Equilibrium point / Transfer function (p1)

Defining Variables

```
clear; clc; close all;

syms G X I U D
Gb = 4.5;
Ib = 15;
P1 = 0;
P2 = 0.025;
P3 = 0.000013;
V = 12;
n = 5.54/60;
F = [ -P1*G-X*(G+Gb)+D;
      -P2*X+P3*I;
      -n*(I+Ib)+U/V ];
y = G;

St = [G;X;I];
In = [U;D];
```

Equilibrium State

```
St_e = [0;0;0];
In_e = [n*Ib*V;0];
Fe = subs(subs(F,In,In_e),St,St_e);
```

Linearize

```
% f(x) = f(xe,ue) + (grad(f,x)|x=xe,u=ue)*(x-xe) + (grad(f,u)|x=xe,u=ue)*(u-ue)
grad_F_St = [ diff(F(1),St(1)), diff(F(1),St(2)), diff(F(1),St(3));
              diff(F(2),St(1)), diff(F(2),St(2)), diff(F(2),St(3));
              diff(F(3),St(1)), diff(F(3),St(2)), diff(F(3),St(3)) ];
grad_F_In = [ diff(F,In(1)) , diff(F,In(2)) ];
A = double(subs(subs(grad_F_St,In,In_e),St,St_e))
```

```
A = 3×3
       0   -4.5000         0
       0   -0.0250    0.0000
       0         0   -0.0923
```

```
B = double(subs(subs(grad_F_In,In,In_e),St,St_e));
B = B(:,1)
```

```
B = 3×1
       0
       0
  0.0833
```

```
C = [1,0,0]
```

```
C = 1×3
    1     0     0
```

```
D = 0
```

```
D = 0
```

Transfer Function

```
syms s
n = size(A,1);
Gp = C*inv(s*eye(n)-A)*B;
ExpFun = matlabFunction(simplifyFraction(Gp,'Expand',true));
ExpFun = str2func(regexprep(func2str(ExpFun), '\.([/^\\*])', '$1'));
Gp = tf(ExpFun(tf('s')));
for i = 1:length(Gp)
    [num,den] = tfdata(Gp(i));
    Gp(i) = tf(num{1}/den{1}(1),den{1}/den{1}(1));
end
Gp = Gp(1)
```

```
Gp =

           -4.875e-06
  -----------------------------
  s^3 + 0.1173 s^2 + 0.002308 s

Continuous-time transfer function.
```

```
[Gp_Num, Gp_Den] = tfdata(Gp); Gp_Num = Gp_Num{1}; Gp_Den = Gp_Den{1};
```

Analog Control (p2)

```
%pidTuner
```

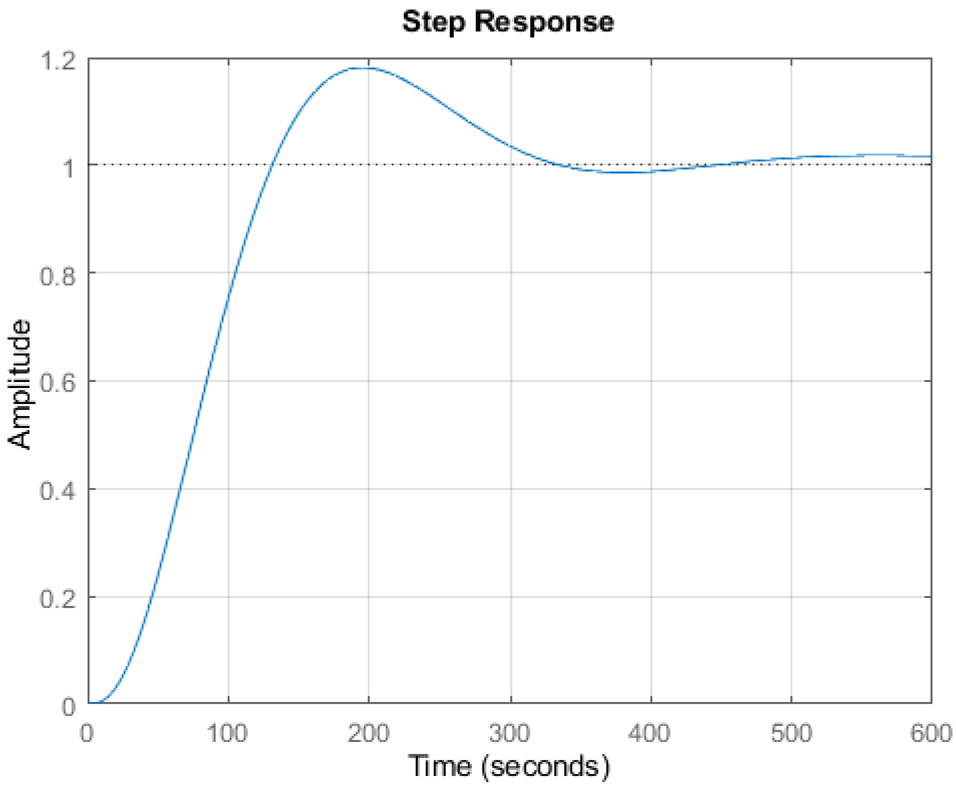
```
Gc = tf([-7.83 -0.001947],[1 0])
```

```
Gc =

  -7.83 s - 0.001947
  ------------------
          s

Continuous-time transfer function.
```

```
trans_info = stepinfo(feedback(series(Gc,Gp),1))
```

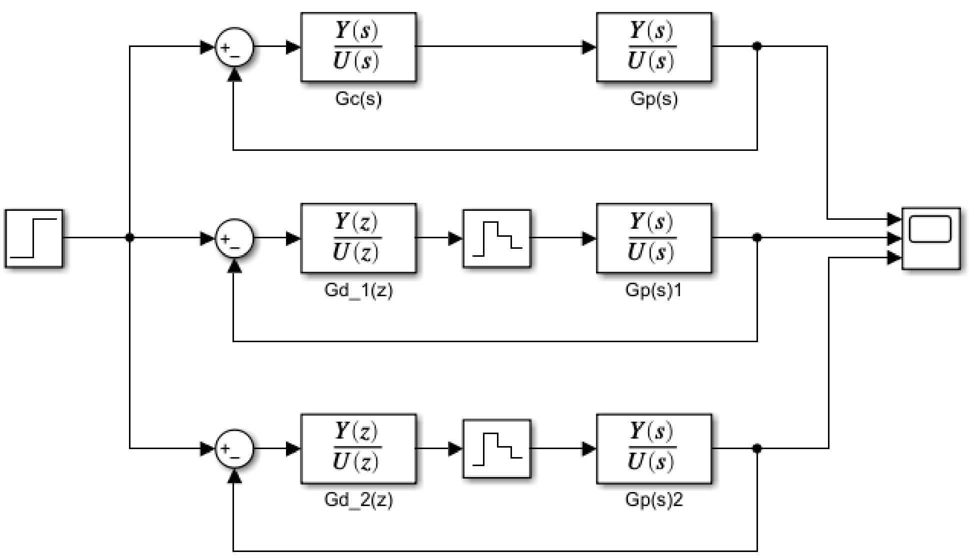
```
trans_info = struct with fields:
        RiseTime: 83.7907
    SettlingTime: 312.4786
     SettlingMin: 0.9142
     SettlingMax: 1.1813
       Overshoot: 18.1258
      Undershoot: 0
            Peak: 1.1813
        PeakTime: 194.7882
```

```
step(feedback(series(Gc,Gp),1))
grid on
```




```
[Gc_Num, Gc_Den] = tfdata(Gc); Gc_Num = Gc_Num{1}; Gc_Den = Gc_Den{1};
```

Digital Control (P3)




```
% 2 < tr/Ts < 10
Ts = round(trans_info.RiseTime/6)
```

```
Ts = 14
```

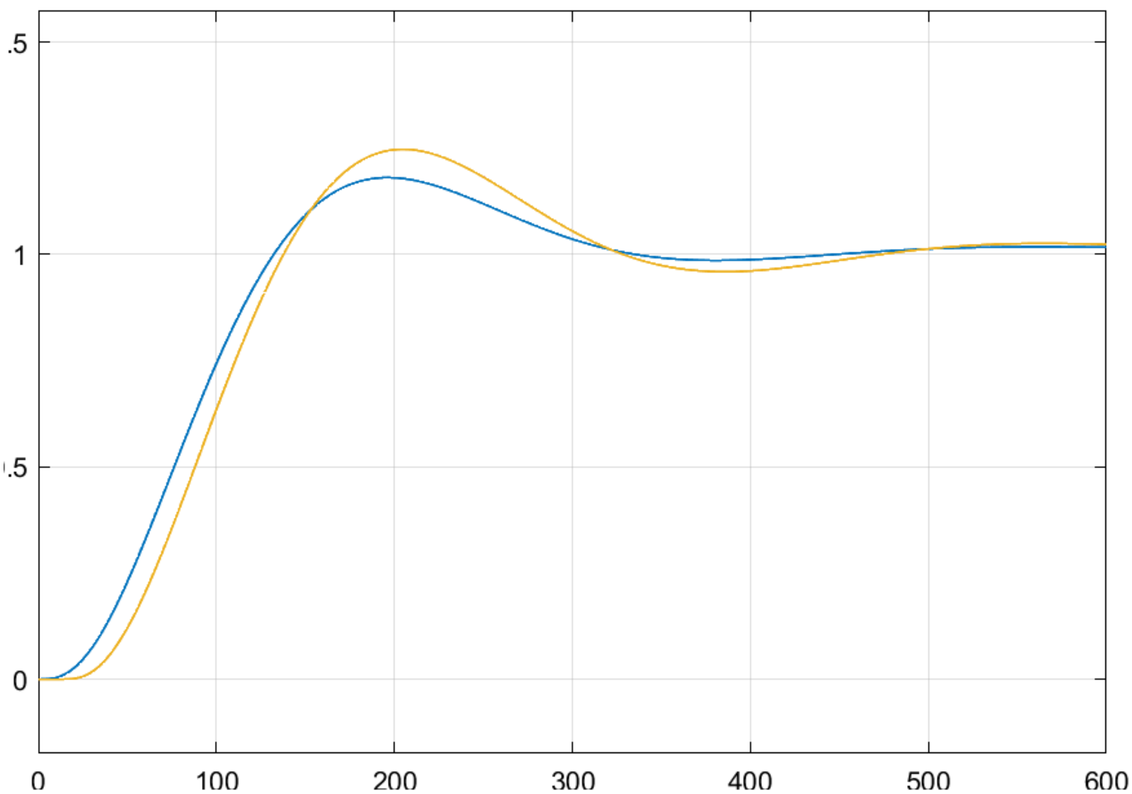
```
Gd_1 = c2d(Gc,Ts,'tastin')
```

```
Gd_1 =

  -7.844 z + 7.816
  ----------------
       z - 1

Sample time: 14 seconds
Discrete-time transfer function.
```

```
Gd_2 = c2d(Gc,Ts,'matched')
```

```
Gd_2 =

  -7.844 z + 7.816
  ----------------
       z - 1

Sample time: 14 seconds
Discrete-time transfer function.
```

```
[Gd_Num_1, Gd_Den_1] = tfdata(Gd_1); Gd_Num_1 = Gd_Num_1{1}; Gd_Den_1 = Gd_Den_1{1};
[Gd_Num_2, Gd_Den_2] = tfdata(Gd_2); Gd_Num_2 = Gd_Num_2{1}; Gd_Den_2 = Gd_Den_2{1};
```




```
Ts = 4
```

```
Ts = 4
```

```
Gd_1 = c2d(Gc,Ts,'tastin')
```

