Matrix Theory (EE5609) Assignment 5

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***Abstract*—This document proves the co-linearity of three points in X-Y plane.**

The code to plot the figure of this problem can be found at

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https://github.com/Arko98/EE5609/blob/master/
    Assignment_5/Codes/Figure.py
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1 Problem

Show that the points $\mathbf{A} = \begin{pmatrix} a & b+c \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} b & c+a \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} c & a+b \end{pmatrix}$ are colinear.

2 Solution

We know that the triangle formed by 3 points i.e $\mathbf{A} = \begin{pmatrix} x_1 & y_1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} x_2 & y_2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} x_3 & y_3 \end{pmatrix}$ is given by,

$$\alpha = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \tag{2.0.1}$$

Now if the three points are colinear, then the area of the triangle formed by the three points is 0, i.e from (2.0.1) we get,

$$\alpha = 0 \tag{2.0.2}$$

$$\implies \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \tag{2.0.3}$$

Then, to proof the colinearity of the given points **A**,**B** and **C** we need to prove (2.0.3).
Putting values of **A**,**B** and **C** in left hand side of (2.0.3) we get,

$$\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = \begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix} \tag{2.0.4}$$

$$\implies \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix} \tag{2.0.5}$$

$$\implies \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = 0 \qquad [\because C1 = C3] \tag{2.0.6}$$

Hence, from (2.0.3) and (2.0.6) proved that, **A**,**B** and **C** are colinear.

3 Example

We illustrate the concept by an example. Let a=1, b=2 and c=3. The points are **A**=$\begin{pmatrix} 1 & 5 \end{pmatrix}$,**B**=$\begin{pmatrix} 2 & 4 \end{pmatrix}$ and **C**=$\begin{pmatrix} 3 & 3 \end{pmatrix}$. Below is the diagram of the line passing through the points **A**, **B** and **C**.

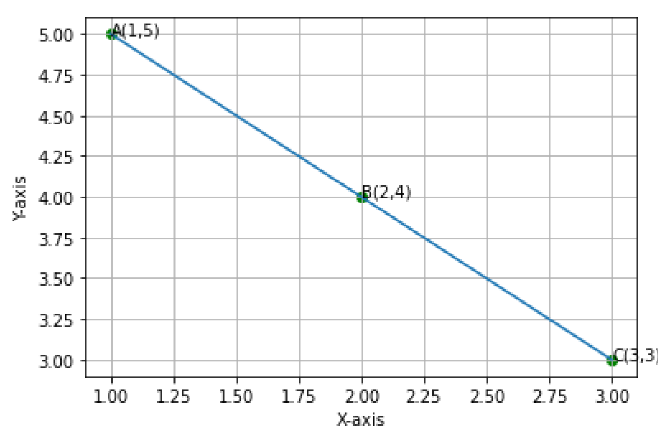

Fig. 1: Line passing through points **A**, **B** and **C**