reachability graph, which tends to become complete rapidly as size/density increase. See Krackhardt (1994) for a useful simulation study.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Krackhardt, David. (1994). "Graph Theoretical Dimensions of Informal Organizations." In K. M. Carley and M. J. Prietula (Eds.), *Computational Organization Theory*, 89-111. Hillsdale, NJ: Lawrence Erlbaum and Associates.

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

### See Also

connectedness, efficiency, hierarchy, lubness, grecip, mutuality, dyad.census

#### **Examples**

```
#Get hierarchy scores for graphs of varying densities
hierarchy(rgraph(10,5,tprob=c(0.1,0.25,0.5,0.75,0.9)),
    measure="reciprocity")
hierarchy(rgraph(10,5,tprob=c(0.1,0.25,0.5,0.75,0.9)),
    measure="krackhardt")
```

infocent

Find Information Centrality Scores of Network Positions

### Description

infocent takes one or more graphs (dat) and returns the information centralities of positions (selected by nodes) within the graphs indicated by g. This function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

### Usage

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#### **Arguments**

dat	one or more input graphs.
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g==1.
nodes	list indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. This is currently ignored.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
cmode	the rule to be used by symmetrize when symmetrizing dichotomous data; must be one of "weak" (for an OR rule), "strong" for an AND rule), "upper" (for a max rule), or "lower" (for a min rule). Set to "weak" by default, this parameter obviously has no effect on symmetric data.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
rescale	if true, centrality scores are rescaled such that they sum to 1.
tol	tolerance for near-singularities during matrix inversion (see solve).

#### **Details**

Actor information centrality is a hybrid measure which relates to both path-length indices (e.g., closeness, graph centrality) and to walk-based eigenmeasures (e.g., eigenvector centrality, Bonacich power). In particular, the information centrality of a given actor can be understood to be the harmonic average of the "bandwidth" for all paths originating with said individual (where the bandwidth is taken to be inversely related to path length). Formally, the index is constructed as follows. First, we take G to be an undirected (but possibly valued) graph – symmetrizing if necessary – with (possibly valued) adjacency matrix A. From this, we remove all isolates (whose information centralities are zero in any event) and proceed to create the weighted connection matrix

$$C = B^{-1}$$

where **B** is a pseudo-adjacency matrix formed by replacing the diagonal of  $1 - \mathbf{A}$  with one plus each actor's degree. Given the above, let T be the trace of  $\mathbf{C}$  with sum  $S_T$ , and let  $S_R$  be an arbitrary row sum (all rows of  $\mathbf{C}$  have the same sum). The information centrality scores are then equal to

$$C_I = \frac{1}{T + \frac{S_T - 2S_R}{|V(G)|}}$$

(recalling that the scores for any omitted vertices are 0).

In general, actors with higher information centrality are predicted to have greater control over the flow of information within a network; highly information-central individuals tend to have a large number of short paths to many others within the social structure. Because the raw centrality values can be difficult to interpret directly, rescaled values are sometimes preferred (see the rescale

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option). Though the use of path weights suggest information centrality as a possible replacement for closeness, the problem of inverting the  ${\bf B}$  matrix poses problems of its own; as with all such measures, caution is advised on disconnected or degenerate structures.

#### Value

A vector, matrix, or list containing the centrality scores (depending on the number and size of the input graphs).

#### Note

The theoretical maximum deviation used here is not obtained with the star network; rather, the maximum occurs for an empty graph with one complete dyad, which is the model used here.

### Author(s)

```
David Barron <david.barron@jesus.ox.ac.uk>
Carter T. Butts <buttsc@uci.edu>
```

### References

Stephenson, K., and Zelen, M. (1989). "Rethinking Centrality: Methods and Applications." *Social Networks*, 11, 1-37.

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

#### See Also

```
evcent, bonpow, closeness, graphcent, centralization
```

## **Examples**

```
#Generate some test data
dat<-rgraph(10,mode="graph")
#Compute information centrality scores
infocent(dat)</pre>
```

interval.graph

Convert Spell Data to Interval Graphs

## **Description**

Constructs one or more interval graphs (and exchangeability vectors) from a set of spells.

### Usage

```
interval.graph(slist, type="simple", diag=FALSE)
```

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#### **Arguments**

slist A spell list. This must consist of an nxmx3 array, with n being the number of

actors, m being the maximum number of spells (one per row) and with the three columns of the last dimension containing a (categorical) spell type code, the time of spell onset (any units), and the time of spell termination (same units),

respectively.

type One of "simple", "overlap", "fracxy", "fracyx", or "jntfrac".

diag Include the dyadic entries?

## **Details**

Given some ordering dimension T (usually time), a "spell" is defined as the interval between a specified onset and a specified termination (with onset preceding the termination). An interval graph, then, on spell set V, is  $G = \{V, E\}$ , where  $\{i, j\} \in E$  iff there exists some point  $t \in T$  such that  $t \in i$  and  $t \in j$ . In more prosaic terms, an interval graph on a given spell set has each spell as a vertex, with vertices adjacent iff they overlap. Such structures are useful for quantifying life history data (where spells might represent marriages, periods of child custody/co-residence, periods of employment, etc.), organizational history data (where spells might reflect periods of strategic alliances, participation in a particular product market, etc.), task scheduling (with spells representing the dedication of a particular resource to a given task), etc. By giving complex historical data a graphic representation, it is possible to easily perform a range of analyses which would otherwise be difficult and/or impossible (see Butts and Pixley (2004) for examples).

In addition to the simple interval graph (described above), interval.graph can also generate valued interval graphs using a number of different edge definitions. This is controlled by the type argument, with edge values as follows:

- 1. simple: dichotomous coding based on simple overlap (i.e., (x,y)=1 iff x overlaps y)
- 2. overlap: edge value equals the total magnitude of the overlap between spells
- 3. fracxy: the (x,y) edge value equals the fraction of the duration of y which is covered by x
- 4. fracyx: the (x,y) edge value equals the fraction of the duration of x which is covered by y
- 5. jntfrac: edge value equals the total magnitude of the overlap between spells divided by the mean of the spells' lengths

Note that "simple," "overlap," and "jntfrac" are symmetric relations, while "fracxy" and "fracyx" are directed. As always, the specific edge type used should reflect the application to which the interval graph is being put.

## Value

A data frame containing:

graph A graph stack containing the interval graphs

exchange.list Matrix containing the vector of spell types associated with each interval graph

## Author(s)

Carter T. Butts <buttsc@uci.edu>

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### References

Butts, C.T. and Pixley, J.E. (2004). "A Structural Approach to the Representation of Life History Data." *Journal of Mathematical Sociology*, 28(2), 81-124.

West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, NJ: Prentice Hall.

is.connected

Is a Given Graph Connected?

## **Description**

Returns TRUE iff the specified graphs are connected.

## Usage

```
is.connected(g, connected = "strong", comp.dist.precomp = NULL)
```

#### **Arguments**

g one or more input graphs.

connected definition of connectedness to use; must be one of "strong", "weak", "unilateral",

or "recursive".

comp.dist.precomp

a component.dist object precomputed for the graph to be analyzed (optional).

### **Details**

is.connected determines whether the elements of g are connected under the definition specified in connected. (See component.dist for details.) Since is.connected is really just a wrapper for component.dist, an object created with the latter can be supplied (via comp.dist.precomp) to speed computation.

### Value

TRUE iff g is connected, otherwise FALSE

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, N.J.: Prentice Hall.

#### See Also

```
component.dist, components
```

is.isolate

## **Examples**

```
#Generate two graphs:
g1<-rgraph(10,tp=0.1)
g2<-rgraph(10)

#Check for connectedness
is.connected(g1) #Probably not
is.connected(g2) #Probably so</pre>
```

is.isolate

Is Ego an Isolate?

# **Description**

Returns TRUE iff ego is an isolate in graph g of dat.

## Usage

```
is.isolate(dat, ego, g=1, diag=FALSE)
```

# Arguments

dat one or more input graphs.

ego index of the vertex (or a vector of vertices) to check.

g which graph(s) should be examined?

diag boolean indicating whether adjacency matrix diagonals (i.e., loops) contain mean-

ingful data.

#### **Details**

In the valued case, any non-zero edge value is taken as sufficient to establish a tie.

### Value

A boolean value (or vector thereof) indicating isolate status

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, NJ: Prentice Hall.

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### See Also

```
isolates, add. isolates
```

### **Examples**

isolates

List the Isolates in a Graph or Graph Stack

# Description

Returns a list of the isolates in the graph or graph set given by dat.

### Usage

```
isolates(dat, diag=FALSE)
```

### **Arguments**

dat one or more input graphs.

diag boolean indicating whether adjacency matrix diagonals (i.e., loops) contain mean-

ingful data.

# Value

A vector containing the isolates, or a list of vectors if more than one graph was specified

### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

### References

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, NJ: Prentice Hall.

#### See Also

```
is.isolate, add.isolates
```

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### **Examples**

kcores

Compute the k-Core Structure of a Graph

## **Description**

kcores calculates the k-core structure of the input network, using the centrality measure indicated in cmode.

## Usage

```
kcores(dat, mode = "digraph", diag = FALSE, cmode = "freeman",
    ignore.eval = FALSE)
```

## **Arguments**

dat one or more (possibly valued) graphs.

mode "digraph" for directed data, otherwise "graph".

diag logical; should self-ties be included in the degree calculations? cmode the degree centrality mode to use when constructing the cores. logical; should edge values be ignored when computing degree?

#### **Details**

Let G = (V, E) be a graph, and let f(v, S, G) for  $v \in V, S \subseteq V$  be a real-valued *vertex property function* (in the language of Batagelj and Zaversnik). Then some set  $H \subseteq V$  is a *generalized k-core* for f if H is a maximal set such that  $f(v, H, G) \ge k$  for all  $v \in H$ . Typically, f is chosen to be a degree measure with respect to S (e.g., the number of ties to vertices in S). In this case, the resulting k-cores have the intuitive property of being maximal sets such that every set member is tied (in the appropriate manner) to at least K others within the set.

Degree-based k-cores are a simple tool for identifying well-connected structures within large graphs. Let the *core number* of vertex v be the value of the highest-value core containing v. Then, intuitively, vertices with high core numbers belong to relatively well-connected sets (in the sense of sets with high minimum internal degree). It is important to note that, while a given k-core need not be connected, it is composed of subsets which are themselves well-connected; thus, the k-cores can be thought of as unions of relatively cohesive subgroups. As k-cores are nested, it is also natural to think of each k-core as representing a "slice" through a hypothetical "cohesion surface" on G. (Indeed, k-cores are often visualized in exactly this manner.)

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The kcores function produces degree-based k-cores, for various degree measures (with or without edge values). The return value is the vector of core numbers for V, based on the selected degree measure. Missing (i.e., NA) edge are removed for purposes of the degree calculation.

#### Value

A vector containing the maximum core membership for each vertex.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Batagelj, V. and Zaversnik, M. (2002). "An O(m) Algorithm for Cores Decomposition of Networks." arXiv:cs/0310049v1

Batagelj, V. and Zaversnik, M. (2002). "Generalized Cores." arXiv:cs/0202039v1

Wasserman, S. and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

## See Also

degree

## **Examples**

```
#Generate a graph with core-periphery structure
cv<-runif(30)
g<-rgraph(30,tp=cv%o%cv)

#Compute the k-cores based on total degree
kc<-kcores(g)
kc

#Plot the result
gplot(g,vertex.col=kc)</pre>
```

kpath.census

Compute Path or Cycle Census Information

### **Description**

kpath.census and kcycle.census compute k-path or k-cycle census statistics (respectively) on one or more input graphs. In addition to aggregate counts of paths or cycles, results may be disaggregated by vertex and co-membership information may be computed.

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### Usage

```
kcycle.census(dat, maxlen = 3, mode = "digraph",
    tabulate.by.vertex = TRUE, cycle.comembership = c("none", "sum",
    "bylength"))

kpath.census(dat, maxlen = 3, mode = "digraph",
    tabulate.by.vertex = TRUE, path.comembership = c("none", "sum",
    "bylength"), dyadic.tabulation = c("none", "sum", "bylength"))
```

### Arguments

cycle.comembership

the type of cycle co-membership information to be tabulated, if any. "sum" returns a vertex by vertex matrix of cycle co-membership counts; these are disaggregated by cycle length if "bylength" is used. If "none" is given, no co-membership information is computed.

dat one or more input graphs.

maxlen the maximum path/cycle length to evaluate.

mode "digraph" for directed graphs, or "graph" for undirected graphs.

tabulate.by.vertex

logical; should path or cycle incidence counts be tabulated by vertex?

path.comembership

as per cycle. comembership, for paths rather than cycles.

dyadic.tabulation

the type of dyadic path count information to be tabulated, if any. "sum" returns a vertex by vertex matrix of source/destination path counts, while "bylength" disaggregates these counts by path length. Selecting "none" disables this computation.

#### **Details**

There are several equivalent characterizations of paths and cycles, of which the following is one example. For an arbitrary graph G, a path is a sequence of distinct vertices  $v_1, v_2, \ldots, v_n$  and included edges such that  $v_i$  is adjacent to  $v_{i+1}$  for all  $i \in 1, 2, \ldots, n-1$  via the pair's included edge. (Contrast this with a walk, in which edges and/or vertices may be repeated.) A cycle is the union of a path and an edge making  $v_n$  adjacent to  $v_i$ . k-paths and k-cycles are respective paths and cycles having k edges (in the former case) or k vertices (in the latter). The above definitions may be applied in both directed and undirected contexts, by substituting the appropriate notion of adjacency. (Note that authors do not always employ the same terminology for these concepts, especially in older texts – it is wise to verify the definitions being used in any particular context.)

A *subgraph census statistic* is a function which, for any given graph and subgraph, gives the number of copies of the latter contained in the former. A collection of subgraph census statistics is referred to as a *subgraph census*; widely used examples include the dyad and triad censuses, implemented in sna by the dyad.census and triad.census functions (respectively). kpath.census and kcycle.census compute a range of census statistics related to k-paths and k-cycles, including:

• Aggregate counts of paths/cycles by length (i.e., k).

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- Counts of paths/cycles to which each vertex belongs (when tabulate.byvertex==TRUE).
- Counts of path/cycle co-memberships, potentially disaggregated by length (when the appropriate co-membership argument is set to bylength).
- For path. census, counts of the total number of paths from each vertex to each other vertex, possibly disaggregated by length (if dyadic.tabulation=="bylength").

The length of the maximum-length path/cycle to compute is given by maxlen. These calculations are intrinsically expensive (path/cycle computation is NP complete in the general case), and users should hence be wary when increasing maxlen. On the other hand, it may be possible to enumerate even long paths or cycles on a very sparse graph; scaling is approximately  $c^k$ , where k is given by maxlen and c is the size of the largest dense cluster.

The paths or cycles computed by this function are directed if mode=="digraph", or undirected if mode=="graph". Failing to set mode correctly may result in problematic behavior.

#### Value

For kpath.census, a list with the following elements:

path.count If tabulate.byvertex==FALSE, a vector of aggregate counts by path length.

Otherwise, a matrix whose first column is a vector of aggregate path counts, and whose succeeding columns contain vectors of path counts for each vertex.

path.comemb If path.comembership!="none", a matrix or array containing co-membership

in paths by vertex pairs. If path.comembership=="sum", only a matrix of comemberships is returned; if bylength is used, however, co-memberships are returned in a maxlen by n by n array whose i,j,kth cell is the number of paths

of length i containing j and k.

paths.bydyad If dyadic.tabulation!="none", a matrix or array containing the number of

paths originating at a particular vertex and terminating. If bylength is used, dyadic path counts are supplied via a maxlen by n by n array whose i, j, kth cell is the number of paths of length i starting at j and ending with k. If sum is used instead, only a matrix whose i, j cell contains the total number of paths

from i to j is returned.

For kcycle.census, a similar list:

cycle.count If tabulate.byvertex==FALSE, a vector of aggregate counts by cycle length.

Otherwise, a matrix whose first column is a vector of aggregate cycle counts, and whose succeeding columns contain vectors of cycle counts for each vertex.

in cycles by vertex pairs. If cycle.comembership=="sum", only a matrix of comemberships is returned; if bylength is used, however, co-memberships are returned in a maxlen by n by n array whose i,j,kth cell is the number of cycles

of length i containing j and k.

# Warning

The computational cost of calculating paths and cycles grows very sharply in both maxlen and network density. Be wary of setting maxlen greater than 5-6, unless you know what you are doing. Otherwise, the expected completion time for your calculation may exceed your life expectancy (and those of subsequent generations).

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T. (2006). "Cycle Census Statistics for Exponential Random Graph Models." IMBS Technical Report MBS 06-05, University of California, Irvine.

West, D.B. (1996). Introduction to Graph Theory. Upper Saddle River, N.J.: Prentice Hall.

#### See Also

```
dyad.census, triad.census, clique.census, geodist
```

## **Examples**

```
g < -rgraph(20, tp=1.5/19)
#Obtain paths by vertex, with dyadic path counts
pc<-kpath.census(g,maxlen=5,dyadic.tabulation="sum")</pre>
pc$path.count
                                                #Examine path counts
pc$paths.bydyad
                                                #Examine dyadic paths
#Obtain aggregate cycle counts, with co-membership by length
cc<-kcycle.census(g,maxlen=5,tabulate.by.vertex=FALSE,</pre>
    cycle.comembership="bylength")
cc$cycle.count
                                             #Examine cycle counts
cc$cycle.comemb[1,,]
                                             #Co-membership for 2-cycles
cc$cycle.comemb[2,,]
                                             #Co-membership for 3-cycles
cc$cycle.comemb[3,,]
                                             #Co-membership for 4-cycles
```

lab.optimize

Optimize a Bivariate Graph Statistic Across a Set of Accessible Permutations

### **Description**

lab.optimize is the front-end to a series of heuristic optimization routines (see below), all of which seek to maximize/minimize some bivariate graph statistic (e.g., graph correlation) across a set of vertex relabelings.

## Usage

```
lab.optimize(d1, d2, FUN, exchange.list=0, seek="min",
    opt.method=c("anneal", "exhaustive", "mc", "hillclimb",
        "gumbel"), ...)
lab.optimize.anneal(d1, d2, FUN, exchange.list=0, seek="min",
        prob.init=1, prob.decay=0.99, freeze.time=1000,
        full.neighborhood=TRUE, ...)
```

## **Arguments**

d1	a single graph.	
d2	another single graph.	
FUN	a function taking two graphs as its first two arguments, and returning a numeric value.	
exchange.list	information on which vertices are exchangeable (see below); this must be a single number, a vector of length n, or a nx2 matrix.	
seek	"min" if the optimizer should seek a minimum, or "max" if a maximum should be sought.	
opt.method	the particular optimization method to use.	
prob.init	initial acceptance probability for a downhill move (lab.optimize.anneal only).	
prob.decay	the decay (cooling) multiplier for the probability of accepting a downhill move (lab.optimize.anneal only).	
freeze.time	number of iterations at which the annealer should be frozen (lab.optimize.anneal only).	
full.neighborhood		
	should all moves in the binary-exchange neighborhood be evaluated at each iteration? (lab.optimize.anneal only).	
tol	tolerance for estimation of gumbel distribution parameters (lab.optimize.gumbel only).	
estimator	Gumbel distribution statistic to use as optimal value prediction; must be one of "mean", "median", or "mode" (lab.optimize.gumbel only).	
draws	number of draws to take for gumbel and mc methods.	
	additional arguments to FUN.	

## **Details**

lab.optimize is the front-end to a family of routines for optimizing a bivariate graph statistic over a set of permissible relabelings (or equivalently, permutations). The accessible permutation set is determined by the exchange.list argument, which is dealt with in the following manner. First, exchange.list is expanded to fill an nx2 matrix. If exchange.list is a single number, this is trivially accomplished by replication; if exchange.list is a vector of length n, the matrix is formed by cbinding two copies together. If exchange.list is already an nx2 matrix, it is left as-is. Once the nx2 exchangeability matrix has been formed, it is interpreted as follows: columns refer to graphs 1 and 2, respectively; rows refer to their corresponding vertices in the original adjacency matrices; and vertices are taken to be theoretically exchangeable iff their corresponding exchangeability matrix values are identical. To obtain an unlabeled graph statistic (the default),

then, one could simply let exchange.list equal any single number. To obtain the labeled statistic, one would use the vector 1:n.

