University Mathematics Bridging Courses: MathStart, MathTrack, A Review of Existing Approaches and Recommendations for Moving Forward.

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Glossary

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fMRI functional magnetic resonance imaging. v, 3, 6

MARS Maths Anxiety Rating Scale. v
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MAS-R Maths Anxiety Scale — Revised. v, 6

NAPLAN National Assessment Program — Literacy and Numeracy. v, 3

OECD Organisation for Economic Co-operation and Development. v, 3

PISA Programme for International Student Assessment. v, 3

PTSD Post-Traumatic Stress Disorder. v

STEM Science, Technology, Engineering and Mathematics. v, 3

Abstract

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I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint award of this degree.

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Acknowledgements

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Chapter 1

Introduction

University mathematics bridging courses serve an important stop-gap role in the Australian educational system, and other educational systems internationally.

This project can be thought of as consisting of two broad categories of work:

- A literature review of Australian mathematics bridging courses, a state of the field of research in this area, and commentary on the role, purpose, and approaches important to implementing effective and impactful mathematics bridging courses in Australia and internationally.
- Reflection on the mathematics bridging courses offered by the University of Adelaide: MathStart and MathTrack with a focus on existing strengths, and directions for potential improvement. This will be broken into two sub-sections of work, with one of the bigger contributions I offer being a curriculum mapping from the Australian Curriculum (AC) to South Australian Certificate of Education (SACE) through to MathStart and MathTrack. This curriculum mapping suggests potential areas for modification of the mathematics bridging courses to more closely align them with the AC and SACE. The second sub-section of work will be on the non-content aspects of the bridging courses (structure, assessment, timing, feedback, etc.), the strengths of their approaches in comparison to others, potential weaknesses, and reccomendations moving forward both for MathStart and MathTrack, and for university mathematics bridging courses more broadly.

This thesis will be structured as follows:

- The remainder of this introductory chapter (Chapter 1), I will give a broad overview of the concepts, challenges, and setting for this project.
- In Chapter 2 I will provide a indepth discussion of the existing literature, what is known, approaches attempted in the past both in Australia and internationally, and some deeper discssion on some of the particularly relevant related concepts, such as maths anxiety.
- One of the major contributions of this work is the curriculum mapping of the AC to SACE, to the content currently in MathStart and MathTrack, with commentary on how this mapping connects with typical first-year university mathematics courses. This mapping is discussed in Chapter 3, and will identify gaps and mis-alignment, discuss the tension between different perspectives on

the role of university mathematics bridging courses and how this impacts on content decisions, and potential modifications to the bridging courses content that would allow them to be more closely aligned with the AC should that be desirable.

• Finally, I will wrap up with commentary on what is being done well, reccomendations for how to improve, and a summary of the work I have done outside of this thesis to generate resources and content that can be used to improve these programs moving forward in Chapter 4.

1.1 The Role of University Mathematics Bridging Courses

Students will usually enroll in university mathematics bridging courses because they are required to demonstrate a certain level of mathematical knowledge/ competance before commencing study at university, but either do not meet those requirements, or do but feel a lack of confidence in their abilities and feel like they need to refresh/revise/ learn the mathematics prior to commencing their studies.

Many of these students will be adult-entry students, and reasons why these students do not either meet the entry requirements, or feel a lack of confidence in their abilities can be quite varied:

- A long period of time may have passed since they last studied mathematics (or studied at all).
- They may have performed poorly in mathematics in highschool.
- They may have chosen not to study mathematics at a higher level in highschool.
- They may suffer from maths anxiety (which would make them likely to fit into the above two categories as well).

The role of mathematics bridging courses is to take these students, and:

- Bridge their content knowledge so they are prepared for university entry.
- Support the growth of their confidence and self-efficacy surrounding mathematics.
- Ultimately prepare them to be successful in a university context.

From the perspective of content, what content should be taught in a university bridging course is actually a question that has dramatically different answers from different perspectives on the role of such a course:

• If you take the perspective that the role of such a course is to fill in the gaps in student's knowledge left from an incomplete or maths-light highschool education, then the content that should be taught should be up to and including the advanced year 12 australian curriculum. This is particularly appropriate if you do not know the direction of the students, or if they are potentially just doing the bridging course with you and they are planning on studying a degree at a different university say, interstate.

• If you take the perspective that the role of such a course is to prepare students for entry into the particular courses they are about to commence studying, the content relevant to them will be dramatically different. The senior mathematics australian curriculum is extremely generalist and contains many topics that would be completely irrelevant to any particular field of study.

In terms of choosing what content to teach in a university bridging course, the above two competing perspectives will often be at odds with each other.

