The Changing Profile of Entrants to Mathematics at A Level and to Mathematical Subjects in Higher Education

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ABSTRACT *Over the past decade most, if not all, university departments of mathematics, science and engineering have talked about a decline in the mathematical capabilities of their entry cohorts. Anecdotal evidence from the universities and the press has blamed this drop on a lowering of standards at A level. However, it may be due to the lowering of admission requirements to allow for the expansion in university places. This article attempts to quantify the present situation by looking at the characteristics of the student population at both the interface between the General Certificate of Secondary Education and A level, and A level and higher education over time. As well as analysing official published data and hitherto unpublished data from the Universities and Colleges Admissions Service, it builds on some initial work carried out at the Mechanics in Action Project at the Universities of Manchester and Leeds over the past 6 years as part of the evaluation of the effects of the Project's mathematics curriculum development work post-16. An explanation for the universities' perception of declining standards is found in (a) the changing A level cohort, (b) the universities' policy on recruitment, and (c) able students choosing not to study mathematical subjects.*

Introduction

> *There is an overwhelming consensus among University teachers of Mathematics, Physics and Engineering that the mathematical achievement of students entering University with given grades is substantially worse than those entering with the same grades five or ten years ago. ([School Curriculum and Assessment Authority, SCAA], 1996a, p. 7.1)*

> *The clear decline in analytical powers, algebraic manipulation and technical fluency of many undergraduates to a broad range of courses, including*

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> *specialist mathematics and statistics courses, must be addressed. (SCAA, 1996a, p. 9.1)*
>
> *In our University, as at many institutions, there is a commonly held belief amongst academic staff in Mathematics and Engineering that students' skills in algebraic manipulation have deteriorated significantly. (SCAA, 1996a, p. 15.2)*

These comments are from papers submitted to the SCAA Seminar on A level mental mathematics by an invited group of academics. They typify the perceptions that the mathematical capabilities of many of the students going into mathematics, science and engineering at university level are inadequate and that the situation is worse now than formerly (London Mathematical Society, [LMS], 1995; Sutherland & Pozzi, 1995). Such a decline would have a very negative effect on both teaching and learning at degree level. Several faculties are introducing remedial mathematics teaching to their first year undergraduates and one mathematics department (Warwick) is asking for an additional mathematics qualification to an A level in mathematics or further mathematics. In addition, as stated in the report on science and mathematics in full-time education (Cheng *et al.*, 1995), the rapid growth of new technologies in modern industries suggests that as we approach the new century Britain will face an increasing demand for scientists and mathematicians. Thus, any reduction in quality or quantity of such undergraduates will affect the companies, businesses and institutions that need to recruit high calibre graduates from these disciplines. There is far less agreement on the causes of such shortcomings. Some blame has been levelled at new teaching and assessment methods used in A level courses since the early 1990s, but preliminary research suggested that the root cause may lie in the lack of suitably qualified students applying for these disciplines (Kitchen, 1996). In a report from the Association of Teachers of Mathematics (ATM, 1996), the Post-16 Group identified several factors that were considered anecdotally to have had an effect on enrolment and achievement in mathematics post-16. They called for research to be carried out in this field.

There are two main boundaries where students choose subjects for future study. These occur at the General Certificate of Secondary Education (GCSE)/Advanced level (A level) boundary and the A level/higher education (HE) boundary. It is obvious that there could be many causes for the perceived decline in mathematical attainment of students at each of these boundaries. Many of these are interrelated. The current debate means that there is a need for relevant data to be published. It is the aim of this article to make available some such data and draw conclusions from them.

Methodology and Data Sources

Much of the data used here comes from previously unpublished sources. One is from the Universities and Colleges Admissions Service (UCAS) database; the second from my work with the Mechanics in Action Project (MAP) in the 6 years funded by the Gatsby Charitable Trust, when I worked in association with the School Mathematics Project (SMP) and the Northern Examinations and Assessment Board (NEAB) on the development of a new A level syllabus, SMP 16–19 Mathematics. Although much of my work was in developing the syllabus and producing both teaching and in service education and training (INSET) materials, a significant part was in evaluating the work in progress. As part of that work, over 500 centres, including 11–18 schools, sixth form colleges and

TABLE I. Surveys carried out between 1992 and 1996 for the Mechanics in Action Project.

Date	n	Response rate (%)	Population	Method
1992	254	85	All SMP 16–19 Mathematics centres	Questionnaire
1993	400	60	All SMP 16–19 Mathematics centres	Questionnaire
1995	500	40	All SMP 16–19 Mathematics centres	Questionnaire
1996	102	98	SMP 16–19 Mathematics centres attending the 1996 SMP summer conference	Questionnaire
1995/96	5	100	University mathematics departments	Collection or raw admissions data

further education colleges, were surveyed and relevant discussions were held with over 2000 teachers in small groups. The results of these were used to inform the design of both the syllabus and the materials and evaluate the success of the work in achieving its published aims. The evaluation was disseminated by means of internal reports to our sponsors and partners, by papers at conferences such as the International Conference of the Teaching of Mathematics and its Applications (ICTMA) (Kitchen, 1993a) and by means of articles published in newsletters, professional journals and books (SMP, 1991–96); (MAP, 1991–97); (Kitchen, 1993b; Kitchen & Lord, 1996). The data and information collected related mainly to the situation at ages 16–19, although some related to the qualifications of the students on entry to post-16 education. However, many of the questions asked had a much wider application. In addition, in order to ensure the reliability of the samples and measure the overall comparability of the SMP 16–19 Mathematics A level with others, comparable national data were collected for the population as a whole. This study attempts to synthesise the data collected from centres for the period 1991–97 with national data from such bodies as the SCAA, now the Qualifications and Assessment Authority (QCA), and UCAS. While in most cases, research is based on a questionnaire designed solely for the hypotheses to be tested, it is my contention that it is no less valid to analyse responses taken for another purpose. This is common when researchers use census data for example.