Assuming a non-degenerate set of accessible permutations/relabelings, optimization proceeds via the algorithm specified in opt.method. The optimization routines which are currently implemented use a variety of different techniques, each with certain advantages and disadvantages. A brief summary of each is as follows:

- 1. exhaustive search ("exhaustive"): Under exhaustive search, the entire space of accessible permutations is combed for the global optimum. This guarantees a correct answer, but at a very high price: the set of all permutations grows with the factorial of the number of vertices, and even substantial exchangeability constraints are unlikely to keep the number of permutations from growing out of control. While exhaustive search is possible for small graphs, unlabeled structures of size approximately 10 or greater cannot be treated using this algorithm within a reasonable time frame.
  - Approximate complexity: on the order of  $\prod_{i \in L} |V_i|!$ , where L is the set of exchangeability classes.
- 2. hill climbing ("hillclimb"): The hill climbing algorithm employed here searches, at each iteration, the set of all permissible binary exchanges of vertices. If one or more exchanges are found which are superior to the current permutation, the best alternative is taken. If no superior alternative is found, then the algorithm terminates. As one would expect, this algorithm is guaranteed to terminate on a local optimum; unfortunately, however, it is quite prone to becoming "stuck" in suboptimal solutions. In general, hill climbing is not recommended for permutation search, but the method may prove useful in certain circumstances.
  - Approximate complexity: on the order of  $|V(G)|^2$  per iteration, total complexity dependent on the number of iterations.
- 3. simulated annealing ("anneal"): The (fairly simple) annealing procedure here employed proceeds as follows. At each iteration, the set of all permissible binary exchanges (if full.neighborhood==TRUE) or a random selection from this set is evaluated. If a superior option is identified, the best of these is chosen. If no superior options are found, then the algorithm chooses randomly from the set of alternatives with probability equal to the current temperature, otherwise retaining its prior solution. After each iteration, the current temperature is reduced by a factor equal to prob.decay; the initial temperature is set by prob.init. When a number of iterations equal to freeze.time have been completed, the algorithm "freezes." Once "frozen," the annealer hillclimbs from its present location until no improvement is found, and terminates. At termination, the best permutation identified so far is utilized; this need not be the most recent position (though it sometimes is).
  - Simulated annealing is sometimes called "noisy hill climbing" because it uses the introduction of random variation to a hill climbing routine to avoid convergence to local optima; it works well on reasonably correlated search spaces with well-defined solution neighborhoods, and is far more robust than hill climbing algorithms. As a general rule, simulated annealing is recommended here for most graphs up to size approximately 50. At this point, computational complexity begins to become a serious barrier, and alternative methods may be more practical. Approximate complexity: on the order of  $|V(G)|^2$ \*freeze.time if full.neighborhood==TRUE, otherwise complexity scales approximately linearly with freeze.time. This can be misleading, however, since failing to search the full neighborhood generally requires that freeze.time be greatly increased.)
- 4. blind monte carlo search ("mc"): Blind monte carlo search, as the name implies, consists of randomly drawing a sample of permutations from the accessible permutation set and selecting

the best. Although this not such a bad option when A) a large fraction of points are optimal or nearly optimal and B) the search space is largely uncorrelated, these conditions do not seem to characterize most permutation search problems. Blind monte carlo search is not generally recommended, but it is provided as an option should it be desired (e.g., when it is absolutely necessary to control the number of permutations examined).

Approximate complexity: linear in draws.

5. extreme value estimation ("gumbel"): Extreme value estimation attempts to estimate a global optimum via stochastic modeling of the distribution of the graph statistic over the space of accessible permutations. The algorithm currently proceeds as follows. First, a random sample is taken from the accessible permutation set (as with monte carlo search, above). Next, this sample is used to fit an extreme value (gumbel) model; the gumbel distribution is the limiting distribution of the extreme values from samples under a continuous, unbounded distribution, and we use it here as an approximation. Having fit the model, an associated statistic (the mean, median, or mode as determined by estimator) is then used as an estimator of the global optimum.

Obviously, this approach has certain drawbacks. First of all, our use of the gumbel model in particular assumes an unbounded, continuous underlying distribution, which may or may not be approximately true for any given problem. Secondly, the inherent non-robustness of extremal problems makes the fact that our prediction rests on a string of approximations rather worrisome: our idea of the shape of the underlying distribution could be distorted by a bad sample, our parameter estimation could be somewhat off, etc., any of which could have serious consequences for our extremal prediction. Finally, the prediction which is made by the extreme value model is *nonconstructive*, in the sense that *no permutation need have been found by the algorithm which induces the predicted value*. On the bright side, this *could* allow one to estimate the optimum without having to find it directly; on the dark side, this means that the reported optimum could be a numerical chimera.

At this time, extreme value estimation should be considered *experimental*, and *is not recommended for use on substantive problems*. lab.optimize.gumbel is not guaranteed to work properly, or to produce intelligible results; this may eventually change in future revisions, or the routine may be scrapped altogether.

Approximate complexity: linear in draws.

This list of algorithms is itself somewhat unstable: some additional techniques (canonical labeling and genetic algorithms, for instance) may be added, and some existing methods (e.g., extreme value estimation) may be modified or removed. Every attempt will be made to keep the command format as stable as possible for other routines (e.g., gscov, structdist) which depend on lab.optimize to do their heavy-lifting. In general, it is not expected that the end-user will call lab.optimize directly; instead, most end-user interaction with these routines will be via the structural distance/covariance functions which used them.

#### Value

The estimated global optimum of FUN over the set of relabelings permitted by exchange.list

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

lnam

### References

Butts, C.T. and Carley, K.M. (2005). "Some Simple Algorithms for Structural Comparison." *Computational and Mathematical Organization Theory*, 11(4), 291-305.

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS Working Paper, Carnegie Mellon University.

#### See Also

```
gscov, gscor, structdist, sdmat
```

# Examples

lnam

Fit a Linear Network Autocorrelation Model

### **Description**

lnam is used to fit linear network autocorrelation models. These include standard OLS as a special case, although lm is to be preferred for such analyses.

### Usage

```
lnam(y, x = NULL, W1 = NULL, W2 = NULL, theta.seed = NULL,
    null.model = c("meanstd", "mean", "std", "none"), method = "BFGS",
    control = list(), tol=1e-10)
```

## **Arguments**

У	a vector of responses.
X	a vector or matrix of covariates; if the latter, each column should contain a single covariate.
W1	one or more (possibly valued) graphs on the elements of y.
W2	one or more (possibly valued) graphs on the elements of y.
theta.seed	an optional seed value for the parameter vector estimation process.

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null.model the null model to be fit; must be one of "meanstd", "mean", "std", or "none".

method method to be used with optim.

control optional control parameters for optim.

tol convergence tolerance for the MLE (expressed as change in deviance).

#### **Details**

1nam fits the linear network autocorrelation model given by

$$y = W_1 y + X\beta + e$$
,  $e = W_2 e + \nu$ 

where y is a vector of responses, X is a covariate matrix,  $\nu \sim N(0, \sigma^2)$ ,

$$W_1 = \sum_{i=1}^{p} \rho_{1i} W_{1i}, \quad W_2 = \sum_{i=1}^{q} \rho_{2i} W_{2i},$$

and  $W_{1i}$ ,  $W_{2i}$  are (possibly valued) adjacency matrices.

Intuitively,  $\rho_1$  is a vector of "AR"-like parameters (parameterizing the autoregression of each y value on its neighbors in the graphs of  $W_1$ ) while  $\rho_2$  is a vector of "MA"-like parameters (parameterizing the autocorrelation of each *disturbance* in y on its neighbors in the graphs of  $W_2$ ). In general, the two models are distinct, and either or both effects may be selected by including the appropriate matrix arguments.

Model parameters are estimated by maximum likelihood, and asymptotic standard errors are provided as well; all of the above (and more) can be obtained by means of the appropriate print and summary methods. A plotting method is also provided, which supplies fit basic diagnostics for the estimated model. For purposes of comparison, fits may be evaluated against one of four null models:

- 1. meanstd: mean and standard deviation estimated (default).
- 2. mean: mean estimated; standard deviation assumed equal to 1.
- 3. std: standard deviation estimated; mean assumed equal to 0.
- 4. none: no parameters estimated; data assumed to be drawn from a standard normal density.

The default setting should be appropriate for the vast majority of cases, although the others may have use when fitting "pure" autoregressive models (e.g., without covariates). Although a major use of the lnam is in controlling for network autocorrelation within a regression context, the model is subtle and has a variety of uses. (See the references below for suggestions.)

#### Value

An object of class "lnam" containing the following elements:

y the response vector used.

x if supplied, the coefficient matrix.

W1 if supplied, the W1 array. W2 if supplied, the W2 array. lnam

model a code indicating the model terms fit.

infomat the estimated Fisher information matrix for the fitted model.

acvm the estimated asymptotic covariance matrix for the model parameters.

null.model a string indicating the null model fit.

lnlik.null the log-likelihood of y under the null model.

df.null.resid the residual degrees of freedom under the null model.

df.null the model degrees of freedom under the null model.

null.param parameter estimates for the null model.

lnlik.model the log-likelihood of y under the fitted model.

df.model the model degrees of freedom.

df.residual the residual degrees of freedom.

df.total the total degrees of freedom.

rho1 if applicable, the MLE for rho1.

rho1. se if applicable, the asymptotic standard error for rho1.

rho2 if applicable, the MLE for rho2.

rho2. se if applicable, the asymptotic standard error for rho2.

sigma the MLE for sigma.

sigma.se the standard error for sigma beta if applicable, the MLE for beta.

beta. se if applicable, the asymptotic standard errors for beta.

fitted.values the fitted mean values.

residuals the residuals (response minus fitted); note that these correspond to  $\hat{e}$  in the model

equation, not  $\hat{\nu}$ .

disturbances the estimated disturbances, i.e.,  $\hat{\nu}$ .

call the matched call.

## Note

Actual optimization is performed by calls to optim. Information on algorithms and control parameters can be found via the appropriate man pages.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Leenders, T.Th.A.J. (2002) "Modeling Social Influence Through Network Autocorrelation: Constructing the Weight Matrix" *Social Networks*, 24(1), 21-47.

Anselin, L. (1988) Spatial Econometrics: Methods and Models. Norwell, MA: Kluwer.

loadcent 135

### See Also

```
1m, optim
```

### **Examples**

```
#Construct a simple, random example:
w1<-rgraph(100)
                               #Draw the AR matrix
w2<-rgraph(100)
                               #Draw the MA matrix
x<-matrix(rnorm(100*5),100,5) #Draw some covariates
r1 < -0.2
                               #Set the model parameters
r2 < -0.1
sigma < -0.1
beta<-rnorm(5)
#Assemble y from its components:
nu<-rnorm(100,0,sigma)</pre>
                                 #Draw the disturbances
e<-qr.solve(diag(100)-r2*w2,nu) #Draw the effective errors
y<-qr.solve(diag(100)-r1*w1,x%*%beta+e) #Compute y
#Now, fit the autocorrelation model:
fit < -lnam(y,x,w1,w2)
summary(fit)
plot(fit)
## End(Not run)
```

loadcent

Compute the Load Centrality Scores of Network Positions

### **Description**

loadcent takes one or more graphs (dat) and returns the load centralities of positions (selected by nodes) within the graphs indicated by g. Depending on the specified mode, load on directed or undirected geodesics will be returned; this function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

## Usage

```
loadcent(dat, g = 1, nodes = NULL, gmode = "digraph", diag = FALSE,
    tmaxdev = FALSE, cmode = "directed", geodist.precomp = NULL,
    rescale = FALSE, ignore.eval = TRUE)
```

### Arguments

dat one or more input graphs.

g integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g=1.

loadcent

nodes vector indicating which nodes are to be included in the calculation. By default,

all nodes are included.

gmode string indicating the type of graph being evaluated. digraph indicates that edges

should be interpreted as directed; graph indicates that edges are undirected.

gmode is set to digraph by default.

diag logical; should self-ties be treated as valid data. Set this true if and only if the

data can contain loops. diag is FALSE by default.

tmaxdev logical; return the theoretical maximum absolute deviation from the maximum

nodal centrality (instead of the observed centrality scores)? By default, tmaxdev==FALSE.

cmode string indicating the type of load centrality being computed (directed or undi-

rected).

geodist.precomp

a geodist object precomputed for the graph to be analyzed (optional).

rescale logical; if true, centrality scores are rescaled such that they sum to 1.

ignore.eval logical; ignore edge values when computing shortest paths?

#### **Details**

Goh et al.'s load centrality (as reformulated by Brandes (2008)) is a betweenness-like measure defined through a hypothetical flow process. Specifically, it is assumed that each vertex sends a unit of some commodity to each other vertex to which it is connected (without edge or vertex capacity constraints), with routing based on a priority system: given an input of flow x arriving at vertex y with destination y', y divides y equally among all neigbors of minumum geodesic distance to the target. The total flow passing through a given y via this process is defined as y's load. Load is a potential alternative to betweenness for the analysis of flow structures operating well below their capacity constraints.

#### Value

A vector of centrality scores.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Brandes, U. (2008). "On Variants of Shortest-Path Betweenness Centrality and their Generic Computation." *Social Networks*, 30, 136-145.

Goh, K.-I.; Kahng, B.; and Kim, D. (2001). "Universal Behavior of Load Distribution in Scale-free Networks." *Physical Review Letters*, 87(27), 1-4.

#### See Also

betweenness

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## **Examples**

lower.tri.remove

Remove the Lower Triangles of Adjacency Matrices in a Graph Stack

# Description

Returns the input graph set, with the lower triangle entries removed/replaced as indicated.

# Usage

```
lower.tri.remove(dat, remove.val=NA)
```

## **Arguments**

dat one or more input graphs.

remove.val the value with which to replace the existing lower triangles.

### **Details**

lower.tri.remove is simply a convenient way to apply g[lower.tri(g)]<-remove.val to an entire stack of adjacency matrices at once.

### Value

The updated graph set.

# Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

### See Also

```
lower.tri, upper.tri.remove, diag.remove
```

## **Examples**

```
#Generate a random graph stack
g<-rgraph(3,5)
#Remove the lower triangles
g<-lower.tri.remove(g)</pre>
```

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lubness

Compute Graph LUBness Scores

#### **Description**

lubness takes a graph set (dat) and returns the Krackhardt LUBness scores for the graphs selected by g.

## Usage

lubness(dat, g=NULL)

## Arguments

dat one or more input graphs.

g index values for the graphs to be utilized; by default, all graphs are selected.

#### **Details**

In the context of a directed graph G, two actors i and j may be said to have an  $upper\ bound$  iff there exists some actor k such that directed ki and kj paths belong to G. An upper bound  $\ell$  is known as a  $least\ upper\ bound$  for i and j iff it belongs to at least one ki and kj path (respectively) for all i,j upper bounds k; let L(i,j) be an indicator which returns 1 iff such an  $\ell$  exists, otherwise returning 0. Now, let  $G_1, G_2, \ldots, G_n$  represent the weak components of G. For convenience, we denote the cardinalities of these graphs' vertex sets by |V(G)| = N and  $|V(G_i)| = N_i, \forall i \in 1, \ldots, n$ . Given this, the Krackhardt LUBness of G is given by

$$1 - \frac{\sum_{i=1}^{n} \sum_{v_j, v_k \in V(G_i)} \left(1 - L(v_j, v_k)\right)}{\sum_{i=1}^{n} \frac{1}{2} (N_i - 1)(N_i - 2)}$$

Where all vertex pairs possess a least upper bound, Krackhardt's LUBness is equal to 1; in general, it approaches 0 as this condition is broached. (This convergence is problematic in certain cases due to the requirement that we sum violations across components; where a graph contains no components of size three or greater, Krackhardt's LUBness is not well-defined. lubness returns a NaN in these cases.)

LUBness is one of four measures (connectedness, efficiency, hierarchy, and lubness) suggested by Krackhardt for summarizing hierarchical structures. Each corresponds to one of four axioms which are necessary and sufficient for the structure in question to be an outtree; thus, the measures will be equal to 1 for a given graph iff that graph is an outtree. Deviations from unity can be interpreted in terms of failure to satisfy one or more of the outtree conditions, information which may be useful in classifying its structural properties.

#### Value

A vector of LUBness scores

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#### Note

The four Krackhardt indices are, in general, nondegenerate for a relatively narrow band of size/density combinations (efficiency being the sole exception). This is primarily due to their dependence on the reachability graph, which tends to become complete rapidly as size/density increase. See Krackhardt (1994) for a useful simulation study.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Krackhardt, David. (1994). "Graph Theoretical Dimensions of Informal Organizations." In K. M. Carley and M. J. Prietula (Eds.), *Computational Organization Theory*, 89-111. Hillsdale, NJ: Lawrence Erlbaum and Associates.

#### See Also

```
connectedness, efficiency, hierarchy, lubness, reachability
```

### **Examples**

```
#Get LUBness scores for graphs of varying densities lubness(rgraph(10,5,tprob=c(0.1,0.25,0.5,0.75,0.9)))
```

make.stochastic

Make a Graph Stack Row, Column, or Row-column Stochastic

#### **Description**

Returns a graph stack in which each adjacency matrix in dat has been normalized to row stochastic, column stochastic, or row-column stochastic form, as specified by mode.

### Usage

```
make.stochastic(dat, mode="rowcol", tol=0.005,
    maxiter=prod(dim(dat)) * 100, anneal.decay=0.01, errpow=1)
```

## **Arguments**

dat a collection of input graphs.

mode one of "row," "col," or "rowcol".

tol tolerance parameter for the row-column normalization algorithm.

maxiter maximum iterations for the rwo-column normalization algorithm.

anneal.decay probability decay factor for the row-column annealer.

errpow power to which absolute row-column normalization errors should be raised for

the annealer (i.e., the penalty function).

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#### **Details**

Row and column stochastic matrices are those whose rows and columns sum to 1 (respectively). These are quite straightforwardly produced here by dividing each row (or column) by its sum. Row-column stochastic matrices, by contrast, are those in which each row *and* each column sums to 1. Here, we try to produce row-column stochastic matrices whose values are as close in proportion to the original data as possible by means of an annealing algorithm. This is probably not optimal in the long term, but the results seem to be consistent where row-column stochasticization of the original data is possible (which it is not in all cases).

### Value

The stochasticized adjacency matrices

### Warning

Rows or columns which sum to 0 in the original data will generate undefined results. This can happen if, for instance, your input graphs contain in- or out-isolates.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

## **Examples**

```
#Generate a test matrix
g<-rgraph(15)

#Make it row stochastic
make.stochastic(g,mode="row")

#Make it column stochastic
make.stochastic(g,mode="col")

#(Try to) make it row-column stochastic
make.stochastic(g,mode="rowcol")</pre>
```

maxflow

Calculate Maximum Flows Between Vertices

### Description

maxflow calculates a matrix of maximum pairwise flows within a (possibly valued) input network.

#### **Usage**

```
maxflow(dat, src = NULL, sink = NULL, ignore.eval = FALSE)
```

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## Arguments

dat	one or more input graphs.
src	optionally, a vector of source vertices; by default, all vertices are selected.
sink	optionally, a vector of sink (or target) vertices; by default, all vertices are selected.

ignore.eval logical; ignore edge values (i.e., assume unit capacities) when computing flow?

#### **Details**

maxflow computes the maximum flow from each source vertex to each sink vertex, assuming infinite vertex capacities and limited edge capacities. If ignore.eval==FALSE, supplied edge values are assumed to contain capacity information; otherwise, all non-zero edges are assumed to have unit capacity.

Note that all flows computed here are pairwise – i.e., when computing the flow from v to v', we ignore any other flows which could also be taking place within the network. As a result, it should not be assumed that these flows can be realized *simultaneously*. (For the latter purpose, the values returned by maxflow can be treated as upper bounds.)

#### Value

A matrix of pairwise maximum flows (if multiple sources/sinks selected), or a single maximum flow value (otherwise).

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Edmonds, J. and Karp, R.M. (1972). "Theoretical Improvements in Algorithmic Efficiency for Network Flow Problems." *Journal of the ACM*, 19(2), 248-264.

