MathsStart and MathsTrack: Mathematics Bridging at the University of Adelaide.

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SCHOOL OF EDUCATION

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Abstract

In this thesis the University of Adelaide's mathematics bridging courses: MathsStart and MathsTrack are discussed in detail. Their context and purpose is reviewed, their content is aligned to national and state highschool curriculums, as well as appropriate entry-level university courses. The pedagogy and excellence in educational practice demonstrated through the University of Adelaide's maths learning center, which offers these bridging courses, is examined with an eye to learning from their exemplary approach to supporting what can be one of the most vunerable cohorts of students attempting to enter into tertiary education. Ultimately, reccomendations are made both in terms of exemplary practice that can be generalised to other such courses accross Australia, and also in terms of restructuring of the courses to better align them with national and state curriculums, while also tailoring their content to better suit the cohort of students enrolling in these courses.

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Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

I give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines.

Signed:

Date:

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Acknowledgements

First, my thanks go to David Butler and Nicholas Crouch from the maths learning center at the University of Adelaide. Not only did their exemplary educatational expertise inspire me to try my hand at education in the first place, but they continue to inspire me on a daily basis with the way the help and support students. Even while dealing with a university review Nicholas still found time (somehow) to sit with me and discuss improvements that could be made to their bridging courses.

My thanks also extend to Igusti Darmawan, my supervisor from inside the school of education, for his guidance but mostly support and his willingness to allow me to pursue a somewhat unorthodox project of my own construction, despite it being quite outside of his speciality area of educational research, none-the-less supporting me in my project.

- Lynda
- Linda
- Brendan, for doing his best,
- Ed
- Saman?
- Jason?
- Amanda?
- Trev?
- Students?
- ...

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Chapter 1

Introduction

This thesis is concerned with the mathematics bridging courses offered at the University of Adelaide (UofA) through the Maths Learning Centre (MLC): MathsStart and MathsTrack. These courses are intended to take students with very little mathematics background and help them transition into tertiary education (which will generally have some mathematics pre-requisites or assumed knowledge) and be successful. MathsStart and MathsTrack are structured to be student-paced, with no deadlines having been set *apriori*, in a deliberate attempt to both alleiviate test-anxiety, and to aclimitise students to a learning environment in which the primary onus for motivation is on the students. Although the bridging courses are offered through the MLC at the UofA, the cohort of students come from a very diverse set of backgrounds and have a variety of needs. The majority of students enrolling in these courses are planning to continue into tertiary education at the UofA, but many students do not intend to continue into tertiary education at all, but instead enroll in the bridging courses to meet the requisite assumed knowledge for other pathways — pilot training in the defence forces, for example.

The continuing decline of interest in mathematics along with the sustained trend of increased demand for mathematically skilled graduates by several major leading industries in Australia and globally (enginneering, science, technology, ...) means that bridging courses playing an increasingly important role in our education system (see Chapter 3 for a more detailed discussion NOTE: or maybe reference Section 4.1 here instead?). Although MathsStart and MathsTrack have been very successful so far, as educators we are continually engaging in reflective practice and looking for ways to improve our teaching practice. It is in this frame of mind, and with the knowledge that comprehensive review of the purpose, structure, and content of MathsStart and MathsTrack has not been done before, the purpose of this thesis is to examine the "guiding question":

> How can MathsStart and MathsTrack be improved?

Naturally, this vague statement invites the question "how is improvement measured?", or "improvment in what outcome?". The question is left deliberately vague in this way because part of this work will be dedicated to teasing apart the different possible interpretations, the importance of taking care when interpreting improvement in an educational context, and the consequences of different interpretations. A solution is not proposed, only alternatives and their consequences, with comments on the stakeholders in each case. One important perspective of this question that will be represented heavily in this thesis can be shown by a re-phrasing of the question:

How can MathsStart and MathsTrack best help the students enrolling in them to be successful going forward?

Of course this does not neccessarily make the question any less vague — determining what is in the best interests of our students, is not always clear or straightforward. But nonetheless, this question will be the primary focus of this thesis and despite not being able to give any definitive answers, some suggestions will be made, and at the very least, some context will be given for better understanding the question. Also important to note that although the focus of this work is the courses MathsStart and MathsTrack, much of the contextual background presented herein is relevant to bridging courses accross Australia and internationally. Although it is not the primary goal of this work, giving some broad structure and perhaps a framework for understanding the needs of students going into mathematics bridging programs globally could be thought of as a secondary (perhaps ancillary) objective of this work.

In order to address this guiding question, this thesis will be structured as follows:

- The remainder of this introductory chapter (Chapter 1) is broken into two broad topics. First, in Section 1.1, a broad educational framework is introduced which can be used to give large-scale context for the work that will be done in this thesis, motivating the structure of the work and outlining the key areas of importance. Second, the remainder of this introductory chapter will be spent exploring in more detail the purpose, structure, and context of mathematics bridging courses in general (Section 4.1) and specifically at the UofA (Section 4.2)

- Chapter 2 provides a brief description of the methodology employed in the research that will be presented in Chapters 3 and 5.

- An indepth discussion of the existing literature is presented in Chapter 3: what is known, approaches attempted in the past both in Australia and internationally, frameworks proposed for understanding the secondary-tertiary transition and the maths anxiety-performance link, and some deeper discussion on some of the particularly relevant related concepts.

- One of the major contributions of this thesis is the detailed curriculum mapping which is the focus on Chapter 5 in which the top two levels of senior highschool mathematics in the Australian Curriculum (AC) and South Australian Certificate of Education (SACE) are mapped to each other, and to the content currently in MathsStart and MathsTrack. Detailed discussion of this mapping also includes commentary on how the content relates to typical entry-level university mathematics courses, and other relevant curriculums.

- Finally, in Chapter 6 the conclusions from this work are summarisd, and in particular the interactions between the differnet avenues of research are consolidated. Additional work done outside of this thesis is discussed, and future work is outlined.

1.1 The Curriculum-Assessment Diamond Framework

When considering improvements to the bridging courses, one of the key concepts that comes immediately to mind is content — curriculum. The content of the courses is one of the things that can be most readily modified, and niavely one might think that in this way, improvements to the course could be easily implemneted. However, as discussed by (Mohandas, Wei, & Keeves, 2003) and will be explored in much more depth in Chapter 3, content does not live alone, and cannot be considered independantly of the broader environment. Specifically, there are bidirectional relationships between curriculum (content), learning experiences (the experiences students have while learning), and evaluation (an umbrella term containing several meaningfully different concepts that will be discussed below), as shown in the curriculum triangle of Tyler (1949) in Figure 1.1. To give some simplified examples:

- A test is informed by the content as it must not contain content not taught in the course, and might aim to cover all of (or most of) the content taught in the course. But the results of the test, or even the fact that there is a test at all, can (and should) also influence decisions about what content to include in the course in the first place.
- The learning exeriences students have depends on the content, obviously. But in the other direction, student's experiences should also inform decisions about curriculum.
- If students with a specific concept in a test, perhaps the learning experiences they have surrounding that concept should be re-examined. On the other hand, if the learning experiences students have surrounding a particular concept are framed in a particular way, then the way those concepts are tested should take that into account.

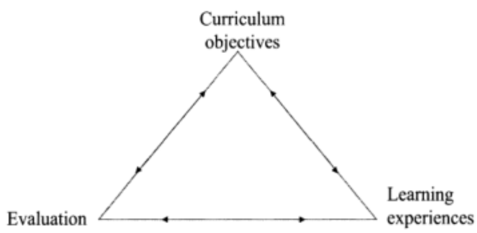

Figure 1.1: The curriculum triangle of Tyler (1949) as visualised in Figure 1 of Mohandas et al. (2003)

Although each of these areas can be considered individually to some degree, it is important that when decisions are made that the bigger picture with all the interactions is taken into account. Mohandas et al. (2003) also make the good point that Evaluation needs to be thought of more granularly, as different forms of evaluation serve very different purposes, and very different roles in both the learning and teaching processes. They expand the curriculum triangle to the "curriculum-evaluation diamond" shown in Figure 1.2, which is of course no diamond at all, but rather an triangular bipyramid with its axis of $\frac{2\pi}{3}$ rotation symmetry representing the fully connected graph of 5 nodes. Mathematical pedantry aside, Mohandas et al. (2003) make the important point that two critical changes should be made to the curriculum triangle model:

- (Student) assessment should be distinguished from evaluation and accountability (Mohandas et al. (2003) also present definitons for each of these terms in order to help distinguish them, of which a concise summary will be included below).
- Standards of performance and how they interact with the other elements play an important role.

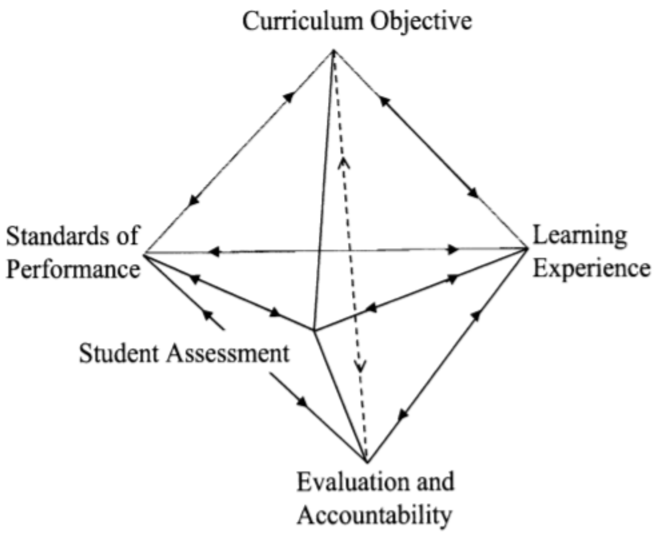

Figure 1.2: The curriculum-assessment diamond as shown in Figure 2 of Mohandas et al. (2003)

The definitions of the terms “assessment”, “evaluation”, and “accoutability” according to Mohandas et al. (2003) and hence as used in Figure 1.2 are useful in order to distinguish between these concepts, and very concisely can be summarised as:

- Assessment usually refers to indivudual students, and it's goal is generally to understand what/ how much learning has occured. It can be performed by eductors, or importantly by students themselves, and it can be formal (tests, exams, assignments) or informal (discussion, practice questions, etc.)
- Evaluation usually refers to some a decision making process: A university evaluates a student to decide if they should be allowed to enroll in a particular degree, for example.
- Accountability usually refers to a responsibility held by an educator or organisation, and is often associated to reporting to some stakeholders.

All three of these terms are important and hold important but very different roles in the context of improving the bridging courses. Assessment is the most important, and in particular student self-assessment as will be discussed in more detail in Chapter 3, but also in terms of the self-pacing of the assignments in the bridging courses, which act as all three: assessmnet (because of the feedback cycle used to help students through the assignments, they are initially used to assess the learning that has occurred and use this information to inform students about how to proceed through feedback), evaluation because the assignments are used to gate students from completion of the courses, and accountability to ensure students are at the required level of knowledge and satisfy the responsibility of ensuring they are adequately prepared for their future studies.

There are two key concepts in which the standards of performance are important. First, because it is important to establish standards based assessmnet in which students are assessed against fixed standards and not against each other (this is widely accepted in the educational literature). Secondly, these fixed standards should be set in clear objectives in mind. It is in this sense that the standards of performance are quite complicated to nail down in the context of the bridging courses. Typically at a university level, standards of performance will be determined by things such as industry standards (for example studying an engineering degree, the industry standards for engineers will apply). Ultimately, the skills and knowledge required of students completing a degree will be determined by the skills and knowledge that the industry hiring those students needs graduates to have. However, with the students in the bridging courses going in so many different directions this is difficult to determine. In Chapter 5 we discuss some of the most common first year subjects students aim to enroll in (which are common to many different degrees), but ultimately as the bridging courses usually fit into the secondary-tertiary transition, i.e. the evaluation of students for university entry, the primary basis for the standards of performance is the senior highshool curriculum, which is discussed in detail in Chapter 5.

This thesis can be thought of as consisting of two broad avenues of research, focusing on different parts of the curriculum-assessment 'diamond' shown in Figure 1.2:

- Chapter 5 explores the curriculum and standards of performance part of the 'diamond' by mapping the national and state curriculums to the current currliculum of the bridging courses, while discussing the various relevant standards of performance to contextualise the advantages and disadvantages of including or excluding particular sections of these curriculums.

- Chapter 3 explores the existing literature in order to make reccomendations around what learning experiences and assessment methodologies are needed in order to facilitate the learning prescribed my the curriculum discussion.

Naturally, and as supported by the 'diamond' framework of Mohandas et al. (2003), neither of these two approaches to improvement of the bridging courses would be successful in isolation, but rather by taking into account both in unison real improvements could be acheived.

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Chapter 2

Methodology

In this chapter the methodology imployed when performing the research presented in the remainder of this thesis is described. There are two main bodies of work in this thesis:

- A literature review, presented in Chapter 3, and
- A curriculum mapping, presented in Chapter 5.

Each of these bodies of work was completed employing a non-overlapping methodology, and so the methodology for each will be described seperately.

2.1 Literature Review

The initial phase of the literature review was performed in an iterative process which given a list of sources (primarily academic papers) involved reading the list of sources and generating a new list repeatedly. In the first iteration, some of the most relevant papers identified included the work of Nicholas and Rylands (2015), Gordon and Nicholas (2013b), and Johnson and O'Keeffe (2016), and these where found by using search engines, including google scholar and the UofA library search with terms such as "mathematics bridging courses" in order to identify several recent, directly relevant, key papers to start with. The iterative process then involved reading the current list of sources, taking notes and quotations for later use, and compiling a new list of sources by:

- Noting relevant references used in the current list,
- Papers referencing these papers (using "cited by" functionality of search engines),
- Additional papers identified by use of seach engines for newly identified key terms, such as "adult-education", "maths anxiety", etc.

Some of the particularly relevant papers which came up in the second and third iterations of this process included the work of Galligan and Taylor (2008) ,Irwin, Baker, and Carter (2018), and Ramirez, Shaw, and Maloney (2018) respectively. This iterative process was performed until the same papers kept coming up more and more frequently, which only took approximately four or five iterations. Then, the notes and quotations made while reading these references where reviewed, and synthesised into a coherent discussion, something akin to a "systematic review", although the individual keywords and search phrases are not explicitly reported, all statements made are traceable (accurately cited). It is important to acknowledge the inherent bias involved in this (and any) literature review methodology. One of the future research directions for this work would be to further expand the literature review to be more comprehensive and less biased in a more systematic way, but the intention of this work was not to provide a comprehensive systematic review, rather a starting point for one to begin from. This starting point is presented in Chapter 3.

2.2 Curriculum Mapping

The curriculum mapping was performed by first establishing the levels of detail of interest, and terminology for these levels of detail. Specifically, there are two levels of detail at which the curriculum mapping is performed: the topic level, and the key concept level. It is important to note that in this work these terms "topic" and "key concept" are used to have the very precise meaning of referring to these two levels of detail. These are discussed in more detail in Chapter 5, but very breifly each curriculum is broken up into approximately 12–24 topics, with each of these topics including typically 6-12 key concepts each.

The first phase of the curriculum mapping methodology was to summarise the key concepts in a concise way such that in the later phases this summary could be used to guide alignment between curriculums. This essentially boils down to the generation of the table presented in Appendix A. This was essentually a "document analysis", and involved carefully reading the curriculum documents associated to each curriculum, and summarising the key concepts in each topic therin. There are three curriculums analysed in this way, and the details of how this phase was performed for each follow:

- For the AC, the curriculum is presented on their website, and both Senior Mathematical Methods and Senior Specialist Mathematics where considered (accessed between Feburary and May 2019). Each of these subjects is broken down into 4 units, and each unit has three components: a description, learning outcomes, and content descriptions. The content descriptions section for each unit is split into three topics, each of these topics corresponds to a topic in our level of detail terminology. The material under these topics in the content descriptions section of each unit was the focus for this curriculum mapping, and this is the material that was read carefully and summarised to generate the key concept list in Appendix A.