Chapter 2

Literature Review

2.1 Bridging Courses

- (Gordon & Nicholas, 2013) describes the perspectives of students enrolled in an Australian university bridging course.
- (Johnson & O'Keeffe, 2016) describes the impact of a bridging course on students maths anxiety and self-efficacy in Ireland.
- (?, ?) Conference proceedings analysing the impact of bridging course on students success in first year university level calculus courses.
- (Nicholas & Rylands, 2015) is very relevant but I can't find the actual paper, just references too it.

2.2 Maths Anxiety

Why is Maths Anxiety Important?

Maths anxiety is hugely prevalent, the 2012 Programme for International Student Assessment (PISA) report states that across Organisation for Economic Co-operation and Development (OECD) countries, over 30% of 15 year old students "get very nervous doing mathematics problems", and over 60% of students "worry about getting poor grades in mathematics" (OECD, 2013). As teachers our foremost concern should be for the wellbeing of our students. It has been shown that students with a high level of maths anxiety often literally experience the anticipation of a maths task as visceral pain (Lyons & Beilock, 2012b). There is a clear and overwhelming moral imperative (and ethical duty of care) on us to do everything in our power to protect students in our care from maths anxiety.

Even if the wellbeing issue was not enough, there is also a clear maths anxiety-performance connection, and all the stakeholders in a students academic success in maths. One example of this is highlighted by Foley et al. (2017) who juxtaposes the internationally rising demand for Science, Technology, Engineering and Mathematics (STEM) professionals with the negative correlation between maths anxiety and performance shown in the 2012 PISA report (OECD, 2013) to highlight the relevance of addressing maths anxiety in filling this demand. The relationship between maths anxiety and maths-qualified professionals in the workforce is supported throughout

the literature: when a student has low self-concept (correlated with high maths anxiety), they will tend not to enroll in maths beyond the minimum requirements for graduation (Ashcraft, Krause, & Hopko, 2007), and students affect towards maths can predict their university major (LeFevre, Kulak, & Heymans, 1992). Beyond this example, the list of stakeholders in a students academic success in maths goes on and on: parents; the student's themselves; schools (which are often funded based on the results of standardised testing such as National Assessment Program — Literacy and Numeracy (NAPLAN)), and teachers amongst them.

Maths Anxiety as Distinct from General Anxiety

The existence of maths anxiety as "emotional disturbances in the presence of mathematics" has been noted as early as the 1950's, Dreger and Aiken Jr (1957) even postulated that what he tentatively designated "Number Anxiety" and later became to be known as Maths Anxiety could be a distinct syndrome from general anxiety. Later the landmark meta-study of Hembree (1990) supported this hypothesis, showing a correlation of only 0.38 between maths anxiety and general anxiety. In more recent times, this hypothesis has also been confirmed by Young, Wu, and Menon (2012) using functional magnetic resonance imaging (fMRI) to show that the brain activity in a person experiencing maths anxiety is measurably distinct from that in a person suffering general anxiety. These later studies, as well as the the work of Kazelskis et al. (2000) and more, have also delineated maths anxiety from test anxiety, and these different anxieties exisiting as meaningfully distinct constructs is now quite well accepted. For more on the history of maths anxiety, Suárez-Pellicioni, Núñez-Peña, and Colomé (2016) offers a more detailed review.

Frameworks for Understanding Maths Anxiety

Only a few studies focus on maths anxiety itself (primarily fMRI studies such as those of Young et al. (2012) or Lyons and Beilock (2012b)). Instead the bulk of the literature is focused on the maths anxiety-performance link. Specifically, there seem to be two distinct theories being pursued and I will adopt the terminology of Ramirez, Shaw, and Maloney (2018) to describe them: the "Disruption Account" and the "Reduced Competency Account". Ramirez et al. (2018) go on to make a convincing argument that although these two theories might seem to compete, they are not actually mutually exclusive and instead quite compatible with each other. Ramirez et al. (2018) suggests a third "Interpretation Account" which encapsulates observations made by both lines of research, see Figure 2.1.