## See Also

flowbet, geodist

### **Examples**

```
g<-rgraph(10,tp=2/9)
maxflow(g)</pre>
```

#Generate a sparse random graph #Compute all-pairs max flow

142 mutuality

mutuality

Find the Mutuality of a Graph

## **Description**

Returns the mutuality scores of the graphs indicated by g in dat.

### Usage

```
mutuality(dat, g=NULL)
```

# Arguments

dat one or more input graphs.

g a vector indicating which elements of dat should be analyzed; by default, all

graphs are included.

#### **Details**

The mutuality of a digraph G is defined as the number of complete dyads (i.e., i<->j) within G. (Compare this to dyadic reciprocity, the fraction of dyads within G which are symmetric.) Mutuality is commonly employed as a measure of reciprocal tendency within the p\* literature; although mutuality can be very hard to interpret in practice, it is much better behaved than many alternative measures.

#### Value

One or more mutuality scores

# Author(s)

Carter T. Butts <buttsc@uci.edu>

### References

Moreno, J.L., and Jennings, H.H. (1938). "Statistics of Social Configurations." *Sociometry*, 1, 342-374.

### See Also

grecip

nacf 143

## **Examples**

```
#Create some random graphs
g<-rgraph(15,3)

#Get mutuality and reciprocity scores
mutuality(g)
grecip(g) #Compare with mutuality</pre>
```

nacf

Sample Network Covariance and Correlation Functions

# Description

nacf computes the sample network covariance/correlation function for a specified variable on a given input network. Moran's I and Geary's C statistics at multiple orders may be computed as well.

## Usage

```
nacf(net, y, lag.max = NULL, type = c("correlation", "covariance",
    "moran", "geary"), neighborhood.type = c("in", "out", "total"),
    partial.neighborhood = TRUE, mode = "digraph", diag = FALSE,
    thresh = 0, demean = TRUE)
```

#### **Arguments**

net one or more graphs.

y a numerical vector, of length equal to the order of net.

lag.max optionally, the maximum geodesic lag at which to compute dependence (defaults

to order net-1).

type the type of dependence statistic to be computed.

neighborhood.type

the type of neighborhood to be employed when assessing dependence (as per

neighborhood).

partial.neighborhood

logical; should partial (rather than cumulative) neighborhoods be employed at

higher orders?

mode "digraph" for directed graphs, or "graph" if net is undirected.

diag logical; does the diagonal of net contain valid data?

thresh threshold at which to dichotomize net.

demean logical; demean y prior to analysis?

144 nacf

#### **Details**

nacf computes dependence statistics for the vector y on network net, for neighborhoods of various orders. Specifically, let  $A_i$  be the *i*th order adjacency matrix of net. The sample network autocovariance of y on  $A_i$  is then given by

$$\sigma_i = \frac{\mathbf{y}^T \mathbf{A}_i \mathbf{y}}{E},$$

where  $E = \sum_{(j,k)} A_{ijk}$ . Similarly, the sample network autocorrelation in the above case is  $\rho_i = \sigma_i/\sigma_0$ , where  $\sigma_0$  is the variance of y. Moran's I and Geary's C statistics are defined in the usual fashion as

$$I_{i} = \frac{N \sum_{j=1}^{N} \sum_{k=1}^{N} (y_{j} - \bar{y})(y_{k} - \bar{y}) A_{ijk}}{E \sum_{j=1}^{N} y_{j}^{2}},$$

and

$$C_i = \frac{(N-1)\sum_{j=1}^{N}\sum_{k=1}^{N}(y_j - y_k)^2 A_{ijk}}{2E\sum_{j=1}^{N}(y - \bar{y})^2}$$

respectively, where N is the order of  $A_i$  and  $\bar{y}$  is the mean of y.

The adjacency matrix associated with the ith order neighborhood is defined as the identity matrix for order 0, and otherwise depends on the type of neighborhood involved. For input graph G = (V, E), let the *base relation*, R, be given by the underlying graph of G (i.e.,  $G \cup G^T$ ) if total neighborhoods are sought, the transpose of G if incoming neighborhoods are sought, or G otherwise. The partial neighborhood structure of order i > 0 on R is then defined to be the digraph on V whose edge set consists of the ordered pairs (j,k) having geodesic distance i in R. The corresponding cumulative neighborhood is formed by the ordered pairs having geodesic distance less than or equal to i in R. For purposes of nacf, these neighborhoods are calculated using neighborhood, with the specified parameters (including dichotomization at thresh).

The return value for nacf is the selected dependence statistic, calculated for each neighborhood structure from order 0 (the identity) through order lag.max (or N-1, if lag.max==NULL). This vector can be used much like the conventional autocorrelation function, to identify dependencies at various lags. This may, in turn, suggest a starting point for modeling via routines such as lnam.

### Value

A vector containing the dependence statistics (ascending from order 0).

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Geary, R.C. (1954). "The Contiguity Ratio and Statistical Mapping." *The Incorporated Statistician*, 5: 115-145.

Moran, P.A.P. (1950). "Notes on Continuous Stochastic Phenomena." Biometrika, 37: 17-23.

neighborhood 145

### See Also

```
geodist, gapply, neighborhood, lnam, acf
```

# Examples

neighborhood

Compute Neighborhood Structures of Specified Order

#### **Description**

For a given graph, returns the specified neighborhood structure at the selected order(s).

#### Usage

```
neighborhood(dat, order, neighborhood.type = c("in", "out", "total"),
   mode = "digraph", diag = FALSE, thresh = 0, return.all = FALSE,
   partial = TRUE)
```

### **Arguments**

dat one or more graphs.

order order of the neighborhood to extract.

neighborhood.type

neighborhood type to employ.

mode "digraph" if dat is directed, otherwise "graph".

diag logical; do the diagonal entries of dat contain valid data?

thresh dichotomization threshold to use for dat; edges whose values are greater than

thresh are treated as present.

return.all logical; return neighborhoods for all orders up to order?

partial logical; return partial (rather than cumulative) neighborhoods?

146 neighborhood

#### **Details**

The adjacency matrix associated with the ith order neighborhood is defined as the identity matrix for order 0, and otherwise depends on the type of neighborhood involved. For input graph G=(V,E), let the base relation, R, be given by the underlying graph of G (i.e.,  $G \cup G^T$ ) if total neighborhoods are sought, the transpose of G if incoming neighborhoods are sought, or G otherwise. The partial neighborhood structure of order i>0 on R is then defined to be the digraph on V whose edge set consists of the ordered pairs (j,k) having geodesic distance i in R. The corresponding cumulative neighborhood is formed by the ordered pairs having geodesic distance less than or equal to i in R.

Neighborhood structures are commonly used to parameterize various types of network autocorrelation models. They may also be used in the calculation of certain types of local structural indices; gapply provides an alternative function which can be used for this purpose.

### Value

An array or adjacency matrix containing the neighborhood structures (if dat is a single graph); if dat contains multiple graphs, then a list of such structures is returned.

### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

### See Also

```
gapply, nacf
```

### **Examples**

```
#Draw a random graph
g<-rgraph(10,tp=2/9)

#Show the total partial out-neighborhoods
neigh<-neighborhood(g,9,neighborhood.type="out",return.all=TRUE)
par(mfrow=c(3,3))
for(i in 1:9)
   gplot(neigh[i,,],main=paste("Partial Neighborhood of Order",i))

#Show the total cumulative out-neighborhoods
neigh<-neighborhood(g,9,neighborhood.type="out",return.all=TRUE,
   partial=FALSE)
par(mfrow=c(3,3))
for(i in 1:9)
   gplot(neigh[i,,],main=paste("Cumulative Neighborhood of Order",i))</pre>
```

netcancor 147

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Canonical Correlation for Labeled Graphs

### **Description**

netcancor finds the canonical correlation(s) between the graph sets x and y, testing the result using either conditional uniform graph (CUG) or quadratic assignment procedure (QAP) null hypotheses.

### Usage

```
netcancor(y, x, mode="digraph", diag=FALSE, nullhyp="cugtie",
    reps=1000)
```

### **Arguments**

٠	2	
	У	one or more input graphs.
	X	one or more input graphs.
	mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.
	diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
	nullhyp	string indicating the particular null hypothesis against which to test the observed estimands. A value of "cug" implies a conditional uniform graph test (see cugtest) controlling for order <i>only</i> ; "cugden" controls for both order and tie probability; "cugtie" controls for order and tie distribution (via bootstrap); and "qap" implies that the QAP null hypothesis (see qaptest) should be used.
	reps	integer indicating the number of draws to use for quantile estimation. (Relevant to the null hypothesis test only - the analysis itself is unaffected by this parameter.) Note that, as for all Monte Carlo procedures, convergence is slower for more extreme quantiles.

### **Details**

The netcancor routine is actually a front-end to the cancor routine for computing canonical correlations between sets of vectors. netcancor itself vectorizes the network variables (as per its graph type) and manages the appropriate null hypothesis tests; the actual canonical correlation is handled by cancor.

Canonical correlation itself is a multivariate generalization of the product-moment correlation. Specifically, the analysis seeks linear combinations of the variables in y which are well-explained by linear combinations of the variables in x. The network version of this technique is performed elementwise on the adjacency matrices of the graphs in question; as usual, the result should be interpreted with an eye to the relationship between the type of data used and the assumptions of the underlying model.

Intelligent printing and summarizing of netcancor objects is provided by print.netcancor and summary.netcancor.

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## Value

An object of class netcancor with the following properties:

xdist	Array containing the distribution of the X coefficients under the null hypothesis test.
ydist	Array containing the distribution of the Y coefficients under the null hypothesis test.
cdist	Array containing the distribution of the canonical correlation coefficients under the null hypothesis test.
cor	Vector containing the observed canonical correlation coefficients.
xcoef	Vector containing the observed X coefficients.
ycoef	Vector containing the observed Y coefficients.
cpgreq	Vector containing the estimated upper tail quantiles (p>=obs) for the observed canonical correlation coefficients under the null hypothesis.
cpleeq	Vector containing the estimated lower tail quantiles (p<=obs) for the observed canonical correlation coefficients under the null hypothesis.
xpgreq	Matrix containing the estimated upper tail quantiles (p>=obs) for the observed X coefficients under the null hypothesis.
xpleeq	Matrix containing the estimated lower tail quantiles (p<=obs) for the observed X coefficients under the null hypothesis.
ypgreq	Matrix containing the estimated upper tail quantiles (p>=obs) for the observed Y coefficients under the null hypothesis.
ypleeq	Matrix containing the estimated lower tail quantiles (p<=obs) for the observed Y coefficients under the null hypothesis.
cnames	Vector containing names for the canonical correlation coefficients.
xnames	Vector containing names for the X vars.
ynames	Vector containing names for the Y vars.
xcenter	Values used to adjust the X variables.
xcenter	Values used to adjust the Y variables.
nullhyp	String indicating the null hypothesis employed.

## Note

This will eventually be replaced with a superior cancor procedure with more interpretable output; the new version will handle arbitrary labeling as well.

# Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS working paper, Carnegie Mellon University.

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### See Also

```
gcor, cugtest, qaptest, cancor
```

### **Examples**

```
#Generate a valued seed structure

cv<-matrix(rnorm(100),nrow=10,ncol=10)

#Produce two sets of valued graphs

x<-array(dim=c(3,10,10))

x[1,,]<-3*cv+matrix(rnorm(100,0,0.1),nrow=10,ncol=10)

x[2,,]<--1*cv+matrix(rnorm(100,0,0.1),nrow=10,ncol=10)

x[3,,]<-x[1,,]+2*x[2,,]+5*cv+matrix(rnorm(100,0,0.1),nrow=10,ncol=10)

y<-array(dim=c(2,10,10))

y[1,,]<--5*cv+matrix(rnorm(100,0,0.1),nrow=10,ncol=10)

y[2,,]<--2*cv+matrix(rnorm(100,0,0.1),nrow=10,ncol=10)

#Perform a canonical correlation analysis

nc<-netcancor(y,x,reps=100)

summary(nc)
```

netlm

Linear Regression for Network Data

## **Description**

netlm regresses the network variable in y on the network variables in stack x using ordinary least squares. The resulting fits (and coefficients) are then tested against the indicated null hypothesis.

### Usage

```
netlm(y, x, intercept=TRUE, mode="digraph", diag=FALSE,
   nullhyp=c("qap", "qapspp", "qapy", "qapx", "qapallx",
   "cugtie", "cugden", "cuguman", "classical"),
   test.statistic = c("t-value", "beta"), tol=1e-7,
   reps=1000)
```

# Arguments

У	dependent network variable. This should be a matrix, for obvious reasons; NAs are allowed, but dichotomous data is strongly discouraged due to the assumptions of the analysis.
х	stack of independent network variables. Note that NAs are permitted, as is dichotomous data.
intercept	logical; should an intercept term be added?
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.

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diag logical; should the diagonal be treated as valid data? Set this true if and only if

the data can contain loops. diag is FALSE by default.

nullhyp string indicating the particular null hypothesis against which to test the observed

estimands.

test.statistic string indicating the test statistic to be used for the Monte Carlo procedures.

tol tolerance parameter for gr. solve.

reps integer indicating the number of draws to use for quantile estimation. (Relevant

to the null hypothesis test only - the analysis itself is unaffected by this parameter.) Note that, as for all Monte Carlo procedures, convergence is slower for

more extreme quantiles. By default, reps=1000.

#### **Details**

netlm performs an OLS linear network regression of the graph y on the graphs in x. Network regression using OLS is directly analogous to standard OLS regression elementwise on the appropriately vectorized adjacency matrices of the networks involved. In particular, the network regression attempts to fit the model:

$$\mathbf{A}_{\mathbf{v}} = b_0 \mathbf{A}_1 + b_1 \mathbf{A}_{\mathbf{x}_1} + b_2 \mathbf{A}_{\mathbf{x}_2} + \ldots + \mathbf{Z}$$

where  $A_y$  is the dependent adjacency matrix,  $A_{x_i}$  is the ith independent adjacency matrix,  $A_1$  is an n x n matrix of 1's, and Z is an n x n matrix of independent normal random variables with mean 0 and variance  $\sigma^2$ . Clearly, this model is nonoptimal when  $A_y$  is dichotomous (or, for that matter, categorical in general); an alternative such as netlogit should be employed in such cases. (Note that netlm will still attempt to fit such data...the user should consider him or herself to have been warned.)

Because of the frequent presence of row/column/block autocorrelation in network data, classical hull hypothesis tests (and associated standard errors) are generally suspect. Further, it is sometimes of interest to compare fitted parameter values to those arising from various baseline models (e.g., uniform random graphs conditional on certain observed statistics). The tests supported by netlm are as follows:

classical tests based on classical asymptotics.

cug conditional uniform graph test (see cugtest) controlling for order.

cugden conditional uniform graph test, controlling for order and density.

cugtie conditional uniform graph test, controlling for order and tie distribution.

qap QAP permutation test (see qaptest); currently identical to qapspp.

qapallx QAP permutation test, using independent x-permutations.

qapspp QAP permutation test, using Dekker's "semi-partialling plus" procedure.

qapx QAP permutation test, using (single) x-permutations.

qapy QAP permutation test, using y-permutations.

The statistic to be employed in the above tests may be selected via test.statistic. By default, the *t*-statistic (rather than estimated coefficient) is used, as this is more approximately pivotal;

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coefficient-based tests are not recommended for QAP null hypotheses, although they are provided here for legacy purposes.

Note that interpretation of quantiles for single coefficients can be complex in the presence of multicollinearity or third variable effects. qapspp is generally recommended for most multivariable analyses, as it is known to be fairly robust to these conditions. Reasonable printing and summarizing of netlm objects is provided by print.netlm and summary.netlm, respectively. No plot methods exist at this time, alas.

### Value

An object of class netlm

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Dekker, D.; Krackhardt, D.; Snijders, T.A.B. (2007). "Sensitivity of MRQAP Tests to Collinearity and Autocorrelation Conditions." *Psychometrika*, 72(4), 563-581.

Dekker, D.; Krackhardt, D.; Snijders, T.A.B. (2003). "Mulicollinearity Robust QAP for Multiple Regression." CASOS Working Paper, Carnegie Mellon University.

Krackhardt, D. (1987). "QAP Partialling as a Test of Spuriousness." Social Networks, 9 171-186.

Krackhardt, D. (1988). "Predicting With Networks: Nonparametric Multiple Regression Analyses of Dyadic Data." *Social Networks*, 10, 359-382.

#### See Also

```
lm, netlogit
```

## **Examples**

```
#Create some input graphs
x<-rgraph(20,4)

#Create a response structure
y<-x[1,,]+4*x[2,,]+2*x[3,,]  #Note that the fourth graph is unrelated

#Fit a netlm model
nl<-netlm(y,x,reps=100)

#Examine the results
summary(nl)</pre>
```

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net	logit

Logistic Regression for Network Data

## **Description**

netlogit performs a logistic regression of the network variable in y on the network variables in set x. The resulting fits (and coefficients) are then tested against the indicated null hypothesis.

## Usage

```
netlogit(y, x, intercept=TRUE, mode="digraph", diag=FALSE,
    nullhyp=c("qap", "qapspp", "qapy", "qapx", "qapallx",
    "cugtie", "cugden", "cuguman", "classical"), test.statistic =
    c("z-value", "beta"), tol=1e-7, reps=1000)
```

## **Arguments**

8	
У	dependent network variable. NAs are allowed, and the data should be dichotomous.
X	the stack of independent network variables. Note that NAs are permitted, as is dichotomous data.
intercept	logical; should an intercept term be fitted?
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
nullhyp	string indicating the particular null hypothesis against which to test the observed estimands.
test.statistic	string indicating the test statistic to be used for the Monte Carlo procedures.
tol	tolerance parameter for qr.solve.
reps	integer indicating the number of draws to use for quantile estimation. (Relevant to the null hypothesis test only – the analysis itself is unaffected by this parameter.) Note that, as for all Monte Carlo procedures, convergence is slower for more extreme quantiles. By default, $reps=1000$ .

### **Details**

netlogit is primarily a front-end to the built-in glm.fit routine. netlogit handles vectorization, sets up glm options, and deals with null hypothesis testing; the actual fitting is taken care of by glm.fit.

Logistic network regression using is directly analogous to standard logistic regression elementwise on the appropriately vectorized adjacency matrices of the networks involved. As such, it is often a more appropriate model for fitting dichotomous response networks than is linear network regression.

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Because of the frequent presence of row/column/block autocorrelation in network data, classical hull hypothesis tests (and associated standard errors) are generally suspect. Further, it is sometimes of interest to compare fitted parameter values to those arising from various baseline models (e.g., uniform random graphs conditional on certain observed statistics). The tests supported by netlogit are as follows:

classical tests based on classical asymptotics.

cug conditional uniform graph test (see cugtest) controlling for order.

cugden conditional uniform graph test, controlling for order and density.

cugtie conditional uniform graph test, controlling for order and tie distribution.

qap QAP permutation test (see qaptest); currently identical to qapspp.

qapallx QAP permutation test, using independent x-permutations.

qapspp QAP permutation test, using Dekker's "semi-partialling plus" procedure.

qapx QAP permutation test, using (single) x-permutations.

qapy QAP permutation test, using y-permutations.

Note that interpretation of quantiles for single coefficients can be complex in the presence of multicollinearity or third variable effects. Although qapspp is known to be robust to these conditions in the OLS case, there are no equivalent results for logistic regression. Caution is thus advised.

The statistic to be employed in the above tests may be selected via test.statistic. By default, the z-statistic (rather than estimated coefficient) is used, as this is more approximately pivotal; coefficient-based tests are not recommended for QAP null hypotheses, although they are provided here for legacy purposes.

Reasonable printing and summarizing of netlogit objects is provided by print.netlogit and summary.netlogit, respectively. No plot methods exist at this time.

#### Value

An object of class netlogit

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS working paper, Carnegie Mellon University.

## See Also

glm, netlm

npostpred npostpred

### **Examples**

npostpred

Take Posterior Predictive Draws for Functions of Networks

# Description

npostpred takes a list or data frame, b, and applies the function FUN to each element of b's net member.

## Usage

```
npostpred(b, FUN, ...)
```

### **Arguments**

b	A list or data frame containing posterior network draws; these draws must take
	the form of a graph stack, and must be the member of b referenced by "net"
FUN	Function for which posterior predictive is to be estimated
	Additional arguments to FUN

#### **Details**

Although created to work with bbnam, npostpred is quite generic. The form of the posterior draws will vary with the output of FUN; since invocation is handled by apply, check there if unsure.

