Matrix Theory (EE5609) Assignment 5

Arkadipta De
MTech Artificial Intelligence
AI20MTECH14002

***Abstract*—This document proves the co-linearity of three points in X-Y plane.**

The code to plot the figure of this problem can be found at

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https://github.com/Arko98/EE5609/blob/master/
    Assignment_5/Codes/Figure.py
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1 Problem

Show that the points $\mathbf{A} = \begin{pmatrix} a & b+c \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} b & c+a \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} c & a+b \end{pmatrix}$ are collinear.

2 Solution

The equation of the line formed by **A** and **B** i.e **BA** and line formed by **B** and **C** i.e **CB** is given by

$$\mathbf{BA} : \mathbf{r_1} = \begin{pmatrix} a \\ b+c \end{pmatrix} + \lambda_1 \begin{pmatrix} a-b \\ b-a \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{CB} : \mathbf{r_2} = \begin{pmatrix} b \\ c+a \end{pmatrix} + \lambda_2 \begin{pmatrix} c-b \\ b-c \end{pmatrix} \tag{2.0.2}$$

So if the three points are collinear then there exists no such non zero λ_1 and λ_2 such that (2.0.1) and (2.0.2) are equal,

$$\begin{pmatrix} a \\ b+c \end{pmatrix} + \lambda_1 \begin{pmatrix} a-b \\ b-a \end{pmatrix} = \begin{pmatrix} b \\ c+a \end{pmatrix} + \lambda_2 \begin{pmatrix} c-b \\ b-c \end{pmatrix} \tag{2.0.3}$$

$$\implies \lambda_1 \begin{pmatrix} a-b \\ b-a \end{pmatrix} - \lambda_2 \begin{pmatrix} c-b \\ b-c \end{pmatrix} = \begin{pmatrix} b \\ c+a \end{pmatrix} - \begin{pmatrix} a \\ b+c \end{pmatrix} \tag{2.0.4}$$

$$\implies \begin{pmatrix} a-b & c-b \\ b-a & b-c \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} b-a \\ a-b \end{pmatrix} \tag{2.0.5}$$

Hence the augmented matrix from (2.0.5) will be,

$$\left(\begin{array}{cc|c} a-b & c-b & b-a \\ b-a & b-c & a-b \end{array}\right) \tag{2.0.6}$$

Using row reduction we get,

$$\left(\begin{array}{cc|c} a-b & c-b & b-a \\ b-a & b-c & a-b \end{array}\right) \tag{2.0.7}$$

$$\xleftrightarrow{R_2 = R_2 - R_1} \left(\begin{array}{cc|c} a-b & c-b & b-a \\ 0 & 0 & 0 \end{array}\right) \tag{2.0.8}$$

Hence, **A**,**B** and **C** are colinear.

3 Example

We illustrate the concept by an example. Let a=1, b=2 and c=3. The points are $\mathbf{A}=\begin{pmatrix} 1 & 5 \end{pmatrix}$,$\mathbf{B}=\begin{pmatrix} 2 & 4 \end{pmatrix}$ and $\mathbf{C}=\begin{pmatrix} 3 & 3 \end{pmatrix}$. Below is the diagram of the line passing through the points **A**, **B** and **C**.

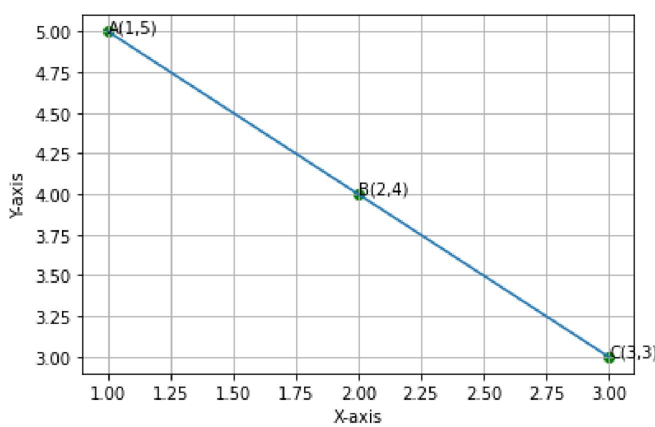

Fig. 1: Line passing through points **A**, **B** and **C**