

Matrix Theory (EE5609) Assignment 7

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Abstract—This finds whether a given second degree equation represents a pair of straight lines or not.

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_7

of straight lines. For the appropriate value of k , (2.0.1) becomes,

$$6x^2 + xy - 12y^2 - 11x + 43y - 35 = 0 \quad (2.0.8)$$

3 GRAPHICAL ILLUSTRATION

1 PROBLEM

Find the value of k so that the following equation may represent a pair of straight lines -

$$6x^2 + xy + ky^2 - 11x + 43y - 35 = 0$$

2 SOLUTION

The given second degree equation is,

$$6x^2 + xy + ky^2 - 11x + 43y - 35 = 0 \quad (2.0.1)$$

Comparing coefficients of (2.0.1) with (??) we get,

$$\mathbf{V} = \begin{pmatrix} 6 & \frac{1}{2} \\ \frac{1}{2} & k \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{u} = \begin{pmatrix} -\frac{11}{2} \\ \frac{43}{2} \end{pmatrix} \quad (2.0.3)$$

$$f = -35 \quad (2.0.4)$$

From (??) the given second degree equation (2.0.1) will represent a pair of straight line if,

$$\begin{vmatrix} 6 & \frac{1}{2} & -\frac{11}{2} \\ \frac{1}{2} & k & \frac{43}{2} \\ -\frac{11}{2} & \frac{43}{2} & -35 \end{vmatrix} = 0 \quad (2.0.5)$$

Expanding the determinant,

$$k + 12 = 0 \quad (2.0.6)$$

$$\Rightarrow k = -12 \quad (2.0.7)$$

Hence, from (2.0.7) we find that for $k = -12$, the given second degree equation (2.0.1) represents pair

Let the pair of straight lines in vector form is given by

$$\mathbf{n}_1^T \mathbf{x} = c_1 \quad (3.0.1)$$

$$\mathbf{n}_2^T \mathbf{x} = c_2 \quad (3.0.2)$$

The pair of straight lines is given by,

$$(\mathbf{n}_1^T \mathbf{x} - c_1)(\mathbf{n}_2^T \mathbf{x} - c_2) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (3.0.3)$$

Putting the values of \mathbf{V} and \mathbf{u} we get,

$$\mathbf{x}^T \begin{pmatrix} 6 & \frac{1}{2} \\ \frac{1}{2} & -12 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -\frac{11}{2} & \frac{43}{2} \end{pmatrix} \mathbf{x} - 35 = 0 \quad (3.0.4)$$

Hence, from (3.0.4) we get,

$$\mathbf{n}_1 * \mathbf{n}_2 = \begin{pmatrix} 6 \\ 1 \\ -12 \end{pmatrix} \quad (3.0.5)$$

$$c_2 \mathbf{n}_1 + c_1 \mathbf{n}_2 = -2 \begin{pmatrix} -\frac{11}{2} \\ \frac{43}{2} \end{pmatrix} \quad (3.0.6)$$

$$c_1 c_2 = -35 \quad (3.0.7)$$

The slopes of the pair of straight lines are given by the roots of the polynomial,

$$cm^2 + 2bm + a = 0 \quad (3.0.8)$$

$$\Rightarrow m_i = \frac{-b \pm \sqrt{-\det(V)}}{c} \quad (3.0.9)$$

$$\mathbf{n}_i = k \begin{pmatrix} -m_i \\ 1 \end{pmatrix} \quad (3.0.10)$$

Substituting the values in above equations (3.0.8) we get,

$$-12m^2 + m + 6 = 0 \quad (3.0.11)$$

$$\Rightarrow m_i = \frac{-\frac{1}{2} \pm \sqrt{-(-\frac{289}{4})}}{-12} \quad (3.0.12)$$

Solving equation (3.0.12) we get ,

$$m_1 = -\frac{2}{3} \quad (3.0.13)$$

$$m_2 = \frac{3}{4} \quad (3.0.14)$$

Hence putting the values of m_1 and m_2 in (3.0.10) we get

$$\mathbf{n}_1 = k_1 \begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix} \quad (3.0.15)$$

$$\mathbf{n}_2 = k_2 \begin{pmatrix} -\frac{3}{4} \\ 1 \end{pmatrix} \quad (3.0.16)$$

