Matrix Theory (EE5609) Assignment 20

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***Abstract*—This document solves a problem on a functional and linear transformation.**

All the codes for the figure in this document can be found at

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https://github.com/Arko98/EE5609/blob/master/
    Assignment_20
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1 Problem

Let $\mathbb{F}$ be a field and let f be the linear functional on $\mathbb{F}^2$ defined by,

$$f(x_1, x_2) = ax_1 + bx_2$$

For the linear operator $T(x_1, x_2) = (x_1, 0)$
Let, $g = T^t y$ and find $g(x_1, x_2)$

2 Solution

The linear functional f on $\mathbb{F}^2$ is defined by,

$$f(x_1, x_2) = \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \forall (x_1, x_2) \in \mathbb{F}^2 \tag{2.0.1}$$

We use the following theorem,
Let $\mathbb{V}$ and $\mathbb{W}$ be vector spaces, over the field F. For each linear transformation $T : \mathbb{V} \to \mathbb{W}$, there is a unique linear transformation $T^t : \mathbb{W}^* \to \mathbb{V}^*$ such that,

$$(T^t g)(\alpha) = g(T\alpha) \tag{2.0.2}$$

The given linear operator T defined as,

$$T(x_1, x_2) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \quad \forall (x_1, x_2) \in \mathbb{F}^2 \tag{2.0.3}$$

Consider the following mapping,

$$g = T^t f \tag{2.0.4}$$

Then, $\forall (x_1, x_2) \in \mathbb{F}^2$ we have,

$$g(x_1, x_2) = T^t f(x_1, x_2) \quad \text{[From (2.0.4)]} \tag{2.0.5}$$
$$= f(T(x_1, x_2)) \quad \text{[From (2.0.2)]} \tag{2.0.6}$$
$$= \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.7}$$
$$= \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \tag{2.0.8}$$
$$= ax_1 \quad \text{[From (2.0.1)]} \tag{2.0.9}$$

Hence, (2.0.9) is the required answer.