Matrix Theory (EE5609) Challenging Problem

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***Abstract*—This document finds the kind of matrix V for which $\mathbf{V} = \mathbf{PDP^T}$, with $\mathbf{P^TP} = \mathbf{I}$.**

Download latex codes from

https://github.com/Arko98/EE5609/tree/master/Challenge_6

1 Problem

$\mathbf{V} = \mathbf{PDP^T}$, with $\mathbf{P^TP} = \mathbf{I}$. So P is an orthogonal matrix. For what matrices V do you get this kind of decomposition where P is an orthogonal ?

2 Proof

Let, $\mathbf{V}$ is an arbitrary $n \times n$ matrix. Now if there exists an orthogonal matrix $\mathbf{P}$ such that, $\mathbf{P^TVP}$ is a diagonal matrix $\mathbf{D}$, then $\mathbf{V}$ is said to be orthogonally diagonalizable. Hence,

$$\mathbf{P^TVP} = \mathbf{D} \tag{2.0.1}$$

Left multiplying (2.0.1) by P we get,

$$\mathbf{PP^TVP} = \mathbf{PD} \tag{2.0.2}$$

Right multiplying (2.0.2) by $\mathbf{P^T}$ we get,

$$\mathbf{PP^TVPP^T} = \mathbf{PDP^T} \tag{2.0.3}$$

Since P is orthogonal, $\mathbf{PP^T} = \mathbf{P^TP} = \mathbf{I}$, we can rewrite (2.0.3) as

$$\implies \mathbf{V} = \mathbf{PDP^T} \tag{2.0.4}$$

Transposing $\mathbf{V}$ in (2.0.4) we get,

$$\mathbf{V^T} = (\mathbf{PDP^T})^\mathbf{T} \tag{2.0.5}$$

$$\implies \mathbf{V^T} = (\mathbf{P^T})^\mathbf{T}\mathbf{D^TP^T} \tag{2.0.6}$$

$$\implies \mathbf{V^T} = \mathbf{PDP^T} \quad [\because \mathbf{D^T} = \mathbf{D}] \implies \mathbf{V^T} = \mathbf{V} \tag{2.0.7}$$

Hence $\mathbf{V}$ is a symmetric matrix.