Matrix Theory (EE5609) Assignment 1

Arkadipta De
MTech Artificial Intelligence
Roll No - AI20MTECH14002

***Abstract*—This assignment solves a problem on checking whether two lines are parallel or perpendicular.**

Below is the link to python code solution of this problem

1 Problem Statement

Show that the line through the points $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}3\\4\\-2\end{pmatrix}$ is parallel to the line through the points $\begin{pmatrix}0\\3\\2\end{pmatrix}$ and $\begin{pmatrix}3\\5\\6\end{pmatrix}$.

2 Theory

The direction vector **A** for a line through the points $\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}$ and $\begin{pmatrix}X_2\\y_2\\z_2\end{pmatrix}$ is given by

$$\mathbf{A} = \begin{pmatrix}x_2 - x_1\\y_2 - y_1\\z_2 - z_1\end{pmatrix} \tag{2.0.1}$$

For two lines having direction vectors **A** and **B** respectively, they will be perpendicular if the scalar product of the two direction vector is 0,

$$\mathbf{AB} = 0 \tag{2.0.2}$$

Where scalar product of two vectors, $\mathbf{A}=\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}$ is defined by

$$\mathbf{AB} = \mathbf{A^T B} = \begin{pmatrix}x_1 & y_1 & z_1\end{pmatrix}\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix} = x_1x_2 + y_1y_2 + z_1z_2 \tag{2.0.3}$$

And the two lines will be parallel if the cross product of the two direction vector is 0,

$$\mathbf{A} \times \mathbf{B} = \mathbf{0} \tag{2.0.4}$$

3 Solution

Let the points be $\mathbf{P}=\begin{pmatrix}1\\-1\\2\end{pmatrix}$, $\mathbf{Q}=\begin{pmatrix}3\\4\\-2\end{pmatrix}$, $\mathbf{R}=\begin{pmatrix}0\\3\\2\end{pmatrix}$ and $\mathbf{S}=\begin{pmatrix}3\\5\\6\end{pmatrix}$. From the theory, using equation 2.0.1, the direction vector for the line through the points **P** and **Q** is

$$\mathbf{A} = \mathbf{P} - \mathbf{Q} \tag{3.0.1}$$

$$\implies \mathbf{A} = \begin{pmatrix}1\\-1\\2\end{pmatrix} - \begin{pmatrix}3\\4\\-2\end{pmatrix} \tag{3.0.2}$$

$$\implies \mathbf{A} = \begin{pmatrix}-2\\-5\\4\end{pmatrix} \tag{3.0.3}$$

Similarly, using equation 2.0.1, the direction vector for the line through the points **R** and **S** is

$$\mathbf{B} = \mathbf{R} - \mathbf{S} \tag{3.0.4}$$

$$\implies \mathbf{B} = \begin{pmatrix}0\\3\\2\end{pmatrix} - \begin{pmatrix}3\\5\\6\end{pmatrix} \tag{3.0.5}$$

$$\implies \mathbf{B} = \begin{pmatrix}-3\\-2\\-4\end{pmatrix} \tag{3.0.6}$$

$$\tag{3.0.7}$$

To check if the two lines are perpendicular, we perform scalar product of the two direction vectors

A and **B** using equation 2.0.3 as follows

$$\mathbf{AB} = \mathbf{A^T B} \tag{3.0.8}$$

$$\implies \mathbf{AB} = \begin{pmatrix} -2 & -5 & 4 \end{pmatrix} \begin{pmatrix} -3 \\ -2 \\ -4 \end{pmatrix} \tag{3.0.9}$$

$$\implies \mathbf{AB} = 6 + 10 - 16 \tag{3.0.10}$$

$$\implies \mathbf{AB} = 0 \tag{3.0.11}$$

Thus the direction vectors of the two lines satisfies the equation 2.0.2, hence proved that the lines are **perpendicular**. Hence they are not **parallel** with each other.

Python Code: The python code for the above solution can be found at - https://github.com/Arko98/EE5609/blob/master/Assignment_1/Codes/Solution_1.py