Matrix Theory (EE5609) Assignment 15

Arkadipta De
MTech Artificial Intelligence
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***Abstract*—This document finds the basis vectors of the subspace of $\mathbf{R}^4$ spanned by some given vectors.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_15

1 Problem

Find a basis for the subspace of $\mathbb{R}^4$ spanned by the four vectors

$$\alpha_{\mathbf{1}} = \begin{pmatrix} 1 & 1 & 2 & 4 \end{pmatrix} \tag{1.0.1}$$

$$\alpha_{\mathbf{2}} = \begin{pmatrix} 2 & -1 & -5 & 2 \end{pmatrix} \tag{1.0.2}$$

$$\alpha_{\mathbf{3}} = \begin{pmatrix} 1 & -1 & -4 & 0 \end{pmatrix} \tag{1.0.3}$$

$$\alpha_{\mathbf{4}} = \begin{pmatrix} 2 & 1 & 1 & 6 \end{pmatrix} \tag{1.0.4}$$

2 Solution

The basis of the given four vectors is equivalent to finding the basis of column-space $C(\mathbf{A})$ of a matrix $\mathbf{A}$ defined as follows,

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & 6 \end{pmatrix} \tag{2.0.1}$$

Now we calculate the row echelon form of $\mathbf{A}$ as follows,

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & -5 & -4 & 1 \\ 4 & 2 & 0 & 6 \end{pmatrix} \xleftrightarrow[R_2=R_2-R_1]{R_2=R_2-R_1} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & -2 & -1 \\ 0 & -9 & -6 & -3 \\ 4 & 2 & 0 & 6 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_4=R_4-R_1} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & -2 & -1 \\ 0 & -9 & -6 & -3 \\ 0 & -6 & -4 & -2 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_2=-\frac{1}{3}R_2} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{3} \\ 0 & -9 & -6 & -3 \\ 0 & -6 & -4 & -2 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_3=R_3-9R_2} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & -6 & -4 & -2 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_4=R_4+6R_2} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.6}$$

From (2.0.6) we can see that the first column and second column of $\mathbf{A}$ contains pivot values. Hence the column 1 and column 2 are the basis of the subspace of $\mathbb{R}^4$ spanned by the given vectors $\alpha_{\mathbf{1}}$, $\alpha_{\mathbf{2}}$, $\alpha_{\mathbf{3}}$, $\alpha_{\mathbf{4}}$
Hence the required basis vectors are,

$$\mathbf{a_1} = \begin{pmatrix} 1 & 1 & 2 & 4 \end{pmatrix} \tag{2.0.7}$$

$$\mathbf{a_2} = \begin{pmatrix} 2 & -1 & -5 & 2 \end{pmatrix} \tag{2.0.8}$$