Road Map: PHYS 521, Fall 2018

Exact solutions of Schroedinger Equation exist only for idealized systems: to solve more complex problems we must resort to approximations

Perturbation Theory
(stationary states)
Problem at hand only slightly different from one that can be solved exactly

Variation Method
for lowest energy states, if form of the wave function is only qualitatively known

WKB method
useful for finding solutions of systems for which the classical limit is valid

Perturbation Theory
(time-dependent)
if Hamiltonian depends on time

TIME - INDEPENDENT PERTURBATION THEORY

[CH. 6 GRIFFITHS]

NON - DEGENERATE PERTURBATION THEORY

UNPERTURBED SOLUTIONS OF TIME - INDEPENDENT SCHRÖDINGER EQUATION FOR SOME POTENTIAL:

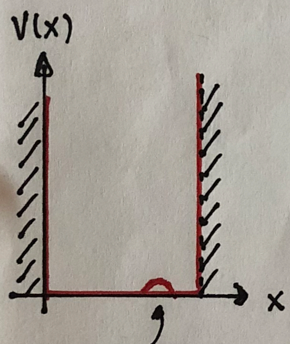
$$H^{(0)} \psi_n^{(0)} = E_n^{(0)} \psi_n^{(0)}$$

EIGENVALUES

COMPLETE SET OF ORTHONORMAL FUNCTIONS

$$\int \psi_n^{(0)}(x)^* \psi_m^{(0)}(x)\, dx = \delta_{nm}$$

INFINITE SQUARE WELL:



SMALL PERTURBATION

INCLUDE NOW "SMALL PERTURBATION" AND SOLVE

$$H \psi_n = E \psi_n$$

- IN GENERAL, EXACT SOLUTION CANNOT BE OBTAINED ANALYTICALLY
- HOWEVER, WE CAN FIND APPROXIMATE SOLUTIONS BY SOMEHOW USING EXACT SOLUTIONS
  [WE DO THIS ALL THE TIME IN RESEARCH]

START WITH : $$H = H^{(0)} + \lambda H'$$

$\lambda = 0$: UNPERTURBED

$\lambda = 1$: PERTURBATION "ON"

NOW APPLY "λ- TRICK" : λ IS SMALL AND ARBITRARY

$$H = H^{(0)} + \underbrace{\lambda H'}_{\text{PERTURBATION}}$$

$$= 5000 + 1 \cdot 10$$

$$= 5000 + 0.01 \cdot 1000$$

$$= 5000 + \underbrace{0.001}_{\lambda} \cdot 10{,}000$$

WE CAN NOW WRITE ψ_n AND E_n AS POWER SERIES IN λ AND SEE IF SERIES CAN BE TRUNCATED AFTER A FEW TERMS

ANSATZ:

$$\psi_n = \psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + \ldots$$

$$E_n = E_n^{(0)} + \underbrace{\lambda E_n^{(1)}}_{\text{FIRST-ORDER CORRECTION}} + \underbrace{\lambda^2 E_n^{(2)}}_{\text{SECOND-ORDER CORRECTION}} + \ldots$$

e.g.

$$E_n^{(1)} = \left[\frac{dE_n}{d\lambda}\right]_{\lambda=0}$$

$$E_n^{(2)} = \frac{1}{2!}\left[\frac{d^2E_n}{d\lambda^2}\right]_{\lambda=0}$$

[λ ALLOWS US TO KEEP TRACK OF PERTURBATION IN POWER SERIES SOLUTION]

$$H \psi_n = E_n \psi_n$$

$$[H^{(0)} + \lambda H'][\psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + \ldots]$$

$$= [E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ldots][\psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + \ldots]$$

COLLECT POWERS OF λ

$$H^{(0)} \psi_n^{(0)} + \lambda [H^{(0)} \psi_n^{(1)} + H' \psi_n^{(0)}] + \lambda^2 [H^{(0)} \psi_n^{(2)} + H' \psi_n^{(1)}] + \ldots$$

$$= E_n^{(0)} \psi_n^{(0)} + \lambda [E_n^{(0)} \psi_n^{(1)} + E_n^{(1)} \psi_n^{(0)}] + \lambda^2 [E_n^{(0)} \psi_n^{(2)} + E_n^{(1)} \psi_n^{(1)} + E_n^{(2)} \psi_n^{(0)}] + \ldots$$

