Contents

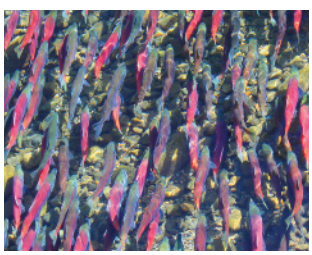
PREFACE xi
TO THE STUDENT xxiii
CALCULATORS, COMPUTERS, AND OTHER GRAPHING DEVICES xxiv
DIAGNOSTIC TESTS xxvi

A Preview of Calculus 1

1 Functions and Models 9

1.1 Four Ways to Represent a Function 10
1.2 Mathematical Models: A Catalog of Essential Functions 23
1.3 New Functions from Old Functions 36
1.4 Exponential Functions 45
1.5 Inverse Functions and Logarithms 55
Review 68
Principles of Problem Solving 71

2 Limits and Derivatives 77



2.1 The Tangent and Velocity Problems 78
2.2 The Limit of a Function 83
2.3 Calculating Limits Using the Limit Laws 95
2.4 The Precise Definition of a Limit 104
2.5 Continuity 114
2.6 Limits at Infinity; Horizontal Asymptotes 126
2.7 Derivatives and Rates of Change 140
Writing Project • Early Methods for Finding Tangents 152
2.8 The Derivative as a Function 152
Review 165
Problems Plus 169

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

3 Differentiation Rules 171

3.1 Derivatives of Polynomials and Exponential Functions 172
Applied Project • Building a Better Roller Coaster 182
3.2 The Product and Quotient Rules 183
3.3 Derivatives of Trigonometric Functions 190
3.4 The Chain Rule 197
Applied Project • Where Should a Pilot Start Descent? 208
3.5 Implicit Differentiation 208
Laboratory Project • Families of Implicit Curves 217
3.6 Derivatives of Logarithmic Functions 218
3.7 Rates of Change in the Natural and Social Sciences 224
3.8 Exponential Growth and Decay 237
Applied Project • Controlling Red Blood Cell Loss During Surgery 244
3.9 Related Rates 245
3.10 Linear Approximations and Differentials 251
Laboratory Project • Taylor Polynomials 258
3.11 Hyperbolic Functions 259
Review 266
Problems Plus 270

4 Applications of Differentiation 275

4.1 Maximum and Minimum Values 276
Applied Project • The Calculus of Rainbows 285
4.2 The Mean Value Theorem 287
4.3 How Derivatives Affect the Shape of a Graph 293
4.4 Indeterminate Forms and l'Hospital's Rule 304
Writing Project • The Origins of l'Hospital's Rule 314
4.5 Summary of Curve Sketching 315
4.6 Graphing with Calculus *and* Calculators 323
4.7 Optimization Problems 330
Applied Project • The Shape of a Can 343
Applied Project • Planes and Birds: Minimizing Energy 344
4.8 Newton's Method 345
4.9 Antiderivatives 350
Review 358
Problems Plus 363

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

5 Integrals 365

5.1 Areas and Distances 366
5.2 The Definite Integral 378
Discovery Project • Area Functions 391
5.3 The Fundamental Theorem of Calculus 392
5.4 Indefinite Integrals and the Net Change Theorem 402
Writing Project • Newton, Leibniz, and the Invention of Calculus 411
5.5 The Substitution Rule 412
Review 421
Problems Plus 425

6 Applications of Integration 427

6.1 Areas Between Curves 428
Applied Project • The Gini Index 436
6.2 Volumes 438
6.3 Volumes by Cylindrical Shells 449
6.4 Work 455
6.5 Average Value of a Function 461
Applied Project • Calculus and Baseball 464
Applied Project • Where to Sit at the Movies 465
Review 466
Problems Plus 468

7 Techniques of Integration 471

7.1 Integration by Parts 472
7.2 Trigonometric Integrals 479
7.3 Trigonometric Substitution 486
7.4 Integration of Rational Functions by Partial Fractions 493
7.5 Strategy for Integration 503
7.6 Integration Using Tables and Computer Algebra Systems 508
Discovery Project • Patterns in Integrals 513
7.7 Approximate Integration 514
7.8 Improper Integrals 527
Review 537
Problems Plus 540

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

8 Further Applications of Integration 543

8.1 Arc Length 544
Discovery Project • Arc Length Contest 550
8.2 Area of a Surface of Revolution 551
Discovery Project • Rotating on a Slant 557
8.3 Applications to Physics and Engineering 558
Discovery Project • Complementary Coffee Cups 568
8.4 Applications to Economics and Biology 569
8.5 Probability 573
Review 581
Problems Plus 583

9 Differential Equations 585

9.1 Modeling with Differential Equations 586
9.2 Direction Fields and Euler's Method 591
9.3 Separable Equations 599
Applied Project • How Fast Does a Tank Drain? 608
Applied Project • Which Is Faster, Going Up or Coming Down? 609
9.4 Models for Population Growth 610
9.5 Linear Equations 620
9.6 Predator-Prey Systems 627
Review 634
Problems Plus 637

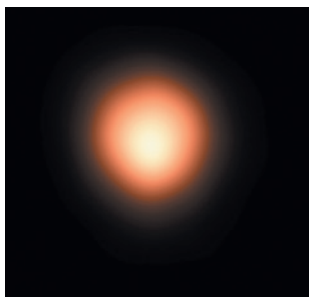
10 Parametric Equations and Polar Coordinates 639