## Value

A series of posterior predictive draws

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### Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Gelman, A.; Carlin, J.B.; Stern, H.S.; and Rubin, D.B. (1995). *Bayesian Data Analysis*. London: Chapman and Hall.

### See Also

bbnam

# **Examples**

```
#Create some random data
g<-rgraph(5)
g.p < -0.8 * g + 0.2 * (1 - g)
dat<-rgraph(5,5,tprob=g.p)</pre>
#Define a network prior
pnet<-matrix(ncol=5,nrow=5)</pre>
pnet[,]<-0.5
#Define em and ep priors
pem<-matrix(nrow=5,ncol=2)</pre>
pem[,1]<-3
pem[,2]<-5
pep<-matrix(nrow=5,ncol=2)</pre>
pep[,1]<-3
pep[,2]<-5
#Draw from the posterior
b<-bbnam(dat,model="actor",nprior=pnet,emprior=pem,epprior=pep,
    burntime=100,draws=100)
#Plot a summary of the posterior predictive of reciprocity
hist(npostpred(b,grecip))
```

nties

Find the Number of Possible Ties in a Given Graph or Graph Stack

## **Description**

nties returns the number of possible edges in each element of dat, given mode and diag.

## Usage

```
nties(dat, mode="digraph", diag=FALSE)
```

numperm

### **Arguments**

dat a graph or set thereof.

mode one of "digraph", "graph", and "hgraph".

diag a boolean indicating whether or not diagonal entries (loops) should be treated as

valid data; ignored for hypergraphic ("hgraph") data.

#### **Details**

nties is used primarily to automate maximum edge counts for use with normalization routines.

### Value

The number of possible edges, or a vector of the same

#### Note

For two-mode (hypergraphic) data, the value returned isn't technically the number of edges per se, but rather the number of edge memberships.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

## **Examples**

```
#How many possible edges in a loopless digraph of order 15?
nties(rgraph(15),diag=FALSE)
```

numperm

Get the nth Permutation Vector by Periodic Placement

# Description

numperm implicitly numbers all permutations of length olength, returning the permuumth of these.

# Usage

```
numperm(olength, permnum)
```

# **Arguments**

olength The number of items to permute

permnum The number of the permutation to use (in 1:olength!)

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### **Details**

The n! permutations on n items can be deterministically ordered via a factorization process in which there are n slots for the first element, n-1 for the second, and n-i for the ith. This fact is quite handy if you want to visit each permutation in turn, or if you wish to sample without replacement from the set of permutations on some number of elements: one just enumerates or samples from the integers on [1,n!], and then find the associated permutation. numperm performs exactly this last operation, returning the permutation on olength items.

### Value

A permutation vector

#### Note

Permutation search is central to the estimation of structural distances, correlations, and covariances on partially labeled graphs. numperm is hence used by structdist, gscor, gscov, etc.

#### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

#### See Also

```
rperm, rmperm
```

## **Examples**

```
#Draw a graph
g<-rgraph(5)

#Permute the rows and columns
p.1<-numperm(5,1)
p.2<-numperm(5,2)
p.3<-numperm(5,3)
g[p.1,p.1]
g[p.2,p.2]
g[p.3,p.3]</pre>
```

plot.bbnam

Plotting for bbnam Objects

## **Description**

Generates various plots of posterior draws from the bbnam model.

## Usage

```
## S3 method for class 'bbnam'
plot(x, mode="density", intlines=TRUE, ...)
```

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### **Arguments**

x A bbnam object
mode "density" for kernel density estimators of posterior marginals; otherwise, his-

tograms are used

intlines Plot lines for the 0.9 central posterior probability intervals?

. . . Additional arguments to plot

## **Details**

plot.bbnam provides plots of the estimated posterior marginals for the criterion graph and error parameters (as appropriate). Plotting may run into difficulties when dealing with large graphs, due to the problem of getting all of the various plots on the page; the routine handles these issues reasonably intelligently, but there is doubtless room for improvement.

### Value

None

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T. (1999). "Informant (In)Accuracy and Network Estimation: A Bayesian Approach." CASOS Working Paper, Carnegie Mellon University.

### See Also

bbnam

# Examples

```
#Create some random data
g<-rgraph(5)
g.p<-0.8*g+0.2*(1-g)
dat<-rgraph(5,5,tprob=g.p)

#Define a network prior
pnet<-matrix(ncol=5,nrow=5)
pnet[,]<-0.5
#Define em and ep priors
pem<-matrix(nrow=5,ncol=2)
pem[,1]<-3
pem[,2]<-5
pep<-matrix(nrow=5,ncol=2)
pep[,1]<-3
pep[,2]<-5
#Draw from the posterior</pre>
```

plot.blockmodel 159

plot.blockmodel

Plotting for blockmodel Objects

## **Description**

Displays a plot of the blocked data matrix, given a blockmodel object.

## Usage

```
## S3 method for class 'blockmodel' plot(x, ...)
```

### **Arguments**

x An object of class blockmodel

... Further arguments passed to or from other methods

#### **Details**

Plots of the blocked data matrix (i.e., the data matrix with rows and columns permuted to match block membership) can be useful in assessing the strength of the block solution (particularly for clique detection and/or regular equivalence).

#### Value

None

### Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

White, H.C.; Boorman, S.A.; and Breiger, R.L. (1976). "Social Structure from Multiple Networks I: Blockmodels of Roles and Positions." *American Journal of Sociology*, 81, 730-779.

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

#### See Also

```
blockmodel, plot.sociomatrix
```

160 plot.cugtest

### **Examples**

plot.cugtest

Plotting for cugtest Objects

### **Description**

Plots the distribution of a CUG test statistic.

#### Usage

```
## S3 method for class 'cugtest'
plot(x, mode="density", ...)
```

## **Arguments**

x A cugtest objectmode "density" for kernel density estimation, "hist" for histogram... Additional arguments to plot

#### **Details**

In addition to the quantiles associated with a CUG test, it is often useful to examine the form of the distribution of the test statistic. plot.cugtest facilitates this.

#### Value

None

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Anderson, B.S.; Butts, C.T.; and Carley, K.M. (1999). "The Interaction of Size and Density with Graph-Level Indices." *Social Networks*, 21(3), 239-267.

plot.equiv.clust 161

### See Also

```
cugtest
```

## **Examples**

```
#Draw two random graphs, with different tie probabilities
dat<-rgraph(20,2,tprob=c(0.2,0.8))

#Is their correlation higher than would be expected, conditioning
#only on size?
cug<-cugtest(dat,gcor,cmode="order")
summary(cug)
plot(cug)

#Now, let's try conditioning on density as well.
cug<-cugtest(dat,gcor)
plot(cug)</pre>
```

plot.equiv.clust

Plot an equiv.clust Object

### **Description**

Plots a hierarchical clustering of node positions as generated by equiv.clust.

### Usage

```
## S3 method for class 'equiv.clust'
plot(x, labels=NULL, ...)
```

## **Arguments**

x An equiv.clust objectlabels A vector of vertex labels... Additional arguments to plot.hclust

## **Details**

plot.equiv.clust is actually a front-end to plot.hclust; see the latter for more additional documentation.

#### Value

None.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

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### References

Breiger, R.L.; Boorman, S.A.; and Arabie, P. (1975). "An Algorithm for Clustering Relational Data with Applications to Social Network Analysis and Comparison with Multidimensional Scaling." *Journal of Mathematical Psychology*, 12, 328-383.

Burt, R.S. (1976). "Positions in Networks." Social Forces, 55, 93-122.

Wasserman, S., and Faust, K. Social Network Analysis: Methods and Applications. Cambridge: Cambridge University Press.

#### See Also

```
equiv.clust, plot.hclust
```

### **Examples**

plot.lnam

Plotting for lnam Objects

## **Description**

Generates various diagnostic plots for lnam objects.

## Usage

```
## S3 method for class 'lnam' plot(x, ...)
```

### Arguments

x an object of class 1nam.... additional arguments to plot.

#### Value

None

### Author(s)

Carter T. Butts <buttsc@uci.edu>

plot.qaptest 163

### See Also

1nam

plot.qaptest

Plotting for qaptest Objects

## **Description**

Plots the Distribution of a QAP Test Statistic.

## Usage

```
## S3 method for class 'qaptest'
plot(x, mode="density", ...)
```

## **Arguments**

x A qaptest object

mode "density" for kernel density estimation, "hist" for histogram

... Additional arguments to plot

## **Details**

In addition to the quantiles associated with a QAP test, it is often useful to examine the form of the distribution of the test statistic. plot.qaptest facilitates this.

## Value

None

### Author(s)

Carter T. Butts <buttsc@uci.edu>

## References

Hubert, L.J., and Arabie, P. (1989). "Combinatorial Data Analysis: Confirmatory Comparisons Between Sets of Matrices." *Applied Stochastic Models and Data Analysis*, 5, 273-325.

Krackhardt, D. (1987). "QAP Partialling as a Test of Spuriousness." Social Networks, 9 171-186.

Krackhardt, D. (1988). "Predicting With Networks: Nonparametric Multiple Regression Analyses of Dyadic Data." *Social Networks*, 10, 359-382.

### See Also

qaptest

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### **Examples**

```
#Generate three graphs
g<-array(dim=c(3,10,10))
g[1,,]<-rgraph(10)
g[2,,]<-rgraph(10,tprob=g[1,,]*0.8)
g[3,,]<-1; g[3,1,2]<-0 #This is nearly a clique

#Perform qap tests of graph correlation
q.12<-qaptest(g,gcor,g1=1,g2=2)
q.13<-qaptest(g,gcor,g1=1,g2=3)

#Examine the results
summary(q.12)
plot(q.12)
summary(q.13)
plot(q.13)</pre>
```

plot.sociomatrix

Plot Matrices Using a Color/Intensity Grid

## **Description**

Plots a matrix, m, associating the magnitude of the i,jth cell of m with the color of the i,jth cell of an nrow(m) by ncol(m) grid.

### Usage

```
## S3 method for class 'sociomatrix'
plot(x, labels=NULL, drawlab=TRUE, diaglab=TRUE,
    drawlines=TRUE, xlab=NULL, ylab=NULL, cex.lab=1, ...)
sociomatrixplot(x, labels=NULL, drawlab=TRUE, diaglab=TRUE,
    drawlines=TRUE, xlab=NULL, ylab=NULL, cex.lab=1, ...)
```

### **Arguments**

X	an input graph.
labels	a list containing the vectors of row and column labels (respectively); defaults to numerical labels.
drawlab	logical; add row/column labels to the plot?
diaglab	logical; label the diagonal?
drawlines	logical; draw lines to mark cell boundaries?
xlab	x axis label.
ylab	y axis label.
cex.lab	optional expansion factor for labels.
	additional arguments to plot.

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### **Details**

plot.sociomatrix is particularly valuable for examining large adjacency matrices, whose structure can be non-obvious otherwise. sociomatrixplot is an alias to plot.sociomatrix, and may eventually supersede it.

### Value

None

### Author(s)

Carter T. Butts <buttsc@uci.edu>

### See Also

```
plot.blockmodel
```

## **Examples**

```
#Plot a small adjacency matrix
plot.sociomatrix(rgraph(5))

#Plot a much larger one
plot.sociomatrix(rgraph(100),drawlab=FALSE,diaglab=FALSE)
```

potscalered.mcmc

Compute Gelman and Rubin's Potential Scale Reduction Measure for a Markov Chain Monte Carlo Simulation

# Description

Computes Gelman and Rubin's (simplified) measure of scale reduction for draws of a single scalar estimand from parallel MCMC chains.

## Usage

```
potscalered.mcmc(psi)
```

### **Arguments**

psi

An nxm matrix, with columns corresponding to chains and rows corresponding to iterations.

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#### **Details**

The Gelman and Rubin potential scale reduction ( $\sqrt{\hat{R}}$ ) provides an ANOVA-like comparison of the between-chain to within-chain variance on a given scalar estimand; the disparity between these gives an indication of the extent to which the scale of the simulated distribution can be reduced via further sampling. As the parallel chains converge  $\sqrt{\hat{R}}$  approaches 1 (from above), and it is generally recommended that values of 1.2 or less be obtained before a series of draws can be considered well-mixed. (Even so, one should ideally examine other indicators of chain mixing, and verify that the properties of the draws are as they should be. There is currently no fool-proof way to verify burn-in of an MCMC, but using multiple indicators should help one avoid falling prey to the idiosyncrasies of any one index.)

Note that the particular estimators used in the  $\sqrt{\hat{R}}$  formulation are based on normal-theory results, and as such have been criticized vis a vis their behavior on other distributions. Where simulating distributions whose properties differ greatly from the normal, an alternative form of the measure using robust measures of scale (e.g., the IQR) may be preferable.

#### Value

The potential scale reduction measure

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Gelman, A.; Carlin, J.B.; Stern, H.S.; and Rubin, D.B. (1995). *Bayesian Data Analysis*. London: Chapman and Hall.

Gelman, A., and Rubin, D.B. (1992). "Inference from Iterative Simulation Using Multiple Sequences." *Statistical Science*, 7, 457-511.

### See Also

bbnam

prestige

Calculate the Vertex Prestige Scores

### **Description**

prestige takes one or more graphs (dat) and returns the prestige scores of positions (selected by nodes) within the graphs indicated by g. Depending on the specified mode, prestige based on any one of a number of different definitions will be returned. This function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

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### Usage

### **Arguments**

dat	one or more input graphs.
g	integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g==1.
nodes	vector indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. gmode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
cmode	one of "indegree", "indegree.rownorm", "indegree.rowcolnorm", "eigenvector", "eigenvector.rownorm", "eigenvector.colnorm", "eigenvector.rowcolnorm", "domain", or "domain.proximity".
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
rescale	if true, centrality scores are rescaled such that they sum to 1.
tol	Currently ignored

### **Details**

"Prestige" is the name collectively given to a range of centrality scores which focus on the extent to which one is nominated by others. The definitions supported here are as follows:

- 1. indegree: indegree centrality
- 2. indegree.rownorm: indegree within the row-normalized graph
- 3. indegree.rowcolnorm: indegree within the row-column normalized graph
- 4. eigenvector: eigenvector centrality within the transposed graph (i.e., incoming ties recursively determine prestige)
- 5. eigenvector.rownorm: eigenvector centrality within the transposed row-normalized graph
- 6. eigenvector.colnorm: eigenvector centrality within the transposed column-normalized graph
- 7. eigenvector.rowcolnorm: eigenvector centrality within the transposed row/column-normalized graph
- 8. domain: indegree within the reachability graph (Lin's unweighted measure)
- 9. domain.proximity: Lin's proximity-weighted domain prestige

Note that the centralization of prestige is simply the extent to which one actor has substantially greater prestige than others; the underlying definition is the same.

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### Value

A vector, matrix, or list containing the prestige scores (depending on the number and size of the input graphs).

# Warning

Making adjacency matrices doubly stochastic (row-column normalization) is not guaranteed to work. In general, be wary of attempting to try normalizations on graphs with degenerate rows and columns.

## Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

#### References

Lin, N. (1976). Foundations of Social Research. New York: McGraw Hill.

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

#### See Also

```
centralization
```

## **Examples**

print.bayes.factor

Printing for Bayes Factor Objects

### **Description**

Prints a quick summary of a Bayes Factor object.

# Usage

```
## S3 method for class 'bayes.factor'
print(x, ...)
```

## Arguments

x An object of class bayes. factor

... Further arguments passed to or from other methods

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# Value

None

# Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

bbnam.bf

print.bbnam

Printing for bbnam Objects

# Description

Prints a quick summary of posterior draws from bbnam.

# Usage

```
## S3 method for class 'bbnam'
print(x, ...)
```

# Arguments

x A bbnam object

... Further arguments passed to or from other methods

## Value

None

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

bbnam

print.cugtest

print.blockmodel

Printing for blockmodel Objects

# Description

Prints a quick summary of a blockmodel object.

## Usage

```
## S3 method for class 'blockmodel'
print(x, ...)
```

## **Arguments**

x An object of class blockmodel

. . . Further arguments passed to or from other methods

### Value

None

### Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

blockmodel

print.cugtest

Printing for cugtest Objects

# Description

Prints a quick summary of objects produced by cugtest.

# Usage

```
## S3 method for class 'cugtest' print(x, ...)
```

## **Arguments**

x An object of class cugtest

... Further arguments passed to or from other methods

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# Value

None.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

cugtest

print.lnam

Printing for lnam Objects

# Description

Prints an objsect of class 1nam

# Usage

```
## S3 method for class 'lnam'
print(x, digits = max(3, getOption("digits") - 3), ...)
```

# Arguments

x an object of class 1nam.digits number of digits to display.additional arguments.

## Value

None.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

# See Also

1nam

print.netlm

print.netcancor

Printing for netcancor Objects

# Description

Prints a quick summary of objects produced by netcancor.

### Usage

```
## S3 method for class 'netcancor'
print(x, ...)
```

## **Arguments**

x An object of class netcancor

... Further arguments passed to or from other methods

### Author(s)

Carter T. Butts <buttsc@uci.edu>

### See Also

netcancor

print.netlm

Printing for netlm Objects

# Description

Prints a quick summary of objects produced by netlm.

# Usage

```
## S3 method for class 'netlm'
print(x, ...)
```

### **Arguments**

x An object of class netlm

... Further arguments passed to or from other methods

# Author(s)

Carter T. Butts <buttsc@uci.edu>

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## See Also

netlm

print.netlogit

Printing for netlogit Objects

# **Description**

Prints a quick summary of objects produced by netlogit.

## Usage

```
## S3 method for class 'netlogit' print(x, ...)
```

# Arguments

x An object of class netlogit

... Further arguments passed to or from other methods

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

netlogit

print.qaptest

Printing for qaptest Objects

# Description

Prints a quick summary of objects produced by qaptest.

## Usage

```
## S3 method for class 'qaptest' print(x, ...)
```

# Arguments

x An object of class qaptest

... Further arguments passed to or from other methods

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### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### See Also

qaptest

```
print.summary.bayes.factor
```

Printing for summary.bayes.factor Objects

# Description

Prints an object of class summary.bayes.factor.

# Usage

```
## S3 method for class 'summary.bayes.factor' print(x, ...)
```

## **Arguments**

x An object of class summary.bayes.factor

... Further arguments passed to or from other methods

# Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

```
summary.bayes.factor
```

print.summary.bbnam

Printing for summary.bbnam Objects

## **Description**

Prints an object of class summary.bbnam.

## Usage

```
## S3 method for class 'summary.bbnam' print(x, ...)
```

## **Arguments**

x An object of class summary.bbnam

... Further arguments passed to or from other methods

### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

## See Also

bbnam

```
print.summary.blockmodel
```

Printing for summary.blockmodel Objects

# Description

Prints an object of class summary.blockmodel.