Putting values of \mathbf{n}_1 and \mathbf{n}_2 in (3.0.5) we get,

$$\mathbf{n}_1 * \mathbf{n}_2 = \begin{pmatrix} -\frac{3k_2}{4} & 0 \\ k_2 & -\frac{3k_2}{4} \\ 0 & k_2 \end{pmatrix} \begin{pmatrix} \frac{2k_1}{3} \\ 1 \\ k_1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ -12 \end{pmatrix} \quad (3.0.17)$$

$$\Rightarrow \begin{pmatrix} -\frac{1}{2}k_1k_2 \\ -\frac{1}{12}k_1k_2 \\ k_1k_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ -12 \end{pmatrix} \quad (3.0.18)$$

Thus, from (3.0.18), $k_1k_2 = -12$. Possible combinations of (k_1, k_2) are $(6, -2)$, $(-6, 2)$, $(3, -4)$, $(-3, 4)$. Let's assume $k_1 = 3$, $k_2 = -4$, then we get,

$$\mathbf{n}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (3.0.19)$$

$$\mathbf{n}_2 = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (3.0.20)$$

From equation (3.0.6) we get

$$\begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{pmatrix} \begin{pmatrix} c_2 \\ c_1 \end{pmatrix} = -2\mathbf{u} \quad (3.0.21)$$

$$\begin{pmatrix} 2 & 3 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} c_2 \\ c_1 \end{pmatrix} = -2 \begin{pmatrix} -\frac{11}{2} \\ \frac{43}{2} \end{pmatrix} \quad (3.0.22)$$

Hence we get the following equations,

$$2c_2 + 3c_1 = 11 \quad (3.0.23)$$

$$3c_2 - 4c_1 = -43 \quad (3.0.24)$$

The augmented matrix of (3.0.23), (3.0.24) is,

$$\begin{pmatrix} 2 & 3 & 11 \\ 3 & -4 & -43 \end{pmatrix} R_1 = \frac{1}{2} R_1 \begin{pmatrix} 1 & \frac{3}{2} & \frac{11}{2} \\ 3 & -4 & -43 \end{pmatrix} \quad (3.0.25)$$

$$R_2 = R_2 - 3R_1 \begin{pmatrix} 1 & \frac{3}{2} & \frac{11}{2} \\ 0 & -\frac{17}{2} & -\frac{119}{2} \end{pmatrix} \quad (3.0.26)$$

$$R_2 = -\frac{2}{17} \begin{pmatrix} 1 & \frac{3}{2} & \frac{11}{2} \\ 0 & 1 & 7 \end{pmatrix} \quad (3.0.27)$$

$$R_1 = R_1 - \frac{3}{2} R_2 \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 7 \end{pmatrix} \quad (3.0.28)$$

$$\quad (3.0.29)$$

Hence we get,

$$c_1 = -5 \quad (3.0.30)$$

$$c_2 = 7 \quad (3.0.31)$$

Hence (3.0.1), (3.0.2) can be modified as follows,

$$(2 \ 3)\mathbf{x} = -5 \quad (3.0.32)$$

$$(3 \ -4)\mathbf{x} = 7 \quad (3.0.33)$$

The figure below corresponds to the pair of straight lines represented by (3.0.32) and (3.0.33).

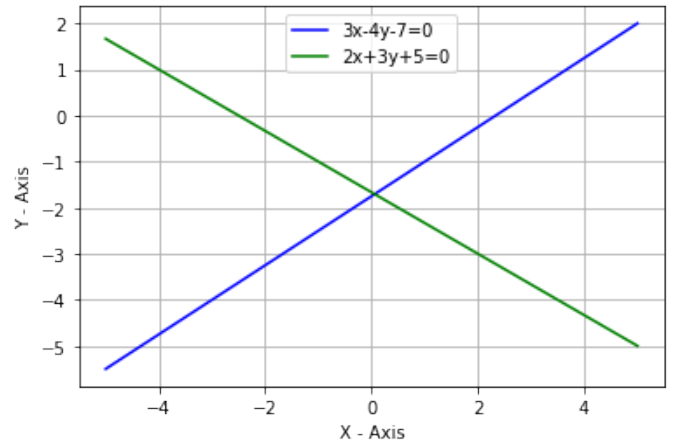


Fig. 1: Pair of Straight Lines