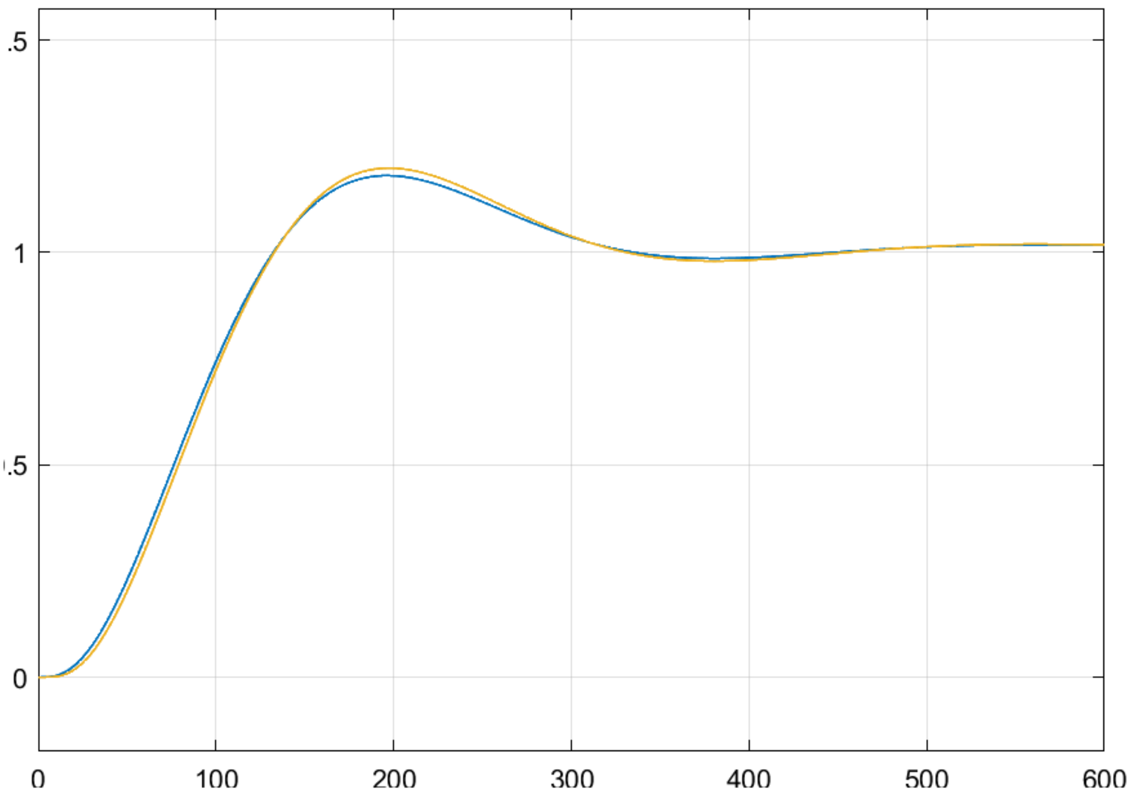
```
Gd_1 =
```

```
  -7.834 z + 7.826
  ----------------
       z - 1

Sample time: 4 seconds
Discrete-time transfer function.
```

```
Gd_2 = c2d(Gc,Ts,'matched')
```

```
Gd_2 =

  -7.834 z + 7.826
  ----------------
       z - 1

Sample time: 4 seconds
Discrete-time transfer function.
```

```
[Gd_Num_1, Gd_Den_1] = tfdata(Gd_1); Gd_Num_1 = Gd_Num_1{1}; Gd_Den_1 = Gd_Den_1{1};
[Gd_Num_2, Gd_Den_2] = tfdata(Gd_2); Gd_Num_2 = Gd_Num_2{1}; Gd_Den_2 = Gd_Den_2{1};
```




```
Ts = 40
```

```
Ts = 40
```

```
Gd_1 = c2d(Gc,Ts,'tastin')
```

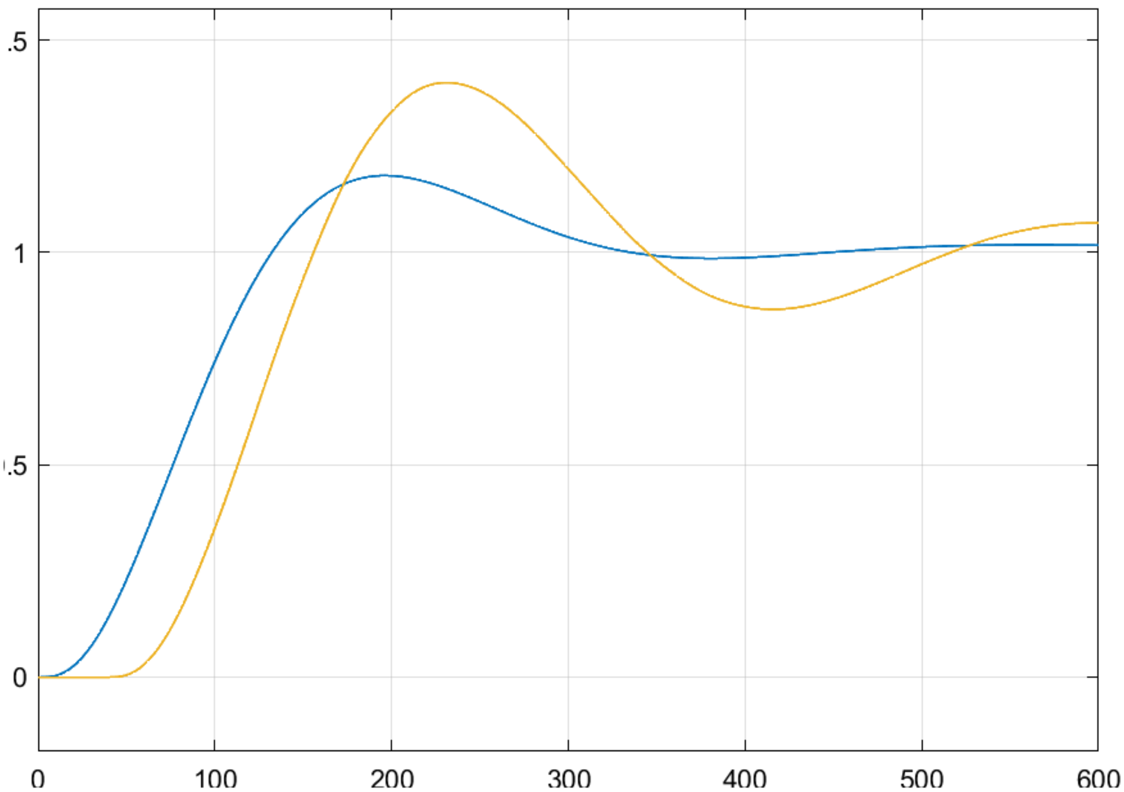
```
Gd_1 =

  -7.869 z + 7.791
  ----------------
```

```
      z - 1

Sample time: 40 seconds
Discrete-time transfer function.
```

```
Gd_2 = c2d(Gc,Ts,'matched')
```

```
Gd_2 =

  -7.869 z + 7.791
  ----------------
      z - 1

Sample time: 40 seconds
Discrete-time transfer function.
```

```
[Gd_Num_1, Gd_Den_1] = tfdata(Gd_1); Gd_Num_1 = Gd_Num_1{1}; Gd_Den_1 = Gd_Den_1{1};
[Gd_Num_2, Gd_Den_2] = tfdata(Gd_2); Gd_Num_2 = Gd_Num_2{1}; Gd_Den_2 = Gd_Den_2{1};
```



Discrete Transfer Function (p4)

Using Theoretical Calculations :

Using Mathematica

Simplify[InverseLaplaceTransform[$-4.875 * 10$^ $-$ $6/(s$^$2 * (s$^$2 + 0.1173 * s + 0.002308)), s, t]//N$]

$-0.00211t + 0.0085e^{-0.0923\,t} - 0.115856e^{-0.025\,t} + 0.1073$

Simplify[$(1 - z$^$-1) * (0.10735/(1 - z$^$-1) - 0.0021122 * z/(z - 1)$^$2 + 0.00850593/(1 -$ Exp$[-0.0923 * 4] * z$^$-1) - 0.11585$

$$-7 * 10^{-8}\frac{z^3 + 705.031 + 2280.422z + 559.37687}{(z - 1.)\,(z - 0.904837)\,(z - 0.691287)}$$

In[33]:= g[z]

Out[33]= $-\frac{7\,(559.377 + 2280.42\,z + 705.031\,z^2 + 1.\,z^3)}{100000\,(-1. + z)\,(-0.904837 + z)\,(-0.691287 + z)}$

In[31]:= T = 4;

Simplify[g[z] /. z → (1 + T / 2 * w) / (1 - T / 2 * w)]

Out[32]= $\frac{-0.00481527 + 0.00882848\,w + 0.00708513\,w^2 - 0.0110489\,w^3}{w\,(0.00227974 + 0.116245\,w + 1.\,w^2)}$

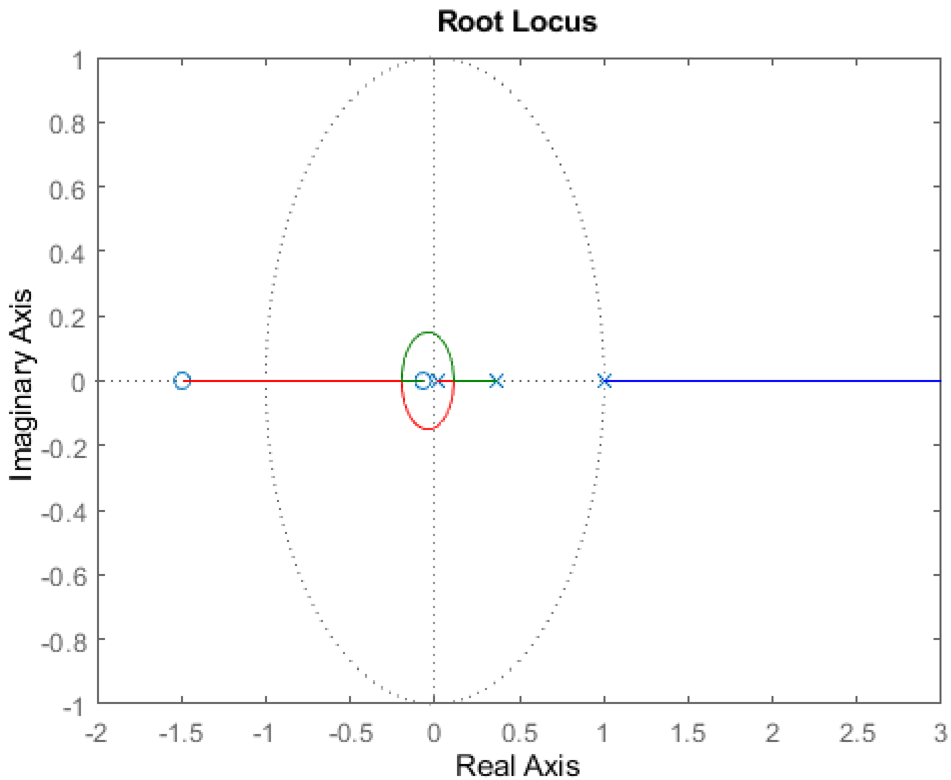
Using c2d in matlab :

```
Gp_d = c2d(Gp,Ts,'zoh')
```

