Matrix Theory (EE5609) Assignment 6

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Abstract—**This proves a theorem on triangle.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_6

1 Problem

$\triangle ABC$ is a triangle right angled at $\mathbf{C}$. A line through the mid-point $\mathbf{M}$ of hypotenuse $\mathbf{AB}$ and parallel to $\mathbf{BC}$ intersects $\mathbf{AC}$ at $\mathbf{D}$. Show that -

(i) $\mathbf{D}$ is the mid-point of $\mathbf{AC}$
(ii) $\mathbf{MD} \perp \mathbf{AC}$
(iii) $\mathbf{CM} = \mathbf{MA} = \frac{1}{2}\ \mathbf{AB}$

2 Solution

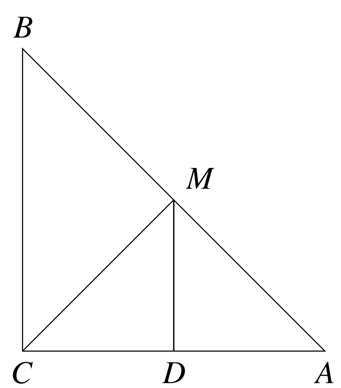

Fig. 1: Right Angled Triangle by Latex-Tikz

In $\triangle ABC$, $\mathbf{M}$ is midpoint of $\mathbf{AB}$ and $\mathbf{MD}$ is parallel to $\mathbf{BC}$, hence,

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{2.0.1}$$
$$\mathbf{MD} \parallel \mathbf{BC} \tag{2.0.2}$$

Let $\mathbf{m_{MD}}$ and $\mathbf{m_{BC}}$ are direction vectors of $\mathbf{MD}$ and $\mathbf{BC}$ respectively. Then,

$$\mathbf{m_{MD}} = \mathbf{M} - \mathbf{D} = \frac{\mathbf{A}+\mathbf{B}}{2} - \mathbf{D} \tag{2.0.3}$$
$$\mathbf{m_{BC}} = \mathbf{B} - \mathbf{C} \tag{2.0.4}$$

Now from (2.0.2) we get,

$$\mathbf{m_{MD}} = k\mathbf{m_{BC}} \tag{2.0.5}$$
$$\implies \frac{\mathbf{A}+\mathbf{B}}{2} - \mathbf{D} = k(\mathbf{B}-\mathbf{C}) \tag{2.0.6}$$

Let $\mathbf{D} = \frac{m\mathbf{A}+\mathbf{C}}{m+1}$, then from (2.0.6) we get,

$$\frac{\mathbf{A}+\mathbf{B}}{2} - \frac{m\mathbf{A}+\mathbf{C}}{m+1} = k(\mathbf{B}-\mathbf{C}) \tag{2.0.7}$$

$$\implies \left(\frac{1}{2} - \frac{m}{m+1}\right)\mathbf{A} + \left(\frac{1}{2} - k\right)\mathbf{B} + \left(k - \frac{1}{m+1}\right)\mathbf{C} = 0 \tag{2.0.8}$$

Since $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are linearly dependent as they form $\triangle ABC$ then

$$\frac{1}{2} - \frac{m}{m+1} = 0 \tag{2.0.9}$$
$$\frac{1}{2} - k = 0 \tag{2.0.10}$$
$$k - \frac{1}{m+1} = 0 \tag{2.0.11}$$

Solving (2.0.9), (2.0.10) and (2.0.11) we get $k = \frac{1}{2}$ and $m = 1$. Hence, substituting value of m in $\mathbf{D}$ we get,

$$\mathbf{D} = \frac{\mathbf{A}+\mathbf{C}}{2} \tag{2.0.12}$$

Hence Proved.

From figure 1,

$$(\mathbf{M}-\mathbf{D})(\mathbf{A}-\mathbf{D}) = \left(\frac{\mathbf{A}+\mathbf{B}}{2} - \frac{\mathbf{A}+\mathbf{C}}{2}\right)(\mathbf{A}-\mathbf{C}) \tag{2.0.13}$$

$$\implies (\mathbf{M}-\mathbf{D})(\mathbf{A}-\mathbf{D}) = \frac{1}{2}(\mathbf{B}-\mathbf{C})(\mathbf{A}-\mathbf{C}) \tag{2.0.14}$$

$$\implies (\mathbf{M}-\mathbf{D})(\mathbf{A}-\mathbf{D}) = 0 \quad [\because \mathbf{BC} \perp \mathbf{AC}] \tag{2.0.15}$$

From (2.0.15), it is proved that $\mathbf{MD} \perp \mathbf{AC}$
Again we get,

$$\mathbf{C}-\mathbf{M} = \mathbf{C}-\mathbf{D}+\mathbf{D}-\mathbf{M} \tag{2.0.16}$$

$$\implies \mathbf{C}-\mathbf{M} = \mathbf{A}-\mathbf{D}+\mathbf{D}-\mathbf{M} \quad \text{[From (2.0.12)]} \tag{2.0.17}$$

$$\implies \mathbf{C}-\mathbf{M} = \mathbf{A}-\mathbf{M} \tag{2.0.18}$$

$$\implies \mathbf{C}-\mathbf{M} = \mathbf{A}-\frac{\mathbf{A}+\mathbf{B}}{\mathbf{2}} \quad \text{[From (2.0.1)]} \tag{2.0.19}$$

$$\implies \mathbf{C}-\mathbf{M} = \frac{1}{2}(\mathbf{A}-\mathbf{B}) \tag{2.0.20}$$

$$\implies \|\mathbf{C}-\mathbf{M}\| = \frac{1}{2}\|\mathbf{A}-\mathbf{B}\| \tag{2.0.21}$$

Hence from (2.0.18) and (2.0.21) proved,
$\mathbf{CM} = \mathbf{MA} = \frac{1}{2}\,\mathbf{AB}$