SIMULATION SCIENCE

First task: Finite Volume Scheme for Periodic Problems

We want to implement finite volume schemes. Therefore we consider the following problem:

$$\begin{aligned} u_t + f(u)_x = 0 \quad \forall (t,x) \in (0,T) \times (0,1), \\ u(0,x) = u_0(x) \quad \forall x \in (0,1). \end{aligned}$$

We restrict ourselves to periodic problems, i.e.

$$u(t,0) = u(t,1).$$

We discretize $[0,1]$ by N cells $I_j := (x_j, x_{j+1})$, $j = 0, \dots, N-1$, where $x_j := \frac{j}{N}$. Let u_i^n denote the approximate mean value on the cell I_j at a discrete time level t_n, i.e.,

$$u_i^n \approx \frac{1}{\Delta x} \int_{I_j} u(t_n, x)\, dx.$$

For this kind of problems implement in a language of your choice (e.g., Matlab or C++) a 3-point finite-volume scheme of the form:

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x} \left(g(u_i^n, u_{i+1}^n) - g(u_{i-1}^n, u_i^n) \right).$$

Hint: Use the periodicity at the boundaries, i.e., $u_{-1}^n = u_{N-1}^n$ and $u_N^n = u_0^n$.