2 Unit Bridging Course - Day 3

Quadratic Equations

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Solving Quadratic Equations

A quadratic equation has a standard form of:

$$ax^2 + bx + c = 0$$

where a, b, c are constants $a \neq 0$.

If we can factorise the expression $ax^2 + bx + c$, we do so.

As the product of the factors are equal to zero, then one or other of the factors must be equal to zero. We then solve the resulting equations for x.

Solving Quadratic Equations

For example, solve $x^2 + 2x - 3 = 0$.

$$\begin{aligned} x^2 + 2x - 3 &= 0 \\ (x+3)(x-1) &= 0 \end{aligned}$$

Either $(x+3) = 0$ or $(x-1) = 0$. Hence $x = -3$ or $x = 1$.
Substitute to check:

$$x^2 + 2x - 3 = (-3)^2 + 2 \times -3 - 3 = 9 - 6 - 3 = 0$$

$$x^2 + 2x - 3 = 1^2 + 2 \times 1 - 3 = 1 + 2 - 3 = 0$$

Example

Example

Solve $x^2 + 4x + 9 = 5$

First we rewrite the equation as

$$x^2 + 4x + 4 = 0$$

then factorise to give

$$(x + 2)(x + 2) = 0$$

so $x = -2$ is the solution.

Note that this quadratic equation only has 1 solution.

Example 2

Example

Solve $4x^2 - 16 = 0$.

$$\begin{aligned} 4x^2 - 16 &= 0 \\ (2x - 4)(2x + 4) &= 0 \end{aligned}$$

so either $2x - 4 = 0$ or $2x + 4 = 0$.

Therefore the solution is $x = 2$ or $x = -2$.

Quadratic Formula

Some quadratics are difficult or cannot be factorised.
If a solution exists, we can solve it using the quadratic formula.

Quadratic Formula

For $ax^2 + bx + c = 0$ where a, b and c are the constants and $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the formula by simply plugging in the values from the quadratic equation.

Note that if $b^2 - 4ac < 0$, then the equation has no real solutions.

Example

Example

Solve $2x^2 + 4x - 2 = 0$.

For this equation $a = 2$, $b = 4$, $c = -2$.
Plug them into the formula.

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{4^2 - 4(2)(-2)}}{2(2)} = \frac{-4 \pm \sqrt{32}}{4} \\ &= \frac{-4 \pm \sqrt{16}\sqrt{2}}{4} = -1 \pm \sqrt{2} \end{aligned}$$

Therefore $x = -1 + \sqrt{2}$ or $x = -1 - \sqrt{2}$.

Further Example

Example

Solve $4x^2 + 2x + 5 = 0$.

For this equation $a = 4$, $b = 2$, $c = 5$.

Notice that $b^2 - 4ac = 2^2 - 4(4)(5) = -76$, which is a negative number.

Therefore $4x^2 + 2x + 5 = 0$ has no real solutions.

Practice Questions

Practice Questions

Solve the following.

1. $x^2 + 4x + 3 = 0$
2. $x^2 - 9 = 0$
3. $x^2 - 6x - 4 = -13$
4. $3x^2 + 2x - 1 = 0$
5. $4m^2 + 5m + 1 = 0$
6. $9x^2 - 2x - 5 = -1 - 2x$
7. $-2n^2 + 2x + 1 = 0$
8. $2x^2 - 3 = 0$
9. $4x^2 + x + 2 = 0$
10. $3x^2 - 5x = 0$

Answers

Answers to the practice questions.

1. $x = -3$ or $x = -1$
2. $x = 3$ or $x = -3$
3. $x = 3$
4. $x = \frac{1}{3}$ or $x = -1$
5. $m = -\frac{1}{4}$ or $m = -1$
6. $x = \frac{2}{3}$ or $x = -\frac{2}{3}$
7. $n = \frac{1+\sqrt{3}}{2}$ or $n = \frac{1-\sqrt{3}}{2}$
8. $x = \frac{\sqrt{6}}{2}$ or $x = -\frac{\sqrt{6}}{2}$
9. No real solutions.
10. $x = 0$ or $x = \frac{5}{3}$