

Trends in the mathematical competency of university entrants in Ireland by leaving certificate mathematics grade

FIONA FAULKNER^{†*}, AILISH HANNIGAN[†] AND OLIVIA GILL[‡]

[†]Mathematics and Statistics Department, University of Limerick, Ireland and [‡]National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL), and Department of Mathematics and Statistics, University of Limerick, Ireland

*Email: fiona.faulkner@ul.ie

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This article reports on the mathematical competency of students entering third-level education in the University of Limerick (UL). Data from diagnostic testing, gathered on 5,949 students between the years 1998 and 2008, are used to demonstrate the changes in competency levels of students entering UL. There has been a significant decline in the performance of students in the diagnostic test over time. The performance, on average, of students with the same leaving certificate mathematics grade on entry into university has not changed significantly over time. However, the profile of the entrants has changed considerably and this is contributing to the overall decline in mathematical competency.

I. Introduction

Diagnostic testing has become an increasingly popular tool in third level education, both in Ireland and abroad, to help identify weaknesses in basic mathematical skills in students in mathematics courses (Tall & Razali, 1993; Edwards, 1995, 1996; Hunt & Lawson, 1996; Lawson, 1997; Todd, 2001; Engineering council, 2002; Malcolm and McCoy, 2007). Diagnostic testing has been used in the University of Limerick (UL), Ireland since 1997 to help identify students who may be¹ ‘at risk’ of failing end of semester mathematics examinations and since then there has been a significant decline in mathematical competency (Gill, 2006).

1.1 Entry into third level: The Irish educational system

Second level education in Ireland is divided into the Junior Certificate program, which takes three years to complete, and the Leaving Certificate program, which takes two years to complete. The Junior Certificate program provides the pre-requisite knowledge needed for successful completion of

¹ Students who score 19 or less out of 40 in the UL diagnostic test are defined as ‘at risk’ of failing service mathematics (Gill, 2006). This will be discussed in greater detail in Section 2.4.

the Leaving Certificate program and both are examined through a state examination at the end of the 3- and 2-year periods, respectively. Mathematics examinations in both Junior and Leaving Certificate programs can be taken at three levels: Higher, Ordinary and Foundation Level. Students who take mathematics at Foundation level are not eligible for direct entry into third-level education

The minimum mathematics entry requirement for direct entry to universities in Ireland is a grade C or higher in Ordinary Level Leaving Certificate mathematics. The minimum entry requirement for Institutes of Technology in Ireland is a grade D or higher in Ordinary Level Leaving Certificate mathematics. The Higher Level curriculum, as its name suggests, is the most advanced level and includes more topics than the Ordinary Level curriculum, for example, integration.

Entry into third level education in Ireland is based on points received in the Leaving Certificate examination; the final state examination that students, aged ~16–19 years, sit before leaving second level education. Typically, students take seven subjects at Leaving Certificate one of which must be mathematics, providing students do not have an exemption from mathematics. Approximately 82% of Irish students study mathematics from the age of 9 to at least 17 and 96% of Leaving Certificate students study mathematics (Breen *et al.*, 2008).

Each grade received in the Leaving Certificate examination is awarded a specific number of points (Table 1). The sum of a student's highest six grades is calculated to give their CAO (Central Applications Office, 2009) points. The CAO processes all standard applications to first year undergraduate courses in higher level institutions in Ireland. Entry to these courses is competitive and is based solely on a student's CAO points. The CAO points required for entry into different degree programs can vary from year to year depending on a number of factors including number of places for a particular course together with the level of demand for the course in a given year.

Assessment at all levels of the Leaving Certificate mathematics examination is carried out by means of a 100% terminal written examination. The mathematics Leaving Certificate examination consists of two papers each covering different topics on the syllabus. The average of the two papers is calculated to determine the students' Leaving Certificate mathematics grade. This method of assessment has been said to encourage examination-focused teaching to the detriment of learning essential mathematical knowledge and skills vital to students' future success in education and life (NCCA, 2006).

TABLE 1. Irish Leaving Certificate examination points calculation grid

Leaving Certificate grade	Higher level	Ordinary level	Bonus ^a
A1	100	60	40
A2	90	50	35
B1	85	45	30
B2	80	40	25
B3	75	35	20
C1	70	30	15
C2	65	25	10
C3	60	20	5
D1	55	15	
D2	50	10	
D3	45	5	

^aBonus points are awarded by a certain colleges in Ireland, such as UL, for the achievement of an honours grade (A1–C3) in Higher Level Leaving Certificate mathematics.

This article is timely because the first major review since the 1960s of the Leaving Certificate mathematics syllabus, and the manner in which it is taught, is currently underway. The new curriculum entitled ‘Project Maths’ has been piloted in 24 schools around Ireland since September 2008. The aim of the curriculum is to place a greater emphasis on students’ understanding and the development of problem solving skills. The changes in curriculum will be met with appropriate changes in assessment in which the level of understanding of the students is assessed.

2. Diagnostic testing in the University of Limerick

2.1 *Why was the test introduced?*

Diagnostic testing was first introduced in UL in 1997 as a response to the growing concerns of mathematics lecturers about students’ mathematical preparedness. Lecturers reported students’ unease when faced with problems in slightly different formats to those they were familiar with. On examination of students’ work, evidence of difficulties with basic algebra and arithmetic emerged as well as difficulties in approaches to trigonometry, calculus and complex numbers. In an attempt to avoid diminishing degree standards and increased failure rates as well as diagnose where problems existed, the diagnostic test was introduced. The main aim of the test is to help identify those who are struggling with basic mathematical concepts needed for third level mathematics study, in an attempt to help both lecturers, to understand the ability of the students with whom they are dealing, and students, to see if/where they need help (Gill, 2006).