Data have been used from four surveys of the centres taking SMP 16–19 Mathematics, which I carried out. The centres doing SMP 16–19 Mathematics A level came from all over England and Wales and had a similar spread between 11–18 schools, sixth form colleges and colleges of further education to the population as a whole. Three of the surveys used all the centres and one a specific sample of the centres. The surveys covered both factual questions such as class size and sex of students and open-ended questions concerning the suitability of content and teaching methods. In addition, several university mathematics departments were questioned in 1995 about the entry profile of their students. The populations and response rates are given in Table I.

My research (at present funded by the Zochonis Trust) is ongoing.

There is one major problem, that must be recognised and addressed, when attempting any such study. This is the variability of any statistical data, especially grouped data, collected over time from a variety of sources. This is true of some of the data used here. For example, at age 16, the standard 'pass' criterion has changed from a 'C' at Ordinary (O) level through to either a 'C' at O level or a '1' at the Certificate of Secondary

Education (CSE) to the present 'C' at GCSE. This latter can be awarded on either a basic syllabus at intermediate tier, or on a more demanding syllabus at the higher tier. Furthermore, the Department for Education and Employment (DfEE) or its predecessors varied between including and excluding Wales in the official statistics. In some SCAA publications, the data relating to proportions have as denominator the number of the cohort in England, while the numerator is the number of students attaining a certain qualification in England, Wales and Northern Ireland. While this still enables a picture of the situation as a whole to be obtained, it must be interpreted with care. As well as this variability in the actual cohort measured, the change to modularity at A level has meant that such terms as 'Double Mathematics' no longer have the meaning understood by many in higher education. In the 1980s the term implied that the student had studied an enlarged, more difficult, pure mathematics syllabus. This is still the case for syllabuses such as SMP 16–19 Mathematics where further mathematics has a compulsory 50% of more advanced pure mathematics. It is not the case for many of the modular syllabuses where a student can get a double award which has no more pure mathematics in it than that specified for a single A level.

The problem of variability is extended to statistics from the higher education sector. Prior to 1993 the University Central Council for Admissions (UCCA) and the Polytechnics Central Admissions System (PCAS) had their own statistical services and did not publish joint data. When many institutions changed their status from polytechnics to universities in 1993 the two admissions authorities merged to form UCAS. The new system incorporated a sophisticated computer database on the entry profile of students. However, this amalgamation of two different systems means that it is difficult to interpret longitudinal data for entry qualifications for degree subjects such as engineering, though the two main degree courses for mathematics (G1) and physics (F3) have remained mainly the preserve of the 'older universities' and thus their entrants can be compared over a longer time span than 1994–96. Much of the disquiet voiced over the standards of A level mathematics has come from these departments. Most of the statistical investigations into entrance to higher education, therefore, will centre on these two courses. The data taken from my own research and the UCAS raw data do not suffer from any of the above limitations.

The article will look at statistics covering the years from 1980. I have also attempted to cross-check data from other sources where possible. It is all too easy to draw erroneous conclusions from a figure that might be a typing error in a report.

I have had the cooperation of UCAS, who have made available to me some of their unpublished university entrance data for the 1994-96 cohorts, and it is analysis of these data that led me to focus on the lack of a sufficiency of well-qualified mathematicians who choose to study mathematically related degrees, in mathematics, physics and engineering in particular.

The GCSE/A Level Boundary: who chooses mathematics?

Most students give a variety of reasons for subject choice at A level. In the study by Cheng *et al.* (1995), it was found that a good GCSE result in mathematics strongly increased the probability of studying mathematics at A level. This matches with my own experience both when teaching A level and when interviewing students during the course of my research. Success at GCSE seems to be measured by a grade of C or above as far as league tables, teachers and parents are concerned. Hence, most of the statistics available are for numbers achieving a C or greater. Since the early 1980s these numbers

TABLE II. Students achieving an A*–C at O level, GCSE or level 1 in CSE in mathematics for 1980–95.

	1980	1985	1990	1995
Students (000s)	212	228	211	231
Year 11 cohort (000s)*	787	734	580	580
Students as a % of cohort	26.9	30.0	36.4	39.8

Data from: Dearing, 1996 and SCAA, 1996b.
*England.

have increased. Some, but not all, of this increase is in line with the demographic trends. Table II gives the numbers achieving an A*–C at 5-year intervals.

