International Journal of Mathematical Education in Science and Technology

ISSN: 0020-739X (Print) 1464-5211 (Online) Journal homepage: https://www.tandfonline.com/loi/tmes20

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To cite this article: Ann Kajander & Miroslav Lovric (2005) Transition from secondary to tertiary mathematics: McMaster University experience, International Journal of Mathematical Education in Science and Technology, 36:2-3, 149-160, DOI: 10.1080/00207340412317040

To link to this article: https://doi.org/10.1080/00207340412317040

International Journal of Mathematical Education in Science and Technology, Vol. 36, Nos. 2–3, 2005, 149–160

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Transition from secondary to tertiary mathematics: McMaster University experience

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The transition ('gap') between secondary and tertiary education in mathematics is a complex phenomenon covering a vast array of problems and issues. The aim of this paper is to present the ways in which the issues of mathematics transition have been dealt with at McMaster University. Roughly, the process of transition has been broken into three stages: students' voluntary preparation for university mathematics courses facilitated by the Mathematics Review Manual; administration of Mathematics Background Survey; and redesign of the first-year Calculus (and, subsequently, other mathematics courses).

1. Introduction

The transition ('gap') between secondary and tertiary education in mathematics is a complex phenomenon covering a vast array of problems and issues. Although there is evidence of similar 'gaps' in other disciplines in science and beyond, it seems that the transition in mathematics is by far the most serious and the most problematic. Compared to other subjects, mathematics in elementary and high schools enjoys a unique position, in terms of time devoted to it – both in the classroom, and outside. There has been no single reform of (elementary or high school) education anywhere that has not, in one way or another, affected the way mathematics is taught. However, in spite of all efforts and energy ventured into the pre-tertiary mathematics education, the knowledge and skills of incoming university students are far from satisfactory [1].

Students coming to tertiary institutions are more numerous and have more diverse backgrounds than previously; they have different and often unclear views of mathematics and its role in their future career, and in their life. Their views echo a seemingly contradictory (but, unfortunately, true) fact that, although the importance of acquiring mathematical skills has been rising,[1] the lack

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[1]'Mathematical skills in the workplace are changing, with increasing numbers of people invovled in mathematics-related work, and with such involving increasingly sophisticated mathematical activities.' (from the UK report: Hoyles, C. *et al.* Mathematical Skills in the Workplace. Final Report to the Science, Technology and Mathematics Council, June 2002).

International Journal of Mathematical Education in Science and Technology
ISSN 0020–739X print/ISSN 1464–5211 online
http://www.tandf.co.uk/journals
DOI: 10.1080/00207340412317040

of appreciation of mathematics on the part of the public is evident [2]. Curricular changes may also have influenced characteristics of incoming students.

University teachers may largely be unaware of or unwilling to accept the magnitude of these changes. Moreover, the amount of research on mathematics education at the tertiary level is still modest [3]. Inquiry into both the social and the cognitive backgrounds of students may provide insight into how the two factors relate to and affect one another [3]. Barnard [4] studied barriers that inhibit improved outcomes in students' learning of mathematics based on a survey of first-year university students. Suggestions for improvements, with evidence, seem to be emerging [5].

The transitional stage in education is just one instance in the sequence of major changes that every person experiences in her/his life. These changes – known in anthropology as rites of passage – are events that, in a major way, influence one's decisions about the future. Anthropology teaches us that we need to devote a large amount of care and attention to the issues related to mathematics transition. Moreover, our evidence suggests that students' high school experience with mathematics is correlated to their success in university mathematics courses.

2. Aim and context

The aim of this paper is to present the ways in which we have dealt with the issues of mathematics transition at McMaster University. Roughly, the process of transition was broken into three stages: students' voluntary preparation for university mathematics courses facilitated by the Mathematics Review Manual; administration of Mathematics Background Survey in the first week of classes; redesign of the first-year Calculus course (and, subsequently, other mathematics courses).

In 2001, the authors started surveying incoming first-year students on a yearly basis, collecting information on their background mathematics knowledge and skills, as well as their high school experience and expectations of university mathematics courses. A number of results and several important messages from the survey are presented in this paper. Based on our previous experience teaching first-year Calculus, and learning from the surveys, we have been publishing yearly updates of the Mathematics Review Manual. A brief description of the contents and purpose of the 'Manual,' together with a few comments, is given later in this paper. Several important features of the redesign of first-year courses are discussed. We close the paper with a few final comments and remarks.

