```
/*************************************************************************
  File name:    R-ate.h
  Version:
  Date:         Dec 15,2016
  Description:  this code is achieved according to ake12bnx.cpp in MIRCAL C++ source file.
                so,see ake12bnx.cpp for details.
                this code gives calculation of R-ate pairing
  Function List:
        1.zzn2_pow                //regular zzn2 powering
        2.set_frobenius_constant  //calculate frobenius_constant X
        3.q_power_frobenius
        4.line
        5.g
        6.fast_pairing
        7.ecap

 Notes:
*************************************************************************/

#include "zzn12_operation.h"

/*************************************************************
  Function:       zzn2_pow
  Description:    regular zzn2 powering
                  see zzn2.cpp for details in MIRACL c++ source file
  Calls:          MIRACL functions
  Called By:      set_frobenius_constant
  Input:          zzn2 x,big k
  Output:         null
  Return:         zzn2
  Others:
*************************************************************/
zzn2 zzn2_pow(zzn2 x,big k)
{
    int i,j,nb,n,nbw,nzs;
    big zero;
    zzn2 res,u2,t[16];

    zero=mirvar(0);
    res.a=mirvar(0);res.b=mirvar(0);
    u2.a=mirvar(0);u2.b=mirvar(0);

    if (zzn2_iszero(&x))
    {zzn2_zero(&res); return res;}
```

```
    if ( size(k)==0 )
    { zzn2_from_int(1,&res);return res;}
    if (size(k)==1) return x;

// Prepare table for windowing
     zzn2_mul(&x,&x,&u2);
     t[0].a=mirvar(0);
     t[0].b=mirvar(0);
     zzn2_copy(&x,&t[0]);
    for(i=1;i<16;i++)
    {   t[i].a=mirvar(0);
        t[i].b=mirvar(0);
        zzn2_mul(&t[i-1],&u2,&t[i]);
    }

   // Left to right method - with windows
    zzn2_copy(&x,&res);
    nb=logb2(k);
    if (nb>1) for (i=nb-2;i>=0;)
    {
        //Note new parameter of window_size=5. Default to 5, but reduce to 4 (or even 3) to save
RAM
        n=mr_window(k,i,&nbw,&nzs,5);
        for(j=0;j<nbw;j++) zzn2_mul(&res,&res,&res);
        if (n>0) zzn2_mul(&res,&t[n/2],&res);
        i-=nbw;
        if (nzs)
        {
            for (j=0;j<nzs;j++) zzn2_mul(&res,&res,&res);
            i-=nzs;
        }
    }
    return res;
}
/*************************************************************
  Function:       set_frobenius_constant
  Description:    calculate frobenius_constant X
                  see ake12bnx.cpp for details in MIRACL c++ source file
  Calls:          MIRACL functions,zzn2_pow
  Called By:      SM9_init
  Input:          NULL
  Output:         zzn2 *X
  Return:         NULL
  Others:
```

```
*****************************************************************/
void set_frobenius_constant(zzn2 *X)
{
    big p,zero,one,two;
    p=mirvar(0);
    zero=mirvar(0);
    one=mirvar(0);
    two=mirvar(0);

    convert(0,zero);
    convert(1,one);
    convert(2,two);

    mip=get_mip();
    copy(mip->modulus,p);

    switch (get_mip()->pmod8)
    {
    case 5:
        zzn2_from_bigs(zero,one,X);// = (sqrt(-2)^(p-1)/2
        break;
    case 3:                     // = (1+sqrt(-1))^(p-1)/2
        zzn2_from_bigs(one,one,X);
         break;
   case 7:
       zzn2_from_bigs( two, one,X);// = (2+sqrt(-1))^(p-1)/2
    default: break;
    }

    decr(p,1,p);
    subdiv(p,6,p);

    *X=zzn2_pow(*X,p);
}
/*****************************************************************
  Function:       q_power_frobenius
  Description:    F is frobenius_constant X
                  see ake12bnx.cpp for details in MIRACL c++ source file
