# Matrix Theory (EE5609) Assignment 5

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***Abstract*—This document proves the co-linearity of three points in X-Y plane.**

The code to plot the figure of this problem can be found at

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https://github.com/Arko98/EE5609/blob/master/Assignment_5/Codes/Figure.py
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## 1 Problem

Show that the points $\mathbf{A} = \begin{pmatrix} a \\ b+c \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} b \\ c+a \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} c \\ a+b \end{pmatrix}$ are collinear.

## 2 Solution

The points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ will be collinear if

$$\begin{pmatrix} \mathbf{A}^{\mathbf{T}} \\ \mathbf{B}^{\mathbf{T}} \\ \mathbf{C}^{\mathbf{T}} \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.1}$$

$$\implies \begin{pmatrix} a & b+c \\ b & c+a \\ c & a+b \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.2}$$

So the augmented matrix of (2.0.2) is given by

$$\begin{pmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{pmatrix} \tag{2.0.3}$$

Using row reduction we get,

$$\begin{pmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_2 = R_2 - \frac{b}{a}R_1} \begin{pmatrix} 1 & \frac{b+c}{a} & \frac{1}{a} \\ 0 & \frac{(a-b)(a+b+c)}{a} & \frac{a-b}{a} \\ c & a+b & 1 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_3 = R_3 - cR_1} \begin{pmatrix} 1 & \frac{b+c}{a} & \frac{1}{a} \\ 0 & \frac{(a-b)(a+b+c)}{a} & \frac{a-b}{a} \\ 0 & \frac{(a-c)(a+b+c)}{a} & \frac{a-c}{a} \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_2 = \frac{a}{(a-b)(a+b+c)}R_2} \begin{pmatrix} 1 & \frac{b+c}{a} & \frac{1}{a} \\ 0 & 1 & \frac{1}{a+b+c} \\ 0 & \frac{(a-c)(a+b+c)}{a} & \frac{a-c}{a} \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_1 = R_1 - \frac{b+c}{a}R_2} \begin{pmatrix} 1 & 0 & \frac{1}{a+b+c} \\ 0 & 1 & \frac{1}{a+b+c} \\ 0 & \frac{(a-c)(a+b+c)}{a} & \frac{a-c}{a} \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_3 = R_3 - \frac{(a-b)(a+b+c)}{a}R_2} \begin{pmatrix} 1 & 0 & \frac{1}{a+b+c} \\ 0 & 1 & \frac{1}{a+b+c} \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.9}$$

From (2.0.9) we see that the rank of the augmented matrix is less than 3, hence $\mathbf{A}$,$\mathbf{B}$ and $\mathbf{C}$ are colinear.

## 3 Example

We illustrate the concept by an example. Let $a$=1, $b$=2 and $c$=3. The points are $\mathbf{A}$=$\begin{pmatrix} 1 & 5 \end{pmatrix}$,$\mathbf{B}$=$\begin{pmatrix} 2 & 4 \end{pmatrix}$ and $\mathbf{C}$=$\begin{pmatrix} 3 & 3 \end{pmatrix}$.
Below is the diagram of the line passing through the points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$.

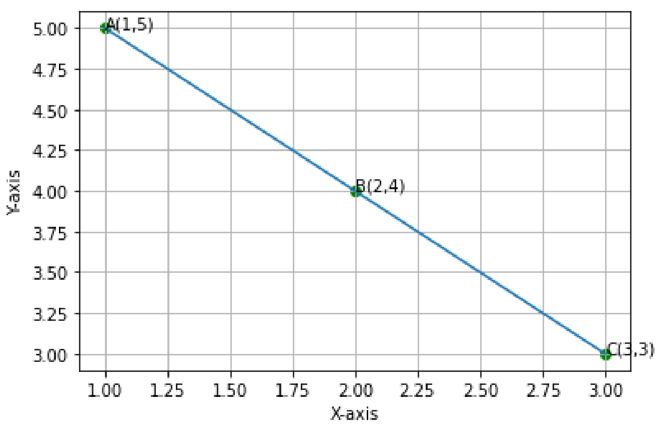


Fig. 1: Line passing through points **A**, **B** and **C**