# Usage

```
## S3 method for class 'summary.blockmodel'
print(x, ...)
```

# Arguments

- x An object of class summary.blockmodel
- ... Further arguments passed to or from other methods

# Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

### See Also

```
summary.blockmodel
```

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```
print.summary.cugtest Printing for summary.cugtest Objects
```

# Description

Prints an object of class summary.cugtest.

### Usage

```
## S3 method for class 'summary.cugtest'
print(x, ...)
```

# Arguments

x An object of class summary.cugtest

... Further arguments passed to or from other methods

### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

#### See Also

```
summary.cugtest
```

print.summary.lnam

Printing for summary.lnam Objects

## **Description**

Prints an object of class summary. 1nam.

## Usage

# Arguments

```
x an object of class summary.lnam.digits number of digits to display.signif.stars show significance stars?additional arguments.
```

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# Value

None

# Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

```
summary.lnam, lnam
```

```
print.summary.netcancor
```

Printing for summary.netcancor Objects

# Description

Prints an object of class summary.netcancor.

# Usage

```
## S3 method for class 'summary.netcancor' print(x, ...)
```

## Arguments

x An object of class summary.netcancor

... Further arguments passed to or from other methods

## Author(s)

Carter T. Butts <buttsc@uci.edu>

# See Also

```
summary.netcancor
```

print.summary.netlm

Printing for summary.netlm Objects

# Description

Prints an object of class summary.netlm.

## Usage

```
## S3 method for class 'summary.netlm' print(x, ...)
```

## **Arguments**

x An object of class summary.netlm

... Further arguments passed to or from other methods

### Author(s)

Carter T. Butts <buttsc@uci.edu>

### See Also

```
summary.netlm
```

```
print.summary.netlogit
```

Printing for summary.netlogit Objects

# Description

Prints an object of class summary.netlogit.

## Usage

```
## S3 method for class 'summary.netlogit' print(x, ...)
```

## **Arguments**

x An object of class summary.netlogit~

... Further arguments passed to or from other methods

# Author(s)

Carter T. Butts <buttsc@uci.edu>

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### See Also

```
summary.netlogit
```

```
print.summary.qaptest Printing for summary.qaptest Objects
```

### **Description**

Prints an object of class summary.qaptest.

### Usage

```
## S3 method for class 'summary.qaptest'
print(x, ...)
```

## **Arguments**

x An object of class summary.qaptest

Further arguments passed to or from other methods

### Author(s)

Carter T. Butts <buttsc@uci.edu>

### See Also

summary.qaptest

pstar

Fit a p\*/ERG Model Using a Logistic Approximation

## **Description**

Fits a p\*/ERG model to the graph in dat containing the effects listed in effects. The result is returned as a glm object.

### Usage

```
pstar(dat, effects=c("choice", "mutuality", "density", "reciprocity",
    "stransitivity", "wtransitivity", "stranstri", "wtranstri",
    "outdegree", "indegree", "betweenness", "closeness",
    "degcentralization", "betcentralization", "clocentralization",
    "connectedness", "hierarchy", "lubness", "efficiency"),
    attr=NULL, memb=NULL, diag=FALSE, mode="digraph")
```

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#### **Arguments**

dat a single graph

effects a vector of strings indicating which effects should be fit.

attr a matrix whose columns contain individual attributes (one row per vertex) whose differences should be used as supplemental predictors.

memb a matrix whose columns contain group memberships whose categorical similarities (same group/not same group) should be used as supplemental predictors.

diag a boolean indicating whether or not diagonal entries (loops) should be counted as meaningful data.

mode "digraph" if dat is directed, else "graph"

#### **Details**

p\* (also called the Exponential Random Graph (ERG) family) is an exponential family specification for network data. Under p\*, it is assumed that

$$p(G = g) \propto \exp(\beta_0 \gamma_0(g) + \beta_1 \gamma_1(g) + \ldots)$$

for all g, where the betas represent real coefficients and the gammas represent functions of g. Unfortunately, the unknown normalizing factor in the above expression makes evaluation difficult in the general case. One solution to this problem is to operate instead on the edgewise log odds; in this case, the p\* MLE can be approximated by a logistic regression of each edge on the *differences* in the gamma scores induced by the presence and absence of said edge in the graph (conditional on all other edges). It is this approximation (known as autologistic regression, or maximum pseudo-likelihood estimation) which is employed here.

Using the effects argument, a range of different potential parameters can be estimated. The network measure associated with each is, in turn, the edge-perturbed difference in:

- 1. choice: the number of edges in the graph (acts as a constant)
- 2. mutuality: the number of reciprocated dyads in the graph
- 3. density: the density of the graph
- 4. reciprocity: the edgewise reciprocity of the graph
- 5. stransitivity: the strong transitivity of the graph
- 6. wtransitivity: the weak transitivity of the graph
- 7. stranstri: the number of strongly transitive triads in the graph
- 8. wtranstri: the number of weakly transitive triads in the graph
- 9. outdegree: the outdegree of each actor (IVI parameters)
- 10. indegree: the indegree of each actor (|V| parameters)
- 11. betweenness: the betweenness of each actor (IVI parameters)
- 12. closeness: the closeness of each actor (IVI parameters)
- 13. degcentralization: the Freeman degree centralization of the graph
- 14. betcentralization: the betweenness centralization of the graph
- 15. clocentralization: the closeness centralization of the graph

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- 16. connectedness: the Krackhardt connectedness of the graph
- 17. hierarchy: the Krackhardt hierarchy of the graph
- 18. efficiency: the Krackhardt efficiency of the graph
- 19. lubness: the Krackhardt LUBness of the graph

(Note that some of these do differ somewhat from the common p\* parameter formulation, e.g. quantities such as density and reciprocity are computed as per the gden and grecip functions rather than via the unnormalized "choice" and "mutual" quantities one often finds in the p\* literature.) *Please do not attempt to use all effects simultaneously!!!* In addition to the above, the user may specify a matrix of individual attributes whose absolute dyadic differences are to be used as predictors, as well as a matrix of individual memberships whose dyadic categorical similarities (same/different) are used in the same manner.

Although the p\* framework is quite versatile in its ability to accommodate a range of structural predictors, it should be noted that the *substantial* collinearity of many of the standard p\* predictors can lead to very unstable model fits. Measurement and specification errors compound this problem; thus, it is somewhat risky to use p\* in an exploratory capacity (i.e., when there is little prior knowledge to constrain choice of parameters). While raw instability due to multicollinearity should decline with graph size, improper specification will still result in biased coefficient estimates so long as an omitted predictor correlates with an included predictor. Caution is advised.

#### Value

A glm object

#### WARNING

Estimation of p\* models by maximum pseudo-likelihood is now known to be a dangerous practice. Use at your own risk.

#### Note

In the long run, support will be included for p\* models involving arbitrary functions (much like the system used with cugtest and qaptest).

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Anderson, C.; Wasserman, S.; and Crouch, B. (1999). "A p\* Primer: Logit Models for Social Networks. *Social Networks*, 21,37-66.

Holland, P.W., and Leinhardt, S. (1981). "An Exponential Family of Probability Distributions for Directed Graphs." *Journal of the American statistical Association*, 81, 51-67.

Wasserman, S., and Pattison, P. (1996). "Logit Models and Logistic Regressions for Social Networks: I. An introduction to Markov Graphs and p\*." *Psychometrika*, 60, 401-426.

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# See Also

```
eval.edgeperturbation
```

## **Examples**

```
#Create a graph with expansiveness and popularity effects
in.str < -rnorm(20,0,3)
out.str<-rnorm(20,0,3)
tie.str<-outer(out.str,in.str,"+")</pre>
tie.p<-apply(tie.str,c(1,2),function(a)\{1/(1+exp(-a))\})
g<-rgraph(20,tprob=tie.p)</pre>
#Fit a model with expansiveness only
p1<-pstar(g,effects="outdegree")</pre>
#Fit a model with expansiveness and popularity
p2<-pstar(g,effects=c("outdegree","indegree"))</pre>
#Fit a model with expansiveness, popularity, and mutuality
p3<-pstar(g,effects=c("outdegree","indegree","mutuality"))</pre>
#Compare the model AICs -- use ONLY as heuristics!!!
extractAIC(p1)
extractAIC(p2)
extractAIC(p3)
```

qaptest

Perform Quadratic Assignment Procedure (QAP) Hypothesis Tests for Graph-Level Statistics

# Description

qaptest tests an arbitrary graph-level statistic (computed on dat by FUN) against a QAP null hypothesis, via Monte Carlo simulation of likelihood quantiles. Note that fair amount of flexibility is possible regarding QAP tests on functions of such statistics (see an equivalent discussion with respect to CUG null hypothesis tests in Anderson et al. (1999)). See below for more details.

### Usage

```
qaptest(dat, FUN, reps=1000, ...)
```

# **Arguments**

dat	graphs to be analyzed. Though one could in principle use a single graph, this is rarely if ever sensible in a QAP-test context.
FUN	function to generate the test statistic. FUN must accept dat and the specified g arguments, and should return a real number.
reps	integer indicating the number of draws to use for quantile estimation. Note that, as for all Monte Carlo procedures, convergence is slower for more extreme quantiles. By default, reps=1000.
	additional arguments to FUN.

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#### **Details**

The null hypothesis of the QAP test is that the observed graph-level statistic on graphs  $G_1, G_2, \ldots$  was drawn from the distribution of said statistic evaluated (uniformly) on the set of all relabelings of  $G_1, G_2, \ldots$  Pragmatically, this test is performed by repeatedly (randomly) relabeling the input graphs, recalculating the test statistic, and then evaluating the fraction of draws greater than or equal to (and less than or equal to) the observed value. This accumulated fraction approximates the integral of the distribution of the test statistic over the set of unlabeled input graphs.

The qaptest procedure returns a qaptest object containing the estimated likelihood (distribution of the test statistic under the null hypothesis), the observed value of the test statistic on the input data, and the one-tailed p-values (estimated quantiles) associated with said observation. As usual, the (upper tail) null hypothesis is rejected for significance level alpha if p>=observation is less than alpha (or p<=observation, for the lower tail); if the hypothesis is undirected, then one rejects if either p<=observation or p>=observation is less then alpha/2. Standard caveats regarding the use of null hypothesis testing procedures are relevant here: in particular, bear in mind that a significant result does not necessarily imply that the likelihood ratio of the null model and the alternative hypothesis favors the latter.

In interpreting a QAP test, it is important to bear in mind the nature of the QAP null hypothesis. The QAP test should *not* be interpreted as evaluating underlying structural differences; indeed, QAP is more accurately understood as testing differences induced by a particular vertex labeling *controlling* for underlying structure. Where there is substantial automorphism in the underlying structures, QAP will tend to given non-significant results. (In fact, it is impossible to obtain a one-tailed significance level in excess of  $\max_{g \in \{G,H\}} \frac{|Aut(g)|}{|Perm(g)|}$  when using a QAP test on a bivariate graph statistic f(G, H), where Aut(g) and Perm(g) are the automorphism and permutation groups on g, respectively. This follows from the fact that all members of Aut(g) will induce the same values of f().) By turns, significance under QAP does not necessarily imply that the observed structural relationship is unusual relative to what one would expect from typical structures with (for instance) the sizes and densities of the graphs in question. In contexts in which one's research question implies a particular labeling of vertices (e.g., "within this group of individuals, do friends also tend to give advice to one another"), QAP can be a very useful way of ruling out spurious structural influences (e.g., some respondents tend to indiscriminately nominate many people (without regard to whom), resulting in a structural similarity which has nothing to do with the identities of those involved). Where one's question does not imply a labeled relationship (e.g., is the *shape* of this group's friendship network similar to that of its advice network), the QAP null hypothesis is inappropriate.

#### Value

An object of class qaptest, containing

testval The observed value of the test statistic.

dist A vector containing the Monte Carlo draws.

pgreq The proportion of draws which were greater than or equal to the observed value.

The proportion of draws which were less than or equal to the observed value.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

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### References

Anderson, B.S.; Butts, C.T.; and Carley, K.M. (1999). "The Interaction of Size and Density with Graph-Level Indices." *Social Networks*, 21(3), 239-267.

Hubert, L.J., and Arabie, P. (1989). "Combinatorial Data Analysis: Confirmatory Comparisons Between Sets of Matrices." *Applied Stochastic Models and Data Analysis*, 5, 273-325.

Krackhardt, D. (1987). "QAP Partialling as a Test of Spuriousness." Social Networks, 9 171-186.

Krackhardt, D. (1988). "Predicting With Networks: Nonparametric Multiple Regression Analyses of Dyadic Data." *Social Networks*, 10, 359-382.

#### See Also

cugtest

# **Examples**

reachability

Find the Reachability Matrix of a Graph

#### **Description**

reachability takes one or more (possibly directed) graphs as input, producing the associated reachability matrices.

# Usage

reachability(dat, geodist.precomp=NULL, return.as.edgelist=FALSE, na.omit=TRUE)

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# Arguments

```
dat one or more graphs (directed or otherwise).

geodist.precomp

optionally, a precomputed geodist object.

return.as.edgelist

logical; return the result as an sna edgelist?

na.omit logical; omit missing edges when computing reach?
```

#### **Details**

For a digraph G=(V,E) with vertices i and j, let  $P_{ij}$  represent a directed ij path. Then the (di)graph

$$R = (V(G), \{(i,j) : i, j \in V(G), P_{ij} \in G\})$$

is said to be the *reachability graph* of G, and the adjacency matrix of R is said to be G's *reachability matrix*. (Note that when G is undirected, we simply take each undirected edge to be bidirectional.) Vertices which are adjacent in the reachability graph are connected by one or more directed paths in the original graph; thus, structural equivalence classes in the reachability graph are synonymous with strongly connected components in the original structure.

Bear in mind that – as with all matters involving connectedness – reachability is strongly related to size and density. Since, for any given density, almost all structures of sufficiently large size are connected, reachability graphs associated with large structures will generally be complete. Measures based on the reachability graph, then, will tend to become degenerate in the large |V(G)| limit (assuming constant positive density).

By default, reachability will try to build the reachability graph using an internal sparse graph approximation; this is no help on fully connected graphs (but not a lot worse than using an adjacency matrix), but will result in considerable savings for large graphs that are heavily fragmented. (The intended design tradeoff is thus that one pays a small cost on the usually cheap cases, in exchange for much greater efficiency on the cases that would otherwise be prohibitively expensive.) If geodist.precomp is given, however, the  $O(N^2)$  cost of an adjacency matrix representation has already been paid, and we simply employ what we are given – so, if you want to force the internal use of adjacency matrices, just pass a geodist object. Because the internal representation used is otherwise list based, using return.as.edgelist=TRUE will save resources; if you are using reachability as part of a more complex series of calls, it is thus recommended that you both pass and return sna edgelists unless you have a good reason not to do so.

When set, na.omit results in missing edges (i.e., edges with NA values) being removed prior to computation. Since paths are not recomputed when geodist.precomp is passed, this option is only active when geodist.precomp==NULL; if this behavior is desired and precomputed distances are being used, such edges should be removed prior to the geodist call.

#### Value

A reachability matrix, or the equivalent edgelist representation

## Author(s)

Carter T. Butts <buttsc@uci.edu>

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### References

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

### See Also

```
geodist
```

# **Examples**

```
#Find the reachability matrix for a sparse random graph
g<-rgraph(10,tprob=0.15)
rg<-reachability(g)
g  #Compare the two structures
rg

#Compare to the output of geodist
all(rg==(geodist(g)$counts>0))
```

read.dot

Read Graphviz DOT Files

### **Description**

Reads network information in Graphviz's DOT format, returning an adjacency matrix.

# Usage

```
read.dot(...)
```

### **Arguments**

... The name of the file whence to import the data, or else a connection object (suitable for processing by readLines.

# Details

The Graphviz project's DOT language is a simple but flexible tool for describing graphs. See the included reference for details.

## Value

The imported graph, in adjacency matrix form.

# Author(s)

Matthijs den Besten <matthijs.denbesten@gmail.com>

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### References

Graphviz Project. "The DOT Language." http://www.graphviz.org/doc/info/lang.html

#### See Also

```
read.nos, write.nos, write.dl
```

read.nos

Read (N)eo-(O)rg(S)tat Input Files

# **Description**

Reads an input file in NOS format, returning the result as a graph set.

# Usage

```
read.nos(file, return.as.edgelist = FALSE)
```

# **Arguments**

```
file the file to be imported return.as.edgelist
```

logical; should the resulting graphs be returned in sna edgelist format?

# **Details**

NOS format consists of three header lines, followed by a whitespace delimited stack of raw adjacency matrices; the format is not particularly elegant, but turns up in certain legacy applications (mostly at CMU). read.nos provides a quick and dirty way of reading in these files, without the headache of messing with read.table settings.

The content of the NOS format is as follows:

```
<m>
<n> <o>
<hr/>
<n> <o>
<kr1> <kr2> ... <krn> <kc1> <kc2> ... <kcn>
<a111> <a112> ... <a110>
<a121> <a122> ... <a120>
...
<a1n1> <a1n2> ... <a1n0>
<a211> <a212> ... <a210>
...
<a2n1> <a2n2> ... <a2n0>
...
<amn1> <amn2> ... <amno>
```

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where <abcd> is understood to be the value of the c->d edge in the bth graph of the file. (As one might expect, m, n, and o are the numbers of graphs (matrices), rows, and columns for the data, respectively.) The "k" line contains a list of row and column "colors", categorical variables associated with each row and column, respectively. Although originally intended to communicate exchangability information, these can be used for other purposes (though there are easier ways to deal with attribute data these days).

### Value

The imported graph set (in adjacency array or edgelist form).

#### Note

read. nos currently ignores the coloring information.

### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

#### See Also

```
write.nos, scan, read.table
```

redist

Find a Matrix of Distances Between Positions Based on Regular Equivalence

# Description

redist uses the graphs indicated by g in dat to assess the extent to which each vertex is regularly equivalent; method determines the measure of approximate equivalence which is used (currently, only CATREGE).

### Usage

```
redist(dat, g = NULL, method = c("catrege"), mode = "digraph",
    diag = FALSE, seed.partition = NULL, code.diss = TRUE, ...)
```

# Arguments

dat	a graph or set thereof.
g	a vector indicating which elements of dat should be examined (by default, all are used).
method	method to use when assessing regular equivalence (currently, only "catrege").
mode	"digraph" for directed data, otherwise "graph".
diag	logical; should diagonal entries (loops) should be treated as meaningful data?

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```
seed.partition optionally, an initial equivalence partition to "seed" the CATREGE algorithm.

code.diss logical; return as dissimilarities (rather than similarities)?

additional parameters (currently ignored).
```

#### **Details**

redist provides a basic tool for assessing the (approximate) regular equivalence of actors. Two vertices i and j are said to be regularly equivalent with respect to role assignment r if  $\{r(u):u\in N^+(i)\}=\{r(u):u\in N^+(j)\}$  and  $\{r(u):u\in N^-(i)\}=\{r(u):u\in N^-(j)\}$ , where  $N^+$  and  $N^-$  denote out- and in-neighborhoods (respectively). RE similarity/difference scores are computed by method, currently Borgatti and Everett's CATREGE algorithm (which is based on the multiplex maximal regular equivalence on G and its transpose). The "distance" between positions in this case is the inverse of the number of iterative refinements of the initial equivalence (i.e., role) structure required to allocate the positions to regularly equivalent roles (with 0 indicating positions which ultimately belong in the same role). By default, the initial equivalence structure is one in which all vertices are treated as occupying the same role; the seed partition option can be used to impose alternative constraints. From this initial structure, vertices within the same role having non-identical mixes of neighbor types are re-allocated to different roles (where "neighbor type" is initially due to the pattern of (possibly valued) in- and out-ties, cross-classified by current alter type). This procedure is then iterated until no further division of roles is necessary to satisfy the regularity condition.

Once the similarities/differences are calculated, the results can be used with a clustering routine (such as equiv.clust) or an MDS (such as cmdscale) to identify the underlying role structure.

# Value

A matrix of similarity/difference scores.

#### Note

The maximal regular equivalence is often very uninteresting (i.e., degenerate) for unvalued, undirected graphs. An exogenous constraint (e.g., via the seed.partition) may be required to uncover a more useful refinement of the unconstrained maximal equivalence.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

# References

Borgatti, S.P. and Everett, M.G. (1993). "Two Algorithms for Computing Regular Equivalence." *Social Networks*, 15, 361-376.

## See Also

```
sedist, equiv.clust
```

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### **Examples**

rgbn

Draw from a Skvoretz-Fararo Biased Net Process

# Description

Produces a series of draws from a Skvoretz-Fararo biased net process using a (pseudo) Gibbs sampler or exact sampling procedure.

# Usage

```
rgbn(n, nv, param = list(pi=0, sigma=0, rho=0, d=0.5, delta=0),
  burn = nv*nv*5*100, thin = nv*nv*5, maxiter = 1e7,
  method = c("mcmc","cftp"), dichotomize.sib.effects = FALSE,
  return.as.edgelist = FALSE)
```

# **Arguments**

n	number of draws to take.
nv	number of vertices in the graph to be simulated.
param	a list containing the biased net parameters (as described below); $d$ may be given as a scalar or as an nv $$ x $$ nv matrix of edgewise baseline edge probabilities.
burn	for the Gibbs sampler, the number of burn-in draws to take (and discard).
thin	the thinning parameter for the Gibbs sampler.
maxiter	for the CFTP method, the number of iterations to try before giving up.
method	"mcmc" for the Gibbs sampler, or "cftp" for the exact sampling procedure.
dichotomize.sib.effects	
	logical; should sibling and double role effects be dichotomized?
return.as.edgelist	
	logical; should the simulated draws be returned in edgelist format?