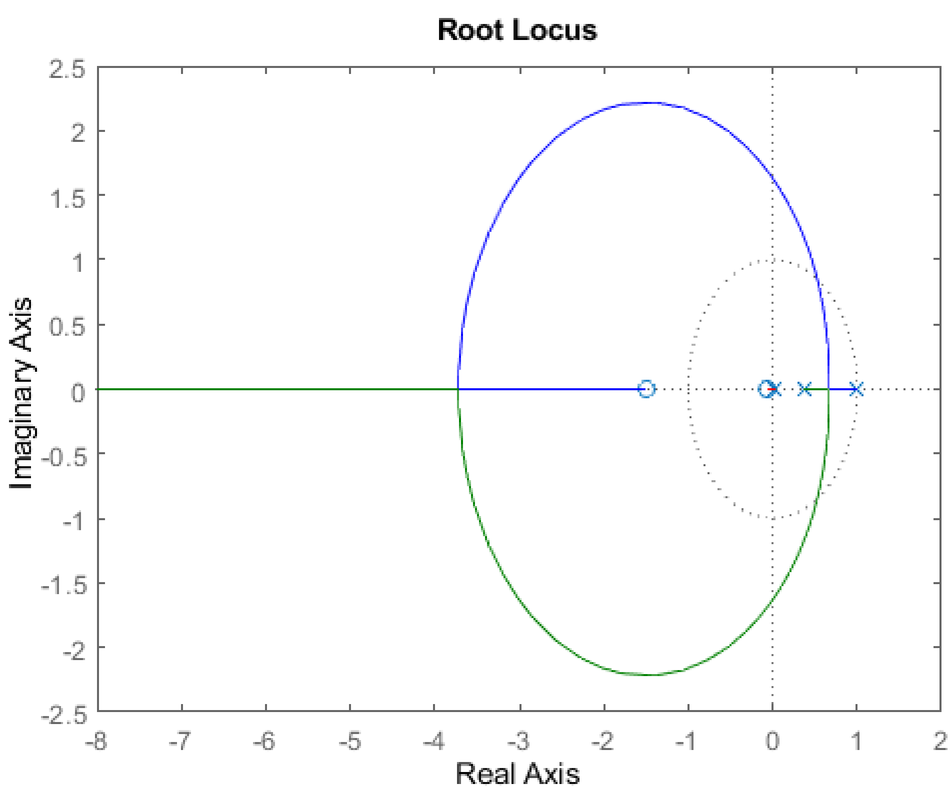
```
Gp_d =

   -0.01953 z^2 - 0.03054 z - 0.001997
  -------------------------------------
  z^3 - 1.393 z^2 + 0.4019 z - 0.009156

Sample time: 40 seconds
Discrete-time transfer function.
```

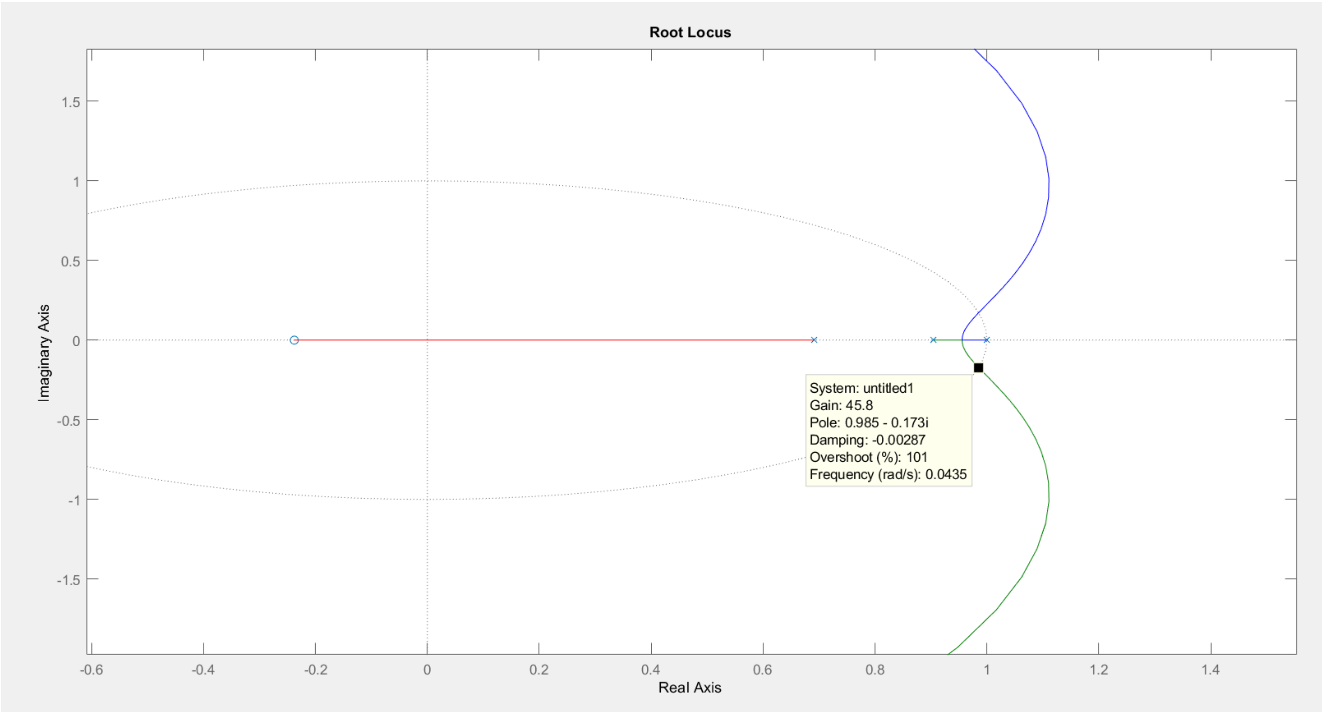
Root Locus

```
rlocus(Gp_d)
```


Root Locus
Imaginary Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3

```
rlocus(-Gp_d)
```

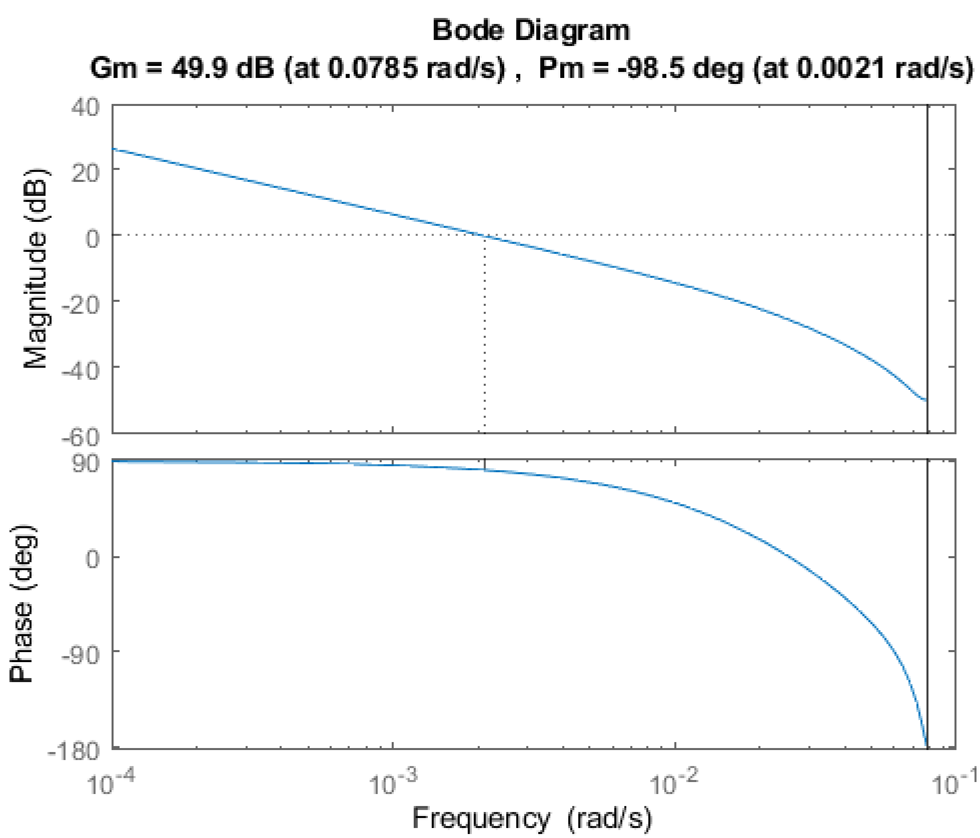

Root Locus
Imaginary Axis
Real Axis
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2



Therefore => -45.8<k<0

Bode plot (Z domain)

```
margin(Gp_d)
```



```
[Gm,Pm,Wgm,Wpm] = margin(Gp_d)
```

```
Warning: The closed-loop system is unstable.
Gm = 311.1157
Pm = -98.5242
Wgm = 0.0785
Wpm = 0.0021
```

Bode plot (Frequency domian)

Using Mathematica :