- For SACE, the Subject Outline (for teaching in 2019) document was retreived from the SACE website for each of the three relevant subjects: Stage 1 Mathematics, Stage 2 Mathematical Methods, and Stage 2 Specialist Mathematics. In each of these documents, the "LEARNING SCOPE AND REQUIREMENTS" section contains a summary of the curriculum by topic, and each of these topics correspond to a topic in our level of detail terminology. Within each topic, SACE often has subtopics, but we do not consider this level of detail in this curriculum mapping, instead treating each entire topic as a whole. Within each topic, the left-hand column "Key questions and key concepts" was read carefully and summarised to generate the key concept list in Appendix A. As discussed in Chapter 5, the focus of this curriculum mapping is primarily on the content itself, rather than the surrounding concepts involved in how the content is taught (which is more what the right-hand column, "Considerations for developing teaching and learning strategies" is relevant too).

- For the bridging courses, the content is available on their website in the form of a number of booklets for each of the courses: MathsStart and MathsTrack. Each of these booklets will constitute a topic in our level of detail terminology, and these entire booklets where carefully read and summarised to generate the key concept list in Appendix A.

Once the first draft of the key concept summary presented in Appendix A was completed, two mappings where produced: one at the topic level, and one at the key-concept level. The topic level mapping, shown in Figure 5.1, was produced by comparing the broad concepts covered in the topics, with little concern for the details involved with particular alignment of key concepts. For example an "introductory calculus" topic would be mapped to another "introductory calculus" topic, even if a specific concept such as anti-derivitives is introduced in one but not the other. The purpose of this mapping is to provide a high-level view of the mapping between the curriculums in order to help structure the more detailed discussion of key concept alignment. The key concept alignment was then performed by going topic by topic, and aligning every single key concept listed in Appendix A, then in any mismatching cases, referring back to the original curriculum document to check for mistakes and validate any conclusions made. This mapping is obviously too complex to be able to meaningfully represent it graphically, and so instead the conclusions thereof are presented in the form of discussion in Chapter 5. No major mistakes where discovered in this process, but some small modifications where made to Appendix A all of which had to do simply with harmonising terminology used. For example, "slope of a line" vs "gradient of a line", etc. This key concept level mapping was also used to make adjustments to the topic level mapping shown in Figure 5.1. No major changes where made, but single key-concept links where added as dashed lines as a result of the key concept mapping.

Although it was not part of the initial intent, it became apparent in the process of completing the mappings described above that particularly due to the very different structure of the curriculums it would be useful to add another level of detail in which topics where grouped under broad areas of mathematics, and to reproduce another version of Figure 5.1 in which the tipics where rearranged into these broad areas, so this was done producing Figure 5.2.

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Chapter 3

Literature Review

3.1 Introduction

In this chapter the literature surrounding mathematics bridging courses is explored both in an Australian context as well as internationally. The literature reviewed also extends into some directly relevant areas such as general perceptions of mathematics, the secondary-tertiary transition more broadly than just in the context of mathematics education, relevant frameworks that have been proposed, and some of the key areas that prevent students from being successful such as maths anxiety.

Remembering the purpose statement of this thesis, and the clarifying secondary questions that it raises, it is interesting to note that these questions are by no means new questions, although they do not neccessarily have any consensus on how to answer them. In particular, Poladian and Nicholas (2013) offer an insightful discussion of two key (unanswered) questions within bridging mathematics posed by Galligan and Taylor (2008):

- How is success defined in bridging mathematics activities?
- Are successful bridging mathematics students successful university students?

which Poladian and Nicholas (2013) address with the following comments:

- "there are inherent difficulties in defining and measuring success in bridging courses. Godden and Pegg (1993) suggest that formal evaluation of bridging mathematics programs may be contrary to the aims of the programs, and undermine their major strengths of flexibility and student-centred approach. They argue that traditional evaluative techniques are 'just not possible' and 'risk losing the essence of the support and assistance so necessary for these students'. "
- "internationally, bridging mathematics programs have been shown to be highly effective at resolving skill deficiencies for some students (Kajander & Lovric, 2005; Bahr, 2008). In a large US study, (Bahr, 2008, p.442) found that 'remediation has the capacity to fully resolve the academic disadvantage of math skill deficiency' for the quarter of students who 'remediated successfully', but the likelihood of successful remediation declined sharply as the 'depth of remedial need' increased. The latter finding echoes (Wood, 2001)'s remark that bridging programs do not work for very mathematically weak students."

respectively.

TODO: I should comment more on the quotes above, and update intro to literature review once I've finished the lit review, to better reflect the content of the chapter.

3.2 "The Mathematics Problem"

"The mathematics problem" is a term originally coined by Howson et al. (1995) but that has continued to be relevant to the present day, receiving even greater attention and research in recent times. It refers to the trend of declining interest and participation of final year highschool students in mathematics. It also refers to the carry-over effects this has on the success of students in tertiary education (both in mathematics, but also notably in other areas). "The mathematics problem" is a term now used also to describe the downstream impacts these trends have on the economy: modern industries are dominated by a need for mathematically skilled graduates (engineering, science, technology, ...), but the importance of mathematics in these fields is often overlooked from the general populations perspective (King & Cattlin, 2015; Gordon & Nicholas, 2013b).

Barrington and Evans (2016) shows that in Australia, although the number of both advanced and intermediate mathematics year 12 students was increasing over the ten years from 2006 to 2015 (as the overall population of total year 12 students increased), the percentage participation in these subjects steadily declined. James (2019) updates the figures of Barrington and Evans (2016) with data up to 2017, showing a continuation of the same steady trend. These reports also highlight the significant gender gap that exists in mathematics participation in final year highschool students. The gender gap is more dramatic in advanced level mathematics than in the intermediate level, with 37.8% of advanced mathematics year 12 students identifying as female, especially when considering that 51.8% of year 12 students of that year where female. 2017 saw a significant jump in intermediate level mathematics participation by female students, with there being more female students than males for the first time in recorded history (James, 2019). The gender gap in mathematics education is a significant issue that needs to be taken into account when considering unviersity mathematics entry, particularly as the gap is most pronounced in the advanced level subjects which are targetted at university entry. It is an issue recognised by the Australian Mathematical Sciences Institute (AMSI), who have commited significant resources towards programs intended to address this inequity over the past two decades in particular. Perhaps the uptick in female student participation in intermediate level mathematics in 2017 could be partly attributed to some of these programs, such as the CHOOSE**MATHS** project. Brown (2009) gives a shocking wider-view picture of this overall trend, specifically that the proportion of year 12 students studying intermediate or advanced level mathematics has declined by 22% and 27% respectively from 1995 to 2007.

Amongt other reasons, this decline in participation in mathematics is a problem in Australia because mathematical skills are essential to just about all the key future industries (Croft, Harrison, & Robinson, 2009), and hence the Australian economy. The key economic importance of mathematics is widely acknowledged amongst the academy and industry, but it's importance is often overlooked and difficult to communicate to the wider community because of it's indirect importance through what are perceived to be other fields: engineering, science, etc. all of which require a deep level of mathematical skills, but aren't associated to mathematics in the general populations view. Thomas, Muchatuta, and Wood (2009) argues that one of the most influential factors in the declining participation in mathematics is the "community's perception that mathematics is not useful in the marketplace". Gordon and Nicholas (2013b) go on to emphasise the carry-on effects of negative community perceptions of mathematics leading to highscool students choosing not to participate in higher-level mathematics impacting on not only their success in university, but on whether they continue to study mathematics at all. This obviously has implicatons for mathematics bridging courses at universities — students who previously had de-prioritised their own mathematics education in favour of pursuing these other fields, notably engineering for example, will often turn to bridging courses when they reaslise the importance of mathematics in being successful in the field of their interest.

Observation, concern surrounding, and research of this decline in mathematics participation in senior highschools are not limited to Australia (Hourigan & O'Donoghue, 2007; Hoyles, Newman, & Noss, 2001). Hoyles et al. (2001), as well as Luk (2005) further connect this trend to another: the apparent divergence of content (curriculum) between senior secondary and tertiary education. This divergence of curriculum is a point that will be explored extensively in Chapter 5. In a landmark study, Kajander and Lovric (2005) identified a gap between secondary and tertiary mathematics education in Canada. In the United Kingdom Tariq (2002) noted a decline in numeracy skills among first-year bioscience students. This trend is neither limited to Australia, nor new. Universities around the world have recognised this continuing problem for some time, but oppinions on how to address it vary. Robinson (2003) suggested that the standard for highschool mathematics should be raised, but even if there where consensus amongst the academy that this was appropriate (which there is not), this is beyond the power of universities to control (although, the setting of pre-requisites is a topic that will be explored in more detail below). Within the power of universities to implement are solutions such as to introduce "remedial mathematics" into first-year teaching programmes as highlighted by Kitchen (1999). More recently, as Moses et al. (2011) suggest, universities have been increasing their reliance on "advanced and targeted preparatory programmes" — i.e. briding courses. As an example of this from outside Australia, Faulkner, Hannigan, and Gill (2010) note that at the University of Limerick in Ireland

> "there has been a 20–25% reduction in students attending their first service mathematics lecture, a 12–16% reduction in the number of students entering service mathematics modules with higher level mathematics and an 8–12% increase in the number of non-standard students. Such changes place additional pressure on support services like MLCs whose primary function is to provide the necessary and appropriate support to all university students." (Johnson & O'Keeffe, 2016)

3.3 The Secondary-Tertiary Education Transition

A key step we are interested in from the perspective of bridging courses is university entry, or more broadly: the transition from secondary to tertiary education. It may seem obvious that students engagement and performance in mathematics in secondary education is a strong predictor of their success in tertiary mathematics education, but the exact relationship has some important subtleties. Specifically, it has been shown that the level of mathematics completed in highschool (advanced, intermediate, etc.) is substantially worse at predicting success in tertiary mathematics education than when combined with the level of acheivement in secondary school (Kajander & Lovric, 2005; Nicholas, Poladian, Mack, & Wilson, 2015). Students having completed a lower level of mathematics in secondary school to a higher degree of acheivement can in some cases have a higher chance of success in tertiary education than students who completed a higher level of mathematics in secondary school but to a lower level of acheivement. Although this might seem intuitive, it is not entirely obvious when looking at it in terms of content — curriculum — alone. It should not be understated that although it has been shown quite clearly that the effect of bridging courses is smaller than the effect of highschool engagement in mathematics education, that bridging courses have been shown to have a substantial effect nonetheless, and even more importantly have been shown to fill a critical gap in addressing student needs (MacGillivray, 2009). This is important to acknowledge, and will come into the discussion surrounding university entry requirements below, but engaging students in mathematics in secondary school is beyond the scope of this work, although it is clearly a very important aspect of "the mathematics problem". For now, we consider that one of the roles of bridging courses is to make tertiary mathematics education accessible to all students, including those that where disengaged with mathematics in highschool and therefore are in particularly high risk in tertiary education.

Rite of Passage Model

Very little has been done in terms of developing educational frameworks for understanding the secondary-tertiary transition more systematically, but Clark and Lovric (2008) suggest using the pre-existing and well-undestood literature surrounding the concept of a 'rite of passage' from anthropology and culture studied (relating concepts such as culture shock) to help structure our thinking about about the difficulties and evaluating strategies to address difficulties with the secondary-tertiary transition. Clark and Lovric (2008) propose using the seminal work of Arnold van Gennep and thinking about a "life crisis" event as consisting of three phrases: separation, liminal, and incorporation. One of key and important implications this perspective has is that this transition does not only involve difficulty for the individuals (students), but the broader community (their family, teachers, etc.). The wider communities negative perceptions of mathematics are widely acknowledged to have a substantial effect on students attitudes, and hence success (King & Cattlin, 2015; Gordon & Nicholas, 2013b), and it is important to take this into account. One of the immediate consequences the "rite-of-passage" model implies is that "It is normal to feel discomfort during a rite of passage but much easier to deal with if this is expected." (Clark & Lovric, 2008). This is a key take-away: setting clear expectations is critical for students to be able to cope with the difficulty of this transition, they need to know that it will be difficult, so they can expect that difficulty and come into it prepared.

NOTE: There are also some suggestions made by Clark and Lovric (2008) that I disagree with. Specifically, that we abandon imprecise language and descriptions of concepts, in favour of rigorous explicit anguage. I should probably add some discussin of this here.

None-the-less, the "rite-of-passage" model of Clark and Lovric (2008) aligns well with the broader literature and research surrounding bridging courses and the secondary-tertiary education transition. Specifically, the concept of being socially isolated and needing to adapt to a new environment with different expectations and social norms is reflected widely in the academic writing. Gordon and Nicholas (2013b) disuccses how one of the key valuble experiences students got out of the bridging courses at the Unviersity of Sydney was the interactions with peers and teachers. This experience is supported by literature discussing the importance of social and interactive learning as a formative element of early university experience that is highly predictive of retention (Peat, Dalziel, & Grant, 2001; Trotter & Roberts, 2006) particularly for students whose family or friends are for example from a "working class" background (Leese, 2010), or from a cultural background less familiar with the social norms and expectations associated with university education. In particular, self-motivation and independant learning are expectations that consistently come up as being shock factors for students transitioning from secondary to tertiary education (Murtagh, 2010).

Assumed Knowledge and Conditions of Entry

Contributing to the problem of expectations not being set explicitly, in recent years Australia universities have been moving away from prerequisites for entry towards a "assummed knowledge" approach. What this means is that instead of requiring students to have completed certain subjects in highschool in order to allow them to enroll in a course at university, they instead put the content from those subjects under "assumed knowledge", allow students to enrol in the subject even if they have not completed the highschool subject, and put the onus for having that knowledge on the students. That is how the universities see it, anyway. How the students see it is quite different, as demonstrated by the work of Gordon and Nicholas (2015), who show substantial variance in student perceptions of "'assumed knowledge' ranging from perceiving it as vague and pointless 'stuff' to a cohesive body of foundational knowledge for tertiary study". One of the consequences of this is the increasing under-preparedness of first year undergraduate students.

The issue of entry requirements into university and prerequisites being moved into "assumed knowledge" is an even more complex issue than it might at first appear. Varsavsky (2010) discuss how in Australia the way university entry is managed may infact be contributing to the problem of low participation in higher level senior highschool mathematics. Specifically, the absence of prerequisite subjects in many universities means the only condition of entry to university is the acheivement of a sufficiently high "tertiary entrance rank", a score calculated based on acheivement in all final highschool year subjects, with some adjustments for the combination of difficulties of the subjects. A substantial amount of effort is gone too by final year highschool students, teachers, and councellors to optimse students performance on this tertiary entrance rank through very careful choice of which subjects to take in their final year of highschool. Often this will result in creating a tension between acheiving a high tertiary entrance rank and hence being accepted into university, and having the required knowledge to be successful in university because the subjects chosen are not those containing the content relevant to the degree the student is enrolling in (Gordon & Nicholas, 2013a; Poladian & Nicholas, 2013). This is of course an issue that generalises far beyond mathematics, but to every area of study. Gordon and Nicholas (2013b) claim that: "the major reasons for students taking lower levels of mathematics in senior year(s), or dropping mathematics, include finding enough time for non-mathematics subjects, confidence in mathematical capability, advice and maximizing potential ranking for university admission". Rylands and Coady (2009) demonstrated that what a student studied in senior highschool predicted their performance at university, whole their tertiary entrance rank did not. The result in the bridging course literature that although bridging courses can help, their effect cannot compare with engagement in highschool is a result that has been reproduced many times in the literature accross many countries (Kajander & Lovric, 2005; Nicholas et al., 2015; Tariq, 2002). This is likely, as suggested by Kajander and Lovric (2005), due to the time-period typically involved. A bridging course is usually a short preparatory course covered in an interim before beggining tertiary study, while highschool engagement is a learning and teaching experience spanning several years. Despite Australia's Chief Scientist reccomending moving back to pre-requisites (Chubb, Findlay, Du, Burmester, & Kusa, 2012), there is no sign of this being on the table: the commercial aspect of universities demands increased enrollment of students, and that means relaxing entry conditions.

NOTE: (King & Cattlin, 2015) has more to say on this topic, I should review that paper and maybe adjust/ add some more discussion here.

3.4 Maths Anxiety

Why is Maths Anxiety Important?