  Calls:          MIRACL functions
  Called By:      fast_pairing
  Input:          ecn2 A,zzn2 F
  Output:         zzn2 A
  Return:         NULL
  Others:
```

```
*****************************************************************/
void q_power_frobenius(ecn2 A,zzn2 F)
{
   // Fast multiplication of A by q (for Trace-Zero group members only)
    zzn2 x,y,z,w,r;
    x.a=mirvar(0);x.b=mirvar(0);
    y.a=mirvar(0);y.b=mirvar(0);
    z.a=mirvar(0);z.b=mirvar(0);
    w.a=mirvar(0);w.b=mirvar(0);
    r.a=mirvar(0);r.b=mirvar(0);

    ecn2_get(&A,&x,&y,&z);
    zzn2_copy(&F,&r);//r=F
    if (get_mip()->TWIST==MR_SEXTIC_M) zzn2_inv(&r);  // could be precalculated
    zzn2_mul(&r,&r,&w);//w=r*r
    zzn2_conj(&x,&x);zzn2_mul(&w,&x,&x);
    zzn2_conj(&y,&y);zzn2_mul(&w,&r,&w);zzn2_mul(&w,&y,&y);
    zzn2_conj(&z,&z);
    ecn2_setxyz(&x,&y,&z,&A);
}
/*****************************************************************
  Function:       line
  Description:    Line from A to destination C. Let A=(x,y)
                  Line Y-slope.X-c=0, through A, so intercept c=y-slope.x
                  Line Y-slope.X-y+slope.x = (Y-y)-slope.(X-x) = 0
                  Now evaluate at Q -> return (Qy-y)-slope.(Qx-x)
                  see ake12bnx.cpp for details in MIRACL c++ source file
  Calls:          MIRACL functions,zzn12_init
  Called By:      g
  Input:          ecn2 A,ecn2 *C,ecn2 *B,zzn2 slope,zzn2 extra,BOOL Doubling,big Qx,big Qy
  Output:
  Return:         zzn12
  Others:
*****************************************************************/
zzn12 line(ecn2 A,ecn2 *C,ecn2 *B,zzn2 slope,zzn2 extra,BOOL Doubling,big Qx,big Qy)
{
    zzn12 res;
    zzn2 X,Y,Z,Z2,U,QY,CZ;
    big QX;

    QX=mirvar(0);
    X.a=mirvar(0);X.b=mirvar(0);
    Y.a=mirvar(0);Y.b=mirvar(0);
    Z.a=mirvar(0);Z.b=mirvar(0);
```

```
Z2.a=mirvar(0);Z2.b=mirvar(0);
U.a=mirvar(0);U.b=mirvar(0);
QY.a=mirvar(0);QY.b=mirvar(0);
CZ.a=mirvar(0);CZ.b=mirvar(0);
zzn12_init(&res);

ecn2_getz(C,&CZ);
// Thanks to A. Menezes for pointing out this optimization...
if (Doubling)
{
  ecn2_get(&A,&X,&Y,&Z);
  zzn2_mul(&Z,&Z,&Z2); //Z2=Z*Z

  //X=slope*X-extra
  zzn2_mul(&slope,&X,&X);
  zzn2_sub(&X,&extra,&X);

  zzn2_mul(&CZ,&Z2,&U);

  //(-(Z*Z*slope)*Qx);
  nres(Qx,QX);
  zzn2_mul(&Z2,&slope,&Y);
  zzn2_smul(&Y,QX,&Y);
  zzn2_negate(&Y,&Y);

    if (get_mip()->TWIST==MR_SEXTIC_M)
     { // "multiplied across" by i to simplify
        zzn2_from_big(Qy,&QY);
        zzn2_txx(&QY);
        zzn2_mul(&U,&QY,&QY);
        zzn4_from_zzn2s(&QY,&X,&res.a);
        zzn2_copy(&Y,&(res.c.b));
     }
     if (get_mip()->TWIST==MR_SEXTIC_D)
     {
          zzn2_smul(&U,Qy,&QY);
        zzn4_from_zzn2s(&QY,&X,&res.a);
        zzn2_copy(&Y,&(res.b.b));
     }
}
else
{   //slope*X-Y*Z
    ecn2_getxy(B,&X,&Y);
    zzn2_mul(&slope,&X,&X);
```

```
        zzn2_mul(&Y,&CZ,&Y);
        zzn2_sub(&X,&Y,&X);

        //(-slope*Qx)
        nres(Qx,QX);
        zzn2_smul(&slope,QX,&Z);
        zzn2_negate(&Z,&Z);

         if (get_mip()->TWIST==MR_SEXTIC_M)
         {
              zzn2_from_big(Qy,&QY);
           zzn2_txx(&QY);
           zzn2_mul(&CZ,&QY,&QY);

           zzn4_from_zzn2s(&QY,&X,&res.a);
           zzn2_copy(&Z,&(res.c.b));
         }
         if (get_mip()->TWIST==MR_SEXTIC_D)
         {
              zzn2_smul(&CZ,Qy,&QY);
           zzn4_from_zzn2s(&QY,&X,&res.a);
           zzn2_copy(&Z,&(res.b.b));
         }
    }
     return res;
}

/*************************************************************
  Function:       g
  Description:    Add A=A+B  (or A=A+A),Return line function value
