Matrix Theory (EE5609) Assignment 7

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***Abstract*—This finds whether a given second degree equation represents a pair of straight lines or not.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_7

1 Problem

Find the value of k so that the following equation may represent a pair of straight lines -

$$6x^2 + xy + ky^2 - 11x + 43y - 35 = 0$$

2 Theory

The general equation of second degree is given by,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

(2.0.1) can be written as,

$$\tag{2.0.2}$$

$$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0 \tag{2.0.3}$$

where,

$$\mathbf{V} = \mathbf{V}^{\mathbf{T}} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \tag{2.0.5}$$

(2.0.3) represents a pair of straight lines if,

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^{\mathbf{T}} & f \end{vmatrix} = 0 \tag{2.0.6}$$

Otherwise, (2.0.3) represents a conic section.

3 Solution

The given second degree equation is,

$$6x^2 + xy + ky^2 - 11x + 43y - 35 = 0 \tag{3.0.1}$$

Comparing coefficients of (3.0.1) with (2.0.1) we get,

$$\mathbf{V} = \begin{pmatrix} 6 & \frac{1}{2} \\ \frac{1}{2} & k \end{pmatrix} \tag{3.0.2}$$

$$\mathbf{u} = \begin{pmatrix} -\frac{11}{2} \\ \frac{43}{2} \end{pmatrix} \tag{3.0.3}$$

$$f = -35 \tag{3.0.4}$$

From (2.0.6) the given second degree equation (3.0.1) will represent a pair of straight line if,

$$\begin{vmatrix} 6 & \frac{1}{2} & -\frac{11}{2} \\ \frac{1}{2} & k & \frac{43}{2} \\ -\frac{11}{2} & \frac{43}{2} & -35 \end{vmatrix} = 0 \tag{3.0.5}$$

Expanding the determinant,

$$k + 12 = 0 \tag{3.0.6}$$

$$\implies k = -12 \tag{3.0.7}$$

Hence, from (3.0.7) we find that for $k = -12$, the given second degree equation (3.0.1) represents pair of straight lines. For the appropriate value of k, (3.0.1) becomes,

$$6x^2 + xy - 12y^2 - 11x + 43y - 35 = 0 \tag{3.0.8}$$

4 Graphical Illustration

We obtain the linear equation of the form $(ax + by + c)$ from (3.0.8) by considering

$$6x^2 + xy - 12y^2 = 6x^2 - 8xy + 9xy - 12y^2 \tag{4.0.1}$$

$$= 2x(3x - 4y) + 3y(3x - 4y) \tag{4.0.2}$$

$$= (3x - 4y)(2x + 3y) \tag{4.0.3}$$

Again (3.0.8) can be written using (4.0.3) as,

$$(3x - 4y + m)(2x + 3y + n) = 0 \tag{4.0.4}$$

$$\implies 6x^2 + xy - 12y^2 + (2m + 3n)x + (3m - 4n)y + mn = 0 \tag{4.0.5}$$

Equating coefficients of (3.0.1) and (4.0.5),

$$2m + 3n = -11 \tag{4.0.6}$$
$$3m - 4n = 43 \tag{4.0.7}$$

The equations can be written as follows,

$$\begin{pmatrix} 2 & 3 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} -11 \\ 43 \end{pmatrix} \tag{4.0.8}$$

The augmented matrix of (4.0.8) is,

$$\begin{pmatrix} 2 & 3 & -11 \\ 3 & -4 & 43 \end{pmatrix} \xleftrightarrow{R_1 = \frac{1}{2}R_1} \begin{pmatrix} 1 & \frac{3}{2} & -\frac{11}{2} \\ 3 & -4 & 43 \end{pmatrix} \tag{4.0.9}$$

$$\xleftrightarrow{R_2 = R_2 - 3R_1} \begin{pmatrix} 1 & \frac{3}{2} & -\frac{11}{2} \\ 0 & -\frac{17}{2} & \frac{119}{2} \end{pmatrix} \tag{4.0.10}$$

$$\xleftrightarrow{R_2 = -\frac{2}{17}R_2} \begin{pmatrix} 1 & \frac{3}{2} & -\frac{11}{2} \\ 0 & 1 & -7 \end{pmatrix} \tag{4.0.11}$$

$$\xleftrightarrow{R_1 = R_1 - \frac{3}{2}R_2} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & -7 \end{pmatrix} \tag{4.0.12}$$

(4.0.13)

Hence we get,

$$m = 5 \tag{4.0.14}$$
$$n = -7 \tag{4.0.15}$$

Substituting (4.0.14) and (4.0.15) in (4.0.4), we obtain

$$(3x - 4y + 5)(2x + 3y - 7) = 0 \tag{4.0.16}$$

Hence (4.0.16) represents equation of a pair of straight lines.
The figure below corresponds to the pair of straight lines represented by (4.0.16).

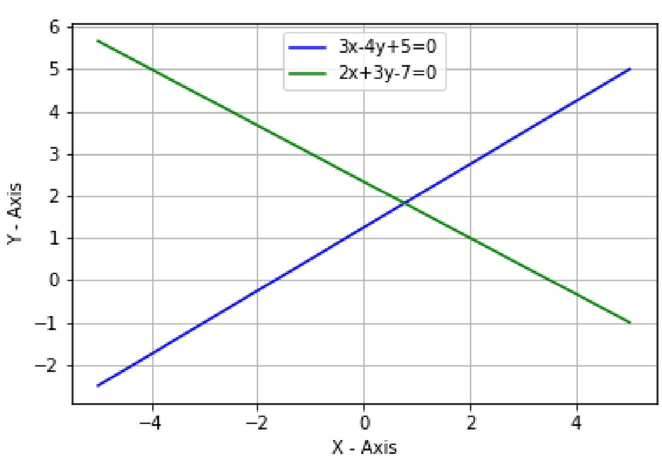

Fig. 1: Pair of Straight Lines