2.2 *When was the test introduced?*

The diagnostic test was introduced at the beginning of the academic year in 1997 for all first year Technological mathematics students—these are generally students enrolled in a technology-based degree who are required to take at least one course in mathematics. This was not the first time a mathematics diagnostic test was carried out in Ireland—Hurley and Stynes (1986) used diagnostic testing to assess the mathematical skills of students in University College Cork. However, the scale of the UL test, which currently has been used on almost 6,000 students, is much larger than any other in Ireland. In 1998, the test was carried out on first year Technological and Science students and since then the test has been used for both of these groups. It is now presented to students in their first lecture, without prior warning, with the intention of increasing the numbers taking it and measuring the level of students’ mathematical knowledge on entry to UL.

2.3 *How was the test designed?*

The diagnostic test, which is paper-based, was developed by O’Donoghue, a Professor of mathematics education in the University of Limerick, in 1997. The test was originally developed with a view to assessing the level of mathematical knowledge held by those in technology/engineering programmes in UL. The test, therefore, has a clear technology/engineering focus but was also considered appropriate for Science students.

A number of controls were used in the design of the test to ensure it fulfilled its function. The Ordinary Level Leaving Certificate mathematics syllabus, the SEFI core level zero syllabus for engineers (Barry & Steele, 1993) and an extensive literature review were all used to inform its content and structure.

The test consists of a rough work column as well as a ‘don’t know’ box to aid students in answering questions and also to offer further information regarding student competency levels to lecturers.

After feedback on the prototype from six mathematics lecturers was received, the test was finalized (see Appendix 1). Each test is marked by hand so as to enable closer inspection of a script if required (O'Donoghue, 1999).

2.4 What does the test consist of?

The diagnostic test consists of 40 questions covering nine topics: arithmetic (13 questions), algebra (8 questions), geometry (4 questions), trigonometry (3 questions), co-ordinate geometry (4 questions), complex numbers (2 questions), differentiation (3 questions), integration (2 questions) and modelling (1 question). The majority of questions on the test are aimed at Ordinary Level Leaving Certificate mathematics standard or below with the exception of six questions: two questions on integration, two questions on logarithms in the arithmetic section and two questions on differentiation. These six questions are only covered on the Higher Level syllabus. The diagnostic test has remained the same since it was implemented in 1997.

Students are advised that if they receive 19/40 or below in the test that they should avail of the support services provided by the university. These students are categorized as being 'at risk' (O'Donoghue, 1999).

3. Methodology

The statistical software package SPSS for Windows (Version 15) was used for analysing the database which consisted of data on 5,949 students collected over the time period 1998–2008. Independent samples *t*-tests were used to test for significant differences between the means of two groups. A 5% level of significance was used for all tests and no adjustment was made for multiple testing. Due to the large number of students in the database, small differences between the means can be statistically significant. The standard deviation and coefficient of variation (standard deviation/mean \times 100) are used as measures of the spread/dispersion of results. Levene's test for equality of variances was used to investigate significant differences between the variances of two groups. The equality of variances assumption was checked for all independent samples *t* tests.²

4. Decline in diagnostic test results over time

The mean diagnostic test score (expressed as a percentage of correct answers out of the 40 questions) for 507 students in 1998 was 59.3 (SD=16.5). This had declined to 50.8 (SD=17.7) for the 542 students who sat the test in 2008. The difference between these means is statistically significant ($P < 0.001$) and corresponds to a 14% reduction in performance from the 1998 baseline. There is also more variation in the scores in 2008 compared with 1998 but the difference between the variances is not statistically significant.

The decline is more dramatic for Science students (Table 2). The mean diagnostic test score for 202 Science students in 1998 was 63.1 (SD=16.3). This had declined to 50.5 (SD=17.9) for the 241 Science students who sat the test in 2008. The difference between these means is statistically significant ($P < 0.001$) and corresponds to a 20% reduction in performance in 2008 from

²Throughout the article Higher and Ordinary Level Leaving Certificate grades are referred to such as HA1 which represents a Higher Level A1 grade and OB1 which represents an Ordinary Level B1 grade

TABLE 2. Mean diagnostic test score (standard deviation) and sample size by year and group (Science and Technology)

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Science	63.1 (16.3) <i>n</i> = 202	62.1 (15.0) <i>n</i> = 189	60.6 (17.5) <i>n</i> = 193	58.5 (16.4) <i>n</i> = 135	58.9 (17.5) <i>n</i> = 141	58.2 (14.7) <i>n</i> = 150	60.0 (15.8) <i>n</i> = 181	55.4 (17.5) <i>n</i> = 227	49.9 (17.4) <i>n</i> = 266	50.5 (17.6) <i>n</i> = 263	50.5 (17.9) <i>n</i> = 241
Tech	56.8 (16.2) <i>n</i> = 305	57.8 (15.8) <i>n</i> = 267	53.6 (14.2) <i>n</i> = 304	53.6 (15.9) <i>n</i> = 211	54.7 (15.2) <i>n</i> = 212	52.6 (17.3) <i>n</i> = 187	54.0 (16.5) <i>n</i> = 225	51.2 (16.0) <i>n</i> = 270	50.6 (18.0) <i>n</i> = 360	54.9 (17.1) <i>n</i> = 317	51.2 (17.3) <i>n</i> = 301
Total	<i>n</i> = 507	<i>n</i> = 456	<i>n</i> = 497	<i>n</i> = 345	<i>n</i> = 353	<i>n</i> = 337	<i>n</i> = 406	<i>n</i> = 497	<i>n</i> = 626	<i>n</i> = 580	<i>n</i> = 542

