2 Unit Bridging Course - Day 11

Differentiating the Logarithm Function

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The Derivative of $y = \ln x$

Recall from the previous module that the logarithm function

$$y = \ln(x)$$

is the inverse function of the exponential function

$$y = e^x.$$

Here is the derivative of the logarithm function, which you should commit to memory.

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The Derivative of $y = \ln x$ (cont.)

Example

Suppose we wish to differentiate the composite function $y = \ln(x^2)$.

Setting $u = x^2$, we thus have $y = \ln u$.

The Chain Rule hence gives:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u} \times 2x = \frac{2x}{x^2} = \frac{2}{x}.$$

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Practice Questions

Differentiate the following:

- $\ln(2x)$
- $\ln(x^3 + x)$
- $x \ln x$

The Derivative of $y = \ln x$ (cont.)

Answers

- $\dfrac{1}{x}$ (via. the Chain rule).
- $\dfrac{3x^2+1}{x^3+x}$ (via. the Chain rule).
- $\ln x + 1$ (via. the Product rule).

The Derivative of $y = \ln f(x)$

Given sufficient experience differentiating composite functions of the form

$$y = \ln f(x),$$

with f being some function, you may have observed a pattern:

$$\boxed{\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}.}$$

Never forget this shortcut comes from the Chain Rule!

I.e. letting $u = f(x)$, we can hence write $y = \ln u$.

The Chain Rule then gives:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u} \times f'(x) = \frac{1}{f(x)} \times f'(x) = \frac{f'(x)}{f(x)}.$$

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The Derivative of $y = \ln f(x)$ (cont.)

Example

Let's now try to differentiate our original example $y = \ln(x^2)$ by using the shortcut instead of the full chain rule.

Here,

$$f(x) = x^2,$$

and so

$$f'(x) = 2x.$$

Hence

$$\frac{d}{dx}\ln(x^2) = \frac{f'(x)}{f(x)} = \frac{2x}{x^2} = \frac{2}{x}.$$

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Summary

- If $y = \ln x$, then $\dfrac{dy}{dx} = \dfrac{1}{x}$.
- If $y = \ln f(x)$, then $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$ (via. the Chain Rule).