2 Unit Bridging Course – Day 6

Applications of the second derivative: A moving body

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Velocity and acceleration

The first derivative and second derivative can be used to calculate the velocity and acceleration of a moving body.

If $s = f(t)$ gives the distance of the body from a fixed point at time t, then

- the first derivative, $\dfrac{ds}{dt}$, gives the velocity at time t; and
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Example

The distance, in meters, of a moving car from a certain point at time t is given by $s = 2t^2 + 5$.

Find:

(a) The distance of the car from the point after 5 seconds.

(b) The velocity of the car after 3 seconds.

(c) The acceleration after 5 seconds.

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(c) The acceleration after 5 seconds.

Example: $s = 2t^2 + 5$

Solution

(a) The distance of the car from the point after 5 seconds. After 5 seconds the car is $2 \times 5^2 + 5 = 55$m from the point.

(b) The velocity of the car after 3 seconds.

Velocity, $v = \dfrac{ds}{dt} = 4t$.

At $t = 3$, the velocity is $v = 12$m/s.

(c) The acceleration after 5 seconds.

Acceleration $= a = \dfrac{d^2s}{dt^2} = \dfrac{dv}{dt} = 4$.

At $t = 5$, the acceleration is 4m/s^2.

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If a cannon is fired directly upwards, the cannon ball reaches a height of $s = 98t - 4.9t^2$ meters after t seconds.

Find:

(a) The initial velocity.

(b) The acceleration of the cannon ball.

(c) How high the cannon ball goes.

(d) How long it takes to hit the ground.

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Example 2: $s = 98t - 4.9t^2$

Solution

The velocity is: $v = \dfrac{ds}{dt} = 98 - 9.8t$.

The acceleration is: $a = -9.8$.

(a) The initial velocity, that is when $t = 0$, is 120m/s.

(b) The acceleration is -9.8m/s^2.

Notice: Here the acceleration is negative, and is the acceleration due to gravity.

Both velocity and acceleration have direction. If the cannon ball has a positive velocity, it is moving upwards. Negative velocity means the cannon ball is moving downwards.

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(c) Find how high the cannon ball goes.

Think about the physical situation.

The cannon ball starts moving upwards, ie has positive velocity, but since acceleration is negative, the cannon ball is slowing down until it is momentarily at rest. Then the cannon ball starts moving downwards.

Thus the cannon ball reaches its peak height when $v = 0$, that is when $v = 98 - 9.8t = 0$.

Solving for t this gives: $t = \frac{98}{9.8} = 10$s.

Substituting it back to the original equation gives:

$$s = 120(10) - 4.9(10)^2 = 710\text{m} \qquad \text{the maximum height.}$$

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(d) Find when the cannon ball hits the ground.

The cannon ball hits the ground when $s = 0$, ie when $s = 98t - 4.9t^2 = 0$.

Solving for t, we get $t = 0$ or 20.

Since $t = 0$ is when the cannon ball is initially fired, the ball hits the ground at $t = 20$s.

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