# 2 Unit Bridging Course – Day 6

## The second derivative

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# Second derivatives

The second derivative of a function is simply the derivative of the derivative of the function.

The second derivative of the function $y = f(x)$ is denoted:

$$f''(x) \text{ or } \frac{d^2y}{dx^2}$$

We find the second derivative, $f''(x)$, by differentiating the first derivative, $f'(x)$.

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## Example

Find the second derivative of $f(x) = x^3 + 4x^2 - 3x + 5$.

Differentiate to find the first derivative,

$$f'(x) = 3x^2 + 8x - 3.$$

Differentiate $f'(x)$ for the second derivative,

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Find the second derivative of $y = 2x^4 - x^3 - x^2 + 2x + 3$.

Find the first derivative,

$$f'(x) = 8x^3 - 3x^2 - 2x + 2.$$

Differentiate $f'(x)$ to get the second derivative,

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## Practice questions

Find the second derivative of the following:

1. $f(x) = 4x^3 - 4x$
2. $f(x) = x^2 + 6x - 6$
3. $y = 3x^4 + 3x^3 - 2x$
4. $f(x) = -3x^4 + 2x^2 - 5$
5. $y = 2x - 5$
6. $f(t) = 1 - 4t$
7. $g(t) = 3t^3 - 2t - 1$
8. $y = 5(x^2 - x + 2)$
9. $f(x) = (x + 1)^2$
10. $f(x) = 6 - 8x - 2x^3 - x^4$.

## Answers to practice questions

1. $f''(x) = 24x$
2. $f''(x) = 2$
3. $\frac{d^2y}{dx^2} = 36x^2 + 18x$
4. $f''(x) = -36x^2 + 4$
5. $\frac{d^2y}{dx^2} = 0$
6. $\frac{d^2f}{dt^2} = 0$
7. $g''(t) = 18t$
8. $\frac{d^2y}{dx^2} = 10$
9. $f''(x) = 2$
10. $f''(x) = -12x - 12x^2$

# Second derivatives and concavity

Some properties of the derivatives:

- $f'(x)$ is the rate of change of $f(x)$ with respect to $x$.
- $f''(x)$ is the rate of change of $f'(x)$ with respect to $x$.
- If $f''(x) > 0$ then $f'(x)$ is increasing and the curve is concave up.




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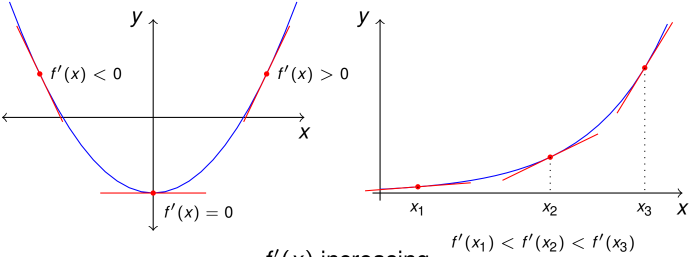
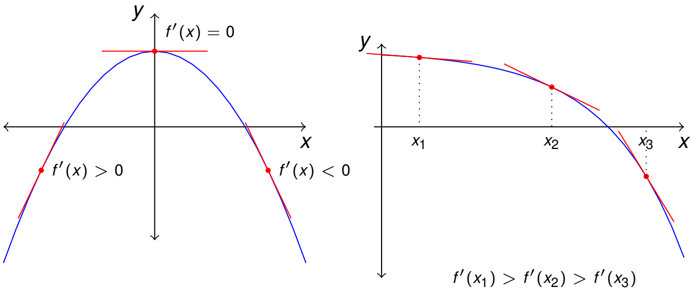



$f'(x)$ increasing.

# Second derivatives and concavity

- If $f''(x) < 0$ then $f'(x)$ is decreasing and the curve is concave down.




$f'(x)$ decreasing.

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# Inflexion points

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- A point of inflexion is a point where the curve changes concavity, ie the curve goes from concave down to concave up or vice versa.

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