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Physics 331
Hw 9

1a)
The plot has real positive values with a slope that is not steep or sporadic (does not fluctuate much). These qualities will make the numerical integration easier to handle than other graphs that have slopes which are sporadic. However, the point f(0) is undefined in the original function, so this may make the integral harder to compute.

The error tends to decrease as N increases. Both the trapezoidal and Simpson's 3/8ths method have more "predictable" error plots (meaning that they are always constantly decreasing in a beneficial way). This means it is easier to predict when a given N will cause a desired tolerance. Both of the Simpson's Methods seem to clearly have less error than the trapezoidal method. However, the Simpson's 1/3 and 3/8ths methods are comparable in error.

Unless we need a tolerance much smaller than 10^-5, there is no need to increase N, but increasing N will continue to decrease the error.

At n = 100, the Simpson's 1/3 seems to be the most accurate, with the Simpson's 3/8ths as the second most accurate and the trapezoidal method being the least accurate. This shows that even though the Simpson's 3/8ths should theoretically have less error, there are always functions that are exceptions to this rule. (However, at other n values Simpson's 3/8ths is more accurate than Simpson's 1/3).

2a)

$$y = (b-x)^{1-\gamma}$$

$$x = b - y^{\frac{1}{1-\gamma}}$$

$$dx = -\left(\frac{1}{\gamma-1}\right) \cdot y^{\frac{\gamma}{1-\gamma}}\, dy$$

$$x = a \to y = (b-a)^{1-\gamma}$$

$$x = b \to y = (b-b)^{1-\gamma} = 0$$

$$\int_a^b f(x)\,dx = \int_{(b-a)^{1-\gamma}}^{0} -\frac{1}{\gamma-1} \cdot y^{\frac{\gamma}{1-\gamma}} f\left(b - y^{\frac{1}{1-\gamma}}\right) dy$$

$$= \frac{1}{\gamma-1} \int_{(b-a)^{1-\gamma}}^{0} y^{\frac{\gamma}{1-\gamma}} f\left(b - y^{\frac{1}{1-\gamma}}\right) dy$$

Explain

2b)

$$f(x) = \sqrt{1-x^2}$$

$$\gamma = 1/2 \rightarrow \frac{1}{(-1/2)} \int_{(1)^{1/2}}^{0} y^{(1)} f(1-y^2)\,dy$$

$$= -2 \int_{1}^{0} y\, f(1-y^2)\,dy$$

$$= -2 \int_{1}^{0} y \left(y\sqrt{2-y^2}\right) dy$$

$$= -2 \int_{1}^{0} y^2 \sqrt{2-y^2}\,dy$$

$$= 2 \int_{0}^{1} y^2 \sqrt{2-y^2}\,dy$$

You can put the 2 inside the integral our multiply the integral shown by a factor of 2. In my code I chose to multiply the integrand by 2.

Our modified integral (which I put in terms of y instead of x), should be easier to numerically compute. This is because the original integral had many points at which the slope were near either 0 or infinity (which are difficult things to numerically estimate). Even though larger meshes could attempt to solve this issue, it would require a lot more computation time. Our modified integral should be able to achieve more accurate results much faster.

3a)

$$W(x) = \frac{1}{\sqrt{1-x^2}} \qquad F(x) = \sqrt{1-x^2}$$

$$f(x) = \frac{F(x)}{W(x)} = 1-x^2$$

Absassas: $x_1 = 0$, $x_{2,3} = \frac{\pm\sqrt{3}}{2}$

$P_0 = 1$, $P_1 = x$, $P_2 = 2x^2 - 1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -1 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \\ w \\ \\ \end{bmatrix} = \begin{bmatrix} \pi \\ 0 \\ 0 \end{bmatrix}$$

$$w_1 = w_2 = w_3 = \pi/3 \qquad (w = \pi/n)$$

$$\sum_{i=1}^{N} w_i f(x_i) = \sum_{i=1}^{3} \frac{\pi}{3} \cdot (1 - x_i^2)$$

Even though N is only 3, it is still a very good approximation.
This only requires 3 (“N”) function calls. The other methods require as many function calls as the density of the mesh we choose for the domain. We used 10, 30, 100, 300, 1000, 3000, 10000, 30000, 100000 (“N”).

3b)

At N = 10 the solution is accurate to 6 decimal places. (The tolerance is less than 10^-6)
The funcCheb seems to be right in between both the wolfram solution and the scipy solution. However it is slightly closer to the scipy solution.