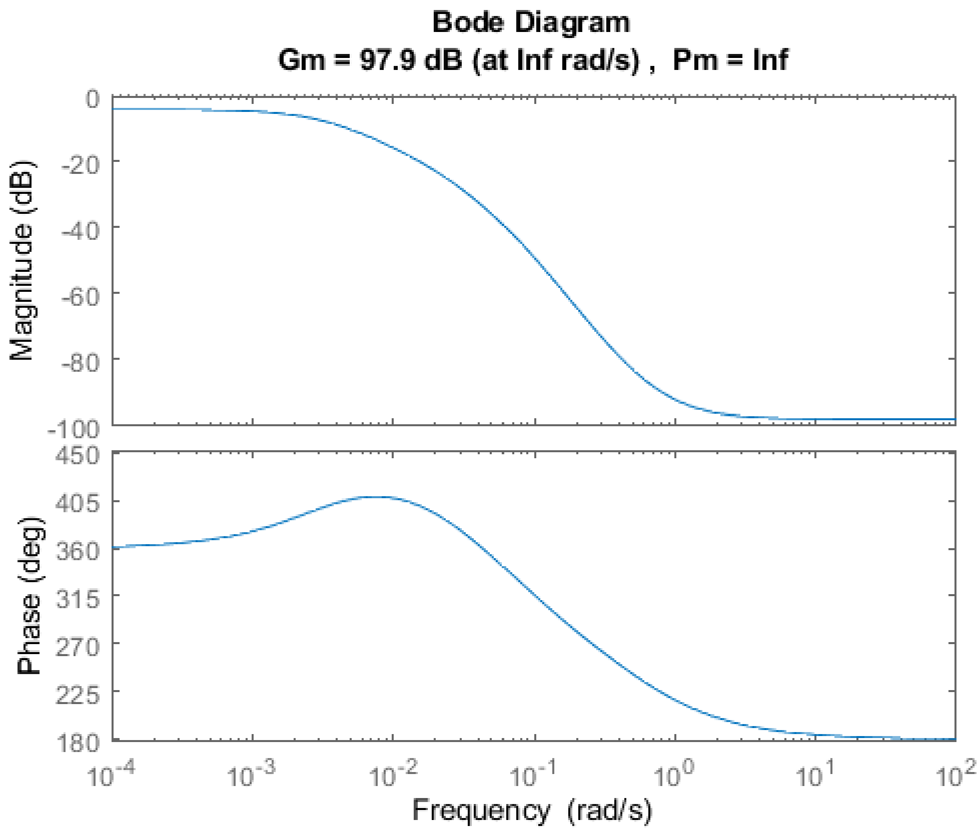
$$G[\mathrm{w\_}] := (-4.635 * 10\hat{}-5 * z\hat{}2 - 0.0001652 * z\hat{}1 - 3.666 * 10\hat{}-5)/(z\hat{}3 - 2.596 * z\hat{}2 + 2.221 * z - 0.6254)/.\,z \to \;((T/2 *$$

$$= \frac{0.0000048159 - 0.0000088798w - 0.000007883w^2 + 0.00001276w^3}{0.0000077611 - 0.002297281w - 0.116323w^2 - 1.w^3}$$

```
num = [0.00001276 , -0.000007883 , -0.0000088798 , 0.0000048159];
den = [-1 , -0.116323 , -0.00229728 , 0.0000077611];
gw = tf(num,den)
```

```
gw =

 
  -1.276e-05 s^3 + 7.883e-06 s^2 + 8.88e-06 s

                                           - 4.816e-06

  ------------------------------------------------------

        s^3 + 0.1163 s^2 + 0.002297 s - 7.761e-06

 
Continuous-time transfer function.
```

```
margin(gw)
```




```
[Gm,Pm,Wgm,Wpm] = margin(gw)
```

```
Warning: The closed-loop system is unstable.
Gm = 7.8370e+04
Pm = Inf
Wgm = Inf
Wpm = NaN
```

Design a controler (p5)

```
Ts = 10;
Gp
```

```
Gp =

           -4.875e-06
  -----------------------------
  s^3 + 0.1173 s^2 + 0.002308 s

Continuous-time transfer function.
```

```
Gp_d = c2d(Gp,Ts,'zoh')
```

```
Gp_d =

  -0.0006143 z^2 - 0.00186 z - 0.000342
  -------------------------------------
   z^3 - 2.176 z^2 + 1.485 z - 0.3093
```

```
Sample time: 10 seconds
Discrete-time transfer function.
```

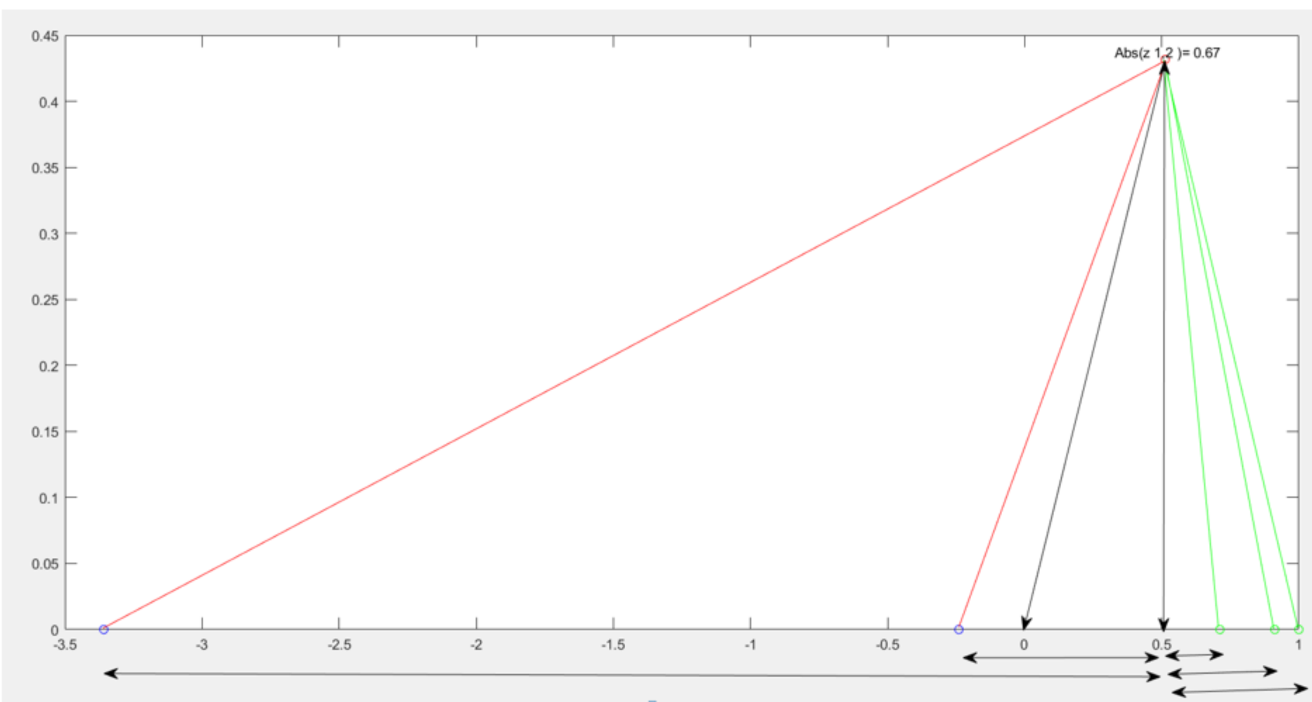
```
[num,den]=tfdata(Gp_d);
[z,p,k] = tf2zp(num{1},den{1})
```

```
z = 2×1
   -2.8306
   -0.1967
p = 3×1
    1.0000
    0.7788
    0.3972
k = -6.1431e-04
```

$$t_s = \frac{4}{\zeta\omega_n}\,, MP = \mathrm{e}^{-\frac{\zeta\omega_n}{\sqrt{1-\zeta^2}}} \rightarrow\ \zeta^2 = 1 - \frac{16}{(t_s \ln MP)^2}$$

$$t_s = 100\,, MP = 0.2 \rightarrow\ \zeta = \pm 0.9994\,, \omega_n = 0.04\,, if\ T = 10 \rightarrow \frac{w_s}{w_d} \cong 453 \gg 8$$

$$\rightarrow \begin{cases} |z| = e^{-\zeta\omega_n T} = 0.4493 \\ < z = T\, w_n \sqrt{1-\zeta^2} = 0.0277\ rad = 1.587^{\circ} \end{cases}$$



$$
\begin{cases}
\theta_1 = 180 - \tan^{-1}\left(\dfrac{0.4493\sin(0.0277)}{0.7788 - 0.4493\cos(0.0277)}\right) * \dfrac{180}{\pi} = 177.838^{\circ} \\
\theta_2 = 180 - \tan^{-1}\left(\dfrac{0.4493\sin(0.0277)}{1 - 0.4493\cos(0.0277)}\right) * \dfrac{180}{\pi} = 178.706^{\circ} \\
\theta_3 = \tan^{-1}\left(\dfrac{0.67\sin(0.0277)}{0.4493\cos(0.0277) - 0.3972}\right) * \dfrac{180}{\pi} = 19.6646^{\circ}
\end{cases}
,
\begin{cases}
\varphi_1 = \tan^{-1}\left(\dfrac{0.4493\sin(0.0277)}{0.1967 + 0.4493\cos(0.0277)}\right) * \dfrac{180}{\pi} = 1.104^{\circ} \\
\varphi_2 = \tan^{-1}\left(\dfrac{0.4493\sin(0.0277)}{2.8306 + 0.4493\cos(0.0277)}\right) * \dfrac{180}{\pi} = 0.217^{\circ}
\end{cases}
$$

$$\varphi_1 + \varphi_2 - \theta_1 - \theta_2 - \theta_3 = -349.717 \rightarrow -374.8876 + 360 = -14.8876^{\circ} \rightarrow reduced\ phase$$

$$G_D(z) = \frac{k(z+\alpha)}{z+\beta}\ , Assume \rightarrow z + \alpha = z + 0.7123 \rightarrow \alpha = 0.7123$$

$$\rightarrow G_D(z)G(z) = -\frac{6.1431 * 10^{-4}\, k\,(z+0.1967)(z+2.8306)}{(z-\beta)(z-0.3972)(z-0.7788)} \rightarrow \varphi_1 + \varphi_2 - \theta' - \theta_2 - \theta_3 = -360$$