                  see ake12bnx.cpp for details in MIRACL c++ source file
  Calls:          MIRACL functions,zzn12_init,line
  Called By:
  Input:          ecn2 *A,ecn2 *B,big Qx,big Qy
  Output:
  Return:         zzn12
  Others:
*************************************************************/
zzn12 g(ecn2 *A,ecn2 *B,big Qx,big Qy)
{
    zzn2 lam,extra;
    BOOL Doubling;
    ecn2 P;
    zzn12 res;
```

```
    lam.a=mirvar(0);lam.b=mirvar(0);
    extra.a=mirvar(0);extra.b=mirvar(0);
    P.x.a=mirvar(0); P.x.b=mirvar(0); P.y.a=mirvar(0);P.y.b=mirvar(0);
    P.z.a=mirvar(0); P.z.b=mirvar(0); P.marker=MR_EPOINT_INFINITY;

    zzn12_init(&res);
    ecn2_copy(A,&P);
    Doubling=ecn2_add2(B,A,&lam,&extra);
    if(A->marker==MR_EPOINT_INFINITY)
        {
            zzn4_from_int(1,&res.a);
            res.miller=FALSE;
            res.unitary=TRUE;
        }
    else res=line(P,A,B,lam,extra,Doubling,Qx,Qy);
    return res;
}
/*************************************************************
  Function:       fast_pairing
  Description:    R-ate Pairing G2 x G1 -> GT
                  P is a point of order q in G1. Q(x,y) is a point of order q in G2.
                  Note that P is a point on the sextic twist of the curve over Fp^2,
                  Q(x,y) is a point on the curve over the base field Fp
                  see ake12bnx.cpp for details in MIRACL c++ source file
  Calls:          MIRACL functions,zzn12_init,g,q_power_frobenius
                  zzn12_copy,zzn12_conj,zzn12_div,zzn12_powq,zzn12_inverse
  Called By:      ecap
  Input:          ecn2 P,big Qx,big Qy,big x,zzn2 X
  Output:         zzn12 *r
  Return:         FALSE: r=0
                  TRUE: correct calculation
  Others:
*************************************************************/
BOOL fast_pairing(ecn2 P,big Qx,big Qy,big x,zzn2 X,zzn12 *r)
{
    int i,nb;
    big n,zero,negify_x;
    ecn2 A,KA;
    zzn12 t0,x0,x1,x2,x3,x4,x5,res;

    zero=mirvar(0);n=mirvar(0);negify_x=mirvar(0);
    A.x.a=mirvar(0); A.x.b=mirvar(0); A.y.a=mirvar(0);A.y.b=mirvar(0);
    A.z.a=mirvar(0); A.z.b=mirvar(0); A.marker=MR_EPOINT_INFINITY;
    KA.x.a=mirvar(0); KA.x.b=mirvar(0); KA.y.a=mirvar(0);KA.y.b=mirvar(0);
```

```
KA.z.a=mirvar(0); KA.z.b=mirvar(0); KA.marker=MR_EPOINT_INFINITY;
zzn12_init(&t0);zzn12_init(&x0);zzn12_init(&x1);zzn12_init(&x2);
zzn12_init(&x3);zzn12_init(&x4);zzn12_init(&x5);zzn12_init(&res);

premult(x,6,n);incr(n,2,n);//n=(6*x+2);
 if (mr_compare(x,zero)<0)  //x<0
    negify(n,n);            //n=-(6*x+2);

ecn2_copy(&P,&A);
nb=logb2(n);
zzn4_from_int(1,&res.a);res.unitary=TRUE; //res=1
// Short Miller loop
res.miller=TRUE;

for (i=nb-2;i>=0;i--)
{
     zzn12_mul(res,res,&res);
    zzn12_mul(res,g(&A,&A,Qx,Qy),&res);
    if(mr_testbit(n,i))
        zzn12_mul(res,g(&A,&P,Qx,Qy),&res);
}
// Combining ideas due to Longa, Aranha et al. and Naehrig
 ecn2_copy(&P,&KA);
q_power_frobenius(KA,X);
if(mr_compare(x,zero)<0)
  {
      ecn2_negate(&A,&A);
      zzn12_conj(&res,&res);
  }
zzn12_mul(res,g(&A,&KA,Qx,Qy),&res);
q_power_frobenius(KA,X);
ecn2_negate(&KA,&KA);
zzn12_mul(res,g(&A,&KA,Qx,Qy),&res);

if(zzn4_iszero(&res.a)&&zzn4_iszero(&res.b)&&zzn4_iszero(&res.c)) return FALSE;

// The final exponentiation
zzn12_copy(&res,&t0);//t0=r;
zzn12_conj(&res,&res);
zzn12_div(res,t0,&res);

res.miller=FALSE;res.unitary=FALSE;

zzn12_copy(&res,&t0);//t0=r;
```

```
    zzn12_powq(X,&res);
    zzn12_powq(X,&res);
    zzn12_mul(res,t0,&res);// r^[(p^6-1)*(p^2+1)]
    res.miller=FALSE;res.unitary=TRUE;

    // Newer new idea...
