Matrix Theory (EE5609) Assignment 4

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Abstract—**This document solves an equation on matrix.**

The code for the solution of this problem can be found at

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https://github.com/Arko98/EE5609/blob/master/
    Assignment_4/Codes/Solution.py
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1 Problem

If $\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $\mathbf{A^3}-6\mathbf{A^2}+7\mathbf{A}+2\mathbf{I}=0$

2 Solution

The equation in the problem can be modified as follows

$$\mathbf{A^3} - 6\mathbf{A^2} + 7\mathbf{A} + 2\mathbf{I} = 0 \tag{2.0.1}$$

$$\implies \mathbf{A^2}(\mathbf{A} - \mathbf{6I}) + \mathbf{7A} + \mathbf{2I} = 0 \tag{2.0.2}$$

So we need to prove equation 2.0.2. Now, at first we calculate the value of $\mathbf{A^2}$ as follows

$$\mathbf{A^2} = \mathbf{A} \cdot \mathbf{A} \tag{2.0.3}$$

$$\implies \mathbf{A^2} = \mathbf{A^T A} \tag{2.0.4}$$

$$\implies \mathbf{A^2} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \tag{2.0.5}$$

$$\implies \mathbf{A^2} = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} \tag{2.0.6}$$

Next we calculate, $\mathbf{A} - 6\mathbf{I}$ where I is identity matrix of order 3, as follows

$$\mathbf{A} - 6\mathbf{I} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} - 6 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{A} - 6\mathbf{I} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \tag{2.0.8}$$

$$\implies \mathbf{A} - 6\mathbf{I} = \begin{pmatrix} -5 & 0 & 2 \\ 0 & -4 & 1 \\ 2 & 0 & -3 \end{pmatrix} \tag{2.0.9}$$

Now we compute $\mathbf{A^2}(\mathbf{A} - 6\mathbf{I})$ by putting values of $\mathbf{A^2}$ from equation 2.0.6 and value of $\mathbf{A} - 6\mathbf{I}$ from 2.0.9 as follows

$$\mathbf{A^2} \cdot (\mathbf{A} - 6\mathbf{I}) = (\mathbf{A^2})^{\mathbf{T}}(\mathbf{A} - 6\mathbf{I}) \tag{2.0.10}$$

$$\implies \mathbf{A^2} \cdot (\mathbf{A} - 6\mathbf{I}) = \begin{pmatrix} 5 & 2 & 8 \\ 0 & 4 & 0 \\ 8 & 5 & 13 \end{pmatrix} \begin{pmatrix} -5 & 0 & 2 \\ 0 & -4 & 1 \\ 2 & 0 & -3 \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{A^2} \cdot (\mathbf{A} - 6\mathbf{I}) = \begin{pmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{pmatrix} \tag{2.0.12}$$

Next we compute $7\mathbf{A} + 2\mathbf{I}$ as follows

$$7\mathbf{A} + 2\mathbf{I} = 7 \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.13}$$

$$\implies 7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{pmatrix} \tag{2.0.14}$$

Now putting the value of $\mathbf{A^2}(\mathbf{A} - 6\mathbf{I})$ from equation 2.0.12 and value of $7\mathbf{A} + 2\mathbf{I}$ from equation 2.0.14

into the left hand side of equation 2.0.2 we get

$$\mathbf{A}^2(\mathbf{A}-6\mathbf{I})+7\mathbf{A}+2\mathbf{I}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix} \tag{2.0.15}$$

$$\implies \mathbf{A}^2(\mathbf{A}-6\mathbf{I})+7\mathbf{A}+2\mathbf{I}=0 \tag{2.0.16}$$

Thus from equation 2.0.16 we arrive at the right hand side of equation 2.0.2, Hence proved.