The pool of candidates from whom A level students can be expected to be drawn would seem from Table II to be increasing in size again. However, this may well not be the case. Very few students with a grade 1 at CSE were allowed to take A level, and until quite recently the number of students going on to A level with a grade C in GCSE mathematics was very small (in 1994, for example, around 7% of those taking an A level in mathematics had entered with a C at GCSE [Dearing, 1996]). There were fewer students getting an A*, A or B at GCSE in 1995 than got Cs (21.6% of the entry cohort compared to 23.1%) (SCAA, 1995).

There has been much discussion over the appropriate GCSE qualification needed as entry for an A level. Both the 1993 (SCAA, 1993) and the proposed 1997 (SCAA, 1997) cores discuss this. In the 1993 core it was suggested that students should have mastered content at level 7 of the National Curriculum and met several of the topics at level 8, the equivalent of a C at GCSE. The 1997 core suggests that students should have mastered level 7 and also a specified list of content from higher levels. This means that without additional teaching in year 11, many students who have been prepared for the intermediate tier at GCSE will not have covered all the content that is required as a prerequisite to A level study. While many teachers and academics are keen to dissuade students with a C grade on either the intermediate or higher tier from attempting an A level in mathematics, the required GCSE grade has fallen over the past 5 years for many institutions. In my survey of attitudes towards A level teaching in the summer of 1996, I asked a sample of 100 A level mathematics teachers from different institutions to give the entry grades required by their institution in 1991 and 1996. Table III gives the percentages of institutions requiring each grade.

The change in those accepting students with a C grade at the intermediate tier is significant. It shows a definite change in attitude over the past 5 years. Obviously many centres will have very few students at the lowest grade even if they are willing to

TABLE III. Centres by required GCSE grade for entry to A level mathematics, 1991 and 1996.

Entry grade	B	C (higher)	C (intermediate)	Open access
1991* (n = 88)	19	26	40	3
1996 (n = 100)	12	13	72	3

*Not all teachers knew the 1991 grades or were teaching then.
MAP survey, June 1996.

TABLE IV. Proportions of year 11 cohort achieving an A*–C at O level, GCSE, or level 1 in CSE in mathematics and English by sex for 1980–95.

	1980	1985	1990	1995
Mathematics				
Males (%)	29	33	36	40
Females (%)	23	28	33	40
English				
Males (%)	31	34	40	42
Females (%)	41	46	57	60

Data from: SCAA, 1996c.

accept students with a grade C at intermediate level. There are no national data on the success of students at A level according to the *tier* of GCSE they took. Anecdotal evidence from responses to the aforementioned survey suggests that students can succeed if they are well motivated. This is substantiated by data I collected from several colleges who accept students with a grade C. However, students need considerable help and encouragement if they are not to fall by the wayside. The new tier arrangements for the 1998 GCSE, with a B being available for all those taking the intermediate tier, may well have a great effect on both recruitment and success at A level.

Many 11–16 schools are talking openly about entering all their students for the intermediate tier to maximise their number of A–C grades. This has been confirmed by the findings of the joint working group looking at algebra (Royal Society, 1997), who state that many schools are not entering even their most able students for the higher tier as pupils have a higher chance of obtaining a B or C at the intermediate tier. This means that many sixth form and further education (FE) colleges who at the moment accept mainly students who have followed the higher tier with its increased content, especially in algebra, may find that they have to accept students, who may well be able, but who through no fault of their own have covered less content. Early data from my ongoing research into achievement in pure mathematics at A level show that at least one FE college has over 40% of its 1997 entry to its A level mathematics course with a GCSE mathematics award at intermediate level. This looks as though it will become more common in the future.

Boys outperformed girls in mathematics at GCSE (and previously at GCE ordinary level) throughout the 1980s (SCAA, 1996b). This meant that fewer girls had the basic qualifications needed to go on to study A level mathematics. However, the performance of girls seems to have gradually increased to match that of the boys. Looking at the statistics in Table IV for those who gained an A*–C in 1995, gender does not now appear to be a factor. However, this may be misleading. The equivalent data for GCSE English show that the girls outperform the boys. It may be that more of the girls perceive their strengths to be in the arts and humanities and go on to study these subjects at A level. In addition, the statistics for 1995 show that girls are still underrepresented in the awards for A* (1.4% of the girls to 2.3% of the boys) and A (6.0% of the girls to 7.2% of the boys) (SCAA, 1996b).

It appears that there are sufficient well-qualified students available to go on to study A level mathematics and that this is true of both sexes. Whether this will continue in the future depends on the number of able students choosing to, or having no option but to, take GCSE at the intermediate tier.

TABLE V. The number of students taking more than one A level in mathematics, England and Wales, 1980–95.

	1980*	1985*	1990	1995
Numbers doing 'double' mathematics	13,400	11,900	6500	7600

Figures for 1980–90 from *Tackling the Mathematics Problem*, (LMS, 1995) and for 1995 from Associated Examining Board interboard statistics. *Students from England only.

TABLE VI. The number of students accepted at university through UCAS with a 'double' A level award in mathematics, 1994–96.

	1994	1995	1996
Number of students	7579	7280	6494

Source: UCAS statistical services.

The A Level Cohort: what changes have actually occurred?

There has certainly been a dramatic decline in the numbers of students offering a 'double' award in mathematics over the past 15 years. As can be seen from Table V, the numbers of students taking a 'double' A level award in mathematics has almost halved. (In all the following statistics, a 'double' award is taken to mean one and a half or two A level awards in mathematics.)

