1)

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Homework #4

4a.
$$P \propto \prod_{i=0}^{n-1} \exp\left(-\frac{1}{2}\left(\frac{y_i - y(x_i: a,b)}{\sigma_i}\right)^2\right)$$

$$\chi^2 = \sum_{i=0}^{n-1}\left(\frac{y_i - y(x_i: a,b)}{\sigma_i}\right)^2,$$

$\Rightarrow P \propto \exp\left(-\frac{1}{2}\chi^2\right)$ since multiplication of exponents of the same base results in sum of the powers.

So, P is maximized when χ^2 is minimized.

4b.
$$0 \equiv \frac{\partial}{\partial a}\chi^2 = \frac{\partial}{\partial a}\sum_{i=0}^{N-1}\left(\frac{y_i - (a+bx_i)}{\sigma_i}\right)^2$$

$$= -2\left(\sum_{i=0}^{N-1}\frac{y_i - a - bx_i}{\sigma_i^2}\right)$$

$$= -2\left(\sum\frac{y_i}{\sigma_i^2} - \sum\frac{a}{\sigma_i^2} - \sum\frac{bx_i}{\sigma_i^2}\right)$$

$$= -2\left(\sum\frac{y_i}{\sigma_i^2} - a\sum\frac{1}{\sigma_i^2} - b\sum\frac{x_i}{\sigma_i^2}\right)$$

$$= \boxed{-2(S_y - aS - bS_x)} = 0$$

$$0 \equiv \frac{\partial}{\partial b}\chi^2 = \frac{\partial}{\partial b}\sum_{i=0}^{N-1}\left(\frac{y_i - (a+bx_i)}{\sigma_i}\right)^2$$

$$= -2\left(\sum_{i=0}^{N-1}\frac{x_iy_i - ax_i - bx_i^2}{\sigma_i^2}\right)$$

$$= -2\left(\sum\frac{x_iy_i}{\sigma_i^2} - \sum\frac{ax_i}{\sigma_i^2} - \sum\frac{bx_i^2}{\sigma_i^2}\right)$$

$$= -2\left(S_{xy} - a\sum\frac{x_i}{\sigma_i^2} - b\sum\frac{x_i^2}{\sigma_i^2}\right)$$

$$= \boxed{-2(S_{xy} - aS_x - bS_{xx})}$$

$$-2(S_y - aS - bS_x) = 0 \quad , \quad -2(S_{xy} - aS_x - bS_{xx}) = 0$$

$$aS = S_y - bS_x \qquad bS_{xx} = S_{xy} - aS_x$$

$$a = \frac{S_y - bS_x}{S} \qquad b = \frac{S_{xy} - aS_x}{S_{xx}}$$

$$a = \frac{S_y - \left(\frac{S_{xy} - aS_x}{S_{xx}}\right)S_x}{S} = \frac{1}{S}\left(S_y - \frac{S_{xy} - aS_x}{S_{xx}}\right)S_x$$

$$a = \frac{S_x S_y}{S} - \frac{(S_{xy} - aS_x)S_x}{S \cdot S_{xx}} \Rightarrow a + \frac{(S_{xy} - aS_x)S_x}{S \cdot S_{xx}} = \frac{S_x S_y}{S}$$

$$a + \frac{S_x S_{xy} - S_x^2 a}{S \cdot S_{xx}} = \frac{S_x S_y}{S} \Rightarrow a\left(S_{xx} - S_x^2\right) = S_{xx}S_y - S_{xy}S_x$$

$$\Rightarrow a(\Omega) = S_{xx}S_y - S_{xy}S_x \Rightarrow \boxed{a = \frac{S_{xx}S_y - S_{xy}S_x}{\Omega}}$$

$$b = \frac{1}{S_{xx}}\left(S_{xy} - \left(\frac{S_y - bS_x}{S}\right)S_x\right) = \frac{1}{S_{xx}}\left(S_{xy} - \frac{S_x S_y - bS_x S_x}{S}\right)$$

$$b = \frac{S_{xy}}{S_{xx}} - \frac{S_{xy} - bS_x S_x}{S} \Rightarrow b\left(S_{xx} - S_x^2\right) = S_{xy} - S_x S_y$$

$$b(\Omega) = S_{xy} - S_x S_y \Rightarrow \boxed{b = \frac{S_{xy} - S_x S_y}{\Omega}}$$

