Matrix Theory (EE5609) Challenging Problem

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***Abstract*—This document proves that orthogonal vectors are linearly independent.**

Download latex codes from

https://github.com/Arko98/EE5609/tree/master/Challenge_2

1 Problem

Suppose that a set of nonzero vectors $\mathbf{v}_1, \mathbf{v}_2, \dots \mathbf{v}_n$ are mutually orthogonal, i.e., $\mathbf{v_i^T}\mathbf{v}_j = 0$ for $i \neq j$. Prove that these vectors are also linearly independent.

2 Proof

Let us consider the following linear combination

$$c_1\mathbf{v_1} + c_2\mathbf{v_2} + \dots + c_n\mathbf{v_n} = 0 \tag{2.0.1}$$

We have to show that in (2.0.1), $c_1 = 0$, $c_2 = 0$ and so on upto $c_n = 0$.
We compute the dot product of (2.0.1) with $\mathbf{v_i}$ as follows -

$$\mathbf{v_i^T}\left(c_1\mathbf{v_1} + c_2\mathbf{v_2} + \dots + c_n\mathbf{v_n}\right) = 0 \tag{2.0.2}$$

$$\implies c_1\mathbf{v_i^T}\mathbf{v_1} + c_2\mathbf{v_i^T}\mathbf{v_2} + \dots + c_n\mathbf{v_i^T}\mathbf{v_n} = 0 \tag{2.0.3}$$

As $\mathbf{v_i^T}\mathbf{v}_j = 0$ for all $i \neq j$

$$\implies c_i\mathbf{v_i^T}\mathbf{v_i} = 0 \tag{2.0.4}$$

$$\implies c_i\left\|\mathbf{v_i}\right\| = 0 \tag{2.0.5}$$

As the set of vectors are non zero, $\|\mathbf{v_i}\| \neq 0$, hence

$$\implies c_i = 0 \tag{2.0.6}$$

(2.0.6) is true for all the vectors in the orthogonal set of vectors, hence,

$$c_1 = c_2 = \dots = c_n = 0 \tag{2.0.7}$$

Hence, from (2.0.7), the set of orthogonal vectors are linearly independent.