2 Unit Bridging Course – Day 5

The Derivative of a Polynomial

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Polynomials

A polynomial is an expression that has the form

$$c_0 + c_1 x + c_2 x^2 + ... + c_n x^n$$

where n is a positive whole number and c_0,...,c_n are constants.

For example the following are polynomials:

- $x^5 + 4x^4 + 2x^3 - 4x^2 - 2x + 4$
- $2n^3 - 3n + 1$

However the following is not a polynomial since it has some negative powers:

- $3x^2 - \frac{5}{x} + \frac{3}{x^2} = 3x^2 - 5x^{-1} + 3x^{-2}$.

Note that a quadratic is also a polynomial (of degree 2).

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Derivative of a polynomial

In Day 4, we learnt how to differentiate quadratics.

Now we want to differentiate polynomials of higher degree. To do this we use the following general result.

Derivative of x^n

For positive integers n

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

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Differentiate $y = x^4 + 5x^3 - 2x^2 + 4x - 7$.

$$\begin{aligned}\frac{dy}{dx} &= 4 \times x^{4-1} + 5 \times 3 \times x^{3-1} - 2 \times 2 \times x^{2-1} + 4 \\ &= 4x^3 + 15x^2 - 4x + 4.\end{aligned}$$

Differentiate $f(x) = \dfrac{x^3}{10} + 3x^2 - 1$.

$$\begin{aligned}f'(x) &= \frac{1}{10} \times 3x^2 + 3 \times 2x \\ &= \frac{3x^2}{10} + 6x.\end{aligned}$$

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Practice Questions

Differentiate the following:

1. $y = 2x^2 + 3x + 1$
2. $m = n^3 - n^2 + 4n - 1$
3. $f(x) = 4x^{10} - 3x^7 + 5$
4. $f(x) = \frac{2x^3}{3} - 10x + 6$
5. $f(a) = a^4 - a^2 + 2a + 4$
6. $f(x) = x^4 + 2x^3 + x^2 + 9$
7. $f(x) = 1 + 2x - x^3$
8. $y = 2x^5 + 3x^4 + 3x^2$
9. $f(n) = 2n^5 - 4n^3 + 2$
10. $y = x^2 - 2x^5$.

Answers

Answers to practice questions

1. $\frac{dy}{dx} = 4x + 3$
2. $\frac{dm}{dn} = 3n^2 - 2n + 4$
3. $f'(x) = 40x^9 - 21x^6$
4. $f'(x) = 2x^2 - 10$
5. $f'(a) = 4a^3 - 2a + 2$
6. $f'(x) = 4x^3 + 6x^2 + 2x$
7. $f'(x) = 2 - 3x^2$
8. $\frac{dy}{dx} = 10x^4 + 12x^3 + 6x$
9. $f'(n) = 10n^4 - 12n^2$
10. $\frac{dy}{dx} = 2x - 10x^4$.