2 Unit Bridging Course – Day 10

Circular Functions III – The cosine function, identities and derivatives

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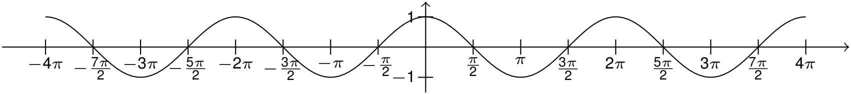

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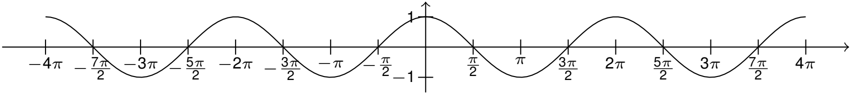

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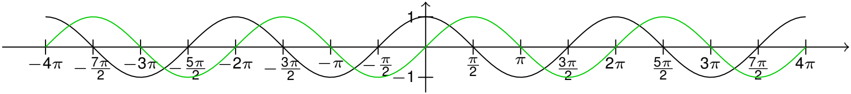

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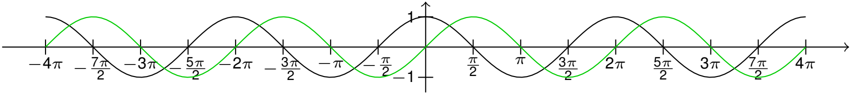

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Properties of cosine

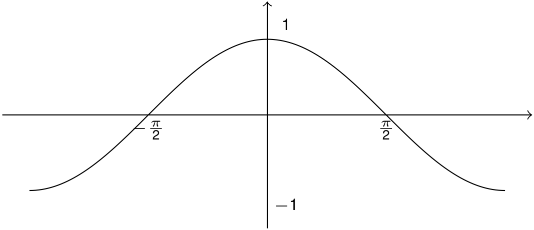
cos shares the following properties with sin:

(i) $-1 \leq \cos x \leq 1$ for all x.

(ii) $\cos(x + 2\pi) = \cos x$ for all x, i.e. $\cos x$ is periodic with period 2π, just like $\sin x$.

Unlike sin, however, cos is not odd:

(iii) $\cos(-x) = \cos(x)$.



$y = \cos x$ is symmetric about the y-axis – we say it is an even function.

Practice questions

See if you can sketch the following cosine curves, using the same ideas we used to sketch sine curves.

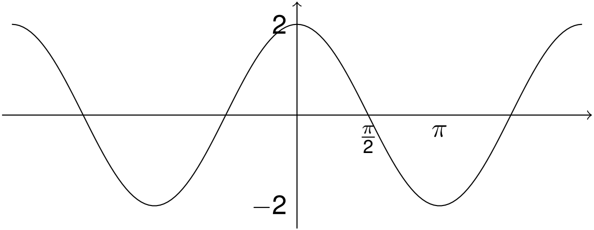
(i) $y = 2\cos x$

(ii) $y = \cos(2x)$

(iii) $y = 3\cos(2x)$.

Answers

(i) $y = 2\cos x$



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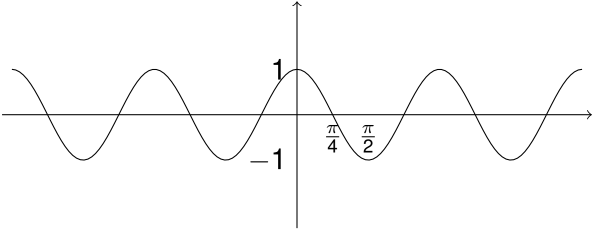

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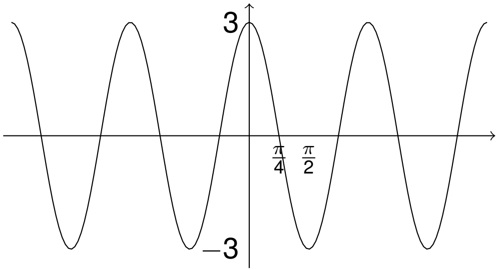
(iii) $y = 3\cos(2x)$



Identities involving circular functions

Together, sin and cos are called the circular functions.

There are many important identities involving circular functions which you should remember.

(i) $\sin^2 x + \cos^2 x = 1$ (where $\sin^2 x = (\sin x)^2$)

(ii) $\sin(x + y) = \sin x \cos y + \cos x \sin y$

(iii) $\cos(x + y) = \cos x \cos y - \sin x \sin y$

(ii) and (iii) are known as double angle formulas. You can find plenty more such identities, for example on Wikipedia.

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Derivatives of circular functions

The circular functions, sin and cos, have particularly simple derivatives.

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$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x.$$

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$$\begin{aligned}\frac{df}{dx} &= 3\cos(2x) \times \frac{d}{dx}(2x) \\ &= 3\cos(2x) \times 2 \\ &= 6\cos(2x).\end{aligned}$$

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$$\begin{aligned} \frac{dy}{dx} &= u\frac{dv}{dx} + v\frac{du}{dx} \\ &= \sin x \times (-\sin x) + \cos x \times (\cos x) \\ &= -\sin^2 x + \cos^2 x. \end{aligned}$$

Derivatives of circular functions

Practice questions

Find the derivatives of the following functions:

(i) $f(x) = \sin^2 x$

(ii) $f(x) = x \cos x$

(iii) $f(x) = \sin(x^2)$

(iv) $f(x) = \dfrac{\sin x}{\cos x}$ (usually written $\tan x$).

Answers to practice questions

(i) $\frac{df}{dx} = 2\sin x \cos x$

(ii) $\frac{df}{dx} = -x\sin x + \cos x$

(iii) $\frac{df}{dx} = 2x\cos(x^2)$

(iv) $\frac{df}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$.