Maths anxiety is hugely prevalent, the 2012 Programme for International Student Assessment (PISA) report states that across Organisation for Economic Co-operation and Development (OECD) countries, over 30% of 15 year old students "get very nervous doing mathematics problems", and over 60% of students "worry about getting poor grades in mathematics" (OECD, 2013). This not only impacts on those students in terms of their academic performance and subject choice, but given this has been an ongoing issue for many decades with literature documenting it dating back to the 1950's (Dreger & Aiken Jr, 1957), it is also a community issue — parents, and teachers also suffering from such anxieties and hence both normalising the behaviour as well as actively passing it down. It has been shown that students with a high level of maths anxiety often literally experience the anticipation of a maths task as visceral pain (Lyons & Beilock, 2012), this is no small issue.

Even if the wellbeing issue was not enough, there is also a clear maths anxiety-performance connection, which is where it holds particular relevance to enrollments in bridging courses. Students enrolling in bridging courses are more likely to have performed poorly in highschool and given the prevalence of maths anxiety and the strength of the maths anxiety-performance link, are more likely to suffer from maths anxiety. This inference is supported by the survey studies of bridging course students by Nicholas, Gordon and Polodian. One example of this is highlighted by Foley et al. (2017) who juxtaposes the internationally rising demand for Science, Technology, Engineering and Mathematics (STEM) professionals with the negative correlation between maths anxiety and performance shown in the 2012 PISA report (OECD, 2013) to highlight the relevance of addressing maths anxiety in filling this demand, aligning with 'the mathematics problem" discussed earlier in this chapter. The relationship between maths anxiety and maths-qualified professionals in the workforce is supported throughout the literature: when a student has low self-concept (correlated with high maths anxiety), they will tend not to enroll in maths beyond the minimum requirements for graduation (Ashcraft, Krause, & Hopko, 2007), and students affect towards maths can predict their university major (LeFevre, Kulak, & Heymans, 1992). Beyond this example, the list of stakeholders in a students academic success in maths goes on and on: parents; the student's themselves; schools (which are often funded based on the results of standardised testing such as National Assessment Program — Literacy and Numeracy (NAPLAN)), and teachers amongst them. From the perspective of bridging courses, this link is important because A) it motivates supporting these maths anxious students to pursue a tertiary mathematics educations, but also B) because industry is an important stakeholder in tertiary education, inlcuding bridging courses.

Maths Anxiety as Distinct from General Anxiety

The existence of maths anxiety as "emotional disturbances in the presence of mathematics" has been noted as early as the 1950's, Dreger and Aiken Jr (1957) even postulated that what he tentatively designated "Number Anxiety" and later became to be known as Maths Anxiety could be a distinct syndrome from general anxiety. Later the landmark meta-study of Hembree (1990) supported this hypothesis, showing a correlation of only 0.38 between maths anxiety and general anxiety. In more recent times, this hypothesis has also been confirmed by Young, Wu, and Menon (2012) using functional magnetic resonance imaging (fMRI) to show that the brain activity in a person experiencing maths anxiety is measurably distinct from that in a person suffering general anxiety. These later studies, as well as the the work of Kazelskis et al. (2000) and more, have also delineated maths anxiety from test anxiety, and these different anxieties exisitng as meaningfully distinct constructs is now quite well accepted. For more on the history of maths anxiety, Suárez-Pellicioni, Núñez-Peña, and Colomé (2016) offers a more detailed review.

Frameworks for Understanding Maths Anxiety

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Only a few studies focus on maths anxiety itself (primarily fMRI studies such as those of Young et al. (2012) or Lyons and Beilock (2012)). Instead the bulk of the literature is focused on the maths anxiety-performance link. Specifically, there seem to be two distinct theories being pursued and I will adopt the terminology of Ramirez et al. (2018) to describe them: the "Disruption Account" and the "Reduced Competency Account". Ramirez et al. (2018) go on to make a convincing argument that although these two theories might seem to compete, they are not actually mutually exclusive and instead quite compatible with each other. Ramirez et al. (2018) suggests a third "Interpretation Account" which encapsulates observations made by both lines of research, see Figure 3.1.

First, a little more detail on the existing theories. The "Disruption Account", spearheaded by the work of Ashcraft et al., is centered around the concept of working memory (Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007). Specifically that anxiety about maths takes up students working memory, which prevents them from

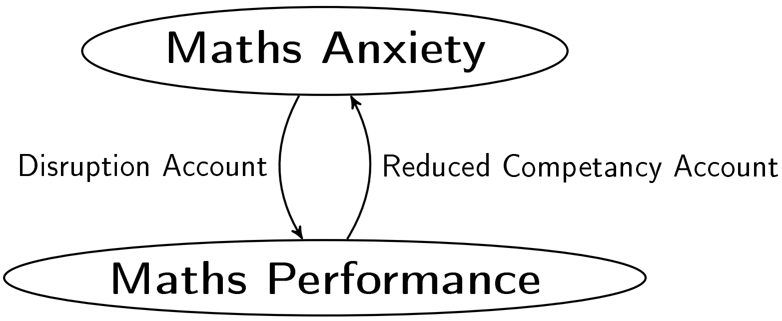

Figure 3.1: The Interpretation Account of Ramirez et al. (2018) for the maths anxiety-performance link showing how the Disruption Account and the Reduced Competency Account can be compatible.

using that working memory to complete maths tasks and thereby impacts their performance. The "Reduced Competency Account" on the other hand proposes the opposite causality: that lower ability in maths leads to negative experiences associated to maths, which in turn cause maths anxiety to develop. There is also a significant body of work to support this hypothesis, including the milestone meta-analysis of Hembree (1990) and the longitudinal study of Ma and Xu (2004) which found that although past maths anxiety was correlated with future maths performance it was a small effect, while past maths performance had a strong effect on future maths anxiety.

Complexities in Finding Effective Interventions

These theoretical views are of course broad oversimplifications of what is an incredibly complex and interconnected topic. They also imply very different approaches for intervention. The "Reduced Competency Account" would imply interventions to boost maths performance and hence allow students to experience success in math should also help to reduce maths anxiety. The results of Supekar, Iuculano, Chen, and Menon (2015) seem to support this hypothesis as when students are given an intensive 8-week tutoring program to boost their maths skills, this is associated to a reduction in maths anxiety. The earlier work by Faust (1996) further supports this by demonstrating an anxiety-complexity effect in which low and high maths anxiety groups performed similarly on low complexity problems, but in high complexity problems the high anxiety groups performance was impacted. On the other hand, Jansen et al. (2013) showed that it is not neccessarily that simple, by showing that when students experience more success they attempt more problems and perform better. However their improved performance is almost completely predicted by the number of problems they attempted, not their experience of success, and their level of maths anxiety was not affected in a significant way which raises a lot of interesting but unanswered questions about this approach.

On the other side of attempted interventions are those in line with the "Disruption Account", in which the maths anxiety itself is addressed in the hopes that will free up extra working memory and hence boost students performance. Park, Ramirez, and Beilock (2014) demonstrate a direct and successful attempt at this in which they used expressive writing exercises to help guide students self-perceived narratives about their maths experiences and thereby reduce their maths anxiety. Notably the approach of Park et al. (2014) is in line with successful treatments for clinical anxiety disorders (see McNally (2007); Becker, Darius, and Schaumberg (2007); Foa et al. (2005)). Another approach that has shown success in this vein does not attempt to directly reduce the anxiety experienced, but rather reappraise it's symptoms (Jamieson, Peters, Greenwood, & Altose, 2016). This is another technique from clinical psychology in which stress is reconceptualised as a coping tool, an evolutionary method for heightening performance in response to a challenge to be overcome, instead of a symptom of exposure to something to be feared and avoided. This change in the perspective of stress is also very much in line with the "Interpretation Account" of Ramirez et al. (2018).

The work of Wang et al. (2015) showed the role that intrinsic motivation has mediating the relationship between maths anxiety and performance, and suggested the importance of a mindset centred on viewing the process of learning maths as one of "productive struggle". This reconceptualisation to a 'productive struggle' model is supported by other literature as well, Lin-Siegler, Ahn, Chen, Fang, and Luna-Lucero (2016) exposes students in a classroom to struggles experienced by famous scientists in order to help normalise the concept of productive struggle, and Hiebert and Grouws (2007) discuss the importance of this same concept in a maths context.

One of the implications of the "Interpretation Account" is that if an intervention targets only one of these two possible links in the cycle (see Figure 3.1), the cycle may re-establish itself after the intervention is over and negate any potential longterm effects. However there is only a very limited amount of research out there on such longterm effects, and several authors have discussed the need for further research into this (Suárez-Pellicioni et al., 2016; Chang & Beilock, 2016). My hypothesis is that a multi-faceted approach targetting both directions simultaneously could disrupt the cycle shown in Figure 3.1 and result in significant longterm effects.

3.5 Self-Efficacy

Much of the early research into self-efficacy has been structured based on the "social cognitive theory" of Bandura, so it seems appropriate to begin with a quote. Bandura (1997, pg391) defines self-efficacy as

> "people's judgement of their capabilities to organize and execute courses of action required to attain designated types of performance"

The connection between student's mathematical self-efficacy and their success in university preparatory mathematics courses has been well established in the literature (Burton, 1987; Klinger, 2006, 2011).

To quote Johnson and O'Keeffe (2016):

> "Self-efficacy is vital among all students but particularly among adult learners as an individual's beliefs of self-capability has been shown to affect motivation, performance, achieve- ment, effort, willingness to persist with a task, as well as the anxiety they experience (Bandura, 1997; Pajares & Miller, 1994; Pajares, 1996; Pajares & Miller, 1997; Pajares & Graham, 1999). Woodley (1987) (cited in (McGivney, 1996)) noted that the main personal factors that contribute to dropout are: self-perception,

being disorganised, not having sufficient study skills and lacking in self-confidence. This suggests that an indi- vidual's self-efficacy plays a role in their decision with regard to dropping out.

(Hackett & Betz, 1989) and (Pajares & Miller, 1994) and Pajares and Miller (1995) also found that self- efficacy can have an impact on career choice. In these studies, it was found that math- ematical self-efficacy is a stronger predictor of students' mathematical interest and choice of degree programmes than either prior mathematical achievement or math- ematical outcome expectations. Self-efficacy also influences how often mathematics is used, as well as an individual'swillingness to pursue advancedwork in mathematics, and even the choice of prospective occupations (Dutton and Dutton 1991). Engineers Ireland (2010) highlight that this avoidance of mathematics, and mathematics-related courses, at university will eventually prove detrimental when attempting to build a knowledge economy. This point was also stressed decades before by (Hembree, 1990, pg34) when he stated that 'when otherwise capable students avoid the study of mathematics, their options regarding careers are reduced, eroding the country's resource base in science and technology'."

It is also well established that mathematical self-efficacy is strongly correlated to a combination of current knowledge/skills and current performance, with females and those who have not studied in a longer period of time generally having lower self-efficacy (C. S. Carmichael, Dunn, & Taylor, 2006; C. Carmichael & Taylor, 2005). In a group of so-called "adult learners", Klinger (2006) confirmed a result from several previous studies and showed that although the negative perceptions of mathematics widely held by the general population and demonstrated to negatively impact on mathematics performance where represented strongly on initial enrollment into a mathematics bridging course, that these negative perceptions changed dramatically during the course. The conclusion being that although yes, these negative perceptions of mathematics are highly predictive of performance, they can also be substantially influenced by early learning experiences, and should certainly not be thought of as fixed variables. In a later study, Klinger (2008) replicated a similar study but accross several diciplines of study and showed that arts/humanities had substantially lower mahtematical self-efficacy and more negative perceptions towards mathematics than the science students. Notably, Klinger (2008) also showed a strong link between their quantitative results around mathemaical self-efficacy/ negative perceptions of mathematics and gender — with female students scoring worse than males. All this research only further supports that weak mathematical skills cannot be addressed with content alone, but that the students negative preconceptions towards mathematics and poor mathematical self-efficacy views must also be addressed in order to support students on their way to success in a variety of fields of study, not only the study of mathematics. These results are all strongly in support of the Ramirez et al. (2018) "Interpretation Account" framework shown in Figure 3.1 as well as the "curriculum-diamond" model shown in Figure 1.2 in the sense that they imply a joint approach is required: simultaneously improving students knowledge/ skills and their self-efficacy/ affect towards mathematics. These two are so inextricably linked, that one cannot hope to successfully address one without also addressing the other. (Taylor & Galligan, 2006) used conversation theory framework to design an approach that was intended to simultaneously develop students' mathematical knowledge/ skills and improve their mathematics self-efficacy/ confidence, which was shown to be effective.

To summarise in the words of Galligan and Taylor (2008):

> "... although attitudes and beliefs about mathematics are important for students enrolled in bridging programs, the programs can change students' attitudes and beliefs about mathematics as well as their achievement."

3.6 Implications for Bridging Courses

One of the primary roles of bridging courses is to facilitate students secondary-tertiary education transtition. Often the students enrolling in bridging courses will have either:

- Performed poorly in mathematics in secondary school,
- Chosen to study mathematics at a intermediate or elementary level in secondary school, or
- Had a substantial time gap between completeing secondary school and engaging in tertiary education,

or some combination thereof. All of these possibilities will be associated with higher than average levels of mathematics anxiety, and negative preconceptions of mathematics. So, in this context, the question here is

> How can a bridging course best support students through their transition into tertiary education?

At a fundamental level, there are two key barriers that these students must overcome to be successful in their tertiary education:

- Developing sufficient mathematics skills, capabilities, and knowledge. This can be addressed through content — curriculum, and traditional teaching practices.
- Overcoming/ changing negative perceptions/affect/anxiety towards mathematics. This is difficult to address, but there are a number of approaches suggested in the literature.

One conclusion that is consistent with all the literature reviewed in this chapter, and is an implication of two of the major frameworks considered in this work: the curriculum-assessment diamond framework shown in Figure 1.2, and the interpretation account of shown in Figure 3.1, is that these two key barriers must both be addressed simultaneously in order to have an effective and longlasting impact on student's success.

Students having completed bridging courses have commented on the importance of this kind of combined approach. In the survey of Gordon and Nicholas (2013b),

> "students are aware of the value of the bridging courses not only to ameliorate prior difficulties with mathemat- ics and improve their approaches to learning mathematics but, less trans- parently, as an important opportunity to facilitate their transition into higher education, meet fellow students and help realise their potential."

Core to addressing the first of these two key barriers is the content, and what the appropriate content to teach in the bridging courses will be the focus of Chapter 5. In terms of how to best address the second these key barriers, there are a number of points on which there is broad agreement amongst the literature reviewed in this chapter, but there is a single message that draws together most of these points, which is to:

SET CLEAR EXPECTATIONS.

To give some examples of how this is featured in the literature, the "rite-of-passage" model of Clark and Lovric (2008) suggests it is critical to set clear expectations around the new (tertiary) learning environment, as students are transitioning into a new and unfamiliar social environment/ community, it is critical to be explicit with them about the expectations in this new environment (i.e. independant learning, didactic lectures, etc.). The "rite-of-passage" model also suggests it is important that expectations be set for students about the difficulty of this transition beforehand (in the years prior to them making the transition to tertiary education) so that they come into the transition expecting it to be difficult and therefore being prepared for that difficulty. This perspective is further supported by the literature on how the perspective of viewing the process of learning mathematics (or learning in general) through the lens/ expectation of "productive struggle" particularly in the context of intervening to help maths anxious students (Wang et al., 2015; Lin-Siegler et al., 2016; Hiebert & Grouws, 2007; Carlson, 1999). Similarly, approaches taken from clinical psychology for the treatment of generalised anxiety disordershave been successful in helping maths anxious students, and reflect the same principles behind the concept of "productive struggle". Universities relaxing pre-requisites to "assumed knowledge" is a good (bad) example of *not* setting clear expectations, and this impacting directly on students (Gordon & Nicholas, 2015). Some of these points are beyond the scope of this work, but there are also actions that can be taken from the perspective of teaching a bridging course to mitigate some of these concerns: even if students come into university without the expectation of it being a difficult culture-shock event and are not prepared, being clear and explicit with them about how it will be difficult, but that that is ok, can help. Similarly, changing university entry requirements is beyond the scope of this work, but even so when students come into the bridging courses with misconceptions such as "mathematics is not important to being successful in science", correcting these misconceptions can be very beneficial for them in terms of their success in pursuing their goals.

Additionally, although most of the reccomendations fall under "set clear expectations", some do not, these additional reccomendations include:

- Helping students meet other students, make friends, and develop a social support network in their new environment is critical to supporting them to be successful, this is implied by the "rite-of-passage" framework of (Clark & Lovric, 2008), but also by a swath of other literature (Trotter & Roberts, 2006; Peat et al., 2001; Leese, 2010; Gordon & Nicholas, 2013b).

