# MathsStart and MathsTrack: Mathematics Bridging at the University of Adelaide.

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# Contents

# Acronyms

**AC** Australian Curriculum. iv, 2, 10, 27–33, 36–46, 48, 50–52, 56, 57

**AMSI** Australian Mathematical Sciences Institute. iv, 14

**ECMS** Faculty of Engineering, Computer and Mathematical Sciences. iv, 5, 55

**fMRI** functional magnetic resonance imaging. iv, 19

**MLC** Maths Learning Centre. iv, vii, xi, 1, 4, 5, 15, 24, 25, 28, 32, 53–56

**NAPLAN** National Assessment Program — Literacy and Numeracy. iv, 18

**OECD** Organisation for Economic Co-operation and Development. iv, 18

**PISA** Programme for International Student Assessment. iv, 18

**SACE** South Australian Certificate of Education. iv, 2, 4, 10, 27–30, 32, 33, 36–52, 57

**STEM** Science, Technology, Engineering and Mathematics. iv, 4, 18, 44, 56

**UofA** University of Adelaide. iv, vii, xi, 1, 2, 4, 9, 24, 25, 28, 30, 32, 33, 49, 53, 56

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# Abstract

Mathematics bridging courses serve an increasingly critical stop-gap role in educational systems globally, providing a pathway into mathematical higher education for students who may otherwise be blocked from pursuing such education. This is particularly critical for already disadvantaged students who may be at increased risk of being denied access to mathematical higher education. Mathematical higher education, including fields of study other than mathematics that require a degree of mathematical competency such as science, engineering, and medicine, is rising in demand. Yet participation in higher level mathematics education in high school has been in steady decline for over three decades, in no small way due to negative perceptions of mathematics help by the public.

In this work, the challenges faced by mathematics bridging courses are considered in general, and in the specific Australian context of the University of Adelaide (UofA) mathematics bridging courses offered through the Maths Learning Centre (MLC): MathsStart and MathsTrack. Potential improvements to the bridging courses are investigated through two avenues of research. First, a literature review puts bridging courses into context: their role in education and in society more broadly, challenges they face, and potential approaches to addressing these challenges. Secondly the content taught in the Australian senior secondary high school curricula is mapped to the content of the bridging courses, and alignment of this content to the actual future needs of the students in their continued study is discussed. Finally, recommendations are made for improvements that could be made to the bridging courses, and good practices that could be generalised to bridging courses more broadly across Australia and internationally.

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# Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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# Chapter 1

# Introduction

This thesis is concerned with the mathematics bridging courses offered at the University of Adelaide (UofA) through the Maths Learning Centre (MLC): MathsStart and MathsTrack. These courses are intended to take students with very little mathematics background and help them transition into tertiary education (which will generally have some mathematics pre-requisites or assumed knowledge) and to be successful there. MathsStart and MathsTrack are structured to be student-paced, with no deadlines having been set *apriori*, in a deliberate attempt to both alleviate test-anxiety, and to acclimatise students to a learning environment in which the primary onus for motivation is on the students. Although the bridging courses are offered through the MLC at the UofA, the cohort of students come from a very diverse set of backgrounds and have a variety of needs. The majority of students enrolling in these courses are planning to continue into tertiary education at the UofA, but many students do not intend to continue into tertiary education at all, and instead enrol in the bridging courses to meet the requisite assumed knowledge for other pathways — pilot training in the defence forces, for example.

The continuing decline of interest in mathematics along with the sustained trend of increased demand for mathematically skilled graduates by several major leading industries in Australia and globally (engineering, science, medicine, ...) means that bridging courses are playing an increasingly important role in our education system. See Chapter 3 for a more detailed discussion. Although MathsStart and MathsTrack have been successful so far, as educators we are continually engaging in reflective practice and looking for ways to improve our teaching practice. It is in this frame of mind, and with the knowledge that a comprehensive review of the purpose, structure, and content of MathsStart and MathsTrack has not yet been done, that in collaboration with the MLC we constructed the "guiding question" for this work:

> How can MathsStart and MathsTrack be improved?

Naturally, this vague statement invites questions such as "how is improvement measured?", and "improvement in what outcome?". The question is left deliberately vague in this way because part of this work will be dedicated to teasing apart the different possible interpretations, the importance of taking care when interpreting improvement in an educational context, and the consequences of different interpretations. A solution is not proposed, only alternatives and their consequences, with comments on the stakeholders in each case. One important perspective of this question that will be represented heavily in this thesis can be shown by a re-phrasing of the question:

> How can MathsStart and MathsTrack best help the students enrolling in them to be successful going forward?

Of course this does not necessarily make the question any less vague — determining what is in the best interests of our students, is not always clear or straightforward. Nonetheless, this question will be the primary focus of this thesis and despite not being able to give any definitive answers, some suggestions and recommendations will be made. At the very least, some context will be given for better understanding the question. It is important to note that although the focus of this work is the courses MathsStart and MathsTrack, much of the contextual background presented herein is relevant to bridging courses across Australia and internationally. Although it is not the primary goal of this work, giving some broad structure and perhaps a framework for understanding the needs of students going into mathematics bridging programs globally could be thought of as a secondary (perhaps ancillary) objective of this work.

In order to address the guiding question above, this thesis will be structured as follows:

- The remainder of this introductory chapter (Chapter 1) is broken into two sections. First, the purpose, structure, and context of mathematics bridging courses is explored both in general, and in the specific context of the UofA (Section 1.1). Second, a broad educational framework is introduced in Section 1.2 which can be used to give high-level context for the work that will be done in this thesis, motivating the structure of the work and outlining the key areas of importance and how they interact with one another.

- Chapter 2 provides a brief description of the methodology employed in the research that will be presented in Chapters 3 and 4.

- An in-depth discussion of the existing literature is presented in Chapter 3: what is known, approaches attempted in the past both in Australia and internationally, frameworks proposed for understanding the secondary-tertiary transition and the maths anxiety-performance link, and some deeper discussion on some of the particularly relevant related concepts.

- One of the major contributions of this thesis is the detailed curriculum mapping which is the focus on Chapter 4. In this curriculum mapping the content of the top two levels of senior high school mathematics in the Australian Curriculum (AC) and South Australian Certificate of Education (SACE) are mapped to each other and to the content currently in MathsStart and MathsTrack. Detailed discussion of this mapping also includes commentary on how the content relates to typical entry-level university mathematics courses, as this is particularly relevant for bridging course students.

- Finally, in Chapter 5 the conclusions from this work are summarised, and in particular the interactions between the different avenues of research are consolidated. Additional work done outside of this thesis is discussed, and potential future research directions are outlined.

## 1.1 Context

### 1.1.1 Bridging Courses in General

Students will usually enrol in university mathematics bridging courses because they are required to demonstrate a certain level of mathematical knowledge/ competence before commencing study at university, but either do not meet those requirements, or do but feel a lack of confidence in their abilities and feel like they need to refresh/ revise/ learn some mathematics prior to commencing their studies.

Reasons why these students do not either meet the entry requirements, or feel a lack of confidence in their abilities can be quite varied:

- A long period of time may have passed since they last studied mathematics (or studied at all). The number and proportion of so-called "adult learners" has been steadily increasing for well over three decades now (Johnson & O'Keeffe, 2016; Hardin, 2008; Murtaugh, Burns, & Schuster, 1999).
- They may have performed poorly in mathematics in high school.
- They may have chosen not to study mathematics at a higher level in high school.
- They may suffer from maths anxiety (which would make them likely to fit into the above two categories as well).

The role of mathematics bridging courses is to take these students, and:

- Bridge their content knowledge so they are prepared for entry level university courses, or other tertiary programs required/ assumed knowledge.
- Support the growth of their confidence and self-efficacy surrounding mathematics.
- Ultimately prepare them to be successful in their continued tertiary study.

The question "what content should be taught in a university bridging course?" has dramatically different answers depending on the perspective one takes on what the role of such a bridging course is. Even restricting the question to purely knowledge-based content (excluding critical affective aspects such as self-efficacy):

- If you take the perspective that the role of such a course is to fill in the gaps in student's knowledge left from an incomplete or maths-light high school education, then the content that should be taught would include the final year of high school curriculum. This approach can be particularly appropriate if you do not know the direction of the students, or if they are potentially planning on continuing study at an interstate university, for example.
- If you take the perspective that the role of such a course is to prepare students for the further study they plan on engaging in however, this is quite different. The content relevant to any one student will be dramatically different depending on their planned direction. The senior high school mathematics curriculum is quite general and would certainly contain many topics that would be completely irrelevant to any particular student.

In terms of choosing what content to teach in a university bridging course, the above two competing perspectives will often create tension between each other. There are advantages, disadvantages, and important stakeholders of both perspectives and this unfortunately means that this tension cannot be resolved, but rather that it must be balanced — a compromise found.

As though that wasn't difficult enough, the question of "what content should be taught in a university bridging course?" is only one part of the issue. As mentioned above, affective aspects such as self-efficacy are crucially important, and it has been shown that, particularly in maths-heavy subject areas such as Science, Technology, Engineering and Mathematics (STEM) significant numbers of students drop out within the first two years (House, 2000; Tsui, 2007), and that engagement with mathematics support services such as the MLC drop-in centre and bridging courses is an approach proven to be effective in addressing this issue (Lee, Harrison, Pell, & Robinson, 2008). Consideration of impacts on these affective aspects of perspectives of mathematics is critical for preparing students to be successful. Content cannot be considered in isolation of these factors, and vice versa, they are fundamentally entwined such that both must be considered in order to achieve the desired outcomes.

### 1.1.2 The Maths Learning Centre

The MLC in it's current form is part of the "Student Engagement and Success team" within the "Division of the Deputy Vice-Chancellor & Vice-President (Academic)" of the UofA, but it has been through many iterations since it's initial opening in 1992 with the incredible support of Liz Cousins and Alison Wolff. Geoff Coates was the heart and soul of the MLC for many years, notably hosting the Australasian Bridging Mathematics Network conference in 1996. From the mid 2000s to present, the MLC has been in the capable hands of David Butler and Nicholas Crouch, who have helped the MLC become what it is today. David Butler and Nicholas Crouch were awarded a Commendation for Excellence in Support of the Student Experience by the UofA in 2013, and a Citation for Outstanding Contribution to Student Learning frm the 2014 Australian Awards for University Teaching, Office for Learning & Teaching. They have consistently represented expertise and a commitment to mathematics education, to the process of learning mathematics, learning to learn mathematics, and learning to teach mathematics.

The primary service the MLC at the UofA provides is a drop-in centre where students can come and get help learning maths. It "exists to help all students at the UofA succeed in learning and using maths relating to their coursework". However in addition to running the drop-in centre, the MLC also offers mathematics bridging courses.

### 1.1.3 MathsStart and MathsTrack

The UofA offers a mathematics bridging course through the MLC called MathsTrack, which can be used as a prerequisite at the University of Adelaide in place of SACE Stage 2 Mathematical Methods. MathsStart is another mathematics bridging course that was previously also offered through the MLC at the UofA, covering various topics from high school mathematics up to the end of Year 11 (SACE Stage 1 Mathematics). Although MathsStart is no longer offered, the resources are still available for personal study/ revision, and is useful for the purposes of reconsidering the content taught in MathsTrack as it gives a broader context for the content, and the required knowledge.

### Student Cohort

According to the staff at the maths learning centre, the vast majority of students enrolling in their bridging courses are aiming to end up in one of three places:

- Studying a tertiary degree at The University of Adelaide (approximately $60$–$70$% of bridging courses at any one time),
- Studying a tertiary degree at James Cook University, or
- In the defence forces.

Only about a single student will not fit into any of the three categories above at any one time, so thinking of this as being the complete cohort of students is fairly close to being accurate. The distribution within these categories can also be broken down and the most common trends considered:

- Of the students aiming to enrol in a tertiary degree at the University of Adelaide, about $50$% are aiming to study something in the Faculty of Engineering, Computer and Mathematical Sciences (ECMS) (i.e. Engineering, Mathematics, Computer Science, etc.), and about $10$% are aiming to study something in the sciences, often veterinary science or oral health.
- Of the students aiming to enrol at James Cook University, most are aiming to enrol in medical degrees, with some interested in marine biology or veterinary science — broadly biological science in large.
- Of the students aiming to enlist in the defence forces, the majority of those enrolled in the bridging courses are doing so to meet their pre-requisite mathematics knowledge criteria for air force pilot training.

The diversity of goals amongst the student cohort of the bridging courses makes it challenging to tailor the content of the bridging courses to the students particular needs without substantial resources unavailable to the MLC at this time. With this in mind, the focus of the curriculum mapping presented in Chapter 4 will be on aligning the content of the bridging courses to the senior high school mathematics curriculum, as this is an achievable goal. That said, being aware of this diversity and taking it into account when making decisions and structuring the content is still critical, and will be revisited in Chapter 5.

### Assessment

The assessment in the bridging courses is self-paced, meaning that students access the resources themselves when they are ready, there are no classes or timetables, students can start at any time and can take as long as they need. This should not be understated, as it's impact on reducing test-anxiety and making mathematics education more accessible to people to whom it might otherwise feel inaccessible is absolutely crucial.

## 1.2 The Curriculum-Assessment Framework

When considering improvements to the bridging courses, one of the key factors is curriculum — content. The content of the courses is one of the things that can be most readily modified, and naively one might think that in this way, improvements to the course could be easily implemented. However, as discussed by (Mohandas, Wei, & Keeves, 2003) and will be explored in much more depth in Chapter 3, content does not live alone, and cannot be considered independently of the broader environment. Specifically, there are bidirectional relationships between curriculum (content), learning experiences (the experiences students have while learning), and evaluation (an umbrella term containing several meaningfully different concepts that will be discussed below), as shown in the curriculum triangle of Tyler (1949) in Figure 1.1. To give some simplified examples:

- A test is informed by the content as it must not contain content not taught in the course, and might aim to cover all of (or most of) the content taught in the course. But the results of the test, or even the fact that there is a test at all, can (and should) also influence decisions about what content to include in the course in the first place.
- The learning experiences that students have depends on the content, obviously. But in the other direction, student's experiences should also inform decisions about curriculum.
- If students struggle with a specific concept in a test, perhaps the learning experiences they have surrounding that concept should be re-examined. On the other hand, if the learning experiences students have surrounding a particular concept are framed in a particular way, then the way those concepts are tested should take that into account.

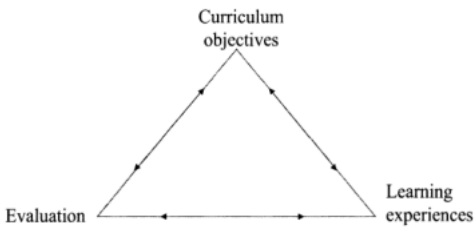


Figure 1.1: The curriculum triangle of Tyler (1949) as visualised in Figure 1 of Mohandas et al. (2003)

Although each of these areas can be considered individually to some degree, it is important that when decisions are made that the bigger picture with all the interactions is taken into account. Mohandas et al. (2003) also make the good point that Evaluation needs to be thought of more granularly, as different forms of evaluation serve very different purposes, and very different roles in both the learning and teaching processes. They expand the curriculum triangle to the "curriculum-evaluation diamond" shown in Figure 1.2, which is of course no diamond at all, but rather an triangular bi-pyramid with its axis of $\frac{2\pi}{3}$ rotation symmetry representing the fully connected graph of $5$ nodes. Mathematical pedantry aside, Mohandas et al. (2003) make the important point that two critical changes should be made to the curriculum triangle model:

- (Student) assessment should be distinguished from evaluation and accountability (Mohandas et al. (2003) also present definitions for each of these terms in order to help distinguish them, of which a concise summary will be included below).
- Standards of performance and how they interact with the other elements play an important role.

