

# Matrix Theory (EE5609) Assignment 14

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**Abstract**—This document proves the existence of inverse of Hilbert Matrix.

All the codes for the figure in this document can be found at

[https://github.com/Arko98/EE5609/blob/master/Assignment\\_14](https://github.com/Arko98/EE5609/blob/master/Assignment_14)

## 1 PROBLEM

Prove that the following matrix is invertible and  $\mathbf{A}^{-1}$  has integer entries.

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{1}{2} & \cdots & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{n+1} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{1}{n} & \frac{1}{n+1} & \cdots & \frac{1}{2n-1} \end{pmatrix}$$

## 2 SOLUTION

Let  $\mathbf{H}_n$  be the  $n$ -th Hilbert matrix given by

$$\mathbf{H}_n = \left[ \frac{1}{i+j-1} \right]_{i,j} \quad (2.0.1)$$

Then  $\mathbf{H}_{n+1}$  is given by,

$$\mathbf{H}_{n+1} = \begin{pmatrix} \mathbf{H}_n & \mathbf{u} \\ \mathbf{u}^T & \frac{1}{2n-1} \end{pmatrix} \quad (2.0.2)$$