It might be thought that the upswing shown between 1990 and 1995 shows that the situation is improving steadily. However, this is not the case. It must be realised that the figures in Table V are for those taking more than one award in mathematics, not for those that gain a pass in both. The figures available from UCAS shown in Table VI for those passing 'double' mathematics and going on to university over the past 3 years suggest that any upsurge is over and the figures are starting to decline again.

The proportions of boys and girls gaining A*–C at GCSE are now roughly similar (see Table IV). Does this hold true for students gaining a 'double' award at A level? Once again the easiest way to obtain figures by gender for such candidates is from the UCAS statistics.

Table VII shows clearly that the profile for the 'double' award is not the same as that for the single award. Of the 46,881 entrants to higher education who have an A level in mathematics, 64% or almost two-thirds of them are male. If we look at the subset of those who have a 'double' award in mathematics, 72%, an increase of 8%, are male. This is a highly significant increase. Given that the sex ratio of all entrants to higher education

TABLE VII. Sex distribution of all entrants to HE offering A level mathematics by mathematics point score by sex, 1996.

	A level mathematics score			
	double award	single 8–10	single 1–7	Total
Number of students	6494	21,792	18,595	46,881
Male (%)	72.0	61.2	63.5	63.6
Female (%)	28.0	38.8	36.5	36.4

Source: UCAS statistical services.

TABLE VIII. Number of students studying for A level in mathematics, 1985–96.

Year	1985	1990	1994	1995	1996*
Number	75,800	69,500	56,000	56,500	61,400

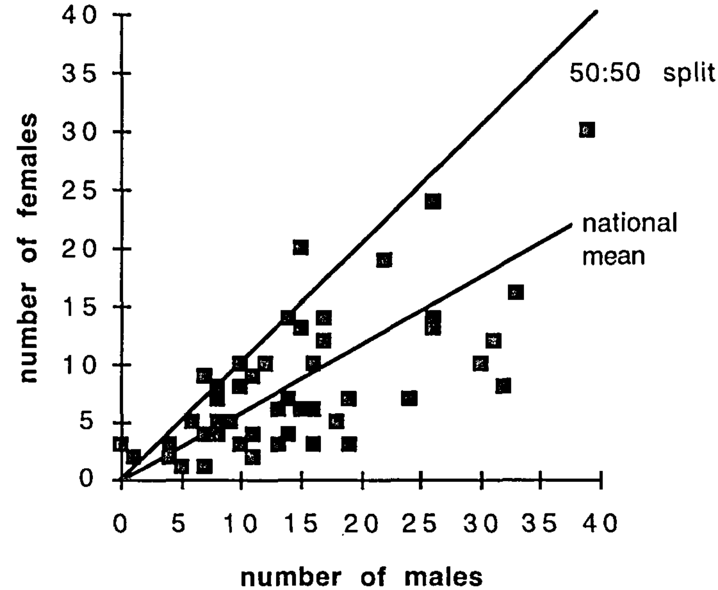
Source: Kitchen 1996, *SCAA 1996d (allowance has been made for those during two mathematics awards).

is 50 : 50, it appears that a much larger proportion of girls do not have the background in mathematics that would allow them to choose to do physics, engineering or mathematics. An even greater proportion have chosen not to study a 'double' mathematics award at A level. The overrepresentation of girls in the A and B band simply reinforces the fact that a number of girls who are capable of studying for and passing a double award in mathematics do not choose to do so. Collecting data over time for the number of students studying mathematics at A level is more problematic. National statistics aggregate all A level entries for both mathematics and further mathematics, thus counting many students twice. In order to allow for this it is necessary to subtract the number of further mathematics students from the total in order to get the number of students studying mathematics at A level. These figures, shown in Table VIII, have, like the 'double mathematics' figures, been decreasing, although they have shown a slight increase in 1995 and 1996.

Looking at the sex ratio of students studying for A level mathematics, the national average is around 62% male : 38% female (SCAA, 1996c). The difference between these figures and the figures from Table VII is that these relate to students studying for an A level award, not those who have passed it. Is this true for most mixed sex institutions? The data collected from my surveys showed that the ratio of these centres was also around 62% male : 38% female. However, the ratio varied widely from centre to centre. A random sample of 50 centres was chosen from the responses to the 1995 survey and the male/female ratios for each are shown in Fig. 1 to illustrate this.

The Youth Cohort Study, 'Science and Mathematics in Full Time Education after 16' (Cheng *et al.*, 1995) looked at those students who went on to study A level mathematics. They found that there was a strong correlation between good GCSE results in mathematics and studying A level mathematics, and there was a significant negative correlation between taking mathematics at A level and the number of A–C grades gained in arts, humanities and social sciences. However, there was no obvious statistical reason for the sex difference in the uptake of mathematics at A level between the schools that pupils attended in Year 11. There was also a significant variation in the level of take-up of A level mathematics by the school attended in Year 11 after controlling for GCSE results, home background, sex and whether the school was single sex or mixed. A large-scale study was carried out by the National Foundation for Educational Research for SCAA (Sharp *et al.*, 1996). Surveying over 700 centres, they found that centres positively associated with high mathematics take-up at A level are independent, selective, single sex and have a high proportion of A*–C passes in GCSE.

