

Matrix Theory (EE5609) Assignment 3

Arkadipta De
MTech Artificial Intelligence
Roll No - AI20MTECH14002

Abstract—This assignment proves that matrix multiplication is not commutative.

The code for this solution can be found from

https://github.com/Arko98/EE5609/blob/master/Assignment_3/Codes/Solution_3.py

1 PROBLEM STATEMENT

Show that

$$\begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$$

2 SOLUTION

Let the two matrices be $\mathbf{A} = \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. From the problem we have to prove the following

$$\mathbf{AB} \neq \mathbf{BA} \quad (2.0.1)$$

At first we compute left hand side of 2.0.1.

$$\mathbf{AB} = \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \quad (2.0.2)$$

$$\Rightarrow \mathbf{AB} = \begin{pmatrix} 5 \times 2 - 1 \times 3 & 5 \times 1 - 1 \times 4 \\ 6 \times 2 + 7 \times 3 & 6 \times 1 + 7 \times 4 \end{pmatrix} \quad (2.0.3)$$

$$\Rightarrow \mathbf{AB} = \begin{pmatrix} 7 & 1 \\ 33 & 34 \end{pmatrix} \quad (2.0.4)$$

Next, we compute right hand side of 2.0.1.

$$\mathbf{BA} = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \quad (2.0.5)$$

$$\Rightarrow \mathbf{BA} = \begin{pmatrix} 2 \times 5 + 1 \times 6 & 2 \times (-1) + 1 \times 7 \\ 3 \times 5 + 4 \times 6 & 3 \times (-1) + 4 \times 7 \end{pmatrix} \quad (2.0.6)$$

$$\Rightarrow \mathbf{BA} = \begin{pmatrix} 16 & 5 \\ 39 & 25 \end{pmatrix} \quad (2.0.7)$$

Clearly we can see from equation 2.0.4 and 2.0.7 that the resultant matrices are not equal. Hence proved,

$$\begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$$

3 EXPLANATION

Matrix multiplication between two matrices \mathbf{A} and \mathbf{B} is the linear combination of the rows of matrix \mathbf{B} using the elements of matrix \mathbf{A} .

If \mathbf{R}_{B1} and \mathbf{R}_{B2} are first row and second row of the matrix \mathbf{B} and \mathbf{R}_{AB1} and \mathbf{R}_{AB2} are first row and second row of the matrix \mathbf{AB} from equation 2.0.2 then rows of \mathbf{AB} is given by

$$\mathbf{R}_{AB1} = 5\mathbf{R}_{B1} - \mathbf{R}_{B2} \quad (3.0.1)$$

$$\mathbf{R}_{AB2} = 6\mathbf{R}_{B1} + 7\mathbf{R}_{B2} \quad (3.0.2)$$

Similarly if \mathbf{R}_{A1} and \mathbf{R}_{A2} are first row and second row of the matrix \mathbf{A} and \mathbf{R}_{BA1} and \mathbf{R}_{BA2} are first row and second row of the matrix \mathbf{BA} from equation 2.0.2 then rows of \mathbf{BA} is given by

$$\mathbf{R}_{BA1} = 2\mathbf{R}_{A1} + \mathbf{R}_{A2} \quad (3.0.3)$$

$$\mathbf{R}_{BA2} = 3\mathbf{R}_{A1} + 4\mathbf{R}_{A2} \quad (3.0.4)$$

Clearly we can see from equations 3.0.1 and 3.0.3 that $\mathbf{R}_{AB1} \neq \mathbf{R}_{BA1}$ and from equations 3.0.2 and 3.0.4 that $\mathbf{R}_{AB2} \neq \mathbf{R}_{BA2}$. Hence matrix multiplication is generally not commutative.