

Matrix Theory (EE5609) Assignment 6

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Abstract—This proves a theorem on triangle.

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_6

1 PROBLEM

$\triangle ABC$ is a triangle right angled at C . A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D . Show that -

- (i) D is the mid-point of AC
- (ii) $MD \perp AC$
- (iii) $CM = MA = \frac{1}{2} AB$

2 SOLUTION

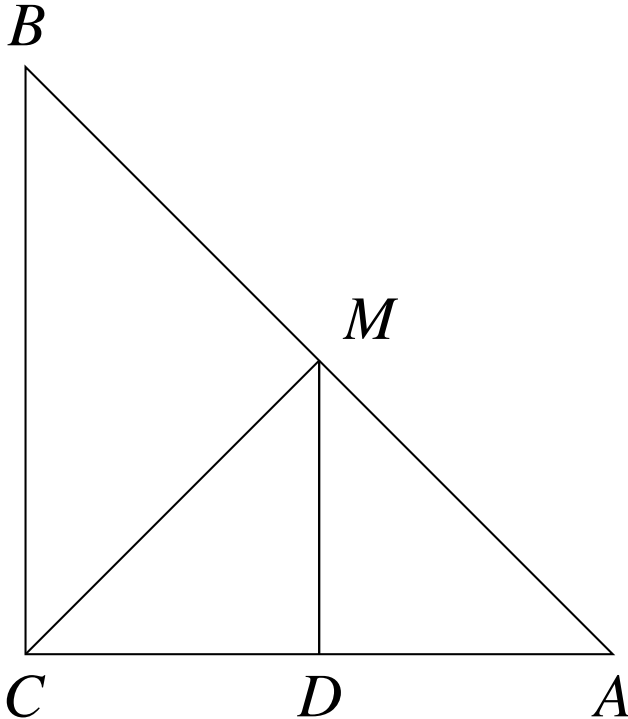


Fig. 1: Right Angled Triangle by Latex-Tikz

In $\triangle ABC$, M is midpoint of AB and MD is parallel to BC , hence,

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (2.0.1)$$

$$\mathbf{MD} \parallel \mathbf{BC} \quad (2.0.2)$$

Let \mathbf{m}_{MD} and \mathbf{m}_{BC} are direction vectors of MD and BC respectively. Then,

$$\mathbf{m}_{MD} = \mathbf{M} - \mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2} - \mathbf{D} \quad (2.0.3)$$

$$\mathbf{m}_{BC} = \mathbf{B} - \mathbf{C} \quad (2.0.4)$$

Now from (2.0.2) we get,

$$\mathbf{m}_{MD} = k\mathbf{m}_{BC} \quad (2.0.5)$$

$$\Rightarrow \frac{\mathbf{A} + \mathbf{B}}{2} - \mathbf{D} = k(\mathbf{B} - \mathbf{C}) \quad (2.0.6)$$

Let $\mathbf{D} = \frac{m\mathbf{A} + \mathbf{C}}{m+1}$, then from (2.0.6) we get,

$$\frac{\mathbf{A} + \mathbf{B}}{2} - \frac{m\mathbf{A} + \mathbf{C}}{m+1} = k(\mathbf{B} - \mathbf{C}) \quad (2.0.7)$$

$$\Rightarrow \left(\frac{1}{2} - \frac{m}{m+1}\right)\mathbf{A} + \left(\frac{1}{2} - k\right)\mathbf{B} + \left(k - \frac{1}{m+1}\right)\mathbf{C} = \mathbf{0} \quad (2.0.8)$$

Since \mathbf{A} , \mathbf{B} and \mathbf{C} are linearly dependent as they form $\triangle ABC$ then

$$\frac{1}{2} - \frac{m}{m+1} = 0 \quad (2.0.9)$$

$$\frac{1}{2} - k = 0 \quad (2.0.10)$$

$$k - \frac{1}{m+1} = 0 \quad (2.0.11)$$

Solving (2.0.9), (2.0.10) and (2.0.11) we get $k = \frac{1}{2}$ and $m = 1$. Hence, substituting value of m in \mathbf{D} we get,

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{C}}{2} \quad (2.0.12)$$

Hence Proved.

From figure 1,

$$(\mathbf{M} - \mathbf{D})(\mathbf{A} - \mathbf{D}) = \left(\frac{\mathbf{A} + \mathbf{B}}{2} - \frac{\mathbf{A} + \mathbf{C}}{2} \right) (\mathbf{A} - \mathbf{C}) \quad (2.0.13)$$

$$\Rightarrow (\mathbf{M} - \mathbf{D})(\mathbf{A} - \mathbf{D}) = \frac{1}{2} (\mathbf{B} - \mathbf{C}) (\mathbf{A} - \mathbf{C}) \quad (2.0.14)$$

$$\Rightarrow (\mathbf{M} - \mathbf{D})(\mathbf{A} - \mathbf{D}) = 0 \quad [\because \mathbf{BC} \perp \mathbf{AC}] \quad (2.0.15)$$

From (2.0.15), it is proved that $\mathbf{MD} \perp \mathbf{AC}$

Again we get,

$$\mathbf{C} - \mathbf{M} = \mathbf{C} - \mathbf{D} + \mathbf{D} - \mathbf{M} \quad (2.0.16)$$

$$\Rightarrow \mathbf{C} - \mathbf{M} = \mathbf{A} - \mathbf{D} + \mathbf{D} - \mathbf{M} \quad [\text{From (2.0.12)}] \quad (2.0.17)$$

$$\Rightarrow \mathbf{C} - \mathbf{M} = \mathbf{A} - \mathbf{M} \quad (2.0.18)$$

$$\Rightarrow \mathbf{C} - \mathbf{M} = \mathbf{A} - \frac{\mathbf{A} + \mathbf{B}}{2} \quad [\text{From (2.0.1)}] \quad (2.0.19)$$

$$\Rightarrow \mathbf{C} - \mathbf{M} = \frac{1}{2} (\mathbf{A} - \mathbf{B}) \quad (2.0.20)$$

$$\Rightarrow \|\mathbf{C} - \mathbf{M}\| = \frac{1}{2} \|\mathbf{A} - \mathbf{B}\| \quad (2.0.21)$$

Hence from (2.0.18) and (2.0.21) proved,

$$\mathbf{CM} = \mathbf{MA} = \frac{1}{2} \mathbf{AB}$$