$a + \lambda b = c + \lambda d$

$\lambda = 0$: $a = c$; $\lambda \neq 0$: $b = d$

λ IS SMALL, BUT ARBITRARY; EQUATION MUST BE FULLFILLED FOR COEFFICIENTS OF EACH POWER IN λ SEPARATELY

YIELDS 3 EQUATIONS:

ZERO ORDER $\quad H^{(0)} \psi_n^{(0)} = E_n^{(0)} \psi_n^{(0)}$ $\quad$ UNPERTURBED

FIRST ORDER $\quad H^{(0)} \psi_n^{(1)} + H' \psi_n^{(0)} = E_n^{(0)} \psi_n^{(1)} + E_n^{(1)} \psi_n^{(0)}$

SECOND ORDER $\quad H^{(0)} \psi_n^{(2)} + H' \psi_n^{(1)} = E_n^{(0)} \psi_n^{(2)} + E_n^{(1)} \psi_n^{(1)} + E_n^{(2)} \psi_n^{(0)}$

[ADDING THESE 3 EQUATIONS GIVES PRECEDING EQUATION]

WE ARE DONE WITH λ

FIRST - ORDER THEORY

$$H^{(0)} \psi_n^{(1)} + H' \psi_n^{(0)} = E_n^{(0)} \psi_n^{(1)} + E_n^{(1)} \psi_n^{(0)}$$

REVIEW

RECALL DIRAC BRACKET NOTATION :

$$\langle \psi_1 | \psi_2 \rangle \equiv \int \psi_1^*(\vec{r})\, \psi_2(\vec{r})\, d\vec{r}$$

$$\langle \psi_1 | \psi_2 \rangle = \langle \psi_2 | \psi_1 \rangle^*$$

$$\langle \psi_1 | (\hat{A} \psi_2) \rangle \equiv \langle \psi_1 | \hat{A} | \psi_2 \rangle$$

RECALL HERMETIAN OPERATORS :

$$\langle \psi_1 | \hat{A} \psi_2 \rangle \overset{!}{=} \langle \hat{A} \psi_1 | \psi_2 \rangle$$

$$= \langle \psi_2 | \hat{A} \psi_1 \rangle^*$$

- THEIR EXPECTATION VALUES ARE REAL
- REPRESENT OBSERVABLES ; SEC. 3.2

REWRITE :

$$[H^{(0)} - E_n^{(0)}]\,\psi_n^{(1)} = -[H' - E_n^{(1)}]\,\psi_n^{(0)}$$

ANSATZ : $\boxed{\psi_n^{(1)} = \sum_m c_{nm}^{(1)}\,\psi_m^{(0)}}$

FORM COMPLETE SET OF ORTHONORMAL EIGENFUNCTIONS

SUM OVER DISCRETE PART, INTEGRATION OVER CONTINUOUS PART

$$\psi_n = \psi_n^{(0)} + \sum_m c_{nm}^{(1)}\,\psi_m^{(0)}$$

$$H^{(0)}\psi_n^{(0)} = E_n^{(0)}\psi_n^{(0)}$$

$$\langle \psi_\ell^{(0)} | \psi_m^{(0)} \rangle = \delta_{\ell m} \qquad \text{"ORTHONORMAL"}$$

SUBSTITUTE :

$$[H^{(0)} - E_n^{(0)}]\sum_m c_{nm}^{(1)}\,\psi_m^{(0)} + [H' - E_n^{(1)}]\,\psi_n^{(0)} = 0$$

