2 Unit Bridging Course - Day 3

Quadratics

Emi Tanaka

THE UNIVERSITY OF SYDNEY

Quadratics

A quadratic expression is an expression where it takes a standard form of:

$$ax^2 + bx + c$$

where a, b, c are constants, $a \neq 0$.

For example, the following are quadratics.

- $x^2 + 4x + 4$ (in this case $a = 1$, $b = 4$, $c = 4$)
- $2x^2 - 3x + 1$ (in this case $a = 2$, $b = -3$, $c = 1$)
  Notice $-3x = +(-3)x$
- $4x^2 - 9$ (in this case $a = 4$, $b = 0$, $c = -9$)

Notice that in all of them the highest power of x is 2.

Factorising Quadratics

Factorising a quadratic means to find 2 factors that will give you the quadratic when multiplied.
Note: not all quadratic expressions can be factorised.

For example when you factorise

$$x^2 + 2x + 1$$

you get

$$(x + 1)(x + 1)$$

Factorising Quadratics

To factorise the quadratic, $ax^2 + bx + c$, use the following method:

1. Write down $a \times c$ and b taking care to get the signs correct.
2. Write down all the factors of $a \times c$.
3. Find factors of $a \times c$, r_1 and r_2 which when added together equal b, ie $r_1 + r_2 = b$.
4. Replace bx with $r_1 x + r_2 x$.
5. Factorise the first 2 terms and the last 2 terms separately.
6. Take out the common factor to finish factorising.

Example

Factorise $2x^2 - 3x + 1$

First find $a \times c$ and b.

$$a \times c = 2 \times 1 = 2 \text{ and } b = -3$$

The factors of $a \times c$ are

- 2 and 1
- -2 and -1

Now $-2 + (-1) = -3 = b$, therefore we will use -2 and -1.

THE UNIVERSITY OF SYDNEY

Example continued

Replacing $-3x$ with $-2x - x$.

$$\begin{aligned} 2x^2 - 3x + 1 &= 2x^2 - 2x - 1x + 1 \\ &= 2x(x-1) - (x-1) \\ &= (x-1)(2x-1) \end{aligned}$$

Check by expanding $(x-1)(2x-1)$.

$$\begin{aligned} (x-1)(2x-1) &= 2x^2 - x - 2x + 1 \\ &= 2x^2 - 3x + 1 \end{aligned}$$

Example

Factorise $x^2 + 6x + 9$.

Find $a \times c$ and b.

$$a \times c = 1 \times 9 = 9 \text{ and } b = 6$$

The factors of $a \times c$ are

- 1 and 9
- -1 and -9
- 3 and 3
- -3 and -3

Now $3 + 3 = 6 = b$, therefore we will use 3 and 3.

Example 2 continued

Replace $6x$ with $3x + 3x$.

$$\begin{aligned} x^2 + 6x + 9 &= x^2 + 3x + 3x + 9 \\ &= x(x+3) + 3(x+3) \\ &= (x+3)(x+3) \\ &= (x+3)^2 \end{aligned}$$

Check by expanding $(x+3)^2$.

$$\begin{aligned} (x+3)(x+3) &= x^2 + 3x + 3x + 9 \\ &= x^2 + 6x + 9 \end{aligned}$$

Practice Questions

Factorise the following.

1. $x^2 + 6x + 8$
2. $n^2 + 3n - 4$
3. $x^2 - 2x - 8$
4. $x^2 - 5x + 6$
5. $x^2 + 6x + 9$
6. $a^2 + 5a + 4$
7. $x^2 - 2x - 24$
8. $2x^2 + 3x + 1$
9. $2n^2 - 4n + 2$
10. $3x^2 - 2x - 1$

Answers

Answers to the practice questions.

1. $(x+2)(x+4)$
2. $(n+4)(n-1)$
3. $(x-4)(x+2)$
4. $(x-2)(x-3)$
5. $(x+3)^2$
6. $(a+4)(a+1)$
7. $(x-6)(x+4)$
8. $(2x+1)(x+1)$
9. $2(n-1)(n-1)$
10. $(3x+1)(x-1)$

Difference of two squares

Notice that

$$(a+b)(a-b) = a^2 - b^2.$$

From this we can derive a special case of factorisation.

If the quadratic is a square minus a square, such that:

$$a^2 - b^2$$

then we can factorise as follows:

$$a^2 - b^2 = (a+b)(a-b).$$

This is called the difference of two squares method.

Example

Factorise $9x^2 - 16$.

$$\begin{aligned} 9x^2 - 16 &= (3x)^2 - 4^2 \\ &= (3x+4)(3x-4) \end{aligned}$$

Example

Factorise $4x^2 - 4$.

$$\begin{aligned} 4x^2 - 4 &= 4(x^2 - 1) \\ &= 4(x+1)(x-1) \end{aligned}$$

Practice Questions

Factorise the following.

1. $x^2 - 4$
2. $4n^2 - 9$
3. $9x^2 - 16$
4. $x^2 - a^2$
5. $a^2b^2 - 4$
6. $4p^2q^2 - 16$

Answers

Answers to the practice questions.

1. $(x+2)(x-2)$
2. $(2n+3)(2n-3)$
3. $(3x+4)(3x-4)$
4. $(x+a)(x-a)$
5. $(ab+2)(ab-2)$
6. $4(pq+2)(pq-2)$