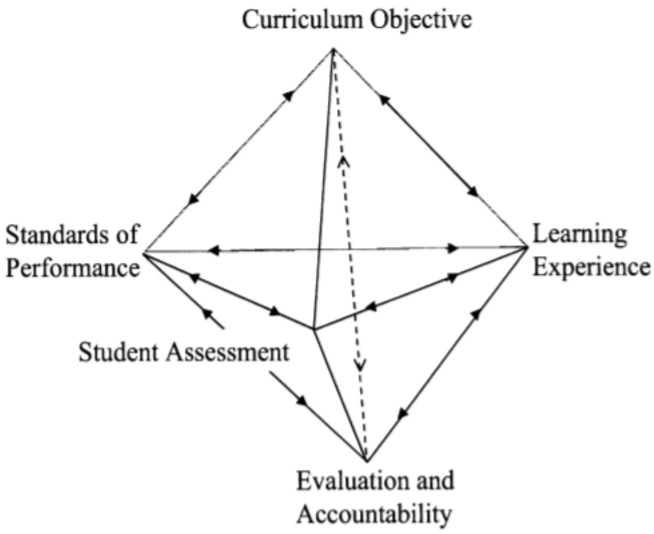


Figure 1.2: The curriculum-assessment diamond as shown in Figure 2 of Mohandas et al. (2003)

The definitions of the terms "assessment", "evaluation", and "accountability" according to Mohandas et al. (2003) and hence as used in Figure 1.2 are useful in order to distinguish between these concepts, and very concisely can be summarised as:

- Assessment usually refers to individual students, and it's goal is generally to understand what/ how much learning has occurred. It can be performed by educators, or importantly by students themselves, and it can be formal (tests, exams, assignments) or informal (discussion, practice questions, etc.)
- Evaluation usually refers to some a decision making process: A university evaluates a student to decide if they should be allowed to enrol in a particular degree, for example.
- Accountability usually refers to a responsibility held by an educator or organisation, and is often associated to reporting to some stakeholders.

All three of these terms are important but serve very different roles in the context of improving the bridging courses. Assessment is the most important, and in particular student self-assessment as will be discussed in more detail in Chapter 3, but also in terms of the self-pacing of the assignments in the bridging courses, which act as all three: assessment (because of the feedback cycle used to help students through the assignments, they are initially used to assess the learning that has occurred and use this information to inform students about how to proceed through feedback), evaluation because the assignments are used to gate students from completion of the courses, and accountability to ensure students are at the required level of knowledge and satisfy the responsibility of ensuring they are adequately prepared for their future studies.

There are two key concepts in which the standards of performance are important. First, because it is important to establish standards based assessment in which students are assessed against fixed standards and not against each other (this is widely accepted in the educational literature). Secondly, these fixed standards should be set in clear objectives in mind. It is in this sense that the standards of performance are quite complicated to nail down in the context of the bridging courses. Typically at a university level, standards of performance will be determined by things such as industry standards (for example studying an engineering degree, the industry standards for engineers will apply). Ultimately, the skills and knowledge required of students completing a degree will be determined by the skills and knowledge that the industry hiring those students needs graduates to have. However, with the students in the bridging courses going in so many different directions this is difficult to determine. In Chapter 4 we discuss some of the most common first year subjects students aim to enrol in (which are common to many different degrees), but ultimately as the bridging courses usually fit into the secondary-tertiary transition, i.e. the evaluation of students for university entry, the primary basis for the standards of performance is the senior high school curriculum, which is discussed in detail in Chapter 4.

This thesis can be thought of as consisting of two broad avenues of research, focusing on different parts of the curriculum-assessment 'diamond' shown in Figure 1.2:

- Chapter 4 explores the curriculum and standards of performance part of the 'diamond' by mapping the national and state curricula to the current curriculum of the bridging courses, while discussing the various relevant standards of performance to contextualise the advantages and disadvantages of including or excluding particular sections of these curricula.

- Chapter 3 explores the existing literature in order to make recommendations around what learning experiences and assessment methodologies are needed in order to facilitate the learning prescribed my the curriculum discussion.

Naturally, and as supported by the 'diamond' framework of Mohandas et al. (2003), neither of these two approaches to improvement of the bridging courses would be successful in isolation, but rather by taking into account both in unison real improvements could be achieved.

# Chapter 2

# Methodology

In this chapter the methodology employed when performing the research presented in the remainder of this thesis is described. There are two main bodies of work in this thesis:

- A literature review, presented in Chapter 3, and
- A curriculum mapping, presented in Chapter 4.

Each of these bodies of work was completed employing a non-overlapping methodology, and so the methodology for each will be described separately.

## 2.1 Literature Review

The initial phase of the literature review was performed in an iterative process which given a list of sources (primarily academic papers) involved reading the list of sources and generating a new list repeatedly. In the first iteration, some of the most relevant papers identified included the work of Nicholas and Rylands (2015), Gordon and Nicholas (2013b), and Johnson and O'Keeffe (2016), and these where found by using search engines, including google scholar and the UofA library search with terms such as "mathematics bridging courses" in order to identify several recent, directly relevant, key papers to start with. The iterative process then involved reading the current list of sources, taking notes and quotations for later use, and compiling a new list of sources by:

- Noting relevant references used in the current list,
- Papers referencing these papers (using "cited by" functionality of search engines),
- Additional papers identified by use of search engines for newly identified key terms, such as "adult-education", "maths anxiety", etc.

Some of the particularly relevant papers which came up in the second and third iterations of this process included the work of Galligan and Taylor (2008) ,Irwin, Baker, and Carter (2018), and Ramirez, Shaw, and Maloney (2018) respectively. This iterative process was performed until the same papers kept coming up more and more frequently, which only took approximately four or five iterations. Then, the notes and quotations made while reading these references where reviewed, and synthesised into a coherent discussion, something akin to a "systematic review", although the individual keywords and search phrases are not explicitly reported, all statements made are traceable (accurately cited). It is important to acknowledge the inherent bias involved in this (and any) literature review methodology. One of the future research directions for this work would be to further expand the literature review to be more comprehensive and less biased in a more systematic way, but the intention of this work was not to provide a comprehensive systematic review, rather a starting point for one to begin from. This starting point is presented in Chapter 3.

## 2.2 Curriculum Mapping

The curriculum mapping was performed by first establishing the levels of detail of interest, and terminology for these levels of detail. Specifically, there are two levels of detail at which the curriculum mapping is performed: the topic level, and the key concept level. It is important to note that in this work these terms "topic" and "key concept" are used to have the very precise meaning of referring to these two levels of detail. These are discussed in more detail in Chapter 4, but very breifly each curriculum is broken up into approximately $12$–$24$ topics, with each of these topics including typically $6$-$12$ key concepts each.

The first phase of the curriculum mapping methodology was to summarise the key concepts in a concise way such that in the later phases this summary could be used to guide alignment between curricula. This essentially boils down to the generation of the table presented in Appendix A. This was essentially a "document analysis", and involved carefully reading the curriculum documents associated to each curriculum, and summarising the key concepts in each topic therein. There are three curricula analysed in this way, and the details of how this phase was performed for each follow:

- For the AC, the curriculum is presented on their website, and both Senior Mathematical Methods and Senior Specialist Mathematics where considered (accessed between February and May 2019). Each of these subjects is broken down into 4 units, and each unit has three components: a description, learning outcomes, and content descriptions. The content descriptions section for each unit is split into three topics, each of these topics corresponds to a topic in our level of detail terminology. The material under these topics in the content descriptions section of each unit was the focus for this curriculum mapping, and this is the material that was read carefully and summarised to generate the key concept list in Appendix A.

- For SACE, the Subject Outline (for teaching in 2019) document was retrieved from the SACE website for each of the three relevant subjects: Stage 1 Mathematics, Stage 2 Mathematical Methods, and Stage 2 Specialist Mathematics. In each of these documents, the "LEARNING SCOPE AND REQUIREMENTS" section contains a summary of the curriculum by topic, and each of these topics correspond to a topic in our level of detail terminology. Within each topic, SACE often has subtopics, but we do not consider this level of detail in this curriculum mapping, instead treating each entire topic as a whole. Within each topic, the left-hand column "Key questions and key concepts" was read carefully and summarised to generate the key concept list in Appendix A. As discussed in Chapter 4, the focus of this curriculum mapping is primarily on the content itself, rather than the surrounding concepts involved in how the content is taught (which is more what the right-hand column, "Considerations for developing teaching and learning strategies" is relevant too).

- For the bridging courses, the content is available on their website in the form of a number of booklets for each of the courses: MathsStart and MathsTrack. Each of these booklets will constitute a topic in our level of detail terminology, and these entire booklets where carefully read and summarised to generate the key concept list in Appendix A.

Once the first draft of the key concept summary presented in Appendix A was completed, two mappings where produced: one at the topic level, and one at the key-concept level. The topic level mapping, shown in Figure 4.1, was produced by comparing the broad concepts covered in the topics, with little concern for the details involved with particular alignment of key concepts. For example an "introductory calculus" topic would be mapped to another "introductory calculus" topic, even if a specific concept such as anti-derivatives is introduced in one but not the other. The purpose of this mapping is to provide a high-level view of the mapping between the curricula in order to help structure the more detailed discussion of key concept alignment. The key concept alignment was then performed by going topic by topic, and aligning every single key concept listed in Appendix A, then in any mismatching cases, referring back to the original curriculum document to check for mistakes and validate any conclusions made. This mapping is obviously too complex to be able to meaningfully represent it graphically, and so instead the conclusions thereof are presented in the form of discussion in Chapter 4. No major mistakes where discovered in this process, but some small modifications where made to Appendix A all of which had to do simply with harmonising terminology used. For example, "slope of a line" versus "gradient of a line", etc. This key concept level mapping was also used to make adjustments to the topic level mapping shown in Figure 4.1. No major changes where made, but single key-concept links where added as dashed lines as a result of the key concept mapping.

Although it was not part of the initial intent, it became apparent in the process of completing the mappings described above that particularly due to the very different structure of the curricula it would be useful to add another level of detail in which topics where grouped under broad areas of mathematics, and to reproduce another version of Figure 4.1 in which the topics where rearranged into these broad areas, so this was done producing Figure 4.2.

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# Chapter 3

# Literature Review

## 3.1 Introduction

In this chapter the literature surrounding mathematics bridging courses is explored both in an Australian context as well as internationally. The literature reviewed also extends into some directly relevant areas such as general perceptions of mathematics, the secondary-tertiary transition more broadly than just in the context of mathematics education, relevant frameworks that have been proposed, and some of the key areas that prevent students from being successful such as maths anxiety.

Remembering the purpose statement of this thesis, and the clarifying secondary questions that it raises, it is interesting to note that these questions are by no means new questions, although they do not necessarily have any consensus on how to answer them. In particular, Poladian and Nicholas (2013) offer an insightful discussion of two key (unanswered) questions within bridging mathematics posed by Galligan and Taylor (2008):

- How is success defined in bridging mathematics activities?
- Are successful bridging mathematics students successful university students?

which Poladian and Nicholas (2013) address with the following comments:

- "there are inherent difficulties in defining and measuring success in bridging courses. Godden and Pegg (1993) suggest that formal evaluation of bridging mathematics programs may be contrary to the aims of the programs, and undermine their major strengths of flexibility and student-centred approach. They argue that traditional evaluative techniques are 'just not possible' and 'risk losing the essence of the support and assistance so necessary for these students'. "
- "internationally, bridging mathematics programs have been shown to be highly effective at resolving skill deficiencies for some students (Kajander & Lovric, 2005; Bahr, 2008). In a large US study, (Bahr, 2008, p.442) found that 'remediation has the capacity to fully resolve the academic disadvantage of math skill deficiency' for the quarter of students who 'remediated successfully', but the likelihood of successful remediation declined sharply as the 'depth of remedial need' increased. The latter finding echoes (Wood, 2001)'s remark that bridging programs do not work for very mathematically weak students."

respectively.

## 3.2 "The Mathematics Problem"

"The mathematics problem" is a term originally coined by Howson et al. (1995) but that has continued to be relevant to the present day, receiving even greater attention and research in recent times. It refers to the trend of declining interest and participation of final year high school students in mathematics. It also refers to the carry-over effects this has on the success of students in tertiary education (both in mathematics, but also notably in other areas). "The mathematics problem" is a term now used also to describe the downstream impacts these trends have on the economy: modern industries are dominated by a need for mathematically skilled graduates (engineering, science, technology, ...), but the importance of mathematics in these fields is often overlooked from the general populations perspective (King & Cattlin, 2015; Gordon & Nicholas, 2013b).

Barrington and Evans (2016) shows that in Australia, although the number of both advanced and intermediate mathematics year 12 students was increasing over the ten years from 2006 to 2015 (as the overall population of total year 12 students increased), the percentage participation in these subjects steadily declined. James (2019) updates the figures of Barrington and Evans (2016) with data up to 2017, showing a continuation of the same steady trend. These reports also highlight the significant gender gap that exists in mathematics participation in final year high school students. The gender gap is more dramatic in advanced level mathematics than in the intermediate level, with 37.8% of advanced mathematics year 12 students identifying as female, especially when considering that 51.8% of year 12 students of that year where female. 2017 saw a significant jump in intermediate level mathematics participation by female students, with there being more female students than males for the first time in recorded history (James, 2019). The gender gap in mathematics education is a significant issue that needs to be taken into account when considering university mathematics entry, particularly as the gap is most pronounced in the advanced level subjects which are targeted at university entry. It is an issue recognised by the Australian Mathematical Sciences Institute (AMSI), who have committed significant resources towards programs intended to address this inequity over the past two decades in particular. Perhaps the up tick in female student participation in intermediate level mathematics in 2017 could be partly attributed to some of these programs, such as the CHOOSE**MATHS** project. Brown (2009) gives a shocking wider-view picture of this overall trend, specifically that the proportion of year 12 students studying intermediate or advanced level mathematics has declined by 22% and 27% respectively from 1995 to 2007.

Amongst other reasons, this decline in participation in mathematics is a problem in Australia because mathematical skills are essential to just about all the key future industries (Croft, Harrison, & Robinson, 2009), and hence the Australian economy. The key economic importance of mathematics is widely acknowledged amongst the academy and industry, but it's importance is often overlooked and difficult to communicate to the wider community because of it's indirect importance through what are perceived to be other fields: engineering, science, etc. all of which require a deep level of mathematical skills, but aren't associated to mathematics in the general populations view. Thomas, Muchatuta, and Wood (2009) argues that one of the most influential factors in the declining participation in mathematics is the "community's perception that mathematics is not useful in the marketplace". Gordon and Nicholas (2013b) go on to emphasise the carry-on effects of negative community perceptions of mathematics leading to high school students choosing not to participate in higher-level mathematics impacting on not only their success in university, but on whether they continue to study mathematics at all. This obviously has implications for mathematics bridging courses at universities — students who previously had de-prioritised their own mathematics education in favour of pursuing these other fields, notably engineering for example, will often turn to bridging courses when they realise the importance of mathematics in being successful in the field of their interest.

