2 Unit Bridging Course – Day 4

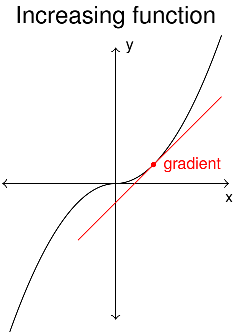
Stationary points and quadratic functions

Emi Tanaka

THE UNIVERSITY OF SYDNEY

Increasing function

A positive gradient indicates an increasing function.



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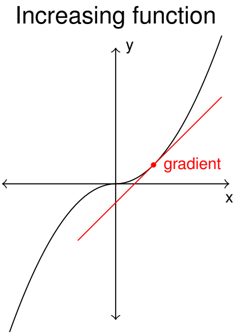

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Decreasing function

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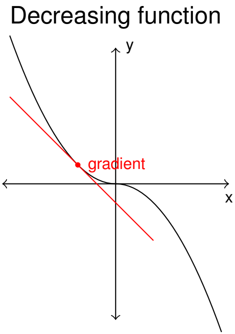

If $f'(x) < 0$ on an interval, then the function is decreasing on that interval.

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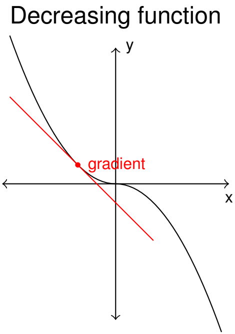

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Stationary points

A gradient of 0, ie $\frac{dy}{dx} = 0$, indicates that the curve is flat at that point. This point is called a stationary point.

There are 2 types of stationary points, *turning points* and *points of inflexion*, and they can be found by setting $\frac{dy}{dx} = 0$.

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A turning point is a point at which $\dfrac{dy}{dx} = 0$ and where the derivative changes sign (positive to negative or negative to positive).

A point of inflexion is where the curve is flat but does not change sign.

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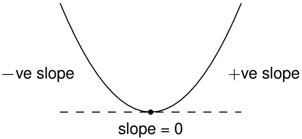
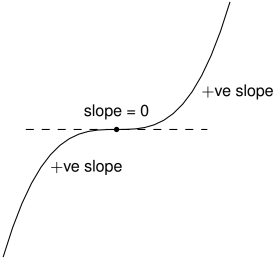


Point of Inflexion



The quadratic function

The graph of a quadratic function $y = ax^2 + bx + c$ is a parabola, which has one turning point and no inflexion points.

The turning point can either be a *maximum or minimum point*, that is, a point where the function takes the maximum or minimum value.

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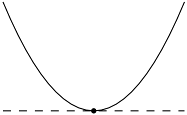
If $a > 0$, the parabola is upright and the function has a minimum point.



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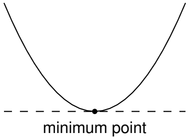

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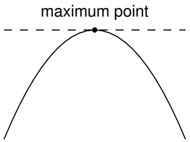

Example

Find the minimum value of $y = x^2 + 2x + 3$. First find the point where $\frac{dy}{dx} = 0$.

$$\begin{aligned}\frac{dy}{dx} &= 2x + 2\\ 2x + 2 &= 0\\ x &= -1\end{aligned}$$

Substitute $x = -1$ back into the original equation to find y, $y = (-1)^2 + 2(-1) + 3 = 2$.

Since $a > 0$, $(-1, 2)$ is the minimum turning point. Hence the minimum value of $y = x^2 + 2x + 3$ is 2.

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$$\begin{aligned} f'(x) &= -4x \\ -4x &= 0 \\ x &= 0. \end{aligned}$$

There is a stationary point when $x = 0$. When $x = 0$, $y = 6 - 2 \times (0)^2 = 6$. Since $a = -2 < 0$, $(0, 6)$ is the maximum turning point. Therefore the maximum value is 6.

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Practice Questions

Find the maximum or minimum of the following:

1. $f(x) = x^2 - 1$
2. $f(x) = 4 - 2x^2$
3. $f(x) = 3x - 5$
4. $f(x) = x^2 - 4x - 6$
5. $f(x) = 6x - 3x^2$
6. $f(x) = 5.$

7. If you enclose a rectangular area with 40m of fencing, what is the maximum area that can be enclosed.
8. Find 2 non-negative numbers whose sum is 12 and product is a maximum.

Answers to practice questions

1. min $= -1$
2. max $= 4$
3. no min or max
4. min $= -10$
5. max $= 3$
6. min and max $= 5$. (horizontal line)
7. $100m^2$.
8. 6 and 6.

Sketching quadratic functions

To sketch the graph of a quadratic function we find the turning point and the intercepts with the axes.

For $y = ax^2 + bx + c$, to find the y-intercept (where the function crosses the y-axis, ie. when $x = 0$) let $x = 0$ and solve for y.

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Example

Sketch $y = x^2 - x - 2$.

Since $a > 0$ the parabola is upright and has a minimum point.

The minimum value occurs when $\dfrac{dy}{dx} = 0$, ie $2x - 1 = 0$ or $x = 0.5$. $y = (0.5)^2 - (0.5) - 2 = -2.25$, so the minimum point is $(0.5, -2.25)$.

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