The major new factor contributing to the need for a good understanding of the mathematics transition in Ontario (Canada) lies in the massive reform of secondary education: not only did the secondary curriculum change from a five-year to a four-year programme but significant changes took place in the curriculum itself [6]. Much of the initial research on mathematics reform has been at the elementary level, but as students continue to progress through new curricula, further research is needed to connect this early work with subsequent outcomes. In Ontario, a new elementary curriculum was introduced for grades one to eight in 1997, with the new secondary curriculum subsequently introduced, one year at a time. The change in the secondary curriculum resulted in universities experiencing the so-called 'double cohort' (two years of high school students entering university at the same time).

The university year 2003–2004 in Ontario contained students from both the old and new Ontario curricula. Studying the differences in skills of the two cohorts, in Ontario, is inseparable from studying transition issues.

3. Background survey

The Mathematics Background Questionnaire (henceforth referred to as 'the survey') has been given to a sample of between 300 and 400 students (depending on the year when it is administered) who take the first year calculus course at McMaster University. The survey was field tested with high school students to ensure the terminology and the questions were appropriate.[2]

The survey was designed to help us better understand the specific features of mathematics preparation of our incoming students. It has two parts, roughly described as 'narrative' and 'mathematical.' The survey is administered very early in students' university 'life,' usually during their first week of university classes. Students are given 50 minutes to complete the survey: 10 minutes to fill in the demographic data and to answer narrative questions, and 40 minutes to do the mathematics part.

Besides inquiring about basic demographic data, the 'narrative' part of the survey asks students to describe their experiences with high school mathematics and their expectations of their university mathematics courses. The 'mathematics' part aims to identify students' strengths and weaknesses in the following areas:

- Basic technical and computational skills (fractions, equations).
- Basic notions of functions (range, composition).
- Familiarity with transcendental functions (exponential, logarithm, trigonometric functions).
- Written communication of mathematical ideas ('explain' type of questions).
- Proficiency in multi-step problems.
- Drawing and interpreting graphs of functions.
- Applied problems (computation and interpretation).

The survey was given to incoming calculus students in September 2001, and again with minor changes in September 2002. In September 2003 a slightly revised survey was used with the so-called 'double cohort' students taking the first-year calculus course.

Information on students' experiences with mathematics in high school was also collected in the survey (in a narrative form) and is being examined for correlations with their success in their first-year mathematics course. It is hoped that having the two perspectives of students from both the old and new curricula in Ontario will give us insight into changing features of students' expectations, and their mathematics knowledge and skills as they undergo the process of transition to university. The information on students' background preparation and experience in mathematics should be very valuable to university faculties, some of whom are not in touch with changes in the incoming student population.

[2]A copy of the survey used in 2003 is in the appendix to this paper.

Table 1. Performance of the 4-year and the 5-year cohorts on the Mathematics Background Survey 2003 questions.

Question		Correct answers (%)	
No	Type	5-year cohort	4-year cohort
1a	Polynomial factorization formula (identify as correct or not correct)	85.1	86.0
1b	Formula involving exponential function (identify as correct or not correct)	78.4	80.7
1c	Formula involving natural logarithm (identify as correct or not correct)	48.6	54.4
2a	Application (revenue) – determine the rate of change	37.8	49.1
2b	Application (revenue) – interpret the results from 2a	8.1	14.0
3a	Intercepts of a quadratic function (identify given statement as true or false and explain reasoning)	25.7	40.4
3b	Inequality involving fractions (identify given statement as true or false and explain reasoning)	8.1	5.3
4a	Application (throwing a ball) – technical/calculus type question	56.8	41.2
4b	Application (throwing a ball) – give a physical interpretation	52.0	65.8
5	Identify the range of a function	43.2	29.8
6a	Graph (of a position function) – determine property (velocity) related to the graph	55.4	62.3
6b	Graph (of a position function) – determine property (acceleration) related to the graph	38.5	42.1
7	Solve an equation involving a rational function	85.8	79.8
8	Solve an exponential equation	73.0	68.4
9	Determine composition of two functions	74.3	70.2
10	Sketch the graph of a quadratic function	73.0	57.9
11	Solve a logarithmic equation	22.3	11.4
12	Solve a two-step analytic geometry problem (line perpendicular to the given line)	45.9	47.4

4. Survey results

In this section, we present results of the survey from 2003, and discuss some issues arising from it.[3] Table 1 shows the performance of both cohorts on all survey questions.