First, a little more detail on the existing theories. The "Disruption Account", spearheaded by the work of Ashcraft et al., is centered around the concept of working memory (Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007). Specifically that anxiety about maths takes up students working memory, which prevents them from using that working memory to complete maths tasks and thereby impacts their performance. The "Reduced Competency Account" on the other hand proposes the opposite causality: that lower ability in maths leads to negative experiences associated to maths, which in turn cause maths anxiety to develop. There is also a significant body of work to support this hypothesis, including the milestone meta-analysis of Hembree (1990) and the longitudinal study of Ma and Xu (2004) which found that

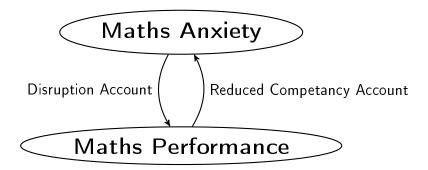


Figure 2.1: The Interpretation Account of Ramirez et al. (2018) for the maths anxiety-performance link showing how the Disruption Account and the Reduced Competency Account can be compatible.

although past maths anxiety was correlated with future maths performance it was a small effect, while past maths performance had a strong effect on future maths anxiety.

Complexities in Finding Effective Interventions

These theoretical views are of course broad oversimplifications of what is an incredibly complex and interconnected topic. They also imply very different approaches for intervention. The "Reduced Competency Account" would imply interventions to boost maths performance and hence allow students to experience success in math should also help to reduce maths anxiety. The results of Supekar, Iuculano, Chen, and Menon (2015) seem to support this hypothesis as when students are given an intensive 8-week tutoring program to boost their maths skills, this is associated to a reduction in maths anxiety. The earlier work by Faust (1996) further supports this by demonstrating an anxiety-complexity effect in which low and high maths anxiety groups performed similarly on low complexity problems, but in high complexity problems the high anxiety groups performance was impacted. On the other hand, Jansen et al. (2013) showed that it is not neccessarily that simple, by showing that when students experience more success they attempt more problems and perform better. However their improved performance is almost completely predicted by the number of problems they attempted, not their experience of success, and their level of maths anxiety was not affected in a significant way which raises a lot of interesting but unanswered questions about this approach.

On the other side of attempted interventions are those in line with the "Disruption Account", in which the maths anxiety itself is addressed in the hopes that will free up extra working memory and hence boost students performance. Park, Ramirez, and Beilock (2014) demonstrate a direct and successful attempt at this in which they used expressive writing exercises to help guide students self-perceived narratives about their maths experiences and thereby reduce their maths anxiety. Notably the approach of Park et al. (2014) is in line with successful treatments for clinical anxiety disorders (see McNally (2007); Becker, Darius, and Schaumberg (2007); Foa et al. (2005)). Another approach that has shown success in this vein does not attempt to directly reduce the anxiety experienced, but rather reappraise it's symptoms (Jamieson, Peters, Greenwood, & Altose, 2016). This is another technique from clin-

ical psychology in which stress is reconceptualised as a coping tool, an evolutionary method for heightening performance in response to a challenge to be overcome, instead of a symptom of exposure to something to be feared and avoided. This change in the perspective of stress is also very much in line with the "Interpretation Account" of Ramirez et al. (2018).

The work of Wang et al. (2015) showed the role that intrinsic motivation has mediating the relationship between maths anxiety and performance, and suggested the importance of a mindset centred on viewing the process of learning maths as one of "productive struggle". This reconceptualisation to a 'productive struggle' model is supported by other literature as well, Lin-Siegler, Ahn, Chen, Fang, and Luna-Lucero (2016) exposes students in a classroom to struggles experienced by famous scientists in order to help normalise the concept of productive struggle, and Hiebert and Grouws (2007) discuss the importance of this same concept in a maths context.

One of the implications of the "Interpretation Account" is that if an intervention targets only one of these two possible links in the cycle (see Figure 2.1), the cycle may re-establish itself after the intervention is over and negate any potential longterm effects. However there is only a very limited amount of research out there on such longterm effects, and several authors have discussed the need for further research into this (Suárez-Pellicioni et al., 2016; Chang & Beilock, 2016). My hypothesis is that a multi-faceted approach targetting both directions simultaneously could disrupt the cycle shown in Figure 2.1 and result in significant longterm effects.

Instruments for Measuring Maths Anxiety

In order to track the effectiveness of these interventions, we will be collating assessment results as a measure of performance, but will also want to measure maths anxiety and maths affect/ self-concept. Significant work has been done over the years to develop psychometrics to measure maths anxiety, almost exclusively consisting of self-reporting surveys (with the exception of some more modern fMRI work, such as that of Lyons and Beilock (2012a)). We will use a recently developed scale: the Maths Anxiety Scale — Revised (MAS-R) of Bai, Wang, Pan, and Frey (2009), which has been shown to be remarkably self consistent by incorporating both positive and negative affect items (Bai, 2011). It is short, easy to implement, and cheap in comparison to fMRI methods. In order to measure maths self-concept, Jansen et al. (2013) modified the Perceived Competence Scale for Children of Harter (1982) to measure "Math Competance". The methodological process imployed by Jansen et al. (2013) was quite rigorous and so we will use their instrument, or a minor modification thereof (we will do it in English), to measure maths self-concept.

