

Matrix Theory (EE5609) Assignment 4

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Abstract—This document solves an equation on matrix.

Next we calculate, $\mathbf{A} - 6\mathbf{I}$ where \mathbf{I} is identity matrix of order 3, as follows

The code for the solution of this problem can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_4/Codes/Solution.py

$$\mathbf{A} - 6\mathbf{I} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} - 6 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.7)$$

$$\Rightarrow \mathbf{A} - 6\mathbf{I} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \quad (2.0.8)$$

$$\Rightarrow \mathbf{A} - 6\mathbf{I} = \begin{pmatrix} -5 & 0 & 2 \\ 0 & -4 & 1 \\ 2 & 0 & -3 \end{pmatrix} \quad (2.0.9)$$

1 PROBLEM

If $\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + 2\mathbf{I} = 0$

Now we compute $\mathbf{A}^2(\mathbf{A} - 6\mathbf{I})$ by putting values of \mathbf{A}^2 from equation 2.0.6 and value of $\mathbf{A} - 6\mathbf{I}$ from 2.0.9 as follows

$$\mathbf{A}^2 \cdot (\mathbf{A} - 6\mathbf{I}) = (\mathbf{A}^2)^T (\mathbf{A} - 6\mathbf{I}) \quad (2.0.10)$$

$$\Rightarrow \mathbf{A}^2 \cdot (\mathbf{A} - 6\mathbf{I}) = \begin{pmatrix} 5 & 2 & 8 \\ 0 & 4 & 0 \\ 8 & 5 & 13 \end{pmatrix} \begin{pmatrix} -5 & 0 & 2 \\ 0 & -4 & 1 \\ 2 & 0 & -3 \end{pmatrix} \quad (2.0.11)$$

$$\Rightarrow \mathbf{A}^2 \cdot (\mathbf{A} - 6\mathbf{I}) = \begin{pmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{pmatrix} \quad (2.0.12)$$

Next we compute $7\mathbf{A} + 2\mathbf{I}$ as follows

$$7\mathbf{A} + 2\mathbf{I} = 7 \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow 7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{pmatrix} \quad (2.0.14)$$

Now putting the value of $\mathbf{A}^2(\mathbf{A} - 6\mathbf{I})$ from equation 2.0.12 and value of $7\mathbf{A} + 2\mathbf{I}$ from equation 2.0.14

$$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A} \quad (2.0.3)$$

$$\Rightarrow \mathbf{A}^2 = \mathbf{A}^T \mathbf{A} \quad (2.0.4)$$

$$\Rightarrow \mathbf{A}^2 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \quad (2.0.5)$$

$$\Rightarrow \mathbf{A}^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} \quad (2.0.6)$$

The equation in the problem can be modified as follows

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + 2\mathbf{I} = 0 \quad (2.0.1)$$

$$\Rightarrow \mathbf{A}^2(\mathbf{A} - 6\mathbf{I}) + 7\mathbf{A} + 2\mathbf{I} = 0 \quad (2.0.2)$$

So we need to prove equation 2.0.2. Now, at first we calculate the value of \mathbf{A}^2 as follows

into the left hand side of equation 2.0.2 we get

$$\mathbf{A}^2(\mathbf{A} - 6\mathbf{I}) + 7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.15)$$

$$\Rightarrow \mathbf{A}^2(\mathbf{A} - 6\mathbf{I}) + 7\mathbf{A} + 2\mathbf{I} = 0 \quad (2.0.16)$$

Thus from equation 2.0.16 we arrive at the right hand side of equation 2.0.2, Hence proved.

minant from the equation 4.0.8 we get the following polynomial equation of λ

$$(1 - \lambda)(2 - \lambda)(3 - \lambda) - 4(2 - \lambda) = 0 \quad (4.0.9)$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \quad (4.0.10)$$

Hence, equation 4.0.10 is the required characteristic equation of matrix \mathbf{A} for λ .

3 PROBLEM 2

Find the characteristic equation of the matrix \mathbf{A}

$$= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

4 SOLUTION

For a general order k square matrix \mathbf{A} , the characteristic equation in variable λ is defined by

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \quad (4.0.1)$$

where \mathbf{I} is identity matrix of order k . Now we compute $\lambda\mathbf{I}$ as follows

$$\lambda\mathbf{I} = \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4.0.2)$$

$$\Rightarrow \lambda\mathbf{I} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \quad (4.0.3)$$

$$\Rightarrow \lambda \det \mathbf{I} = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} \quad (4.0.4)$$

So in equation 4.0.1, putting the values from 4.0.4 we get,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \quad (4.0.5)$$

$$\Rightarrow \det \mathbf{A} - \lambda \det \mathbf{I} = 0 \quad (4.0.6)$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0 \quad (4.0.7)$$

$$\Rightarrow \begin{vmatrix} 1 - \lambda & 0 & 2 \\ 0 & 2 - \lambda & 1 \\ 2 & 0 & 3 - \lambda \end{vmatrix} = 0 \quad (4.0.8)$$

Hence, equation 4.0.8 is the required characteristic equation of matrix \mathbf{A} . Further expanding the deter-