2 Unit Bridging Course – Day 2

Linear functions II: Finding equations

Clinton Boys

Finding equations of lines

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(i) the gradient of a line
(ii) the coordinates of a point on the line

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Let's think about how we would go about doing this.

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Suppose we know a line has gradient 2 and passes through the point with co-ordinates $(-1, 2)$.

Since $m = 2$, we know the line must have the equation

$$y = 2x + b$$

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$$2 = 2 \times (-1) + b.$$

Solving this equation for b gives

$$b = 2 + 2 = 4,$$

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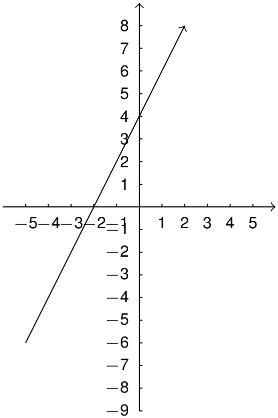

$$y = 2x + 4$$

Practice questions

Find the equations of the following lines:

(i) The line with gradient -1 which passes through $(3, 1)$.

(ii) The line with gradient 2 which passes through $(-1, -4)$.

Answers to practice questions

(i) $y = -x + 4$

(ii) $y = 2x - 2$

Rise-over-run formula

If we know two points (x_1, y_1) and (x_2, y_2) on a line, we can use the following formula, known as the rise-over-run formula to compute the gradient of the line between the two points:

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Let's see how we can use this formula to find the equation of a line.

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Equations of lines, continued

If we know the coordinates of two points which lie on a straight line, we can:

(i) Use the rise-over-run formula to find the gradient.

(ii) Use the gradient and one of the points to find the equation (as before).

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Example

Let's find the equation of the straight line which joins the points $(-1, 2)$ and $(1, 3)$.

Using the rise-over-run formula we can calculate the gradient:

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{2-3}{-1-1} = \frac{-1}{-2} = \frac{1}{2}.$$

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We can now use either of the points $(-1, 2)$ or $(1, 3)$ to find the equation. Let's use $(1, 3)$.

If the equation is $y = \frac{1}{2}x + b$, then the point $(1, 3)$ must satisfy this equation, and so we must have

$$3 = \frac{1}{2} \times 1 + b,$$

i.e. $b = 3 - \frac{1}{2} = 2\frac{1}{2} = \frac{5}{2}$.

So the equation of the line is

$$y = \frac{1}{2}x + \frac{5}{2} = \frac{1}{2}(x + 5).$$

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Practice question

Check that you can also get this equation using the other point $(-1, 2)$.

Graphs of linear functions

The graph of a linear function is a straight line, as we have already seen. In order to sketch the graph of a straight line from its equation, we need to know two points which lie on the line.

Two points which are particularly easy to find are the x-intercept and the y-intercept.

The x-intercept, which is where the line crosses the x-axis, is found by *setting* $y = 0$ (because $y = 0$ along the entire x-axis).

The y-intercept, which is where the line crosses the y-axis, is found by *setting* $x = 0$ (because $x = 0$ along the entire y-axis).

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Example

Draw the graph of the straight line $2y + x - 4 = 0$.

The x-intercept is found by setting $y = 0$:

$$x - 4 = 0, \quad \text{i.e.} \quad x = 4.$$

The y-intercept is found by setting $x = 0$:

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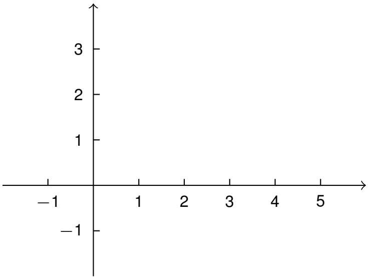
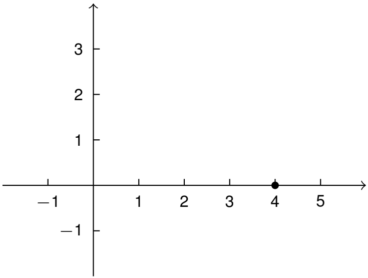

3
2
1
−1
1
2
3
4
5
−1

x-intercept $x = 4$



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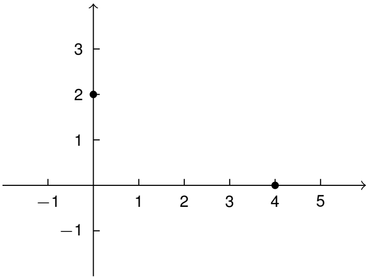

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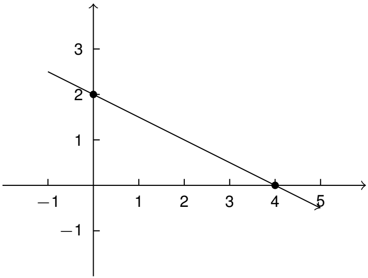

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