- An emphasise on "learning-to-learn" programmes has been shown to be effective (Zeegers & Martin, 2001)

Finally, there are some other important discussion points to be aware of, although no specific actionable reccomendations come from them:

- Success in secondary school mathematics is highly predictive of success and even participation in tertiary mathematics education. This predictive effect is larger tha n the effect of any bridging course on retention and success in tertirary eduction (Kajander & Lovric, 2005; Nicholas et al., 2015). This is important to be aware of, but unfortunately falls outside of the scope of a bridging course to address. Instead, we have to rely on secondary school educators to continue working to improve this.

- Negative community perceptions of mathematics influence rates of maths anxiety, engagement and ultimately success in mathematics educations of our students (King & Cattlin, 2015; Gordon & Nicholas, 2013b; Clark & Lovric, 2008). Again, negative community perceptions is (somewhat) beyond the scope of a bridging course to address but it is critical to be aware of the impact it has, and to be fair it does fall on all mathematicians but even more so non-mathematician mathematically skilled people and educators (including those teaching a bridging course) to gradually create the social change needed to adjust such widespread community perceptions. Broad cultural change is somewhat beyond the scope of this work, however.

3.7 Temporary Section: TODO List

TODO List (Papers to read in more detail, and incorporate into discussion):

-

- (Irwin et al., 2018) Talks about the importance of alternate avenues to access education. — read and summarise again.

- Some of the books I found would be good to read more thoroughly too, (Volmink, 1994) for example, and/or (McGivney, 1996) for example.

- Quote from (Galligan & Taylor, 2008) covering some concepts that would be nice to incorporate into the discussion somewhere:

  > Miller-Reilly (2006), in a significant long term analysis of two large bridging courses and one individual bridging program in New Zealand, used quantitative and qualitative measures to compare students' reactions. These multiple strands of evidence provided a complex overall picture of three largely successful teaching approaches. A one-to-one supervised course focused on understanding fear of mathematics and early mathematics experiences. The course empowered the student who came to believe that mathematics was a creative and enjoyable process. A second course (100 students) focused on the mathematization of realistic situations. Here, students came to regard mathematics as useful, interesting, and relevant to real life. The third course (100 students) was a carefully structured reintroduction of mathematics. The students appreciated the course and were pleased that they could now do mathematics that they could not do in schqol. Students in all programs were highly motivated, mature, and had not seen formal mathematics for some years. One surprising result from the study was that if students were unsuccessful they were, in fact, worse otT than before, and often confused. A significant component of the study was the focus placed on dealing with students' mathematics anxiety or fear. Quantitative measures and qualitative descriptors indicated a decrease in mathematics anxiety throughout the duration of the three programs, and in the larger courses this correlated with achievement. Beliefs about mathematics in general, however, did not necessarily change, although students in the larger courses did see the practical nature of mathematics.

- Student wellbeing is a research interest, with high strain in the first semester at uni (Bewick, Koutsopoulou, Miles, Slaa, & Barkham, 2010)

Chapter 4

Context

4.1 Bridging Courses

Students will usually enroll in university mathematics bridging courses because they are required to demonstrate a certain level of mathematical knowledge/ competence before commencing study at university, but either do not meet those requirements, or do but feel a lack of confidence in their abilites and feel like they need to refresh/ revise/ learn some mathematics prior to commencing their studies.

Reasons why these students do not either meet the entry requirements, or feel a lack of confidence in their abilities can be quite varied:

- A long period of time may have passed since they last studied mathematics (or studied at all). Adult-entry students are over-represented in bridging courses (REFERENCE?).
- They may have performed poorly in mathematics in highschool.
- They may have chosen not to study mathematics at a higher level in highschool.
- They may suffer from maths anxiety (which would make them likely to fit into the above two categories as well).

The role of mathematics bridging courses is to take these students, and:

- Bridge their content knowledge so they are prepared for university entry.
- Support the growth of their confidence and self-efficacy surrounding mathematics.
- Ultimately prepare them to be successful in a university context.

What content should be taught in a university bridging course is actually a question that has dramatically different answers from different perspectives on the role of such a course, even when restricting the question to purely knowledge-based content (and excluding the teaching of self-efficacy etc.):

- If you take the perspective that the role of such a course is to fill in the gaps in student's knowledge left from an incomplete or maths-light highschool education, then the content that should be taught should be up to and including the advanced year 12 australian curriculum. This is particularly appropriate if you do not know the direction of the students, or if they are potentially just doing the bridging course with you and they are planning on studying a degree at a different university say, interstate.

- If you take the perspective that the role of such a course is to prepare students for entry into the particular courses they are about to commence studying, the content relevant to them will be dramatically differnet. The senior mathematics australian curriculum is extremely generalist and contains many topics that would be completely irrelevant to any particular field of study.

In terms of choosing what content to teach in a university bridging course, the above two competing perspectives will often create tension between each other, making finding a happy compromise a difficult endevour.

Quote from (Johnson & O'Keeffe, 2016):

> (Hardin, 2008) highlights that in recent years, the 'face of higher education' has changed, with a more diverse range of leaners now entering third-level education. (Hardin, 2008) notes that in 1987, the number of adult learners 2 in College or University in the U.S. had increased to 4.9 million and the 2010 projections were set at 6.8 million. In the Irish context, the National Adult Learning Organisation (Aontas) identified that in 2012, adult learners accounted for 15% of all third-level full-time students and 96% of all part-time students. According to Aontas (2012), these percentages equate to circa 6000 full-time and 1500 part-time adult learners each year. (Murtaugh, Burns, & Schuster, 1999) point out that increases in the number of adult learners can cause additional retention worries for university policy-makers, as research shows that attrition rates have been found to increase with age. Further to this statistic, studies such as those conducted by (House, 2000) and (Tsui, 2007) indicate that significant numbers of students dropout of STEM degree programmes within the first two years, which highlights the importance of addressing this retention issue as early as possible in a student's career. One approach that has proven effective in addressing this issue is to encourage students to engage with mathematics learner support provisions. (Lee, Harrison, Pell, & Robinson, 2008) advocate that appropriate engagement with mathematics learner support can have a positive impact on student retention and progression.

4.2 MathsStart and MathsTrack

The Univesity of Adelaide offers two mathematics bridging courses, MathsStart and MathsTrack, through their maths learning center. According to the staff at the maths learning center, the vast majority of students enrolling in their bridging courses are aiming to end up in one of three places:

- Studying a tertiary degree at The University of Adelaide (approximately 60–70% of bridging courses at any one time),
- Studying a tertirary degree at James Cook University, or

- In the defence forces.

Only about a single student will not fit into any of the three categories above at any one time, so thinking of this as being the complete cohort of students is fairly close to being accurate. The distribution within these categories can also be broken down and the most common trends considered:

- Of the students aiming to enroll in a tertiary degree at the University of Adelaide, about 50% are aiming to study something in the Faculty of Engineering, Computer and Mathematical Sciences (ECMS) (i.e. Engineering, Mathematics, Computer Science, etc.), and about 10% are aiming to study something in the sciences, often vetenary science or oral health.

- Of the students aiming to enroll at James Cook University, most are aiming to enroll in medical degrees, with some interested in marine biology or vetenary science — broadly biological science in large.

- Of the studets aiming to enlist in the defence forces, the majority of those enrolled in the briding courses are doing so to meet their pre-requisite mathematics knowledge criteria for airforce pilot training.

To meet the needs of these cohort of students, it is clear there are two broad areas of content knowledge that different students will need:

- Calculus focused maths, differentiation, integration, and understanding of functions and graphs are fundamental to the students aiming to study in ECMS at the University of Adelaide, as most entry level mathematics and engineering subjects are very calculus-focused, and this calculus-emphasis is carried through both engineering and mathematics degrees.

- Probability/ Statistics are an important focus of the science, with particularly biological science students having been found to struggle with the mathematical (largely statistical) requirements of their degrees (ADD REFERENCE HERE?).

Both of these focus-areas require a base of content knowledge of fundamentals, things like re-arranging equations, fractions, log-laws, etc. So it makes sense to structure the content of the bridging courses in a way aligned with this thinking, in order to best meet the needs of the students enrolled in the programs.

4.3 Current Strengths of MathsStart and MathsTrack

Moving foward is a two-part process:

- Recognise what is being done well, encourage and recognise it, and continue to support its ongoing excellence.

- Recognise what can be improved on, gaps that may exist, and address them with specific actionable changes.

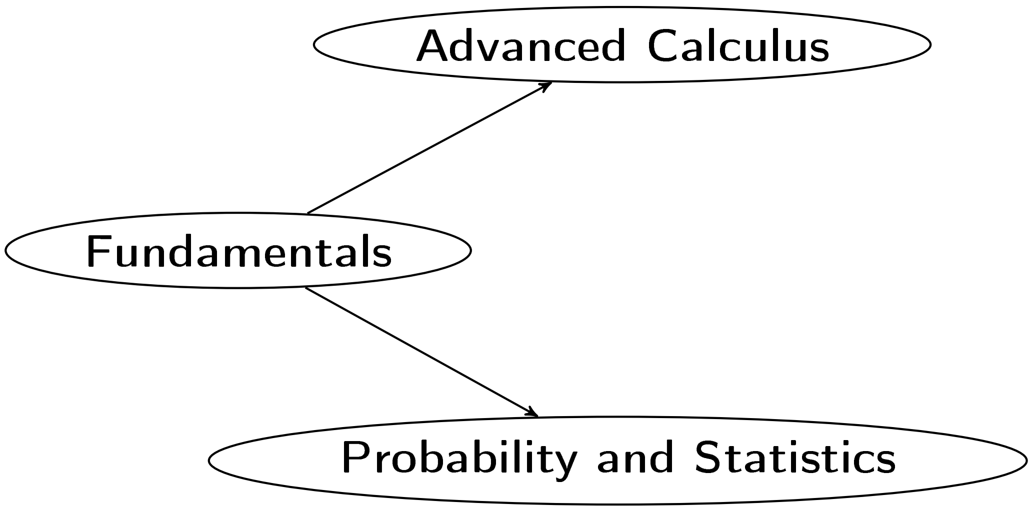

Figure 4.1: Ideal High-Level Content Structure for the University of Adelaide Bridging Courses

4.3.1 SQWIGLES

SQWIGLES is an abbreviation (see Figure 4.2) developed at the University of Adelaide maths learning centre with several purposes in mind:

- As a way to guide tutors working at the maths learning center on how to best help the students.
- To scaffold self-reflection in teaching and education when working one-on-one with students by providing specific actions one can focus on paying attention to and reflecting on.
- As a tool to focus efforts on self-improvement, by choosing one action to improve on at a time it can provide an avenue towards improvement that feels acheivable, and can provide concrete progression.

SQWIGLES also had some ancillary and unintended benefits, as once an educator is engaged actively in self-reflection, even if it is prompted by paying attention to one specific action, they notice other things perhaps even things unrelated to SQWIGLES itself. David Butler, the current maths learning center coordinator at the University of Adelaide, wrote a blog post about SQWIGLES which is quite informative and has more detail, but here I will provide a breif overview as it is a very beneficial tool that generalises well beyond mathematics education.

As teachers we are constantly encouraged to reflect on our practice and continually aim to improve and develop our skills, and rightfully so. Teaching is a process of continual improvement. Often however, we can reflect on our practice and either be overwhelmed by the amount of observations we make and not know where to start, or be unsure what to reflect on, exactly — what aspects of our practice to spend the time to critically analyse. Teaching is an incredibly broad and diverse profession with many aspects, and improvement is always possible in every aspect of it. However pursuing improvement takes time (a resource we as teachers are very low on), and attention and energy, and so it is completely untenable to pursue improvement in all aspects of teaching simultaneously. In the face of this, we often suffer from either

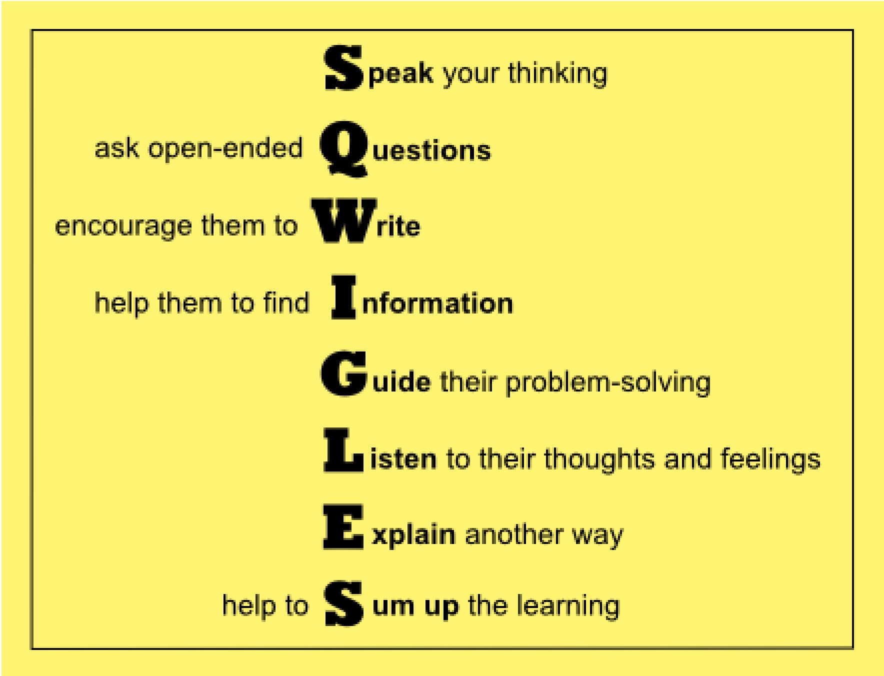

Figure 4.2: SQWIGLES

decision paralysis, or choosing relatively insignificant aspects of our teaching to focus on. SQWIGLES can serve as a guide for both where to focus our self-reflective attention, and as a list of suggestions for individual aspects of our teaching practice to focus on improving one at a time.

Speak Your Thinking

- Mathematics as language, speaking to translate.
- Mathematics as the process of reasoning, speaking to illustrate that process.

Ask Open-Ended Questions

Encourage Them to Write

Help Them Find Information

Guide Their Problem-Solving

Listen to Their Thoughts and Feelings

Explain Another Way

Help to Sum Up the Learning

4.3.2 Staff Culture at the Maths Learning Center

4.3.3 Self-Paced Assessment and Content Speed (!!!)

Link to Maths Anxiety literature review.

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Chapter 5

Curriculum Mapping

TODO: I should add a comment here somewhere about how initially I will be simply comparing content, but that by the end of this chapter I will also consider requirements/ expectations of the bridging course RE MathsStart aliging roughly to year 11 material and MathsTrack covering Maths Methods but not neccessarily Maths Specialist, etc.

One of the important roles of university mathematics bridging courses (such as MathsStart and MathsTrack) is to fill the content knowledge gap for students who wish to commence study that has some required knowledge and skills in mathematics, but either did not complete mathematics to a sufficiently high level in highschool, or completed it long enough ago that they need to re-learn the skills. Ultimately, the content of such bridging courses needs to align with the "Industry Standards" or "Standards of Performance" and in particular the "Curriculum Objective" of the curriculum assessment diamond model (Figure 1.2).

There are two key perspectives that can be taken on what the "Curriculum Objective" of a mathematics bridging course is: the knowledge required for the future (tertiary) study the students are going to engage in, and knowledge expected from highschool graduates. As we will come to see, these two angles or perspectives can be quite dramatically different. From the perspective of knowledge expected from highschool graduates, in Australia the AC serves as a good guide, but even so the exact content knowledge expected of students having completed highschool in Australia varies for a number of reasons:

- To begin with, the AC specifies four levels of mathematics: essential mathematics, general mathematics, mathematical methods, and specialist mathematics. Our focus will be on the higher two of these: mathematical methods and specialist mathematics, as these are the ones targetted at university entry into mathematics-intensive courses.

- Different states within Australia teach different curriculums, with varying degrees of alignment to the AC. In South Australia the primary curriculum taught in senior secondary school is SACE, and so we will focus on that.

