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 Phys 331
 Homework 4

1.

a)

$$g_1(x,y) = \text{Re}[f(x)] = x^3 - 3xy^2 - 1$$

$$g_2(x,y) = \text{Im}[f(x)] = 3iyx^2 - iy^3$$

b)

Jacobian:

$$\begin{bmatrix} 3x^2 - 3y^2 & -6xy \\ 6xy & -3y^2 - 3x^2 \end{bmatrix}$$

Inverse Jacobian:

$$\begin{aligned} & (1/(9x^2 - 9y^2 - 12)) * \\ & \begin{bmatrix} 3 & -2/(xy) \\ 2/(xy) & (y^2 - x^2)/(x^2y^2) \end{bmatrix} \end{aligned}$$

c)

(x1, y1) converges to the root (1.00000260e+00, 1.97334115e-04) which is the root (1,0i)

(x2, y2) converges to the root (-0.50005022, 0.8659397) which is the root (-1/2, sqrt(3)/2)

(x3, y3) converges to the root (-0.50005022, -0.8659397) which is the root (-1/2, -sqrt(3)/2)

They all achieve the desired tolerance after one iteration of updating the values.

This is because the guesses are so close to the values of the roots, and the tolerance is only 10^{-3} .

2.

a)

We need to write the function in the form $f(x)=0$

R = vector of known radii

0 = vector of known angles

$$f(x) = C/(e*\sin(0 + a)) - R = 0$$

The input would be a vector of [C, e, a]

b) In the Code

c) In the Code

3.

a)

At $x = 0$, $f'(x)$ does not exist. This would probably make it difficult to interpolate $f(x)$ at this point.

This statement holds true for all values of x in which $f(x)$ is approaching 0.

b)

The function hits 0 whenever $x = k * (\pi)$ where k is an integer.

However, since our data points are only in the range -10 to 10 with the step size of 0.1, we only see the function equal 0 at $x=0$ (between -10 and 10).

At other places (near $x = k * (\pi)$) the values of $f(x)$ get close to 0, but does not equal 0.

c)

The error of the cubic spline increases as $f(x)=0$.

This is because the cubic spline forces all values to be differentiable, and the "correct" function is not differentiable as $f(x)=0$, so the error becomes larger.