Observation, concern surrounding, and research of this decline in mathematics participation in senior high schools are not limited to Australia (Hourigan & O'Donoghue, 2007; Hoyles, Newman, & Noss, 2001). Hoyles et al. (2001), as well as Luk (2005) further connect this trend to another: the apparent divergence of content (curriculum) between senior secondary and tertiary education. This divergence of curriculum is a point that will be explored extensively in Chapter 4. In a landmark study, Kajander and Lovric (2005) identified a gap between secondary and tertiary mathematics education in Canada. In the United Kingdom Tariq (2002) noted a decline in numeracy skills among first-year bioscience students. This trend is neither limited to Australia, nor new. Universities around the world have recognised this continuing problem for some time, but opinions on how to address it vary. Robinson (2003) suggested that the standard for high school mathematics should be raised, but even if there where consensus amongst the academy that this was appropriate (which there is not), this is beyond the power of universities to control (although, the setting of pre-requisites is a topic that will be explored in more detail below). Within the power of universities to implement are solutions such as to introduce "remedial mathematics" into first-year teaching programmes as highlighted by Kitchen (1999). More recently, as Moses et al. (2011) suggest, universities have been increasing their reliance on "advanced and targeted preparatory programmes" — i.e. bridging courses. As an example of this from outside Australia, Faulkner, Hannigan, and Gill (2010) note that at the University of Limerick in Ireland

> "there has been a 20–25% reduction in students attending their first service mathematics lecture, a 12–16% reduction in the number of students entering service mathematics modules with higher level mathematics and an 8–12% increase in the number of non-standard students. Such changes place additional pressure on support services like MLCs whose primary function is to provide the necessary and appropriate support to all university students." (Johnson & O'Keeffe, 2016)

## 3.3 The Secondary-Tertiary Education Transition

A key step we are interested in from the perspective of bridging courses is university entry, or more broadly: the transition from secondary to tertiary education. It may seem obvious that students engagement and performance in mathematics in secondary education is a strong predictor of their success in tertiary mathematics education, but the exact relationship has some important subtleties. Specifically, it has been shown that the level of mathematics completed in high school (advanced, intermediate, etc.) is substantially worse at predicting success in tertiary mathematics education than when combined with the level of achievement in secondary school (Kajander & Lovric, 2005; Nicholas, Poladian, Mack, & Wilson, 2015). Students having completed a lower level of mathematics in secondary school to a higher degree of achievement can in some cases have a higher chance of success in tertiary education than students who completed a higher level of mathematics in secondary school but to a lower level of achievement. Although this might seem intuitive, it is not entirely obvious when looking at it in terms of content — curriculum — alone. It should not be understated that although it has been shown quite clearly that the effect of bridging courses is smaller than the effect of high school engagement in mathematics education, that bridging courses have been shown to have a substantial effect nonetheless, and even more importantly have been shown to fill a critical gap in addressing student needs (MacGillivray, 2009). This is important to acknowledge, and will come into the discussion surrounding university entry requirements below, but engaging students in mathematics in secondary school is beyond the scope of this work, although it is clearly a very important aspect of "the mathematics problem". For now, we consider that one of the roles of bridging courses is to make tertiary mathematics education accessible to all students, including those that where disengaged with mathematics in high school and therefore are in particularly high risk in tertiary education.

## Rite of Passage Model

Very little has been done in terms of developing educational frameworks for understanding the secondary-tertiary transition more systematically, but Clark and Lovric (2008) suggest using the pre-existing and well-understood literature surrounding the concept of a 'rite of passage' from anthropology and culture studied (relating concepts such as culture shock) to help structure our thinking about about the difficulties and evaluating strategies to address difficulties with the secondary-tertiary transition. Clark and Lovric (2008) propose using the seminal work of Arnold van Gennep and thinking about a "life crisis" event as consisting of three phrases: separation, liminal, and incorporation. One of key and important implications this perspective has is that this transition does not only involve difficulty for the individuals (students), but the broader community (their family, teachers, etc.). The wider communities negative perceptions of mathematics are widely acknowledged to have a substantial effect on students attitudes, and hence success (King & Cattlin, 2015; Gordon & Nicholas, 2013b), and it is important to take this into account. One of the immediate consequences the "rite-of-passage" model implies is that "It is normal to feel discomfort during a rite of passage but much easier to deal with if this is expected." (Clark & Lovric, 2008). This is a key take-away: setting clear expectations is critical for students to be able to cope with the difficulty of this transition, they need to know that it will be difficult, so they can expect that difficulty and come into it prepared.

NOTE: There are also some suggestions made by Clark and Lovric (2008) that I disagree with. Specifically, that we abandon imprecise language and descriptions of concepts, in favour of rigorous explicit language. I should probably add some discussion of this here.

None-the-less, the "rite-of-passage" model of Clark and Lovric (2008) aligns well with the broader literature and research surrounding bridging courses and the secondary-tertiary education transition. Specifically, the concept of being socially isolated and needing to adapt to a new environment with different expectations and social norms is reflected widely in the academic writing. Gordon and Nicholas (2013b) discuses how one of the key valuable experiences students got out of the bridging courses at the University of Sydney was the interactions with peers and teachers. This experience is supported by literature discussing the importance of social and interactive learning as a formative element of early university experience that is highly predictive of retention (Peat, Dalziel, & Grant, 2001; Trotter & Roberts, 2006) particularly for students whose family or friends are for example from a "working class" background (Leese, 2010), or from a cultural background less familiar with the social norms and expectations associated with university education. In particular, self-motivation and independent learning are expectations that consistently come up as being shock factors for students transitioning from secondary to tertiary education (Murtagh, 2010).

## Assumed Knowledge and Conditions of Entry

Contributing to the problem of expectations not being set explicitly, in recent years Australia universities have been moving away from prerequisites for entry towards a "assumed knowledge" approach. What this means is that instead of requiring students to have completed certain subjects in high school in order to allow them to enrol in a course at university, they instead put the content from those subjects under "assumed knowledge", allow students to enrol in the subject even if they have not completed the high school subject, and put the onus for having that knowledge on the students. That is how the universities see it, anyway. How the students see it is quite different, as demonstrated by the work of Gordon and Nicholas (2015), who show substantial variance in student perceptions of "'assumed knowledge' ranging from perceiving it as vague and pointless 'stuff' to a cohesive body of foundational knowledge for tertiary study". One of the consequences of this is the increasing under-preparedness of first year undergraduate students.

The issue of entry requirements into university and prerequisites being moved into "assumed knowledge" is an even more complex issue than it might at first appear. Varsavsky (2010) discuss how in Australia the way university entry is managed may in fact be contributing to the problem of low participation in higher level senior high school mathematics. Specifically, the absence of prerequisite subjects in many universities means the only condition of entry to university is the achievement of a sufficiently high "tertiary entrance rank", a score calculated based on achievement in all final high school year subjects, with some adjustments for the combination of difficulties of the subjects. A substantial amount of effort is gone too by final year high school students, teachers, and counsellors to optimise students performance on this tertiary entrance rank through very careful choice of which subjects to take in their final year of high school. Often this will result in creating a tension between achieving a high tertiary entrance rank and hence being accepted into university, and having the required knowledge to be successful in university because the subjects chosen are not those containing the content relevant to the degree the student is enrolling in (Gordon & Nicholas, 2013a; Poladian & Nicholas, 2013). This is of course an issue that generalises far beyond mathematics, but to every area of study. Gordon and Nicholas (2013b) claim that: "the major reasons for students taking lower levels of mathematics in senior year(s), or dropping mathematics, include finding enough time for non-mathematics subjects, confidence in mathematical capability, advice and maximising potential ranking for university admission". Rylands and Coady (2009) demonstrated that what a student studied in senior high school predicted their performance at university, whole their tertiary entrance rank did not. The result in the bridging course literature that although bridging courses can help, their effect cannot compare with engagement in high school is a result that has been reproduced many times in the literature across many countries (Kajander & Lovric, 2005; Nicholas et al., 2015; Tariq, 2002). This is likely, as suggested by Kajander and Lovric (2005), due to the time-period typically involved. A bridging course is usually a short preparatory course covered in an interim before beginning tertiary study, while high school engagement is a learning and teaching experience spanning several years. Despite Australia's Chief Scientist recommending moving back to pre-requisites (Chubb, Findlay, Du, Burmester, & Kusa, 2012), there is no sign of this being on the table: the commercial aspect of universities demands increased enrolment of students, and that means relaxing entry conditions.

NOTE: (King & Cattlin, 2015) has more to say on this topic, I should review that paper and maybe adjust/ add some more discussion here.

## 3.4 Maths Anxiety

### Why is Maths Anxiety Important?

In 2012 Programme for International Student Assessment (PISA) reported that across Organisation for Economic Co-operation and Development (OECD) countries, 60% of students "worry about getting poor grades in mathematics" (OECD, 2013), and over 30% of 15 year old students "get very nervous doing mathematics problems". This impacts on students' academic performance, but at a more fundamental level also impacts on their subject choice in the first place. This has been recognised widely as being an ongoing issue for many decades, with literature discussing maths anxiety dating back as far as the 1950's (Dreger & Aiken Jr, 1957). Maths anxiety is also a community issue — parents and teachers also suffer from maths anxiety and hence both normalise the behaviour for students and actively create a new generation of maths-anxious people. As though the problem was not already severe enough, recent research has shown that students with a high level of maths anxiety often experience the anticipation of a maths task literally as visceral pain (Lyons & Beilock, 2012). Maths anxiety is a serious well-being issue, beyond being simply an academic and economic issue.

Beyond well-being however the maths anxiety-performance connection has also been robustly and repeatedly demonstrated, this this connection is particularly relevant when considering students coming into bridging courses. Students enrolling in bridging courses are more likely to have performed poorly in high school and given the prevalence of maths anxiety and the strength of the maths anxiety-performance link, are more likely to suffer from maths anxiety. This inference is supported by the survey studies of bridging course students by Nicholas, Gordon and Polodian. One example of this is highlighted by Foley et al. (2017) who juxtaposes the internationally rising demand for STEM professionals with the negative correlation between maths anxiety and performance shown in the 2012 PISA report (OECD, 2013) to highlight the relevance of addressing maths anxiety in filling this demand, aligning with 'the mathematics problem" discussed earlier in this chapter. The relationship between maths anxiety and maths-qualified professionals in the workforce is supported throughout the literature: when a student has low self-concept (correlated with high maths anxiety), they will tend not to enrol in maths beyond the minimum requirements for graduation (Ashcraft, Krause, & Hopko, 2007), and students affect towards maths can predict their university major (LeFevre, Kulak, & Heymans, 1992). Beyond this example, the list of stakeholders in a students academic success in maths goes on and on: parents; the student's themselves; schools (which are often funded based on the results of standardised testing such as National Assessment Program — Literacy and Numeracy (NAPLAN)), and teachers amongst them. From the perspective of bridging courses, this link is important because A) it motivates supporting these maths anxious students to pursue a tertiary mathematics educations, but also B) because industry is an important stakeholder in tertiary education, including bridging courses.

## Maths Anxiety as Distinct from General Anxiety

The existence of maths anxiety as "emotional disturbances in the presence of mathematics" has been noted as early as the 1950's, Dreger and Aiken Jr (1957) even postulated that what he tentatively designated "Number Anxiety" and later became to be known as Maths Anxiety could be a distinct syndrome from general anxiety. Later the landmark meta-study of Hembree (1990) supported this hypothesis, showing a correlation of only $0.38$ between maths anxiety and general anxiety. In more recent times, this hypothesis has also been confirmed by Young, Wu, and Menon (2012) using functional magnetic resonance imaging (fMRI) to show that the brain activity in a person experiencing maths anxiety is measurably distinct from that in a person suffering general anxiety. These later studies, as well as the the work of Kazelskis et al. (2000) and more, have also delineated maths anxiety from test anxiety, and these different anxieties existing as meaningfully distinct constructs is now quite well accepted. For more on the history of maths anxiety, Suárez-Pellicioni, Núñez-Peña, and Colomé (2016) offers a more detailed review.

## Frameworks for Understanding Maths Anxiety

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Very little research has been conducted on maths anxiety in isolation, although some of the research that has shows some interesting results (Young et al., 2012; Lyons & Beilock, 2012). Instead, the bulk of the literature is investigates the maths anxiety-performance connection. Specifically, there are two popular theories being pursued and I will use the terminology of Ramirez et al. (2018) to describe them: the "Disruption Account" and the "Reduced Competency Account". Ramirez et al. (2018) make a convincing argument that although it might seem at initial glance that these two theories contradict one another, they are not actually mutually exclusive and in actual fact are compatible with each other. Ramirez et al. (2018) suggests a third "Interpretation Account" which encapsulates observations made by both lines of research into a single framework, see Figure 3.1.

First, a brief summary of the two popular existing theories. The "Disruption Account", spearheaded by the work of Ashcraft et al., is focused on the concept of working memory (Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007). Specifically, Ashcraft et al. claim that anxiety takes up working memory capacity in a student's mind, which prevents them from using that working memory to complete maths tasks and in that way impacts on their performance in said tasks. In seeming contrast, the "Reduced Competency Account" proposes the opposite causality: that low maths

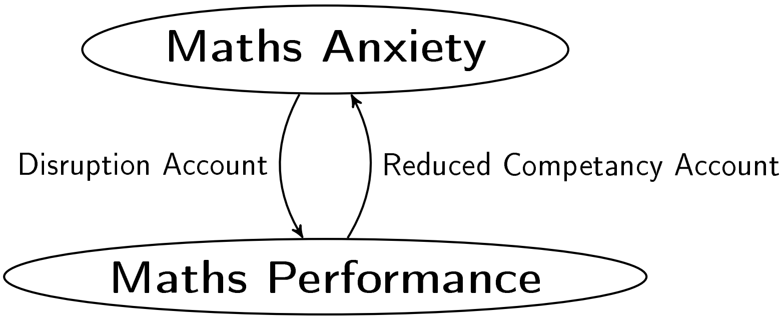


Figure 3.1: The Interpretation Account of Ramirez et al. (2018) for the maths anxiety-performance link showing how the Disruption Account and the Reduced Competency Account can be compatible.

ability leads to poor performance and affectively negative experiences of maths, which leads to an association between maths and negative affect to be formed in a students mind, i.e. causing maths anxiety to develop. There is a substantial body of work to support this hypothesis, including the landmark meta-analysis of Hembree (1990) and the longitudinal study of Ma and Xu (2004) which found that although past maths anxiety was correlated with future maths performance it was a small effect, while past maths performance had a strong effect on future maths anxiety.

## Complexities in Finding Effective Interventions

The models for the causation structure in the maths anxiety-performance connection presented above are of course broad oversimplifications of what is an incredibly complex and interconnected topic. They do however imply very different approaches for intervention. The "Reduced Competency Account" suggests interventions that boost maths performance and hence allow students to experience success in maths should also help to reduce maths anxiety. The results of Supekar, Iuculano, Chen, and Menon (2015) seem to support this hypothesis as when students are given an intensive 8-week tutoring program to boost their maths skills, this is associated to a reduction in maths anxiety. The earlier work by Faust (1996) further supports via an anxiety-complexity effect in which low and high maths anxiety groups performed similarly on low complexity problems, but in high complexity problems the high anxiety groups performance was impacted. However, Jansen et al. (2013) showed that it is not necessarily as simple as these studies suggest — they showed that when students experience success they attempt more problems and perform better but that when the confounding factors are modelled accurately enough it can be shown that the improved performance is almost completely predicted by the number of problems attempted, not their experience of success. Furthermore, Jansen et al. (2013) showed that the level of maths anxiety was not affected in a significant way when experience of success was varied, which raises a lot of interesting but as yet unanswered questions.