the 1998 baseline. There is also more variation in the scores for Science students in 2008 compared to 1998 but the difference between the variances is not statistically significant. Technological students had a smaller decline from a mean diagnostic test performance of 56.8 (SD = 16.2) in 1998 for 305 students to 51.2 (SD = 17.3) in 2008 for 301 students. The difference between the means is also statistically significant ($P < 0.001$). There is also more variation in the scores for Technological students in 2008 compared to 1998 but the difference between the variances is not statistically significant.

Both groups perform to a very similar mean standard in 2008. This is in spite of Science students being traditionally of a higher mathematical competency level entering third level to Technological students and having a higher minimum entry requirement. The minimum entry requirement to degree courses which study Science mathematics is an Ordinary Level B3 compared to Technological students who have a minimum entry requirement of an Ordinary Level C3. The overall decline over time is coupled with an increase in variability of performances which can be seen in Table 2. The variability fluctuates over time for both cohorts of students; however, the overall trend is that it increases. Students in 2008 are performing further from the mean value, above and below, in the diagnostic test than they were in 1998.

The differences between the variances for both Science and Technological students' scores are not statistically significant over the 10-year period. The trend, however, particularly in the last four years (2005–2008), seems to suggest that variation of performances from the mean may continue to increase in years to come. An increase in spread such as this may reduce the ability of lecturers and other staff to group students into ability categories depending on their performance in the diagnostic test. Consequently, 'it is impossible to identify topics and prerequisite knowledge that all students are guaranteed to be familiar with' (Hourigan & O'Donoghue, 2007, p. 463). Decisions regarding issues such as the pace of lectures may also be made more complicated.

5. Aim of this study

There have been no changes to the minimum mathematics entry requirement of standard entrants to these courses from 1998 to 2008; so, what has caused the decline in the diagnostic test scores over time? The aim of this study is to investigate if the performance of students in the diagnostic test with the same Leaving Certificate mathematics grade has remained the same over time or is there evidence of grade dilution? Research which concludes that grade dilution is occurring in

the Leaving Certificate examination has been carried out (O'Grady, 2009) and will be discussed further in Section 6. The performance in the UL diagnostic test of both Science and Technological students in the core mathematical areas of arithmetic and algebra will also be investigated over the time period 1998–2008. Questions on arithmetic and algebra represent over half of the questions on the diagnostic test. The database consists of records on 5,949 students in total. The most commonly occurring grades are OA2 (13.5% of students), OB1 (11.9%) and OA1 (11.8%). The most commonly occurring higher level grade is HC1 (6.3%) and the minimum entry requirement for most courses is OB3 (5.5%). The investigation of grade dilution will focus on these five grades.

6. Investigation into students' performance over time by Leaving Certificate grade

When the most commonly occurring grades in the database are analysed, there is little change in performance in the diagnostic test, on average, for the five grades between 1998 and 2008 (Fig. 1). There is also no cross over between performances in the diagnostic test for each Leaving Certificate grade over time. This lack of cross over suggests that the diagnostic test has the ability to differentiate between students with different Leaving Certificate grades.

Figure 1 shows some slight changes in performance over time for all grades; however, the change in mean performance between 1998 and 2008 are minimal and are not statistically significant. There is, however, one exception in the case of OA1 grade students whose means are statistically significantly different between 1998 and 2008 ($P=0.03$). The mean for OA1 students went from 56.3% in 1998 to 52.8% in 2008; however, this represents just over one fewer correct questions in the diagnostic test in 2008 (Table 3).

There has been a slight increase in the spread of diagnostic test results over time for each Leaving Certificate grade indicating more variability in students' results in the diagnostic test in 2008 than in 1998 (Table 3). For example, the coefficient of variation, which is a measure of dispersion of results,

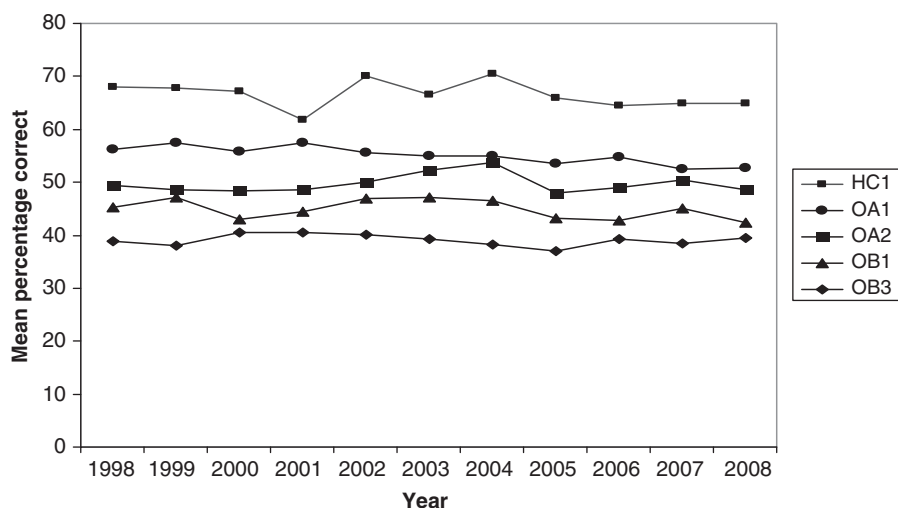


FIG. 1. Mean diagnostic test score (expressed as a percentage of correct answers out of 40 questions) from 1998 to 2008 for all students with grades HC1, OA1, OA2, OB1 and OB3.

