

Matrix Theory (EE5609) Assignment 5

Arkadipta De
MTech Artificial Intelligence
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Abstract—This document proves the co-linearity of three points in X-Y plane. Using row reduction we get,

The code to plot the figure of this problem can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_5/Codes/Figure.py

$$\begin{pmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{pmatrix} \quad (2.0.4)$$

$$\xleftrightarrow{R_3 = cR_3 - bR_2} \begin{pmatrix} a & b+c & 1 \\ b & c+a & 1 \\ 0 & (b-c)(a+b+c) & b-c \end{pmatrix} \quad (2.0.5)$$

$$\xleftrightarrow{R_3 = \frac{1}{(b-c)}R_3} \begin{pmatrix} a & b+c & 1 \\ b & c+a & 1 \\ 0 & a+b+c & 1 \end{pmatrix} \quad (2.0.6)$$

1 PROBLEM

Show that the points $\mathbf{A} = \begin{pmatrix} a \\ b+c \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} b \\ c+a \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} c \\ a+b \end{pmatrix}$ are collinear.

$$\xleftrightarrow{R_2 = aR_2 - bR_1} \begin{pmatrix} a & b+c & 1 \\ 0 & (a-b)(a+b+c) & a-b \\ 0 & a+b+c & 1 \end{pmatrix} \quad (2.0.7)$$

$$\xleftrightarrow{R_2 = \frac{1}{(a-b)}R_2} \begin{pmatrix} a & b+c & 1 \\ 0 & a+b+c & 1 \\ 0 & a+b+c & 1 \end{pmatrix} \quad (2.0.8)$$

2 SOLUTION

$$\xleftrightarrow{R_3 = R_3 - R_2} \begin{pmatrix} a & b+c & 1 \\ 0 & a+b+c & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.9)$$

The points \mathbf{A} , \mathbf{B} and \mathbf{C} will be collinear if

(2.0.10)

$$\begin{pmatrix} \mathbf{A}^T \\ \mathbf{B}^T \\ \mathbf{C}^T \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (2.0.1)$$

$$\Rightarrow \begin{pmatrix} a & b+c \\ b & c+a \\ c & a+b \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (2.0.2)$$

So the augmented matrix of (2.0.2) is given by

$$\begin{pmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{pmatrix} \quad (2.0.3)$$

From (2.0.10) we see that the rank of the augmented matrix is less than 3, hence \mathbf{A} , \mathbf{B} and \mathbf{C} are collinear.

3 EXAMPLE

We illustrate the concept by an example. Let $a=1$, $b=2$ and $c=3$. The points are $\mathbf{A}=\begin{pmatrix} 1 & 5 \end{pmatrix}$, $\mathbf{B}=\begin{pmatrix} 2 & 4 \end{pmatrix}$ and $\mathbf{C}=\begin{pmatrix} 3 & 3 \end{pmatrix}$.

Below is the diagram of the line passing through the points \mathbf{A} , \mathbf{B} and \mathbf{C} .

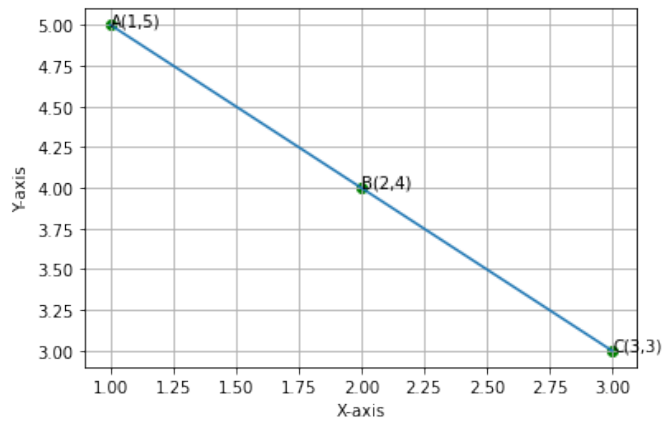


Fig. 1: Line passing through points **A**, **B** and **C**