In contrast to the "Reduced Competency Account", the "Disruption Account" implies that maths anxiety itself should be the target of interventions, and that if the maths anxiety is successfully reduced this will result in more free working memory and hence boost student performance. Following this line of reasoning, Park, Ramirez, and Beilock (2014) demonstrate a direct and successful attempt at this type of intervention where they used expressive writing exercises to guide students self-perceived narratives about their experiences with maths and reduce their maths anxiety. Notably the approach of Park et al. (2014) is similar to successful approaches to the treatment of clinical anxiety disorders (McNally, 2007; Becker, Darius, & Schaumberg, 2007; Foa et al., 2005). Another approach drawn from the clinical psychology literature is that of reappraisal (Jamieson, Peters, Greenwood, & Altose, 2016). In this approach stress is re-conceptualised as a coping tool, a physiological response the purpose of which is to improve physical and mental performance in response to a challenge to be overcome. By reconceptualising stress in this productive way, rather than as a symptom of exposure to something to be feared and avoided, this approach does not necessarily aim to directly reduce stress and anxiety levels in students, but instead allow them to perform despite the stress. This change in the perspective of stress is also very much in line with other parts of the literature discussing "productive struggle" (Wang et al., 2015), including some of the discussion of the "Interpretation Account" by Ramirez et al. (2018).

Intrinsic motivation has been shown to have an important mediating role in the relationship between Maths anxiety and performance (Wang et al., 2015), and serves as a crucial link in the "productive struggle" mindset. This reconceptualisation to a 'productive struggle' model is not an isolated occurrence in the literature. Lin-Siegler, Ahn, Chen, Fang, and Luna-Lucero (2016) tells stories about the struggles experienced by famous scientists to students in an attempt to help normalise the concept of productive struggle in the context of a science classroom. Hiebert and Grouws (2007) similarly provide a discussion on the importance of the "productive struggle" concept but specifically in a maths context.

One of the implications of the "Interpretation Account" proposed by Ramirez et al. (2018)is that if an intervention targets only one causal direction in the cycle of Figure 3.1, the cycle is likely to re-establish and negate any potential long-term effects. However the research of such long-term effects is very limited, and several authors have discussed the need for further research into this (Suárez-Pellicioni et al., 2016; Chang & Beilock, 2016). None-the-less, the limited literature that does exist discussing this seems to agree that the approach most likely to be successful is one that targets both causal directions simultaneously, i.e. that focuses on both improving maths performance and reducing maths anxiety, simultaneously. This is very much in line with several of the other frameworks discussed in this work, not least of ll the curriculum-assessment diamond of Figure 1.2 in which it is emphasised that while content is important, it is only equally as important as the learning experience. If students are not taught the required content they will not succeed, but if they are taught the required content and their experience of learning it is filled with anxiety and stress, they will also not succeed. The path to achieving student success is clearly one in which both aspects are considered.

## 3.5 Self-Efficacy

Much of the early research into self-efficacy has been structured based on the "social cognitive theory" of Bandura, so it seems appropriate to begin with a quote. Bandura (1997, pg391) defines self-efficacy as

> "people's judgement of their capabilities to organize and execute courses

> of action required to attain designated types of performance"

The connection between student's mathematical self-efficacy and their success in university preparatory mathematics courses has been well established in the literature (Burton, 1987; Klinger, 2006, 2011).

To quote Johnson and O'Keeffe (2016):

> "Self-efficacy is vital among all students but particularly among adult learners as an individual's beliefs of self-capability has been shown to affect motivation, performance, achievement, effort, willingness to persist with a task, as well as the anxiety they experience (Bandura, 1997; Pajares & Miller, 1994; Pajares, 1996; Pajares & Miller, 1997; Pajares & Graham, 1999). Woodley (1987) (cited in (McGivney, 1996)) noted that the main personal factors that contribute to dropout are: self-perception, being disorganised, not having sufficient study skills and lacking in self-confidence. This suggests that an individual's self-efficacy plays a role in their decision with regard to dropping out.
>
> (Hackett & Betz, 1989) and (Pajares & Miller, 1994) and Pajares and Miller (1995) also found that self-efficacy can have an impact on career choice. In these studies, it was found that mathematical self-efficacy is a stronger predictor of students' mathematical interest and choice of degree programmes than either prior mathematical achievement or mathematical outcome expectations. Self-efficacy also influences how often mathematics is used, as well as an individual's willingness to pursue advanced work in mathematics, and even the choice of prospective occupations (Dutton and Dutton 1991). Engineers Ireland (2010) highlight that this avoidance of mathematics, and mathematics-related courses, at university will eventually prove detrimental when attempting to build a knowledge economy. This point was also stressed decades before by (Hembree, 1990, pg34) when he stated that 'when otherwise capable students avoid the study of mathematics, their options regarding careers are reduced, eroding the country's resource base in science and technology'."

It is also well established that mathematical self-efficacy is strongly correlated to a combination of current knowledge/skills and current performance, with females and those who have not studied in a longer period of time generally having lower self-efficacy (C. S. Carmichael, Dunn, & Taylor, 2006; C. Carmichael & Taylor, 2005). In a group of so-called "adult learners", Klinger (2006) confirmed a result from several previous studies and showed that although the negative perceptions of mathematics widely held by the general population and demonstrated to negatively impact on mathematics performance where represented strongly on initial enrolment into a mathematics bridging course, that these negative perceptions changed dramatically during the course. The conclusion being that although yes, these negative perceptions of mathematics are highly predictive of performance, they can also be substantially influenced by early learning experiences, and should certainly not be thought of as fixed variables. In a later study, Klinger (2008) replicated a similar study but across several disciplines of study and showed that arts/humanities had substantially lower mathematical self-efficacy and more negative perceptions towards mathematics than the science students. Notably, Klinger (2008) also showed a strong link between their quantitative results around mathematical self-efficacy/

negative perceptions of mathematics and gender — with female students scoring worse than males. All this research only further supports that weak mathematical skills cannot be addressed with content alone, but that the students negative preconceptions towards mathematics and poor mathematical self-efficacy views must also be addressed in order to support students on their way to success in a variety of fields of study, not only the study of mathematics. These results are all strongly in support of the Ramirez et al. (2018) "Interpretation Account" framework shown in Figure 3.1 as well as the "curriculum-diamond" model shown in Figure 1.2 in the sense that they imply a joint approach is required: simultaneously improving students knowledge/ skills and their self-efficacy/ affect towards mathematics. These two are so inextricably linked, that one cannot hope to successfully address one without also addressing the other. (Taylor & Galligan, 2006) used conversation theory framework to design an approach that was intended to simultaneously develop students' mathematical knowledge/ skills and improve their mathematics self-efficacy/ confidence, which was shown to be effective.

To summarise in the words of Galligan and Taylor (2008):

> "... although attitudes and beliefs about mathematics are important for students enrolled in bridging programs, the programs can change students' attitudes and beliefs about mathematics as well as their achievement."

## 3.6 Implications for Bridging Courses

One of the primary roles of bridging courses is to facilitate students secondary-tertiary education transition. Often the students enrolling in bridging courses will have either:

- Performed poorly in mathematics in secondary school,
- Chosen to study mathematics at a intermediate or elementary level in secondary school, or
- Had a substantial time gap between completing secondary school and engaging in tertiary education,

or some combination thereof. All of these possibilities will be associated with higher than average levels of mathematics anxiety, and negative preconceptions of mathematics. So, in this context, the question here is

> How can a bridging course best support students through their transition into tertiary education?

At a fundamental level, there are two key barriers that these students must overcome to be successful in their tertiary education:

- Developing sufficient mathematics skills, capabilities, and knowledge. This can be addressed through content — curriculum, and traditional teaching practices.
- Overcoming/ changing negative perceptions/affect/anxiety towards mathematics. This is difficult to address, but there are a number of approaches suggested in the literature.

These two key barriers must both be addressed simultaneously in order to have an effective and long-lasting impact on student's success. This conclusion is consistent with all the literature reviewed in this chapter, and is an implication of two of the major frameworks considered in this work: the curriculum-assessment diamond framework discussed in Section 1.2 and shown in Figure 1.2, and the "Interpretation Account" of Ramirez et al. (2018) shown in Figure 3.1. These two frameworks come from completely different perspectives and literatures, i.e. curriculum design and maths anxiety respectively, and yet they are both consistent in the conclusion that neither content nor learning experience — performance nor affect, can be considered alone when designing effective curriculum/ interventions. Both aspects must be taken into account in order to design effective curriculum/ interventions.

Students having completed bridging courses have commented on the importance of this kind of combined approach. In the survey of Gordon and Nicholas (2013b),

> "students are aware of the value of the bridging courses not only to ameliorate prior difficulties with mathematics and improve their approaches to learning mathematics but, less transparently, as an important opportunity to facilitate their transition into higher education, meet fellow students and help realise their potential."

Core to addressing the first of these two key barriers is the content, and what the appropriate content to teach in the bridging courses will be the focus of Chapter 4. In terms of how to best address the second of these key barriers, there are a number of points on which there is broad agreement amongst the literature reviewed in this chapter, but there is a single message that draws together most of these points, which is to:

SET CLEAR EXPECTATIONS.

To give some examples of how this is featured in the literature, the "rite-of-passage" model of Clark and Lovric (2008) suggests it is critical to set clear expectations around the new (tertiary) learning environment, as students are transitioning into a new and unfamiliar social environment/ community, it is critical to be explicit with them about the expectations in this new environment (i.e. independent learning, didactic lectures, etc.). The "rite-of-passage" model also suggests it is important that expectations be set for students about the difficulty of this transition beforehand (in the years prior to them making the transition to tertiary education) so that they come into the transition expecting it to be difficult and therefore being prepared for that difficulty. This perspective is further supported by the literature on how the perspective of viewing the process of learning mathematics (or learning in general) through the lens/ expectation of "productive struggle" particularly in the context of intervening to help maths anxious students — a concept that has been extensively demonstrated to be both critical and effective for supporting students (Wang et al., 2015; Lin-Siegler et al., 2016; Hiebert & Grouws, 2007; Carlson, 1999). Similarly, approaches taken from clinical psychology for the treatment of generalised anxiety disorders have been successful in helping maths anxious students, and reflect the same principles behind the concept of "productive struggle". Universities relaxing pre-requisites to "assumed knowledge" is a good (bad) example of *not* setting clear expectations, and this impacting directly on students (Gordon & Nicholas, 2015). Some of these points are beyond the scope of this work, but there are also actions that can be taken from the perspective of teaching a bridging course to mitigate some of these concerns: even if students come into university without the expectation of it being a difficult culture-shock event and are not prepared, being clear and explicit with them about how it will be difficult, but that that is ok and pointing them towards support services, can still have a positive impact. Similarly, changing university entry requirements is beyond the scope of this work, but even so when students come into the bridging courses with misconceptions such as "mathematics is not important to being successful in science", correcting these misconceptions can be very beneficial for them in terms of their success in pursuing their goals in the longer term.

Additionally, although the generalisation "set clear expectations" does bring together many of the recommendations implied by the literature, it does not cover everything. Some additional recommendations not covered by "set clear expectations" include:

- Helping students meet other students, make friends, and develop a social support network in their new environment is critical to supporting them to be successful, this is implied by the "rite-of-passage" framework of (Clark & Lovric, 2008), but also by a swath of other literature (Trotter & Roberts, 2006; Peat et al., 2001; Leese, 2010; Gordon & Nicholas, 2013b).

- An emphasis on "learning-to-learn" programmes has been shown to be effective (Zeegers & Martin, 2001). This is already ingrained into the culture of the MLC at the UofA thanks in large part to the work of Dr. David Butler and Nicholas Crouch over the past decade, and if anything this is a point that other bridging course programme directors could learn something from the way that the MLC at the UofA emphasises "learning-to-learn". It is a subtle, but powerful, mechanism for supporting students.

Finally, there are some other important discussion points to be aware of, although no specific actionable recommendations come from them:

- Success in secondary school mathematics is highly predictive of success and even participation in tertiary mathematics education. This predictive effect is larger than the effect of any bridging course on retention and success in tertiary eduction (Kajander & Lovric, 2005; Nicholas et al., 2015). This is important to be aware of, but unfortunately falls outside of the scope of a bridging course to address. Instead, we have to rely on secondary school educators to continue working to improve this.

- Negative community perceptions of mathematics influence rates of maths anxiety, engagement and ultimately success in mathematics educations of our students (King & Cattlin, 2015; Gordon & Nicholas, 2013b; Clark & Lovric, 2008). Again, negative community perceptions is (somewhat) beyond the scope of a bridging course to address but it is critical to be aware of the impact it has, and to be fair it does fall on all mathematicians but even more so non-mathematician mathematically skilled people and educators (including those teaching a bridging course) to gradually create the social change needed to adjust such widespread community perceptions. Broad cultural change is somewhat beyond the scope of this work, however.

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# Chapter 4

# Curriculum Mapping

One of the important roles of university mathematics bridging courses (such as MathsStart and MathsTrack) is to fill the content knowledge gap for students who wish to commence study that has some required knowledge and skills in mathematics, but either did not complete mathematics to a sufficiently high level in high school, or completed it long enough ago that they need to re-learn the skills. Ultimately, the content of such bridging courses needs to align with the "Industry Standards" or "Standards of Performance" and in particular the "Curriculum Objective" of the curriculum assessment diamond model (Figure 1.2).

There are two key perspectives that can be taken on what the "Curriculum Objective" of a mathematics bridging course is: the knowledge required for the future (tertiary) study the students are going to engage in, and knowledge expected from high school graduates. As we will come to see, these two angles or perspectives can be quite dramatically different. From the perspective of knowledge expected from high school graduates, in Australia the AC serves as a good guide, but even so the exact content knowledge expected of students having completed high school in Australia varies for a number of reasons:

- To begin with, the AC specifies four levels of mathematics: essential mathematics, general mathematics, mathematical methods, and specialist mathematics. Our focus will be on the higher two of these: mathematical methods and specialist mathematics, as these are the ones targeted at university entry into mathematics-intensive courses.

- Different states within Australia teach different curricula, with varying degrees of alignment to the AC. In South Australia the primary curriculum taught in senior secondary school is SACE, and so we will focus on that.

- Occasionally, students will do the bridging course and then travel internationally to study in which case the expectations placed on them will be based on an entirely different curriculum. This is comparatively rare, but also the modern international education system has a remarkable level of homogeneity partially as a carry-over consequence of the western colonial era (Mohandas et al., 2003). Regardless, it is beyond the scope of this work to consider alignment of content to high school curricula internationally, although this would make for interesting future research.

