Matrix Theory (EE5609) Assignment 5

Arkadipta De
MTech Artificial Intelligence
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***Abstract*—This document proves the co-linearity of three points in X-Y plane.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_6

1 Problem

$\triangle ABC$ is a triangle right angled at $\mathbf{C}$. A line through the mid-point $\mathbf{M}$ of hypotenuse $\mathbf{AB}$ and parallel to $\mathbf{BC}$ intersects $\mathbf{AC}$ at $\mathbf{D}$. Show that -

(i) $\mathbf{D}$ is the mid-point of $\mathbf{AC}$
(ii) $\mathbf{MD} \perp \mathbf{AC}$
(iii) $\mathbf{CM} = \mathbf{MA} = \frac{1}{2}\ \mathbf{AB}$

2 Solution

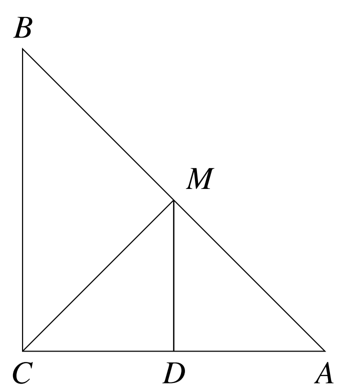

Fig. 1: Right Angled Triangle by Latex-Tikz

In $\triangle ABC$, $\mathbf{M}$ is midpoint of $\mathbf{AB}$ and $\mathbf{MD}$ is parallel to $\mathbf{BC}$, hence,

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{2.0.1}$$

$$\mathbf{MD} \parallel \mathbf{BC} \tag{2.0.2}$$

As line drawn through mid point of one side of triangle parallel to other side bisects third side, hence proved from (2.0.1) and (2.0.2), $\mathbf{D}$ is midpoint of $\mathbf{A}$ and $\mathbf{C}$ i.e

$$\mathbf{D} = \frac{\mathbf{A}+\mathbf{C}}{2} \tag{2.0.3}$$

From figure 1, direction vectors of $\mathbf{MD}$ and $\mathbf{AC}$ are given by,

$$\mathbf{m_{MD}} = \mathbf{M} - \mathbf{D} \tag{2.0.4}$$

$$\implies \mathbf{m_{MD}} = \frac{\mathbf{A}+\mathbf{B}}{2} - \frac{\mathbf{A}+\mathbf{C}}{2} \tag{2.0.5}$$

$$\implies \mathbf{m_{MD}} = \frac{\mathbf{B}-\mathbf{C}}{2} \tag{2.0.6}$$

$$\mathbf{m_{AC}} = \mathbf{A} - \mathbf{C} \tag{2.0.7}$$

Hence,

$$\mathbf{m_{MD}}\mathbf{m_{AC}} = (\frac{\mathbf{B}-\mathbf{C}}{2})(\mathbf{A}-\mathbf{C}) \tag{2.0.8}$$

$$\implies \mathbf{m_{MD}}\mathbf{m_{AC}} = (\frac{\mathbf{m_{BC}}}{2})(\mathbf{m_{AC}}) \tag{2.0.9}$$

$$\implies \mathbf{m_{MD}}\mathbf{m_{AC}} \ = 0 \quad [\because \mathbf{BC} \perp \mathbf{AC},\ \angle BCA = 90°] \tag{2.0.10}$$

From (2.0.10), it is proved that $\mathbf{MD} \perp \mathbf{AC}$
If $\mathbf{m_{CM}}$, $\mathbf{m_{CD}}$, $\mathbf{m_{DM}}$, $\mathbf{m_{AD}}$, $\mathbf{m_{AM}}$ and $\mathbf{m_{AB}}$ are direction vectors of $\mathbf{CM}$, $\mathbf{CD}$, $\mathbf{DM}$, $\mathbf{AD}$, $\mathbf{AM}$ and $\mathbf{AB}$ respectively then from the figure 1, after joining $\mathbf{M}$ and $\mathbf{C}$, in $\triangle AMD$ and $\triangle CMD$ we get,

$$\mathbf{m_{CM}} = \mathbf{m_{CD}} + \mathbf{m_{DM}} \quad [\text{From } \triangle CDM] \tag{2.0.11}$$

$$\implies \mathbf{m_{CM}} = \mathbf{m_{AD}} + \mathbf{m_{DM}} \quad [\text{Proved in (2.0.3)}] \tag{2.0.12}$$

$$\implies \mathbf{m_{CM}} = \mathbf{m_{AM}} \quad [\text{From } \triangle ADM] \tag{2.0.13}$$

$$\implies \mathbf{m_{CM}} = \mathbf{m_{AM}} = \frac{\mathbf{m_{AB}}}{2} \quad [\text{From (2.0.1)}] \tag{2.0.14}$$

Hence proved, $\mathbf{CM} = \mathbf{MA} = \frac{1}{2}\ \mathbf{AB}$