- Occasionally, students will do the bridging course and then travel internationally to study in which case the expectations placed on them will be based on an entirely different curriculum. This is comparatively rare, but also the modern international education system has a remarkable level of homogeniety partially as a carry-over consequence of the western colonial era (Mohandas et al., 2003). Regardless, it is beyond the scope of this work to consider alignment of content to highschool curriculums internationally, although this would make for interesting future research.

The other perspective is of course also important, but also far more difficult to address: the knowledge required for entry level university mathematics courses. This will vary hugely from course to course: a entry level calculus course will require very different knowledge than an entry level statistics course, for example. Even within one discipline of mathematics, different universities will have very different expectations of entry level students: in particular, South Australian universities will often structure their entry level mathematics courses to align with SACE even though not all their students have completed SACE, because of the majority who have it is still useful for them to do so. For example, the University of Adeliade re-structured it's first year mathematics courses in 2018 to match changes in SACE. Similarly, universities interstate will often structure their entry level courses to align with their local senior highschool curriculum. That is only within dedicated mathematics courses, other courses that also require mathematics knowledge and skills, such as engineering, psychology, health science, and medicine will all have their own requirements. Bridging courses will enroll students aiming to study a wide variety of such topics, and even more broadly also to go into non-university further study (notably pilot training in the defence force, for example), which have different requirements again.

This places a difficult tension on mathematics bridging courses as to what content to teach. Although many of the students enrolling in the mathematics bridging courses at the UofA do so with the intention to begin study at the UofA (and hence might benefit from SACE structured content), many do not. Even amongst those that do, some may end up going to a different university interstate or even overseas — plans change. So it is important to try and maintain some connection to a broader set of knowledge expected in general and not neccessarily remain laser focussed on the requirements of the particular university courses most students are going to be attempting. This is one of the reasons why the AC is a useful construct as even though some states do not align to the AC as well as others, it still forms a guiding structure at a national level and individually considering the curriculum taught in each state is beyond the scope of this work. Tailoring the content of the bridging courses more narrowly to target entry into particular disciplines (say calculus/ matrix algebra/ statistics for example) could potentially still be of interest down the line, but is likely to be unrealistic with the current resources available to the UofA MLC.

Because of the difficulty of aligning the content to the future requirements of students due to their variety of different directions and needs, and considering that the majority (over half) of students enrolling in MathsStart and MathsTrack plan on enrolling in tertiary study at the UofA, the focus of this curriculum mapping will be on aligning the content of the bridging courses to the SACE curriculum. That said, the alignment of the SACE curriculum to the AC will also be considered, in order to give some idea of the national alignment (and it turns out the AC is very closely aligned to the SACE curriculum). Now that said, the different directions students are going is still very important to consider, and although a direct alignment to the plethora of options students pursue is not realistic, the needs of the most common of these options will be incorporated into the discussion surrounding the curriculum alignment presented in this chapter. Even if direct alignment to these needs cannot be acheived, it is none-the-less important to be aware of these needs, as these are critical to students future success, and while it might not be realistic to tailor the content of the bridging courses to each of these contexts, differentiation can still be acheived through an awareness of these needs and individual interactions with students each of which will often have a particular future direction in mind.

This chapter will be structured as follows. First, in Section 5.1, some notation will be introduced and the content of each of the three curriculums that will be systematically reviewed:

- The AC senior mathematics subjects mathematical methods and specialist mathematics,
- The SACE curriculum stage 1 mathematics, stage 2 mathematical methods, and stage 2 specialist mathematics,
- The University of Adelaide's bridging courses: MathsStart, and MathsTrack.

Note that the alignment done here is entirely on the content of these curriculums, nothing else. The focus of this chapter is entirely on content. The alignment between the content in these curriculums will be considered in Section 5.2 (see Figure 5.1), and alignments/ misalignments discussed. Finally, the discussion throughout around alignment and gaps between the content of these curriculums and courses will be summarised, explanations and reasons for these discrepancies discussed, and potential modifications to content suggested.

Beyond that, this chapter will also briefly discuss the alignment of these bridging courses to first year university mathematics courses and bridging courses offered by other universities in Australia, and discuss the relationship between the gaps in alignment between the AC/SACE and the bridging courses and the requirements of these first year university courses.

5.1 Content

The curriculum alignment in this chapter is presented at two levels of detail — the topic level, and the key concept level. The terms "topic" and "key concept" are reserved in the context of this discussion to specifically refer to these levels of detail. The content of each of the senior highschool curriculums, as well as the university bridging courses, is broken down into topics, and each topic can be summarised as covering a number of key concepts. In Section 5.2, the alignment between these curriculums and bridging courses will be considered thoroughly at both a topic-level, and to the finer detail of particular key concepts. Although the key concept alignment is in essence the core of the work, as this is what allows for concrete changes to be made and content to be planned, the purpose of the topic level comparison is to help structure the overall alignment and discussion.

5.1.1 Notation

In order to provide a useful curriculum-wide topic-level alignment to structure our thinking, it is important to be able to present this alignment in a comprehensible form that can be viewed on a single page. In order to acheive this, the topic-level description (identification of topics) needs to be summarised concisely enough. This is acheived in Figures 5.1 and 5.2, by identifying each topic with an abbreviated code. These abbreviated codes are presented in Table 5.1 and will be used for the remainder of this chapter to help refer to and identify topics. Each topic in each of the curriculums being considered is assigned a unique identifying code in Table 5.1, and the curriculum (and subject within it) can be easily seen from the structure of the code.

Table 5.1: Abbreviated codes for topics within the AC and SACE senior mathematics subjects: Mathematical Methods nd Specialist Mathematics, as well as the Unviersity of Adelaides bridging courses: MathsStart and MathsTrack. Square brackets ([]) are used to indicate numeric values that can vary.

Code	Meaning
MMu[#1]t[#2]	AC Senior Mathematical Methods Unit [#1], Topic [#2]
MMu[#1]t[#2]	AC Senior Specialist Mathematics Unit [#1], Topic [#2]
S1M[#]	SACE Stage 1 Mathematics, Topic [#]
S2MM[#]	SACE Stage 2 Mathematical Methods, Topic [#]
S2SM[#]	SACE Stage 2 Specialist Mathematics, Topic [#]
MS[#]	Maths Start, Topic (Booklet) [#]
MT[#]	Maths Track, Topic (Booklet) [#]

5.1.2 Key Concepts

Appendix A provides a description of each topic in each of the curriculums considered here: the AC Mathematical Methods and Specialist Mathematics, SACE stage 1 mathematics, stage 2 mathematical methods and stage 2 specialist mathematics, and the UofA MathsStart and MathsTrack programs. For brevity, the codes from Table 5.1 are used to identify each topic. The name of each topic is given in bold, followed by a list of the key concepts covered in that topic seperated by commas. These are discussed at length for the remainder of this chapter, and the table presented in Appendix A is intended to be used as reference material while reading the content of this chapter.

Some notes on the way the key concepts are summarised:

- The key concepts listed for each topic are intended for a reader deeply familiar with the content, and as such it is heavily condensed and uses standard mathematical notation and terminology without the usually appropriate rigorous definitions.

- Concepts relating to "interpretation" and application in a general sense are ommited from the key concepts of a topic. The assumption is that to the intended readers, these should go without saying. For example, in S1M2 the key concept "Quadratic Equations in Vertex and Factorised Form" is included, but this implies a variety of auxillary knowledge which is not explicity included in the key concept summary: the interpretation of roots and vertices, deducing vertices and roots from the equation of a quadratic, or deducing the equation of a quadratic given these bits of information, etc. These are skills directly and universally associated to the key concept, and it is assumed that an experienced mathematics educator (which is the intended audience for this text) should be able to easily deduce such surrounding associated skills from the key concepts listed.

These restrictions in the key concept summaries are neccessary in order to be able to present this curriculum alignment concisely enough that it can be useful. The curriculum documents used to generate these summaries contain all the additional detail if required, but the purpose of this work is to align the content in those documents to identify gaps and mis-alignments, and as such it is beneficial to be as concise and dense as possible both to make the alignment a tractable problem and also to make the discussion thereof comprehensible. That said, it is a delicate balance between being broad and vague in order to be able to present the entire curriculum mapping within a single frame of view, and yet still be granular enough so that specific content is clear and explicity and useful actionable reccomendations can be made. It is this tension that led to the development of the methodology which split the two levels of detail:

- The topic level description is intended to give the broad strokes, to show the entire mapping in a single frame of view (a page, in this case). It is also intended to be reference material for the following more detailed discussion, to aid the reader in structuring the information contained in the more detialed discussion and place each peice of information into where it belongs in the bigger picture. Being able to structure the detailed discussion into this larger concept is critical for being able to reach broad overall conclusions.

- The key concept level is what comprises the bulk of the discussion, and this is intended to be the grandular level at which content is presented specifically enough that reccomended actions can be understood explicitly and implemented easily. Note that although the key concept level is much more granular than the topic level discussion, it is still intended as a summary and does not include every single detail of the content, as discussed above.

5.1.3 Curriculum Structure

The AC

The AC is separated into it's F-10 curriculum, and senior secondary curriculum. In this work we are only concerned with the senior secondary curriculum. The senior secondary AC for mathematics is split into four subjects, corresponding to different "levels" of mathematics: Essential Mathemaics, General Mathematics, Mathematical Methods, and Specialist Mathematics. In this work we are concerned only with Mathematical Methods and Specialist Mathematics, and will be considering the mathematical content of these subjects not any other aspects (such as cross-curricular priorities, for example). Importantly, the senior secondary AC does not make any distinction between years 11 and 12 (typically the final year of highschool in Australia). So the senior secondary AC subject "Mathematical Methods" for example, covers content that is in practice taught accross both years 11 and 12. Each of the two subjects we are concerned with in this work, Mathematical Methods and Specialist

Mathematics, are split into four "units" of content, and each of these units is split into three topics, for a total of 12 topics per subject, and a total of 24 topics that we will consider from the AC. At no point do we consider the unit structure of the AC, partly because it does not have any analog in the other curriculums we are aligning too, but mostly because it does not give a useful level of detail for our purposes.

SACE

SACE, in comparison to the AC, does distinguish between year 11 and year 12 content, although to allow for some alternative senior highschool teaching structures they have a different naming convention, calling them stage 1 and stage 2 respectively. In the majority of mainstream cases in Australia, SACE stage 1 will correspond to year 11, and SACE stage 2 will correspond to year 12. To further complicate matters, stage 1 SACE has only three levels of mathematics: Essential Mathematics, General Mathematics, and Mathematics, while stage 2 SACE has four: Essential Mathematics, General Mathemaitcs, Mathematical Methods, and Specialist Mathematics. In this work we will only be concerned with SACE stage 1 Mathematics, stage 2 Mathematical Methods, and stage 2 Specialist Mathematics. SACE stage 1 Mathematics is broken down into 12 topics, while stage 2 Mathematical Methods and Specialist Mathematics are broken down into 6 topics each. This makes for a total of 24 topics from senior highschool SACE mathematics subjects that we will be considering in the curriculum alignment presented in this chapter.

UofA Bridging Courses

The UofA offers two bridging courses through their MLC: MathsStart and MathsTrack. These are both taught through a series of booklets which conveniently each contain roughly on "topic" worth of content, and so these booklets will be used as the topic-level structure of these courses. Both courses are currently structured into 8 topics (booklets) each, although MathsTrack used to have 9, and the fifth was removed some time ago, so the numbering of the MathsTrack topics have a gap (they are numbered 1, 2, 3, 4, 6, 7, 8, 9). So there are a total of 16 topics (booklets) accross both bridging courses that will be considered in the curriculum alignment presented in this chapter.

Topic Grouping

In order to help structure the discussion to follow, it will be useful to think about one broader level of detail, which will loosely be refered to as "content areas". From a very low level of detail perspective, the topics in each of the curriculums being considered can be grouped into the following five broad content areas:

- Functions and Graphs,
- Calculus,
- Geometry and Linear Algebra,
- (Complex) Numbers, and
- Probability and Statistics

There is also some nested hierachicial structures within these content areas that are useful to understand. For example, both "Functions and Graphs" and "Calculus" can be further separated into three sub-areas, corresponding to different categories of functions. Specifically:

- Linear, Polynomial, and Rational Functions,
- Exponential and Logarithmic Functions, and
- Trigonometric Functions.

"Calculus" is naturally divided into differentiation and integration, "Probability and Statistics" can be divided into probability and statistics as seperate content sub-areas, although more commonly is divided into discrete and continuous random variables. Geometry and Linear Algebra covers perhaps the widest variety of topics, from vectors to matrices to systems of equations as well as more traditional geometry topics such as circle theorems.

The two areas ("Functions and Graphs" and "Calculus") are also often taught together, with new categories of functions being introduced/ revised together with concepts around how to do calculus with these functions, so this particular pair of content areas are very closely linked. Although there are some notable connections between the other content areas, such as for example:

- Complex numbers providing a method for finding roots to polynomials that could not otherwise be found,
- Applying calculus to parameterised vector equations,
- Integration being used to understand probabilities as areas under distribution functions,

Broadly speaking they stand comparatively apart from each other, particularly Probabilty and Statistics.

This broad content area grouping of topics covers almost all of the content in all the curriculums considered here. The only notable exceptions being MMu2t2 from the AC and S1M7 from SACE both covering primarily sequences (geometric and arithmetic) as recurrance relations, and S2SM1 from the SACE covering inductive proof, neither of which fit neatly into any of the content areas above. With that having been clarified, these content areas will be used to help structure the discussion for the remainder of this chapter.

5.2 Curriculum Mapping

Figure 5.1 shows the topic-level alignment between the AC, SACE, and bridging courses, organised by subject/ course. Each node (ellipse) in Figure 5.1 corresponds to a topic, and is identified by the abbreviated code as per Table 5.1. What "organised by subject/ course" means in this context is that while the curriculla (the AC, SACE, and the UofA bridging courses) are arranged as columns in Figure 5.1, within each of these columns topics are grouped by subject. So for example in the AC column topics are grouped into Mathematical Methods topics, and then Specialist Mathematics topics.