The fact that, in 2003, we surveyed two 'cohorts' of students crystallized one of our major conclusions: students' performance is strongly correlated to the time they spend doing mathematics in high school. The 5-year cohort students not only spent one extra year in high school, but they also took more mathematics courses during their final year in high school (an average of 2.39 courses per student for a 5-year cohort student group, versus an average of 1.81 courses for a 4-year cohort student group). The 5-year cohort students, on average, were (not surprisingly!) one year older than the average 4-year cohort student. This one-year difference is also believed to play a significant role in transition for a number of reasons (difficulties in adjustment, maturity, requirements of social life, etc.).

[3]A more comprehensive analysis is the subject of a forthcoming paper by the authors.

We will focus on those issues in the survey that are directly related to the transition, namely, the lack of adequate technical knowledge and computational proficiency, and problems in mathematical maturity: thinking and logical skills and formation of a logical argument (i.e. mathematical narratives).

Our survey shows that the 4-year cohort students had more difficulties with questions that required basic techniques and computational skills. For instance, although the 4-year cohort students were slightly better than the 5-year cohort at identifying the formula $\ln(2x) = 2\ln(x)$ (question 1c) as incorrect, they did very poorly on the question that required the use of properties of logarithms to solve the equation $\log(x) + \log(x+7) = \log(4) + \log(2)$ (question 11). Only about 11% gave the correct answer, whereas 22.3% of the 5-year cohort students solved the equation correctly.[4]

A similar performance was observed on the two questions involving quadratics: the 4-year cohort students were better at identifying the intercepts of the function $f(x) = (x+a)(x+\mathrm{b})$ (question 3a), whereas the 5-year cohort students were superior in sketching the graph of $y = (x-1)^2 + 2$ (question 10). It is unclear what influence the increased use of graphing calculators in the classroom may have had on these results.

Further evidence from the survey suggests that the 5-year cohort students were better prepared for technical questions directly related to traditional calculus content, whereas the 4-year cohort students were somewhat better at interpretive or conceptual questions.

How did this difference play out in the university? A good answer is obtained by considering students' responses to the (university calculus course) test question 'Using the definition (i.e. starting from first principles), compute the derivative of the function $f(x) = 1/x$.' (In successive iterations of the test, functions such as square root, or a cubic function, have been used.) Usually, students have no problems setting the difference quotient $(f(x+h) - f(x))/h$. However, a great number of them get stuck in computing a common denominator (or, manipulating square roots). Not surprisingly, students from the 4-year cohort have more problems with this question. Although they might solve a similar question correctly on an assignment, they will still have problems on a test. Thus observation points to the source of the problem, which is not in any way conceptual in nature, but simply an absence of adequate routine. We have also noticed that the total number of students who have problems with this type of question has increased over the years.

Let us look a bit closer at the surveyed students' university performance. Table 2 shows the success of the two cohorts in the first-term, first year calculus course

Table 2. Performance of the 4-year and the 5-year cohorts in Calculus I.

	5-year cohort	4-year cohort
Quizzes	83.3	78
Test 1	77.4	75.8
Test 2	71.7	71.5
Final exam	70.4	68.7
Final mark	73.9	69.1

[4]The fact that both percentages are so low is of significant concern.

(Calculus I). Assessment in the course was based on five 30-minute quizzes, two one-hour tests and a three-hour final exam.

Each quiz consisted of five technical and/or computational questions (such as computing limits or derivatives, evaluating definite integrals, testing knowledge of integration methods, etc.). The success rate of 83.3% for the 5-year cohort students is about 5% better than for the 4-year cohort group, which was not a surprise – our survey showed that the 5-year cohort students were better prepared for these types of questions – once university classes start, students have very little time to properly review high school material or brush up on their algebra and arithmetics.

The two remaining assessment tools (two tests and the final exam) contained both technical/computational questions and conceptual/interpretative questions. Although the 4-year cohort students performed better on conceptual/interpretative questions in the survey, that difference did not show in their Calculus I work. It is probably too early to fully explain the reasons why this is so, but a number of questions come to mind. For example, what relative emphasis was placed in the calculus course on computational versus conceptual ideas, and what influence did this emphasis have on the further development of the students in university? How did the emphasis compare to the high school experience, and how did this affect the students' proficiencies? Bluntly put, is it possible that the more traditional university calculus course experience redirected the students back to a more computational value system in which they concentrated more on traditional skills?