PRE-MULTIPLY BY $\psi_\ell^{(0)*}$ AND INTEGRATE :

$$\underbrace{\langle \psi_\ell^{(0)} | H^{(0)} | \sum_m c_{nm}^{(1)}\psi_m^{(0)} \rangle}_{\langle \psi_\ell^{(0)} | \sum_m E_m^{(0)} c_{nm}^{(1)}\psi_m^{(0)} \rangle = E_\ell^{(0)} c_{n\ell}^{(1)}} - \underbrace{\langle \psi_\ell^{(0)} | E_n^{(0)} | \sum_m c_{nm}^{(1)}\psi_m^{(0)} \rangle}_{E_n^{(0)} c_{n\ell}^{(1)}}$$

$$+ \langle \psi_\ell^{(0)} | H' | \psi_n^{(0)} \rangle - \underbrace{\langle \psi_\ell^{(0)} | E_n^{(1)} | \psi_n^{(0)} \rangle}_{E_n^{(1)}\delta_{\ell n}} = 0$$

$$c_{n\ell}^{(1)}\left[E_\ell^{(0)} - E_n^{(0)}\right] + \langle \psi_\ell^{(0)} | H' | \psi_n^{(0)} \rangle - E_n^{(1)} \delta_{\ell n} = 0$$

FOR $\ell = n$:

$$\boxed{E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle}$$

IMPORTANT: FIRST-ORDER CORRECTION TO ENERGY IS EQUAL TO EXPECTATION VALUE OF PERTURBATION H' IN UNPERTURBED STATE; EXPANSION COEFFICIENTS $c_{nm}^{(1)}$ DO NOT ENTER IN EXPRESSION

$$E_n = E_n^{(0)} + \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$$

FOR $\ell \neq n$:

$$c_{n\ell}^{(1)} = \frac{\langle \psi_\ell^{(0)} | H' | \psi_n^{(0)} \rangle}{E_n^{(0)} - E_\ell^{(0)}}$$

SWITCH $\ell \to m$

?

$$\boxed{\psi_n^{(1)} = \sum_m c_{nm}^{(1)} \psi_m^{(0)} = \sum_{m \neq n} \frac{\langle \psi_m^{(0)} | H' | \psi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} \psi_m^{(0)}}$$

IMPORTANT: Σ SPECIFIES HOW MUCH EACH UNPERTURBED EIGEN FUNCTION $\psi_m^{(0)}$ IS MIXED IN WITH DOMINANT UNPERTURBED EIGENFUNCTION $\psi_n^{(0)}$ TO FORM PERTURBED EIGENFUNCTION ψ_n

$$\psi_n = \psi_n^{(0)} + \sum_{m \neq n} \frac{\langle \psi_m^{(0)} | H' | \psi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} \psi_m^{(0)}$$

THE ABOVE EQUATION GIVES NO INFORMATION ON $c_{nn}^{(1)}$!

THE COEFFICIENT $c_{nn}^{(1)}$ CAN BE FOUND FROM THE NORMALIZATION [IN FIRST ORDER OF λ] OF PERTURBED SOLUTION ψ_n :

$$\underbrace{\langle \psi_n | \psi_n \rangle}_{=1} = \langle \psi_n^{(0)} + \lambda \psi_n^{(1)} | \psi_n^{(0)} + \lambda \psi_n^{(1)} \rangle$$

$$= \underbrace{\langle \psi_n^{(0)} | \psi_n^{(0)} \rangle}_{=1} + \lambda \left[\langle \psi_n^{(1)} | \psi_n^{(0)} \rangle + \langle \psi_n^{(0)} | \psi_n^{(1)} \rangle \right] + \mathcal{O}(\lambda^2)$$

$$\underbrace{\langle \psi_n^{(1)} | \psi_n^{(0)} \rangle}_{\langle \sum_m c_{nm}^{(1)} \psi_m^{(0)} | \psi_n^{(0)} \rangle = c_{nn}^{(1)}} + \underbrace{\langle \psi_n^{(0)} | \psi_n^{(1)} \rangle}_{\langle \psi_n^{(0)} | \sum_m c_{nm}^{(1)} \psi_m^{(0)} \rangle = c_{nn}^{(1)}} = 0$$

$$\Rightarrow \quad c_{nn}^{(1)} = 0$$

COMMENTS:

- SUFFICIENT CONDITIONS FOR APPLICABILITY OF FIRST-ORDER APPROXIMATION:

$$c_{nm}^{(1)} \ll 1 \quad \text{OR} \quad |\langle \psi_m^{(0)} | H' | \psi_n^{(0)} \rangle| \ll |E_n^{(0)} - E_m^{(0)}|$$

