

Matrix Theory (EE5609) Assignment 9

Arkadipta De
MTech Artificial Intelligence
AI20MTECH14002

Abstract—This document performs QR decomposition on a given matrix.

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/Assignment_9

1 PROBLEM

Find QR decomposition of $\begin{pmatrix} 2 & 3 \\ 3 & -4 \end{pmatrix}$

2 SOLUTION

Let \mathbf{a} and \mathbf{b} be transpose of column vectors of the given matrix.

$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (2.0.2)$$

The column vectors can be expressed as follows,

$$\mathbf{a} = k_1 \mathbf{u}_1 \quad (2.0.3)$$

$$\mathbf{b} = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \quad (2.0.4)$$

Here,

$$k_1 = \|\mathbf{a}\| \quad (2.0.5)$$

$$\mathbf{u}_1 = \frac{\mathbf{a}}{k_1} \quad (2.0.6)$$

$$r_1 = \frac{\mathbf{u}_1^T \mathbf{b}}{\|\mathbf{u}_1\|^2} \quad (2.0.7)$$

$$\mathbf{u}_2 = \frac{\mathbf{b} - r_1 \mathbf{u}_1}{\|\mathbf{b} - r_1 \mathbf{u}_1\|} \quad (2.0.8)$$

$$k_2 = \mathbf{u}_2^T \mathbf{b} \quad (2.0.9)$$

The (2.0.3) and (2.0.4) can be written as,

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = \mathbf{Q} \mathbf{R} \quad (2.0.11)$$

Now, \mathbf{R} is an upper triangular matrix and also,

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \quad (2.0.12)$$

Now using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{2^2 + 3^2} = \sqrt{13} \quad (2.0.13)$$

$$\mathbf{u}_1 = \frac{1}{\sqrt{13}} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (2.0.14)$$

$$r_1 = \left(\frac{2}{\sqrt{13}} \quad \frac{3}{\sqrt{13}} \right) \begin{pmatrix} 3 \\ -4 \end{pmatrix} = -\frac{6}{\sqrt{13}} \quad (2.0.15)$$

$$\mathbf{u}_2 = \frac{1}{\sqrt{13}} \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (2.0.16)$$

$$k_2 = \left(\frac{3}{\sqrt{13}} \quad -\frac{2}{\sqrt{13}} \right) \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \frac{17}{\sqrt{13}} \quad (2.0.17)$$

Thus putting the values from (2.0.13) to (2.0.17) in (2.0.11) we obtain QR decomposition,

$$\begin{pmatrix} 2 & 3 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \\ \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \end{pmatrix} \begin{pmatrix} \sqrt{13} & -\frac{6}{\sqrt{13}} \\ 0 & \frac{17}{\sqrt{13}} \end{pmatrix} \quad (2.0.18)$$