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#### **Details**

The biased net model stems from early work by Rapoport, who attempted to model networks via a hypothetical "tracing" process. This process may be described loosely as follows. One begins with a small "seed" set of vertices, each member of which is assumed to nominate (generate ties to) other members of the population with some fixed probability. These members, in turn, may nominate new members of the population, as well as members who have already been reached. Such nominations may be "biased" in one fashion or another, leading to a non-uniform growth process.

While the original biased net model depends upon the tracing process, a local interpretation has been put forward by Skvoretz and colleagues in recent years. Using the standard four-parameter process, the conditional probability of an (i,j) edge given all other edges in a random graph G can be approximated as

$$\Pr(i \to j | G_{-ij}) \approx 1 - (1 - \rho)^z (1 - \sigma)^y (1 - \pi)^x (1 - d_{ij})$$

where x=1 iff  $j\to i$  (and 0 otherwise), y is the number of vertices  $k\neq i, j$  such that  $k\to i, k\to j$ , and z=1 iff x=1 and y>0 (and 0 otherwise). Thus, x is the number of potential parent bias events, y is the number of potential sibling bias events, and z is the number of potential double role bias events.  $d_{ij}$  is the probability of the baseline edge event; note that an edge arises if the baseline event or any bias event occurs, and all events are assumed conditionally independent. Written in this way, it is clear that the edges of G are conditionally independent if they share no endpoint. Thus, a model with the above structure should be a subfamily of the Markov graphs.

One potential problem with the above structure is that the hypothetical probabilities implied by the model are not guaranteed to be consistent - that is, the conditions under which there exists a joint pmf with the implied full conditionals are currently unknown (and may be restrictive). The interpretation of the above as exact conditional probabilities is thus potentially problematic. However, a well-defined process can be constructed by interpreting the above as transition probabilities for a Markov chain that evolves by updating a randomly selected edge variable at each time point; this is a Gibbs sampler for the implied joint pmf where it exists, and otherwise an irreducible and aperiodic Markov chain with a well-defined equilibrium distribution.

In the above process, all events act to promote the formation of edges; it is also possible to define events that inhibit them. For instance, consider a *satiation* event that, if it occurs, forbids the creation of an  $i \to j$  edge; we assume that a potential satiation event occurs every time i emits an edge to some other vertex. The associated approximate conditional (i.e., transition probability) is

$$\Pr(i \to j | G_{-ij}) \approx (1 - \delta)^w \left(1 - (1 - \rho)^z (1 - \sigma)^y (1 - \pi)^x (1 - d_{ij})\right)$$

where w is the outdegree of i in  $G_{-ij}$  and  $\delta$  is the probability of the satiation event. The net effect of satiation is to suppress edge formation (in roughly geometric fashion) on high degree nodes. This may be useful in preventing degeneracy when using sigma and rho effects. Degeneracy can also be reduced by employing the dichotomize.sib.effects argument, which counts only the first shared partner's contribution towards sibling and double role effects.

It should be noted that the above process is not entirely consistent with the tracing-based model, which is itself not uniformly well-specified in the literature. For this reason, the local model is referred to here as a Skvoretz-Fararo graph process. One significant advantage of this process is that it is well-defined, and easily simulated: the above equation can be used to form the basis of a (pseudo-) Gibbs sampler, which is used by rgbn to take draws from the (local) biased net

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model. Burn-in and thinning are controlled by the corresponding arguments; since degeneracy is common with models of this type, it is advisable to check for adequate mixing. An alternative simulation strategy is the exact sampling procedure of Butts (2009), which employs a form of coupling from the past (CFTP). The CFTP method generates exact, independent draws from the equilibrium distribution of the biased net process (up to numerical limits), but can be slow to attain coalescence (and does not currently support satiation events). Setting maxiter to smaller values limits the search depth employed, at the possible cost of biasing the resulting sample.

#### Value

An adjacency array containing the simulated graphs.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T. (2009). "A Perfect Sampling Method for Exponential Random Graph Models". Working paper, University of California, Irvine.

Rapoport, A. (1957). "A Contribution to the Theory of Random and Biased Nets." *Bulletin of Mathematical Biophysics*, 15, 523-533.

Skvoretz, J.; Fararo, T.J.; and Agneessens, F. (2004). "Advances in Biased Net Theory: Definitions, Derivations, and Estimations." *Social Networks*, 26, 113-139.

#### See Also

bn

# **Examples**

```
#Generate draws with low density and no biases g1<-rgbn(50,10,param=list(pi=0, sigma=0, rho=0, d=0.17)) apply(dyad.census(g1),2,mean) #Examine the dyad census

#Add a reciprocity bias g2<-rgbn(50,10,param=list(pi=0.5, sigma=0, rho=0, d=0.17)) apply(dyad.census(g2),2,mean) #Compare with g1

#Alternately, add a sibling bias g3<-rgbn(50,10,param=list(pi=0.0, sigma=0.3, rho=0, d=0.17)) mean(gtrans(g3)) #Compare transitivity scores mean(gtrans(g1))
```

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rgnm

Draw Density-Conditioned Random Graphs

## **Description**

rgnm generates random draws from a density-conditioned uniform random graph distribution.

### Usage

```
rgnm(n, nv, m, mode = "digraph", diag = FALSE,
    return.as.edgelist = FALSE)
```

## **Arguments**

n the number of graphs to generate.

nv the size of the vertex set (|V(G)|) for the random graphs.

m the number of edges on which to condition.

mode "digraph" for directed graphs, or "graph" for undirected graphs.

diag logical; should loops be allowed?

return.as.edgelist

logical; should the resulting graphs be returned in edgelist form?

#### **Details**

rgnm returns draws from the density-conditioned uniform random graph first popularized by the famous work of Erdos and Renyi (the G(N,M) process). In particular, the pmf of a G(N,M) process is given by

$$p(G = g|N, M) = \left(\frac{E_m}{M}\right)^{-1}$$

where  $E_m$  is the maximum number of edges in the graph. ( $E_m$  is equal to nv\*(nv-diag)/(1+(mode=="graph")).)

The G(N,M) process is one of several process which are used as baseline models of social structure. Other well-known baseline models include the Bernoulli graph (the G(N,p) model of Erdos and Renyi) and the UlMAN model of dyadic independence. These are implemented within sna as rgraph and rgnm, respectively.

### Value

A matrix or array containing the drawn adjacency matrices

#### Note

The famous mathematicians referenced in this man page now have misspelled names, due to R's difficulty with accent marks.

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### Author(s)

Carter T. Butts <buttsc@uci.edu>

### References

Erdos, P. and Renyi, A. (1960). "On the Evolution of Random Graphs." *Public Mathematical Institute of Hungary Academy of Sciences*, 5:17-61.

#### See Also

```
rgraph, rguman
```

# **Examples**

```
#Draw 5 random graphs of order 10
all(gden(rgnm(5,10,9,mode="graph"))==0.2) #Density 0.2
all(gden(rgnm(5,10,9))==0.1) #Density 0.1
#Plot a random graph
gplot(rgnm(1,10,20))
```

rgnmix

Draw Mixing-Conditioned Random Graphs

# **Description**

rgnmix generates random draws from a mixing-conditioned uniform random graph distribution.

### Usage

```
rgnmix(n, tv, mix, mode = "digraph", diag = FALSE,
    method = c("probability", "exact"), return.as.edgelist = FALSE)
```

# **Arguments**

n	the number of graphs to generate.	
tv	a vector of types or classes (one entry per vertex), corresponding to the rows and columns of mix. (Note that the total number of vertices generated will be length(tv).)	
mix	a class-by-class mixing matrix, containing either mixing rates (for method=="probability") or edge counts (for method=="exact").	
mode	"digraph" for directed graphs, or "graph" for undirected graphs.	
diag	logical; should loops be allowed?	
method	the generation method to use. "probability" results in a Bernoulli edge distribution (conditional on the underlying rates), while "exact" results in a uniform draw conditional on the exact per-block edge distribution.	
return.as.edgelist		

logical; should the resulting graphs be returned in sna edgelist form?

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#### **Details**

The generated graphs (in either adjacency or edgelist form).

#### Value

rgnmix draws from a simple generalization of the Erdos-Renyi N,M family (and the related N,p family), generating graphs with fixed expected or realized mixing rates. Mixing is determined by the mix argument, which must contain a class-by-class matrix of mixing rates (either edge probabilities or number of realized edges). The class for each vertex is specified in tv, whose entries must correspond to the rows and columns of mix. The resulting functionality is much like blockmodel.expand, although more general (and in some cases more efficient).

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Wasserman, S. and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

### See Also

```
rguman, rgnm, blockmodel.expand
```

# **Examples**

```
#Draw a random mixing matrix
mix<-matrix(runif(9),3,3)

#Generate a graph with 4 members per class
g<-rgnmix(1,rep(1:3,each=4),mix)
plot.sociomatrix(g)  #Visualize the result

#Repeat the exercise, using the exact method
mix2<-round(mix*8)  #Draw an exact matrix
g<-rgnmix(1,rep(1:3,each=4),mix2,method="exact")
plot.sociomatrix(g)</pre>
```

rgraph

Generate Bernoulli Random Graphs

### **Description**

rgraph generates random draws from a Bernoulli graph distribution, with various parameters for controlling the nature of the data so generated.

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#### Usage

### **Arguments**

n The size of the vertex set (|V(G)|) for the random graphs

m The number of graphs to generate

Information regarding tie (edge) probabilities; see below mode "digraph" for directed data, "graph" for undirected data

diag Should the diagonal entries (loops) be set to zero?

replace Sample with or without replacement from a tie list (ignored if tielist==NULL

tielist A vector of edge values, from which the new graphs should be bootstrapped

return.as.edgelist

logical; should the resulting graphs be returned in edgelist form?

#### **Details**

rgraph is a reasonably versatile routine for generating random network data. The graphs so generated are either Bernoulli graphs (graphs in which each edge is a Bernoulli trial, independent conditional on the Bernoulli parameters), or are bootstrapped from a user-provided edge distribution (very handy for CUG tests). In the latter case, edge data should be provided using the tielist argument; the exact form taken by the data is irrelevant, so long as it can be coerced to a vector. In the former case, Bernoulli graph probabilities are set by the tprob argument as follows:

- 1. If tprob contains a single number, this number is used as the probability of all edges.
- 2. If tprob contains a vector, each entry is assumed to correspond to a separate graph (in order). Thus, each entry is used as the probability of all edges within its corresponding graph.
- 3. If tprob contains a matrix, then each entry is assumed to correspond to a separate edge. Thus, each entry is used as the probability of its associated edge in each graph which is generated.
- 4. Finally, if tprob contains a three-dimensional array, then each entry is assumed to correspond to a particular edge in a particular graph, and is used as the associated probability parameter.

Finally, note that rgraph will symmetrize all generated networks if mode is set to "graph" by copying down the upper triangle. The lower half of tprob, where applicable, must still be specified, however.

### Value

A graph stack

## Note

The famous mathematicians referenced in this man page now have misspelled names, due to R's difficulty with accent marks.

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### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Erdos, P. and Renyi, A. (1960). "On the Evolution of Random Graphs." *Public Mathematical Institute of Hungary Academy of Sciences*, 5:17-61.

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

#### See Also

```
rmperm, rgnm, rguman
```

# **Examples**

```
#Generate three graphs with different densities
g<-rgraph(10,3,tprob=c(0.1,0.9,0.5))

#Generate from a matrix of Bernoulli parameters
g.p<-matrix(runif(25,0,1),nrow=5)
g<-rgraph(5,2,tprob=g.p)</pre>
```

rguman

Draw Dyad Census-Conditioned Random Graphs

# Description

rguman generates random draws from a dyad census-conditioned uniform random graph distribution.

## Usage

```
rguman(n, nv, mut = 0.25, asym = 0.5, null = 0.25,
    method = c("probability", "exact"), return.as.edgelist = FALSE)
```

# **Arguments**

n	the number of graphs to generate.
nv	the size of the vertex set $( V(G) )$ for the random graphs.
mut	if method=="probability", the probability of obtaining a mutual dyad; otherwise, the number of mutual dyads.
asym	if method=="probability", the probability of obtaining an asymmetric dyad; otherwise, the number of asymmetric dyads.
null	if method=="probability", the probability of obtaining a null dyad; otherwise, the number of null dyads.

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method

the generation method to use. "probability" results in a multinomial dyad distribution (conditional on the underlying rates), while "exact" results in a uniform draw conditional on the exact dyad distribution.

return.as.edgelist

logical; should the resulting graphs be returned in edgelist form?

#### **Details**

A simple generalization of the Erdos-Renyi family, the UlMAN distributions are uniform on the set of graphs, conditional on order (size) and the dyad census. As with the E-R case, there are two UlMAN variants. The first (corresponding to method=="probability") takes dyad states as independent multinomials with parameters m (for mutuals), a (for asymmetrics), and n (for nulls). The resulting pmf is then

$$p(G = g|m, a, n) = \frac{(M + A + N)!}{M!A!N!} m^M a^A n^N,$$

where M, A, and N are realized counts of mutual, asymmetric, and null dyads, respectively. (See dyad.census for an explication of dyad types.)

The second UlMAN variant is selected by method=="exact", and places equal mass on all graphs having the specified (exact) dyad census. The corresponding pmf is

$$p(G = g|M, A, N) = \frac{M!A!N!}{(M+A+N)!}.$$

UlMAN graphs provide a natural baseline model for networks which are constrained by size, density, and reciprocity. In this way, they provide a bridge between edgewise models (e.g., the E-R family) and models with higher order dependence (e.g., the Markov graphs).

#### Value

A matrix or array containing the drawn adjacency matrices

## Note

The famous mathematicians referenced in this man page now have misspelled names, due to R's difficulty with accent marks.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

# References

Holland, P.W. and Leinhardt, S. (1976). "Local Structure in Social Networks." In D. Heise (Ed.), *Sociological Methodology*, pp 1-45. San Francisco: Jossey-Bass.

Wasserman, S. and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

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### See Also

```
rgraph, rgnm, dyad.census
```

# **Examples**

```
#Show some examples of extreme U|MAN graphs
gplot(rguman(1,10,mut=45,asym=0,null=0,method="exact")) #Clique
gplot(rguman(1,10,mut=0,asym=45,null=0,method="exact")) #Tournament
gplot(rguman(1,10,mut=0,asym=0,null=45,method="exact")) #Empty

#Draw a sample of multinomial U|MAN graphs
g<-rguman(5,10,mut=0.15,asym=0.05,null=0.8)

#Examine the dyad census
dyad.census(g)</pre>
```

rgws

Draw From the Watts-Strogatz Rewiring Model

## **Description**

rgws generates draws from the Watts-Strogatz rewired lattice model. Given a set of input graphs, rewire.ws performs a (dyadic) rewiring of those graphs.

# Usage

```
rgws(n, nv, d, z, p, return.as.edgelist = FALSE)
rewire.ud(g, p, return.as.edgelist = FALSE)
rewire.ws(g, p, return.as.edgelist = FALSE)
```

## **Arguments**

n	the number of draws to take.	
nv	the number of vertices per lattice dimension.	
d	the dimensionality of the underlying lattice.	
z	the nearest-neighbor threshold for local ties.	
p	the dyadic rewiring probability.	
g	a graph or graph stack.	
return.as.edgelist		
	logical; should the resulting graphs be returned in edgelist form?	

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#### **Details**

A Watts-Strogatz graph process generates a random graph via the following procedure. First, a d-dimensional uniform lattice is generated, here with nv vertices per dimension (i.e., nv^d vertices total). Next, all z neighbors are connected, based on geodesics of the underlying lattice. Finally, each non-null dyad in the resulting augmented lattice is "rewired" with probability p, where the rewiring operation exchanges the initial dyad state with the state of a uniformly selected null dyad sharing exactly one endpoint with the original dyad. (In the standard case, this is equivalent to choosing an endpoint of the dyad at random, and then transferring the dyadic edges to/from that endpoint to another randomly chosen vertex. Hence the "rewiring" metaphor.) For p==0, the W-S process generates (deterministic) uniform lattices, approximating a uniform G(N,M) process as p approaches 1. Thus, p can be used to tune overall entropy of the process. A well-known property of the W-S process is that (for large nv^d and small p) it generates draws with short expected mean geodesic distances (approaching those found in uniform graphs) while maintaining high levels of local "clustering" (i.e., transitivity). It has thus been proposed as one potential mechanism for obtaining "small world" structures.

rgws produces independent draws from the above process, returning them as an adjacency matrix (if n==1) or array (otherwise). rewire.ws, on the other hand, applies the rewiring phase of the W-S process to one or more input graphs. This can be used to explore local perturbations of the original graphs, conditioning on the dyad census. rewire.ud is similar to rewire.ws, save in that all dyads are eligible for rewiring (not just non-null dyads), and exchanges with non-null dyads are permitted. This process may be easier to work with than standard W-S rewiring in some cases.

#### Value

A graph or graph stack containing draws from the appropriate W-S process.

#### Warning

Remember that the total number of vertices in the graph is nv^d. This can get out of hand *very* quickly.

### Note

rgws generates non-toroidal lattices; some published work in this area utilizes underlying toroids, so users should check for this prior to comparing simulations against published results.

# Author(s)

Carter T. Butts <buttsc@uci.edu>

### References

Watts, D. and Strogatz, S. (1998). "Collective Dynamics of Small-world Networks." *Nature*, 393:440-442.

#### See Also

rgnm, rgraph

rmperm 201

# **Examples**

```
#Generate Watts-Strogatz graphs, w/increasing levels of rewiring
gplot(rgws(1,100,1,2,0))  #No rewiring
gplot(rgws(1,100,1,2,0.01)) #1% rewiring
gplot(rgws(1,100,1,2,0.05)) #5% rewiring
gplot(rgws(1,100,1,2,0.1)) #10% rewiring
gplot(rgws(1,100,1,2,1)) #100% rewiring

#Start with a simple graph, then rewire it
g<-matrix(0,50,50)
g[1,]<-1; g[,1]<-1  #Create a star
gplot(g)
gplot(rewire.ws(g,0.05)) #5% rewiring</pre>
```

rmperm

Randomly Permute the Rows and Columns of an Input Matrix

# **Description**

Given an input matrix (or stack thereof), rmperm performs a (random) simultaneous row/column permutation of the input data.

# Usage

```
rmperm(m)
```

# **Arguments**

m

a matrix, or stack thereof (or a graph set, for that matter).

#### **Details**

Random matrix permutations are the essence of the QAP test; see qaptest for details.

# Value

The permuted matrix (or matrices)

## Author(s)

Carter T. Butts <buttsc@uci.edu>

### See Also

rperm

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### **Examples**

```
#Generate an input matrix
g<-rgraph(5)
g  #Examine it

#Examine a random permutation
rmperm(g)</pre>
```

rperm

Draw a Random Permutation Vector with Exchangeability Constraints

# Description

Draws a random permutation on 1:length(exchange.list) such that no two elements whose corresponding exchange.list values are different are interchanged.

## Usage

```
rperm(exchange.list)
```

# **Arguments**

exchange.list A vector such that the permutation vector may exchange the ith and jth positions iff exchange.list[i]==exchange.list[j]

### **Details**

rperm draws random permutation vectors given the constraints of exchangeability described above. Thus, rperm(c(0,0,0,0)) returns a random permutation of four elements in which all exchanges are allowed, while rperm(c(1,1,"a","a")) (or similar) returns a random permutation of four elements in which only the first/second and third/fourth elements may be exchanged. This turns out to be quite useful for searching permutation spaces with exchangeability constraints (e.g., for structural distance estimation).

# Value

A random permutation vector satisfying the given constraints

# Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

rmperm

sdmat 203

# **Examples**

```
rperm(c(0,0,0,0)) #All elements may be exchanged rperm(c(0,0,0,1)) #Fix the fourth element rperm(c(0,0,1,1)) #Allow \{1,2\} and \{3,4\} to be swapped rperm(c("a",4,"x",2)) #Fix all elements (the identity permutation)
```

sdmat

Estimate the Structural Distance Matrix for a Graph Stack

# Description

Estimates the structural distances among all elements of dat using the method specified in method.

### Usage

```
sdmat(dat, normalize=FALSE, diag=FALSE, mode="digraph",
    output="matrix", method="mc", exchange.list=NULL, ...)
```

# **Arguments**

dat	graph set to be analyzed.
normalize	divide by the number of available dyads?
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.
output	"matrix" for matrix output, "dist" for a dist object.
method	method to be used to search the space of accessible permutations; must be one of "none", "exhaustive", "anneal", "hillclimb", or "mc".
exchange.list	information on which vertices are exchangeable (see below); this must be a single number, a vector of length $n$ , or a $nx2$ matrix.
	additional arguments to lab.optimize.

