Armaan Sethi
HW 10
1.
a)
i) y = 1 - abs(x) , on the interval (-1,1)
b)
i) x’ = c * x
ii) x’ = pi*x, c = pi
iii) dx’ = pi*dx
iv)

$x' = cx$, $x' = \pi x$, $c = \pi$

$dx' = \pi dx$

$$A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx = \cancel{\frac{1}{\pi}}\int_{-1}^{1} f(x)\cos(k\pi x)\,dx \cdot \cancel{\pi}$$

$$\boxed{A_k = \int_{-1}^{1} f(x)\cos(k\pi x)\,dx}$$

$B_k = 0$ since the sawtooth function is even and would not need any "sin" terms. However, if it did $B_k = \int_{-1}^{1} f(x)\sin(k\pi x)\,dx$ (which always $= 0$ when $f(x) = 1 - |x|$)

$$\boxed{f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k\cos(k\pi x) + 0}$$

c)
i) A_0 = 1

$$A_0 = \int_{-1}^{1} (1-|x|)\cos(0\pi x)\,dx$$

$$= \int_{-1}^{1} (1-|x|)\,dx = 1 \text{ due to geometry.}$$

$$A_k = \int_{-1}^{1} (1-|x|)\cos(k\pi x)\,dx$$

$$= -2\int_0^1 (x-1)\cos(\pi k x)\,dx$$

$$= -2\int_0^1 \left(x\cos(\pi k x) - \cos(\pi k x)\right) dx$$

$$= -2\int_0^1 x\cos(\pi k x)\,dx + 2\int_0^1 \cos(\pi k x)\,dx$$

$$= \left(\frac{-2x\sin(\pi k x)}{\pi k}\right)\Bigg|_{x=0}^{x=1} = \frac{-2\sin(\pi k)}{\pi k}$$

$$= -\frac{2\sin(\pi k)}{\pi k} + \frac{2}{\pi k}\int_0^1 \sin(\pi k x)\,dx + 2\int_0^1 \cos(\pi k x)\,dx$$

$$= \frac{-2\sin(\pi k)}{\pi k} + \left(\frac{-2\cos(u)}{\pi^2 k^2}\right)\Bigg|_{u=0}^{u=\pi k} + 2\int_0^1 \cos(\pi k x)\,dx$$

$$= \frac{2-2\cos(\pi k)}{\pi^2 k^2} - \frac{2\sin(\pi k)}{\pi k} + 2\int_0^1 \cos(\pi k x)\,dx$$

$$= \frac{2-2\cos(\pi k)}{\pi^2 k^2} - \frac{2\sin(\pi k)}{\pi k} + \frac{2\sin(u)}{\pi k}\Bigg|_{u=0}^{u=\pi k}$$

$$= \frac{2-2\cos(\pi k)}{\pi^2 k^2} - \cancel{\frac{2\sin(\pi k)}{\pi k}} + \cancel{\frac{2\sin(\pi k)}{\pi k}}$$

$$= \frac{2-2\cos(\pi k)}{\pi^2 k^2}$$

ii)

iii) $B_k = 0$ (this makes sense since the function is even, so there will be no sin terms in the Fourier Series) . The integral is symmetrical, so it is equal to 0.

d) In the Code

2.

a)

i) $k_{max} = k_{(n/2)-1} = k_{nyquist} = 1/(2\Delta x)$
ii) So, $2\Delta x * k_{max} = 1$, since $k_{max} = 30$
iii) $\Delta x = 1/60$
iv) $\Delta k = 1/(N\Delta x)$
v) $N = 120$

b) All even Ak values (other than A0) are equal to 0. The ratio of the FFT/Ak then is approximately $-1/\Delta x$ when Ak is odd, and undefined when Ak is even (divide by 0). However as we increase k, we see that this holds less and less true. I believe this is because there is error accumulating in my calculations.

3.

I set the DC = 0

The Nyquist Frequency = 500s

There is a peak at 500s where the frequency is 0.5Hz.

They should be worried if the peak at frequency 0.5Hz can lead to a collapse.

4.

a)

$$y''(x) + (2x+3)y'(x) + 6xy = x$$

$$y''(x) = -(2x+3)y'(x) - 6xy + x$$

$$y_1 = y$$
$$y_2 = y'$$

$$y_1' = y' = y_2$$

$$y_2' = y'' = -(2x+3)y'(x) - 6xy + x$$

$$y_1(0) = y(0) = 1$$
$$y_2(0) = y'(0) = 1$$

$$y' + 2xy = x$$

$$\frac{dy}{dx} = x - 2xy$$

$$dy/dx = x(1-2y)$$

$$\frac{dy/dx}{1-2y(x)} = x$$

$$\int \frac{dy/dx}{1-2y(x)}\,dx = \int x\,dx$$

$$-\tfrac{1}{2}\ln(1-2y(x)) = \frac{x^2}{2} + c$$

$$y(x) = -\tfrac{1}{2}e^{-x^2-2c_1} + \frac{1}{2}$$

Solve for constants:

$$y(x) = \frac{c_1}{e^{x^2}} + \frac{1}{2}, \quad y(0) = 4$$

$$c_1 + 1/2 = 4, \quad c_1 = 7/2$$

$$\boxed{y(x) = \frac{7}{2}e^{-x^2} + \frac{1}{2}}$$

5.

a)

i) As the h value decreases, the mesh becomes finer and the curves become more smooth. When h = 1, the step size is 1 which is extremely large. This is why it is very inaccurate and not smooth (it only samples about times on the graph).

b) The Runge Kutta should be extremely accurate.

6.

$$\frac{d^2\theta}{d\tau^2} = -\sin\theta$$

(from the textbook)

→ $$\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\sin\theta = 0$$ (converted in terms of theta and t), g = 9.8m/s^2, l = 1.0m