Matrix Theory (EE5609) Assignment 11

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Abstract—**This document proves that, each field of the characteristic zero contains a copy of the rational number field.**

All the codes for the figure in this document can be found at

https://github.com/Arko98/EE5609/blob/master/
Assignment_11

1 PROBLEM

Prove that, each field of the characteristic zero contains a copy of the rational number field.

2 SOLUTION

The characteristic of a field is defined to be the smallest number of times one must use the field's multiplicative identity (1) in a sum to get the additive identity. If this sum never reaches the additive identity (0), then the field is said to have characteristic zero.

Let $\mathbb{Q}$ be the rational number field. Hence,

$$0 \in \mathbb{Q} \qquad \text{[Additive Identity]} \qquad (2.0.1)$$

$$1 \in \mathbb{Q} \qquad \text{[Multiplicative Identity]} \qquad (2.0.2)$$

As addition is defined on $\mathbb{Q}$ hence we have,

$$1 \neq 0 \qquad (2.0.3)$$

$$1 + 1 = 2 \neq 0 \qquad (2.0.4)$$

And so on,

$$1 + 1 + \cdots + 1 = n \neq 0 \qquad (2.0.5)$$

From the definition of characteristic of a field and from (2.0.3), (2.0.4) and so on up-to (2.0.5), the rational number field, $\mathbb{Q}$ has characteristic 0.