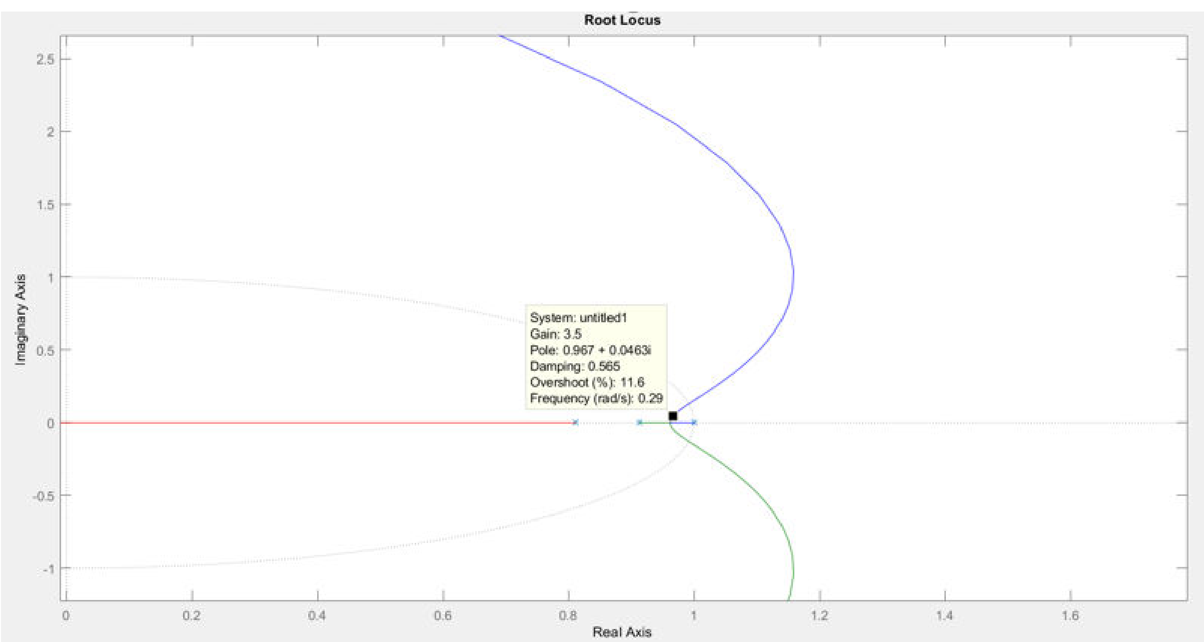
$$\rightarrow 360 + 1.104 + 0.217 - 177.838 - 19.6646 = \theta' = 163.8184^{\circ} \rightarrow \tan^{-1}\left(\frac{0.4493\sin(0.0277)}{\beta - 0.4493\cos(0.0277)}\right) * \frac{180}{\pi} = 16.1816^{\circ}$$

$$\rightarrow \beta = 0.492$$

So, Reconstructing the Open loop Transfer function : Gp(z)*G(z)

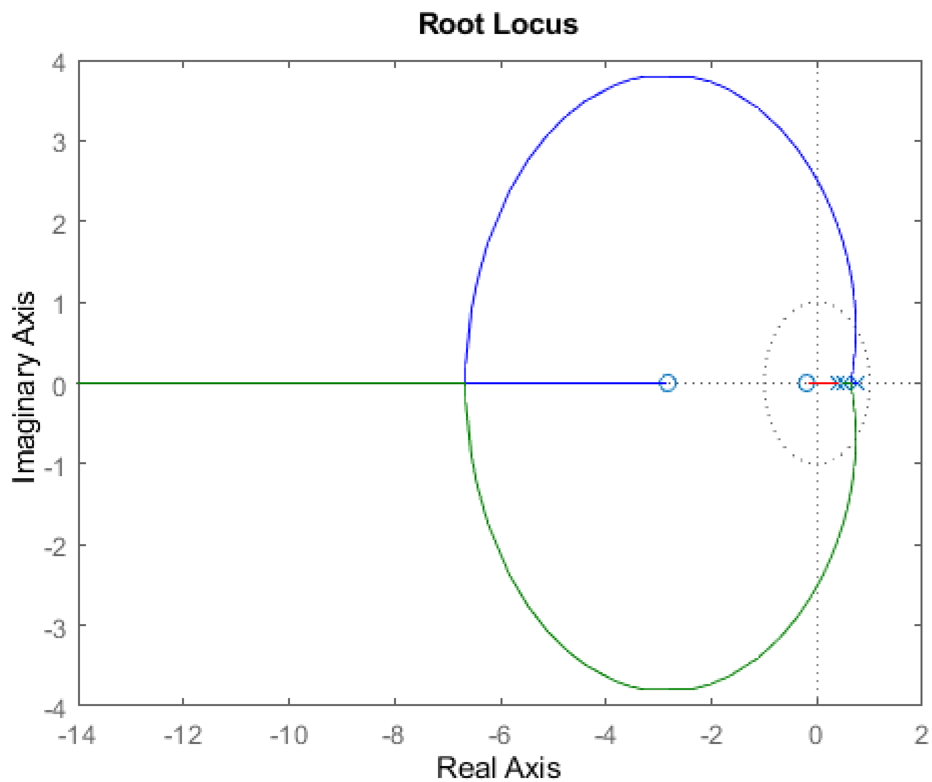
```
GC_pre = tf(-6.1431*10^-4*conv([1 0.1967],[1 2.8306]),conv(conv([1,-0.492],[1,-0.3972]), [1,-0.
```

```
GC_pre =

  -0.0006143 z^2 - 0.00186 z - 0.000342
  -------------------------------------
   z^3 - 1.668 z^2 + 0.8879 z - 0.1522

Sample time: 10 seconds
Discrete-time transfer function.
```



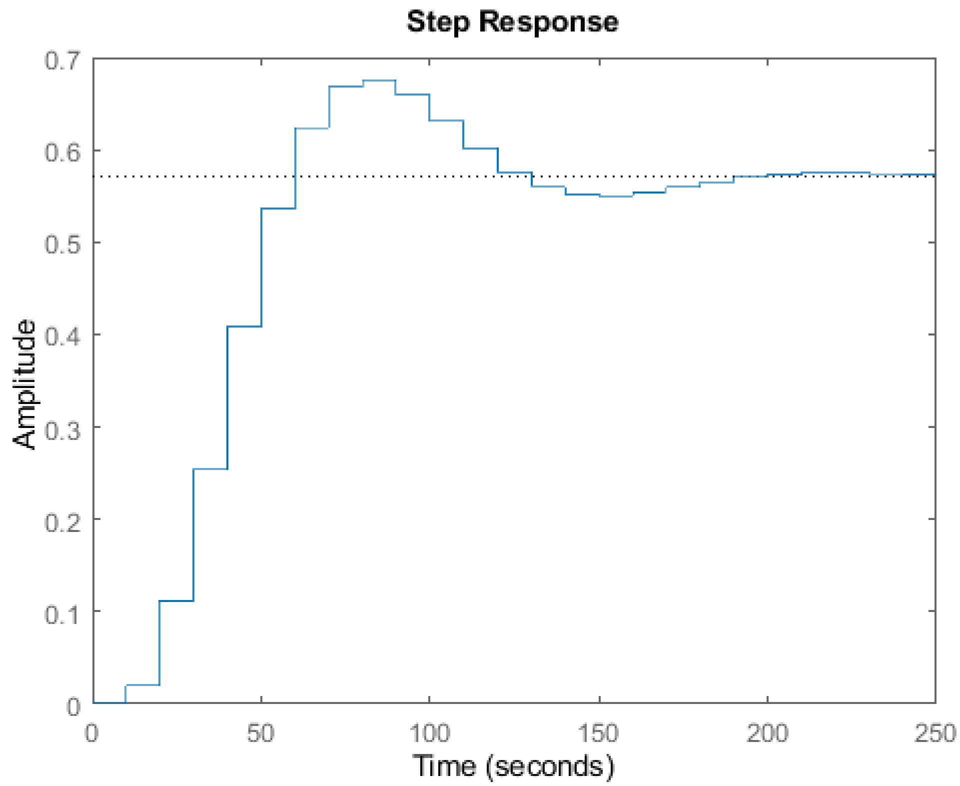
Decide to choose the controller coeffient form root locus

```
rlocus(-GC_pre)
```



choosing k = -3.5, then plot the step response

```
step(feedback(-32*GC_pre,1))
```




```
stepinfo(step(feedback(-32*GC_pre,1)))
```

```
ans = struct with fields:
        RiseTime: 3.4343
    SettlingTime: 18.5526
     SettlingMin: 0.5367
     SettlingMax: 0.6758
       Overshoot: 17.6908
      Undershoot: 0
            Peak: 0.6758
        PeakTime: 9
```

```
GC = -32*GC_pre
```

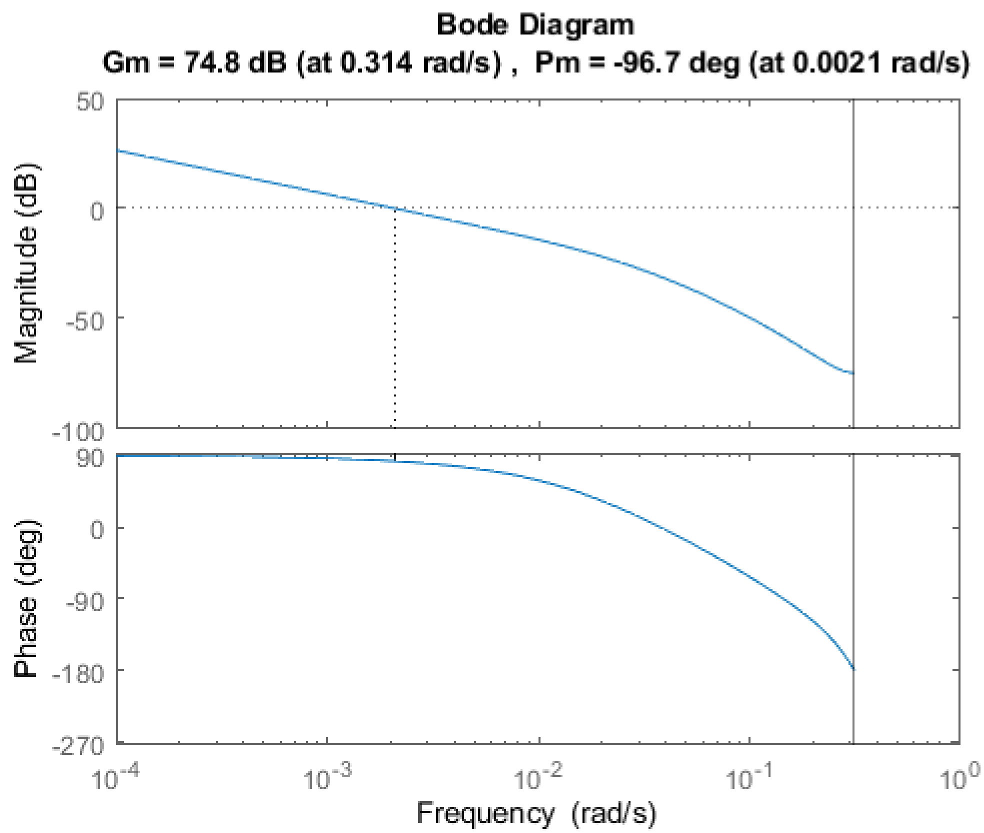
```
GC =
 
     0.01966 z^2 + 0.05951 z + 0.01095
   -----------------------------------
   z^3 - 1.668 z^2 + 0.8879 z - 0.1522
 
Sample time: 10 seconds
Discrete-time transfer function.
```

Comparing the features per plot (p6)

Bode plot (Frequency Domain)

(Uncontrolled system)

```
margin(Gp_d)
```




```
[Gm,Pm,Wgm,Wpm] = margin(Gp_d)
```

