Matrix Theory (EE5609) Assignment 13

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***Abstract*—This document proves if AB = I then BA = I given that both A and B are square matrices.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_13

1 Problem

Let $\mathbf{A}$ and $\mathbf{B}$ be $n \times n$ matrices such that $\mathbf{AB} = \mathbf{I}$. Prove that $\mathbf{BA} = \mathbf{I}$

2 Solution

Let $\mathbf{BX} = 0$ be a system of linear equation with n unknowns and n equations as $\mathbf{B}$ is $n \times n$ matrix. Hence,

$$\mathbf{BX} = 0 \tag{2.0.1}$$
$$\implies \mathbf{A}(\mathbf{BX}) = 0 \tag{2.0.2}$$
$$\implies (\mathbf{AB})\mathbf{X} = 0 \tag{2.0.3}$$
$$\implies \mathbf{IX} = 0 \quad [\because \mathbf{AB} = \mathbf{I}] \tag{2.0.4}$$
$$\implies \mathbf{X} = 0 \tag{2.0.5}$$

From (2.0.5) since $\mathbf{X} = 0$ is the only solution of (2.0.1), hence $rank(\mathbf{B}) = n$. Which implies all columns of $\mathbf{B}$ are linearly independent. Hence $\mathbf{B}$ is invertible. Therefore, every left inverse of $\mathbf{B}$ is also a right inverse of $\mathbf{B}$. Hence there exists a $n \times n$ matrix $\mathbf{C}$ such that,

$$\mathbf{BC} = \mathbf{CB} = \mathbf{I} \tag{2.0.6}$$

Or $\mathbf{C} = \mathbf{B^{-1}}$. Again given that $\mathbf{AB} = \mathbf{I}$. Hence from the given equation and (2.0.6) we can say,

$$\mathbf{CB} = \mathbf{AB} \tag{2.0.7}$$
$$\implies \mathbf{C} = \mathbf{A} \quad \text{[Where A is the left inverse of B]} \tag{2.0.8}$$

Hence $\mathbf{A}$ is also the right inverse of $\mathbf{B}$. Therefore,

$$\mathbf{BC} = \mathbf{BA} = \mathbf{I} \tag{2.0.9}$$
$$\implies \mathbf{BA} = \mathbf{I} \tag{2.0.10}$$

Hence Proved.