Homework 1

Discrete Structures 2

Due Thursday, Jan 17, 2019, 12:00 pm

Instructions: You should submit your homework online on webcourse. Only 3 problems will be graded, each graded problem is worth 5 points. There will be 3 effort points in total for the ungraded problems, 1 for each problem. Hence the maximum score for this homework is 3 × 5 + 3 = 18.

1. Let A and B be arbitrary sets. Prove that $(A \times B) \cap (B \times A) = (A \cap B)^2$.
2. Let A and B be arbitrary sets. Prove that $\mathrm{P}(A) \cap \mathrm{P}(B) = \mathrm{P}(A \cap B)$. (Note: P stands for power set.)
3. Let $n \in \mathbb{N}$ and let $S_n = \{(x,y) \in \mathbb{N}^2 | x < y \leq n\}$. List all the elements of S_3. How many elements are there in S_n? Prove your answer.
4. Determine which functions below define surjections from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$. Give a one-line explanation.
   - $f(x,y) = x+y$
   - $f(x,y)=xy$
5. Given that a, b, and c are odd integers, prove that equation $ax2 + bx + c = 0$ cannot have a rational root.
6. Prove that $\exists x,y$ irrational such that x^y is rational. Use only the fact that $\sqrt{2}$ is irrational. (Rational numbers are real numbers that can be expressed as a fraction.)