PHYS358: Session 01, Group A

Ordinary Differential Equations I

The lunar landing problem is a classic optimization problem: How should the throttle (i.e. the burning rate) of the engine be adjusted such that (a) the lander hits the ground smoothly, and (b) the lander uses as little fuel as possible?

Derive the set of coupled ODEs describing the vertical motion (coordinate z) of a lunar lander of mass $M_{tot} = M_{ship} + M_{fuel}$, under the influence of a constant thrust. The thrust should be parameterized as $T_{max}k$, where T_{max} [N] is the constant maximum thrust, and the throttle $k \in [0, 1]$. The lunar lander will be subject to a constant gravitational acceleration $-g$ [m s^{-2}]. Fuel is burnt at a rate $T_{max}k/V_{nozz}$, where V_{nozz} is the (constant) exhaust velocity.

1. Write down the forces acting on the lander.
2. Derive the ODEs from the force equation, and from the fuel burning rate.
3. Sketch the time evolution of (a) the vertical position z, (b) the vertical velocity v_z, and (c) the fuel mass M_{fuel} for $k = 0$.
4. Sketch z, v_z and M_{fuel} for $k = 1$. Assume that $T_{max}/M_{tot} > g$. *Hint:* Calculate the time at which the lunar lander runs out of fuel.