FACULTY OF INFORMATION TECHNOLOGY

Probability and Statistics

Marking scheme for assignment II

Lecturer:
Dr.Pacifique Nizeyimana

August 17, 2022

Question I :

a) $P(x > 40) = P(z > \frac{40-35}{3})$

$= P(z > 1.67) = 1 - P(z < 1.67)$

$= 1 - 0.9525 = 0.0475$

b) $P(x_1 > 40) \cap P(x_2 > 40)$

Since x_1 and x_2 are independent

$P(x_1 > 40) \cap P(x_2 > 40) = P(x_1 > 40) \cdot P(x_2 > 40)$

$= P(z > \frac{40-35}{3}) \cdot P(z > \frac{40-35}{3})$

$= (0.0475)^2 = 0.00226$

c) we know that 95% of all measurements for normal random variable lie within 1.96 Standard deviations of the mean So, the interval needed is

$\mu \pm 1.96\sigma = 35 \pm 1.96(3)$

$= 35 \pm 5.88$

Interval will Start from 29.12 to 40.88

d) the p^{th} percentile of standard normal distribution is a value of z which has area $P/100$ to its left.

the area to the left of 90^{th} percentile is 0.9.

by looking on table the appropriate value of z is close to $z = 1.28$ with area 0.8997

& thus, 90^{th} percentile is approximately $z = 1.28$

we know that $z = \frac{x - \mu}{\sigma}$, thus

$x = \mu + \sigma z$.

$x = 35 + 1.28(3)$

$x = 38.84$

Question 2:

a) Sampling distribution of mean $\bar{x}$ is normal with mean $\mu = 64571$

$S.D = \sigma/\sqrt{n} = \frac{4000}{\sqrt{60}} = 516.39$

b) $p(\bar{x} > 66,000)$

$p\left(z > \frac{66,000 - 64571}{516.39}\right)$

$p(z > 2.77) = 1 - p(z < 2.77)$

$= 1 - 0.9972$

$= 0.0028$

c) $z = \frac{66,000 - 64571}{516.39} = 2.77$

thus, by assuming that $\mu = 64,571$, it is moderate unusual to get $\bar{x} = 66,000$, because it lies in 2.77 above the mean. therefore, perhaps sample was not Random sample or average salary of 64571 is not correct.

Note: z score is not unusual if $[-2 < z < 2]$

a) Question 3

a) yes $n = 120$

$p = 0.2$

$Q = 1 - 0.2$

$Q = 0.8$

$np = 0.2 \times 120 = 24 > 5$

$nQ = 0.8 \times 120 = 96 > 5$

thus, distribution of $\hat{p}$ have an approximate normal distribution (i.e: you can confirm this by using central limit theorem since our $n \geq 30$)

mean : $p = 0.2$

S.D : $\cancel{\frac{PQ}{\sqrt{n}}}$ $\sqrt{\frac{PQ}{n}} = 0.0365$

b) $P(\hat{p} > 0.25) = P\left(Z > \frac{0.25 - 0.2}{0.0365}\right)$

$= P(Z > 1.37) = 1 - P(Z < 1.37)$

$= 1 - 0.9147 = 0.0853$

c) $P(0.25 < \hat{p} < 0.3) = P\left(\frac{0.25 - 0.2}{0.0365} < Z < \frac{0.3 - 0.2}{0.0365}\right)$

$P(1.37 < Z < 2.74) = P(Z < 2.74) - P(Z < 1.37)$

$= 0.9969 - 0.9147$

$= 0.0822$

d) $\cancel{P(\hat{p})}$ $Z = \frac{0.3 - 0.2}{0.0365} = 2.73$

thus, the value is moderate unusual because $\hat{p} = 0.3$ lies in 2.73 above the mean.

Remember: Z is not unusual if $[-2 < Z < 2]$

Question 4 :

$n = 130$
$\bar{x} = 98.25$
$s = 0.73$

a) 99% CI:

$1 - \alpha = 99\%, \quad 1 - \alpha = 0.99$

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$Z_{\frac{\alpha}{2}} = Z_{0.005} = 2.58$ (look on table)

99% CI: $\bar{x} \pm 2.58\, S.E.$

$98.25 \pm 2.58 \frac{S}{\sqrt{n}}$

$98.25 \pm 2.58 \times \frac{0.73}{\sqrt{130}}$

98.25 ± 0.165185

$[98.08481 < \mu < 98.41519]$

b) No. the confidence interval constructed in part (a) does not contain value 98.6 degrees

CCL: perhaps 98.6 is not true average body temperature for healthy person.

Question 5:

$H_0: \mu = 50$

$H_1: \mu \neq 50$

$\alpha = 0.05$

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{51.3 - 50}{2/\sqrt{25}} = 3.25$$

CCL: since $z > 1.96$, we reject H_0 $\mu = 50$ at the 0.05 level of significance. we conclude that mean burning rate exceeds 50 centimeters per second.