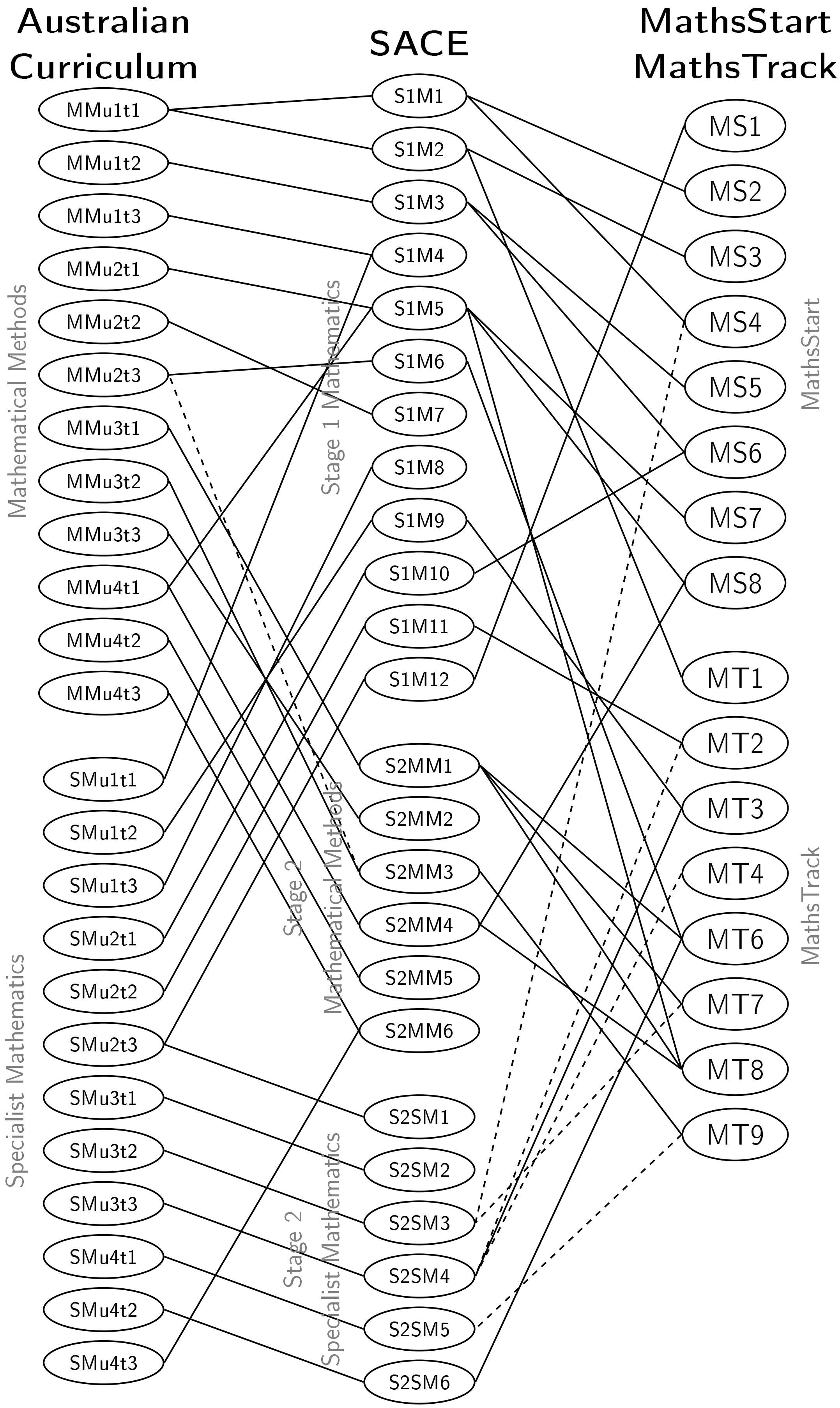

Figure 5.1: Curriculum Mapping by Subject/ Course

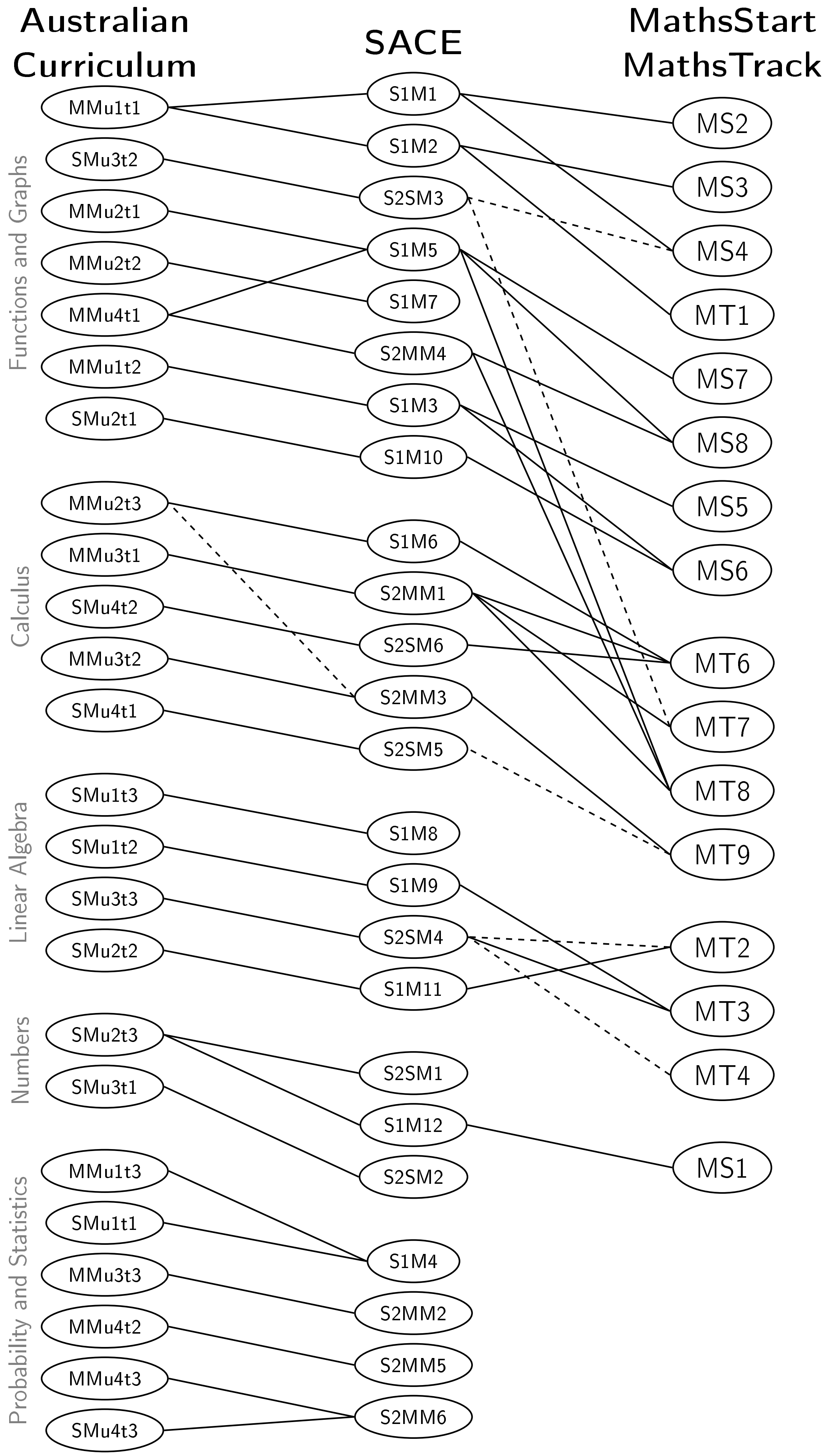

Figure 5.2: Curriculum Mapping by Content Area

The topic-level alignment organised by subject/ course shown in Figure 5.1 is the broad, eagle-eye, view of the alignment between the content in these topics, but even from this wide-view the way in which the curriculum structures of the AC and SACE discussed in Section 5.1.3 align to each other is reasonably clear. It can be seen for example, that roughly speaking the first 6 topics of senior secondary AC Mathematical Methods and Specialist Mathematics map to the 12 topics of stage 1 SACE Mathematics, while the last 6 topics in each of these AC subjects map to the 6 topics of the corresponding stage 2 SACE subject. At a key concept level this alignment is imperfect (and this is discussed in more detail below) to varying degrees of imperfection, and some of these imperfections are visible in the imperfect alignment shown in Figure 5.1, but broadly speaking the AC and SACE are actually very closely aligned (and this makes sense given SACE was recently modified with the explicit purpose of aligning it more closely to the AC). In contrast, it is much less obvious how to interpret the alignment between SACE and the bridging courses shown in Figure 5.1.

However, rearranging the topics shown in Figure 5.1 (permuting the columns) and grouping them into the five broad content areas discussed above in Section 5.1.3:

- Functions and Graphs,
- Calculus,
- Geometry and Linear Algebra,
- (Complex) Numbers, and
- Probability and Statistics

gives us a much clearer picture, which is shown in Figure 5.2. Even though the allocation of some topics into these content areas can be somewhat arguable particularly in a few edge cases (as mentioned above in Section 5.1.3), Figure 5.2 shows a very clear picture in terms of the SACE-bridging course alignment: the bridging courses do not contain any probability or statistics what-so-ever, and very very little on complex numbers.

Before moving on to discuss the details of the key concept level alignment within each of these topic alignments shown, a quick note on interpreting the visualisations of Figures 5.1 and 5.2. While solid lines connecting topic-nodes represent almost complete or substantial key-concept level alignment, dashed lines are used to represent tenuous connections with only a small overlap in key concept terms, usually just a single key concept. To briefly cover which concepts these correspond too:

- The one dashed line between the AC and SACE essentially represents the concept of anti-differentition,
- The dahsed line between S2SM3 and MT7, as well as the one between S2SM3 and MS4 essentially represents sketching rational functions, although in MS4 only reciprocal functions and transformations thereof are considered. The ideas surrounding the sketching of these graphs and the properties of these graphs (asymptotes, etc.) are heavily emphasised as a way to explore them in both cases.

- The dashed line between S2SM5 and MT9 essentially represents integration by substitution.
- The dashed line between S2SM4 and MT2 essentially represents row operations, in MT2 introduced on matrices, but in S2SM4 it is introduced explicitly in the context of solving 3×3 systems of linear equations. Similarly the dashed line between S2SM4 and MT4 represents essentially the same concept in S2SM4, but in MT4 the system of equations perspective/ application is explored, which is not really done as much in MT2.

5.2.1 AC to SACE

At a glance, there appears to be a very good one-to-one alignment at the topic level between the AC and SACE. Broadly speaking the biggest difference between these two curriculums is their structure, as discussed in Section 5.1.3. As usual however, the devil is in the details. In this section, a detailed discussion of the key concept level alignent between the AC and SACE will be presented. This discussion will be structured by the broad content areas as introduced in Section 5.1.3.

Functions and Graphs

The content area "Functions and Graphs" can be reasonably spit into three content sub-areas in both the AC and SACE: Polynomials and Rational Functions, Exponential and Logarithmic functions, and Trigonometric Functions, as discussed in Section 5.1.3, with the notable additional comment that in both the AC and SACE general concepts and notation are strongly emphasised and introduced through the Polynomials and Rational Functions topics. The content in this area aligns almost perfectly between the AC and SACE, with only minor differences in notation, emphasise, and how key concepts are split into topics. Despite this very close alignment, a more detailed topic by topic discussion of the key-concept level alignment is included for completeness:

- **Polynomials and Rational Functions:** In both the AC and SACE this area is split into two: basic introduction and advanced concepts.The basic introduction topics align well (MMu1t1 to S1M1 and S1M2), with only slight differences in terminology (AC refers to inverse proportion while SACE refers to reciprocal for example) and focus (SACE puts much more of an emphasis on polynomials, seperating it into it's own topic (S1M2) and breaking it down into much more granular concepts). The advanced concepts are covered in SMu3t2 and S2SM3 are essentially identical.
- **Exponentials and Logarithims:** There is essentially perfect alignment between the concepts for logarithms between MMu4t1 and S2MM4. Similarly, MMu2t2 is almost exactly the same as S1M7, they are both centered on the introduction of recurrance relations, partial sums, and linking this back to exponential functions. I include these topics under exponentials as they link to and are used to introduce those concepts, but really the bulk of the content in these topics is focussed on sequences and series. The alignment between MMu2t1 and S15 has a notable difference however: S15 includes Log-Laws, while MM2t1 does not, focusing only on Index Laws. This is not actually a

difference in content between the AC and SACE as the log laws are covered in the AC in MMu4t1, but a difference at the topics level. The log laws are actually repeated in the SACE curriculum, covered both in S1M5 and then again in S2MM4, while they are not repeated in this way in the AC.

- **Trigonometry:** MMu1t2 matches almost identically to S1M3, with the biggest difference being that in the AC the unit circle interpretations/ definitions of $\sin(x)$, $\cos(x)$, and $\tan(x)$ are emphasised, where in SACE $\tan(x)$ in particular is introduced instead as $\frac{\sin(x)}{\cos(x)}$. That being the biggest difference between the two should emphasise how similar they are in terms of content. Similarly, SMu2t1 and S1M10 align just about perfectly.

Calculus

Similarly to Functions and Graphs, there is very good alignment between the AC and SACE in the Calculus content area but for completeness a detailed discussion of the key concept level alignment on a topic-to-topic basis in included:

- SMu4t1 aligns perfectly with S2SM5, both covering integration by parts, by substitution, inverse trig substitutions in integration problems, volume of solids of revolution, partial fractions and area between two curves.

- SMu4t2 aligns well to S2SM6, both covering implicit differentiation, solving first-order seperable differential equations, and the logistic equation. However there are some differences in that the AC goes on to focus on rates of change, while SACE instead decides to focus on parameterised curves — trigonometric parameterisations and such.

- MMu2t3 and S1M6 both introduce differentiation by leading in with the concept of average rate of change, first principles and lead into linearity of differentiation, derivitives of polynomials, slope of the tangent and optimisation but in SACE S1M6 introduces the terms "increasing" and "decreasing" and sign diagrams, which are not mentioned in MMu2t3, while MMu2t3 introduces the concept of an antiderivative (which is only introduced in S2MM3 of SACE and is represented by a dashed line in Figure 5.1 and Figure 5.2).

- MMu3t1 and S2MM1 align perfectly introducing the chain, product, and quotient rule. Introducing $e = 2.718\ldots$ in the same way (using first principles to explore $\frac{d}{dx}a^x$ for different a, derivitives of $\sin(x)$ and $\cos(x)$, and second derivatives.

- MMu3t2 and S2MM3 are very closely aligned, both introducing definite and indefinite integrals of polynomials, exponentials, and trigonometric functions, linearity of integration and the fundamental theorem of calculus. They do however diverge slightly in their approach to definite integrals. In particular, SACE S2MM3 introduces the concepts of upper and lower sums and the definite integral as the unique number between the two as the size of the rectangles approaches zero, while in the AC MMu3t2 this is not discussed. Also, S2MM3 introduces anti-differentiation, a concept introduced in the AC MMu2t3 but not introduced in SACE S1M6, instead being covered here in S2MM3.

Note how most derivatives are introduced in differentiation (i.e. calculus) specific topics. The exception is $\frac{d}{dx}\ln(x)$, which is introduced in a seperate topic entirely about logarithm functions in both the AC (MMu4t1) and SACE (S2MM4), and in this content area sturcturing these topics are categorised under 'Functions and Graphs' because they introduce logarithmic functions, but it should be noted that they do also contain concepts around calculus (of logarithm functions).

Geometry and Linear Algebra

The content area "Geometry and Linear Algebra", similarly to "Functions and Graphs", can be split into four content sub-areas in both the AC and SACE: Vectors in $\mathbb{R}^2$ (in the Plane), Circle Theorems, Matrices and Vectors in $\mathbb{R}^3$ (in 3D).

- **Vectors in the Plane** are covered in SMu1t2 and S1M9, with the content being very well aligned and the only notable difference being the inclusion of geometric vector proofs in SACE S1M9 which is not included in SMu1t2, instead being restricted to topics such as SMu1t3.

- **Circle Theorems and Proof** are covered in SMu1t3 to S1M8. Both these cover the same "content" in the sense of theorems: circle theorems, but they also both attempt to broach the difficult topic of proof, methods of proof, and some of the language around proof, and they take quite different approaches to this. The AC SMu1t2 is quite explicit specifing the introduction of language around formal logic, defining the terms: "implication", "equivalence", "converse", "negative", "contrapositive", "contradiction", "for all", "there exists", and "counter-example". On the other hand, SACE S1M8 simply specifies proof to be investigated as "justification of properties of circles", and only breifly mentions specifics of language and methods as suggestions not specificying them as being required components of the curriculum and instead leaving the approach and specific content chosen to be used to introduce the concept of proof much more open to interpretation by the teacher.

- **Matrices**, covered in SMu2t2 and S1M11 are essentially identical in content covering matrix notation, linear combinations of matrices, matrix multiplication, matrix indentity and inverses (and determinants), and the perspective of matrices as linear transformations.

- **Vectors in 3D** in SMu3t3 and S2SM4 are also introduced very similarly in terms of content: cross product, equations for lines and planes, systems of equations and geometric interpretation of their solutions. One difference however is in how they apply these concepts, the AC SMu3t3 includes a focus on parameterised vector equations, the equation for a sphere, and in particular kinematics: projectile and circular motion in 3D, which are not coverted in SACE S2SM4. Instead S2SM4 remains more abstract with these concepts, and on the other hand the examples required are less complex to interpret.

(Complex) Numbers

Complex Numbers are introduced in two topics, a basic an advanced topic, in both curriculums. The basic topics, SMu2t3 in the AC and S1M12 in SACE are quite similar in their base content: rational/ irrational numbers, i, complex arithmetic, conjugates, and complex roots of polynomials. However there are a couple of key differences between the two: first, induction is introduced in the AC SMu2t3 while in SACE it is seperated into it's own seperate topic: S2SM1. The second key difference is that interval notation is explicitly introduced in SACE S1M12, while in the AC interval notation seems to be neglected. The advanced topics SMu3t1 and S2SM2 on the other hand align almost perfectly in content.

Probability and Statistics

Probability and Statistics is the content area in which the most substantial differences in content exist between the two curriculums. Similarly to "Functions and Graphs" and "Geometry and Linear Algebra", the content covered in "Probbility and Statistics" can be organised into three content sub-areas: Combinatorics, Random Variables, and Confidence Intervals.

