

Matrix Theory (EE5609) Assignment 1

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Abstract—This assignment solves a problem on checking whether two lines are parallel or perpendicular.

Below is the link to python code solution of this problem

1 PROBLEM STATEMENT

Show that the line through the points $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ is parallel to the line through the points $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$.

2 THEORY

The direction vector \mathbf{A} for a line through the points $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ is given by

$$\mathbf{A} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix} \quad (2.0.1)$$

For two lines having direction vectors \mathbf{A} and \mathbf{B} respectively, they will be perpendicular if the scalar product of the two direction vector is 0,

$$\mathbf{AB} = 0 \quad (2.0.2)$$

Where scalar product of two vectors, $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and

$\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ is defined by

$$\mathbf{AB} = \mathbf{A}^T \mathbf{B} = \begin{pmatrix} x_1 & y_1 & z_1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = x_1 x_2 + y_1 y_2 + z_1 z_2 \quad (2.0.3)$$

And the two lines will be parallel if the cross product of the two direction vector is 0,

$$\mathbf{A} \times \mathbf{B} = \mathbf{0} \quad (2.0.4)$$

3 SOLUTION

From the theory, the direction vector for the line through the points $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ is $\mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$ (using equations 2.0.1 and 2.0.2). Similarly, the direction vector for the line through the points $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$ is

$$\mathbf{B} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \text{ (using equations 2.0.1 and 2.0.2).}$$

To check if the two lines are perpendicular, we perform scalar product of the two direction vectors \mathbf{A} and \mathbf{B} using equation 2.0.1 and 2.0.2 as follows

$$\mathbf{AB} = \mathbf{A}^T \mathbf{B} \quad (3.0.1)$$

$$\Rightarrow \mathbf{AB} = \begin{pmatrix} 2 & 5 & -4 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \quad (3.0.2)$$

$$\Rightarrow \mathbf{AB} = 6 + 10 - 16 = 0 \quad (3.0.3)$$

Thus the direction vectors of the two lines satisfies the equation 2.0.1, hence proved that the lines are **perpendicular**. Hence they are not **parallel** with each other.

Python Code: The python code for the above solution can be found at - https://github.com/Arko98/EE5609/blob/master/Assignment_1/Codes/Solution_1.py