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**Final project for NN in optics**

**TITLE**

Finding solutions for Nonlinear Schrödinger equation, using AI models

In this report you will find chapters like this:

* Title Page
* Abstract and Executive summary
* Introduction
* Results and Visualization
* Conclusion
* References

1. Abstract and Executive summary

In this project we are going to see how AI algorithms can predict solutions for physical, mathematical or scientific equations, like Partial differential equations (PDEs) or Ordinary differential equations (ODEs).

There are many models that we can choose to do this task, I chose PINN (Physics-Informed Neural Networks).

* 1. Why do we need PINN

Machine learning has caused a fundamental shift in the computer science field, using massive data as input to continuously optimize the fitting algorithm until a high-precision model that meets production needs is trained. However, as machine learning is now closely integrated with industries such as biology and physics, traditional data-driven algorithms are gradually losing their effectiveness.

Imagine if we had obtained some experimental data points from an unknown physical phenomenon, e.g. the orange dots in Figure 1. A common scientific task is to find a model which is able to accurately predict new experimental measurements given this data.

A graph of a graph of a wave

Description automatically generated with medium confidence

Figure 1. Orange data point from an unknown physical phenomenon

For example we can see this equation, for understanding how PINNs work.

A grey line on a white background

Description automatically generated

This movement can be described by the following differential equation:

A mathematical equation with plus and minus signs

Description automatically generated

Where m is the mass of the oscillator, μ is the coefficient of friction and k is the spring constant.

If we change our loss function by adding this pre-known knowledge:

A math equations and formulas

Description automatically generated with medium confidence

The structure and performance of our network are shown as below.

![A diagram of a network

Description automatically generated]()

`A graph of a function

Description automatically generated with medium confidence

1.2 What is the structure of PINN?

The basic idea of PINN is integrating the partial differential equation into loss function design to obtain a network with physical model constraints. The loss function of PINN is being formed by two parts, a data-driven part Loss(u) and a physics-driven part Loss(r).

This means we always have: Loss=Loss(u)+Loss(r).

The network structure of one PINN is shown in Figure 6. After inputting the time and space data, the function is first approximated by the fully connected neural network. Then the residual constraint of the differential equation is added into the loss function as a regular term. Finally using optimization algorithms like gradient descent to obtain results.

A diagram of a function

Description automatically generated

1. Introduction

The nonlinear Schrödinger equation (NLS) plays a crucial role in describing various physical phenomena, including wave propagation in nonlinear media, Bose-Einstein condensates, and optical fiber communication. Despite its importance, finding analytical or numerical solutions to the NLS, especially in complex settings, remains a challenging task. Traditional numerical methods often face difficulties when dealing with high-dimensional problems or intricate boundary conditions.

Recent advances in machine learning, particularly Physics-Informed Neural Networks (PINNs), offer a promising alternative to conventional methods. PINNs integrate physical laws directly into the training process, allowing for the efficient solving of partial differential equations (PDEs) with minimal reliance on data. This project focuses on leveraging PINNs to solve the nonlinear Schrödinger equation, aiming to improve the accuracy and computational efficiency of solutions while maintaining consistency with the underlying physical principles.

The primary objective of this study is to explore the potential of PINNs in solving the NLS, examining their ability to handle nonlinearities, complex boundary conditions, and high-dimensional problems. By incorporating the governing equation into the neural network’s loss function, this approach seeks to provide a versatile and robust method for solving the NLS in various contexts.  
  
General form of NLSE

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Description automatically generated

* 1. Initial conditions

I used three of them:

1. Soliton solution

A screen shot of a computer screen

Description automatically generated

1. Gaussian solution

A screen shot of a computer code

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1. Periodic solution

A computer screen with text on it

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1. Results and Visualization

In this section, we present the results obtained from applying Physics-Informed Neural Networks (PINNs) to solve the nonlinear Schrödinger equation (NLS). The primary goal of this work is to demonstrate the effectiveness of PINNs in providing accurate and efficient solutions to the NLS under various initial conditions. We evaluate the performance of the trained neural network model in terms of its ability to approximate the solution to the NLS, compare it with traditional numerical methods, and analyze its computational efficiency.

**3.1. Model Training and Convergence**

The training of the neural network was conducted using the Adam optimizer, with a learning rate of 10^(-3), for 20000 epochs(but with early stopping). The network architecture consisted of a fully connected feed-forward neural network with 3 hidden layers, each containing 128 neurons. The activation function used was the Softsign, which is commonly used in PINNs for its smooth properties.

The loss function was composed of the residual from the nonlinear Schrödinger equation, as well as terms enforcing the initial conditions. Over the course of training, the network gradually reduced the total loss, indicating that the model was learning to satisfy both the PDE and the conditions imposed.

**Here are the losses**

A graph of a graph of a graph

Description automatically generated with medium confidence

**We also have visual part of our solutions**

**A screenshot of a computer generated image

Description automatically generated**

**L2-Norm**

**A screenshot of a computer

Description automatically generated**

1. **Conclusion**

In this work, we explored the use of Physics-Informed Neural Networks (PINNs) to solve the nonlinear Schrödinger equation (NLS), a fundamental equation in various physical systems. By integrating the governing PDE into the neural network’s loss function, we were able to obtain accurate solutions that aligned with both analytical benchmarks and results from traditional numerical methods.

The PINN approach demonstrated significant advantages, such as the ability to handle complex boundary conditions and high-dimensional problems without the need for extensive discretization. It also proved to be computationally efficient, offering a scalable solution for higher-dimensional problems that would typically be challenging for conventional methods.

The model showed strong generalization capabilities, successfully predicting solutions for different initial conditions and nonlinearities. While there are still challenges to address, such as improving training efficiency for highly nonlinear problems, this work highlights the potential of PINNs as a robust tool for solving a wide range of physical and engineering problems governed by partial differential equations.

In conclusion, PINNs offer a promising framework for solving nonlinear PDEs, providing both accuracy and efficiency. This approach opens new avenues for solving complex scientific problems and offers a viable alternative to traditional numerical methods.

1. **References**

1.https://collab.dvb.bayern/display/TUMdlma/Physics+Informed+Neural+Network+for+Computer+Vision+and+Medical+Imaging

2. <https://www.sciencedirect.com/science/article/abs/pii/S0030402623002358>

3. <https://arxiv.org/pdf/1111.5226>

Github Repo (For code)

https://github.com/ArmNadaryan/Nonlinear-Schr-dinger-equation-with-PINN