# **Details**

The structural distance between two graphs G and H is defined as

$$d_{S}\left(G,H\left|L_{G},L_{H}\right.\right)=\min_{L_{G},L_{H}}d\left(\ell\left(G\right),\ell\left(H\right)\right)$$

where  $L_G$  is the set of accessible permutations/labelings of G, and  $\ell(G)$  is a permuation/relabeling of the vertices of G ( $\ell(G) \in L_G$ ). The set of accessible permutations on a given graph is determined by the *theoretical exchangeability* of its vertices; in a nutshell, two vertices are considered to be theoretically exchangeable for a given problem if all predictions under the conditioning theory are invariant to a relabeling of the vertices in question (see Butts and Carley (2001) for a more

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formal exposition). Where no vertices are exchangeable, the structural distance becomes the its labeled counterpart (here, the Hamming distance). Where *all* vertices are exchangeable, the structural distance reflects the distance between unlabeled graphs; other cases correspond to distance under partial labeling.

The accessible permutation set is determined by the exchange.list argument, which is dealt with in the following manner. First, exchange.list is expanded to fill an nx2 matrix. If exchange.list is a single number, this is trivially accomplished by replication; if exchange.list is a vector of length n, the matrix is formed by cbinding two copies together. If exchange.list is already an nx2 matrix, it is left as-is. Once the nx2 exchangeability matrix has been formed, it is interpreted as follows: columns refer to graphs 1 and 2, respectively; rows refer to their corresponding vertices in the original adjacency matrices; and vertices are taken to be theoretically exchangeable iff their corresponding exchangeability matrix values are identical. To obtain an unlabeled distance (the default), then, one could simply let exchange.list equal any single number. To obtain the Hamming distance, one would use the vector 1:n.

Because the set of accessible permutations is, in general, very large (o(n!)), searching the set for the minimum distance is a non-trivial affair. Currently supported methods for estimating the structural distance are hill climbing, simulated annealing, blind monte carlo search, or exhaustive search (it is also possible to turn off searching entirely). Exhaustive search is not recommended for graphs larger than size 8 or so, and even this may take days; still, this is a valid alternative for small graphs. Blind monte carlo search and hill climbing tend to be suboptimal for this problem and are not, in general recommended, but they are available if desired. The preferred (and default) option for permutation search is simulated annealing, which seems to work well on this problem (though some tinkering with the annealing parameters may be needed in order to get optimal performance). See the help for lab. optimize for more information regarding these options.

Structural distance matrices may be used in the same manner as any other distance matrices (e.g., with multidimensional scaling, cluster analysis, etc.) Classical null hypothesis tests should not be employed with structural distances, and QAP tests are almost never appropriate (save in the uniquely labeled case). See cugtest for a more reasonable alternative.

# Value

A matrix of distances (or an object of class dist)

### Warning

The search process can be *very slow*, particularly for large graphs. In particular, the *exhaustive* method is order factorial, and will take approximately forever for unlabeled graphs of size greater than about 7-9.

### Note

For most applications, sdmat is dominated by structdist; the former is retained largely for reasons of compatibility.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

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### References

Butts, C.T. and Carley, K.M. (2005). "Some Simple Algorithms for Structural Comparison." *Computational and Mathematical Organization Theory*, 11(4), 291-305.

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS Working Paper, Carnegie Mellon University.

#### See Also

hdist, structdist

# **Examples**

sedist

Find a Matrix of Distances Between Positions Based on Structural Equivalence

### **Description**

sedist uses the graphs indicated by g in dat to assess the extent to which each vertex is structurally equivalent; joint.analysis determines whether this analysis is simultaneous, and method determines the measure of approximate equivalence which is used.

# Usage

```
sedist(dat, g=c(1:dim(dat)[1]), method="hamming",
    joint.analysis=FALSE, mode="digraph", diag=FALSE, code.diss=FALSE)
```

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#### **Arguments**

dat a graph or set thereof.

g a vector indicating which elements of dat should be examined. method one of "correlation", "euclidean", "hamming", or "gamma".

joint.analysis should equivalence be assessed across all networks jointly (TRUE), or individu-

ally within each (FALSE)?

mode "digraph" for directed data, otherwise "graph".

diag boolean indicating whether diagonal entries (loops) should be treated as mean-

ingful data.

code.diss reverse-code the raw comparison values.

#### **Details**

sedist provides a basic tool for assessing the (approximate) structural equivalence of actors. (Two vertices i and j are said to be structurally equivalent if i->k iff j->k for all k.) SE similarity/difference scores are computed by comparing vertex rows and columns using the measure indicated by method:

1. correlation: the product-moment correlation

2. euclidean: the euclidean distance

3. hamming: the Hamming distance

4. gamma: the gamma correlation

Once these similarities/differences are calculated, the results can be used with a clustering routine (such as equiv.clust) or an MDS (such as cmdscale).

#### Value

A matrix of similarity/difference scores

### Note

Be careful to verify that you have computed what you meant to compute, with respect to similarities/differences. Also, note that (despite its popularity) the product-moment correlation can give rather strange results in some cases.

### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Breiger, R.L.; Boorman, S.A.; and Arabie, P. (1975). "An Algorithm for Clustering Relational Data with Applications to Social Network Analysis and Comparison with Multidimensional Scaling." *Journal of Mathematical Psychology*, 12, 328-383.

Burt, R.S. (1976). "Positions in Networks." Social Forces, 55, 93-122.

Wasserman, S., and Faust, K. Social Network Analysis: Methods and Applications. Cambridge: Cambridge University Press.

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#### See Also

```
equiv.clust, blockmodel
```

## **Examples**

sna

Tools for Social Network Analysis

# **Description**

sna is a package containing a range of tools for social network analysis. Supported functionality includes node and graph-level indices, structural distance and covariance methods, structural equivalence detection, p\* modeling, random graph generation, and 2D/3D network visualization (among other things).

#### **Details**

Network data for sna routines can (except as noted otherwise) appear in any of the following forms:

- adjacency matrices (dimension N x N);
- arrays of adjacency matrices, aka "graph stacks" (dimension m x N x N);
- sna edge lists (see below);
- sparse matrix objects (from the SparseM package);
- network objects (from the network package); or
- lists of adjacency matrices/arrays, sparse matrices, and/or network objects.

Within the package documentation, the term "graph" is used generically to refer to any or all of the above (with multiple graphs being referred to as a "graph stack"). Note that usage of sparse matrix objects requires that the SparseM package be installed. (No additional packages are required for use of adjacency matrices/arrays or lists thereof, though the network package, on which sna depends as of 2.4, is used for network objects.) In general, sna routines attempt to make intelligent decisions regarding the processing of multiple graphs, but common sense is always advised; certain functions, in particular, have more specific data requirements. Calling sna functions with inappropriate input data can produce "interesting" results.

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One special data type supported by the sna package (as of version 2.0) is the *sna edgelist*. This is a simple data format that is well-suited to representing large, sparse graphs. (As of version 2.0, many - now most - package routines also process data in this form natively, so using it can produce significant savings of time and/or memory. Prior to 2.0, all package functions coerced input data to adjacency matrix form.) An sna edgelist is a three-column matrix, containing (respectively) senders, receivers, and values for each edge in the graph. (Unvalued edges should have a value of 1.) Note that this form is invariant to the number of edges in the graph: if there are no edges, then the edgelist is a degenerate matrix of dimension 0 by 3. Edgelists for undirected graphs should be coded as fully mutual digraphs (as would be the case with an adjacency matrix), with two edges per dyad (one (i,j) edge, and one (j,i) edge). Graph size for an sna edgelist matrix is indicated by a mandatory numeric attribute, named "n". Vertex names may be optionally specified by a vector-valued attribute named "vnames". In the case of two-mode data (i.e., data with an enforced bipartition), it is possible to indicate this status via the optional "bipartite" attribute. Vertices in a two-mode edgelist should be grouped in mode order, with "n" equal to the total number of vertices (across both modes) and "bipartite" equal to the number of vertices in the first mode.

Direct creation of sna edgelists can be performed by creating a three-column matrix and using the attr function to create the required "n" attribute. Alternately, the function as.edgelist.sna can be used to coerce data in any of the above forms to an sna edgelist. By turns, the function as.sociomatrix.sna can be used to convert any of these data types to adjacency matrix form.

To get started with sna, try obtaining viewing the list of available functions. This can be accomplished via the command library(help=sna).

#### Note

If you use this package and/or software manual in your work, a citation would be appreciated. References to the current versions are:

Butts, Carter T. (2016). "sna: Tools for Social Network Analysis." R package version 2.4.

Butts, Carter T. (2016). "Software Manual for the R sna Package." R package version 2.4.

See also the following paper, which explores the package in some detail:

Butts, Carter T. (2008). "Social Network Analysis with sna." Journal of Statistical Software, 24(6).

If utilizing a contributed routine, please also consider recognizing the author(s) of that specific function. Contributing authors, if any, are listed on the relevant manual pages. Your support helps to encourage the growth of the sna package, and is greatly valued!

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

sna-coercion

sna Coercion Functions

### **Description**

Functions to coerce network data into one form or another; these are generally internal, but may in some cases be helpful to the end user.

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#### Usage

#### **Arguments**

x network data in any of several acceptable forms (see below).

attrname if x is a network object, the (optional) edge attribute to be used to obtain edge

values.

simplify logical; should output be simplified by collapsing adjacency matrices of identi-

cal dimension into adjacency arrays?

force.bipartite

logical; should the data be interpreted as bipartite (with rows and columns rep-

resenting different data modes)?

as . digraph logical; should network objects be coded as digraphs, regardless of object prop-

erties? (Recommended)

suppress.diag logical; should loops be suppressed?

... additional arguments to sna.edgelist (currently ignored).

#### **Details**

The sna coercion functions are normally called internally within user-level sna functions to convert network data from various supported forms into a format usable by the function in question. With few (if any) exceptions, formats acceptable by these functions should be usable with any user-level function in the sna library.

as.sociomatrix.sna takes one or more input graphs, and returns them in adjacency matrix (and/or array) form. If simplify==TRUE, consolidation of matrices having the same dimensions into adjacency arrays is attempted; otherwise, elements are returned as lists of matrices/arrays.

as.edgelist.sna takes one or more input graphs, and returns them in sna edgelist form – i.e., a three-column matrix whose rows represent edges, and whose columns contain (respectively) the sender, receiver, and value of each edge. (Undirected graphs are generally assumed to be coded as fully mutual digraphs; edges may be listed in any order.) sna edgelists must also carry an attribute named n indicating the number of vertices in the graph, and may optionally contain the attributes vnames (carrying a vector of vertex names, in order) and/or bipartite (optionally, containing the number of row vertices in a two-mode network). If the bipartite attribute is present and non-false, vertices whose numbers are less than or equal to the attribute value are taken to belong to the first mode (i.e., row vertices), and those of value greater than the attribute are taken to belong to the second mode (i.e., column vertices). Note that the bipartite attribute is not strictly necessary to represent two-mode data, and may not be utilized by all sna functions.

is.edgelist.sna returns TRUE if its argument is a sna edgelist, or FALSE otherwise; if called with a list, this check is performed (recursively) on the list elements.

Data for sna coercion routines may currently consist of any combination of standard or sparse (via SparseM) adjacency matrices or arrays, network objects, or sna edgelists. If multiple items are

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given, they must be contained within a list. Where adjacency arrays are specified, they must be in three-dimensional form, with dimensions given in graph/sender/receiver order. Matrices or arrays having different numbers of rows and columns are taken to be two-mode adjacency structures, and are treated accordingly; setting force.bipartite will cause square matrices to be treated in similar fashion. In the case of network or sna edgelist matrices, bipartition information is normally read from the object's internal properties.

#### Value

An adjacency or edgelist structure, or a list thereof.

### Note

For large, sparse graphs, edgelists can be dramatically more efficient than adjacency matrices. Where such savings can be realized, sna package functions usually employ sna edgelists as their "native" format (coercing input data with as.edgelist.sna as needed). For this reason, users of large graphs can often obtain considerable savings by storing data in edgelist form, and passing edgelists (rather than adjacency matrices) to sna functions.

The maximum size of adjacency matrices and edgelists depends upon R's vector allocation limits. On a 64-bit platform, these limits are currently around 4.6e4 vertices (adjacency case) or 7.1e8 edges (edgelist case). The number of vertices in the edgelist case is effectively unlimited (and can technically be infinite), although not all functions will handle such objects gracefully. (Use of vertex names will limit the number of edgelist vertices to around 2e9.)

# Author(s)

Carter T. Butts <buttsc@uci.edu>

# See Also

sna, network

## **Examples**

```
#Produce some random data, and transform it
g<-rgraph(5)
                                  #TRUE
all(g==as.sociomatrix.sna(g))
as.edgelist.sna(g)
                                  #View in edgelist form
as.edgelist.sna(list(g,g))
                                #Double the fun
g2<-as.sociomatrix.sna(list(g,g)) #Will simplify to an array
dim(g2)
g3<-as.sociomatrix.sna(list(g,g),simplify=FALSE) #Do not simplify
                                                  #Now a list
#We can also build edgelists from scratch...
edges<-rbind(
c(1,2,1),
c(2,1,2),
c(1,3,1),
```

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```
c(1,5,2),
c(4,5,1),
c(5,4,1)
attr(edges, "n")<-n
attr(edges, "vnames")<-letters[1:n]</pre>
                                                #Plot the graph
gplot(edges, displaylabels=TRUE)
as.sociomatrix.sna(edges)
                                                #Show in matrix form
#Two-mode data works similarly
n<-6
edges<-rbind(
c(1,4,1),
c(1,5,2),
c(4,1,1),
c(5,1,2),
c(2,5,1),
c(5,2,1),
c(3,5,1),
c(3,6,2),
c(6,3,2)
attr(edges,"n")<-n
attr(edges, "vnames")<-c(letters[1:3],LETTERS[4:6])</pre>
attr(edges, "bipartite") <-3
gplot(edges,displaylabels=TRUE,gmode="twomode") #Plot
as.sociomatrix.sna(edges)
                                                   #Convert to matrix
```

sna-deprecated

Deprecated Functions in sna Package

# Description

These functions are provided for compatibility with older versions of sna only, and may be defunct as soon as the next release.

## **Details**

The following sna functions are currently deprecated:

None.

The original help pages for these functions can be found at help("oldName-deprecated"). Please avoid using them, since they will disappear....

## Author(s)

Carter T. Butts <buttsc@uci.edu>

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# See Also

Deprecated

sna.operators

**Graphical Operators** 

# **Description**

These operators allow for algebraic manupulation of graph adjacency matrices.

# Usage

```
## S3 method for class 'matrix'
e1 %c% e2
```

# **Arguments**

e1 an (unvalued) adjacency matrix.

e2 another (unvalued) adjacency matrix.

# **Details**

Currently, only one operator is supported. x %c% y returns the adjacency matrix of the composition of graphs with adjacency matrices x and y (respectively). (Note that this may contain loops.)

# Value

The resulting adjacency matrix.

## Author(s)

Carter T. Butts <buttsc@uci.edu>

# References

Wasserman, S. and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: University of Cambridge Press.

### **Examples**

```
#Create an in-star
g<-matrix(0,6,6)
g[2:6,1]<-1
gplot(g)

#Compose g with its transpose
gcgt<-g%c%t(g)
gplot(gcgt,diag=TRUE)
gcgt</pre>
```

sr2css 213

sr2css	Convert a Row-wise Self-Report Matrix to a CSS Matrix with Missing Observations
	observations and the second se

# **Description**

Given a matrix in which the ith row corresponds to i's reported relations, sr2css creates a graph stack in which each element represents a CSS slice with missing observations.

# Usage

```
sr2css(net)
```

# **Arguments**

net

an adjacency matrix.

#### **Details**

A cognitive social structure (CSS) is an nxnxn array in which the ith matrix corresponds to the ith actor's perception of the entire network. Here, we take a conventional self-report data structure and put it in CSS format for routines (such as bbnam) which require this.

# Value

An array (graph stack) containing the CSS

# Note

A row-wise self-report matrix doesn't contain a great deal of data, and the data in question is certainly not an ignorable sample of the individual's CSS for most purposes. The provision of this routine should not be perceived as license to substitute SR for CSS data at will.

# Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

# References

Krackhardt, D. (1987). Cognitive Social Structures, 9, 109-134.

### **Examples**

```
#Start with some random reports
g<-rgraph(10)
#Transform to CSS format
c<-sr2css(g)</pre>
```

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stackcount

How Many Graphs are in a Graph Stack?

# **Description**

Returns the number of graphs in the stack provided by d.

# Usage

```
stackcount(d)
```

# **Arguments**

d

a graph or graph stack.

#### Value

The number of graphs in d

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### See Also

nties

# **Examples**

```
stackcount(rgraph(4,8))==8
```

stresscent

Compute the Stress Centrality Scores of Network Positions

# Description

stresscent takes one or more graphs (dat) and returns the stress centralities of positions (selected by nodes) within the graphs indicated by g. Depending on the specified mode, stress on directed or undirected geodesics will be returned; this function is compatible with centralization, and will return the theoretical maximum absolute deviation (from maximum) conditional on size (which is used by centralization to normalize the observed centralization score).

# Usage

```
stresscent(dat, g=1, nodes=NULL, gmode="digraph",
    diag=FALSE, tmaxdev=FALSE, cmode="directed",
    geodist.precomp=NULL, rescale=FALSE, ignore.eval=TRUE)
```

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### **Arguments**

dat	one or more input graphs.
g	Integer indicating the index of the graph for which centralities are to be calculated (or a vector thereof). By default, g==1.
nodes	list indicating which nodes are to be included in the calculation. By default, all nodes are included.
gmode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. gmode is set to "digraph" by default.
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
tmaxdev	boolean indicating whether or not the theoretical maximum absolute deviation from the maximum nodal centrality should be returned. By default, tmaxdev==FALSE.
cmode	string indicating the type of betweenness centrality being computed (directed or undirected geodesics).
geodist.precom	р
	a geodist object precomputed for the graph to be analyzed (optional).
rescale	if true, centrality scores are rescaled such that they sum to 1.

### **Details**

ignore.eval

The stress of a vertex, v, is given by

$$C_S(v) = \sum_{i,j: i \neq j, i \neq v, j \neq v} g_{ivj}$$

logical; should edge values be ignored when calculating density?

where  $g_{ijk}$  is the number of geodesics from i to k through j. Conceptually, high-stress vertices lie on a large number of shortest paths between other vertices; they can thus be thought of as "bridges" or "boundary spanners." Compare this with betweenness, which weights shortest paths by the inverse of their redundancy.

# Value

A vector, matrix, or list containing the centrality scores (depending on the number and size of the input graphs).

# Note

Judicious use of geodist.precomp can save a great deal of time when computing multiple path-based indices on the same network.

# Author(s)

Carter T. Butts <buttsc@uci.edu>

216 structdist

### References

Shimbel, A. (1953). "Structural Parameters of Communication Networks." *Bulletin of Mathematical Biophysics*, 15:501-507.

### See Also

```
centralization
```

### **Examples**

structdist

Find the Structural Distances Between Two or More Graphs

# Description

structdist returns the structural distance between the labeled graphs g1 and g2 in stack dat based on Hamming distance for dichotomous data, or else the absolute (manhattan) distance. If normalize is true, this distance is divided by its dichotomous theoretical maximum (conditional on |V(G)|).

# Usage