increased from 17.9% to 24.8% for OB1 students between 1998 and 2008. It should be noted, however, that there was no statistically significant difference between the variances in 1998 and 2008 for all grades examined.

Table 3 suggests that significant grade dilution is not occurring within the Leaving Certificate mathematics examination and so this cannot be responsible for the declining mathematical standards of university entrants over time as highlighted by the diagnostic test performances in UL. These findings are in contrast to those found by the Network for Irish Educational Standards who have suggested that significant grade inflation in the Leaving Certificate Examinations has occurred between 1992 and 2006. Although O'Grady (2009) admits that 'the case for grade inflation is ... the weakest in Maths with no obvious pattern of grade increase since the early nineties', based on stronger conclusions for grade inflation across the nine other most popular Leaving Certificate subjects, he maintains that there is a strong case to doubt the validity of the increase in A and B grades in Higher Level Mathematics (p. 24). Grade inflation is defined simply as 'higher grades being awarded without improved learning' (O'Grady, 2009, p. 3). However, Fig. 1 shows an almost consistent set of diagnostic test results over the 10-year period studied in this article indicating little change, on average, in the mathematical ability of Leaving Certificate students entering UL with the grades HC1, OA1, OA2, OB1 and OB3. This suggests that the Leaving Certificate Examination is awarding the same grade for a similar average level of mathematical ability as it was ten years ago.

7. Investigation into performance in arithmetic and algebra sections of the diagnostic test

There are nine different topics covered on the UL diagnostic test. Arithmetic and algebra combined make up over half (21/40) of the questions on the test and so have been chosen for separate analyses in

TABLE 3. Mean diagnostic test score (standard deviation) and sample size by year and Leaving Certificate grade

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
HC1	68.1 (10.4) <i>n</i> = 35	67.9 (9.7) <i>n</i> = 36	67.1 (9.8) <i>n</i> = 24	61.8 (11.8) <i>n</i> = 15	70.1 (12.6) <i>n</i> = 23	66.6 (11.9) <i>n</i> = 23	70.4 (10.2) <i>n</i> = 23	65.9 (8.8) <i>n</i> = 31	64.5 (11.1) <i>n</i> = 37	64.9 (13.2) <i>n</i> = 37	64.9 (14.2) <i>n</i> = 36
OA1	56.3 (9.3) <i>n</i> = 92	57.4 (7.9) <i>n</i> = 78	55.8 (9.4) <i>n</i> = 68	57.4 (10.0) <i>n</i> = 34	55.6 (8.5) <i>n</i> = 50	55.0 (12.0) <i>n</i> = 22	55.0 (9.9) <i>n</i> = 44	53.6 (11.7) <i>n</i> = 60	54.8 (9.8) <i>n</i> = 65	52.5 (11.0) <i>n</i> = 57	52.8 (9.6) <i>n</i> = 59
OA2	49.5 (11.2) <i>n</i> = 66	48.5 (8.3) <i>n</i> = 58	48.3 (9.7) <i>n</i> = 81	48.5 (9.7) <i>n</i> = 49	50.0 (8.7) <i>n</i> = 52	52.3 (10.3) <i>n</i> = 41	53.8 (8.2) <i>n</i> = 52	47.9 (8.8) <i>n</i> = 70	48.9 (10.7) <i>n</i> = 79	50.4 (11.0) <i>n</i> = 76	48.5 (11.2) <i>n</i> = 84
OB1	45.3 (8.1) <i>n</i> = 48	47.2 (10.3) <i>n</i> = 35	43.1 (7.7) <i>n</i> = 58	44.5 (9.5) <i>n</i> = 41	47.0 (10.8) <i>n</i> = 44	47.2 (9.6) <i>n</i> = 50	46.5 (10.9) <i>n</i> = 52	43.3 (11.2) <i>n</i> = 62	42.8 (10.3) <i>n</i> = 94	45.1 (10.5) <i>n</i> = 76	42.4 (10.5) <i>n</i> = 68
OB3	38.8 (6.5) <i>n</i> = 15	38.0 (7.0) <i>n</i> = 10	40.6 (7.8) <i>n</i> = 29	40.6 (8.4) <i>n</i> = 16	40.2 (10.9) <i>n</i> = 20	39.3 (9.4) <i>n</i> = 27	38.2 (6.8) <i>n</i> = 20	37.1 (11.8) <i>n</i> = 28	39.3 (10.7) <i>n</i> = 30	38.4 (10.6) <i>n</i> = 39	39.4 (7.7) <i>n</i> = 36
Total	<i>n</i> = 256	<i>n</i> = 217	<i>n</i> = 260	<i>n</i> = 155	<i>n</i> = 189	<i>n</i> = 163	<i>n</i> = 191	<i>n</i> = 251	<i>n</i> = 305	<i>n</i> = 285	<i>n</i> = 283

this section. They are also generic mathematics skills which are pivotal to each other and to all other areas of mathematics covered in third level mathematics. Learning arithmetic is generally a precursor to learning algebra although problems when making this transition have been shown to occur by researchers of mathematics education (Kieran, 1992). A successful transition, however, to an understanding of algebraic concepts provides proven benefits to one's future success with mathematics (Gamoran & Hannigan, 2000).

