

# Matrix Theory (EE5609) Assignment 4

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**Abstract**—This document solves an equation on matrix.

Next we calculate,  $\mathbf{A} - 6\mathbf{I}$  where  $\mathbf{I}$  is identity matrix of order 3, as follows

The code for the solution of this problem can be found at

[https://github.com/Arko98/EE5609/blob/master/Assignment\\_4/Codes/Solution.py](https://github.com/Arko98/EE5609/blob/master/Assignment_4/Codes/Solution.py)

$$\mathbf{A} - 6\mathbf{I} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} - 6 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.7)$$

$$\Rightarrow \mathbf{A} - 6\mathbf{I} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \quad (2.0.8)$$

$$\Rightarrow \mathbf{A} - 6\mathbf{I} = \begin{pmatrix} -5 & 0 & 2 \\ 0 & -4 & 1 \\ 2 & 0 & -3 \end{pmatrix} \quad (2.0.9)$$

## 1 PROBLEM

If  $\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ , prove that  $\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + 2\mathbf{I} = 0$

Now we compute  $\mathbf{A}^2(\mathbf{A} - 6\mathbf{I})$  by putting values of  $\mathbf{A}^2$  from equation 2.0.6 and value of  $\mathbf{A} - 6\mathbf{I}$  from 2.0.9 as follows

$$\mathbf{A}^2 \cdot (\mathbf{A} - 6\mathbf{I}) = (\mathbf{A}^2)^T (\mathbf{A} - 6\mathbf{I}) \quad (2.0.10)$$

$$\Rightarrow \mathbf{A}^2 \cdot (\mathbf{A} - 6\mathbf{I}) = \begin{pmatrix} 5 & 2 & 8 \\ 0 & 4 & 0 \\ 8 & 5 & 13 \end{pmatrix} \begin{pmatrix} -5 & 0 & 2 \\ 0 & -4 & 1 \\ 2 & 0 & -3 \end{pmatrix} \quad (2.0.11)$$

$$\Rightarrow \mathbf{A}^2 \cdot (\mathbf{A} - 6\mathbf{I}) = \begin{pmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{pmatrix} \quad (2.0.12)$$

Next we compute  $7\mathbf{A} + 2\mathbf{I}$  as follows

$$7\mathbf{A} + 2\mathbf{I} = 7 \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow 7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{pmatrix} \quad (2.0.14)$$

Now putting the value of  $\mathbf{A}^2(\mathbf{A} - 6\mathbf{I})$  from equation 2.0.12 and value of  $7\mathbf{A} + 2\mathbf{I}$  from equation 2.0.14

$$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A} \quad (2.0.3)$$

$$\Rightarrow \mathbf{A}^2 = \mathbf{A}^T \mathbf{A} \quad (2.0.4)$$

$$\Rightarrow \mathbf{A}^2 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \quad (2.0.5)$$

$$\Rightarrow \mathbf{A}^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} \quad (2.0.6)$$

## 2 SOLUTION

The equation in the problem can be modified as follows

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + 2\mathbf{I} = 0 \quad (2.0.1)$$

$$\Rightarrow \mathbf{A}^2(\mathbf{A} - 6\mathbf{I}) + 7\mathbf{A} + 2\mathbf{I} = 0 \quad (2.0.2)$$

So we need to prove equation 2.0.2. Now, at first we calculate the value of  $\mathbf{A}^2$  as follows

into the left hand side of equation 2.0.2 we get

$$\mathbf{A}^2(\mathbf{A} - 6\mathbf{I}) + 7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.15)$$

$$\Rightarrow \mathbf{A}^2(\mathbf{A} - 6\mathbf{I}) + 7\mathbf{A} + 2\mathbf{I} = 0 \quad (2.0.16)$$

Thus from equation 2.0.16 we arrive at the right hand side of equation 2.0.2, Hence proved.

### 3 PROBLEM 2

Find the characteristic equation of the matrix  $\mathbf{A}$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

### 4 SOLUTION

For a general order  $k$  square matrix  $\mathbf{A}$ , the characteristic equation in variable  $\lambda$  is defined by

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \quad (4.0.1)$$

where  $\mathbf{I}$  is identity matrix of order  $k$ . Now we compute  $\lambda\mathbf{I}$  as follows

$$\lambda\mathbf{I} = \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4.0.2)$$

$$\Rightarrow \lambda\mathbf{I} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \quad (4.0.3)$$

$$\Rightarrow \lambda \det I = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} \quad (4.0.4)$$

So in equation 4.0.1, putting the values from 4.0.4 we get,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \quad (4.0.5)$$

$$\Rightarrow \det \mathbf{A} - \lambda \det I = 0 \quad (4.0.6)$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0 \quad (4.0.7)$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0 \quad (4.0.8)$$

Hence, equation 4.0.8 is the required characteristic equation of matrix  $\mathbf{A}$ .