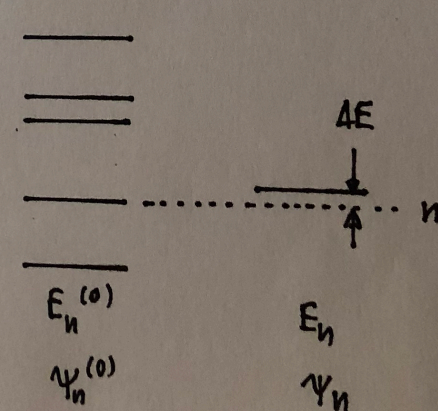
[REMEMBER: PERTURBED AND UNPERTURBED WAVEFUNCTIONS AND ENERGIES DIFFER ONLY SLIGHTLY]

- EASIER RULE OF THUMB: $E_n^{(1)} \ll E_n^{(0)}$

- WAVEFUNCTIONS OF ALL OTHER UNPERTURBED STATES HAVE TO BE KNOWN TO CALCULATE PERTURBED WAVE FUNCTION OF nth STATE

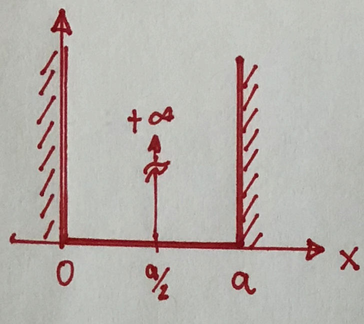
- INFLUENCE OF OTHER WAVE FUNCTIONS DECREASES AS ENERGY SEPARATION INCREASES

- RESULT ONLY APPLIES TO UNPERTURBED ENERGY SPECTRUM THAT IS NON-DEGENERATE! SEE LATER FOR $E_n^{(0)} = E_m^{(0)}$



IN - CLASS EXAMPLE

δ - FUNCTION BUMP IN CENTER OF INFINITE SQUARE WELL



$$H' = \alpha\,\delta\left(x - \frac{a}{2}\right)$$

SOME CONSTANT

PROB. 6.1 IN GRIFFITHS, P.254

(i) FIND FIRST - ORDER ENERGY CORRECTIONS FOR ODD AND EVEN n

(ii) FIND FIRST THREE NON-ZERO TERMS IN EXPANSION $\psi^{(1)}_{n=1}$ FOR THE GROUND STATE [JUST COMPUTE $\langle \psi_m^{(0)} | H' | \psi_1^{(0)} \rangle$]

HELP :

(1) DIRAC δ - FUNCTION

USE MAKES PHYSICAL SENSE ONLY INSIDE INTEGRAL

$$\delta(x) = \begin{cases} +\infty & , x = 0 \\ 0 & , x \neq 0 \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(x)\,dx = 1 \quad ; \quad \int_{-\infty}^{+\infty} f(x)\,\delta(x-c)\,dx = f(c)$$

UNITS:

$\delta(x - \frac{a}{2})$: $1/\text{LENGTH}$

α : ENERGY $\times$ LENGTH

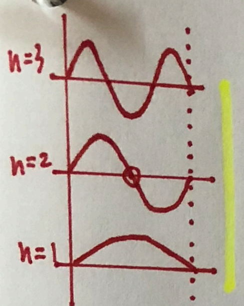
(2) UNPERTURBED SOLUTIONS OF INFINITE SQUARE WELL

$$E_n^{(0)} = \frac{n^2 \pi^2 \hbar^2}{2 M a^2}$$

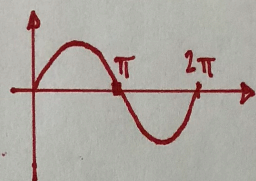
$n = 1, 2, 3, \ldots$

$$\psi_n^{(0)}(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi}{a} x\right)$$

(i) $$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle = \frac{2}{a}\alpha \int_0^a \sin^2\left(\frac{n\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)dx$$



$$= \frac{2}{a}\alpha \sin^2\left(\frac{n\pi}{a}\frac{a}{2}\right) = \frac{2\alpha}{a}\sin^2\left(\frac{n\pi}{2}\right) = \begin{cases} \emptyset & \text{IF } n = \text{EVEN} \\ \frac{2\alpha}{a} & \text{IF } n = \text{ODD} \end{cases}$$