7.1 Arithmetic

There is a small decline over time in arithmetic performance for both Science and Technology students (Table 4). The decline is greater for Science mathematics students, where a drop of 12.3% in the mean percentage of questions answered correctly for this section occurred. The difference in mean values between 1998 and 2008 for Science mathematics students is statistically significant ($P < 0.001$). A drop of 6.5% for Technological mathematics students between 1998 and 2008 occurred; this difference is also significant ($P = 0.018$).

These significant declines in mean percentage of questions answered correctly are accompanied by an increase in variability of results in arithmetic for both Science and Technological mathematics students, respectively. For both groups, there is a statistically significant difference between the variances in 1998 and 2008 ($P = 0.004$ for Science, $P = 0.031$ for Technology).

Upon analyses of arithmetic performance over time in terms of Leaving Certificate grades, it can be seen that there is no crossover of results between grades with one exception in 2006 when Ordinary Level B3 students outperformed Ordinary Level B1 students in this section of the diagnostic test (Fig. 2). There are no statistically significant changes in arithmetic performance over time by Leaving Certificate grades.

7.2 Algebra

Performance in the algebra section of the paper is more varied over time for all grades than was the case for arithmetic (Fig. 3). Similar to the findings for arithmetic, the Science mathematics students decreased more in performance over time compared to the Technological students (Table 5). In 1998, the mean score for Science mathematics students was 74.6% compared with 58.3% in 2008—the difference between the means was statistically significant ($P < 0.001$). For Technological students, the decline over time was 4.4% which is also a statistically significant difference ($P = 0.026$).

TABLE 4. Mean arithmetic score (standard deviation) and sample size by year and group (science and technological)

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Science	66.6 (15.0) $n = 202$	63.1 (14.5) $n = 189$	65.0 (15.3) $n = 193$	61.0 (17.4) $n = 134$	62.7 (16.0) $n = 141$	62.2 (14.3) $n = 150$	62.9 (15.5) $n = 181$	59.6 (17.3) $n = 227$	56.6 (17.3) $n = 266$	55.7 (17.3) $n = 263$	54.3 (18.6) $n = 241$
Tech	62.4 (14.9) $n = 305$	62.5 (15.2) $n = 267$	60.4 (13.7) $n = 304$	59.6 (14.7) $n = 211$	61.8 (14.8) $n = 212$	57.5 (16.5) $n = 187$	58.6 (15.0) $n = 225$	57.4 (15.3) $n = 270$	55.6 (17.6) $n = 360$	59.4 (15.5) $n = 317$	55.9 (16.8) $n = 301$
Total	$n = 507$	$n = 456$	$n = 497$	$n = 345$	$n = 353$	$n = 337$	$n = 406$	$n = 497$	$n = 626$	$n = 580$	$n = 542$

An increase in the variability, as measured by the coefficient of variation, of results over time occurred for both groups of students. For Science students, an increase of 14.9% in the variability of test scores occurred between 1998 and 2008, and an increase of 4.9% occurred for Technological students (Table 5). There was a statistically significant difference between the variances for Science mathematics results in 1998 and 2008 ($P=0.006$); however the difference was not statistically significant for Technological mathematics results.

There is more fluctuation over time and some crossover in performance between grades; in 1999, OB1 grade students outperformed OA2 grade students and, in 2004, OA2 students outperformed OA1 grade students in the algebra section of the test (Fig. 3). There were no statistically significant changes in algebra performance over time by Leaving Certificate grades.

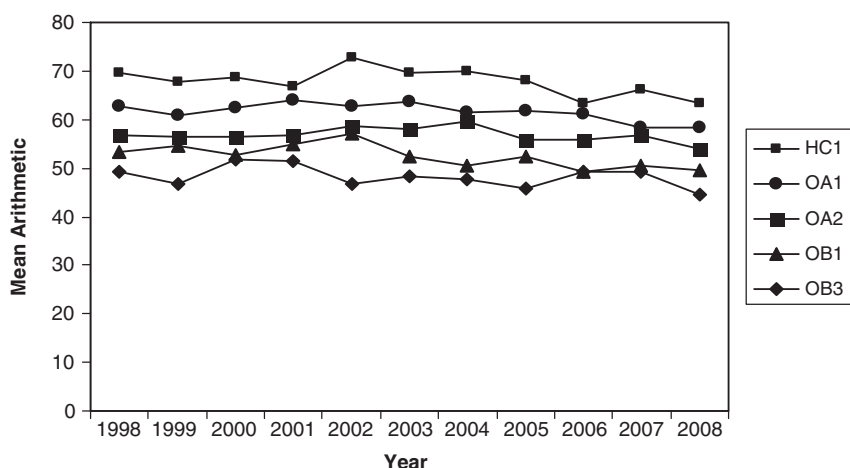


FIG. 2. Mean arithmetic score (expressed as a percentage of correct answers out of 13 questions) from 1998 to 2008 for all students with grades HC1, OA1, OA2, OB1 and OB3 in science and technological mathematics.

