# 2 Unit Bridging Course – Day 2

## Linear functions I: Gradients

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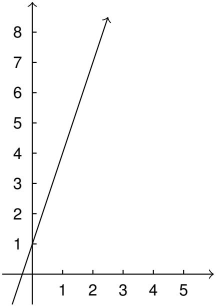
Let's look at the function $y = 3x + 1$:

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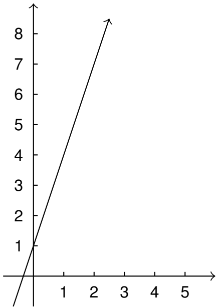


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These are all features shared by linear functions in general.

A linear function is one of the form

$$y = mx + b \tag{1}$$

or

$$ax + by + c = 0. \tag{2}$$

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# Example

Consider the function we discussed earlier, namely $y = 3x + 1$.

This is already in the form $y = mx + b$ with $m = 3$ and $b = 1$.

Can you rearrange this function into the form $ax + by + c = 0$?
If so, what are the values of $a$, $b$ and $c$?

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## Answer

Yes. Moving everything to the left-hand side gives

$$-3x + y - 1 = 0.$$

Comparing this with $ax + by + c = 0$ we see that $a = -3$, $b = 1$ and $c = -1$.

## Practice questions

Convert the following functions into form $ax + by + c = 0$:

(i) $y = 2x - 1$

(ii) $y = -x + 3$

(iii) $2y = x + 3$

Convert the following functions into form $y = mx + b$:

(i) $x + y + 1 = 0$

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# Answers to practice questions

(i) $-2x + y + 1 = 0$

(ii) $x + y - 3 = 0$

(iii) $-x + 2y - 3 = 0$

(i) $y = -x - 1$

(ii) $y = \frac{1}{2}x - \frac{3}{2}$

(iii) $y = x$

# Gradients

Thinking about our example function $y = 3x + 1$ again, let's think about what happens to the $y$-value when we change the $x$-value.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|
| $y$ | $-5$ | $-2$ | 1 | 4 | 7 | 10 | 13 | 16 |

Notice that whenever we change the $x$-value by 1, the corresponding $y$-value changes by 3.

In fact, whatever amount we change the $x$-value by, the $y$-value will change by 3 times as much (you can check this with the table above).

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## Gradients

All linear functions have this property of changing at a constant rate. This constant rate is called the *gradient* or *slope* of the linear function.

There are several ways we can work out the gradient of a linear function.

(a) When the function is written in the form $y = mx + b$, we can simply read the gradient off the equation – the number $m$ will be the gradient of the function (can you think about why this is true?)

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(b) Calculate the quantity

$$\frac{\text{change in } y}{\text{change in } x}$$

between any two points on the graph of the function (can you think about why it doesn't matter which points we choose?)

This is often called the rise over run formula (the change in $y$ is how far up the function has risen, and the change in $x$ is how far along it has run).

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(c) If our function is in form $ax + by + c = 0$, we can convert it into form $y = mx + b$, and then use (a)!

(d) Once we have learnt the basic principles of calculus (see Day 4), we can use them to calculate the gradient. You will see later how this explains all of the above methods.

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Let's find the gradient of a few lines.

## Example

(i) The gradient of our example function $y = 3x + 1$ is 3.

(ii) The gradient of the function $y = 5 - \frac{1}{2}x$ is $-\frac{1}{2}$.

(iii) To find the gradient of the function $2y + 4x - 2 = 0$, we first rearrange it into the form $y = mx + b$:

$$y = -2x + 1$$

and then read off that the gradient is $-2$.

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## Practice questions

Find the gradients of the following functions:

(i) $y + x + 1 = 0$

(ii) $2y - x + 1 = 0$

(iii) $y - x = 0$

What is the gradient of a horizontal line? What about a vertical line?

## Answers

(i) $-1$, (ii) $\frac{1}{2}$, (iii) $1$.

A horizontal line has zero gradient (draw a picture to convince yourself of this!)

The gradient of a vertical line, on the other hand, is not defined. Using the rise-over-run formula between any two points results in dividing by zero, which doesn't make sense (check this!). You may like to think of the gradient of a vertical line being “infinite”.