Matrix Theory (EE5609) Assignment 24

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***Abstract*—This document solves problem on ideals.**

All the codes for the figure in this document can be found at

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https://github.com/Arko98/EE5609/blob/master/
    Assignment_24
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1 Problem

Let $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ \beta \end{pmatrix}$. Then the system $\mathbf{AX} = \mathbf{b}$ over the real numbers has

1) No solution when $\beta \neq 7$
2) Infinite number of solutions when $\alpha \neq 2$
3) Infinite number of solutions when $\alpha = 2$ and $\beta \neq 7$
4) A unique solution if $\alpha \neq 2$

2 Solution

First we derive the Row Reduced Echelon Form (RREF) of the augmented matrix of the system $\mathbf{AX} = \mathbf{b}$ as follows,

$$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 2 & 3 & \alpha & \beta \end{pmatrix} \xleftrightarrow[R_3=R_3-2R_1]{R_2=R_2-R_1} \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 5 & \alpha-2 & \beta-2 \end{pmatrix} \tag{2.0.1}$$

$$\xleftrightarrow{R_2=\frac{1}{2}R_2} \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 5 & \alpha-2 & \beta-2 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_1=R_1+R_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 5 & \alpha-2 & \beta-2 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_3=R_3-5R_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & \alpha-2 & \beta-7 \end{pmatrix} \tag{2.0.4}$$

From the RREF of the augmented matrix of the system $\mathbf{AX} = \mathbf{b}$ in (2.0.4) we make the following observations for different values of α and β,

Values	Observations
$\beta \neq 7$	Then the existence of solution and the number of solutions will entirely depend on value of α
$\alpha = 7$ $\beta \neq 7$	Then RREF in (2.0.4) will contain Zero Row in R_3. Moreover solvability condition will not satisfy. $\implies$ system will have Zero solutions
$\alpha \neq 2$	RREF in (2.0.4) will have all pivots $\implies$ RREF in (2.0.4) will be fullrank $\implies$ $\mathbf{AX} = \mathbf{b}$ have unique solution.

Hence, if $\alpha \neq 2$ then the system $\mathbf{AX} = \mathbf{b}$ has unique solution.