2 Unit Bridging Course – Day 10

Circular Functions II – The general sine function

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THE UNIVERSITY OF SYDNEY

The general sine function

We're now going to consider variations on the sine function.

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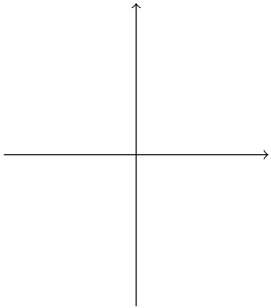

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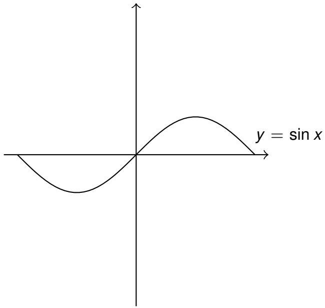

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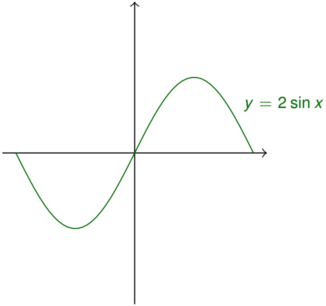

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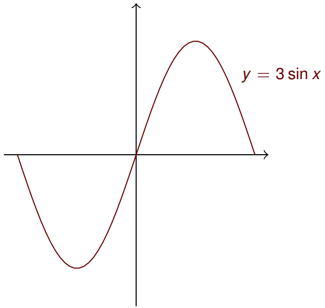

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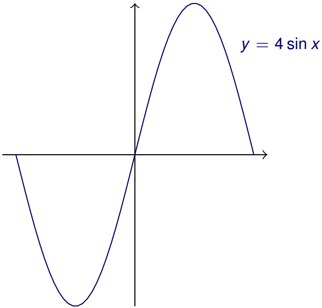

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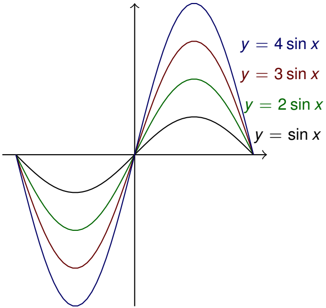

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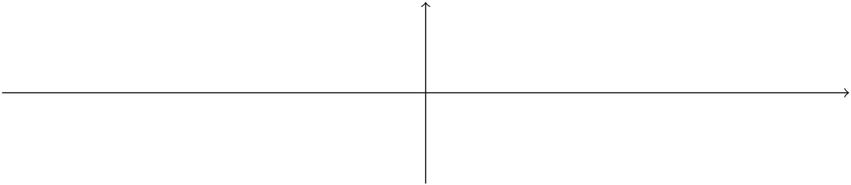

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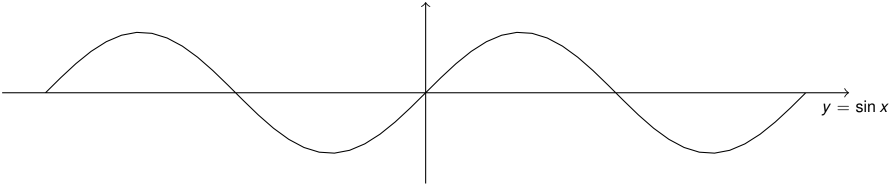

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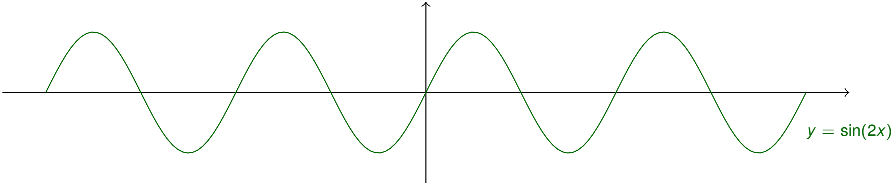

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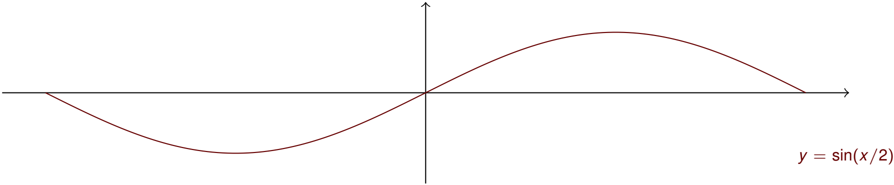

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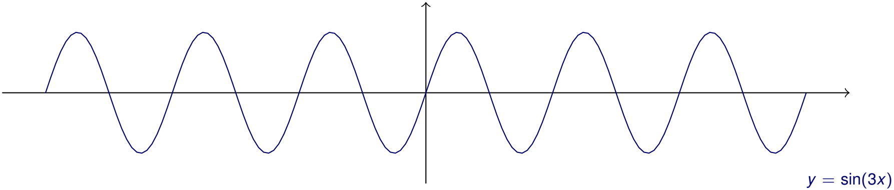

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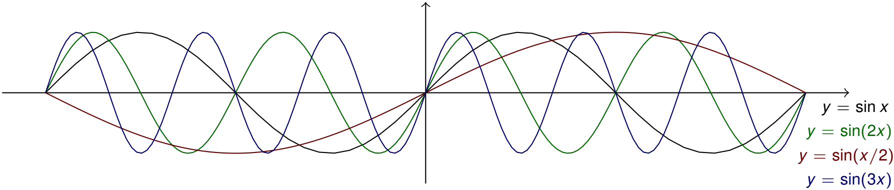

Period

If we change $y = \sin x$ to $y = \sin(\omega x)$, the effect is to stretch or compress the graph horizontally.

If $\omega > 1$, then the graph becomes compressed – we need to now fit in the full sine curve ω times where we previously fitted it in once.

If $\omega < 1$ then the graph becomes stretched – we now have $1/\omega$ times as much space to fit in a full sine curve.

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Sketching sine curves

If we are given a general sine curve $y = A\sin(\omega x)$, we can sketch it without using calculus.

Indeed, if we know that the amplitude is A, and the period is $\dfrac{2\pi}{\omega}$, this is enough information to sketch the curve, since we know that the basic shape of the curve is the same as for $y = \sin x$.

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Sketch the graph of the function $y = 2\sin(3x)$.

The amplitude is 2.

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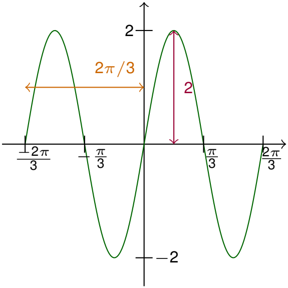

Practice questions

For each of the following sine curves, write down the amplitude, the period, and sketch the curve.

(i) $y = 3\sin(x/2)$
(ii) $y = 2\sin(x/3)$
(ii) $y = (1/2)\sin(2x)$
(iv) $y = 4\sin(\pi x)$.

Answers

(i) amplitude 3, period 4π

(ii) amplitude 2, period 6π

(iii) amplitude $\frac{1}{2}$, period π

(iv) amplitude 4, period $\frac{2\pi}{\pi} = 2$.