Chapter 3

Curriculum Mapping

One of the important roles of university mathematics bridging courses (MathStart and MathTrack) is to fill the content knowledge gap for students who did not complete Mathematical Methods or Specialist Mathematics in highschool but wish to commence study at a university level in subjects that have required knowledge that in contained in these subjects. Although the AC lays out the expected curriculum for senior highschool (years 11 and 12) mathematical methods and specialist mathematics, the exact content knowledge required varies dramatically depending on the university course. Beyond this, different states in Australia teach different curriculums in senior highschool, and some differ significantly from the AC, some do not even use the terms "Mathematical Methods" and "Specialist Mathematics" at all. In South Australia we teach the SACE, and so the expected knowledge when entrying South Australian unviersities tends to be based on the SACE curriculum. A notable example of this was in 2018 when significant changes where made to the SACE senior mathematics curriculum, the University of Adelaide re-modelled the content in it's primary first year mathematics courses to better match the prior knowledge of the majority of new students coming from SACE. That said, it is still of interest to align the bridging courses with not only the SACE curriculum, but also with AC as students enrolling in the bridging courses are not always intending to commence study in south australia. For example, students planning to commence study in a university interstate in the following year but currently living in south australia may be reccomended to the bridging courses by the university they are planning to go too.

This chapter will examine the alignment of the content of MathStart and Math-Track (the mathematics bridging courses offered at the university of adelaide) with the AC and SACE. First, in Section 3.1, some notation will be introduced and the content of each of the three curriculums will be reviewed:

- The AC senior mathematics subjects mathematical methods and specialist mathematics,
- The SACE curriculum stage 1 mathematics, stage 2 mathematical methods, and stage 2 specialist mathematics,
- The University of Adelaide's bridging courses: MathStart, and MathTrack.

Then, these will be mapped to each other in Section 3.2 (see Figure 3.1). Throughout, discussion will be had around alignment and gaps between the content of these

curriculums and courses, explanations and reasons for these discrepancies, and potential alternate approaches.

Beyond that, this chapter will also briefly discuss the alignment of these bridging courses to first year university mathematics courses and bridging courses offered by other universities in Australia, and discuss the relationship between the gaps in alignment between the AC/SACE and the bridging courses and the requirements of these first year university courses.

3.1 Content

3.1.1 Notation

Each of the senior highschool curriculums, as well as the university bridging courses, being considered here is broken down into topics, with each topic containing a number of key concepts. In Section 3.2, the alignment between these curriculums and bridging courses will be considered thoroughly at both a topic-level, and to the finer detail of particular key concepts. In order to abstract away some of the complexity of considering the topic-level alignment, and be able to present the topic-level alignment in a meaningful way abbreviated codes will be used to identify each topic. These abbreviated codes are presented in Table 3.1 and will be used for the remainder of this chapter.

Table 3.1: Abbreviated codes for topics within the AC and SACE senior mathematics subjects: Mathematical Methods and Specialist Mathematics, as well as the Unviersity of Adelaides bridging courses: MathStart and MathTrack. Square brackets ([]) are used to indicate numeric values that can vary.

Code	Meaning
MMu[#1]t[#2] MMu[#1]t[#2]	AC Senior Mathematical Methods Unit [#1], Topic [#2] AC Senior Specialist Mathematics Unit [#1], Topic [#2]
S1M[#] S2MM[#] S2SM[#]	SACE Stage 1 Mathematics, Topic [#] SACE Stage 2 Mathematical Methods, Topic [#] SACE Stage 2 Specialist Mathematics, Topic [#]
MS[#] MT[#]	Maths Start, Topic (Booklet) [#] Maths Track, Topic (Booklet) [#]

3.1.2 Within-Topic Key Concepts

Description of each topic in the AC Mathematical Methods and Specialist Mathematics Topics, SACE stage 1 mathematics, stage 2 mathematical methods and stage 2 specialist mathematics, and the University of Adelaides MathStart and MathTrack programs. For brevity a code is used to identify each topic, see Table 3.1 above, and then for each topic it's name is given in bold followed by a list of the key concepts covered in that topic. These are discussed at length below, and this table is intended to be used as reference material for that discussion.

Note: I generally omit "interpretation" concepts. For example in S1M2 on quadratics I include the key concepts around the Vertex Form and Factorised Form, but I do not include the interpretation fo roots and vertices, deducing these from the equation of a quadratic, or deducing the equation of a quadratic given these bits of information. These are more surrounding skills, which I think it is fair to assume should be taught along with the key concepts. To an experienced maths educator, the key concepts should be enough to deduce the required surrounding interpretative framework.