- **Combinatorics and Introductory Probability**: MMu1t3, SMu1t1, and S1M4. The overlap between the AC and SACE for these topics is essentially concepts around permutations, factorial (and the 'multiplication principle'), combinations. Although it is notable that the AC MMu1t3 extends the concept of combinations to binomial coefficients and Pascal's triangle while SACE does not. Beyond these common concepts, both curriculums have some introductory probability content, but they take very different approaches to this. The AC does this via set thoeretic concepts, union intersection and complement of sets, the pidgeonhole principle, and then probability notation ($P(A)$) for set complement, intersection and union and introduces basic probability concepts from this angle (for example, $0 \leq P(A) \leq 1$), including conditional probabilities ($P(A|B)$). On the other hand, SACE S1M4 has introductory statistics concepts (as opposed to introductory probability concepts). Specifically, S1M4 reviews mean median ad mode, interquartile range, standard deviation, and introduces the basic concepts around the normal distribution. S1M4 also introduces the distinction between discrete and continuous random data/ variables, not quite introducing the concept of a 'random' variable yet, but still. Very big difference in approach between these topics.

- **Random Variables**: Discrete (MMu3t3 and S2MM2), and Continuous (MMu4t2 and S2MM5). There is quite good alignment between these topics actually. For both discrete and continuous general definitions of expected value and variance are given. For discrete the uniform, examples of arbitrary non-uniform, the bernoulli, and binomial distributions are introduced. For continuous the uniform, restricted domain polynomial, and normal distributions are considered, and transformations of normal distributions (in particular to get the standard normal) are considered. The one key difference is that in SACE the central limit theorem is explicitly explored, while it's significance is much less explicit in the AC.

- **Confidence Intervals**: The confidence intervals introduced are the same accross both curricula, specifically the normal approximation to the binomial confidence interval for a proportion (Wald interval, MMu4t3) and the standard normal distribution confidence interval for the mean of a continuous variable

(SMu4t3) are both introduced in SACE S2MM6. However the approach taken to justifying these confidence intervals is a little difference, in SACE the justification is very central limit theorem centric, relying on the introduction to that concept in S2MM5, while in the AC instead many of these concepts (including the central limit theorem itself) are simply stated and students are encouraged to test them by simulation. Althogh SACE also takes this simulation approach to justification it is emphasised less, and the introduction of the concepts around the central limit theorem are much more explicit.

Summary

TODO: Review above dot points, trim/edit down, condense, and write a paragraph here begginng "In summary, ..." or "To summarise, ..."

The only substantial difference in content is the concept of proof by induction, which is in the SACE curriculum but not the AC. This is represented in Figure 5.1 by S2SM1 which is an entire topic on induction with no link to the AC, although induction is also briefly introduced earlier in SACE in S1M12.

Note: Cumulative Distribution Function not mentioned in SACE

Proof is more integrated into SACE in numerous topics, although still almost as an afterthought.

5.2.2 AC and SACE to MathsStart and MathsTrack

In the broad sense of areas of mathematics the topcs can be grouped into naturally, as discussed above in Section 5.1, the topics that are covered in the AC and SACE but not in MathsStart or MathsTrack are complex numbers, and Probability/ Statistics/ Combinatorics. Note that the 'missing' MT5 is a topic on complex numbers that is currently being ommitted from the bridging course. So given that those areas are not covered in MathsStart or MathsTrack currently, lets take a look in more detail (at a key concept level) at the alignment of the topics that are covered in MathsStart and MathsTrack.

- Functions and Graphs. This topic essentially covers the entire of MathsStart, and the first topic of MathsTrack MT1, and it make sense to split it into the usual three sub-topics:
  - General Concepts, Polynomials and Rational Functions.
    * Polynomials and Rational Functions have an interesting binary tree structure, with MMu1t1 splitting into both S1M1 and S1M2 in SACE, which each split into MS2, MS4 and MS3, MT1 respectively in the bridging courses. S1M1 covers mainly linear equations, but also recipricol functions and asymptotes, while in MathsStart these are split into MS2 and MS4 respectively. Similarly S1M2 covers polynomials, including quadratics and related concepts as well as higher order polynomials, while in the bridging course these are separated into an entire topic just on quadratics (MS3) in MathsStart which is still less indepth than the introduction to quadrtics in S1M2, which introduces rearrangments of quadratics into vertex and factorised form, and then later a more indepth topic on polynomials in MathsTrack

(MT1), which covers some of the more details concepts on quadratics missed in MS3 (such as the quadratic formula), and goes on to higher order polynomials.

    * S2SM3 introduces advanced general concepts on functions: domain and range, function composition, one-to-one, inverse functions, graphing more general rational functions (not just reciprocal functions), and the absolute value function. This concepts are not covered in the bridging courses, and could be useful, but on the other hand, are part of the specialist mathematics curriculum, so aren't the highest priority to include.

  – Exponentials and Logarithms. Both the AC and SACE introduce the concept of exponential functions via recurrance relations describing geometric sequences. Although this is certainly not the only (or even neccessarily the best) way to introduce and understand exponential functions, it is the way prescribed by the AC and SACE curriculums, so it might be valuable. On the other hand, the way the number e is introduced is actually identical accross AC, SACE, and the bridging course — which is remarkable given how many different ways this could be done. Interestingly, exponent laws and logarithm laws (as well as basic properties of exponential and logarithmic functions) are introduced in S1M5 of SACE, and are split into the two topics MS7 and MS8, it seems that the granularity of the MathsStart program is roughly a factor of two more granular than the SACE curriculum, which is pretty interesting. Could be a direction for future research on some measure of "degree of granularity" of a course/ program. It is also interesting to note the location of S2MM4 in Figure 5.2 in the Functions and Graphs section, rather than the differentiation section. This is a choice because of how concepts of exponents and logarithms are very distinctly seperated in the AC into MMu2t1 and MMu4t1, while in SACE introductory concepts for both are introduced in S1M5 and S1M7, but while advanced concepts around logarithms (including calculus) have their own topic in SACE — S2MM4, advanced concepts (such as calculus) around exponential functions do not, and are instead lumped into more general calculus topics (S2MM1 in SACE and MMu3t1 in AC). I'll include a more indepth discussion of this under calculus, as this section ought to focus on the function and graph properties, but there is significant overlap between the two in the way the content is arranged (perhaps rightfully so).

  – Trigonometry There is actually fairly good alignment in the trigonometry sections, although they are organised differently (S1M5 and S1M10 in SACE and MS5 and MS6 in MathsStart), the content is fairly well aligned. SACE covers some graphing slightly more comprehensively, talking about translations and dilations for example (a concept from MS3 that could be "translated" here effectively, linking the concepts and chaining them from topic to topic a little more strongly.

• Calculus, it makes sense to organise this under the MathsTrack topics (maybe I should do that for functions and graphs above as well actually).

  – MT6: Differential calculus is introduced very similarly, through first principles etc. in both MT6 and S1M6, MT6 actually goes beyond the content of S1M6, introducing also the product, chain and quotient rules (which are covered in S2MM1) and implicit differentiation (which is covered in S2SM6). On the other hand, MT6 does not cover increasing and decreasing (which is in S1M6), which is instead covered in MT7. Also, the concept of a normal to a curve is introduced in MT6, but it is not really covered anywhere in SACE (apart from sort of in S2SM4 using cross products).

  - MT7: Covers a few concepts from S1M6 that where skimmed over in MT6, as well as some of the more advanced function and graph concepts such as sketching rational functions which is only covered in S2CM3.
  - MT8: Similarly, taking the more advanced concepts from the functions and graphs topics of SACE, and combining them with the introductory calculus concepts MT8 introduced differentiation of exponents (from S2MM1) and logarithms (S2MM4), at the same time as re-hashing concepts from MS7 and MS8 and revising them (such as introducing exponential and logarithm functions/ sketching them, etc.). Notably, surge models and logistic models are introduced as well. Surge models are not covered anywhere in the AC or SACE as far as I cna tell, and logistic models are only introduced in S2SM6.
  - MT9: All the integration is fit into this single topic in MathsTrack, which students inevitably find challenging. This covers essentially all of S2MM3, and then goes a little further with the notable addition being integration by substitution, which in SACE is only covered in S2SM5. Notably summation notation is also introudced (in an appendix) in MT9, an important bit of notation that students often struggle with in first year university, but that isn't anywhere in the SACE curriculum as far as I can tell.

- Geometry and Linear Algebra is where SACE and the briding courses begin to really diverge in earnest.

  - MT2 aligns well to S1M11, although it goes a little further and also introduced row operations, a concept not introduced in S1M11 although it is introduced in S2SM4 in a quite different context, not so much to do with matrices in the pure sense, but instead focussing on the connection to solving 3×3 systems of linear equations. This aspect, of solving systems of equations, in introduced in MT4 and actually gone into in great depth, while the concept seems tacked on and is not gone into in detail at all in S2SM4.
  - MT3 Introduces vectors and vector concepts in both $\mathbb{R}^2$ (concepts covered in S1M9) and $\mathbb{R}^3$ (concepts covered in S2SM4). The overlap between S1M9 and MT3 is substantial, with MT3 covering most of the concepts in S1M9, although S1M9 goes into a little more detail on scalar dot products (a concept covered in both), and also introduces the concept of orthogonal projection, and even throws in a dash of geometric styled proof. S2SM4 also goes significantly further, most notably introducing the concept of the vector cross product which is not covered in MT3, although both introduce equations for planes in $\mathbb{R}^3$.

  - MT4 focuses on systems of linear equations, and although this concept is introduced in S2SM4, it is covered in MT4 in a much more detailed, granular way. Also, MT4 introduces Gauss-Jordan Elimination, an algorithm not explicitly introduced in SACE as far as I can tell (although it is implied).

MS4: Note the link to S2SM3

- MS1: Maybe Introduce Interval Notation along with Intervals?

5.3 Conclusions/ Summary

Broadly the content for functions and graphs, as well as calculus is well aligned between the bridging courses and the AC/ SACE, although the bridging courses tend to mix the two more, merging the more advanced concepts from functions and graphs into the calculus topics. This makes perfect sense to do, especially considering how inter-connected the areas are in the first place. Bigger differences between the content of the bridging courses and the AC and SACE exist in the other area though, with big differences in linear algebra and geometry, complex numbers almost not being covered in the bridging courses (although they used to be covered more, see the discussion relating to MT5), and the biggest difference being that statistics and probability are not covered in the bridging courses at all.

Broadly, a big difference in emphasis between the bridging courses and SACE is the emphasis the bridging courses place on sketching graphs, and explicitly exploring the connection between transformation (translations and dilations primarily) of a graph relate to algebraic changes to functions. Although this is covered in SACE to some degree, it is largely implicit and left to reading between the lines, while it is quite explicit in the bridging courses.

My reccomendations for MathsStart would be to include some work on fractions, index laws, and more emphasise on re-arranging equations as in my experience these are the topics and concepts that students need the most from middle school mathematics (up to year 10) and form foundations for building other concepts with in senior highschool. This is already the implicit purpose of MathsStart in a sense, and this is good. Aside from that, concepts that could be added to the bridging courses to bring them more in alignment with the AC and SACE are:

- I'm not sure if I missed it somewhere, but a formal introduction of the definition of a function (and hence the alternative — a relation) would be good as the concept of a relation is in the AC and SACE, although it is not used much. It is a little useful later when doing implicit differentiation, for example.

- S2SM3 introduces advanced general concepts on functions: domain and range, function composition, one-to-one, inverse functions, graphing more general rational functions (not just reciprocal functions), and the absolute value function. This concepts are not covered in the bridging courses, and could be useful, but on the other hand, are part of the specialist mathematics curriculum, so aren't the highest priority to include.

- I like the way quadratics are introduced in MS3, particularly as the concepts used to introduce them (dilations, translation) are very applicable in the AC and SACE in many places. On the other hand there are a couple of concepts here that are missed in the gap betwen MS3 and MT1, specifically rearranging quadratics algebraically to get them in vertex form and factored form, although these forms are introduced implicitly in MS3 and interpreted, algebraic rearrangment of them could be emphasised more, especially given how this is one of the key concepts we want students to be picking up as they go through MathsStart.

- Both the AC and SACE introduce the concept of exponential functions via recurrance relations describing geometric sequences. Although this is certainly not the only (or even neccessarily the best) way to introduce and understand exponential functions, it is the way prescribed by the AC and SACE curriculums, so it might be valuable.

- Overall having the concepts link from topic to topic more, for example in MS3 dilations and translations of quadratic functions are considered. It would be useful if these concepts where re-visited in MS6 for example when looking at graphing/ sketching trigonometric functions, as this is covered in SACE explicitly but also because it connects the concept to multiple different topics and applications (different kinds of functions).

- Scalar dot product of vectors is introduced in MT3, but SACE goes a little further, also introducing the concept of orthogonal projection in $\mathbb{R}^2$ in S1M9.

- I think it would be a good idea to introduce interval notation along with the concept of intervals when that concept is introduced in MS1.

while concepts currently in the briding courses that could be removed as they are not part of the AC or SACE include:

- Implicit differentiation is technically only in Stage 2 Specialist Mathematics, so could potentially be removed from MT6.

- Normal to a curve is a concept introduced in MT6 but not used anywhere in the SACE curriculum (as far as I can tell), apart from breifly in S2SM4 when it is used in the contect of vector cross products and equations for a plane — a very different context.

- Surge models are not anywhere in the AC or SACE, so could potentially be removed from MT8 entirely.

- Logistic models are only included in Stage 2 Specialist Mathematics, so could potentially be removed from MT8. Alternatively if it was kept, it could be introduced with concepts around differential equations, which is how it is introduced in Stage 2 Specialist Mathematics. At the moment, it is introduced as a model (an equation), not a solution to a differential equation, so even as it is it is very different content.

- Integration by Substitution is only covered in Stage 2 Specialist Mathematics, so could potentially be removed but is also very useful in first year university calculus-based courses (such as Maths IA etc.) so I'm not sure. Depends, as most of these do, on the mindset: it would be useful content for the students to be exposed too early, or you could look at it from the perspective of they are not expected to know it going into a course like Maths IM.

- Concepts covered in S2SM4 on geometry in $\mathbb{R}^3$ are technically only covered in Specialist Mathematics, so could be cut in principal if we are operating under the assumption bridging course students will be doing Maths IM. So concepts like vector cross product, and the equation for a plane, to be specific.

- Technically, the Gauss-Jordan Elimination method introduced in MT4 is not anywhere in SACE, but that said it is somewhat implied a little in the topic that discusses the other concepts in MT4 (around systems of linear equations) — S2SM4. That said, this entire topic and broad concept of systems of linear equations is entirely contained in Stage 2 Specialist Mathematics, so it could be argued to not be included for that reasoning.

Other reccomendations:

- Maybe spread out the introduction of integration? Potentially introduce both differentiation and integration, and then introduce them together on each individual type of function? Not sure.

Chapter 6

Conclusions and Reccomendations

With respect to the bridging courses run through the university of adelaide's maths learning centre: MathsStart and MathsTrack,

- The self-paced and feedback focused approach to assessment is certainly the highlight of the programs, should be continued, encouraged, potentially further resourced, expanded, and reccomended to other bridging course facilitators.
- The role of bridging courses as what is often student's first experience at university implies that potentially students wellbeing and retention could be improved by structuring the programs to provide more opportunities for students to meet each other and work together: either in the maths learning center drop-in area, or a seperate area, but potentially assigning a certain time on a certain day perhaps weekly or fortnightly during which students are encouraged to come and work together, could allow them to make freinds, build social networks, and better aclimitise them to the university environment in order to better prepare them for success in their studies.
- The smallest but perhaps easiest to implement improvement could be to better align the course content with curriculum, both the highschool curriculum (AC/ SACE) in the case of students doing the bridging course to then comence study interstate or overseas, or with specific first year entry level courses, to better match the potential gaps in knowledge students may encounter.

6.1 Further Research

- Complete a more comprehensive systematic review of the literature surrounding mathematics bridging and the secondary-tertiary transition.
- Review of Australian Universities, Bridging courses they offer, and placing the UofA courses into that context.
- Generation of Resources for a Probability and Statistics Topic booklet
- Alignment of final year highschool curriculum content internationally, and accross the other states of Australia in comparison to the AC.

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Appendices

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Appendix A

Key-Concept Level Description of Topics in the AC, SACE, MathsStart and MathsTrack

Note, topics are identified using the code notation introduced in Table 5.1. The full **topic name is given in bold where applicable**, and then key concepts covered in that topic are listed.