The other perspective is of course also important, but also far more difficult to address: the knowledge required for entry level university mathematics courses. This will vary hugely from course to course: a entry level calculus course will require very different knowledge than an entry level statistics course, for example. Even within one discipline of mathematics, different universities will have very different expectations of entry level students: in particular, South Australian universities will often structure their entry level mathematics courses to align with SACE even though not all their students have completed SACE, because of the majority who have it is still useful for them to do so. For example, the University of Adelaide re-structured it's first year mathematics courses in 2018 to match changes in SACE. Similarly, universities interstate will often structure their entry level courses to align with their local senior high school curriculum. That is only within dedicated mathematics courses, other courses that also require mathematics knowledge and skills, such as engineering, psychology, health science, and medicine will all have their own requirements. Bridging courses will enrol students aiming to study a wide variety of such topics, and even more broadly also to go into non-university further study (notably pilot training in the defence force, for example), which have different requirements again.

This places a difficult tension on mathematics bridging courses as to what content to teach. Although many of the students enrolling in the mathematics bridging courses at the UofA do so with the intention to begin study at the UofA (and hence might benefit from SACE structured content), many do not. Even amongst those that do, some may end up going to a different university interstate or even overseas — plans change. So it is important to try and maintain some connection to a broader set of knowledge expected in general and not necessarily remain laser focussed on the requirements of the particular university courses most students are going to be attempting. This is one of the reasons why the AC is a useful construct as even though some states do not align to the AC as well as others, it still forms a guiding structure at a national level and individually considering the curriculum taught in each state is beyond the scope of this work. Tailoring the content of the bridging courses more narrowly to target entry into particular disciplines (say calculus/ matrix algebra/ statistics for example) could potentially still be of interest down the line, but is likely to be unrealistic with the current resources available to the UofA MLC.

Because of the difficulty of aligning the content to the future requirements of students due to their variety of different directions and needs, and considering that the majority (over half) of students enrolling in MathsStart and MathsTrack plan on enrolling in tertiary study at the UofA, the focus of this curriculum mapping will be on aligning the content of the bridging courses to the SACE curriculum. That said, the alignment of the SACE curriculum to the AC will also be considered, in order to give some idea of the national alignment (and it turns out the AC is very closely aligned to the SACE curriculum). Now that said, the different directions students are going is still very important to consider, and although a direct alignment to the plethora of options students pursue is not realistic, the needs of the most common of these options will be incorporated into the discussion surrounding the curriculum alignment presented in this chapter. Even if direct alignment to these needs cannot be achieved, it is none-the-less important to be aware of these needs, as these are critical to students future success, and while it might not be realistic to tailor the content of the bridging courses to each of these contexts, differentiation can still be achieved through an awareness of these needs and individual interactions with students each of which will often have a particular future direction in mind.

This chapter will be structured as follows. First, in Section 4.1, some notation will be introduced and the content of each of the three curricula that will be systematically reviewed:

- The AC senior mathematics subjects mathematical methods and specialist mathematics,
- The SACE curriculum stage 1 mathematics, stage 2 mathematical methods, and stage 2 specialist mathematics,
- The University of Adelaide's bridging courses: MathsStart, and MathsTrack.

Note that the alignment done here is entirely on the content of these curricula, nothing else. The focus of this chapter is entirely on content. The alignment between the content in these curricula will be considered in Section 4.2 (see Figure 4.1), and alignments/ misalignments discussed. Finally, the discussion throughout around alignment and gaps between the content of these curricula and courses will be summarised, explanations and reasons for these discrepancies discussed, and potential modifications to content suggested.

Beyond that, this chapter will also briefly discuss the alignment of these bridging courses to first year university mathematics courses and bridging courses offered by other universities in Australia, and discuss the relationship between the gaps in alignment between the AC/SACE and the bridging courses and the requirements of these first year university courses.

## 4.1 Content

The curriculum alignment in this chapter is presented at two levels of detail — the topic level, and the key concept level. The terms "topic" and "key concept" are reserved in the context of this discussion to specifically refer to these levels of detail. The content of each of the senior high school curricula, as well as the university bridging courses, is broken down into topics, and each topic can be summarised as covering a number of key concepts. In Section 4.2, the alignment between these curricula and bridging courses will be considered thoroughly at both a topic-level, and to the finer detail of particular key concepts. Although the key concept alignment is in essence the core of the work, as this is what allows for concrete changes to be made and content to be planned, the purpose of the topic level comparison is to help structure the overall alignment and discussion.

### 4.1.1 Notation

In order to provide a useful curriculum-wide topic-level alignment to structure our thinking, it is important to be able to present this alignment in a comprehensible form that can be viewed on a single page. In order to achieve this, the topic-level description (identification of topics) needs to be summarised concisely enough. This is achieved in Figures 4.1 and 4.2, by identifying each topic with an abbreviated code. These abbreviated codes are presented in Table 4.1 and will be used for the remainder of this chapter to help refer to and identify topics. Each topic in each of the curricula being considered is assigned a unique identifying code in Table 4.1, and the curriculum (and subject within it) can be easily seen from the structure of the code.

Table 4.1: Abbreviated codes for topics within the AC and SACE senior mathematics subjects: Mathematical Methods and Specialist Mathematics, as well as the UofA bridging courses: MathsStart and MathsTrack. Square brackets ([]) are used to indicate numeric values that can vary.

| Code | Meaning |
|---|---|
| MMu[#1]t[#2] | AC Senior Mathematical Methods Unit [#1], Topic [#2] |
| MMu[#1]t[#2] | AC Senior Specialist Mathematics Unit [#1], Topic [#2] |
| S1M[#] | SACE Stage 1 Mathematics, Topic [#] |
| S2MM[#] | SACE Stage 2 Mathematical Methods, Topic [#] |
| S2SM[#] | SACE Stage 2 Specialist Mathematics, Topic [#] |
| MS[#] | Maths Start, Topic (Booklet) [#] |
| MT[#] | Maths Track, Topic (Booklet) [#] |

### 4.1.2 Key Concepts

Appendix A provides a description of each topic in each of the curricula considered here: the AC Mathematical Methods and Specialist Mathematics, SACE stage 1 mathematics, stage 2 mathematical methods and stage 2 specialist mathematics, and the UofA MathsStart and MathsTrack programs. For brevity, the codes from Table 4.1 are used to identify each topic. The name of each topic is given in bold, followed by a list of the key concepts covered in that topic separated by commas. These are discussed at length for the remainder of this chapter, and the table presented in Appendix A is intended to be used as reference material while reading the content of this chapter.

Some notes on the way the key concepts are summarised:

- The key concepts listed for each topic are intended for a reader deeply familiar with the content, and as such it is heavily condensed and uses standard mathematical notation and terminology without the usually appropriate rigorous definitions.

- Concepts relating to "interpretation" and application in a general sense are omitted from the key concepts of a topic. The assumption is that to the intended readers, these should go without saying. For example, in S1M2 the key concept "Quadratic Equations in Vertex and Factorised Form" is included, but this implies a variety of auxiliary knowledge which is not explicitly included in the key concept summary: the interpretation of roots and vertices, deducing vertices and roots from the equation of a quadratic, or deducing the equation of a quadratic given these bits of information, etc. These are skills directly and universally associated to the key concept, and it is assumed that an experienced mathematics educator (which is the intended audience for this text) should be able to easily deduce such surrounding associated skills from the key concepts listed.

These restrictions in the key concept summaries are necessary in order to be able to present this curriculum alignment concisely enough that it can be useful. The curriculum documents used to generate these summaries contain all the additional detail if required, but the purpose of this work is to align the content in those documents to identify gaps and misalignments, and as such it is beneficial to be as concise and dense as possible both to make the alignment a tractable problem and also to make the discussion thereof comprehensible. That said, it is a delicate balance between being broad and vague in order to be able to present the entire curriculum mapping within a single frame of view, and yet still be granular enough so that specific content is clear and explicitly and useful actionable recommendations can be made. It is this tension that led to the development of the methodology which split the two levels of detail:

- The topic level description is intended to give the broad strokes, to show the entire mapping in a single frame of view (a page, in this case). It is also intended to be reference material for the following more detailed discussion, to aid the reader in structuring the information contained in the more detailed discussion and place each piece of information into where it belongs in the bigger picture. Being able to structure the detailed discussion into this larger concept is critical for being able to reach broad overall conclusions.

- The key concept level is what comprises the bulk of the discussion, and this is intended to be the granular level at which content is presented specifically enough that recommended actions can be understood explicitly and implemented easily. Note that although the key concept level is much more granular than the topic level discussion, it is still intended as a summary and does not include every single detail of the content, as discussed above.

### 4.1.3 Curriculum Structure

#### The AC

The AC is separated into it's F-10 curriculum, and senior secondary curriculum. In this work we are only concerned with the senior secondary curriculum. The senior secondary AC for mathematics is split into four subjects, corresponding to different "levels" of mathematics: Essential Mathematics, General Mathematics, Mathematical Methods, and Specialist Mathematics. In this work we are concerned only with Mathematical Methods and Specialist Mathematics, and will be considering the mathematical content of these subjects not any other aspects (such as cross-curricular priorities, for example). Importantly, the senior secondary AC does not make any distinction between years 11 and 12 (typically the final year of high school in Australia). So the senior secondary AC subject "Mathematical Methods" for example, covers content that is in practice taught across both years 11 and 12. Each of the two subjects we are concerned with in this work, Mathematical Methods and Specialist Mathematics, are split into four "units" of content, and each of these units is split into three topics, for a total of 12 topics per subject, and a total of 24 topics that we will consider from the AC. At no point do we consider the unit structure of the AC, partly because it does not have any analogue in the other curricula we are aligning too, but mostly because it does not give a useful level of detail for our purposes.

### SACE

SACE, in comparison to the AC, does distinguish between year 11 and year 12 content, although to allow for some alternative senior high school teaching structures they have a different naming convention, calling them stage 1 and stage 2 respectively. In the majority of mainstream cases in Australia, SACE stage 1 will correspond to year 11, and SACE stage 2 will correspond to year 12. To further complicate matters, stage 1 SACE has only three levels of mathematics: Essential Mathematics, General Mathematics, and Mathematics, while stage 2 SACE has four: Essential Mathematics, General Mathematics, Mathematical Methods, and Specialist Mathematics. In this work we will only be concerned with SACE stage 1 Mathematics, stage 2 Mathematical Methods, and stage 2 Specialist Mathematics. SACE stage 1 Mathematics is broken down into 12 topics, while stage 2 Mathematical Methods and Specialist Mathematics are broken down into 6 topics each. This makes for a total of 24 topics from senior high school SACE mathematics subjects that we will be considering in the curriculum alignment presented in this chapter.

### UofA Bridging Courses

The UofA offers two bridging courses through their MLC: MathsStart and MathsTrack. These are both taught through a series of booklets which conveniently each contain roughly on "topic" worth of content, and so these booklets will be used as the topic-level structure of these courses. Both courses are currently structured into 8 topics (booklets) each, although MathsTrack used to have 9, and the fifth was removed some time ago, so the numbering of the MathsTrack topics have a gap (they are numbered 1, 2, 3, 4, 6, 7, 8, 9). So there are a total of 16 topics (booklets) across both bridging courses that will be considered in the curriculum alignment presented in this chapter.

### Topic Grouping

In order to help structure the discussion to follow, it will be useful to think about one broader level of detail, which will loosely be referred to as "content areas". From a very low level of detail perspective, the topics in each of the curricula being considered can be grouped into the following five broad content areas:

- Functions and Graphs,
- Calculus,
- Geometry and Linear Algebra,
- (Complex) Numbers, and
- Probability and Statistics

There is also some nested hierarchical structures within these content areas that are useful to understand. For example, both "Functions and Graphs" and "Calculus" can be further separated into three sub-areas, corresponding to different categories of functions. Specifically:

- Linear, Polynomial, and Rational Functions,

- Exponential and Logarithmic Functions, and
- Trigonometric Functions.

"Calculus" is naturally divided into differentiation and integration, "Probability and Statistics" can be divided into probability and statistics as separate content sub-areas, although more commonly is divided into discrete and continuous random variables. Geometry and Linear Algebra covers perhaps the widest variety of topics, from vectors to matrices to systems of equations as well as more traditional geometry topics such as circle theorems.

The two areas ("Functions and Graphs" and "Calculus") are also often taught together, with new categories of functions being introduced/ revised together with concepts around how to do calculus with these functions, so this particular pair of content areas are very closely linked. Although there are some notable connections between the other content areas, such as for example:

- Complex numbers providing a method for finding roots to polynomials that could not otherwise be found,
- Applying calculus to parameterised vector equations,
- Integration being used to understand probabilities as areas under distribution functions,

Broadly speaking they stand comparatively apart from each other, particularly Probability and Statistics.

This broad content area grouping of topics covers almost all of the content in all the curricula considered here. The only notable exceptions being MMu2t2 from the AC and S1M7 from SACE both covering primarily sequences (geometric and arithmetic) as recurrence relations, and S2SM1 from the SACE covering inductive proof, neither of which fit neatly into any of the content areas above. With that having been clarified, these content areas will be used to help structure the discussion for the remainder of this chapter.

## 4.2 Curriculum Mapping

Figure 4.1 shows the topic-level alignment between the AC, SACE, and bridging courses, organised by subject/ course. Each node (ellipse) in Figure 4.1 corresponds to a topic, and is identified by the abbreviated code as per Table 4.1. What "organised by subject/ course" means in this context is that while the curricula (the AC, SACE, and the UofA bridging courses) are arranged as columns in Figure 4.1, within each of these columns topics are grouped by subject. So for example in the AC column topics are grouped into Mathematical Methods topics, and then Specialist Mathematics topics.