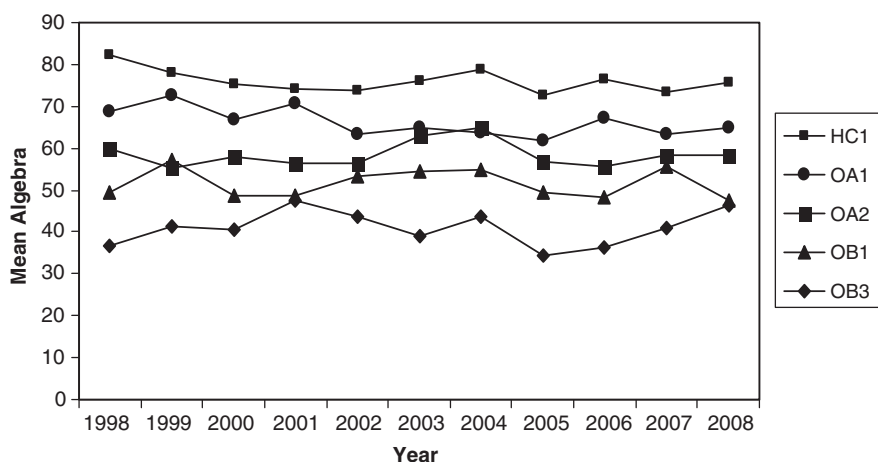


FIG. 3. Mean algebra (expressed as a percentage of correct answers out of 8 questions) from 1998 to 2008 for all students with grades HC1, OA1, OA2, OB1 and OB3.

7.3 Conclusion

There were no significant changes in mean performance between 1998 and 2008 for these two areas of the diagnostic test according to the five grades being analysed. A significant decline over time occurred for Science and Technological mathematics students in UL in the areas of arithmetic and algebra along with an increase in the variability of results. A decline in performance in these particular areas highlights that students in 2008 do not have as good knowledge of algebra and arithmetic as they did in 1998 and this could have detrimental effects on a students' future achievement in mathematics and their ability to successfully complete service mathematics courses in UL.

8. Changing profile of students over time

The changing profile of students attracted to Science and Technological degrees over time is evident from Table 6, and is likely to be a contributing factor to the decline in diagnostic test score.

Both Science and Technological degrees in UL attracted more students in 2008 than in 1998. The number of students in these degrees with Higher Level mathematics has not changed over time but the proportion of students with Higher Level mathematics has decreased from 46.7% in 1998 to 35.3% in 2008. The extra students attracted to these degrees are students with Ordinary level mathematics (from 266 to 381 students) and non-standard² students (from 4 to 57 students). There has been a drop of 7.8% of the total cohort entering Technological mathematics with Higher Level mathematics and a much larger drop of 17.4% between 1998 and 2008 of those entering Science mathematics with Higher Level mathematics (Table 6). The bigger decrease in the proportion of students in Science mathematics with Higher Level mathematics, largely explains why Science students have seen bigger declines over time to Technological students in all of the analyses done so far in this article (Faulkner *et al.*, 2009).

TABLE 5. Mean algebra score (standard deviation) and sample size by year and group (science and technological)

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Science	74.6 (20.5) <i>n</i> = 202	71.4 (22.2) <i>n</i> = 189	68.6 (22.8) <i>n</i> = 193	67.3 (24.1) <i>n</i> = 134	65.0 (24.1) <i>n</i> = 141	65.8 (22.0) <i>n</i> = 150	69.3 (21.3) <i>n</i> = 181	61.0 (21.8) <i>n</i> = 227	56.1 (26.1) <i>n</i> = 266	58.1 (24.5) <i>n</i> = 263	58.3 (24.7) <i>n</i> = 241
Tech	63.8 (23.6) <i>n</i> = 305	66.3 (23.1) <i>n</i> = 267	61.2 (21.7) <i>n</i> = 304	61.0 (24.3) <i>n</i> = 211	60.7 (22.4) <i>n</i> = 212	58.3 (24.6) <i>n</i> = 187	62.0 (24.1) <i>n</i> = 225	56.8 (23.4) <i>n</i> = 270	56.7 (25.9) <i>n</i> = 360	62.4 (24.3) <i>n</i> = 317	59.4 (24.8) <i>n</i> = 301
Total	<i>n</i> = 507	<i>n</i> = 456	<i>n</i> = 497	<i>n</i> = 345	<i>n</i> = 353	<i>n</i> = 337	<i>n</i> = 406	<i>n</i> = 497	<i>n</i> = 626	<i>n</i> = 580	<i>n</i> = 542

TABLE 6. Percentages and numbers of students entering UL with higher level and ordinary level Leaving Certificate mathematics and non-standard students

Year	Whole cohort		Technological maths		Science maths	
	1998	2008	1998	2008	1998	2008
Percentage doing HL	237 (46.7%)	239 (35.3%)	125 (41.0%)	124 (33.1%)	112 (55.4%)	115 (38.0%)
Percentage doing OL	266 (52.5%)	381 (56.3%)	179 (58.7%)	215 (57.5%)	87 (43.1%)	166 (55.0%)
Non-traditional students	4 (0.8%)	57 (8.4%)	1 (0.3%)	35 (9.4%)	3 (1.5%)	21 (7.0%)
Total	507 (100%)	677 (100%)	305 (100%)	374 (100%)	202 (100%)	302 (100%)

A shift in student intake like this has been shown to affect overall mathematical performance in UL, as highlighted by the diagnostic test results over time, as well as in other institutions such as Coventry University (Lawson, 2003). The findings of Barry & Chapman (2007) also state that performance in mathematics in third level institutions has been shown to be better when students have Higher Level mathematics as pre-requisite knowledge.