One suggestion made by many respondents in the discussions during the formulation of both the 1993 and proposed 1997 cores was that the inclusion of mechanics in an A level syllabus would tend to inhibit girls from taking mathematics at A level. Thus, centres where all students had to study some mechanics would tend to have a lower female participation rate and this would affect the national ratio of boys to girls. I attempted to test this hypothesis by looking at the male : female ratio in schools that gave



Source: 1995 MAP survey.

FIG. 1. Scatter graph showing male–female distribution of SMP 16–19 A level mathematics year 12 students for 50 random centres, 1995.

their students a choice between mechanics and statistics and those where students had to take both. Using the same sample of 50 centres offering SMP 16–19 Mathematics, the male : female ratio for both types of centres was 63% : 37%. The data shown in Table IX thus strongly suggest that the inclusion of mechanics did not affect the sex ratio.

Where a choice was possible, girls tended to take statistics and boys mechanics. However, discussion with the centres suggested that the 'choice' was in many cases illusory. Students doing physics, mainly males, were expected to do mechanics and timetabling requirements often meant that those not doing physics had to do statistics. Looking at the scatter graphs for each of these groups shows that once again there were very big differences between individual centres (Fig. 2).

Have Changes to Content and Assessment Criteria at A Level Changed 'Standards'?

Stating, as has been done all too often in the press, that a rise in the percentage of students gaining the top grades at A level proves that standards have fallen, is not defensible. The only valid comparison would be to look at the questions asked, the marks

TABLE IX. Number of students studying mathematics at A level in year 12 by option choice and sex, 1995.

	Male	Female
Centres offering no choice, 50% statistics: 50% mechanics	441 (63%)	261 (37%)
Centres offering a free choice of options	586 (63%)	342 (37%)

Source: Mechanics in Action Project, evaluation questionnaire 1995.

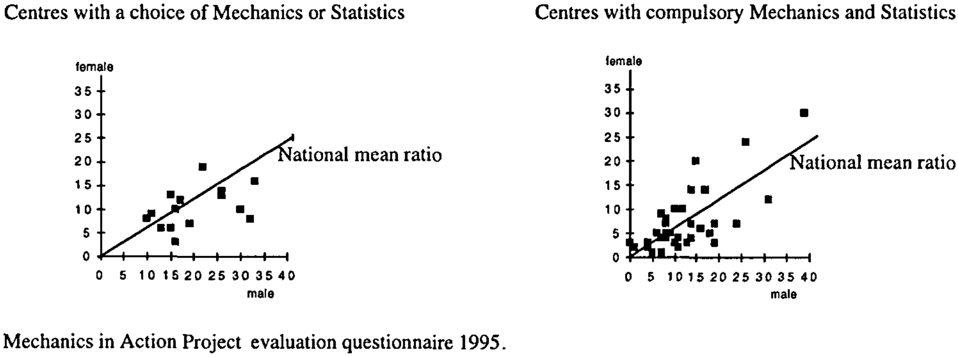


FIG. 2. Scatter graph to show the male/female ratio of A level mathematics students by centre by applications offered.

awarded and the marks needed for the award of a specific grade over time. This is not a simple task and has been attempted by many over the past 10 years (Burghes, 1994; Royal Society, 1997).

One fact, however, does support the contention that the depth of work in pure mathematic achieved by students with a good grade in a second mathematics A level is now much more varied. Up to the 1990s, all students who had studied for two mathematics A levels had covered subjects such as complex numbers, hyperbolic functions, advanced differential equations and matrices, no matter what syllabus they had studied for. These students were therefore well qualified for the first term of a mathematics course at university where such subjects were studied at speed. However, the increase in the number of students taking a modular A level in mathematics, together with the growth in applications areas such as numerical methods and a diversity of statistics and mechanics modules in the 1990s, meant that many students could now do one and a half, or even two A levels in mathematics and be counted as having a 'double' award in mathematics having covered few, or even none, of the pure topics mentioned earlier. This means that while the percentage offering 'double' mathematics at A level has halved over the past 15 years, the pure mathematics content of many such awards may also have diminished.

As far as a single award is concerned, there are several methods used at the moment to attempt to compare standards both over time and between subjects. One such is the subject pair analysis carried out by the awarding bodies. This looks at a cohort of students taking two subjects at A level and measures the differences in performance between subjects. It uses the difference in mean grade as a measure of leniency or difficulty between the two subjects. The reasoning given for this is that a student of a given ability should on average perform equally well at all the A levels they attempt if the standards in each are the same. In the same way a change in the subject pair data over time between two subjects is also used by some to denote a change in standards. The main failing of a subject pair as a measure is that while it measures the mean difference in achievement of students doing the two subjects, it does not measure leniency or severity. It could be that the students are better prepared for one subject. It could be that the assessment was better at measuring achievement of the syllabus content in one subject. Other methods, based on the work of the ALIS project (Fitz-Gibbon & Vincent, 1994), use the GCSE average score of students, the performance across all A levels and the performance in an individual A level subject. A similar national analysis

TABLE X. Difficulty of A level subjects, using correction factors, from subject pairs, 1994.