```
structdist(dat, g1=NULL, g2=NULL, normalize=FALSE, diag=FALSE,
  mode="digraph", method="anneal", reps=1000, prob.init=0.9,
  prob.decay=0.85, freeze.time=25, full.neighborhood=TRUE,
  mut=0.05, pop=20, trials=5, exchange.list=NULL)
```

# **Arguments**

dat	one or more input graphs.
g1	a vector indicating which graphs to compare (by default, all elements of dat).
g2	a vector indicating against which the graphs of g1 should be compared (by default, all graphs).
normalize	divide by the number of available dyads?
diag	boolean indicating whether or not the diagonal should be treated as valid data. Set this true if and only if the data can contain loops. diag is FALSE by default.
mode	string indicating the type of graph being evaluated. "digraph" indicates that edges should be interpreted as directed; "graph" indicates that edges are undirected. mode is set to "digraph" by default.
method	method to be used to search the space of accessible permutations; must be one of "none", "exhaustive", "anneal", "hillclimb", or "mc".
reps	number of iterations for Monte Carlo method.

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prob.init initial acceptance probability for the annealing routine.

prob. decay cooling multiplier for the annealing routine.

freeze.time freeze time for the annealing routine.

full.neighborhood

should the annealer evaluate the full neighborhood of pair exchanges at each

iteration?

mut GA Mutation rate (currently ignored).

pop GA population (currently ignored).

trials number of GA populations (currently ignored).

exchange.list information on which vertices are exchangeable (see below); this must be a

single number, a vector of length n, or a nx2 matrix.

#### **Details**

The structural distance between two graphs G and H is defined as

$$d_{S}\left(G,H\left|L_{G},L_{H}\right.\right)=\min_{L_{G},L_{H}}d\left(\ell\left(G\right),\ell\left(H\right)\right)$$

where  $L_G$  is the set of accessible permutations/labelings of G, and  $\ell(G)$  is a permuation/relabeling of the vertices of G ( $\ell(G) \in L_G$ ). The set of accessible permutations on a given graph is determined by the *theoretical exchangeability* of its vertices; in a nutshell, two vertices are considered to be theoretically exchangeable for a given problem if all predictions under the conditioning theory are invariant to a relabeling of the vertices in question (see Butts and Carley (2001) for a more formal exposition). Where no vertices are exchangeable, the structural distance becomes the its labeled counterpart (here, the Hamming distance). Where *all* vertices are exchangeable, the structural distance reflects the distance between unlabeled graphs; other cases correspond to distance under partial labeling.

The accessible permutation set is determined by the exchange.list argument, which is dealt with in the following manner. First, exchange.list is expanded to fill an nx2 matrix. If exchange.list is a single number, this is trivially accomplished by replication; if exchange.list is a vector of length n, the matrix is formed by cbinding two copies together. If exchange.list is already an nx2 matrix, it is left as-is. Once the nx2 exchangeability matrix has been formed, it is interpreted as follows: columns refer to graphs 1 and 2, respectively; rows refer to their corresponding vertices in the original adjacency matrices; and vertices are taken to be theoretically exchangeable iff their corresponding exchangeability matrix values are identical. To obtain an unlabeled distance (the default), then, one could simply let exchange.list equal any single number. To obtain the Hamming distance, one would use the vector 1:n.

Because the set of accessible permutations is, in general, very large (o(n!)), searching the set for the minimum distance is a non-trivial affair. Currently supported methods for estimating the structural distance are hill climbing, simulated annealing, blind monte carlo search, or exhaustive search (it is also possible to turn off searching entirely). Exhaustive search is not recommended for graphs larger than size 8 or so, and even this may take days; still, this is a valid alternative for small graphs. Blind monte carlo search and hill climbing tend to be suboptimal for this problem and are not, in general recommended, but they are available if desired. The preferred (and default) option for permutation search is simulated annealing, which seems to work well on this problem (though some tinkering

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with the annealing parameters may be needed in order to get optimal performance). See the help for lab.optimize for more information regarding these options.

Structural distance matrices may be used in the same manner as any other distance matrices (e.g., with multidimensional scaling, cluster analysis, etc.) Classical null hypothesis tests should not be employed with structural distances, and QAP tests are almost never appropriate (save in the uniquely labeled case). See cugtest for a more reasonable alternative.

#### Value

A structural distance matrix

#### Warning

The search process can be *very slow*, particularly for large graphs. In particular, the *exhaustive* method is order factorial, and will take approximately forever for unlabeled graphs of size greater than about 7-9.

#### Note

Consult Butts and Carley (2001) for advice and examples on theoretical exchangeability.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Butts, C.T. and Carley, K.M. (2005). "Some Simple Algorithms for Structural Comparison." *Computational and Mathematical Organization Theory*, 11(4), 291-305.

Butts, C.T., and Carley, K.M. (2001). "Multivariate Methods for Interstructural Analysis." CASOS Working Paper, Carnegie Mellon University.

#### See Also

```
hdist, sdmat
```

#### **Examples**

```
#Generate two random graphs
g<-array(dim=c(3,5,5))
g[1,,]<-rgraph(5)
g[2,,]<-rgraph(5)

#Copy one of the graphs and permute it
g[3,,]<-rmperm(g[2,,])

#What are the structural distances between the labeled graphs?
structdist(g,exchange.list=1:5)

#What are the structural distances between the underlying unlabeled</pre>
```

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```
#graphs?
structdist(g,method="anneal", prob.init=0.9, prob.decay=0.85,
    freeze.time=50, full.neighborhood=TRUE)
```

structure.statistics

Compute Network Structure Statistics

## **Description**

Computes the structure statistics for the graph(s) in dat.

## Usage

```
structure.statistics(dat, geodist.precomp = NULL)
```

## **Arguments**

```
dat one or more input graphs.

geodist.precomp

a geodist object (optional).
```

#### **Details**

Let G=(V,E) be a graph of order N, and let d(i,j) be the geodesic distance from vertex i to vertex j in G. The "structure statistics" of G are then given by the series  $s_0,\ldots,s_{N-1}$ , where  $s_i=\frac{1}{N^2}\sum_{j\in V}\sum_{k\in V}I\left(d(j,k)\leq i\right)$  and I is the standard indicator function. Intuitively,  $s_i$  is the expected fraction of G which lies within distance i of a randomly chosen vertex. As such, the structure statistics provide an index of global connectivity.

Structure statistics have been of particular importance to biased net theorists, because of the link with Rapoport's original tracing model. They may also be used along with component distributions or connectedness scores as descriptive indices of connectivity at the graph-level.

#### Value

A vector, matrix, or list (depending on dat) containing the structure statistics.

#### Note

The term "structure statistics" has been used somewhat loosely in the literature, a trend which seems to be accelerating. Users should carefully check references before comparing results generated by this routine with those appearing in published work.

#### Author(s)

220 summary.bayes.factor

#### References

Fararo, T.J. (1981). "Biased networks and social structure theorems. Part I." *Social Networks*, 3, 137-159.

Fararo, T.J. (1984). "Biased networks and social structure theorems. Part II." *Social Networks*, 6, 223-258.

Fararo, T.J. and Sunshine, M.H. (1964). "A study of a biased friendship net." Syracuse, NY: Youth Development Center.

#### See Also

```
geodist, component.dist, connectedness, bn
```

#### **Examples**

```
#Generate a moderately sparse Bernoulli graph
g<-rgraph(100,tp=1.5/99)

#Compute the structure statistics for g
ss<-structure.statistics(g)
plot(0:99,ss,xlab="Mean Coverage",ylab="Distance")</pre>
```

summary.bayes.factor

Detailed Summaries of Bayes Factor Objects

## **Description**

Returns a bayes. factor summary object.

# Usage

```
## S3 method for class 'bayes.factor'
summary(object, ...)
```

## **Arguments**

object An object of class bayes.factor

... Further arguments passed to or from other methods

## Value

An object of class summary.bayes.factor

## Author(s)

summary.bbnam 221

## See Also

bbnam.bf

summary.bbnam

Detailed Summaries of bbnam Objects

# Description

Returns a bbnam summary object

#### Usage

```
## S3 method for class 'bbnam'
summary(object, ...)
```

# Arguments

object An object of class bbnam

... Further arguments passed to or from other methods

#### Value

An object of class summary.bbnam

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### See Also

bbnam

summary.blockmodel

Detailed Summaries of blockmodel Objects

# Description

Returns a blockmodel summary object.

## Usage

```
## S3 method for class 'blockmodel'
summary(object, ...)
```

222 summary.cugtest

## **Arguments**

object An object of class blockmodel

Further arguments passed to or from other methods

#### Value

An object of class summary.blockmodel

## Author(s)

Carter T. Butts <buttsc@uci.edu>

#### See Also

blockmodel

summary.cugtest

Detailed Summaries of cugtest Objects

# Description

Returns a cugtest summary object

# Usage

```
## S3 method for class 'cugtest'
summary(object, ...)
```

## **Arguments**

object An object of class cugtest

... Further arguments passed to or from other methods

## Value

An object of class summary.cugtest

## Author(s)

Carter T. Butts <buttsc@uci.edu>

# See Also

cugtest

summary.lnam 223

summary.lnam

Detailed Summaries of lnam Objects

# Description

Returns a 1nam summary object.

## Usage

```
## S3 method for class 'lnam'
summary(object, ...)
```

# Arguments

```
object an object of class lnam.
... additional arguments.
```

#### Value

An object of class summary.lnam.

#### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

# See Also

1nam

summary.netcancor

Detailed Summaries of netcancor Objects

# Description

Returns a netcancor summary object

## Usage

```
## S3 method for class 'netcancor'
summary(object, ...)
```

## **Arguments**

object An object of class netcancor

... Further arguments passed to or from other methods

224 summary.netlm

# Value

An object of class summary.netcancor

# Author(s)

Carter T. Butts <buttsc@uci.edu>~

## See Also

netcancor

summary.netlm

Detailed Summaries of netlm Objects

# Description

Returns a netlm summary object

## Usage

```
## S3 method for class 'netlm'
summary(object, ...)
```

# Arguments

object An object of class netlm

... Further arguments passed to or from other methods

## Value

An object of class summary.netlm

## Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

netlm

summary.netlogit 225

summary.netlogit

Detailed Summaries of netlogit Objects

# Description

Returns a netlogit summary object~

## Usage

```
## S3 method for class 'netlogit'
summary(object, ...)
```

## **Arguments**

object An object of class netlogit

Further arguments passed to or from other methods

#### Value

An object of class summary.netlogit

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

## See Also

netlogit

summary.qaptest

Detailed Summaries of qaptest Objects

# Description

Returns a qaptest summary object

# Usage

```
## S3 method for class 'qaptest'
summary(object, ...)
```

## **Arguments**

object An object of class qaptest

... Further arguments passed to or from other methods

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## Value

An object of class summary.qaptest

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### See Also

qaptest

symmetrize

Symmetrize an Adjacency Matrix

# Description

Symmetrizes the elements of mats according to the rule in rule.

## Usage

```
symmetrize(mats, rule="weak", return.as.edgelist=FALSE)
```

## **Arguments**

```
mats a graph or graph stack.

rule one of "upper", "lower", "strong" or "weak".

return.as.edgelist
```

logical; should the symmetrized graphs be returned in edgelist form?

# **Details**

The rules used by symmetrize are as follows:

- 1. upper: Copy the upper triangle over the lower triangle
- 2. lower: Copy the lower triangle over the upper triangle
- 3. strong: i<->j iff i->j and i<-j (AND rule)
- 4. weak: i<->j iff i->j or i<-j (OR rule)

## Value

The symmetrized graph stack

#### Author(s)

triad.census 227

#### References

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

## **Examples**

```
#Generate a graph
g<-rgraph(5)

#Weak symmetrization
symmetrize(g)

#Strong symmetrization
symmetrize(g,rule="strong")</pre>
```

triad.census

Compute the Davis and Leinhardt Triad Census

# **Description**

triad.census returns the Davis and Leinhardt triad census of the elements of dat indicated by g.

#### Usage

```
triad.census(dat, g=NULL, mode = c("digraph", "graph"))
```

## **Arguments**

dat a graph or graph stack.

g the elements of dat to process.

mode string indicating the directedness of edges; "digraph" implies a directed struc-

ture, whereas "graph" implies an undirected structure.

## **Details**

The Davis and Leinhardt triad census consists of a classification of all directed triads into one of 16 different categories; the resulting distribution can be compared against various null models to test for the presence of configural biases (e.g., transitivity bias). triad.census is a front end for the triad.classify routine, performing the classification for all triads within the selected graphs. The results are placed in the order indicated by the column names; this is the same order as presented in the triad.classify documentation, to which the reader is referred for additional details.

In the undirected case, the triad census reduces to four states (based on the number of edges in each triad. Where mode=="graph", this is returned instead.

Compare triad.census to dyad.census, the dyadic equivalent.

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#### Value

A matrix whose 16 columns contain the counts of triads by class for each graph, in the directed case. In the undirected case, only 4 columns are used.

#### Warning

Valued data may cause strange behavior with this routine. Dichotomize the data first.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### References

Davis, J.A. and Leinhardt, S. (1972). "The Structure of Positive Interpersonal Relations in Small Groups." In J. Berger (Ed.), *Sociological Theories in Progress, Volume* 2, 218-251. Boston: Houghton Mifflin.

Wasserman, S., and Faust, K. (1994). "Social Network Analysis: Methods and Applications." Cambridge: Cambridge University Press.

## See Also

triad.classify, dyad.census, kcycle.census, kpath.census, gtrans

## **Examples**

```
#Generate a triad census of random data with varying densities triad.census(rgraph(15,5,tprob=c(0.1,0.25,0.5,0.75,0.9)))
```

triad.classify

Compute the Davis and Leinhardt Classification of a Given Triad

## Description

triad.classify returns the Davis and Leinhardt classification of the triad indicated by tri in the gth graph of stack dat.

#### Usage

## **Arguments**

dat	a graph or graph stack.
g	the index of the graph to be analyzed.
tri	a triple containing the indices of the triad to be classified.

mode string indicating the directedness of edges; "digraph" implies a directed struc-

ture, whereas "graph" implies an undirected structure.

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#### **Details**

Every unoriented directed triad may occupy one of 16 distinct states. These states were used by Davis and Leinhardt as a basis for classifying triads within a larger structure; the distribution of triads within a graph (see triad.census), for instance, is linked to a range of substantive hypotheses (e.g., concerning structural balance). The Davis and Leinhardt classification scheme describes each triad by a string of four elements: the number of mutual (complete) dyads within the triad; the number of asymmetric dyads within the triad; the number of null (empty) dyads within the triad; and a configuration code for the triads which are not uniquely distinguished by the first three distinctions. The complete list of classes is as follows.

```
003 a \not\leftrightarrow b \not\leftrightarrow c, a \not\leftrightarrow c
012 a \rightarrow b \not\leftrightarrow c, a \not\leftrightarrow c
102 a \leftrightarrow b \not\leftrightarrow c, a \not\leftrightarrow c
021D a \leftarrow b \rightarrow c, a \not\leftrightarrow c
021U a \rightarrow b \leftarrow c, a \not\leftrightarrow c
\texttt{021C} \ \ a \rightarrow b \rightarrow c, a \not\leftrightarrow c
111D a \not\leftrightarrow b \rightarrow c, a \leftrightarrow c
111U a \not\leftrightarrow b \leftarrow c, a \leftrightarrow c
\texttt{030T} \ a \rightarrow b \leftarrow c, a \rightarrow c
\texttt{030C} \ a \leftarrow b \leftarrow c, a \rightarrow c
201 a \leftrightarrow b \not\leftrightarrow c, a \leftrightarrow c
120D a \leftarrow b \rightarrow c, a \leftrightarrow c
120U a \rightarrow b \leftarrow c, a \leftrightarrow c
120C a \rightarrow b \rightarrow c, a \leftrightarrow c
210 a \rightarrow b \leftrightarrow c, a \leftrightarrow c
300 a \leftrightarrow b \leftrightarrow c, a \leftrightarrow c
```

These codes are returned by triad.classify as strings. In the undirected case, only four triad states are possible (corresponding to the number of edges in the triad). These are evaluated for mode=="graph", with the return value being the number of edges.

#### Value

A string containing the triad classification, or NA if one or more edges were missing

## Warning

Valued data and/or loops may cause strange behavior with this routine. Dichotomize/remove loops first.

#### Author(s)

230 upper.tri.remove

#### References

Davis, J.A. and Leinhardt, S. (1972). "The Structure of Positive Interpersonal Relations in Small Groups." In J. Berger (Ed.), *Sociological Theories in Progress, Volume 2*, 218-251. Boston: Houghton Mifflin.

Wasserman, S., and Faust, K. (1994). *Social Network Analysis: Methods and Applications*. Cambridge: Cambridge University Press.

#### See Also

```
triad.census, gtrans
```

## **Examples**

```
#Generate a random graph
g<-rgraph(10)

#Classify the triads (1,2,3) and (2,3,4)
triad.classify(g,tri=c(1,2,3))
triad.classify(g,tri=c(1,2,3))

#Plot the triads in question
gplot(g[1:3,1:3])
gplot(g[2:4,2:4])</pre>
```

upper.tri.remove

Remove the Upper Triangles of Adjacency Matrices in a Graph Stack

## **Description**

Returns the input graph stack, with the upper triangle entries removed/replaced as indicated.

## Usage

```
upper.tri.remove(dat, remove.val=NA)
```

## Arguments

dat a graph or graph stack.

remove.val the value with which to replace the existing upper triangles.

## Details

upper.tri.remove is simply a convenient way to apply g[upper.tri(g)]<-remove.val to an entire stack of adjacency matrices at once.

#### Value

The updated graph stack.

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#### Author(s)

```
Carter T. Butts <buttsc@uci.edu>
```

#### See Also

```
upper.tri, lower.tri.remove, diag.remove
```

## **Examples**

```
#Generate a random graph stack
g<-rgraph(3,5)
#Remove the upper triangles
g<-upper.tri.remove(g)</pre>
```

write.dl

Write Output Graphs in DL Format

# Description

Writes a graph stack to an output file in DL format.

#### Usage

```
write.dl(x, file, vertex.lab = NULL, matrix.lab = NULL)
```

# Arguments

x a graph or graph stack, of common order.

file a string containing the filename to which the data should be written.

vertex.lab an optional vector of vertex labels.
matrix.lab an optional vector of matrix labels.

#### **Details**

DL format is used by a number of software packages (including UCINET and Pajek) to store network data. write.dl saves one or more (possibly valued) graphs in DL edgelist format, along with vertex and graph labels (if desired). These files can, in turn, be used to import data into other software packages.

## Value

None.

## Author(s)

232 write.nos

## See Also

```
write.nos
```

#### **Examples**

```
## Not run:
#Generate a random graph stack
g<-rgraph(5,10)

#This would save the graphs in DL format
write.dl(g,file="testfile.dl")

## End(Not run)</pre>
```

write.nos

*Write Output Graphs in (N)eo-(O)rg(S)tat Format* 

# Description

Writes a graph stack to an output file in NOS format.

## Usage

```
write.nos(x, file, row.col = NULL, col.col = NULL)
```

## **Arguments**

```
x a graph or graph stack (all graphs must be of common order).

file string containing the output file name.

row.col vector of row labels (or "row colors").

col.col vector of column labels ("column colors").
```

#### **Details**

NOS format consists of three header lines, followed by a whitespace delimited stack of raw adjacency matrices; the format is not particularly elegant, but turns up in certain legacy applications (mostly at CMU). write.nos provides a quick and dirty way of writing files NOS, which can later be retrieved using read.nos.

The content of the NOS format is as follows:

```
<m>
<n> <o>
<hr/>
<n> <o>
<hr/>
<kr1> <kr2> ... <krn> <kc1> <kc2> ... <kcn>
<a111> <a112> ... <a110>
<a121> <a122> ... <a120>
<hr/>
<hr/>
<a120>
```

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```
<a1n1> <a1n2> ... <a1no> <a211> <a212> ... <a21o> ... <a211> <a212> ... <a21o> ... <a2n1> <a2n1> <a2n2> ... <a2no> ... <amn1> <amn1> <amno> ... <amno>
```

where <abcd> is understood to be the value of the c->d edge in the bth graph of the file. (As one might expect, m, n, and o are the numbers of graphs (matrices), rows, and columns for the data, respectively.) The "k" line contains a list of row and column "colors", categorical variables associated with each row and column, respectively. Although originally intended to communicate exchangability information, these can be used for other purposes (though there are easier ways to deal with attribute data these days).

Note that NOS format only supports graph stacks of common order; graphs of different sizes cannot be stored within the same file.

#### Value

None.

#### Author(s)

Carter T. Butts <buttsc@uci.edu>

#### See Also

```
read.nos, write.dl, write.table
```

# **Examples**

```
## Not run:
#Generate a random graph stack
g<-rgraph(5,10)

#This would save the graphs in NOS format
write.nos(g,file="testfile.nos")

#We can also read them back, like so:
g2<-read.nos("testfile.nos")

## End(Not run)</pre>
```

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