Matrix Theory (EE5609) Assignment 18

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Abstract—**This document finds the basis of range and null-space of a certain linear operator.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_18

1 Problem

Let T be a linear operator on $\mathbb{R}^3$, the matrix of which in the standard ordered basis is,

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$$

Find a basis for the range of T and a basis for the null-space of T.

2 Solution

The basis of the range of linear transformation T is the basis of the column-space of $\mathbf{A}$ or basis of $C(\mathbf{A})$. Hence the basis of the range of the linear transformation T is derived by reducing $\mathbf{A}$ into Reduced-Row Echelon form as follows,

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix} \xleftrightarrow[R_1=R_1-2R_2]{R_3=R_3+R_1} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 5 & 5 \end{pmatrix} \tag{2.0.1}$$

$$\xleftrightarrow{R_3=R_3-5R_2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.2}$$

From (2.0.2) the basis of the range of linear operator T are as follows,

$$\mathbf{a_1} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{a_2} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \tag{2.0.4}$$

Again, the basis for null-space of linear operator T or $N(\mathbf{A})$ is a solution of the equation $\mathbf{Ax} = 0$. From (2.0.2) we have,

$$\mathbf{Ax} = 0 \tag{2.0.5}$$

$$\implies \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \tag{2.0.6}$$

Setting the value of the free variable $x_3 = 1$ we get the solution,

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{2.0.7}$$

Hence, the basis of the null-space of the linear operator T is given by,

$$\mathbf{b} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \tag{2.0.8}$$