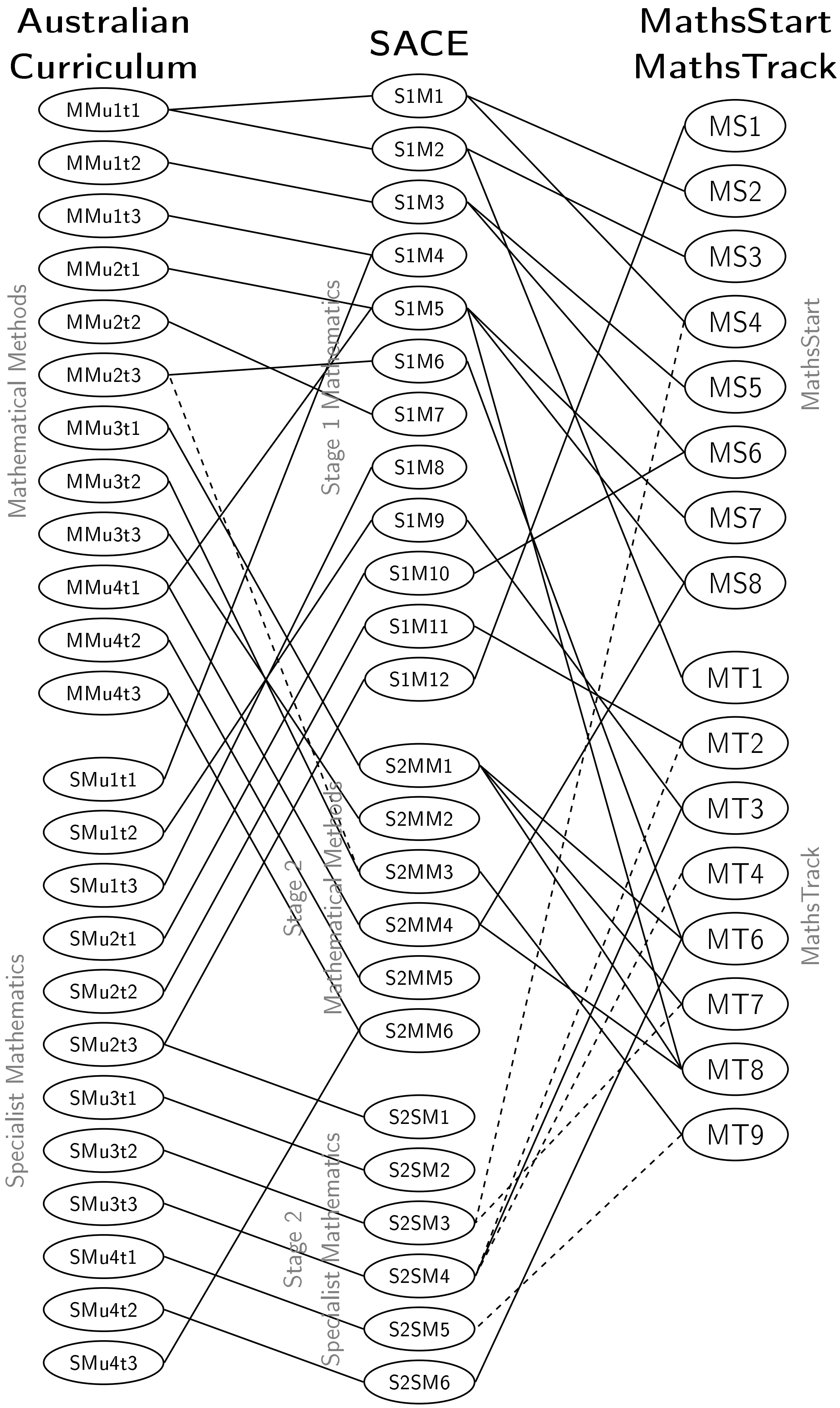


Figure 4.1: Curriculum Mapping by Subject/ Course

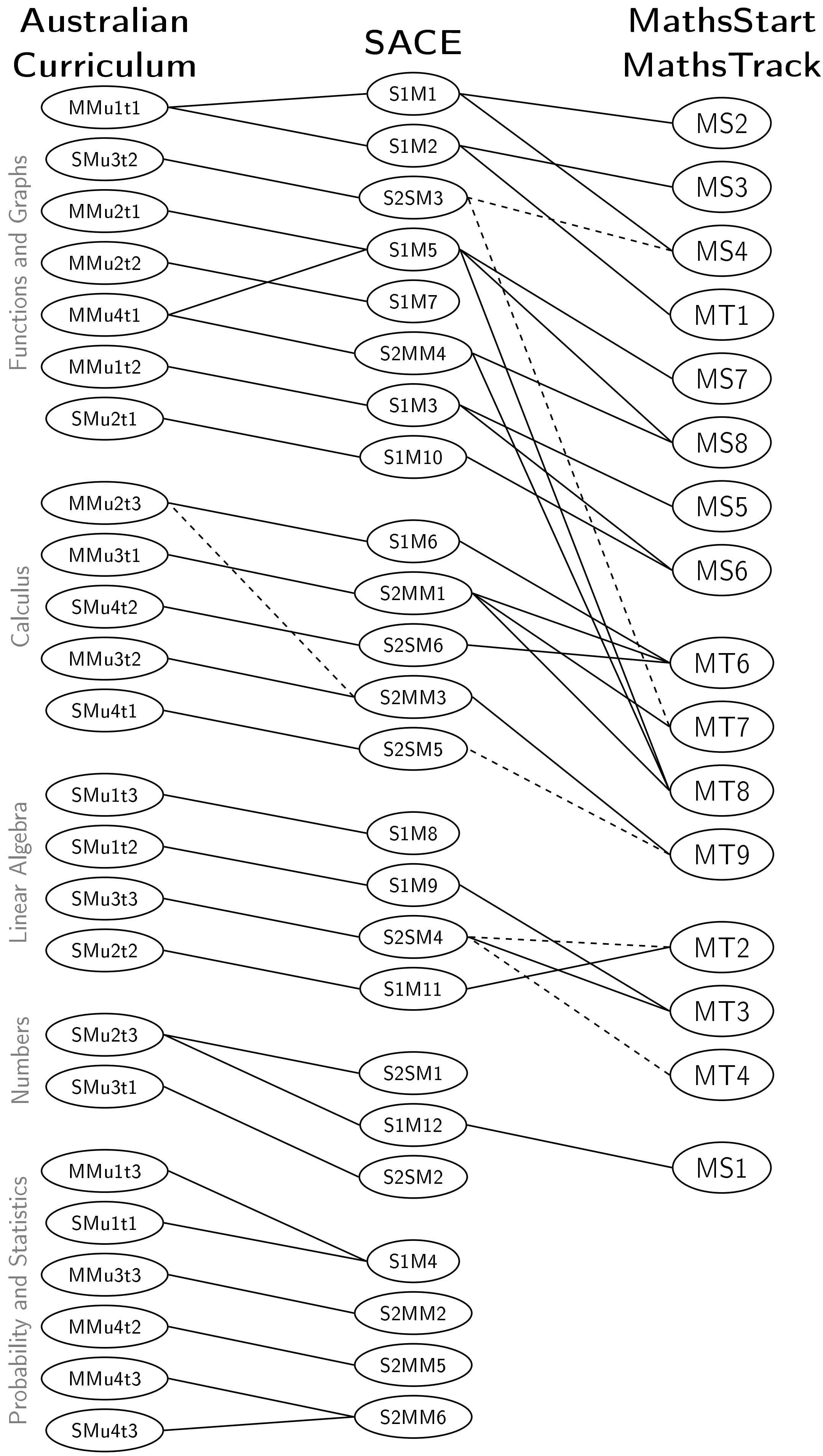


Figure 4.2: Curriculum Mapping by Content Area

The topic-level alignment organised by subject/ course shown in Figure 4.1 is the broad, eagle-eye, view of the alignment between the content in these topics, but even from this wide-view the way in which the curriculum structures of the AC and SACE discussed in Section 4.1.3 align to each other is reasonably clear. It can be seen for example, that roughly speaking the first 6 topics of senior secondary AC Mathematical Methods and Specialist Mathematics map to the 12 topics of stage 1 SACE Mathematics, while the last 6 topics in each of these AC subjects map to the 6 topics of the corresponding stage 2 SACE subject. At a key concept level this alignment is imperfect (and this is discussed in more detail below) to varying degrees of imperfection, and some of these imperfections are visible in the imperfect alignment shown in Figure 4.1, but broadly speaking the AC and SACE are actually very closely aligned (and this makes sense given SACE was recently modified with the explicit purpose of aligning it more closely to the AC). In contrast, it is much less obvious how to interpret the alignment between SACE and the bridging courses shown in Figure 4.1.

However, rearranging the topics shown in Figure 4.1 (permuting the columns) and grouping them into the five broad content areas discussed above in Section 4.1.3:

- Functions and Graphs,
- Calculus,
- Geometry and Linear Algebra,
- (Complex) Numbers, and
- Probability and Statistics

gives us a much clearer picture, which is shown in Figure 4.2. Even though the allocation of some topics into these content areas can be somewhat arguable particularly in a few edge cases (as mentioned above in Section 4.1.3), Figure 4.2 shows a very clear picture in terms of the SACE-bridging course alignment: the bridging courses do not contain any probability or statistics what-so-ever, and very very little on complex numbers.

Before moving on to discuss the details of the key concept level alignment within each of these topic alignments shown, a quick note on interpreting the visualisations of Figures 4.1 and 4.2. While solid lines connecting topic-nodes represent almost complete or substantial key-concept level alignment, dashed lines are used to represent tenuous connections with only a small overlap in key concept terms, usually just a single key concept. To briefly cover which concepts these correspond too:

- The one dashed line between the AC and SACE essentially represents the concept of anti-differentiation,
- The dashed line between S2SM3 and MT7, as well as the one between S2SM3 and MS4 essentially represents sketching rational functions, although in MS4 only reciprocal functions and transformations thereof are considered. The ideas surrounding the sketching of these graphs and the properties of these graphs (asymptotes, etc.) are heavily emphasised as a way to explore them in both cases.

- The dashed line between S2SM5 and MT9 essentially represents integration by substitution.
- The dashed line between S2SM4 and MT2 essentially represents row operations, in MT2 introduced on matrices, but in S2SM4 it is introduced explicitly in the context of solving $3 \times 3$ systems of linear equations. Similarly the dashed line between S2SM4 and MT4 represents essentially the same concept in S2SM4, but in MT4 the system of equations perspective/ application is explored, which is not really done as much in MT2.

### 4.2.1 The AC and SACE

At a glance, there appears to be a very good one-to-one alignment at the topic level between the AC and SACE. Broadly speaking the biggest difference between these two curricula is their structure, as discussed in Section 4.1.3. As usual however, the devil is in the details. In this section, a detailed discussion of the key concept level alignment between the AC and SACE will be presented. This discussion will be structured by the broad content areas as introduced in Section 4.1.3.

#### Functions and Graphs

The content area "Functions and Graphs" can be reasonably spit into three content sub-areas in both the AC and SACE: Polynomials and Rational Functions, Exponential and Logarithmic functions, and Trigonometric Functions, as discussed in Section 4.1.3, with the notable additional comment that in both the AC and SACE general concepts and notation are strongly emphasised and introduced through the Polynomials and Rational Functions topics. The content in this area aligns almost perfectly between the AC and SACE, with only minor differences in notation, emphasise, and how key concepts are split into topics. Despite this very close alignment, a more detailed topic by topic discussion of the key-concept level alignment is included for completeness:

- **Polynomials and Rational Functions:** In both the AC and SACE this area is split into two: basic introduction and advanced concepts.The basic introduction topics align well (MMu1t1 to S1M1 and S1M2), with only slight differences in terminology (AC refers to inverse proportion while SACE refers to reciprocal for example) and focus (SACE puts much more of an emphasis on polynomials, separating it into it's own topic (S1M2) and breaking it down into much more granular concepts). The advanced concepts are covered in SMu3t2 and S2SM3 are essentially identical.
- **Exponentials and Logarithims:** There is essentially perfect alignment between the concepts for logarithms between MMu4t1 and S2MM4. Similarly, MMu2t2 is almost exactly the same as S1M7, they are both centred on the introduction of recurrence relations, partial sums, and linking this back to exponential functions. I include these topics under exponentials as they link to and are used to introduce those concepts, but really the bulk of the content in these topics is focussed on sequences and series. The alignment between MMu2t1 and S15 has a notable difference however: S15 includes Log-Laws, while MM2t1 does not, focusing only on Index Laws. This is not actually a

difference in content between the AC and SACE as the log laws are covered in the AC in MMu4t1, but a difference at the topics level. The log laws are actually repeated in the SACE curriculum, covered both in S1M5 and then again in S2MM4, while they are not repeated in this way in the AC.

- **Trigonometry:** MMu1t2 matches almost identically to S1M3, with the biggest difference being that in the AC the unit circle interpretations/ definitions of $\sin(x)$, $\cos(x)$, and $\tan(x)$ are emphasised, where in SACE $\tan(x)$ in particular is introduced instead as $\frac{\sin(x)}{\cos(x)}$. That being the biggest difference between the two should emphasise how similar they are in terms of content. Similarly, SMu2t1 and S1M10 align just about perfectly.

### Calculus

Similarly to Functions and Graphs, there is very good alignment between the AC and SACE in the Calculus content area but for completeness a detailed discussion of the key concept level alignment on a topic-to-topic basis in included:

- SMu4t1 aligns perfectly with S2SM5, both covering integration by parts, by substitution, inverse trig substitutions in integration problems, volume of solids of revolution, partial fractions and area between two curves.

- SMu4t2 aligns well to S2SM6, both covering implicit differentiation, solving first-order separable differential equations, and the logistic equation. However there are some differences in that the AC goes on to focus on rates of change, while SACE instead decides to focus on parameterised curves — trigonometric parameterisations and such.

- MMu2t3 and S1M6 both introduce differentiation by leading in with the concept of average rate of change, first principles and lead into linearity of differentiation, derivatives of polynomials, slope of the tangent and optimisation but in SACE S1M6 introduces the terms "increasing" and "decreasing" and sign diagrams, which are not mentioned in MMu2t3, while MMu2t3 introduces the concept of an anti-derivative (which is only introduced in S2MM3 of SACE and is represented by a dashed line in Figure 4.1 and Figure 4.2).

- MMu3t1 and S2MM1 align perfectly introducing the chain, product, and quotient rule. Introducing $e = 2.718\ldots$ in the same way (using first principles to explore $\frac{d}{dx}a^x$ for different $a$, derivatives of $\sin(x)$ and $\cos(x)$, and second derivatives.

- MMu3t2 and S2MM3 are very closely aligned, both introducing definite and indefinite integrals of polynomials, exponentials, and trigonometric functions, linearity of integration and the fundamental theorem of calculus. They do however diverge slightly in their approach to definite integrals. In particular, SACE S2MM3 introduces the concepts of upper and lower sums and the definite integral as the unique number between the two as the size of the rectangles approaches zero, while in the AC MMu3t2 this is not discussed. Also, S2MM3 introduces anti-differentiation, a concept introduced in the AC MMu2t3 but not introduced in SACE S1M6, instead being covered here in S2MM3.

Note how most derivatives are introduced in differentiation (i.e. calculus) specific topics. The exception is $\frac{d}{dx}\ln(x)$, which is introduced in a separate topic entirely about logarithm functions in both the AC (MMu4t1) and SACE (S2MM4), and in this content area structuring these topics are categorised under 'Functions and Graphs' because they introduce logarithmic functions, but it should be noted that they do also contain concepts around calculus (of logarithm functions).

### Geometry and Linear Algebra

The content area "Geometry and Linear Algebra", similarly to "Functions and Graphs", can be split into four content sub-areas in both the AC and SACE: Vectors in $\mathbb{R}^2$ (in the Plane), Circle Theorems, Matrices and Vectors in $\mathbb{R}^3$ (in 3D).

- **Vectors in the Plane** are covered in SMu1t2 and S1M9, with the content being very well aligned and the only notable difference being the inclusion of geometric vector proofs in SACE S1M9 which is not included in SMu1t2, instead being restricted to topics such as SMu1t3.

- **Circle Theorems and Proof** are covered in SMu1t3 to S1M8. Both these cover the same "content" in the sense of theorems: circle theorems, but they also both attempt to broach the difficult topic of proof, methods of proof, and some of the language around proof, and they take quite different approaches to this. The AC SMu1t2 is quite explicit specifying the introduction of language around formal logic, defining the terms: "implication", "equivalence", "converse", "negative", "contrapositive", "contradiction", "for all", "there exists", and "counter-example". On the other hand, SACE S1M8 simply specifies proof to be investigated as "justification of properties of circles", and only briefly mentions specifics of language and methods as suggestions not specifying them as being required components of the curriculum and instead leaving the approach and specific content chosen to be used to introduce the concept of proof much more open to interpretation by the teacher.

- **Matrices**, covered in SMu2t2 and S1M11 are essentially identical in content covering matrix notation, linear combinations of matrices, matrix multiplication, matrix identity and inverses (and determinants), and the perspective of matrices as linear transformations.

