Matrix Theory (EE5609) Assignment 4

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***Abstract*—This document solves an equation on matrix. Additionally it finds characteristic equation of a square matrix.**

1 Problem

If $\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $\mathbf{A^3} - 6\mathbf{A^2} + 7\mathbf{A} + 2\mathbf{I} = 0$

2 Solution

The solution code for this problem can be found at: https://github.com/Arko98/EE5609/blob/master/Assignment_4/Codes/Solution.py

3 Problem 2

Find the characteristic equation of the matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$

4 Solution

For a general order k square matrix $\mathbf{A}$, the characteristic equation in variable λ is defined by

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \tag{4.0.1}$$

where $\mathbf{I}$ is identity matrix of order k. Now we compute $\lambda\mathbf{I}$ as follows

$$\lambda\mathbf{I} = \lambda\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.0.2}$$

$$\implies \lambda\mathbf{I} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \tag{4.0.3}$$

$$\implies \lambda\det\mathbf{I} = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} \tag{4.0.4}$$

So in equation 4.0.1, putting the values from 4.0.4 we get,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \tag{4.0.5}$$

$$\implies \det\mathbf{A} - \lambda\det\mathbf{I} = 0 \tag{4.0.6}$$

$$\implies \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0 \tag{4.0.7}$$

$$\implies \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0 \tag{4.0.8}$$

Hence, equation 4.0.8 is the required characteristic equation of matrix $\mathbf{A}$. Further expanding the determinant from the equation 4.0.8 we get the following polynomial equation of λ

$$(1-\lambda)(2-\lambda)(3-\lambda) - 4(2-\lambda) = 0 \tag{4.0.9}$$

$$\implies \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \tag{4.0.10}$$

Hence, equation 4.0.10 is the required characteristic equation of matrix $\mathbf{A}$ for λ.