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HW #2

2a.

$$y''(x) + (\lambda\pi)^2 y(x) = 0 \quad , \quad 0 \le x \le 1 \, , \; y(0) = 0 \, , \; y(1) = 0$$

$$\mathcal{L}[f(x)](s) = \int_0^\infty f(x) e^{-sx} \, dx$$

$$\mathcal{L}[y''(x) + \pi^2\lambda^2 y(x)](s) = \mathcal{L}_x[0](s)$$

$$\mathcal{L}[y''(x)](s) + \pi^2\lambda^2 \left(\mathcal{L}[y(x)](s)\right) = \mathcal{L}[0](s)$$

$$s^2 \mathcal{L}[y(x)](s) - sy(0) - y'(0) + \pi^2\lambda^2 \left(\mathcal{L}[y(x)](s)\right) = 0$$

$$(s^2 + \pi^2\lambda^2)\left(\mathcal{L}[y(x)](s)\right) - sy(0) - y'(0) = 0$$

$$(s^2 + \pi^2\lambda^2)\left(\mathcal{L}[y(x)](s)\right) = sy(0) + y'(0)$$

$$\mathcal{L}[y(x)](s) = \frac{sy(0) + y'(0)}{s^2 + \pi^2\lambda^2}$$

$$\boxed{\mathcal{L}[y(x)](s) = \frac{sy(0)}{s^2 + \pi^2\lambda^2} + \frac{y'(0)}{s^2 + \pi^2\lambda^2}}$$

apply initial conditions

$$\boxed{\mathcal{L}[y(x)](s) = \frac{a}{s^2 + \pi^2\lambda^2}}$$

2b.

Using Wolfram Alpha: $\boxed{\mathcal{L}_s^{-1}\left[\frac{a}{s^2 + \lambda^2\pi^2}\right](x) = \frac{a\sin(\pi\lambda x)}{\pi\lambda}}$

$a = \pi\lambda$, so that Amplitude = 1

$y[0] \Rightarrow \boxed{y(x) = \sin(\pi\lambda x)}$

$y[2] = \lambda$

$y[1] \Rightarrow y'(x) = \pi\lambda\cos(\pi\lambda x)$

dydx[0] = $y' = y[1]$

dydx[1] = y''

~~dydx[2] = $\lambda' = 0$~~

$$y''(x) = -\pi^2\lambda^2 \, y(x)$$