- **Vectors in 3D** in SMu3t3 and S2SM4 are also introduced very similarly in terms of content: cross product, equations for lines and planes, systems of equations and geometric interpretation of their solutions. One difference however is in how they apply these concepts, the AC SMu3t3 includes a focus on parameterised vector equations, the equation for a sphere, and in particular kinematics: projectile and circular motion in 3D, which are not converted in SACE S2SM4. Instead S2SM4 remains more abstract with these concepts, and on the other hand the examples required are less complex to interpret.

### (Complex) Numbers

Complex Numbers are introduced in two topics, an introductory an advanced topic, in both curricula. The introductory topics, SMu2t3 in the AC and S1M12 in SACE are quite similar in their base content: rational/ irrational numbers, $i$, complex arithmetic, conjugates, and complex roots of polynomials. However there are a couple of key differences between the two: first, induction is introduced in the AC SMu2t3 while in SACE it is separated into it's own seperate topic: S2SM1. The second key difference is that interval notation is explicitly introduced in SACE S1M12, while in the AC interval notation seems to be neglected. The advanced topics SMu3t1 and S2SM2 on the other hand align almost perfectly in content.

### Probability and Statistics

Probability and Statistics is the content area in which the most substantial differences in content exist between the two curricula. Similarly to "Functions and Graphs" and "Geometry and Linear Algebra", the content covered in "Probability and Statistics" can be organised into three content sub-areas: Combinatorics, Random Variables, and Confidence Intervals.

- **Combinatorics and Introductory Probability** are introduced in the AC topics MMu1t3, SMu1t1, and the SACE topic S1M4. The content-overlap between the two curricula in these topics is primarily concepts around permutations, the factorial (and the 'multiplication principle'), and combinations. Although it is notable that the AC MMu1t3 extends the concept of combinations to binomial coefficients and Pascal's triangle while SACE does not. Beyond these common concepts, both curricula have some introductory probability content, but they take very different approaches to this. The AC does this via set theoretic concepts: union, intersection and complement of sets, the pigeon-hole principle, and uses probability notation ($P(A)$) to take the set-theoretic ideas of complement, intersection and union into a probability context. In this same frame of mind, the AC also introduces the introductory probability concepts using the same formal notation, i.e. $0 \leq P(A) \leq 1$ and conditional probabilities ($P(A|B)$) for example. On the other hand, SACE S1M4 has introductory statistics concepts (as opposed to introductory probability concepts). Specifically, S1M4 reviews mean median and mode, interquartile range, standard deviation, and introduces the basic concepts around the normal distribution. S1M4 also introduces the distinction between discrete and continuous random data/ variables, not quite introducing the concept of a 'random variable' *per se*, but laying the foundation for that introduction.

- **Random Variables**: Discrete (MMu3t3 and S2MM2), and Continuous (MMu4t2 and S2MM5). There is quite good alignment between these topics actually. For both discrete and continuous general definitions of expected value and variance are given. For discrete the uniform, examples of arbitrary non-uniform (defined values), the Bernoulli, and binomial distributions are introduced. For continuous the uniform, arbitrary function (for example restricted domain polynomial), and normal distributions are considered, and transformations of normal distributions (in particular to get the standard normal) are considered. The key difference between the curricula in these topics is that in SACE the central limit theorem is explicitly explored, while it's significance is implied but not explicitly explored in the AC. It is notable that it appears that while SACE explicitly introduces the concept of a cumulative distribution function, the AC does not (although the AC does still introduce probabilities associated to continuous random variables as areas under, i.e. integrals of, probability density functions).

- **Confidence Intervals**: The confidence intervals introduced are the same across both curricula, specifically the normal approximation to the binomial confidence interval for a proportion (Wald interval, MMu4t3) and the standard normal distribution confidence interval for the mean of a continuous variable (SMu4t3) are both introduced in SACE S2MM6. However the approach taken to justifying these confidence intervals is a little different, in SACE the justification heavily relies on the central limit theorem, relying on the introduction to that concept in S2MM5, while in the AC instead many of these concepts (including the central limit theorem itself) are simply stated and students are encouraged to test them by simulation. Although SACE also takes this simulation approach to justification it is emphasised less, and the introduction of the theory surrounding the central limit theorem is much more explicit.

#### Summary

Overall the AC and SACE are very closely aligned in terms of content, as is to be expected given the focus given by the SACE board to national curriculum alignment. That said, there are some differences between the two, with the biggest of these differences being in the introductory statistics/ probability section, with the AC introducing a substantial amount of combinatorics and set theory notation and terminology, while SACE introduces less combinatorics (although still some), and opts to focus on revising introductory statistics (mean, median, mode, etc.) instead of introductory set-theoretic probability concepts (intersection, union, etc.). This is likely to do with all of these concepts technically being covered in the year 10 AC, and the two curricula relying on different parts of this assumed knowledge to varying degrees. Apart from a small number of differences in precise content alignment, (for example, SACE does not explicitly introduce the concept of a cumulative distribution function while the AC does), the two curricula take quite different approaches to the statistics and probability content area overall, with the AC focusing on the theory much more, while SACE focusses instead on investigating behaviour empirically more. The only other substantial difference between the AC and SACE outside of the probability and statistics content area is in how the concept of proof is integrated into the curriculum. While SACE attempts to introduce the concept of proof in a variety of contexts, geometric proofs in particular, the AC in contrast only introduces proof as a relatively isolated concept, in a more limited number of contexts.

### 4.2.2 MathsStart and MathsTrack

In the broad sense of the five content areas discussed in Section 4.1.3, it can be seen from Figure 4.2 that two of these content areas — Complex Numbers and Probability and Statistics — are essentially not covered by MathsStart or MathsTrack whatsoever. In Figure 4.2 MS1 is grouped into the (Complex) Numbers content area, as it includes an introduction to rational/ irrational numbers, but it does not include any reference to complex numbers at all. On the topic of complex numbers, it is notable that the 'missing' MathsTrack Topic 5, which was part of MathsTrack in the past but is currently being omitted from the course, covered complex numbers. So if including content on complex numbers was of interest, MT5 could be used as a starting point for better aligning the content of the bridging courses to the AC/ SACE in the content area of complex numbers. That said, for the purposes of this curriculum alignment, we are more concerned with the content that is currently in the bridging courses and so this section will discuss the key concept level aligned on a topic-by-topic basis for the remaining three content areas that are currently covered in the bridging courses: Functions and Graphs, Calculus, and Linear Algebra.

### Functions and Graphs

Just about the entire of MathsStart is concerned with this content area, as well as the first topic of MathsTrack MT1. This is the content area used in both the AC and SACE to introduce general concepts around functions which will be used throughout several of the following topics, and at a high level the bridging courses are taking this same approach: using the general content area "Functions and Graphs" to introduce, explore, and acclimatise students to general concepts and ideas. So although specific functions and their properties are introduces as well (i.e. Polynomials and Rational Functions, Exponential and Logarithmic Functions, and Trigonometric Functions much as in the AC and SACE, the intention seems to be less on the specifics of these functions and more on building a solid foundation of general understanding, familiarity, and comfort with the concepts in order to be able to build on in later topics. This implicitly means that underlying the content in these topics there is an intention to prompt students to be practising and revising skills such as rearranging equations, fractions, and arithmetic as these will be foundational for students moving forward from MathsStart.

It is interesting to note how it is intended that MathsStart be, in a very rough sense, equivalent to year 11 mathematics, while MathsTrack is intended in a similarly rough sense to primarily cover the content of year 12 mathematical methods. However because of the importance of establishing these foundational skills and understandings in MathsStart unlike the SACE curriculum which roughly speaking introduces the basics of each new idea in year 11 and then extends this into advanced applications and understandings in year 12, MathsStart completely omits certain topics in favour of establishing foundational skills more concretely, and then MathsTrack, particularly in the later topics, covers new ideas end-to-end in a single topic. To further explore the specifics of this alignment, a topic-by-topic key concept level alignment discussion is presented below, organised in the same way that the "Functions and Graphs" content area has been previously split into content sub-areas: Polynomials and Rational Functions (and General Concepts), Exponential and Logarithmic Functions, and Trigonometric Functions

- **General Concepts, Polynomials and Rational Functions** have an interesting binary tree structure that can be observed in Figure 4.2, with the AC MMu1t1 splitting into both S1M1 and S1M2 in SACE, which each split into MS2, MS4 and MS3, MT1 respectively in the bridging courses. S1M1 covers mainly linear equations, but also reciprocal functions and asymptotes, while in MathsStart these are split: linear functions are covered in MS2 while reciprocal functions and asymptotes are covered in MS4. Similarly S1M2 covers polynomials, including quadratics and related concepts as well as higher order polynomials, while in the bridging course these are separated with MS3 focussing entirely on quadratics (MS3), and MT1 which provides a more in-depth exploration of quadratics and introduces higher order polynomials. It is notable that although the entire of MS3 is dedicated to quadratics, it is still less in-depth than S1M2, which introduces rearrangements of quadratics into vertex and factorised form, but the missing concepts (such as the quadratic formula) are covered in MT1. S2SM3, also part of "Functions and Graphs", introduces advanced general concepts on functions: domain and range, function composition, one-to-one, inverse functions, graphing more general rational functions (not just reciprocal functions), and the absolute value function. These concepts are not covered in the bridging courses, and could be useful, but on the other hand, are part of the specialist mathematics curriculum, and so whether they need to be covered in the bridging courses is open to discussion.

- **Exponentials and Logarithms**: Both the AC and SACE introduce the concept of exponential functions via recurrence relations describing geometric sequences, while the bridging courses do not use recurrence relations whatsoever. Although this is certainly not the only (or even necessarily the best) way to introduce and understand exponential functions, it is the way prescribed by the AC and SACE curricula, and so this difference in approach might lead to a difference (and worst-case scenario a systematic disadvantage) for students coming out of the bridging courses as opposed to students coming out of a SACE high school education, so adjusting the bridging courses to use concepts of recurrence relations to introduce exponential functions might be worth considering. On the other hand, the way the number $e$ is introduced is actually identical across AC, SACE, and the bridging courses — which is remarkable given how many different ways this could be done. Interestingly, exponent laws and logarithm laws (as well as basic properties of exponential and logarithmic functions) are introduced in S1M5 of SACE, and are split into the two topics MS7 and MS8, it seems that the granularity of the MathsStart program is roughly a factor of two more granular than the SACE curriculum, which is interesting. Developing a measure of "granularity" and estimating it's effect on learning could make for an interesting line of future research, but is beyond the scope of this work. It is also interesting to note that S2MM4 is grouped into the "Functions and Graphs" content area in Figure 4.2, rather than being grouped in the "Calculus" content area, as discussed above this is because of how concepts of exponents and logarithms are very distinctly separated in the AC into MMu2t1 and MMu4t1, while in SACE introductory concepts for both are introduced in S1M5 and S1M7, but while advanced concepts around logarithms (including calculus) have their own topic in SACE — S2MM4, advanced concepts (such as calculus) around exponential functions do not, and are instead lumped into more general calculus topics (S2MM1 in SACE and MMu3t1 in AC). A more in-depth discussion of this is included below under the "Calculus" content area, but suffice it to say there is substantial overlap between the "Functions and Graphs" and "Calculus" content areas, which leads to the crossovers seen in Figure 4.2 when these ideas are structured even slightly differently as they are in the bridging courses. This could indicate that the categorisation of concepts into these two content areas is not the most appropriate and maybe more appropriate content area groupings could be found for these topics, or perhaps more likely simply that these concepts are highly interconnected and so are often taught together in an integrated way. Exploring this in a broader sense, in terms of when concepts are taught separately and when they are integrated together could make for very interesting future research and very applicable to recent trends in educational research into STEM.

- **Trigonometry**: Similarly to the rest of the "Functions and Graphs" content area, the trigonometry aligns fairly well between the AC/ SACE and the bridging courses, although it is organised differently (S1M5 and S1M10 in SACE and MS5 and MS6 in MathsStart). SACE covers some graphing slightly more comprehensively, talking about translations and dilations for example (a concept from MS3 that could be "translated" here effectively, linking the concepts and chaining them from topic to topic a little more strongly.

The "Functions and Graphs" content area aligns quite well between the AC, SACE, and the bridging courses in terms of content. One of the most prominent differences is in how the content is structured, both in terms of the "granularity", and in terms of how certain key concepts (for example concepts relating to exponential and logarithmic functions) are grouped with other key concepts (such as grouping them with calculus-concepts, or introducing them separately). Both of these differences could make for interesting future research directions, however analysing the structural differences between these curricula is, despite being very interesting, beyond the scope of this work. It is notable for such future research though, that the methodology employed here to align the content between these curricula seems to have been very effective at identifying structural differences between the curricula, although this was not the intended purpose of this methodology it is likely a result of the use of a rigorously structured approach that yielded this information about structure which could be leveraged by future research that was interested in investigating structural differences between curricula that cover the same content in different structures.

### Calculus

The "Calculus" content area is a major focus on MathsTrack, because of the focus of many entry level university courses on calculus, and so spans four topics: MT6, MT7, MT8, and MT9. As such, it makes sense to structure the key concept level alignment discussion below under these four topics:

- **MT6** introduces differential calculus in a very similar way to S1M6, i.e. through first principles. MT6 actually goes beyond the content of S1M6, introducing also the product, chain and quotient rules (which are covered in S2MM1) and implicit differentiation (which is only covered in S2SM6). On the other hand, MT6 does not cover increasing and decreasing (which is in S1M6), which is instead covered in MT7. All these differences are purely structural, not differences in content between the curricula. A small but notable difference in content between the curricula is that MT6 introduces the concept of a normal to a curve, which is not covered anywhere in SACE (apart from implicitly in S2SM4 in the context of vector cross product).

- **MT7**: covers a few concepts from S1M6 that where skimmed over in MT6, as well as some of the more advanced function and graph concepts such as sketching rational functions which is only covered in S2SM3.

- **MT8**: Similarly to the way that MT7 mixes advanced concepts from the "Functions and Graphs" content area and mixes them with introductory calculus concepts from SACE, MT8 introduces differentiation of exponential functions (covered in S2MM1) and logarithms (covered in S2MM4), at the same time as re-hashing concepts from MS7 and MS8 and revising them (such as sketching exponential and logarithm functions). Notably, surge models and logistic models are introduced in MT8 as well. Surge models are not covered anywhere in the AC or SACE, and logistic models are only introduced in S2SM6 in a somewhat different context.

- **MT9**: All the integration is fit into this single topic in MathsTrack, which students inevitably find challenging. This covers essentially all of S2MM3, and then goes a little further with the notable addition being integration by substitution, which in SACE is only covered in S2SM5. Notably summation notation is also introduced (in an appendix) in MT9, an important bit of notation that students often struggle with in first year university.

Overall the "Calculus" content area aligns fairly well between the AC/ SACE and the bridging courses as it is a major focus of the latter, with only a few notable excepts (Surge Models, for example). In a number of cases, the bridging courses actually go beyond SACE Stage 2 Mathematical Methods and cover a substantial portion of the content of SACE Stage 2 Specialist Mathematics, which could be of interest if reducing the amount of content in the bridging courses was of interest, as strictly speaking it is not necessary for the bridging courses to cover SACE Stage 2 Specialist Mathematics content.