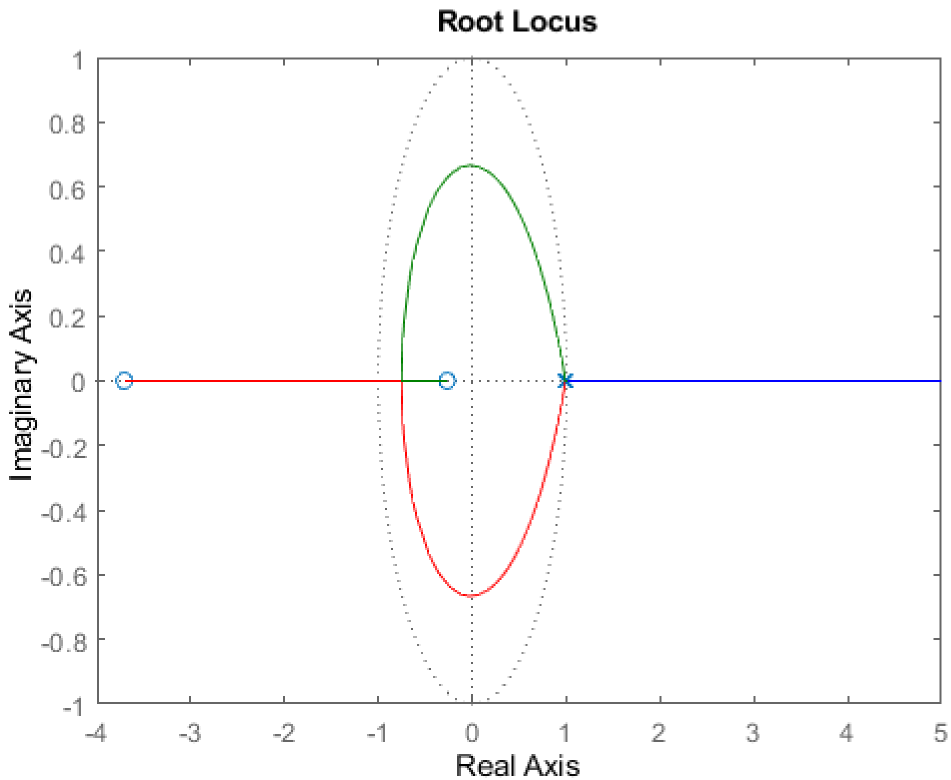
```
Warning: The closed-loop system is unstable.
Gm = 5.5026e+03
Pm = -96.7192
Wgm = 0.3142
Wpm = 0.0021
```

Root Locus (Uncontrolled system)

```
Ts = 0.2;
Gp_d = c2d(Gp,Ts,'zoh')
```

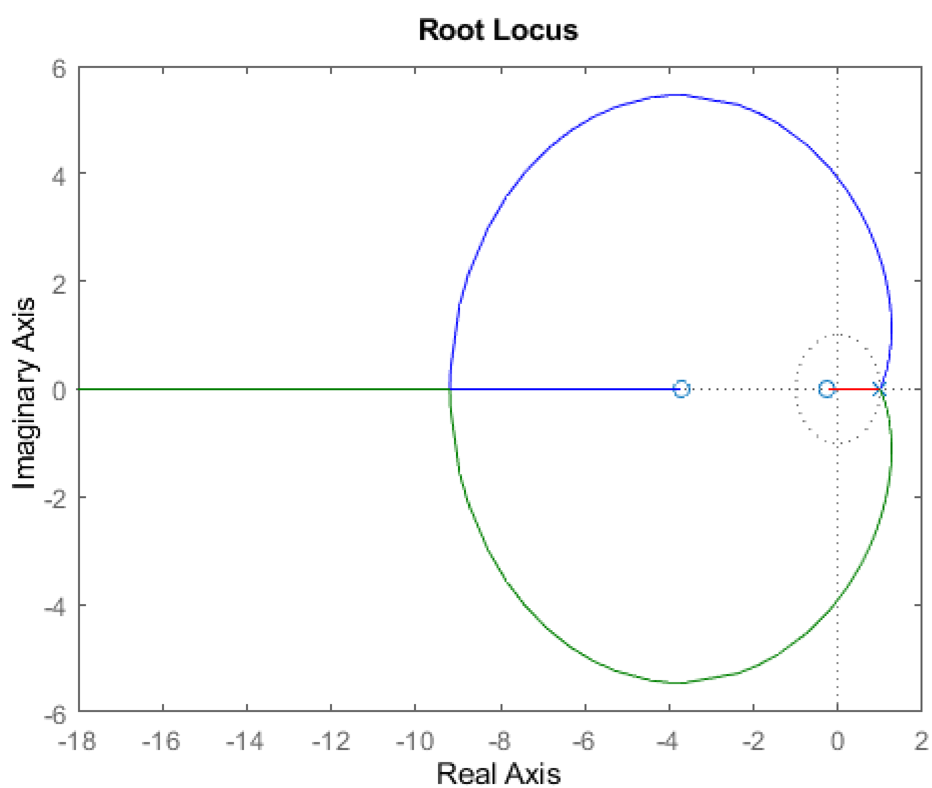
```
Gp_d =

  -6.462e-09 z^2 - 2.57e-08 z - 6.387e-09
  ---------------------------------------
    z^3 - 2.977 z^2 + 2.954 z - 0.9768

Sample time: 0.2 seconds
Discrete-time transfer function.
```

```
rlocus(Gp_d)
```



Root Locus
Imaginary Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-4
-3
-2
-1
0
1
2
3
4
5


```
rlocus(-Gp_d)
```

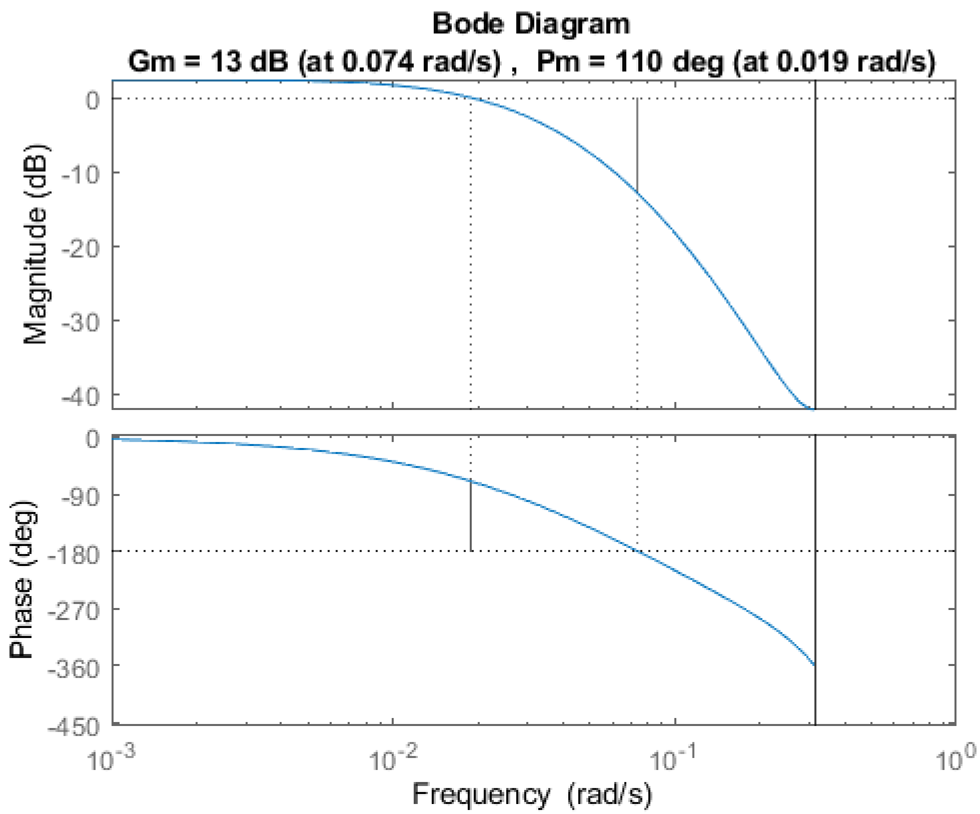


Root Locus
Imaginary Axis
Real Axis
6
4
2
0
-2
-4
-6
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2

Bode plot (Frequency Domain)

(Contolled system)

```
margin(GC)
```

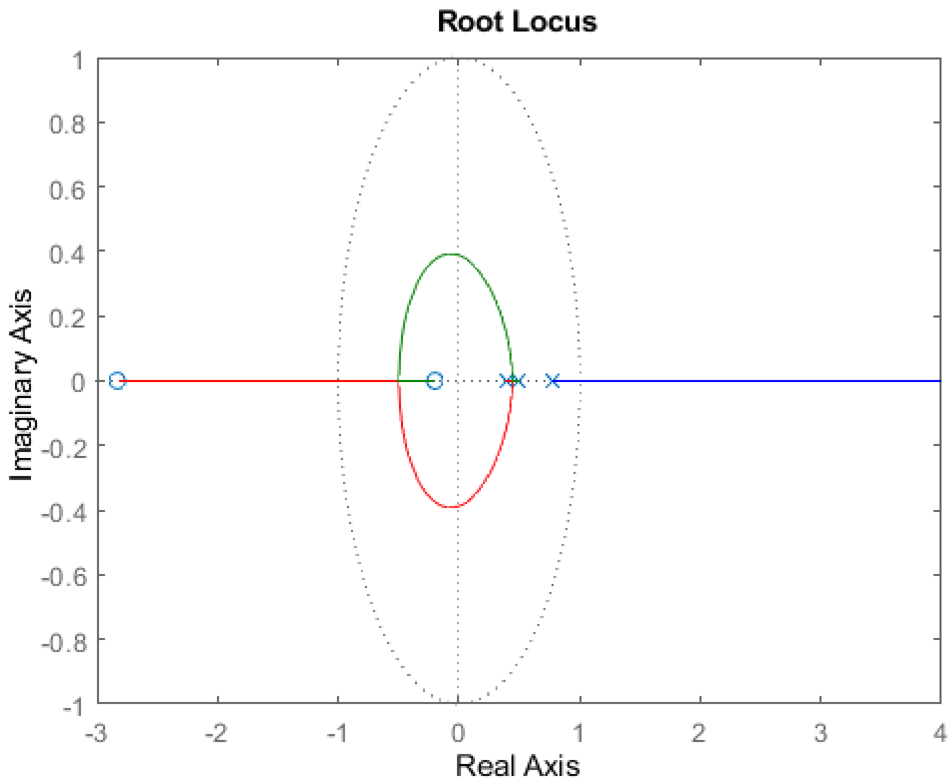



```
[Gm,Pm,Wgm,Wpm] = margin(GC)
```

```
Gm = 4.4548
Pm = 110.0369
Wgm = 0.0740
Wpm = 0.0190
```

Root Locus (Controlled system)

```
rlocus(-GC)
```



DeadBeat controller (p7)

$$G(z) = -6.1431e-04 * \frac{(z+2.8306)(z+0.1967)}{(z-1)(z-0.7788)(z-0.3972)} = a * z^{-1} + \cdots$$

