Matrix Theory (EE5609) Assignment 2

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***Abstract*—This assignment finds the equation of a straight line given two points on that line.**

1 Problem Statement

Find the equation of the line passing through the origin and the point $\begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$.

2 Theory

The direction vector $\mathbf{A}$ for a line through the points $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ y_2 \\ z_2 \end{pmatrix}$ is given by

$$\mathbf{A} = \begin{pmatrix} a \\ b \\ b \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix} \tag{2.0.1}$$

The Cartesian form of a line passing through the two points $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ y_2 \\ z_2 \end{pmatrix}$ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \tag{2.0.2}$$

And the parametric form of the same line is given by

$$x = x_1 + at, y = y_1 + bt, z = z_1 + ct \tag{2.0.3}$$

Where t is a parameter and a, b, c, which are the components of the direction vector, are defined by equation 2.0.1

The vector form of equation of a line passing through a point with position vector $\mathbf{a}$ and along the direction vector $\mathbf{b}$ is given by

$$\mathbf{r} = \mathbf{a} + k\mathbf{b} \tag{2.0.4}$$

where k is a constant multiple.

3 Solution

Let the points be $\mathbf{O}$=$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ which is the origin and $\mathbf{P}$=$\begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$.

From the theory, using equation 2.0.1, the direction vector for the line through the points $\mathbf{O}$ and $\mathbf{P}$ is

$$\mathbf{A} = \mathbf{P} - \mathbf{O} \tag{3.0.1}$$

$$\implies \mathbf{A} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.0.2}$$

$$\implies \mathbf{A} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} \tag{3.0.3}$$

So, by putting the values of the points and the direction vector in equation 2.0.2, the Cartesian form of the line passing through $\mathbf{O}$ and $\mathbf{P}$ is given by

$$\frac{x}{5} = \frac{y}{-2} = \frac{z}{3} \tag{3.0.4}$$

Again the parametric form of the line is obtained from equation 2.0.3 by putting the value of the points and direction vector. The parametric form of the line passing through $\mathbf{O}$ and $\mathbf{P}$ is given by

$$x = 5t, y = -2t, z = 3t \tag{3.0.5}$$

where t is the parameter.

From equation 2.0.4, the vector form of the line passing through $\mathbf{O}$ and $\mathbf{P}$, which is the line passing through the point $\mathbf{O}$ and along direction vector $\mathbf{A}$ is given by

$$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} \tag{3.0.6}$$

$$\implies \mathbf{r} = k \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} \tag{3.0.7}$$

where k is a constant multiple.