Subject	Physics	General Studies	French	Mathematics	Geography	English	Art
Score	0.72	0.5	0.49	0.44	0.10	− 0.40	− 1.15

(Positive is taken to be more difficult.)
Source: Dearing 1996.

for A level data in 1994 was carried out by the Department for Education and Employment (DfEE) (Dearing, 1996). The data in Table X show that mathematics would appear to be of the order of half a grade more difficult than the norm for all subjects.

A different and possibly more accurate method of measuring changes was that attempted by the SCAA/Office for Standards in Education (OFSTED) report (SCAA, 1996e) on the standards in public examinations from 1975 to 1995. They used independent specialists to compare a sample of the syllabuses, examination papers and scripts from 1975, 1985 and 1995. The results of this work stated that there was a broad balance between the gains and losses in syllabus content. They found that there was a change in the style of the papers over the 20 years, with those set in 1975 containing a higher proportion of demanding questions, with comparatively low marks required for each grade.

Drawing together the different strands of the study, the balance of evidence suggests that:

> in pure mathematics, standards set at the grade E boundary have fallen in two of the three syllabuses studied;
>
> at the grade A boundary in pure mathematics, whilst many aspects of performance were comparable with 1985, candidates in 1995 were not required to demonstrate as much competence in the areas of problem solving, reasoning and algebraic manipulation;
>
> standards had risen in the statistics syllabus reviewed;
>
> evidence about standards in mechanics is inconclusive. (SCAA, 1996e, p. 49)

The findings are not surprising and do much to support the complaints about algebraic fluency from those in higher education. How did these changes come about? The problems with recruitment obvious by 1990 led to calls for the provision of new syllabuses. The principles underlying the need for change included the following:

> the need for A level courses to be much broader and to involve a variety of disciplines, skills and cross curricular themes;
>
> the need to improve access to them by increasing available options. (Burton *et al.*, 1992, p. 14)

This widening of the content base led inevitably to a lowering in the depth of content coverage of the topics covered by the old A level syllabuses. This change was exacerbated by the changing content of the 16 + examinations. The change from GCE O level to GCSE had led to a change in emphasis away from algebra:

> While there have been decreases in manipulative algebra, Euclidean geometry and trigonometry there have been increases in number topics, graphical representations of functions. (SCAA, 1996e, p. 25)

TABLE XI. Undergraduate entry by degree subject, 1994–96.

Degree subject	Number of students with a 'double' mathematics A level award		
	1994	1995	1996
Mathematics G1	1159 (33.6)	1162 (32.4)	1338 (34.4)
Physics F3	469 (16.9)	480 (17.9)	454 (16.7)
Engineering (all)	1387 (7.2)	1240 (7.0)	1105 (6.5)
Total all subjects	7579	7280	6494

Source: UCAS statistical service. *Figures in brackets are as a % of the degree subject entry cohort.

It was apparent at that stage that all these changes would have implications for those in higher education. Recommendations made by the study on access into higher education included the following:

(a) There needs to be a greater awareness on the part of mathematics staff in higher education of the changes that have taken place at GCSE, how these in their turn have influenced proposals for new developments at A level mathematics and ultimately, how they will affect the mathematical experience of students entering mathematics courses at higher education in the future.

(b) Teachers of mathematics in higher education will have to respond to these changes by reviewing what they teach and how. (Burton *et al.*, 1992, p. 50)

Entry Standards in Higher Education: how have they changed?

The decrease in numbers doing 'double' mathematics would not matter if the drop in numbers doing a 'double' award from 1980 to 1996 did not affect the entry to Mathematics, Physics and Engineering departments. However, data for the last 3 years in Table XI show that less than half of the students who have studied for a 'double' award in mathematics at A level (including those with an AS in Further Mathematics) have gone on to read for a degree in these areas. It is disappointing that many students who obviously have an aptitude for mathematics should choose not to continue with its study.

Around a third of the cohort for mathematics G1 have a 'double' A level award in mathematics. Study of relevant prospectuses shows that for most universities in the early 1980s, a 'double' award in mathematics was expected from the majority of their students starting an honours mathematics degree similar to G1. This is a dramatic change in entry pattern. Even today the true depth of this change is not realised. The Royal Society report on the teaching and learning of algebra pre-19, states that 'in the past it was assumed that students for mathematics degrees would enter university with a double award in mathematics. Nowadays students are often accepted on such courses with a single mathematics qualification at A level (Royal Society, 1997, p. 19). Such wording is misleading. *Most* entrants now only have a single award at A level. One example is noted by Keith Hirst in his study on the consequences of changes in school mathematics for the university curriculum, when he states that the proportion of entrants to the mathematics faculty at Southampton with a 'double' award in A level mathematics was nearly 95% in 1969, by 1983 it had dropped to 75% and in 1990 it was just over 45% (Hirst, 1991, p. 4). Given the perception of lower A level standards in mathematics by

TABLE XII. Mathematics scores of home students entering degree courses in mathematics, physics and engineering in (1995) 1996 by degree subject.

