Matrix Theory (EE5609) Assignment 9

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***Abstract*—This document performs QR decomposition on a given matrix.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_9

1 Problem

Find QR decomposition of $\begin{pmatrix} 2 & 3 \\ 3 & -4 \end{pmatrix}$

2 Solution

Let $\mathbf{a}$ and $\mathbf{b}$ be the column vectors of the given matrix.

$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{2.0.2}$$

The column vectors can be expressed as follows,

$$\mathbf{a} = k_1 \mathbf{u}_1 \tag{2.0.3}$$

$$\mathbf{b} = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \tag{2.0.4}$$

Here,

$$k_1 = \|\mathbf{a}\| \tag{2.0.5}$$

$$\mathbf{u}_1 = \frac{\mathbf{a}}{k_1} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T \mathbf{b}}{\|\mathbf{u}_1\|^2} \tag{2.0.7}$$

$$\mathbf{u}_2 = \frac{\mathbf{b} - r_1 \mathbf{u}_1}{\|\mathbf{b} - r_1 \mathbf{u}_1\|} \tag{2.0.8}$$

$$k_2 = \mathbf{u}_2^T \mathbf{b} \tag{2.0.9}$$

The (2.0.3) and (2.0.4) can be written as,

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

Now, $\mathbf{R}$ is an upper triangular matrix and also,

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \tag{2.0.12}$$

Now using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{2^2 + 3^2} = \sqrt{13} \tag{2.0.13}$$

$$\mathbf{u}_1 = \frac{1}{\sqrt{13}} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2.0.14}$$

$$r_1 = \begin{pmatrix} \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix} = -\frac{6}{\sqrt{13}} \tag{2.0.15}$$

$$\mathbf{u}_2 = \frac{1}{\sqrt{13}} \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{2.0.16}$$

$$k_2 = \begin{pmatrix} \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \frac{17}{\sqrt{13}} \tag{2.0.17}$$

Thus putting the values from (2.0.13) to (2.0.17) in (2.0.11) we obtain QR decomposition,

$$\begin{pmatrix} 2 & 3 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \\ \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \end{pmatrix} \begin{pmatrix} \sqrt{13} & -\frac{6}{\sqrt{13}} \\ 0 & \frac{17}{\sqrt{13}} \end{pmatrix} \tag{2.0.18}$$