Lack of appropriate mathematical thinking is evident in a number of situations with both cohorts. One of them is a very common assumption that our students make, namely that all (or most) functions are linear: 'formulas' such as $1/(a+b)=1/a+1/b$, or $(a+b)^2=a^2+b^2$ (or equivalent statements for logarithm, exponential or root functions) are found more and more often in students' university mathematics tests and on assignments. Although a student could very easily convince herself/himself that, for instance, $1/(a+b)=1/a+1/b$ does not work since $1/(2+2)$ is not equal to $1/2+1/2$, they seem not to be aware of it, or are simply not being able to make appropriate use of it.

The survey from 2001 shows that only 12% of students were able to reduce the equation $\ln(x)+\ln(x+7)=\ln(4)+\ln(2)$ to the quadratic equation $x^2+7x=8$ (by far, the most common error was using $\ln(x+7)=\ln(x)+\ln(7)$ to simplify the logarithm term on the left side).

Responses to the question 'Give the Statement of the Pythagorean Theorem' revealed that about 24% of students were able to give a correct statement (meaning that they listed all assumptions and gave the correct conclusion).[5] The most common answer involved writing the formula $a^2+b^2=c^2$ only, with no explanation of what a, b or c are, or without giving adequate geometric context. Some students drew a picture, but quite often did not label the right angle. They had lots of problems expressing their thoughts in a narrative form. Here is a sample of their attempts at stating the Pythagorean Theorem:

- The square of a side opposite to hypotenuse is equal...
- The sum of both sides of a triangle squared equals...

[5]The question about the Pythagorean Theorem appears in earlier versions of the survey; it was not included in the 2003 version.

- $x^2 + y^2$ will always give you z when you are dealing with a 90 degrees triangle.
- Pythagorean theorem is an old theorem, still used today, to find out angles. It is used in a triangle only and requires two angles to be known. The formula of it is $a^2 + b^2 = c^2$.

Keeping this in mind, it comes as no surprise that the university calculus test question that has been done most poorly is related to the Intermediate Value Theorem. Students were asked to quote the theorem, and were given a situation where they had to apply it. They were not asked to do anything new, since the theorem was presented and discussed in class.[6]

The cases discussed here point to two important aspects of the transition, that require our special attention: the nature of mathematical objects that we require our students to know how to deal with, and the variety of ways in which they need to reason about them. Some research in this direction has been done recently [7].

5. Mathematics Review Manual and redesign of first year Calculus course

Based on the results of the survey, one of the authors (Miroslav Lovric) wrote a 70-page brochure, called Mathematics Review Manual.[7] The rationale for the Manual is to help students prepare, during the summer months before coming to university, for their mathematics courses. The manual is available online; moreover, a paper version is mailed, free of charge, to all incoming students in science, engineering, and arts and science programmes.

The Manual consists, roughly, of two parts. In the introductory narrative part, students are given all kinds of information about university, and about mathematics courses they will have to take. This part is supposed to motivate a high school student to think a bit about her/his future experiences as a university student. The 'mathematical' part provides a brief review of background material that the students will need in their university mathematics courses. There are a large number of fully solved problems, as well as a list of additional problems that students are encouraged to do. The Manual starts with very basic algebra and geometry (fractions, powers, roots, etc.); the largest part of it is devoted to functions, and in particular, to transcendental functions (exponential, logarithm, and trigonometric functions).

Although students have been heard saying that the idea of the Manual was 'great,' a very large number of them admitted that they 'noticed that they got it in the mail, but have not read it.' On a more positive note, students brought the Manual with them to university, and sporadically used it to review the material when they needed it.

In order to better bridge the difference between high school and university mathematics, we have redesigned the calculus course, mainly in two directions: providing some review, as an introduction to the course, and spending time

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[6]As a matter of fact, a whole 50-minute lecture was devoted to the Intermediate Value Theorem, with a discussion of ways in which the theorem is used; moreover, cases in which the theorem cannot be used (i.e. some/all assumptions are not satisfied) were also discussed.
[7]The online version is at http://www.math.mcmaster.ca/lovric/rm.html

talking about language of mathematics and the good and productive use of the mathematics textbook.[8]

For the first time in the history of calculus courses at McMaster University (we do not offer bridging courses), in 2003, we spent time actually teaching trigonometry – all the way from the basics: discussing degrees and radians, and defining sine, cosine and other trigonometric functions (in a right triangle, and on the unit circle). On top of that, we used every opportunity in the course to practise dealing with various aspects of those functions (graphing, calculations, formulas, etc.).

