Matrix Theory (EE5609) Assignment 20

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Abstract—**This document solves a problem on a functional and linear transformation.**

All the codes for the figure in this document can be found at

1 PROBLEM

Let $\mathbb{F}$ be a field and let f be the linear functional on $\mathbb{F}^2$ defined by,

$$f(x_1, x_2) = ax_1 + bx_2$$

For the linear operator $T(x_1, x_2) = (x_1, 0)$
Let, $g = T^t y$ and find $g(x_1, x_2)$

2 SOLUTION

The linear functional f on $\mathbb{F}^2$ is defined by,

$$f(x_1, x_2) = ax_1 + bx_2 \quad \forall (x_1, x_2) \in \mathbb{F}^2 \qquad (2.0.1)$$

We use the following theorem,
Let $\mathbb{V}$ and $\mathbb{W}$ be vector spaces, over the field F. For each linear transformation $T : \mathbb{V} \to \mathbb{W}$, there is a unique linear transformation $T^t : \mathbb{W}^* \to \mathbb{V}^*$ such that,

$$(T^t g)(\alpha) = g(T\alpha) \qquad (2.0.2)$$

The given linear operator T defined as,

$$T(x_1, x_2) = (x_1, 0) \quad \forall (x_1, x_2) \in \mathbb{F}^2 \qquad (2.0.3)$$

Consider the following mapping,

$$g = T^t f \qquad (2.0.4)$$

Then, $\forall (x_1, x_2) \in \mathbb{F}^2$ we have,

$$
\begin{aligned}
g(x_1, x_2) &= T^t f(x_1, x_2) \quad \text{[From (2.0.4)]} &(2.0.5)\\
&= f(T(x_1, x_2)) \quad \text{[From (2.0.2)]} &(2.0.6)\\
&= f(x_1, 0) \quad \text{[From (2.0.3)]} &(2.0.7)\\
&= ax_1 \quad \text{[From (2.0.1)]} &(2.0.8)
\end{aligned}
$$

Hence, (2.0.8) is the required answer.