## ALAFF Midterm-2

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1. a) Cholesky wing Bordered Algorithm

8 tep 1. Derivation

Using notation from 5.5.1.1, lets consider the loop invariant for Cholesky:

(ATL ATR) = ((L)L)TL ATR N LTL LTL = ATL ABL ABR ABR ABL ABR

meaning principal submation ATL has been overwritten with it's Cholesky factors of rest have not leen to uched.

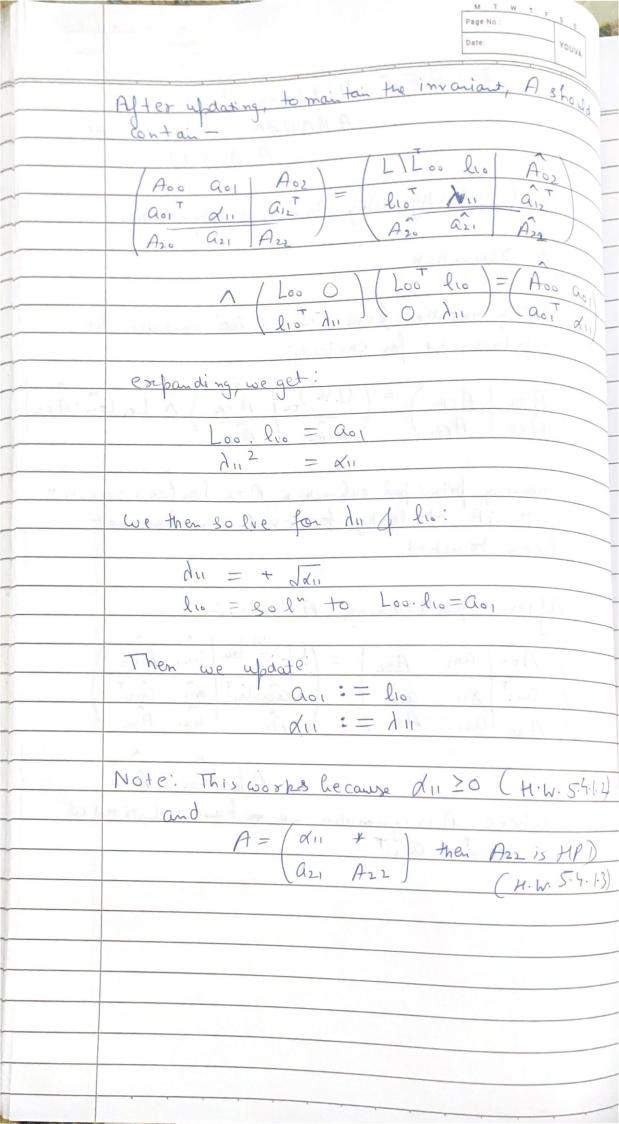
After report troning, A contains

 $\begin{vmatrix} A_{00} & a_{01} & A_{02} \\ a_{01} & A_{11} & a_{12} \end{vmatrix} = \begin{vmatrix} L_{1} & a_{0} & a_{01} & A_{02} \\ a_{01} & A_{11} & a_{12} & a_{01} & a_{01} \end{vmatrix}$   $\begin{vmatrix} A_{20} & a_{21} & A_{22} \\ A_{20} & a_{21} & A_{22} \end{vmatrix} = \begin{vmatrix} A_{20} & a_{21} & A_{22} \\ A_{20} & a_{21} & A_{22} \end{vmatrix}$ 

My Camba summer March Loo Loo = Ac

where, A is egmmetric, we so have substituted

a lead .





	Full algarithm
	AND LOTTED THE STUDY OF THE STU
	A = LL-horeder(A)
	A -> (ATL ATR) ABL ABR
	ATL is OXO
	while n(ATL) < n(A).
31	ATL ATR > / Aoo aoi Aoz
.58	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
	1. Solve Loo.lio = aoi, overwrite aoi  aoi:= &o.lio
	2. $d_{11} = + \int d_{11}$
1. b	Proof: Bordered Cholesky is well defined for an SPD matrix.
	We use induction our above algorithm.  Base Case: if A is size IXI, Ano > 0 & L = JAOI
	$\alpha$
	n: if At step as of the algo, we assume
	ATL = (L/L)TL A LILTE ATL
	tran H.W. S. G. I. )
	Induction step:  From H.W. 5.4.1.2, we know $\alpha_{11} > 0$ (1) => $\lambda_{11} = + \int \alpha_{11}$ is well defined
	Jan 13 well delland

D From H.W. 5.4.1.3, we know that Gorithm

ABA is also MPD, hence the algorithm 3) Also, for solving Loo. 110 = aci we note that Loo has all diag. Elements tre (= JKII I for each step), hence Loo Bis a full rank.

=> Loo. loo = aoi has a definite solt. Given, D, D & B, we see that induction step of algorithm is well defined. flance, proved! The fit stop so at the adique to fit