Note: This is a huge table. I could maybe put it in an apprendix?

Code Name and Key Concepts

- MMu1t1 Functions and graphs: Lines, Quadratics, Inverse Proportions, Polynomials, Relations, Translations and Dilations
- MMu1t2 **Trigonometric functions**: Unit Circle, Radians, SOH CAH TOA, Sine Rule, Exact Values, Amplitude/ Period/ Phase, Sum of Angles Identities
- MMu1t3 Counting and probability: Binomial Coefficients, Set Complement Intersection and Union, Probability, $P(A \cup B) = P(A) + P(B) P(A \cap B)$, Conditional Probability, Independence
- MMu2t1 Exponential functions: Index Laws, Fractional Indices, Functions, Asymptotes, Graphs
- MMu2t2 Arithmetic and geometric sequences and series: Arithmetic and Geometric Sequences as Recurrence Relations, Limiting Behaviour, and Partial Sum Formulae, Growth and Decay
- MMu2t3 Introduction to differential calculus Average Rate of Change, First Principles, Leibniz Notation, Instantaneous Rate of Change, Slope of Tangent, Derivitive of Polynomials, Linearity of Differentiation, Optimisation, Anti-Derivitives, Interpret Position-Time Graphs
- MMu3t1 Further differentiation and applications: Define e as a s.t. $\lim_{h\to 0}\frac{a^h-1}{h}=1$, Derivitives of $e^x\sin(x)$ and $\cos(x)$, Chain Product and Quotient Rules, Second Derivitives
- MMu3t2 Integrals: Integrate Polynomial Exponential and Trigonometric Functions, Linearity of Integration, Determine Displacement given Velocity, Definite Integrals, Fundamental Theorem of Calculus, (signed) Area Under a Curve
- MMu3t3 Discrete random variables: Frequencies, General Properties, Expected Value, Variance, Standard Deviation, Bernoulli and Binomial Distribtions
- MMu4t1 The logarithmic function: Logs as Inverse of Exponentials, Log-Scales, Log Function Graphs, Natural Log, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, $\int \frac{1}{x}dx = \ln(x) + c$ for x > 0
- MMu4t2 Continuous random variables and the normal distribution: Probability Density Function, Cumulative Distribution Function, Probabilites Expected Value, Variance and Standard Deviation as Integrals, Linear Transformation of Random Variables, Normal Distribution using Technology

- MMu4t3 Interval estimates for proportions Simple Random Sampling, Bias, Sample Proportion, Normal Approximation to the Binomial Proportion, Wald Confidence Interval, Trade-Off Between Width and Level of Confidence
- SMu1t1 Combinatorics Multiplication of Possibilities, Factorial Notation, Permutations with and without Repeated Objects, Union of Three Sets, Pigeon-Hole Principle, Combinations, Pascals Triangle
- SMu1t2 Vectors in the plane: Magnetude and Direction, Scalar Multiplication, Addition and Substraction as a Triangle, Vector Notation, $a\mathbf{i} + b\mathbf{j}$ Notation, Scalar Dot Product, Projection, Parallel and Perpendicular Vectors
- SMu1t3 **Geometry**: Notation for Implication (\Rightarrow) and Equivalence (\Leftrightarrow), Converse ($B \Rightarrow A$) Negation ($\neg A \Rightarrow \neg B$) and Contrapositive ($\neg B \Rightarrow \neg A$), Proof by Contradiction, \forall and \exists Notation, Counter-Examples, Circle Theorems, Quadrilateral Proofs in \mathbb{R}^2
- SMu2t1 Trigonometry: Graph and Solve Trig Functions, Prove Various Trig Indentities, Reciprocal Trig Functions
- SMu2t2 Matrices: Notation, Addition and Scalar Multiplication of Matrices, Multiplicative Identity and Inverse, Determinant, Matrices as Transformations
- SMu2t3 Real and complex numbers: Rationality and Irrationality, Induction, $i=\sqrt{-1}$, Complex Numbers a+bi and Arithmetic $(+,-,\times,\div)$, Complex Conjugates, Complex Plane, Complex Conjugate Roots of Polynomials
- SMu3t1 Complex numbers: Modulus and Argument, Arithmetic $(\times, \div, \text{ and } z^n)$ in Polar Form, Convert between Polar and Cartesian Form, De Moivre's Theorem, Roots of Complex Numbers, Factorising Polynomials
- SMu3t2 Functions and sketching graphs: Composition of Functions, Oneto-One, Inverse Functions, Absolute Value Function, Rational Functions
- SMu3t3 Vectors in three dimensions: $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ Notation, Equation for Spheres, Parameterised Vector Equations, Equations of Lines, the Cross Product, Equation for a Plane, Systems of Linear Equation (Elimination Method) and Geometric Interpretation of Solutions, Kinematics via Differentiation of Vector Equations, Projectile and Circular Motion
- SMu4t1 Integration and applications of integration Substitution, $\int \frac{1}{x} dx = \ln |x| + c$ for $x \neq 0$, Inverse Trig Functions and their Derivitives, Integrate $\frac{\pm 1}{\sqrt{a^2 x^2}}$ and $\frac{a}{a^2 + x^2}$, Partial Fractions, Integration by Parts, Volume of Solids of Revolution, Numerical Integration using Technology
- SMu4t2 Rates of change and differential equations: Implicit Differentiation, First-Order Seperable Differential Equations, The Logistic Equation, Kinematics (Rates of Change)
- SMu4t3 Statistical inference: Central Limit Theorem and the Resulting Confidence Interval for a Mean