From Section 6, we can conclude that student performance in the diagnostic test according to Leaving Certificate grade over time has remained largely unchanged; the difference between average performances for each grade between 1998 and 2008 is not statistically significant with the exception of OA1 students. We also saw that, in general, the higher the Leaving Certificate grade, the better the performance in the diagnostic test (Fig. 1). In 2008, the mean diagnostic test score was 65.7 (SD=13.5) for those with Higher Level mathematics, 44.7 (SD=11.8) for those with Ordinary Level mathematics and 30.7 (SD=21.0) for those in the non-standard category.

The increase in students entering UL with lower Ordinary Level grades is therefore one major factor contributing to the decline in performance in the diagnostic test over time. The increase in non-standard students is another contributing factor. Upon analyses of students who perform poorest in the diagnostic test, it is very clear to see that the Ordinary Level Leaving Certificate students together with the non-standard students perform poorest.³ Shifts such as this in the students entering third level have been shown to have negative effects on mathematical performance in UL as well as in other countries such as the UK. Hourigan and O'Donoghue (2007) describe the changing profile of third level mathematics students in Ireland as 'the single most detrimental factor leading to under-preparedness within the mathematics-intensive courses in tertiary level' (p. 463).

The investigation of the non-standard category which have the lowest mean diagnostic test score and also the most variability in scores requires further research.

9. Discussion and conclusions

The information outlined in this article confirms the usefulness of a mathematics diagnostic test in UL. Anecdotal evidence from those involved in mathematics education regarding declining standards needs quantitative evidence in order to see exactly where the problems lie and how effective change could be implemented.

A decline in mean percentage achieved in the diagnostic test of 14% from the 1998 baseline occurred. This decline was more dramatic in the case of Science mathematics students who experienced a 20% decline in performance in 2008 from the 1998 baseline. There was also an increase in the variability of scores over time for both cohorts; however, the change in variability over the ten years was not found to be statistically significant. Possible implications of an increasing variability of diagnostic test scores could be difficult for those involved in mathematics education to establish a common level of competency amongst students resulting in issues with choosing lecture material and pacing of lectures.

A further investigation into this decline examined the change in performance in the diagnostic test of students entering UL with particular Leaving Certificate mathematics grades. This showed that there has been no significant change over time (with the exception of OA1 grades students) in performance on average and there has been no crossover between grades i.e. on no occasion did a lower Leaving Certificate grade outperform a higher one in the diagnostic test. This suggests that the Leaving

³ A non-standard student is defined as a student who did not enter UL through the CAO system. This cohort therefore consists of mature students (i.e. anyone over the age of 23), non-national students and those who have completed previous certificate/diploma/degree and have used these as an entry point to UL.

Certificate is still producing the same level of competency of mathematics for the same Leaving Certificate grades on average and therefore grade dilution is not responsible for the declining standards within Science and Technological mathematics in UL. These findings are in contrast to those found in the UK in which Lawson (2003) describes the decline over time in mathematical competency levels of A level students entering university with the same A level grades. Lawson (2003) concludes that 'A-level mathematics does not produce the same degree of competency as it did formerly'.

Analyses of Science and Technological students' performance in the algebra and arithmetic sections of the diagnostic test showed statistically significant declines in performance over time as well as increases in variability of results; in several cases, the increase in variability was statistically significant. Once again, Science mathematics students experienced the largest decline in both areas of the test. This therefore highlights that students entering UL with the Leaving Certificate grades being examined did not experience a major decline over time for the entire diagnostic test. However, statistically significant declines over time in the algebra and arithmetic sections of the paper occurred for the entire cohort, i.e. when we ignore performance by Leaving Certificate grade, which as previously mentioned make up over half of the questions on the paper and are vital for progression in other areas of mathematics.

Finally, the changing profile of UL mathematics students was investigated. It can be seen that there is a decrease in the percentage of Higher Level Leaving Certificate students, an increase in the number of students entering Science and Technological mathematics with Ordinary Level grades as well as an increase in the percentage of non-standard students. Figure 1 highlights that performance in the diagnostic test is directly related to a students' entry grade to UL. This changing profile within the Science and Technological mathematics students between 1998 and 2008 is therefore the major contributing factor to the declining standards in mathematical competency of students entering UL.

These findings must also be analysed in terms of practical significance to the teaching and learning of mathematics in third level education in Ireland and elsewhere. It is no longer the same profile of students who enter our lecture halls today. Students' mathematical backgrounds are not as strong as they were ten years ago and one can only conclude that some adaptations regarding the starting point of lecture material needs to be made or 'in the absence of any change of starting point, a deterioration in the effectiveness of learning' (Hunt and Lawson, 1996, p. 171) will occur.

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Fiona Faulkner graduated from the University of Limerick with a Bachelor of Science in Physical Education and Mathematics. She started her PhD in Mathematics Education in October 2008.