In one respect we were not successful: the students' fear of trigonometry was still present at the end of the course.[9] Whether it is the lack of technical proficiency on the students' part, or the way we did the review, or something else, we are not sure at the moment. The learning of technical skills and use of memory as parts of overall mathematics learning are areas full of confusion and controversy [8, 9]. Consulting the literature will, most probably, help us at this stage. We need more time to assess to what extent our efforts need to refocus.

A new feature of the present reform in high schools in Ontario is that the students are not taught how to use a textbook; i.e., they do not have a chance to develop experience in reading mathematics text and thinking about it. Needless to say, the new approach to learning mathematics in high school has a serious impact on students' future contacts with mathematics. Aware of this situation, we now discuss, in lectures, the strategies of dealing with a mathematics text. Moreover, a lecture on the 'language' of mathematics (if-then statements, reversing if-then, universal and existential quantifiers, etc.) is now an integral part of the Calculus course. Students also learn what a definition is, and what constitutes a statement of a theorem. We can no longer expect that the incoming students will know this material.

6. Comments

Probably the most important message that we can derive from the McMaster experience is that a review, or a bridging course, or any other experience that aims to ease the transition into tertiary education must be done over a longer period of time. The 4-year cohort students at McMaster University ended up their first year calculus course in December 2003 with an average mark of 6 (passing marks are 1–12 (12 is the highest), zero is a failing mark). The 5-year cohort's final mark average in the same course was 8. We believe that the major reason for the difference in performance was strongly related to the time spent doing mathematics in high school.

We also realized that we could only go some distance to help students with the transition. Students need to realize that it is their responsibility to prepare, as much as possible, for their future as university students. Those that take initiative in the summer before coming to university (by taking 'cram' (intense review) courses, or by using various web sites to practise and/or for self-diagnosis, or by actually working through the Mathematics Review Manual) do significantly better in our

[8]Due to a large class size (about 400), lecturing is the major (quite often the only) mode of classroom instruction.
[9]Anecdotal evidence, obtained from talking to students after their final examination.

courses. The usual complaint is that students work during summer, and have no time for much else. In our opinion, working one hour per day for 3–4 weeks before classes start (as suggested in the Manual) is usually quite sufficient.

We have noticed that most high school students adopt a surface-learning attitude (which, by the way, does not prevent them for obtaining very high marks in high school!). University courses expect them to change their attitude, i.e. to get involved in deep-learning activities [10]. Deep learning involves thinking, and the ability to transfer to new situations. The evidence that most of our incoming students are not deep learners lies in the fact that they usually experience a 'fear of the new.' Talking about the difficulties in solving a problem with inverse trigonometric functions,[10] students often say that 'they did not do this in high school.'

Hyperbolic functions (except for the name), do not contain new material. As a matter of fact, discovering the properties of $\sinh(x)$ and $\cosh(x)$ is one of the best ways to practice the properties of the exponential function e^x. Students, in large part, find the topic quite difficult, lacking the ability to connect to the previously learnt material about e^x.

Furthermore, it is very difficult, in university, to make up for an inadequate high school experience. High school really matters because it gives students an opportunity to better develop a variety of mathematics-related skills over longer periods of time in small groups [5].

Analysing the narrative responses to our survey, we found a correlation between a student's positive experience in high school and their good performance in university mathematics courses. Here is a sample of students' statements:

- Math was never my best subject but I gained confidence in my ability through my high school years.
- I learned a lot in calculus and algebra and geometry through the lectures.
- My experiences were pretty good.
- The work was easy, and it interested me. The math was fun to learn.
- Enjoyed math in high school and found it to be interesting.

Although some of the students with a positive high school experience did not do so well on the survey (implying that they needed to work to improve their background knowledge), they were able to recover, and obtained good marks in calculus. However, students with negative high school experience have, as a rule, a hard time in university. One student said in his survey:

- In grade 11, I had a teacher . . . my mark dropped from 99% to 79% and so did my interest and enthusiasm for math.

We found out that students with a negative high school experience quite often do not fully recover, and end up with mediocre to low marks in university mathematics courses.

It will take some time before we are able to assess the impact of our efforts at easing the transition for our students. Several major issues – time devoted to doing mathematics, mode of learning (surface versus deep), and psychological factors (experiences in high school) – make us fully aware of the fact that, without a good collaboration with high school teachers, our efforts will be of very limited success.

