

# Homework 1

## Discrete Structures 2

Due Thursday, Jan 17, 2019, 12:00 pm

**Instructions:** You should submit your homework online on webcourse. Only 3 problems will be graded, each graded problem is worth 5 points. There will be 3 effort points in total for the ungraded problems, 1 for each problem. Hence the maximum score for this homework is  $3 \times 5 + 3 = 18$ .

1. Let  $A$  and  $B$  be arbitrary sets. Prove that  $(A \times B) \cap (B \times A) = (A \cap B)^2$ .
2. Let  $A$  and  $B$  be arbitrary sets. Prove that  $P(A) \cap P(B) = P(A \cap B)$ . (Note:  $P$  stands for power set.)
3. Let  $n \in \mathbb{N}$  and let  $S_n = \{(x, y) \in \mathbb{N}^2 \mid x < y \leq n\}$ . List all the elements of  $S_3$ . How many elements are there in  $S_n$ ? Prove your answer.
4. Determine which functions below define surjections from  $\mathbb{N} \times \mathbb{N}$  to  $\mathbb{N}$ . Give a one-line explanation.
  - $f(x, y) = x + y$
  - $f(x, y) = xy$
5. Given that  $a$ ,  $b$ , and  $c$  are odd integers, prove that equation  $ax^2 + bx + c = 0$  cannot have a rational root.
6. Prove that  $\exists x, y$  irrational such that  $xy$  is rational. Use only the fact that  $\sqrt{2}$  is irrational. (Rational numbers are real numbers that can be expressed as a fraction.)