Ailish Hannigan is a lecturer in Statistics and consultant statistician to the National of Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL) at the University of Limerick.

Olivia Gill is manager of the Mathematics Learning Centre in the Department of Mathematics. She completed her PhD in Mathematics Education at the University of Limerick in 2006.

Appendix I. The UL diagnostic test

UNIVERSITY OF LIMERICK
DEPARTMENT OF MATHEMATICS AND STATISTICS
DIAGNOSTIC TEST

PURPOSE

The purpose of this test is to ascertain whether there are gaps in your knowledge of basic mathematics. A programme of extra voluntary tutorials is planned for students who need assistance in mathematics during this term.

NOTE

This assessment does not contribute in any way to your grade for this course

INSTRUCTIONS

Please fill in the details requested and proceed to answer questions.
 Attempt each question and if you do not know how to do a question, simply tick the *don't know* box. There is no penalty. Calculators are not allowed.

TIME: 40 mins (approx)

Name _____ I.D. No _____

UL Programme _____

Please tick (✓) the appropriate grade:

LC (Maths): Higher A1 ☐ A2 ☐ B1 ☐ B2 ☐ B3 ☐ C1 ☐ C2 ☐ C3 ☐ D1 ☐ D2 ☐ D3 ☐

Grade

Ordinary A1 ☐ A2 ☐ B1 ☐ B2 ☐ B3 ☐ C1 ☐ C2 ☐ C3 ☐ Other ☐

Mature Students: Please enter grade in appropriate box

LC (Higher) ☐

Ordinary ☐

ARITHMETIC Q1 - Q13**ROUGH WORK**

1. Work out $-(-8) + (-3)$

Ans _____ ☐ Don't know

2. Write down the value of $10 - 8 \div 2 + 9$

Ans _____ ☐ Don't know

3. Work out: $\frac{1}{2} - \frac{1}{3}$

Ans _____ ☐ Don't know

4. Find the mean of the numbers 12, 14, 10

Ans _____ ☐ Don't know

5. Work out: $\frac{2}{3} \times \frac{4}{5}$

Ans _____ ☐ Don't know

6. Find 25% of 500

Ans _____ ☐ Don't know

7. Write down the value of 3^4 .

Ans _____ ☐ Don't know

8. Write down the value of $8^{\frac{1}{3}}$.

Ans _____ ☐ Don't know

9. Write down the value of $\frac{1}{2^{-4}}$.

Ans _____ ☐ Don't know

10. If $x = 10^2$ then write down the value of $\log x$.

Ans _____ ☐ Don't know

11. If $\log x = 5$ then write down the value of $\log(x^2)$.

Ans _____ ☐ Don't know

ROUGHWORK

12. Express 0.01234 in Scientific Notation.

Ans _____ ☐ Don't know

13. Divide 30 in the ratio 3:2

Ans _____ ☐ Don't know

ALGEBRA Q14 - Q21

14. Solve for h : $V = \pi r^2 h$

Ans _____ ☐ Don't know

15. Evaluate $ab + 2bc - 3ac$ when $a = 3$, $b = -2$ and $c = 4$.

Ans _____ ☐ Don't know

16. Solve the equation: $3(x+2) - 24 = 0$

Ans _____ ☐ Don't know

17. Solve for x : $x^2 + x - 6 = 0$

Ans _____ ☐ Don't know

18. Solve the set of equations:

$$2x + y = 7$$

$$x + 2y = 5$$

Ans _____ ☐ Don't know

19. Write out $(x+3y)(a-2b)$ in an equivalent form without brackets.

Ans _____ ☐ Don't know

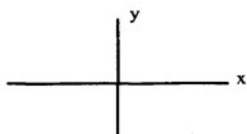
20. Solve for x : $3 - 6x < 21$

Ans _____ ☐ Don't know

21. Simplify $\frac{1}{x-1} - \frac{2}{x+1}$

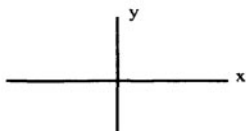
Ans _____ ☐ Don't know

31. Sketch the line $y = 3x + 2$ on the diagram.



☐ Don't know

32. Sketch the curve $y = x^2 + 2$ on the diagram.



☐ Don't know

COMPLEX NUMBERS Q33 - Q34

33. If z is the complex number $1+2i$ find the modulus $|z|$.

Ans _____ ☐ Don't know

34. Simplify: $\frac{1+2i}{2-3i}$

Ans _____ ☐ Don't know

DIFFERENTIATION Q35 - Q37

35. If $y = 2x^2 + 3$ find $\frac{dy}{dx}$

Ans _____ ☐ Don't know

36. If $y = x \sin x$ find $\frac{dy}{dx}$

Ans _____ ☐ Don't know

37. If $y = e^{-2x}$ find $\frac{dy}{dx}$

Ans _____ ☐ Don't know

ROUGH WORK

INTEGRATION Q38 - Q39

38. Evaluate $\int (x^2 + 2x + 3)dx$

Ans _____ ☐ Don't know

39. Evaluate $\int_0^3 x^2 dx$

Ans _____ ☐ Don't know

MODELLING (SPECIAL QUESTION)

40. The ESB charges domestic users £0.92 per unit of electricity used. In addition each customer is also charged a standing charge of £6.50. Devise a formula for calculating the amount A of a customer's monthly bill when she uses x units. (Ignore VAT).

Ans _____ ☐ Don't know

ROUGH WORK