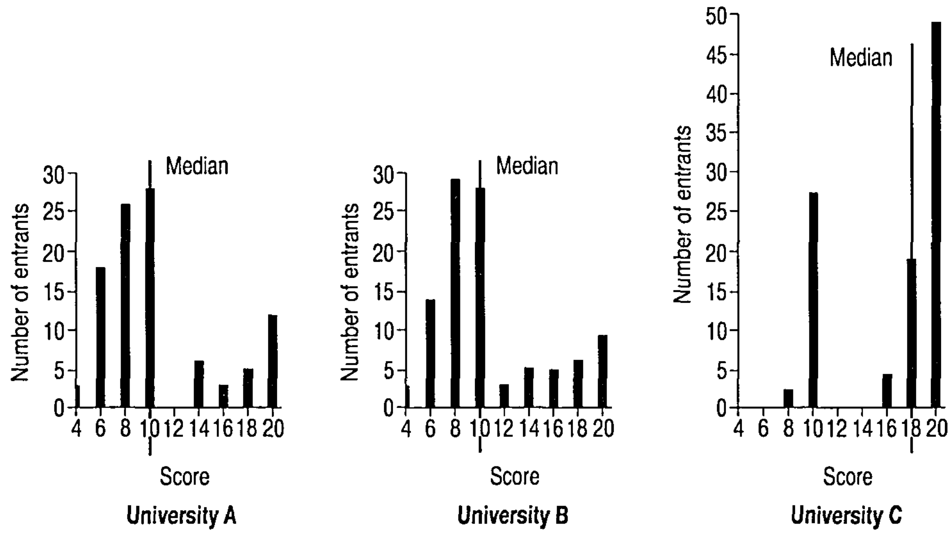
Degree subject	Mathematics (G1) (1995) 1996	Physics (F3) (1995) 1996	Engineering (1995) 1996
Mathematics score			
(Double mathematics)	(1162) 1338	(480) 454	(1240) 1105
8–10 (A or B)	(1339) 1630	(882) 1003	(3015) 3244
1–7	(263) 353	(444) 530	(2977) 3211
Other qualifications	(821) 567	(871) 724	(10,413) 9372
Total	(3585) 3888	(2677) 2711	(17,645) 16,932

Point scores are A level: A = 10, B = 8 etc., AS level: A = 5, B = 4 etc.
Double mathematics can be assumed to give a point score of over 10.
Source: UCAS statistical service.

many in higher education, it might be supposed that most universities would expect an A or possibly a B in single mathematics as the absolute minimum to continue to study mathematics at degree level. Similarly, the importance given to mathematics by academics in both physics and engineering would lead one to suppose that the requirements for entering these disciplines would be nearly as stringent. However, this does not appear to be the case.

Figures in Table XII show that 34% of entrants to G1 had a 'double' award in mathematics and around 42% had an A or B in the single award. This would suggest that in a typical university mathematics department, a third of the first year had mastered more than the contents of a single A level award, just over a third had mastered most of the content of an A level in mathematics and many of the rest, having spent 2 years on their study, were still having difficulty with much of the pure mathematics content in a single A level. One problem in accepting those with a C grade or lower at A level is that there is no necessity for students to get that grade both in their pure mathematics options and those of their applications. The overall grade is decided by a simple aggregation of the marks. Thus, students with a C grade or lower doing pure mathematics with statistics, say, will range from those doing extremely well in statistics and failing in their pure modules to those doing well in their pure modules and failing in their statistics modules. Those achieving an A or B have, obviously, attained a passing grade in both. That this variation in A level grade on entry is spread evenly between most of the universities, with only a handful being able to cream off the top students, is suggested by the results of a survey of five of the 'older universities', which I carried out in 1995 (Kitchen, 1996), in which the mathematics scores of entrants to the mathematics degree course (G1) were collected. The university departments concerned provided the complete A level profile of each of their entrants and a mathematics score for each student was calculated. The line diagrams in Fig. 3 show the variation in three of the institutions concerned. Institution C is recognised as being highly selective. The other two institutions, D and E, resembled institutions A and B respectively. The grades range from a D in single mathematics to a double A in mathematics and further mathematics.

Table XII also shows that this variability in the mathematics A level grades of entrants is true of most physics and engineering departments as well. Even more relevant to the problems of engineering departments is the fact that over a half of the entrants to degree-level courses have not studied A level mathematics. In engineering a surprising 9372 (10,413 in 1995) students or 55% (59%) of all engineering entrants did not have



Source: Mechanics in Action Project Survey, 1995/6

FIG. 3. Mathematics scores of entrants to G1, for three universities, 1995.

a qualification in A level mathematics. What did they have? Around 11% had A levels other than mathematics. They may have studied for mathematics but failed, or not studied A level mathematics at all. Some 19% had Business and Technology Education Council (BTEC) or General National Vocational Qualification (GNVQ) qualifications, 9% had Scottish Highers, 3% came via access courses and 12% had other qualifications or were listed as having none or unidentified. An A or B in single mathematics or any 'double' mathematics award would be a good qualification for an engineering degree. Looking at these figures, it is clear that 74% of the engineering students accepted did not fall into this category. This is worrying both for the university departments concerned and the engineering profession.

Does the Gender Imbalance at A Level Carry through to Higher Education?

Table XIII shows the gender balance for all entrants to mathematics degree courses (G1) over the past 11 years. It is obvious that the gender distribution overall is now not much different from that at A level. Although there was obviously an increase in the proportion of female students from 1985 to 1990, this increase has now levelled off.