[10]Inverse trigonometric functions are not covered in high schools in Ontario.

Deeper thought on the part of university faculty as to what ideas are most important to learn in mathematics may also be important in light of the influence of secondary level curricular changes on students. In particular, the role of conceptual understanding versus technical/computational proficiency may need to be re-examined. This issue, and the questions that we raise in this paper, will be a subject of our subsequent research.

Appendix

Mathematics Background Questionnaire 2003

[The following is the list of questions used in the Mathematics Background Questionnaire, without preserving its original format.]

Name (please print): ____________________

Student No.: __________ Faculty: ____________

Note: The information you are sharing will be used to gain information on your high school mathematics background. It will help your mathematics instructor better plan and design the calculus course. Your responses will be kept confidential and will never be reported individually. **Thank you for taking the time to complete this questionnaire.**

- Your gender: __________ Age (years and months): __________
- How many years did you go to high school? What high school(s) did you attend (for each school, list: name, location (city and country), and for how long you attended it)
- Your high school math marks. Indicate the marks that you got for the courses below. If you do not remember the mark, write 'yes' instead.

  OAC Calculus: ___ OAC Finite Math: ___ OAC Algebra and Geometry: ___
  Advanced Functions and Introductory Calculus U: _____
  Geometry and Discrete Math U: ___ Mathematics of Data Management U: ___

- What language do you speak most often in your parental home(s)?

Describe your experiences with high school mathematics courses that you took.

What are your expectations about the Calculus course that you are taking now?

Try to solve as many problems as you can.
Most probably you will not be able to do all of them

1. Indicate whether each of the following formulas is correct or not. Circle your choice. You do not have to justify your answer.

   (a) $x^2 + y^2 = (x - y)(x + y)$ CORRECT NOT CORRECT

(b) $(e^x)^2 = e^{x^2}$ CORRECT NOT CORRECT
(c) $\ln(2x) = 2\ln x$ CORRECT NOT CORRECT

2. The revenue of a company is modelled by $R(x) = x(50 - x)$, where x is the price per item, $0 <= x <= 50$.
   (a) Determine the rate of change of the revenue with respect to the price when the price is 10 dollars and when the price is 15 dollars.
   (b) Explain what the values of the rate of change above mean to the company.

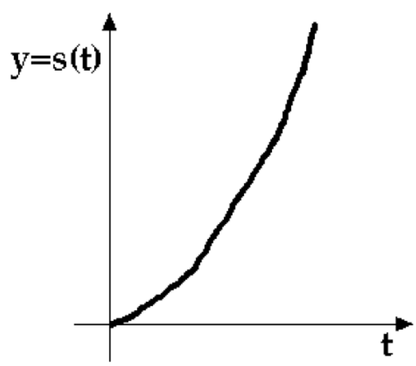
3. Indicate whether each of the following statements is correct or not (circle your choice). Explain your answer.
   (a) If $f(x) = (x + a)(x + b)$, then the graph of $f(x)$ cuts the x-axis at both a and b.

   CORRECT NOT CORRECT

   (b) If $a > b$, then $1/a < 1/b$ for all real numbers a, b not equal to 0.

   CORRECT NOT CORRECT

4. A ball is thrown from a building into the air and falls on the ground below. The height of the ball t seconds after being thrown is $y = -5t^2 + 30t + 35$ metres.
   (a) Determine the maximum height of the ball above ground.
   (b) After how many seconds does the ball hit the ground?
5. What is the range of the function $h(x) = |x|$?
6. A position function $y = s(t)$ is given below.



   (a) Describe the velocity $v(t)$ as increasing or decreasing. Explain how you know.
   (b) Is the acceleration $a(t)$ positive or negative? Explain how you know.
7. Solve the equation $(x^2 + 5x + 6)/(x^2 + 7x + 10) = 2$.
8. Solve the equation $4^x = 16^{2x-2}$.
9. Compute the composition $(g \circ f)(x)$ or $g(f(x))$ of the functions $f(x) = x^2 + 1$ and $g(x) = 1/x + 1$.
10. Sketch the graph of the function $y = (x - 1)^2 + 2$.
11. Solve the equation $\log x + \log(x + 7) = \log 4 + \log 2$.

12. Find an equation of a line perpendicular to the line $2x + y - 4 = 0$ that goes through the point $(1, -2)$.

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