Matrix Theory (EE5609) Assignment 13

Arkadipta De
MTech Artificial Intelligence
AI20MTECH14002

***Abstract*—This document proves if AB = I then BA = I given that both A and B are square matrices.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_13

1 Problem

Let $\mathbf{A}$ and $\mathbf{B}$ be $n \times n$ matrices such that $\mathbf{AB} = \mathbf{I}$. Prove that $\mathbf{BA} = \mathbf{I}$

2 Solution

Let $\mathbf{BX} = 0$ be a system of linear equation with n unknowns and n equations as $\mathbf{B}$ is $n \times n$ matrix. Hence,

$$\mathbf{BX} = 0 \tag{2.0.1}$$
$$\implies \mathbf{A}(\mathbf{BX}) = 0 \tag{2.0.2}$$
$$\implies (\mathbf{AB})\mathbf{X} = 0 \tag{2.0.3}$$
$$\implies \mathbf{IX} = 0 \quad [\because \mathbf{AB} = \mathbf{I}] \tag{2.0.4}$$
$$\implies \mathbf{X} = 0 \tag{2.0.5}$$

From (2.0.5) since $\mathbf{X} = 0$ is the only solution of (2.0.1), hence $rank(\mathbf{B}) = n$. Which implies all columns of $\mathbf{B}$ are linearly independent. Hence $\mathbf{B}$ is invertible. Therefore, every left inverse of $\mathbf{B}$ is also a right inverse of $\mathbf{B}$. Hence there exists a $n \times n$ matrix $\mathbf{C}$ such that,

$$\mathbf{BC} = \mathbf{CB} = \mathbf{I} \tag{2.0.6}$$

Again given that $\mathbf{AB} = \mathbf{I}$. Hence,

$$\mathbf{AB} = \mathbf{I} \tag{2.0.7}$$
$$\implies \mathbf{ABC} = \mathbf{C} \tag{2.0.8}$$
$$\implies \mathbf{A}(\mathbf{BC}) = \mathbf{C} \tag{2.0.9}$$
$$\implies \mathbf{A} = \mathbf{C} \quad [\because \mathbf{BC} = \mathbf{I}] \tag{2.0.10}$$

Hence using (2.0.10) and (2.0.6) we can write,

$$\mathbf{BA} = \mathbf{I} \tag{2.0.11}$$

Hence Proved.