**Assignment 5 COMP/BMME 576, spring 2018 17 April 2018**
**Rotations, translations, ridges Due Thursday, 26 April 2018**

1. This problem will give you practice in producing a rotated image.

   a. Create a 3D 128 ×128 × 128 image of an ellipsoid along the cardinal axes centered at a center voxel c, which you give as input, and width principal radii r, which you also give as input. . To do this, evaluate the function $(x-c_c)^2/r_x^2 + (y-c_y)^2/r_y^2 + (z-c_z)^2/r_z^2 -1$ at each of the voxel corners, and give that corner the value 1 if the function is non-positive (the corner is inside or on the ellipsoid boundary) and the value 0 if the function is positive (the corner is outside the ellipsoid). The 8 corners of voxel i,j,k are at (i±½, j±½, k±½). Give each voxel an intensity I(i,j,k) which is the sum of the values over its corners [Thus, the value will be 8 if all the voxel's corners are inside the ellipsoid, 4 if half of its corners are inside the ellipsoid, zero if all of its corners re outside the ellipsoid, etc.] Display and turn in the 2D slice of this image with $z=c_z$. Also, use the matlab display program xxx provided to see the boundary of your ellipsoid.
   b. Translate your 3D image so that its origin is a center of rotation g, which you also give as input, and use quaternions to rotate it about an axis and by an angle both given by an input quaternion. [Choose the parameters so that the rotated ellipsoid will stay entirely within the image.] This will give new locations of all of your voxel centers. Write down and pass in the quaternion formula you used to compute these positions.
   c. Use trilinear interpolation (you may use matlab functions) to compute the image intensities in the new pixel locations (in the rotated image). Display the 2D slice of this image with $z=c_z$. Also, use the matlab display program xxx provided to see the boundary of your ellipsoid.
   d. Finally, 1) use the first moment formula for each of x, y, and z to produce the center of mass of your translated and rotated ellipsoid. This formula for the variable x is $d_x = (1/\Sigma_{voxels}\, I(voxel))\ \Sigma_{voxels}\, I(voxel)\ x_{voxel}$; and similarly for y and z. Turn in your computed center of mass d as well as the center error, the 3-tuple d-g. 2) Compute the 3×3 matrix A that gives the 2$^{nd}$ moments about the center of mass, the formula for the i,j$^{th}$ element of the matrix is $(1/\Sigma_{voxels}\, I(voxel))\ \Sigma_{voxels}\, I(voxel)\ (x^i_{voxel}-d_i)(x^j_{voxel}-d_j)$, where $x^i$ is x if i=1, y if i=2, and z if i=3). The eigenvectors of A will be the principal axes of your rotated ellipsoid. Compare these to the axes you expected (the comparison has nothing to turn in), and turn these 3 axes in as unit quaternions. The eigenvalues of A will be proportional to the principal radii of your unit quaternion. Turn in the ratio of the eigenvalues to the actual radii you inputted.

2. Multiple choice, choose as many answers as are correct
   a) A maximum convexity 2D height ridge in h(x,y,σ,θ) [where (x,y) is a location, σ is a spatial scale and θ is a direction] at (x,y,σ,θ) is associated with
      - 1 eigenvector of $D^2h$ having a negative eigenvalue
      - 2 eigenvectors of $D^2h$ having a negative eigenvalue
      - 3 eigenvectors of $D^2h$ having a negative eigenvalue
      - 4 eigenvectors of $D^2h$ having a negative eigenvalue
      - In each eigenvector direction **v** chosen above $\mathbf{v}\bullet\nabla f < 0$

   b) A Canny edge in a 2D image I with height function h
      - Uses $\nabla I / |\nabla I|$ as the direction in which $D^1h$ must be zero
      - Uses $|\nabla I|$ as the height function h
      - Requires the maximum eigenvalue of $D^2h$ to be <0
      - Requires the minimum eigenvalue of $D^2h$ to be < 0
      - Requires $\mathbf{v}^T D^2 h \mathbf{v} < 0$ in the chosen direction **v**

**3. True/False**

- 1D Height ridges in 2D cannot end.
- Maximum convexity curves of $D_{\mathbf{v}}f=0$ cannot end.
- Following maximum convexity curves of $D_{\mathbf{v}}f=0$ is useless for finding additional ridge points
- A 2D Canny edge in a 3D image with height function h will require at least 2 orthogonal directions $\mathbf{v}_1$ and $\mathbf{v}_2$ in which $D_{\mathbf{v}_1}h=0$ and $D_{\mathbf{v}_2}h=0$.