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4c.

$$\sigma_a^2 \equiv \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{\partial a}{\partial y_i}\right)^2 = \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{\partial [S_{xx}S_y - S_{xy}S_y/\Omega]}{\partial y_i}\right)^2$$

$$= \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{\partial}{\partial y_i}\left[\frac{S_{xx}S_y - S_{xy}S_y}{SS_{xx} - S_x^2}\right]\right)^2$$

$$= \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{\partial}{\partial y_i}\left[\frac{S_{xx}S_y}{SS_{xx} - S_x^2}\right] - \frac{\partial}{\partial y_i}\left(\frac{S_{xy}S_y}{SS_{xx} - S_x^2}\right)\right)^2$$

$$= \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{1}{\Omega}\frac{\partial}{\partial y_i}\left(S_{xx}S_y - S_{xy}S_y\right)\right)^2$$

$$= \sum_{i=0}^{N-1} \sigma_i^2 \frac{1}{\Omega}\left(S_{xx}\frac{\partial}{\partial y_i}(S_y) - \frac{\partial}{\partial y_i}S_{xy}S_x\right) = \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{S_{xx} - x_i S_x}{\sigma_i^2 \Omega}\right)^2$$

$$= \sum_{i=0}^{N-1} \frac{1}{\sigma_i^2}\cdot\frac{1}{\Omega^2}\left(S_{xx}^2 - 2x_i S_x S_{xx} + x_i S_x^2\right) =$$

$$= \frac{S_{xx}S_{xx}S - 2S_xS_xS_{xx} + S_xS_xS_{xx}}{\Omega^2} = \frac{S_{xx}\Omega}{\Omega^2} = \boxed{\frac{S_{xx}}{\Omega}}$$

$$\sigma_b^2 = \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{\partial b}{\partial y_i}\right)^2 = \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{\partial}{\partial y}\left[\frac{SS_{xy} - S_xS_y}{\Omega}\right]\right)^2$$

$$= \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{1}{\Omega}\frac{\partial}{\partial y}\left[SS_{xy} - S_xS_y\right]\right)^2$$

$$= \sum_{i=0}^{N-1} \sigma_i^2 \left(\frac{1}{\Omega}\cdot\left(S\left(\frac{x_i}{\sigma_i^2}\right) - S_x\left(\frac{1}{\sigma_i^2}\right)\right)\right)^2$$

$$= \sum_{i=0}^{N-1} \sigma_i^2 \frac{1}{\Omega^2}\left[S\cdot S\cdot\left(\frac{x_i^2}{\sigma_i^2}\right)^2 - 2\cdot S_x\cdot S\left(\frac{x_i^2}{\sigma_i^2}\right)^2 + S_xS_x\left(\frac{1}{\sigma_i^2}\right)^2\right]$$

$$= \frac{S\cdot S\cdot S_{xx} - 2S_xS_xS + S_xS_xS}{\Omega^2} = \frac{S\cdot\Omega}{\Omega^2} = \boxed{\frac{S}{\Omega}}$$

4.d/e

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You will need to view the 10 graphs one at a time, in order (sorry). I have pasted the output into this document as well.

