PHYS358: Session 01, Group C

Ordinary Differential Equations I

Chemical reaction networks are classic examples for a numerically very challenging ODE integration: reaction rates often span many orders of magnitude, with the resulting ODE system being very "stiff" (i.e. while one quantity will not change within an hour, another quantity may change within a fraction of a second). Here, we develop a reaction network for molecular hydrogen formation in interstellar gas.

The reactions are

$$\begin{aligned} A + A &\rightarrow B \text{ } H_2 \text{ formation on grains, rate } k_1 \\ B + \gamma &\rightarrow 2A \text{ photodissociation, rate } k_2 \\ A + \gamma &\rightarrow C + e \text{ photoionization, rate } k_3 \\ C + e &\rightarrow A + \gamma \text{ recombination, rate } k_4, \end{aligned}$$

where A, B, C stand for atomic, molecular, and ionized hydrogen (H, H_2, H^+). We assume charge neutrality, i.e the abundances of electrons is that of ionized hydrogen.

1. Write down the ODEs describing the above reaction network. You will need three ODEs for the three species abundances A, B, C. As a rule, "summations" on the left side ("reactants") translate into multiplications, for example $A + A$ translates into $k_1 A^2$.
2. Predict what will happen for $t \rightarrow \infty$.