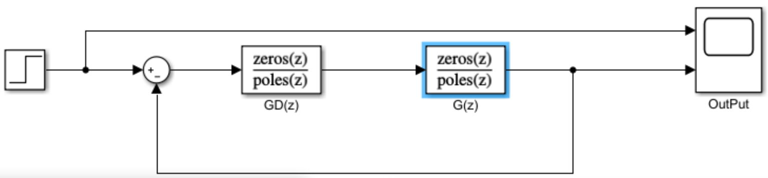
$*$ The First Sentence of G(z) starts with the term $z^{-1} \rightarrow F(z) = f_1 z^{-1} + f_2 z^{-2}$

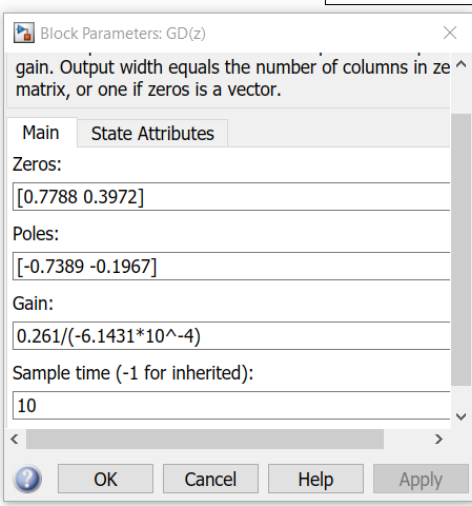
$$\begin{cases} F(z) = (1 + 2.8306z^{-1})(m_1 z^{-1}) \\ 1 - F(z) = (1 - z^{-1})(1 + n_1 z^{-1}) \end{cases} \rightarrow \begin{cases} f_1 + n_1 - 1 = 0 \\ f_2 - n_1 = 0 \\ f_1 - m_1 = 0 \\ f_2 - 2.8306\, m_1 = 0 \end{cases}$$

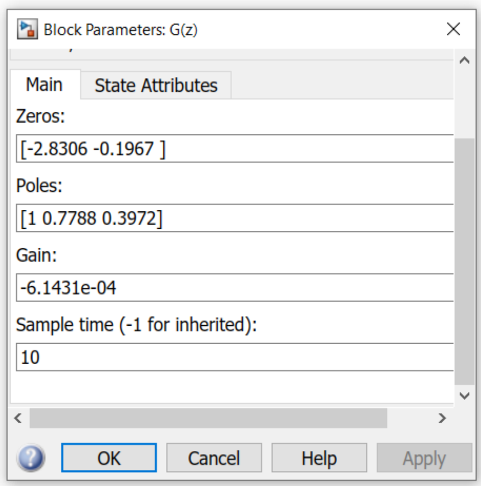
```
Solve[{f1 + n1 - 1 == 0, f2 - n1 == 0, f1 - m1 == 0, f2 - 2.8306 * m1 == 0},
  {f1, n1, f2, m1}]

{{f1 → 0.261056, n1 → 0.738944, f2 → 0.738944, m1 → 0.261056}}
```

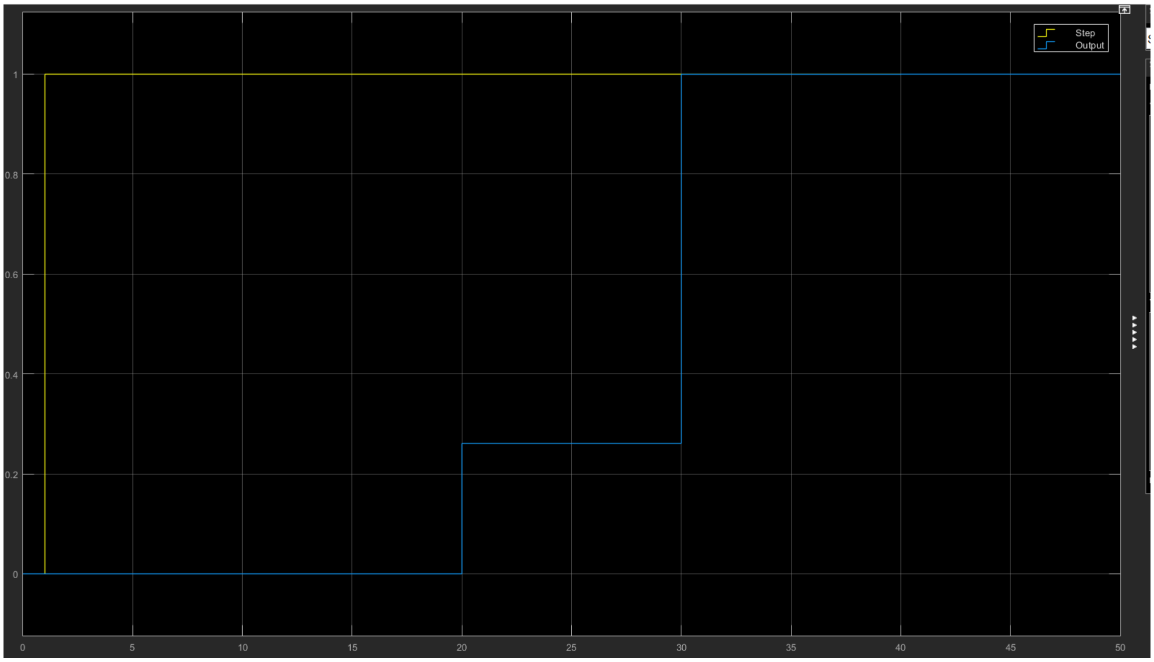
$$\rightarrow F(z) = 0.261z^{-1} + 0.7389z^{-2}$$

$$\rightarrow G_D(z) = \frac{F(z)}{(1 - F(z))G(z)} = \frac{0.261(1 - 0.7788z^{-1})(1 - 0.3972z^{-1})}{-6.1431e - 04(1 + 0.7389z^{-1})(1 + 0.1967z^{-1})}$$









Comparing controllers (p8)

State space (p9)

Discrete State Space

```
Ts = 4;
sys = ss(A,B,C,D)
```

```
sys =

  A =
             x1        x2        x3
   x1         0      -4.5         0
   x2         0    -0.025   1.3e-05
   x3         0         0  -0.09233

  B =
            u1
   x1        0
   x2        0
   x3  0.08333

  C =
       x1  x2  x3
   y1   1   0   0

  D =
       u1
   y1   0

Continuous-time state-space model.
```

```
sys_d = c2d(sys,Ts)
```

```
sys_d =

  A = 
             x1         x2         x3
   x1         1     -17.13  -0.0004014
   x2         0     0.9048   4.125e-05
   x3         0          0      0.6912

  B = 
             u1
   x1  -4.635e-05
   x2   7.434e-06
   x3      0.2787

  C = 
       x1  x2  x3
   y1   1   0   0

  D = 
       u1
   y1   0

Sample time: 4 seconds
Discrete-time state-space model.
```

```
G = sys_d.A; H = sys_d.B;
```

Controllable & Observable

```
n = size(G,1);
M = [];
N = [];
for i = 0:n-1
    M = [M G^i*H];
    N = [N ; C*G^i];
end

M
```

```
M = 3×3
   -0.0000   -0.0003   -0.0007
    0.0000    0.0000    0.0000
    0.2787    0.1926    0.1332
```

```
fprintf('Rank(M):%i\n',rank(M))
```

```
Rank(M):3
```

```
if rank(M) == n
    disp('(G,H) is controllable')
end
```

```
(G,H) is controllable
```

```


N
```

```
N = 3×3
    1.0000         0         0
```

```
    1.0000  -17.1293   -0.0004
    1.0000  -32.6285   -0.0014
```

```
fprintf('Rank(N):%i\n',rank(N))
```

```
Rank(N):3
```

```
if rank(N) == n
    disp('(G,C) is observable')
end
```

```
(G,C) is observable
```

```

% Hankel Matrix
if rank(N*M) == n
    disp('State space is minimal')
end
```

```
State space is minimal
```

DeadBeat controller from the state space (p10)

```
% we can use all of the states
tf_d = tf(sys_d)
```

```
tf_d =

  -4.635e-05 z^2 - 0.0001652 z - 3.666e-05
  ----------------------------------------
     z^3 - 2.596 z^2 + 2.221 z - 0.6254

Sample time: 4 seconds
Discrete-time transfer function.
```

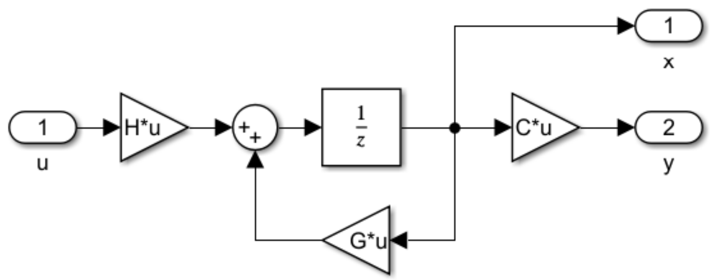
```
[tf_Num,tf_Den] = tfdata(tf_d);
n = size(tf_Den{1},2);
a = zeros(1,n-1);
b = zeros(1,n);
for i = 1:n-1
    a(i) = tf_Den{1}(i+1);
    b(i) = tf_Num{1}(i);
end
W = [];
for i = 0:n-2
    W = [W ; flip(a(1:end-1-i)) 1 zeros(1,i)];
end
T = M*W;
K_deadbeat = -flip(a)*inv(T)
```

K_deadbeat = 1×3

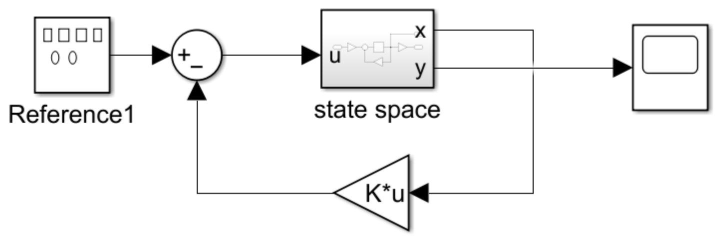
10^5 ×

```
   -0.0403    1.3010    0.0001
```

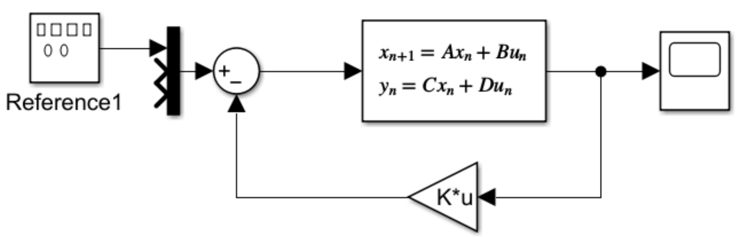
In order to achive the states, we need to implement the system with delay block:



Then simply connect the state feedback controller:



The other way is implement it as discrete state space block and connect the state feedback controller. But we need to set the C matrix equal to eye(3) because we need all the states in the output:



