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# HW #3

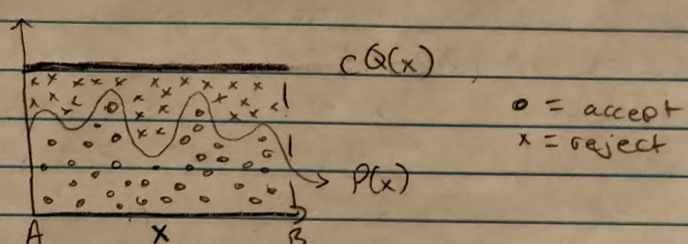
3a $f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\mu = 0$
$\sigma^2 = 1 \Rightarrow f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

$\int_{-1}^{1} f(x)\,dx = 0.682689 \Rightarrow$ so $\approx 68\%$ of samples exist between -1 and 1

$\int_{-2}^{2} f(x)\,dx = 0.9545 \approx$ so $\approx 95\%$ of all samples exist between -2 and 2

3b. $Q(x)$ is constant so




For random values for $x$, as $N_{tot} \to \infty$, $\frac{N_R}{N_{tot}} \to$ points under $P(x)$ curve,

So as all values of $x$ become explored, $R \to \infty$, $\frac{N_R}{N_{tot}}$ = area under the curve

$\Rightarrow \frac{\text{Points Under Curve}}{\text{Points Generated}} \times$ "box" area (when $Q(x)$ is constant) $= \lim_{n\to\infty} \int_A^B P(x)\,dx$

3c.

| #Accepted | P(x) | Mean 1 | Variance 1 | Mean 2 | Variance 2 | Mean 3 | Variance 3 |
|---|---|---|---|---|---|---|---|
| 100 | Normal | -1.339e-1 | 1.065e0 | -1.686e-1 | 7.339e-1 | -1.416e-1 | 7.060e-1 |
| 1000 | | 1.983e-3 | 9.579e-1 | -1.821e-2 | 1.038e0 | 4.104e-3 | 1.076e0 |
| 10000 | | 5.455e-3 | 9.848e-1 | 5.618e-3 | 1.008e0 | -3.232e-3 | 9.793e-1 |
| 100 | Exponential | 1.148e0 | 7.146e-1 | 1.006e0 | 9.118e-1 | 9.400e-1 | 1.068e0 |
| 1000 | | 1.018e0 | 9.026e-1 | 1.006e0 | 8.746e-1 | 1.029e0 | 8.629e-1 |
| 10000 | | 1.012e0 | 8.939e-1 | 9.971e-1 | 9.356e-1 | 1.011e0 | 8.984e-1 |

(3 trials)

No, they do not significantly change, but when R is small the values for the mean & variance vary more.

3d

$$CDF = C(x) = \int_{-\infty}^{x} P(x')\,dx' = \int_{-\infty}^{x} \frac{1}{\pi(1+x^2)}\,dx = \frac{1}{\pi}\tan^{-1}(x)\Big|_{-\infty}^{x}$$

$$= \frac{1}{\pi}\tan^{-1}(x) - \left(\frac{1}{\pi}\left(-\pi/2\right)\right) = \frac{1}{\pi}\tan^{-1}(x) + \frac{1}{2} \Rightarrow \boxed{C(x) = \frac{1}{\pi}\tan^{-1}(x) + \frac{1}{2}}$$

Now we need the inverse of $C(x) \Rightarrow u = \frac{1}{\pi}\tan^{-1}(x) + 1/2$

$$\Rightarrow \pi\left(u - \tfrac{1}{2}\right) = \tan^{-1}(x) \Rightarrow \boxed{x_Q = \tan\left(\pi\left(u - \tfrac{1}{2}\right)\right)}$$

3e.

| #Accepted | P(x) | Proposal | Ratio | Mean | Variance |
|---|---|---|---|---|---|
| 100 | Normal | Uniform | 0.2531 | 0.04721 | 1.0675 |
| 1000 | | | 0.3315 | -0.01478 | 1.0642 |
| 10000 | | | 0.3172 | -0.00797 | 1.0234 |
| 10000 | Normal | Cauchy | 0.65694 | 0.00604 | 1.0058 |
| 1000 | | | 0.65876 | 0.03079 | 0.9515 |
| 100 | | | 0.6993 | 0.0145 | 1.0451 |

The mean and variance are similar to 3c, but the ratio is much better with cauchy.