

# Matrix Theory (EE5609) Assignment 17

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**Abstract**—This document proves the invertibility of a certain linear operator.

All the codes for the figure in this document can be found at

[https://github.com/Arko98/EE5609/blob/master/Assignment\\_17](https://github.com/Arko98/EE5609/blob/master/Assignment_17)

Proof	Conclusion
Let $\mathbf{V}_1, \mathbf{V}_2 \in \mathbb{V}$ then, If $\mathbf{V}_1 \neq \mathbf{V}_2$ then, $T[U(\mathbf{V}_1)] \neq T[U(\mathbf{V}_2)]$	$T$ is one-one function
$T$ is linear operator on finite dimensional vector space	$T$ is onto function

TABLE 1: Proof of Invertibility of transformation

## 1 PROBLEM

Let  $T$  be a linear operator on the finite-dimensional space  $\mathbb{V}$ . Suppose there is a linear operator  $U$  on  $\mathbb{V}$  such that  $TU = I$ . Prove that  $T$  is invertible and  $U = T^{-1}$ . Give an example which shows that this is false when  $\mathbb{V}$  is not finite-dimensional.

## 2 SOLUTION

### 2.1 Proof

Let  $T : \mathbb{V} \rightarrow \mathbb{V}$  be a linear operator, where  $\mathbb{V}$  is a finite dimensional vectors space and  $U : \mathbb{V} \rightarrow \mathbb{V}$  is also a linear operator such that,

$$TU = I \quad (2.1.1)$$

Where,  $I$  is an identity transformation. Now we know that linear transformations are functions. Hence,

$$TU = I \text{ is a function} \quad (2.1.2)$$

$$\Rightarrow I : \mathbb{V} \rightarrow \mathbb{V} \quad (2.1.3)$$

Such that  $T(V) = V$ . Defining  $TU : \mathbb{V} \rightarrow \mathbb{V}$  to be a linear operator, we have

$$T[U(V_i)] = V_i \quad [V_i \in \mathbb{V}] \quad (2.1.4)$$

Now we show in the below Table that  $T$  is one-one and onto as follows,

Hence we get from Table 1 that,  $T$  is invertible. Hence we get the following,

$$TT^{-1} = I \quad (2.1.5)$$

Where  $T^{-1}$  is an inverse function of linear operator  $T$ . Hence,

$$TT^{-1} = I = TU \quad (2.1.6)$$

$$\Rightarrow T^{-1}(TT^{-1}) = T^{-1}(TU) \quad (2.1.7)$$

$$\Rightarrow T^{-1}(I) = IU \quad (2.1.8)$$

$$\Rightarrow T^{-1} = U \quad (2.1.9)$$

Hence from (2.1.9) it is proven that  $T$  is invertible and  $T^{-1} = U$

### 2.2 Example

Let  $D$  be the differential operator  $D : \mathbb{V} \rightarrow \mathbb{V}$  where  $\mathbb{V}$  is a space of polynomial functions in one variable over  $\mathbb{R}$ . Hence,

$$D(c_0 + c_1x + \cdots + c_nx^n) = c_1 + c_2'x + \cdots + c_n'x^{n-1} \quad (2.2.1)$$

And,  $U : \mathbb{V} \rightarrow \mathbb{V}$  is another linear operator such that,

$$U(c_0 + c_1x + \cdots + c_nx^n) = c_0x + c_1\frac{x^2}{2} + \cdots + c_n\frac{x^{n+1}}{n+1} \quad (2.2.2)$$

Then  $UD : \mathbb{V} \rightarrow \mathbb{V}$  is a linear operator such that,

$$UD(c_0 + c_1x + \cdots + c_nx^n) \quad (2.2.3)$$

$$= U[D(c_0x + c_1\frac{x^2}{2} + \cdots + c_n\frac{x^{n+1}}{n+1})] \quad (2.2.4)$$

$$= U[c_1 + c'_2x + \cdots + c'_nx^{n-1}] \quad (2.2.5)$$

$$= c_1x + c_2\frac{x^2}{2} + \cdots + c_n\frac{x^n}{n} \quad (2.2.6)$$

Hence, from (2.2.6),

$$UD \neq I \quad (2.2.7)$$

Again,  $DU : \mathbb{V} \rightarrow \mathbb{V}$  is a linear operator such that,

$$DU(c_0 + c_1x + \cdots + c_nx^n) \quad (2.2.8)$$

$$= D[U(c_0x + c_1\frac{x^2}{2} + \cdots + c_n\frac{x^{n+1}}{n+1})] \quad (2.2.9)$$

$$= D[c_0x + c_1\frac{x^2}{2} + \cdots + c_n\frac{x^{n+1}}{n+1}] \quad (2.2.10)$$

$$= c_0 + c_1\frac{2x}{2} + \cdots + c_n\frac{(n+1)x^n}{n+1} \quad (2.2.11)$$

$$= c_0 + c_1x + \cdots + c_nx^n \quad (2.2.12)$$

Hence, from (2.2.12),

$$DU = I \quad (2.2.13)$$

Hence, from (2.2.7) and (2.2.13),  $D$  is not invertible.