Deadbeat controller with full rank observer (p11)

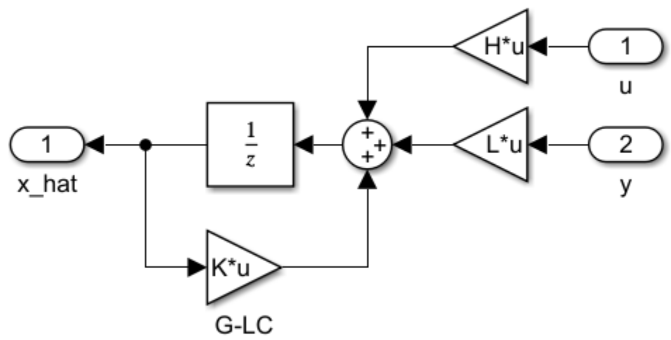
```
% x[k+1] = (G-LC)x[k] + Hu[k] + Ly[k]
G_new = transpose(G);
H_new = transpose(C);
L = transpose(acker(G_new,H_new,zeros(n-1,1)))
```

```
L = 3×1
    2.5960
   -0.0997
 -531.9399
```

$$\dot{\hat{x}} = (A - LC) * \hat{x} + Bu + Ly$$

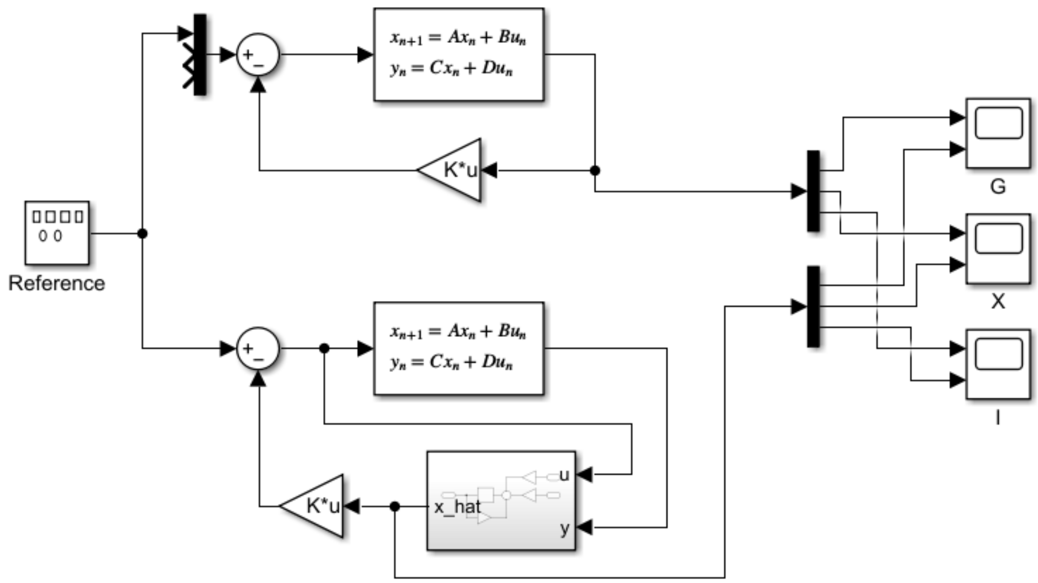
$$\begin{cases} k \to L^T \\ B \to C^T \\ A \to A^T \end{cases} \to \det\big(\lambda I - (A - LC)\big) = \det\left(\lambda I - \left(A^T - L^T C^T\right)\right)$$

So the block of observer is something like this:

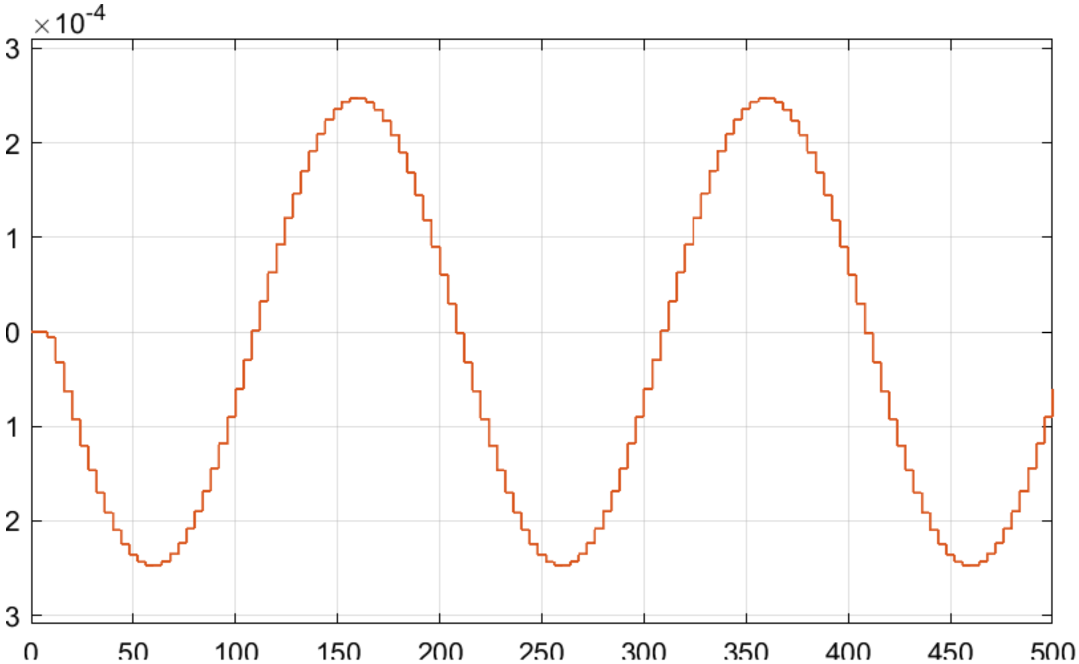


To compare states from system without observer and system with observer we impelement the circuit below.

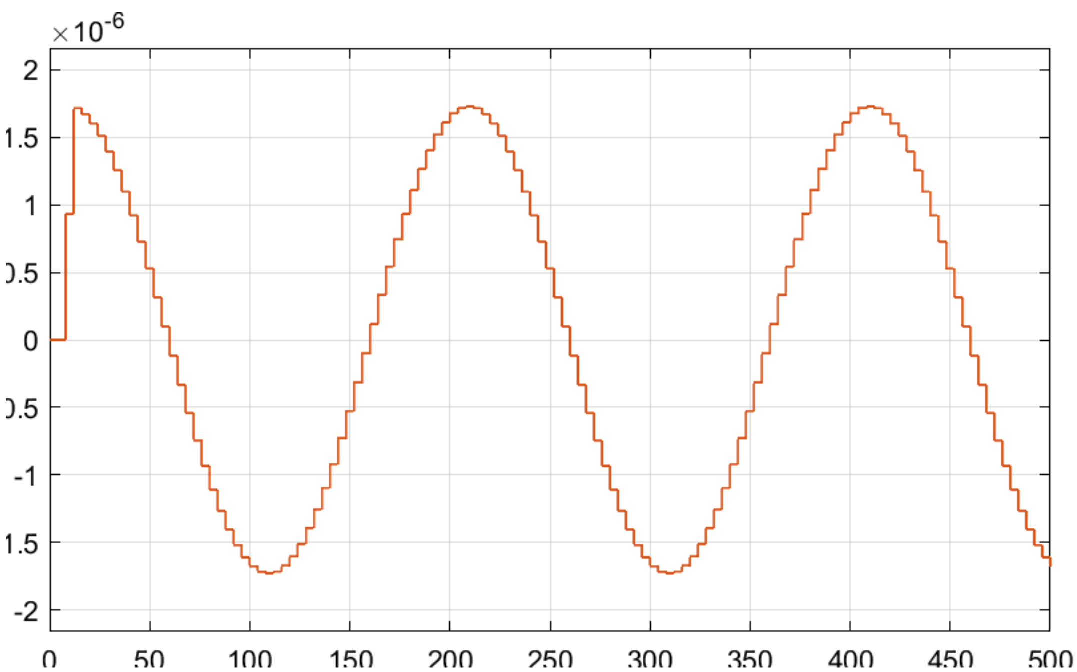
The input is a sinous signal with the frequency of 0.005Hz.



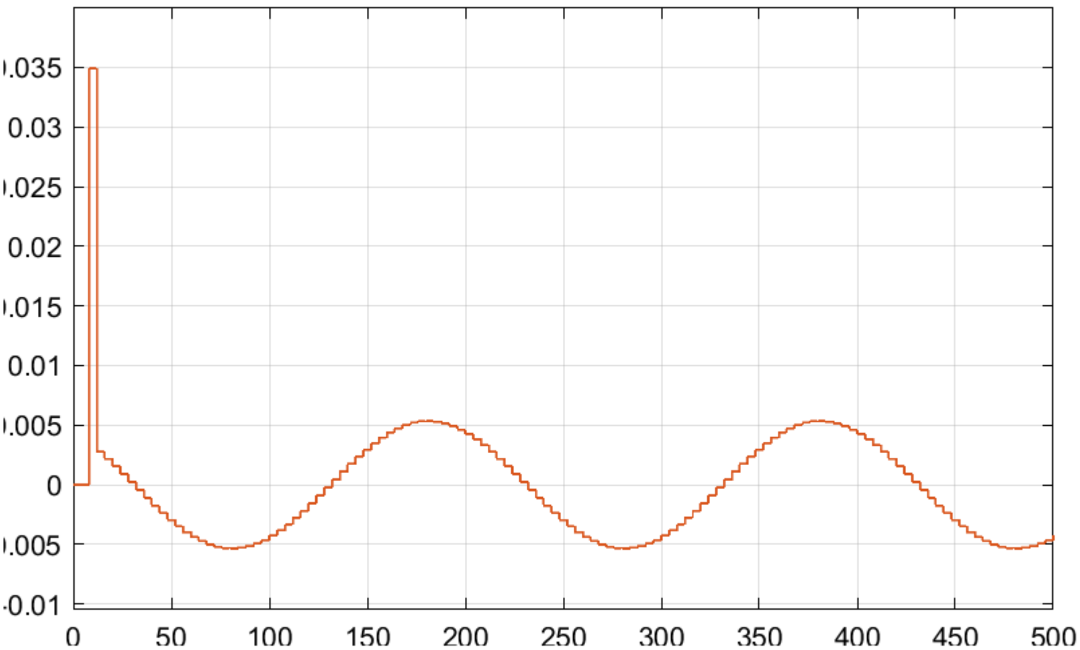
G:


×10^-4
3
2
1
0
1
2
3
0
50
100
150
200
250
300
350
400
450
500

X:


×10^-6
2
1.5
1
0.5
0
0.5
-1
1.5
-2
0
50
100
150
200
250
300
350
400
450
500

I:



As we can see, observer prefectly follows the states. But if we apply the initial conditions, we can see the error between them that converges to zero over the time.

initial condition = 0.1

G: