Matrix Theory (EE5609) Assignment 21

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***Abstract*—This document solves a evaluates a polynomial of a given square matrix.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_21

1 Problem

Let $\mathbb{F}$ be a subfield of the complex numbers and let $\mathbf{A}$ be the following 2×2 matrix over $\mathbb{F}$,

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

For the following polynomial f over $\mathbb{F}$,

$$f = x^3 - 1$$

compute $f(\mathbf{A})$

2 Solution

We first find the eigen values of the $\mathbf{A}$. We get the characteristic equation of $\mathbf{A}$ as follows,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \tag{2.0.1}$$

$$\implies \lambda^2 - 5\lambda + 7 = 0 \tag{2.0.2}$$

Where $\mathbf{I}$ is 2×2 Identity matrix. Now using Cayley Hamilton Theorem we get from (2.0.2) the following,

$$\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} = 0 \tag{2.0.3}$$

Hence the polynomial $f(\mathbf{A})$ can be written using the characteristic function of $\mathbf{A}$ as follows,

$$f(\mathbf{A}) = \mathbf{A}^3 - \mathbf{I} \tag{2.0.4}$$

$$= (\mathbf{A} - \mathbf{I})(\mathbf{A}^2 + \mathbf{A} + \mathbf{I}) \tag{2.0.5}$$

$$= (\mathbf{A} - \mathbf{I})(\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} + 6\mathbf{A} - 6\mathbf{I}) \tag{2.0.6}$$

$$= 6(\mathbf{A} - \mathbf{I})^2 \quad \text{[From (2.0.3)]} \tag{2.0.7}$$

$$= 6\left[\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]^2 \tag{2.0.8}$$

$$= \begin{pmatrix} 0 & 18 \\ -18 & 18 \end{pmatrix} \tag{2.0.9}$$

Here, (2.0.9) is the required answer.