Code	Name and Key Concepts
Couc	Traine and Ney Concepts
S1M1	Functions and graphs: Equations for a Line, Slope, y-intercept, Intersection of Lines, Reciprocal Function, Asymptotes, Functions vs Relations, Domain, Range, Function Notation
S1M2	Polynomials: Quadratic Equations in Vertex and Factorised Forms, Quadratic Formula, Completing the Square, The Leading Coefficient and Degree of a Polynomials, Cubics, Quartics
S1M3	Trigonometry : Pythagoras, SOH CAH TOA, Cosine Rule, Sine Rule, Unit Circle, Sine and Cosine Functions, Radians, Length of Arc, Area of Sector, Amplitude, Period, Phase, $\tan(x) = \frac{\sin(x)}{\cos(x)}$
S1M4	Counting and statistics: Factorial, Permutations, Multiplication Principle, Combinations, Discrete vs Continuous Random Variables, Mean, Median, Mode, Range, Interquartile Range, Standard Deviation, Normal Distribution,
S1M5	Growth and decay : Index and Logarithm Laws, Exponential Functions and their Graphs
S1M6	Introduction to differential calculus: Average Rate of Change, First Principles, Notation $f'(x) = \frac{df}{dx}$, $\frac{d}{dx}x^n = nx^{n-1}$, Linearity of Differentiation, Slope of Tangent, Increasing vs Decreasing, Local and Global Maxima and Minima, Stationary Points, Sign Diagram
S1M7	Arithmetic and geometric sequences and series: Arithmetic and Geometric Series as Recurrence Relations and Explicit Expressions, Partial Sums, Limiting Behaviour
S1M8	Geometry : Circle Properties, Proofs (Direct, Contradiction, and Contrapositive)
S1M9	Vectors in the plane : Component (column) vs $ai + bj$ Notation, Length and Direction, Linear Combinations of Vectors, Scalar Dot Product, Projection, Angle Between Two Vectors and Parallel / Perpendicular, Geometric Proof
S1M10	Further Trigonometry : Sketch Trigonometric Functions with Translations and Dilations, Solve for Angles, Trigonometric Identities, Reciprocal Trigonometric Functions
S1M11	Matrices : Linear Combinations of Matrices, Matrix Multiplication, The Identity, Inverse Matrices, The 2×2 Inverse, The 2×2 Determinant, Linear Transformations (including rotations, reflections and composition)
S1M12	Real and complex numbers: Rationals, Irrationals, Interval Notation, Induction, $i=\sqrt{-1}$, Real and Imaginary Components, Complex Conjugates and Arithmetic, Argand Diagram, Modulus, Complex Roots of Polynomals
S2MM1	Further differentiation and applications: S1M6, Chain Product and Quotient Rules, $e=2.718\ldots, \frac{d}{dx}e^x=e^x, \frac{d}{dx}\sin(x)=\cos(x), \frac{d}{dx}\cos(x)=-\sin(x)$, Second Derivatives, Concavity and Points of Inflection

