Matrix Theory (EE5609) Assignment 16

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***Abstract*—This document proves a given transformation to be linear.**

All the codes for the figure in this document can be found at

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https://github.com/Arko98/EE5609/blob/master/
    Assignment_16
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1 Problem

Let $\mathbf{V}$ be the vector space of all $n \times n$ matrices over the field $\mathbb{F}$, and let $\mathbf{B}$ be a fixed $n \times n$ matrix. If a transformation T defined as follows,

$$T(\mathbf{A}) = \mathbf{AB} - \mathbf{BA}$$

Prove that T is a linear transformation from $\mathbf{V}$ into $\mathbf{V}$

2 Solution

Let, $\mathbf{A_1}$ and $\mathbf{A_2}$

$$\mathbf{A_1} \in \mathbf{V} \tag{2.0.1}$$
$$\mathbf{A_2} \in \mathbf{V} \tag{2.0.2}$$

If c be any scalar of the field $\mathbb{F}$ we get,

$$c\mathbf{A_1} + \mathbf{A_2} \in \mathbf{V} \tag{2.0.3}$$

Applying transformation T on $(c\mathbf{A_1} + \mathbf{A_2})$ we get,

$$T(c\mathbf{A_1} + \mathbf{A_2}) = (c\mathbf{A_1} + \mathbf{A_2})\mathbf{B} - \mathbf{B}(c\mathbf{A_1} + \mathbf{A_2}) \tag{2.0.4}$$
$$= c\mathbf{A_1B} + \mathbf{A_2B} - c\mathbf{BA_1} - \mathbf{BA_2} \tag{2.0.5}$$
$$= c(\mathbf{A_1B} - \mathbf{BA_1}) + (\mathbf{A_2B} - \mathbf{BA_2}) \tag{2.0.6}$$
$$= cT(\mathbf{A_1}) + T(\mathbf{A_2}) \tag{2.0.7}$$

From (2.0.7) we conclude that T is a linear transformation from vector space $\mathbf{V}$ to $\mathbf{V}$.