Blue = True Data
Orange = Best fit line

```
###DATA0###
#####LINFIT
a:      -0.0031605977250549647
 b:     1.051843023270421
 siga:   4.09432984719998e-05
 sigb:   7.952557545411861e-05
 chi2:   41.536288233299416
 q:      0.0005404324027341963
#####


#####General Linfit
a:       [-0.00405687  0.99548935  0.13427558]
 sig:    [ 0.0937076   0.253443   -0.0909083  -0.166434   -0.0381431   0.20491
  0.130804    0.0887457  -0.0429324  -0.00859234 -0.0208498   0.0137064
  0.0489929   0.205131    0.0429683  -0.260178   -0.196854   -0.0482093
  0.165664    0.0814919 ]
 chi2:   16.89154464728326
 q:      0.5531272849976303
#####



###DATA1###
#####LINFIT
a:      0.00953181388556922
 b:     1.0740330155108173
 siga:   3.4929292013424885e-05
 sigb:   0.00014792688975562594
 chi2:   24.980744863030473
 q:      0.15831010348882107
#####


#####General Linfit
a:       [0.00933038 0.99684178 0.13569248]
 sig:    [-0.506573    0.214851   -0.0707153   0.0983307  -0.0442056   0.0948023
```

```
 -0.0890501   0.156709   -0.0305836   0.00815078  0.0105219   0.0218425
  0.070087    0.0866633   0.0687747  -0.140401   -0.311715    0.159044
 -0.280356   -0.710825  ]
 chi2:    15.292644038569957
 q:       0.6619388366317642
#####



###DATA2###
#####LINFIT
a:       0.018573325003274705
 b:      1.0342771491918485
 siga:    3.030220722867111e-05
 sigb:    6.288152626929635e-05
 chi2:    27.813438443920116
 q:       0.07771154349275367
#####


#####General Linfit
a:        [0.00727229 0.97695261 0.12789543]
 sig:     [ 0.247624    0.337749    0.0323221   0.0878533   0.170869   -0.142816
  0.0653684   0.158299   -0.0540792   0.0142778   0.00644936  0.0783494
 -0.033432   -0.15866     0.020871    0.214475   -0.0457534   0.167299
 -0.225609   -0.272143  ]
 chi2:    15.356819948877016
 q:       0.6576873515461921
#####



###DATA3###
#####LINFIT
a:       0.005266135952123236
 b:      1.0266465228776975
 siga:    2.618718324459102e-05
 sigb:    1.2004594627002148e-05
 chi2:    24.63472970823219
 q:       0.17128616640248745
#####
```

```
#####General Linfit
a:      [-0.00230146  1.00906066  0.07822127]
 sig:     [-0.313958    0.242194    0.0689157  -0.57637    -0.15612    -0.31869
 -0.145559   -0.0489576   0.149756    0.0152981  -0.00533004 -0.0461422
 -0.0581547  -0.0342307   0.169297    0.0353678  -0.157182    0.00680665
 -0.4512      0.338566  ]
 chi2:    18.028183974349385
 q:      0.4749776325420444
#####
```

```
###DATA4###
#####LINFIT
a:      -0.006755284998213365
 b:      1.0505186275336869
 siga:    4.4012959776661396e-06
 sigb:    6.684786611615563e-05
 chi2:    66.4685042141113
 q:      4.360827679896626e-10
#####
```

```
#####General Linfit
a:      [0.00182058 0.98572013 0.13779897]
 sig:     [ 0.0461196   0.0714196  -0.123673   -0.150672    0.184139   -0.0354771
  0.0554571   0.127051   -0.0419698  -0.00193617  0.00523097 -0.0450947
  0.0656087   0.0826055   0.0373158  -0.116787    0.284701   -0.238286
 -0.222376    0.423417  ]
 chi2:    15.696391474454115
 q:      0.6349738963650517
#####
```

The general linfit seems to do substantially better than the linear regression. The q values in the linear regression are generally much smaller than the q values of the general linfit. Additionally, the q values are much less consistent with the linear regression ranging from 4.360827679896626e-10 (unacceptable) to 0.17128616640248745 (acceptable). The general linfit has much higher q values that are also much more consistent (around 0.5-0.6).