10.1 Curves Defined by Parametric Equations 640
Laboratory Project • Running Circles Around Circles 648
10.2 Calculus with Parametric Curves 649
Laboratory Project • Bézier Curves 657
10.3 Polar Coordinates 658
Laboratory Project • Families of Polar Curves 668
10.4 Areas and Lengths in Polar Coordinates 669

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

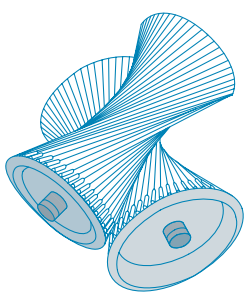
10.5 Conic Sections 674
10.6 Conic Sections in Polar Coordinates 682
Review 689
Problems Plus 692

11 Infinite Sequences and Series 693



11.1 Sequences 694
Laboratory Project • Logistic Sequences 707
11.2 Series 707
11.3 The Integral Test and Estimates of Sums 719
11.4 The Comparison Tests 727
11.5 Alternating Series 732
11.6 Absolute Convergence and the Ratio and Root Tests 737
11.7 Strategy for Testing Series 744
11.8 Power Series 746
11.9 Representations of Functions as Power Series 752
11.10 Taylor and Maclaurin Series 759
Laboratory Project • An Elusive Limit 773
Writing Project • How Newton Discovered the Binomial Series 773
11.11 Applications of Taylor Polynomials 774
Applied Project • Radiation from the Stars 783
Review 784
Problems Plus 787

12 Vectors and the Geometry of Space 791



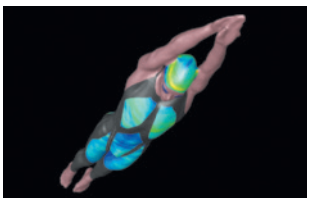
12.1 Three-Dimensional Coordinate Systems 792
12.2 Vectors 798
12.3 The Dot Product 807
12.4 The Cross Product 814
Discovery Project • The Geometry of a Tetrahedron 823
12.5 Equations of Lines and Planes 823
Laboratory Project • Putting 3D in Perspective 833
12.6 Cylinders and Quadric Surfaces 834
Review 841
Problems Plus 844

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

13 Vector Functions 847

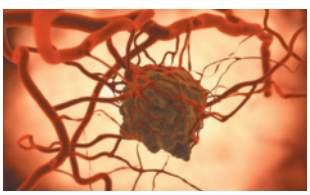
13.1 Vector Functions and Space Curves 848
13.2 Derivatives and Integrals of Vector Functions 855
13.3 Arc Length and Curvature 861
13.4 Motion in Space: Velocity and Acceleration 870
Applied Project • Kepler's Laws 880
Review 881
Problems Plus 884

14 Partial Derivatives 887



14.1 Functions of Several Variables 888
14.2 Limits and Continuity 903
14.3 Partial Derivatives 911
14.4 Tangent Planes and Linear Approximations 927
Applied Project • The Speedo LZR Racer 936
14.5 The Chain Rule 937
14.6 Directional Derivatives and the Gradient Vector 946
14.7 Maximum and Minimum Values 959
Applied Project • Designing a Dumpster 970
Discovery Project • Quadratic Approximations and Critical Points 970
14.8 Lagrange Multipliers 971
Applied Project • Rocket Science 979
Applied Project • Hydro-Turbine Optimization 980
Review 981
Problems Plus 985

15 Multiple Integrals 987

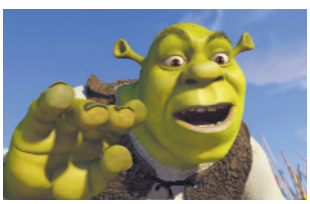


15.1 Double Integrals over Rectangles 988
15.2 Double Integrals over General Regions 1001
15.3 Double Integrals in Polar Coordinates 1010
15.4 Applications of Double Integrals 1016
15.5 Surface Area 1026

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

15.6 Triple Integrals 1029
Discovery Project • Volumes of Hyperspheres 1040
15.7 Triple Integrals in Cylindrical Coordinates 1040
Discovery Project • The Intersection of Three Cylinders 1044
15.8 Triple Integrals in Spherical Coordinates 1045
Applied Project • Roller Derby 1052
15.9 Change of Variables in Multiple Integrals 1052
Review 1061
Problems Plus 1065

16 Vector Calculus 1067



16.1 Vector Fields 1068
16.2 Line Integrals 1075
16.3 The Fundamental Theorem for Line Integrals 1087
16.4 Green's Theorem 1096
16.5 Curl and Divergence 1103
16.6 Parametric Surfaces and Their Areas 1111
16.7 Surface Integrals 1122
16.8 Stokes' Theorem 1134
Writing Project • Three Men and Two Theorems 1140
16.9 The Divergence Theorem 1141
16.10 Summary 1147
Review 1148
Problems Plus 1151

17 Second-Order Differential Equations 1153

17.1 Second-Order Linear Equations 1154
17.2 Nonhomogeneous Linear Equations 1160
17.3 Applications of Second-Order Differential Equations 1168
17.4 Series Solutions 1176
Review 1181

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Appendixes A1

A Numbers, Inequalities, and Absolute Values A2
B Coordinate Geometry and Lines A10
C Graphs of Second-Degree Equations A16
D Trigonometry A24
E Sigma Notation A34
F Proofs of Theorems A39
G The Logarithm Defined as an Integral A50
H Complex Numbers A57
I Answers to Odd-Numbered Exercises A65

Index A139

Copyright 2016 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.