   // See "On the final exponentiation for calculating pairings on ordinary elliptic curves"
   // Michael Scott and Naomi Benger and Manuel Charlemagne and Luis J. Dominguez Perez and Ezekiel
J. Kachisa
    zzn12_copy(&res,&t0);zzn12_powq(X,&t0);
    zzn12_copy(&t0,&x0); zzn12_powq(X,&x0);   //x0=t0

    zzn12_mul(res,t0,&x1);zzn12_mul(x0,x1,&x0);// x0*=(res*t0);
    zzn12_powq(X,&x0);

    x1=zzn12_inverse(res);// just a conjugation!
    negify(x,negify_x);  x4=zzn12_pow(res,negify_x);//negify_x=-x   x is sparse.
    zzn12_copy(&x4,&x3);zzn12_powq(X,&x3);

    x2=zzn12_pow(x4,negify_x);
    x5=zzn12_inverse(x2);
    t0=zzn12_pow(x2,negify_x);

    zzn12_powq(X,&x2);
    zzn12_div(x4,x2,&x4);

    zzn12_powq(X,&x2);
    zzn12_copy(&t0,&res);// res=t0
    zzn12_powq(X,&res);
    zzn12_mul(t0,res,&t0);

    zzn12_mul(t0,t0,&t0);zzn12_mul(t0,x4,&t0);zzn12_mul(t0,x5,&t0);//t0*=t0;t0*=x4;t0*=x5;
    zzn12_mul(x3,x5,&res);zzn12_mul(res,t0,&res);//res=x3*x5;res*=t0;
    zzn12_mul(t0,x2,&t0);//t0*=x2;
    zzn12_mul(res,res,&res);zzn12_mul(res,t0,&res);zzn12_mul(res,res,&res);//res*=res;
res*=t0;res*=res;
    zzn12_mul(res,x1,&t0);//  t0=res*x1;
    zzn12_mul(res,x0,&res);//res*=x0;
    zzn12_mul(t0,t0,&t0);zzn12_mul(t0,res,&t0);//t0*=t0;t0*=res;

    zzn12_copy(&t0,r);//r= t0;

    return TRUE;
}
```

```
/*************************************************************
  Function:       ecap
  Description:    caculate Rate pairing
                  see ake12bnx.cpp for details in MIRACL c++ source file
  Calls:          MIRACL functions,fast_pairing
  Called By:
  Input:          ecn2 P,epoint *Q,big x,zzn2 X
  Output:         zzn12 *r
  Return:         FALSE: calculation error
                  TRUE: correct calculation
  Others:
*************************************************************/
BOOL ecap(ecn2 P,epoint *Q,big x,zzn2 X,zzn12 *r)
{
    BOOL Ok;
    big Qx,Qy;
    Qx=mirvar(0); Qy=mirvar(0);

    ecn2_norm(&P);
    epoint_get(Q,Qx,Qy);

    Ok=fast_pairing(P,Qx,Qy,x,X,r);

    if (Ok) return TRUE;

    return FALSE;
}
/*************************************************************
  Function:       member
  Description:    ctest if a zzn12 element is of order q
                  test r^q = r^(p+1-t) =1, so test r^p=r^(t-1)
                  see ake12bnx.cpp for details in MIRACL c++ source file
  Calls:          MIRACL functions,zzn12_init,zzn12_copy,zzn12_powq
  Called By:
  Input:          zzn12 r,big x,zzn2 F
  Output:         NULL
  Return:         FALSE: zzn12 element is not of order q
                  TRUE: zzn12 element is of order q
  Others:
*************************************************************/
BOOL member(zzn12 r,big x,zzn2 F)
{
     zzn12 w;
    big six;
```

```
    six=mirvar(0);
    zzn12_init(&w);

    convert(6,six);
    zzn12_copy(&r,&w);//w=r
    zzn12_powq(F,&w);
    r=zzn12_pow(r,x);r=zzn12_pow(r,x);r=zzn12_pow(r,six); // t-1=6x^2
    if(zzn4_compare(&w.a,&r.a)&&zzn4_compare(&w.a,&r.a)&&zzn4_compare(&w.a,&r.a)) return
TRUE;
     return FALSE;
}
```