Matrix Theory (EE5609) Assignment 7

Arkadipta De
MTech Artificial Intelligence
AI20MTECH14002

***Abstract*—This finds whether a given second degree equation represents a pair of straight lines or not.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_7

1 Problem

Find the value of k so that the following equation may represent a pair of straight lines -

$$6x^2 + xy + ky^2 - 11x + 43y - 35 = 0$$

2 Theory

The general equation of second degree is given by,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \quad (2.0.1)$$

(2.0.1) can be written as,

$$(2.0.2)$$

$$\mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0 \quad (2.0.3)$$

where,

$$\mathbf{V} = \mathbf{V^T} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad (2.0.4)$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \quad (2.0.5)$$

(2.0.3) represents a pair of straight lines if,

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u^T} & f \end{vmatrix} = 0 \quad (2.0.6)$$

Otherwise, (2.0.3) represents a conic section.

3 Solution

The given second degree equation is,

$$6x^2 + xy + ky^2 - 11x + 43y - 35 = 0 \quad (3.0.1)$$

Comparing coefficients of (3.0.1) with (2.0.1) we get,

$$\mathbf{V} = \begin{pmatrix} 6 & \frac{1}{2} \\ \frac{1}{2} & k \end{pmatrix} \quad (3.0.2)$$

$$\mathbf{u} = \begin{pmatrix} -\frac{11}{2} \\ \frac{43}{2} \end{pmatrix} \quad (3.0.3)$$

$$f = -35 \quad (3.0.4)$$

From (2.0.6) the given second degree equation (3.0.1) will represent a pair of straight line if,

$$\begin{vmatrix} 6 & \frac{1}{2} & -\frac{11}{2} \\ \frac{1}{2} & k & \frac{43}{2} \\ -\frac{11}{2} & \frac{43}{2} & -35 \end{vmatrix} = 0 \quad (3.0.5)$$

Expanding the determinant,

$$k + 12 = 0 \quad (3.0.6)$$

$$\implies k = -12 \quad (3.0.7)$$

Hence, from (3.0.7) we find that for $k = -12$, the given second degree equation (3.0.1) represents pair of straight lines.
The figure below corresponds to the pair of straight lines represented by (3.0.1) when $k = -12$.

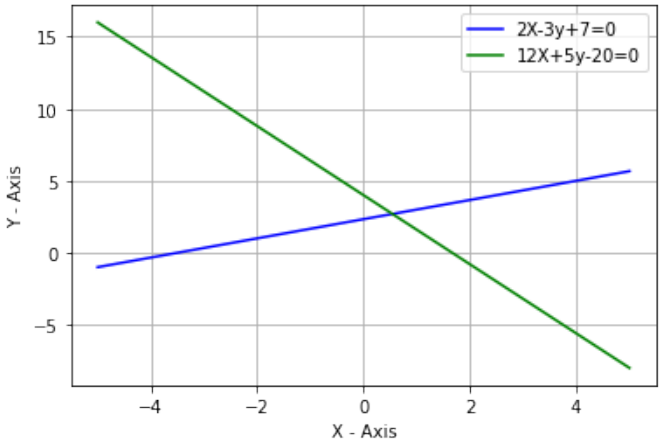

Fig. 1: Pair of Straight Lines