Code	**Name** and Key Concepts
MMu1t1	**Functions and graphs**: Midpoint of a Line, $y = mx + c$, Quadratic Equations in Vertex and Factorised Forms, Inverse Proportions, Polynomials, Relations, Translations and Dilations
MMu1t2	**Trigonometric functions**: Unit Circle, Radians, SOH CAH TOA, Sine Rule, Cosine Rule, Exact Values, Amplitude/ Period/ Phase, Length of Arc, Area of Sector
MMu1t3	**Counting and probability**: Binomial Coefficients, Set Complement Intersection and Union, Probability, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, Conditional Probability, Independance
MMu2t1	**Exponential functions**: Index Laws, Fractional Indices, Functions, Asymptotes, Graphs
MMu2t2	**Arithmetic and geometric sequences and series**: Arithmetic and Geometric Sequences as Recurrence Relations, Limiting Behaviour, and Partial Sum Formulae, Growth and Decay
MMu2t3	**Introduction to differential calculus** Average Rate of Change, First Principles, Leibniz Notation, Instantaneous Rate of Change, Slope of Tangent, Derivitive of Polynomials, Linearity of Differentiation, Stationary Points, Optimisation, Anti-Derivitives, Interpret Position-Time Graphs
MMu3t1	**Further differentiation and applications**: Define e as a s.t. $\lim_{h \to 0} \frac{a^h - 1}{h} = 1$, Derivitives of e^x $\sin(x)$ and $\cos(x)$, Chain Product and Quotient Rules, Second Derivitives
MMu3t2	**Integrals**: Integrate Polynomial Exponential and Trigonometric Functions, Linearity of Integration, Determine Displacement given Velocity, Definite Integrals, Fundamental Theorem of Calculus, (signed) Area Under a Curve

Code	**Name** and Key Concepts
MMu3t3	**Discrete random variables**: Frequencies, General Properties, Expected Value, Variance, Standard Deviation, Bernoulli and Binomial Distribtions
MMu4t1	**The logarithmic function**: Logs as Inverse of Exponentials, Log-Scales, Log Laws, Log Function Graphs, Natural Log, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, $\int \frac{1}{x}dx = \ln(x) + c$ for $x > 0$
MMu4t2	**Continuous random variables and the normal distribution**: Probability Density Function, Cumulative Distribution Function, Probabilites Expected Value, Variance and Standard Deviation as Integrals, Linear Transformation of Random Variables, Normal Distribution using Technology
MMu4t3	**Interval estimates for proportions** Simple Random Sampling, Bias, Sample Proportion, Normal Approximation to the Binomial Proportion, Wald Confidence Interval, Trade-Off Between Width and Level of Confidence
SMu1t1	**Combinatorics** Multiplication of Possibilities, Factorial Notation, Permutations with and without Repeated Objects, Union of Three Sets, Pigeon-Hole Principle, Combinations, Pascals Triangle
SMu1t2	**Vectors in the plane**: Magnetude and Direction, Scalar Multiplication, Addition and Substraction as a Triangle, Vector Notation, $a\mathbf{i} + b\mathbf{j}$ Notation, Scalar Dot Product, Projection, Parallel and Perpendicular Vectors
SMu1t3	**Geometry**: Notation for Implication ($\Rightarrow$) and Equivalence ($\Leftrightarrow$), Converse ($B \Rightarrow A$) Negation ($\neg A \Rightarrow \neg B$) and Contrapositive ($\neg B \Rightarrow \neg A$), Proof by Contradiction, $\forall$ and $\exists$ Notation, Counter-Examples, Circle Theorems, Quadrilateral Proofs in $\mathbb{R}^2$
SMu2t1	**Trigonometry**: Graph and Solve Trig Functions, Prove Various Trig Indentities, Reciprocal Trig Functions
SMu2t2	**Matrices**: Notation, Addition and Scalar Multiplication of Matrices, Multiplicative Identity and Inverse, Determinant, Matrices as Transformations
SMu2t3	**Real and complex numbers**: Rationality and Irrationality, Induction, $i = \sqrt{-1}$, Complex Numbers $a + bi$ and Arithmetic ($+$, $-$, $\times$, $\div$), Complex Conjugates, Complex Plane, Complex Conjugate Roots of Polynomials
SMu3t1	**Complex numbers**: Modulus and Argument, Arithmetic ($\times$, $\div$, and z^n) in Polar Form, Convert between Polar and Cartesian Form, De Moivre's Theorem, Roots of Complex Numbers, Factorising Polynomials
SMu3t2	**Functions and sketching graphs**: Composition of Functions, One-to-One, Inverse Functions, Absolute Value Function, Rational Functions
SMu3t3	**Vectors in three dimensions**: $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ Notation, Equation for Spheres, Parameterised Vector Equations, Equations of Lines, the Cross Product, Equation for a Plane, Systems of Linear Equation (Elimination Method) and Geometric Interpretation of Solutions, Kinematics via Differentiation of Vector Equations, Projectile and Circular Motion

Code	**Name** and Key Concepts
SMu4t1	**Integration and applications of integration** Substitution, $\int \frac{1}{x}dx = \ln\|x\|+c$ for $x \neq 0$, Inverse Trig Functions and their Derivitives, Integrate $\frac{\pm 1}{\sqrt{a^2-x^2}}$ and $\frac{a}{a^2+x^2}$, Partial Fractions, Integration by Parts, Area Between Two Curves, Volume of Solids of Revolution, Numerical Integration using Technology
SMu4t2	**Rates of change and differential equations**: Implicit Differentiation, First-Order Seperable Differential Equations, The Logistic Equation, Kinematics (Rates of Change)
SMu4t3	**Statistical inference**: Central Limit Theorem and the Resulting Confidence Interval for a Mean
S1M1	**Functions and graphs**: Equations for a Line, Slope, y-intercept, Intersection of Lines, Reciprocal Function, Asymptotes, Functions vs Relations, Domain, Range, Function Notation
S1M2	**Polynomials**: Quadratic Equations in Vertex and Factorised Forms, Quadratic Formula, Completing the Square, The Leading Coefficient and Degree of a Polynomials, Cubics, Quartics
S1M3	**Trigonometry**: Pythagoras, SOH CAH TOA, Cosine Rule, Sine Rule, Unit Circle, Exact Values, Sine and Cosine Functions, Radians, Length of Arc, Area of Sector, Amplitude, Period, Phase, $\tan(x) = \frac{\sin(x)}{\cos(x)}$
S1M4	**Counting and statistics**: Factorial, Permutations, Multiplication Principle, Combinations, Discrete vs Continuous Random Variables, Mean, Median, Mode, Range, Interquartile Range, Standard Deviation, Normal Distribution,
S1M5	**Growth and decay**: Index and Logarithm Laws, Exponential Functions and their Graphs
S1M6	**Introduction to differential calculus**: Average Rate of Change, First Principles, Notation $f'(x) = \frac{df}{dx}$, $\frac{d}{dx}x^n = nx^{n-1}$, Linearity of Differentiation, Slope of Tangent, Increasing vs Decreasing, Local and Global Maxima and Minima, Stationary Points, Sign Diagram
S1M7	**Arithmetic and geometric sequences and series**: Arithmetic and Geometric Series as Recurrance Relations and Explicit Expressions, Partial Sums, Limiting Behaviour
S1M8	**Geometry**: Circle Properties, Proofs (Direct, Contradiction, and Contrapositive)
S1M9	**Vectors in the plane**: Component (column) vs $ai + bj$ Notation, Length and Direction, Linear Combinations of Vectors, Scalar Dot Product, Projection, Angle Between Two Vectors and Parallel/ Perpendicular, Geometric Proof
S1M10	**Further Trigonometry**: Sketch Trigonometric Functions with Translations and Dilations, Solve for Angles, Trigonometric Identities, Reciprocal Trigonometric Functions
S1M11	**Matrices**: Linear Combinations of Matrices, Matrix Multiplication, The Identity, Inverse Matrices, The 2×2 Inverse, The 2×2 Determinant, Linear Transformations (including rotations, reflections and composition)

Code	**Name** and Key Concepts
S1M12	**Real and complex numbers**: Rationals, Irrationals, Interval Notation, Induction, $i = \sqrt{-1}$, Real and Imaginary Components, Complex Conjugates and Arithmetic, Argand Diagram, Modulus, Complex Roots of Polynomals
S2MM1	**Further differentiation and applications**: S1M6, Chain Product and Quotient Rules, $e = 2.718\ldots$, $\frac{d}{dx}e^x = e^x$, $\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$, Second Derivatives, Concavity and Points of Inflection
S2MM2	**Discrete random variables**: Random Variables, Discrete vs Continuous, Probability Functions and Distributions, Properties of Probabilities, Frequency, Expected Value $E[X] = \sum xp(x) = \mu_X$, Standard Deviation $\sigma_X = \sqrt{\sum(x-\mu_X)^2 p(x)}$, Uniform Bernoulli and Binomial Distributions
S2MM3	**Integral calculus**: Anti-differentiation, Reversing Chain Rule for $\int f(ax+b)dx$, Linearity of Integration, Finding the Constant of Integration, Area Under the Curve as Upper and Lower Sum Approximations, Definite Integral, Area Between Two Functions and Between a Negative Function and the x-axis, Fundamental Theorem of Calculus,
S2MM4	**Logarithmic functions**: Logs as Inverse of Exponentials, Log-Scales, Log Laws, Sketching $y = a\ln(b(x-c))$, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, For $x > 0$ $\int \frac{1}{x}dx = \ln(x) + c$
S2MM5	**Continuous random variables and the normal distribution**: $P(X = x) = 0$, Probability Density Function, $\mu_X = \int_{-\infty}^{\infty} xf(x)dx$, $\sigma_X = \int_{-\infty}^{\infty}(x-\mu_X)^2 f(x)dx$, $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, Standard Normal $Z = \frac{X-\mu}{\sigma}$, Simple Random Sampling, For $X \sim (\mu, \sigma)$ and $X_i \sim iidX$ Sampling Distributions of $S_n = \Sigma_{i=1}^{n} X_i$ $(n\mu, \sigma\sqrt{n})$ and $\bar{X}_n = \frac{S_n}{n}$ $(\mu, \frac{\sigma}{\sqrt{n}})$, If X is Normally Distributed, then so are S_n and $\bar{X}_n$, Central Limit Theorem (CLT)
S2MM6	**Sampling and confidence intervals**: Confidence Interval for a Mean using CLT $\left(\bar{x} - z^*\frac{s}{\sqrt{n}}\right) \leq \mu \leq \left(\bar{x} + z^*\frac{s}{\sqrt{n}}\right)$, Wald Interval for a Proportion
S2SM1	**Mathematical induction**: Initial Case and Induction Step
S2SM2	**Complex numbers**: Cartesian vs Polar Form, Real and Imaginary Components, Modulus and Argument, Arithmetic in both Cartesian and Polar Forms, de Moivre's Theorem including Negative and Fractional Powers, Geometric Properties of the Argand Plane, Complex Arithmetic as Transformations, n^{th} Roots of a Complex Number, Factorising Polynomials with Complex Roots
S2SM3	**Functions and sketching graphs**: Function Composition, Informal Intro to Domain and Range, One-to-One, Inverse Functions, Absolute Value Function, Graphing Rational Functions

Code	**Name** and Key Concepts
S2SM4	**Vectors in three dimensions**: Notation, Equations of a Line in $\mathbb{R}^3$, Scalar Dot Product, Vector Cross Product, $\lvert\mathbf{a} \times \mathbf{b}\rvert$ is the Area of their Parallelogram, Equation for a Plane in $\mathbb{R}^3$, Systems of Linear Equations, Geometric Interpretation of No/Unique/Infinite Solutions to a System of Linear Equations in $\mathbb{R}^3$
S2SM5	**Integration techniques and applications**: Integration by Substitution, Using Trigonometric Identities for Integration, Derivatives of Inverse Trigonometric Functions (so $\int \frac{\pm 1}{\sqrt{a^2-x^2}}dx$ and $\int \frac{a}{a^2+x^2}dx$, Integration by Parts, Partial Fractions for Integrating Rational Functions, Area Between two Curves, Volume of Solids of Revolution
S2SM6	**Rates of change and differential equations**: Implicit Differentiation, First-Order Seperable Differential Equations, The Logistic Differential Equation, Parameterised Curves, Example: if $\mathbf{v} = \frac{d}{dt}(x(t), y(t))$ is Velocity, $\lvert\mathbf{v}\rvert$ is Speed, and so the Arc Length along the Parameterised Curve is $\int_a^b \sqrt{\mathbf{v} \bullet \mathbf{v}}dt$, Trigonometric Parameterisations (unit circle, and non-circular parameterisations)
MS1	**Numbers & Functions**: Natural Numbers, Integers, Rational Numbers, Real Numbers, Functions, Intervals
MS2	**Linear Functions**: Equation for Linear Functions, Simultaneous Linear Equations, Sketching Linear Inequalities
MS3	**Quadratic Functions**: Sketching a Parabola, General Form of a Quadratic, Translations and Dilations
MS4	**Rational Functions**: Sketching Reciprocal Functions (Hyperbola), Lines of Symmetry, Limits and Asymptotes
MS5	**Trigonometry I**: Pythagoras, Similar Triangles, SOH CAH TOA, Trigonometric and Inverse Trigonometric Functions using Technology, Exact Values
MS6	**Trigonometry II**: Unit Circle, Sketching Trigonometric Functions, Finding all Solutions to Trigonometric Equations, The Sine Rule, The Cosine Rule, Introductory Trigonometric Identities, Radians
MS7	**Exponential Functions**: Index Laws, Sketching Exponential Functions, $e = 2.718\ldots$, Growth and Decay
MS8	**Logarithms**: Natural Logarithm, Logarithm Laws, Using Logarithm to Fit Growth/Decay Functions, Half-Life/ Doubling Time
MT1	**Polynomials**: Polynomial Division and "Remainder Theorem", Factor Theorem Linking Zeros to Factors, Continuous vs Discontinuous Functions, Smoothness, Sketching Factorised Form of Polynomials, Factorising Polynomials, The Quadratic Formula
MT2	**Matrices**: Order, Notation, Linear Combinations of Matrices, Matrix Multiplication (Associative but not Commutative, Distributes across Linear Combinations), The Identity Matrix, Powers of Square Matrices, Matrix Transpose, Systems of Linear Equations, Matrix Inverse, 2×2 determinant, The 2×2 Inverse, $n \times n$ Inverses, Elementary Row Operations,

Code	**Name** and Key Concepts
MT3	**Vectors and Applications**: Directed Line Segment Notation for Vectors, Magnetude/ Length and Direction, Linear Combinations of Vectors, Component and $a\mathbf{i}+b\mathbf{j}$ Notation, Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$, Scalar Dot Product, Equation for a Plane in $\mathbb{R}^3$
MT4	**Systems of Linear Equations**: Augmented Matrix for Systems of Linear Equations, Elementary Row Operations, Row-Echelon Form, Solutions to Systems of Linear Equations and Geometric Interpretations in $\mathbb{R}^2$ and $\mathbb{R}^3$, Matrix Inverses by Gauss-Jordan Elimination
MT6	**Differentiation**: Rates of Change, Gradient, First Principles, Limit Notation, Derivative Notation, $\frac{d}{dx}x^n = nx^{n-1}$ (including $n = 0$ and $n = 1$), Linearity of Differentiation, Product Rule, Quotient Rule, Chain Rule, Implicit Differentiation, Normal to a Curve
MT7	**Applications of Differentiation**: Sketching Polynomials and Rational Functions (Intercepts and Asymptotes), Continuity, Sign Diagrams, Increasing and Decreasing, Stationary Points, Points of Inflection, Concavity, Optimisation,
MT8	**Exponential and Logarithm Functions**: Sketching Exponential Functions, $e = 2.718\ldots$, $\frac{d}{dx}e^x = e^x$, Natural Logarithm, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, Growth and Decay, Surge Models, Logistic Models
MT9	**Integration**: Area Under a Curve, Lower and Upper Sums, Definite Integrals, Definite Integrals of Negative Functions, Linearity of Integration, Properties of Definite Integrals, Fundamental Theorem of Calculus, Antiderivatives, Indefinite Integrals, Integrating by Reversing the Chain Rule, Integration by Substitution, Area Between two Curves, Summation Notation (Appendix)

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