- S2MM2 **Discrete random variables**: Random Variables, Discrete vs Continuous, Probability Functions and Distributions, Properties of Probabilities, Frequency, Expected Value $E[X] = \sum xp(x) = \mu_X$, Standard Deviation $\sigma_X = \sqrt{\sum (x \mu_X)^2 p(x)}$, Uniform Bernoulli and Binomial Distributions
- S2MM3 Integral calculus: Anti-differentiation, If F'(x) = f(x) then $\int f(x)dx = F(x) + c$, Reversing Chain Rule for $\int f(ax+b)dx$, Linearity of Integration, Finding the Constant of Integration, Area Under the Curve as Upper and Lower Sum Approximations, Definite Integral, Area Between Two Functions and Between a Negative Function and the x-axis, Fundamental Theorem of Calculus,
- S2MM4 Logarithmic functions: Sketching $y=a\ln(b(x-c))$, $\frac{d}{dx}\ln(x)=\frac{1}{x}$. For x>0 $\int \frac{1}{x}dx=\ln(x)+c$
- S2MM5 Continuous random variables and the normal distribution: $P(X=x)=0, \text{ Probability Density Function, } \mu_X=\int_{-\infty}^{\infty}xf(x)dx,$ $\sigma_X=\int_{-\infty}^{\infty}(x-\mu_X)^2f(x)dx, \ f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \text{ Standard Normal } Z=\frac{X-\mu}{\sigma}, \text{ Simple Random Sampling, For } X\sim(\mu,\sigma) \text{ and } X_i\sim iidX \text{ Sampling Distributions of } S_n=\sum_{i=1}^nX_i \ (n\mu,\sigma\sqrt{n}) \text{ and } \bar{X}_n=\frac{S_n}{n} \ (\mu,\frac{\sigma}{\sqrt{n}}), \text{ If } X \text{ is Normally Distributed, then so are } S_n \text{ and } \bar{X}_n, \text{ Central Limit Theorem (CLT)}$
- S2MM6 Sampling and confidence intervals: Confidence Interval for a Mean using CLT $\left(\bar{x}-z^*\frac{s}{\sqrt{n}}\right) \leq \mu \leq \left(\bar{x}+z^*\frac{s}{\sqrt{n}}\right)$, Wald Interval for a Proportion
- S2SM1 Mathematical induction: Initial Case and Induction Step
- S2SM2 **Complex numbers**: Cartesian vs Polar Form, Real and Imaginary Components, Modulus and Argument, Arithmetic in both Cartesian and Polar Forms, de Moivre's Theorem including Negative and Fractional Powers, Geometric Properties of the Argand Plane, Complex Arithmetic as Transformations, $n^{\rm th}$ Roots of a Complex Number, Factorising Polynomials with Complex Roots
- S2SM3 Functions and sketching graphs: Function Composition, Informal Intro to Domain and Range, One-to-One, Inverse Functions, Absolute Value Function, Graphing Rational Functions
- S2SM4 **Vectors in three dimensions**: Notation, Equations of a Line in \mathbb{R}^3 , Scalar Dot Product, Vector Cross Product, $|\mathbf{a}\times\mathbf{b}|$ is the Area of their Parallelogram, Equation for a Plane in \mathbb{R}^3 , Systems of Linear Equations, Geometric Interpretation of No/Unique/Infinite Solutions to a System of Linear Equations in \mathbb{R}^3
- S2SM5 Integration techniques and applications: Integration by Substitution, Using Trigonometric Identities for Integration, Derivatives of Inverse Trigonometric Functions (so $\int \frac{\pm 1}{\sqrt{a^2-x^2}} dx$ and $\int \frac{a}{a^2+x^2} dx$, Integration by Parts, Area Between two Curves, Volume of Solids of Revolution

Code	Name and Key Concepts
S2SM6	Rates of change and differential equations: Implicit Differentiation, First-Order Seperable Differential Equations, The Logistic Differential Equation, Parameterised Curves, Example: if $\mathbf{v} = \frac{d}{dt}(x(t),y(t))$ is Velocity, $ \mathbf{v} $ is Speed, and so the Arc Length along the Parameterised Curve is $\int_a^b \sqrt{\mathbf{v} \bullet \mathbf{v}} dt$, Trigonometric Parameterisations (unit circle, and non-circular parameterisations)
MS1	Numbers & Functions: Natural Numbers, Integers, Rational Numbers, Real Numbers, Functions, Intervals
MS2	Linear Functions : Equation for Linear Functions, Simultaneous Linear Equations, Sketching Linear Inequalities
MS3	Quadratic Functions : Sketching a Parabola, General Form of a Quadratic, Translations and Dilations
MS4	Rational Functions: Sketching Reciprocal Functions (Hyperbola), Lines of Symmetry, Limits and Asymptotes
MS5	Trigonometry I : Pythagoras, Similar Triangles, SOH CAH TOA, Trigonometric and Inverse Trigonometric Functions using Technology, Exact Values
MS6	Trigonometry II : Unit Circle, Sketching Trigonometric Functions, Finding all Solutions to Trigonometric Equations, The Sine Rule, The Cosine Rule, Introductory Trigonometric Identities, Radians
MS7	Exponential Functions : Index Laws, Sketching Exponential Functions, $e = 2.718$, Growth and Decay
MS8	Logarithms : Natural Logarithm, Logarithm Laws, Using Logarithm to Fit Growth/Decay Functions, Half-Life/ Doubling Time
MT1 MT2	Polynomials: Matrices:
MT3	Vectors and Applications:
MT4	Systems of Linear Equations:
MT6 MT7	Differentiation: Applications of Differentiation:
MT8	Exponential and Logarithm Functions:
MT9	Integration:

