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Matrix Theory (EE5609) Assignment 1

Arkadipta De MTech Artificial Intelligence Roll No - AI20MTECH14002

Abstract—This assignment solves a problem on checking whether two lines are parallel or perpendicular.

Below is the link to python code solution of this problem

1 PROBLEM STATEMENT

Show that the line through the points $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and

$$\begin{pmatrix} 3\\4\\-2 \end{pmatrix}$$
 is parallel to the line through the points $\begin{pmatrix} 0\\3\\2 \end{pmatrix}$ and $\begin{pmatrix} 3\\5\\6 \end{pmatrix}$.

2 THEORY

The direction vector **A** for a line through the points $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ y_2 \\ z_2 \end{pmatrix}$ is given by $\mathbf{A} = a_1 \hat{i} + b_1 \hat{j} + c_1 \hat{k}$

$$a_1 = x_2 - x_1 \tag{2.0.1}$$

$$b_1 = y_2 - y_1 \tag{2.0.2}$$

$$c_1 = z_2 - z_1 \tag{2.0.3}$$

and \hat{i} , \hat{j} , \hat{k} are unit vectors along x-axis, y-axis and z-axis respectively.

For two lines having direction vectors **A** and **B** respectively, they will be perpendicular if

$$\mathbf{A} \cdot \mathbf{B} = 0 \tag{2.0.4}$$

And the two lines will be parallel if

$$\mathbf{A} \times \mathbf{B} = \mathbf{0} \tag{2.0.5}$$

3 Solution

From the theory, the direction vector for the line through the points $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ is $\mathbf{A} = 2\hat{i} + 5\hat{j} - 4\hat{k}$

(using equations 2.0.1, 2.0.2, 2.0.3). Similarly, the direction ratios for the line through the points $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$

and
$$\begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$$
 is $\mathbf{B} = 3\hat{i} + 2\hat{j} + 4\hat{k}$ (using equations 2.0.1, 2.0.2, 2.0.3).

To check if the two lines are perpendicular, we perform scalar product of the two direction vectors **A** and **B** as follows

$$\mathbf{A} \cdot \mathbf{B} = (2\hat{i} + 5\hat{j} - 4\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 4\hat{k})$$

$$\Rightarrow \mathbf{A} \cdot \mathbf{B} = (2 \cdot 3)(\hat{i} \cdot \hat{i}) + (5 \cdot 2)(\hat{j} \cdot \hat{j}) + (-4 \cdot 4)(\hat{k} \cdot \hat{k})$$

$$\Rightarrow \mathbf{A} \cdot \mathbf{B} = 6 + 10 - 16$$

$$\Rightarrow \mathbf{A} \cdot \mathbf{B} = 0$$

Thus the direction vectors of the two lines satisfies the equation 2.0.4, hence proved that the lines are **perpendicular**. Hence they are not **parallel** with each other.

Python Code: The python code for the above solution can be found at -https://github.com/Arko98/EE5609/blob/master/ Assignment_1/Codes/Solution_1.py