ENERGIES ARE NOT PERTURBED FOR $n =$ EVEN; [UNPERTURBED] WAVE FUNCTION IS ZERO AT LOCATION OF PERTURBATION [$x = a/2$]; IT NEVER FEELS H'

(ii) $$\psi_1^{(1)} = \sum_{m\neq 1} \frac{\langle \psi_m^{(0)} | H' | \psi_1^{(0)} \rangle}{E_1^{(0)} - E_m^{(0)}} \psi_m^{(0)}$$

g.s.

$$\langle \psi_m^{(0)} | H' | \psi_1^{(0)} \rangle = \frac{2}{a}\alpha \int_{-\infty}^{+\infty} \sin\left(\frac{m\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)\sin\left(\frac{\pi}{a}x\right)dx$$

$$= \frac{2}{a}\alpha \sin\left(\frac{m\pi}{a}\frac{a}{2}\right)\underbrace{\sin\left(\frac{\pi}{a}\frac{a}{2}\right)}_{=1} = \frac{2\alpha}{a}\sin\left(\frac{m\pi}{2}\right) = \begin{cases} \emptyset & , m = \text{EVEN} \\ \neq 0 & , m = \text{ODD} \end{cases}$$

$[m \neq n = 1]$

FIRST THREE NON-ZERO TERMS WILL BE $m = 3, 5, 7$

⇩ HERE

FURTHERMORE:

$$E_1^{(0)} - E_m^{(0)} = \frac{\pi^2 \hbar^2}{2Ma^2} - \frac{m^2 \pi^2 \hbar^2}{2Ma^2} = \frac{\pi^2 \hbar^2}{2Ma^2}(1-m^2)$$

$$\psi_1^{(1)} = \sum_{m=3,5,7,\ldots} \frac{\frac{2\alpha}{a} \sin\left(\frac{m\pi}{2}\right)}{\frac{\hbar^2 \pi^2}{2Ma^2}(1-m^2)} \psi_m^{(0)}$$

$$= \frac{2}{a}\alpha \frac{2Ma^2}{\pi^2 \hbar^2}\left[\psi_3^{(0)} \frac{\overbrace{\sin\left(\frac{3}{2}\pi\right)}^{-1}}{1-9} + \psi_5^{(0)} \frac{\overbrace{\sin\left(\frac{5}{2}\pi\right)}^{+1}}{1-25} + \psi_7^{(0)} \frac{\overbrace{\sin\left(\frac{7}{2}\pi\right)}^{-1}}{1-49} + \ldots \right]$$

$$= 4\alpha \frac{Ma}{\pi^2 \hbar^2}\sqrt{\frac{2}{a}} \left[-1\left(-\frac{1}{8}\right)\sin\left(\frac{3\pi}{a}x\right) + 1\left(-\frac{1}{24}\right)\sin\left(\frac{5\pi}{a}x\right) - 1\left(-\frac{1}{48}\right)\sin\left(\frac{7\pi}{a}x\right) + \ldots \right]$$

$$= \frac{\alpha M}{\pi^2 \hbar^2}\sqrt{2a}\left[\frac{1}{2}\sin\left(\frac{3\pi}{a}x\right) - \frac{1}{6}\sin\left(\frac{5\pi}{a}x\right) \underbrace{+ \frac{1}{12}\sin\left(\frac{7\pi}{a}x\right) + \ldots}\right]$$

ON AVERAGE [DEPENDING ON x]
TERMS BECOME SMALLER AND SMALLER

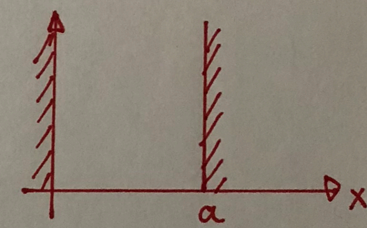
GRIFFITHS, PROB. 6.3

$\boxed{\text{IN-CLASS PROBLEM}}$

[INTEGER SPIN]
PHOTONS, GLUONS, HIGGS BOSON, DEUTERIUM, ^4He

TWO IDENTICAL BOSONS ARE PLACED IN AN INFINITE SQUARE WELL; THEY INTERACT WEAKLY WITH EACH OTHER VIA A δ- POTENTIAL

$$V(x_1, x_2) = -\underset{\substack{\uparrow \\ \text{WIDTH OF WELL}}}{a}\underset{\substack{\uparrow \\ \text{SOME CONSTANT}}}{V_0}\,\delta(x_1 - x_2)$$