3.1.3 AC Mathematical Methods and Specialist Mathematics

AC has four levels of mathmatics, including also essential and general mathematics. Mathematical Methods and Specialist Mathematics are the two highest level mathematics, intended (partly) for preparation to university entry.

3.1.4 SACE Stage 1 Mathematics, Stage 2 Mathematical Methods and Specialist Mathematics

Cumulative Distribution Function not mentioned

3.1.5 MathStart and MathTrack

Note the missing topic 5 in MathTrack, this used to be part of the course some time ago but is now ommitted.

MS4: Note the link to S2SM3

• MS1: Maybe Introduce Interval Notation along with Intervals?

3.2 Curriculum Mapping

Figure 3.1 shows the topic-level alignment between the AC (senior mathematical methods and specialist mathematics), SACE (mathematical methods and specialist mathematics), MathStart and MathTrack. Although sub-topic alignment within these topic alignments is not always perfect, it is fairly strong in each case but individual cases will be discussed in more detail below.

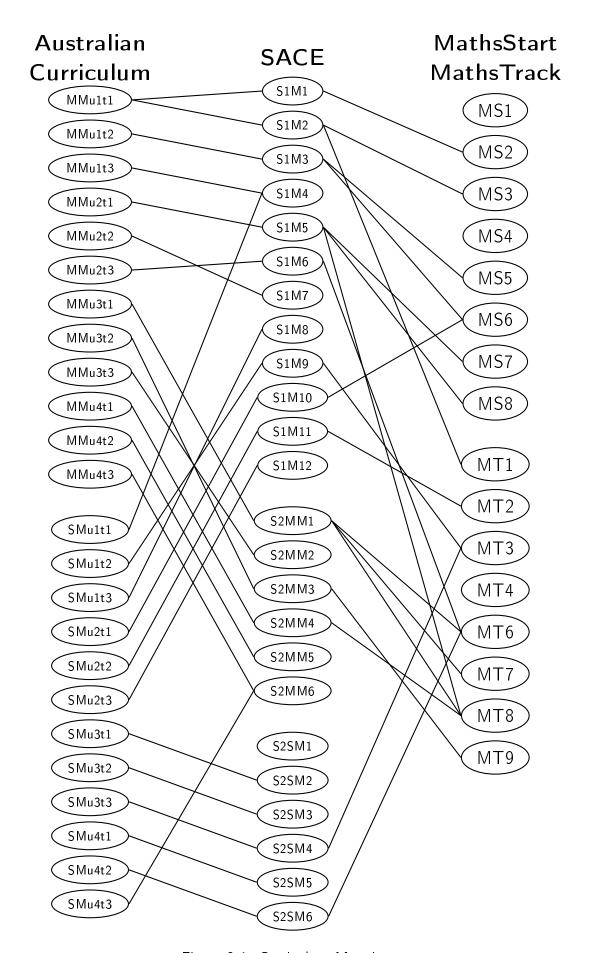


Figure 3.1: Curriculum Mapping

Chapter 4

Moving Forward: Improvements

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Conclusions and Reccomendations

With respect to the bridging courses run through the university of adelaide's maths learning centre: MathsStart and MathsTrack,

- The self-paced and feedback focused approach to assessment is certainly the highlight of the programs, should be continued, encouraged, potentially further resourced, expanded, and reccomended to other bridging course facilitators.
- The role of bridging courses as what is often student's first experience at university implies that potentially students wellbeing and retention could be improved by structuring the programs to provide more opportunities for students to meet each other and work together: either in the maths learning center drop-in area, or a seperate area, but potentially assigning a certain time on a certain day perhaps weekly or fortnightly during which students are encouraged to come and work together, could allow them to make freinds, build social networks, and better aclimitise them to the university environment in order to better prepare them for success in their studies.
- The smallest but perhaps easiest to implement improvement could be to better
 align the course content with curriculum, both the highschool curriculum (AC/
 SACE) in the case of students doing the bridging course to then comence study
 interstate or overseas, or with specific first year entry level courses, to better
 match the potential gaps in knowledge students may encounter.

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