Mathematics Learning Centre

The University of Sydney

Absolute values

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1 The absolute value function

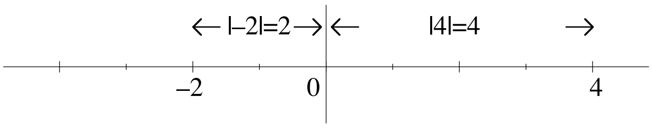
Before we define the absolute value function we will review the definition of the absolute value of a number.

The *Absolute value of a number* x is written $|x|$ and is defined as

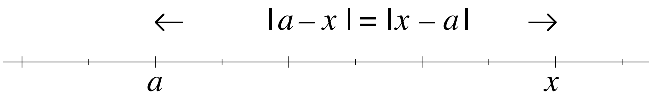
$$|x| = x \text{ if } x \geq 0 \qquad \text{or} \qquad |x| = -x \text{ if } x < 0.$$

That is, $|4| = 4$ since 4 is positive, but $|-2| = 2$ since -2 is negative.

We can also think of $|x|$ geometrically as the distance of x from 0 on the number line.



More generally, $|x - a|$ can be thought of as the distance of x from a on the numberline.



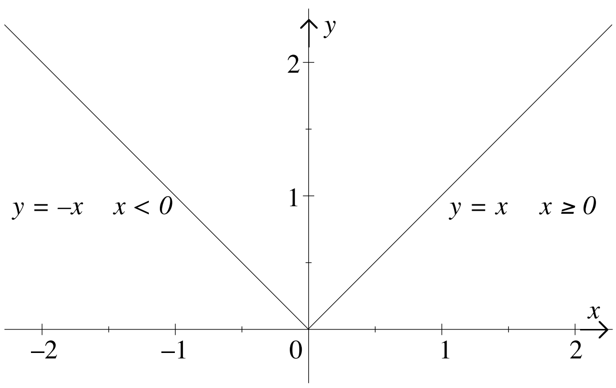
Note that $|a - x| = |x - a|$.

The absolute value *function* is written as $y = |x|$.

We define this function as

$$y = \begin{cases} +x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

From this definition we can graph the function by taking each part separately. The graph of $y = |x|$ is given below.



The graph of $y = |x|$.

Example

Sketch the graph of $y = |x - 2|$.

Solution

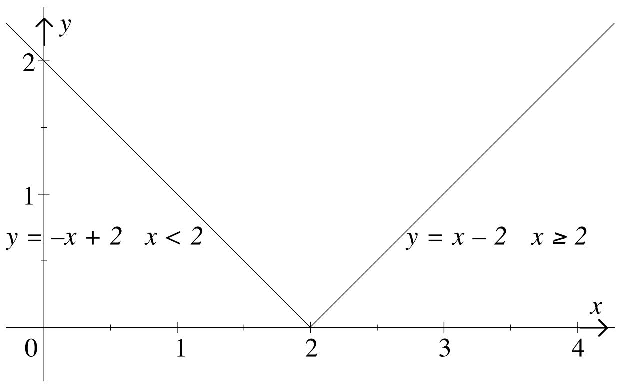
For $y = |x - 2|$ we have

$$y = \begin{cases} +(x-2) & \text{when } x-2 \geq 0 \quad \text{or} \quad x \geq 2 \\ -(x-2) & \text{when } x-2 < 0 \quad \text{or} \quad x < 2 \end{cases}$$

That is,

$$y = \begin{cases} x-2 & \text{for } x \geq 2 \\ -x+2 & \text{for } x < 2 \end{cases}$$

Hence we can draw the graph in two parts.



The graph of $y = |x - 2|$.

We could have sketched this graph by first of all sketching the graph of $y = x - 2$ and then reflecting the negative part in the x-axis.

Example

Find the values of x for which $|x + 3| = 6$.

Solution

First of all note that

$$|x+3| = \begin{cases} +(x+3) & \text{when } x+3 \geq 0 \quad \text{or} \quad x \geq -3 \\ -(x+3) & \text{when } x+3 < 0 \quad \text{or} \quad x < -3. \end{cases}$$

Taking each of these separately.

When $x \leq -3$, $|x+3| = -x-3 = 6$, so $x = -9$.

When $x \geq -3$, $|x+3| = x+3 = 6$, so $x = 3$.

Therefore $|x+3| = 6$ when $x = -9$ or $x = 3$. You can check this by substitution.

Example

For what values of x is $|x-4| = |2x-1|$.

Solution

We know that the values $x = \frac{1}{2}$ and $x = 4$ are important x values here, so we will use them to divide the x axis into three sections and will consider them in turn.

Case 1. For $x < \frac{1}{2}$, $|x-4| = -(x-4) = |2x-1| = -(2x-1)$, so $-x+4 = -2x+1$.

Therefore, $x = -3$.

Case 2. For $\frac{1}{2} \leq x < 4$, $|x-4| = -(x-4) = |2x-1| = 2x-1$, so $-x+4 = 2x-1$.

Therefore, $x = \frac{5}{3}$.

Case 3. For $x \geq 4$, $|x-4| = x-4 = |2x-1| = 2x-1$, so $x-4 = 2x-1$.

Therefore, $x = -3$, but this does not satisfy the assumption $x \geq 4$ so this case does not give us a solution.

The solutions are $x = -3$ and $x = \frac{5}{3}$.