FOR THE GROUND STATE OF TWO-PARTICLE SYSTEM, CALCULATE FIRST-ORDER ENERGY CORRECTION, $E^{(1)}$, WHEN PARTICLE-PARTICLE INTERACTION IS TURNED ON

HELP:

(1) REMEMBER FOR IDENTICAL, INDISTINGUISHABLE, NON-INTERACTING BOSONS [GRIFFITHS, PROB. 5.4]:

BOSONS OCCUPY DIFFERENT STATES $\quad [\langle \psi_a | \psi_b \rangle = 0] : \psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} [\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) \pm \psi_b(\vec{r}_1)\psi_a(\vec{r}_2)]$

BOTH BOSONS OCCUPY SAME STATE $\quad [\langle \psi_a | \psi_a \rangle = 1] : \psi(\vec{r}_1, \vec{r}_2) = \psi_a(\vec{r}_1)\psi_a(\vec{r}_2) = \phi$

FOR FERMIONS

PAULI-EXCLUSION PRINCIPLE

[ψ_a, ψ_b : SINGLE-PARTICLE EIGENFUNCTIONS; $\psi(\vec{r}_1, \vec{r}_2)$: TWO-PARTICLE WAVEFUNCTION]

(2) UNPERTURBED SINGLE-PARTICLE SOLUTIONS:

$$\psi_n^{(0)}(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \quad ; \quad E_n^{(0)} = n^2 \frac{\pi^2 \hbar^2}{2Ma^2}$$

(3) $$\int_a^b \sin^4 y \, dy = \frac{1}{32}[12y - 8\sin(2y) + \sin(4y)]_a^b$$

FOR GROUND STATE OF TWO-PARTICLE SYSTEM, PLACE BOTH BOSONS IN SINGLE-PARTICLE GROUND STATE :

$$\psi_1^{(0)}(x_1,x_2) = \psi_1^{(0)}(x_1)\,\psi_1^{(0)}(x_2) = \frac{2}{a}\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)$$

$$E^{(0)}_{1\,(x_1,x_2)} = 2E_1^{(0)} = \frac{\pi^2\hbar^2}{Ma^2}$$

NON-INTERACTING BOSONS

$$E_1^{(1)} = \langle \psi_1^{(0)} | H' | \psi_1^{(0)} \rangle = -aV_0\left(\frac{2}{a}\right)^2 \int_0^a\int_0^a \sin^2\left(\frac{\pi x_1}{a}\right)\sin^2\left(\frac{\pi x_2}{a}\right)\delta(x_2-x_1)\,dx_1\,dx_2$$

$$= -\frac{4V_0}{a}\int_0^a \sin^2\left(\frac{\pi x_1}{a}\right)dx_1 \int_0^a \underbrace{\sin^2\left(\frac{\pi x_2}{a}\right)}_{f(x)}\,\delta(\underbrace{x_2}_{x}-\underbrace{x_1}_{c})\,dx_2$$

$$= -\frac{4V_0}{a}\int_0^a \sin^2\left(\frac{\pi x_1}{a}\right)\sin^2\left(\frac{\pi x_1}{a}\right)dx_1 = -\frac{4V_0}{a}\int_0^a \sin^4\left(\frac{\pi x}{a}\right)dx$$

