Matrix Theory (EE5609) Assignment 11

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***Abstract*—This document proves that, each field of the characteristic zero contains a copy of the rational number field.**

All the codes for the figure in this document can be found at

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https://github.com/Arko98/EE5609/blob/master/Assignment_11
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1 Problem

Prove that, each field of the characteristic zero contains a copy of the rational number field.

2 Solution

Let F be a field of characteristic zero. Hence

$$0 \in F \tag{2.0.1}$$
$$1 \in F \tag{2.0.2}$$

Since the characteristic of F is zero hence,

$$1 \neq 1+1 \neq 1+1+\cdots \neq 0 \tag{2.0.3}$$

As F is a field, it is closed under addition. Hence for addition of n number of 1 we have,

$$1+1+\cdots+1 = n \in F \tag{2.0.4}$$

And,

$$n \neq 0 \tag{2.0.5}$$

If $\mathbb{Z}$ is the set of integers then we have,

$$\mathbb{Z} \subseteq F \tag{2.0.6}$$

As F is a field, every element in F will have a multiplicative inverse, thus,

$$\frac{1}{n} \in F \tag{2.0.7}$$

Also, F is closed under multiplication and thus,

$$\forall m, n \in \mathbb{Z} \quad \text{and } n \neq 0 \tag{2.0.8}$$
$$m \cdot \frac{1}{n} \in F \tag{2.0.9}$$
$$\implies \frac{m}{n} \in F \tag{2.0.10}$$

Hence, if $\mathbb{Q}$ is the rational number field then,

$$\mathbb{Q} \subseteq F \tag{2.0.11}$$

Hence, proved that field F contains a copy of the rational number field.