Is this distribution the same for each of the attainment groupings? Table XIV shows that there is some variation. While the gender balance of students entering a G1 course

TABLE XIII. *Sex distribution of entrants to all mathematics degree courses (G1).*

Year	1985	1990	1994	1995	1996
Females (%)	31.8	35.0	36.7	37.5	36.7
Males (%)	68.2	65.0	63.3	62.5	63.3

Source: Data for 1985 and 1990 from Hirst (1991); data for 1994–96 from UCAS statistical services.

TABLE XIV. Gender distribution of entrants to all mathematics degree courses (G1), physics degree courses (F3) and engineering degree courses, 1996, by A level mathematics score, females as a percentage of each cohort.

Year	A level mathematics score				
	Double	8–10	1–7	Other	Total
Mathematics	29.5	38.2	54.4	38.1	36.7
Physics	17.8	20.2	15.8	20.6	19.0
Engineering	14.9	12.7	11.3	12.8	12.6

Source: UCAS statistical services.

in HE with an A or B in single mathematics matches the overall profile (37%), those students who might be assumed to be less well prepared with a score of between 1 and 7 are predominantly female. At the other end of the scale the percentage of those entering with a double award in mathematics who are female is only 30%. This variation does not occur in physics, where the female to male ratio is around 1 to 4 in all the attainment bands. Engineering, with only 13% of its cohort female, is a predominantly male preserve even now.

Conclusions

- Increasingly students are being recruited onto A level courses with a weaker mathematical, especially algebraic, experience.
- Many students who have shown high attainment in mathematics at GCSE do not choose to take A level mathematics. This is especially true of girls.
- The numbers gaining a 'double' award in mathematics at A level are declining. Even allowing for the imbalance between girls and boys taking A level mathematics, girls are underrepresented here.
- The amount of pure mathematics content that is compulsory for some modular 'double' mathematics awards is now no more than that of a single mathematics A level.
- The gender balance at A level mathematics in individual centres varies greatly but this does not seem to be affected by the choice of applied options on offer by the centre.
- The relative demands on algebraic fluency have decreased at A level.
- Less than half the students with a 'double' award in mathematics go on to study for a degree in mathematics, physics or engineering at university.
- Many universities recruit their mathematics undergraduates from a much wider spectrum of mathematics A level grades, thus accepting students with lower grades than before.
- There is a wide imbalance between the sexes at undergraduate level in physics and engineering.

We will be at the point of crisis at A level entry if more schools take up the policy of entering all students for intermediate level GCSE in mathematics. This is especially true in areas where there are 11–16 centres, sixth form colleges and further education colleges. In 11–18 schools, teachers would be more likely either to enhance the curriculum for their able candidates or to retain the higher tier.

It is unfortunate that this is exacerbated by the decision of many girls who have gained the qualifications needed to go on to A level to choose subjects other than mathematics. If the same proportion of girls as boys went on to study A level mathematics, the numbers would rise dramatically. This imbalance is especially noticeable in those choosing to study 'double' mathematics.

There is a dilemma for those designing A level mathematics syllabuses. On the one hand A level serves as an entry hurdle to the study of specific mathematical disciplines that require a high degree of fluency in pure mathematical content, and on the other it serves as a subject in its own right, taken by students who do not wish to continue its study at higher education. This was unimportant when entry to higher education was gained through the 'double' mathematics award. It is far more problematic now.

The new modular syllabuses for 'double' mathematics do not enable those in higher education to distinguish between those who have met and understood a substantial pure syllabus and those who have gained a wide variety of applied options together with a basic amount of pure mathematics. The reasons behind the perception of a lowering in standards in A level mathematics on the part of those in higher education are probably twofold. Firstly, many universities have lowered their admissions requirements for mathematics simply to increase their numbers. Lecturers in mathematics and mathematics-related disciplines rely on their students having a mastery of the algebra, calculus and trigonometry that was defined in the 1985 core (Lord *et al.*, 1995). Yet this is not always the case. Students can, and do, gain a C in A level mathematics purely by gaining high marks in certain areas of mathematics with a very low pure mathematical content. Thus, accepting students with lower grades without making allowance for this will inevitably cause problems. These have been faced by some universities and strategies put in place to cope with them. A level mathematics will continue to be the major entry criteria for those going on to read mathematics at university. Universities cannot afford to have low entry standards unless they also address the problems that the wider ability range will bring. Secondly, the decrease in the demand for algebraic fluency on the part of A level awarding bodies together with the widening of the subject content means that those attaining high grades may have studied less of the content assumed and desired by lecturers in higher education, replacing it with other content deemed to be less crucial. Thus, while the standards of the mathematics A level as a whole may not have declined, the standard of attainment of that content deemed most important for progression to higher education is almost certainly less, especially at the lower grades.

The gender imbalance at university in mathematics is the same as that at A level. However, it is much worse for physics, and especially engineering. This may be because girls are even more underrepresented at A level physics, though statistics are not available to test this theory. Ideally, as well as publishing the numbers of students at each of the grade levels for each subject at A level, QCA should make available the numbers for each sex for each subject, as is done periodically for mathematics and English. There is obviously still a need to encourage more of our able female students to consider both physics and engineering as a possible choice of career if this imbalance is to be overcome.

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