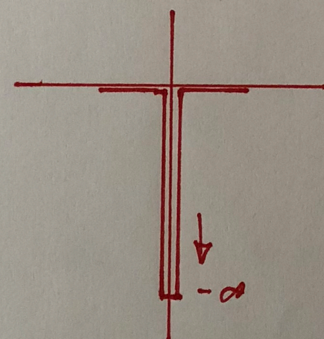
$$y = \frac{\pi x}{a}\ ,\quad \frac{dy}{dx} = \frac{\pi}{a}$$

$$= -\frac{4V_0}{a}\,\frac{a}{\pi}\underbrace{\int_0^\pi \sin^4 y\,dy}_{\frac{1}{32}\left[12y-8\sin(2y)+\sin(4y)\right]_0^\pi} = -\frac{4V_0}{\pi}\,\frac{1}{32}\left[12\pi-0+0-0+0+0\right]$$

$$= -\frac{4V_0}{\pi}\,\frac{12}{32}\pi = -V_0\,\frac{12}{8} = -\frac{3}{2}V_0$$

COMMENT ON NEGATIVE SIGN IN $V(x_1, x_2) = -aV_0\,\delta(x_1 - x_2)$

· DELTA POTENTIAL IS A LIMITING CASE OF THE FINITE POTENTIAL WELL

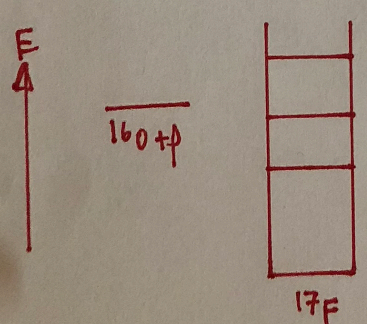


$V(x) = \lambda\,\delta(x)$

IF λ NEGATIVE : DELTA POTENTIAL WELL

IF λ POSITIVE : DELTA POTENTIAL STEP

IN OTHER WORDS : WHEN THIS [NEGATIVE] ATTRACTIVE POTENTIAL IS TURNED ON, THE TOTAL ENERGY OF THE SYSTEM IS LOWERED BECAUSE BINDING ENERGY IS RELEASED



SECOND - ORDER ENERGIES

SECOND - ORDER EQUATION $\quad H^{(0)}\psi_n^{(2)} + H'\psi_n^{(1)} = E_n^{(0)}\psi_n^{(2)} + E_n^{(1)}\psi_n^{(1)} + E_n^{(2)}\psi_n^{(0)}$

$$\psi_n^{(2)} = \sum_m a_{nm}^{(2)} \psi_m^{(0)}$$

$$\psi_n^{(1)} = \sum_m c_{nm}^{(1)} \psi_m^{(0)}$$

SEE EARLIER

PRE - MULTIPLY BY $\psi_l^{(0)*}$ AND INTEGRATE OVER ALL COORDINATES:

$$\underbrace{\langle \psi_l^{(0)} | H^{(0)} | \sum_m a_{nm}^{(2)} \psi_m^{(0)} \rangle}_{E_l^{(0)} a_{nl}^{(2)}} + \boxed{\langle \psi_l^{(0)} | H' | \sum_m c_{nm}^{(1)} \psi_m^{(0)} \rangle}$$

TAKE SUM OUT

$$= \underbrace{E_n^{(0)} \langle \psi_l^{(0)} | \sum_m a_{nm}^{(2)} \psi_m^{(0)} \rangle}_{E_n^{(0)} a_{nl}^{(2)}} + \overbrace{E_n^{(1)} \langle \psi_l^{(0)} | \sum_m c_{nm}^{(1)} \psi_m^{(0)} \rangle}^{E_n^{(1)} c_{nl}^{(1)}}$$

$$+ \underbrace{E_n^{(2)} \langle \psi_l^{(0)} | \psi_n^{(0)} \rangle}_{E_n^{(2)} \delta_{nl}}$$

GIVES

$$E_l^{(0)} a_{nl}^{(2)} + \sum_m c_{nm}^{(1)} \langle \psi_l^{(0)} | H' | \psi_m^{(0)} \rangle = E_n^{(0)} a_{nl}^{(2)} + E_n^{(1)} c_{nl}^{(1)} + E_n^{(2)} \delta_{nl}$$

$$a_{nl}^{(2)} [E_l^{(0)} - E_n^{(0)}] + \sum_m c_{nm}^{(1)} \langle \psi_l^{(0)} | H' | \psi_m^{(0)} \rangle - E_n^{(1)} c_{nl}^{(1)} - E_n^{(2)} \delta_{nl} = 0$$