### Geometry and Linear Algebra

The "Functions and Graphs" and "Calculus" content areas align fairly well between the AC/ SACE and the bridging courses despite being structured quite differently and overlapping substantially. In contrast, the "Geometry and Linear Algebra" content area is one where there are substantial differences in content between the AC/ SACE and the bridging courses. This content area is covered across three topics in the bridging courses: MT2, MT3, and MT4, and so the topic-by-topic key concept level discussion to follow will be structured around these three MathsTrack topics.

- **MT2** covers much of the content in S1M11, although it goes further and also introduced row operations, a concept not introduced in S1M11 although it is introduced almost implicitly in S2SM4 when matrices are used to solve $3 \times 3$ systems of linear equations. This aspect, of solving systems of equations, in introduced in MT4 and actually gone into in great depth, while the concept seems tacked on and is not gone into in detail at all in S2SM4.

- **MT3** Introduces vectors and vector concepts in both $\mathbb{R}^2$ (concepts covered in S1M9) and $\mathbb{R}^3$ (concepts covered in S2SM4). The overlap between S1M9 and MT3 is substantial, with MT3 covering most of the concepts in S1M9, although S1M9 goes into a little more detail on scalar dot products (a concept covered in both), and also introduces the concept of orthogonal projection, and even throws in a dash of geometric styled proof. S2SM4 also goes significantly further, most notably introducing the concept of the vector cross product which is not covered in MT3, although both introduce equations for planes in $\mathbb{R}^3$.

- **MT4** focuses on systems of linear equations, and although this concept is introduced in S2SM4, it is covered in MT4 in a much more detailed, granular way. Also, MT4 introduces Gauss-Jordan Elimination, an algorithm not explicitly introduced in SACE (although it is implied in the sentence "solve a system of equations using row operations").

## 4.3 Summary

Broadly the content for both "Functions and Graphs" as well as "Calculus" content areas is well aligned between the bridging courses and the AC/ SACE. Although the AC and SACE to mix these closely related content areas, particularly for logarithmic functions, the bridging courses tend to mix the two more, merging the more advanced concepts from functions and graphs into the calculus topics in a more systematic way. This makes perfect sense, especially considering how inter-connected the areas are in the first place, and might highlight the inappropriateness of the content area categorisation more than anything, but it does demonstrate an interesting difference in systematic structuring of concepts between the two.

Broadly, a big difference in emphasis between the bridging courses and the AC/ SACE is the emphasis the bridging courses place on sketching graphs, and explicitly exploring the connection between how transformations (translations and dilations primarily) of a graph relate to algebraic changes to functions. Although this is covered in SACE to some degree, it is largely implicit and left to reading between the lines, while it is quite explicit and fairly systematically embedded in the bridging courses. Whether this is an advantageous emphasis or not is beyond the scope of this work, but would make for interesting future research.

Another broad observation is that MathsStart has an implicit focus on practising foundational skills such as rearranging equations, etc. This seems like a good approach, and implies another direction of future research going further backwards into the curriculum and looking at the year 10 and prior AC to identify assumed knowledge and skills that are important foundational cornerstones for building the required knowledge in these courses. Offhand, this would likely include concepts such as fractions, index laws and rearranging equations.

In terms of differences in content rather than just structure, bigger differences between the content of the bridging courses and the AC and SACE exist in the other content areas. With Probability and Statistics being completely omitted from the bridging courses and Complex Numbers almost completely omitted. As discussed in Section 4.2.2, the MT5, not currently included in MathsTrack, covered complex numbers, and as such any attempt to re-incorporate content about complex numbers should begin with examining the content in the old MT5 topic. In addition, the "Complex Numbers" content area has the additional complication of where it fits into the SACE curriculum: It is not touched at all in SACE Stage 2 Mathematical Methods, only SACE Stage 2 Specialist Mathematics, and SACE Stage 1 Mathematics. As it is in the year 11 SACE Stage 1 Mathematics one might assume that is should be covered in MathsStart, but this is not entirely accurate. Although MathsStart includes concepts from high school mathematics up to and including year 11, it is not intended to be comprehensive, and topics that build required knowledge for only SACE Stage 2 Specialist Mathematics in particular and not Mathematical Methods are particularly less important to cover in MathsStart. The decision of to include complex numbers in the bridging courses or not is obviously beyond the scope of this work, here it is simply clarified where in the curricula it is touched on, so that that decision can be made with clarity.

The remainder of this summary will be split into different categories of differences in content that could be useful in terms of re-designing the content of the bridging course moving forward:

- First the content covered in "Probability and Statistics", and
- "Complex Numbers" will be summarised, with notes on which SACE subjects contain which content.
- All other key concepts covered in SACE Stage 2 Specialist Mathematics will be listed briefly, separated into concepts currently already covered in the bridging courses/ not covered in the bridging courses.
- Lastly, and perhaps most importantly, any remaining key concept misalignments between the SACE curriculum and the bridging courses that do not relate to SACE Stage 2 Specialist Mathematics will be listed.

### 4.3.1 Probability and Statistics

The "Probability and Statistics" content area is almost entirely contained in SACE Stage 2 Mathematical Methods, and the key concepts can be summarised as follows:

- The concept of a random variable is a critical foundational concept for probability and statistics, and often serves as the abstract object used to define the difference between "Sample" and "Population" thinking, a fundamentally important difference of perspective that can often be confusing. One approach to introducing the concept of a random variable is to describe it as a type of object distinct from a number, vector, set, or function. One way to do this is to frame a random variable as a measurement, that would result in a number if it were made, but that has not yet been made. Because the process of measurement involves some "random" process that we cannot perfectly predict, such as choosing a person from a crowd or rolling a dice, drawing a card from a deck, or observing if it rains tomorrow, we cannot determine the outcome beforehand but we can describe all the possible outcomes, and understand that they might not all be equally likely. This then leads nicely into the concept of a probability distribution: the probability or "likelihood" of different outcomes is "distributed" across all the different potential outcomes.
- Discrete Random Variables:
  - Uniform distributions,
  - Arbitrary distributions,
  - The Bernoulli and Binomial Distributions.
- Continuous Random Variables:
  - Uniform distributions,
  - Arbitrary function (eg. restricted domain polynomial) distributions,

- The normal distribution.

- Calculating probabilities for all the above,
- Expected Value ($\mathbb{E}[X] = \sum_i x_i p_i$ for discrete random variables for which $P(X = x_i) = p_i$ and $\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x).dx$ for a continuous random variable with probability density function $f(x)$),
- Variance (var$(X) = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$) and Standard Deviation ($\sigma = \sqrt{\text{var}(X)}$),
- The Central Limit Theorem.
- Confidence Intervals:
  - The standard normal ("z-score") confidence interval,
  - The Wald confidence interval for a proportion (z-score normal approximation as per the central limit theorem).

Also important to note that there are some important foundational concepts covered in SACE Stage 1 Mathematics, specifically:

- Calculating mean, median, mode, variance and standard deviation in the sample rather than the population.
- Combinatorics (Combinations, the Factorial, and Permutations).

Also of note is that the AC goes further than SACE in terms of explicitly introducing more foundational concepts here, including:

- Extending combinatorics to explicitly discuss binomial coefficients, Pascals triangle, and the pigeon-hole principle.
- Set-theoretic terminology and notation both in general and applied to probability calculations: Complement, Intersection and Union of sets, Conditional Probabilities, Independence of Random Variables, and Disjoint sets.

### 4.3.2 Complex Numbers

While complex numbers are introduced in two topics in SACE, an introductory topic in Stage 1 Mathematics and an advanced topic in Stage 2 Specialist Mathematics, it is interesting to note that in schools in South Australia year 11 students are often streamed into those planning on taking Stage 2 Specialist Mathematics and those planning on only taking Stage 2 Mathematical Methods, and often the year 11 students not planning on taking Stage 2 Specialist Mathematics will skip topics such as Complex Numbers as they do not appear in Stage 2 Mathematical Methods and instead use the time to focus on the other topics. That said, the key concepts covered in SACE Stage 1 Mathematics are:

- Rational and Irrational Numbers (covered in MS1),
- Introducing the concept of $i = \sqrt{-1}$, complex numbers as $a + ib$.
- Arithmetic ($+$, $-$, $\times$, $\div$) using Conjugates for division.

- Argand Diagram (Complex Plane $\mathbb{C}$).
- Modulus of a complex number.
- Conjugate Roots of Polynomials.

Then these are expended in SACE Stage 2 Specialist Mathematics to also cover:

- Argument and Polar Form, $r\text{cis}(\theta) = r\cos(\theta) + ri\sin(\theta)$ notation.
- de Moivre's Theorem, including negative and fractional indices.
- Geometric properties of the Argand Plane (what happens to complex numbers when you do arithmetic to them — rotations, translations, dilations).
- Using deMoivre's Theorem to find complex roots and factorising polynomials with complex roots.

### 4.3.3 SACE Stage 2 Specialist Mathematics Concepts

#### Key Concepts that are in the Bridging Courses, but are only in SACE Stage 2 Specialist Mathematics and not any other SACE Subject

These are concepts that could in principle be removed from the bridging courses because technically, the bridging courses do not need to cover SACE Stage 2 Specialist Mathematics, only Stage 2 Mathematical Methods as there is a usually an entry level university course whose purpose is to bridge students who did not complete Stage 2 Specialist Mathematics (at the UofA this course is called "Maths IM"). Important to note that although technically these could be removed for the reasons discussed above, often they serve important roles in building understanding and in the purpose, design, and intent of a unit of work and this should be taken into account when making decisions. This list is intended to provide a starting point for that consideration, and in some cases a short note on the importance of a concept to understanding or developing foundational skills will be included. That said, the key concepts covered only in Stage 2 Specialist Mathematics that could potentially be cut from the bridging courses on the technicality discussed above are:

- **Implicit Differentiation**, which is currently covered in MT6 and is only touched on in S2SM6.
- **Normal to a Curve**, introduced in MT6 and is only briefly mentioned in S2SM4 when it is used in the context of vector cross products and equations for a plane — a very different context. It is not covered explicitly in the way it is in MT6 anywhere in SACE, but can be a useful tool for understanding slopes and gradients in the way it is used in MT6.
- **Logistic Models** from MT8. Note that in S2SM6 logistic models are introduced from the perspective of differential equations, quite different to how they are used in MT8, where it is introduced as a model (an equation), not a solution to a differential equation.

- **Integration by Substitution** is covered in MT9 and S2SM5. Note that the calculus focus of many entry level university mathematics courses means it might be beneficial to students to have seen this before when it is covered in those courses, as it often is a difficult concept for them when encountering it for the first time.
- **Vectors in** $\mathbb{R}^3$: MT3 covers vector concepts in both $\mathbb{R}^2$ and $\mathbb{R}^3$, but in SACE the $\mathbb{R}^2$ concepts are covered in Stage 1 Mathematics (S1M9) and the $\mathbb{R}^3$ concepts in Stage 2 Specialist Mathematics (S2SM4). To be specific, the key differentiating $\mathbb{R}^3$ concepts are: vector cross product, and the equation for a plane.
- **Gauss-Jordan Elimination** is introduced in MT4 is not anywhere in SACE. That said, it is implied in S2SM4 in the context of solving $3\times3$ systems of linear equations "using row operations". Note however that basic matrix concepts (matrix multiplication, the $2 \times 2$ determinant and inverse) are introduced in Stage 1 Mathematics (S1M11) however, so some matrix concepts do exist beyond Stage 2 Specialist Mathematics — the concept that is restricted to Stage 2 Specialist is the idea of using matrices to solve systems of equations, specifically.
- **Sketching Rational Functions** (covered in MT7), and some similar advanced "Function and Graph" concepts in S2SM3 are only covered in Stage 2 Specialist Mathematics, but could also serve as useful ways to build understanding of function behaviour and graphical implications of algebraic manipulation.

#### Key Concepts that are in SACE Stage 2 Specialist Mathematics but not in the Bridging Courses

Perhaps less relevant, but for the sake of completeness in this section key concepts that are covered in SACE Stage 2 Specialist Mathematics that are missing from the bridging courses are listed. These are likely less relevant because at the present time the bridging courses are not required to cover the Specialist Mathematics curriculum, but in the future that may change, and in that eventuality this list may be of use:

- **Formal Definition of a Function**, and hence the complement — what is not a function? Relations. The concept of a relation is in the AC and SACE, although it is not used much. The only spots where it would be relevant are restricted to SACE Stage 2 Specialist Mathematics: implicit differentiation (discussed above), parameterised curves, and some advanced function concepts such as one-to-one discussed below.
- **Advanced Function Concepts**: domain and range, function composition, one-to-one, inverse functions, and the absolute value function (covered in S2SM3) are not covered in the bridging courses, and could be useful.

### 4.3.4 Other Key Concept Misalignments Between the Bridging Courses and SACE

In this section, all key concept misalignments between the bridging courses and SACE that are not already covered in one of the sections above will be listed. So that means all concepts that do not align between the Bridging Courses and Stage 1 Mathematics and Stage 2 Mathematical Methods, except those that relate to the content areas of "Probability and Statistics" or "Complex Numbers".

### Concepts Covered in SACE but not in the Bridging Courses

These are concepts that are missing from the bridging courses (listed roughly in decreasing order of importance/ ease of incorporation), and adding them to the bridging courses should probably be considered:

- **Interval Notation** (covered in S1M12): MS1 already introduces the concept of intervals, it would be fairly straightforward to go the one extra step and also introduce interval notation, and MS1 would be the perfect place.

- **Algebraic Rearrangement of Quadratics**: The introduction to quadratics in MS3 is excellent, particularly as the concepts used to introduce them (dilations, translation) are very applicable in the AC and SACE and in introducing many concepts in later topics. However because of how concepts around Polynomials are split between MS3 and MT1, the specific key concept relating to rearranging quadratics algebraically to get them in vertex form and factored form is missed almost halfway between MS3 and MT1. Although these forms are introduced implicitly in MS3 and interpreted, algebraic rearrangement of them could be emphasised more (to better align with S1M2, for example). Adding this to MS3 would also re-enforce the emphasise early in MathsStart on building the foundational skill of rearranging equations, which would serve students well in later topics as well.

- **Orthogonal Projection**: Scalar dot product of vectors is introduced in MT3, but SACE goes a little further, also introducing the concept of orthogonal projection in $\mathbb{R}^2$ in S1M9.

- **Translations and Dilations of Trigonometric Functions**: In MS3 dilations and translations of quadratic functions are considered. It would be useful if these concepts where re-visited in MS6 when looking at graphing/ sketching trigonometric functions, as this is covered in SACE explicitly and also because it connects the concept through multiple different topics and applications (different kinds of functions). This is a concept that is emphasised early in MathsStart and that could be leveraged more by bringing it into each new type of function as they are introduced, with the added benefit that this would also improve the content-alignment to SACE and the AC.

- **Geometric Sequences and Recurrence Relations**: Both the AC and SACE introduce the concept of exponential functions via recurrence relations describing them with geometric sequences. Although this is certainly not the only (or even necessarily the best) way to introduce and understand exponential functions, it is the way prescribed by the AC and SACE curricula, so it might be valuable particularly in avoiding a difference in thinking between students coming out of the bridging courses and those coming straight out of high school, if that is of concern.

### Concepts Covered in the Bridging Courses but not in SACE

There is only one key concept that is left: **Surge Models** are not included in either the AC or SACE, and could potentially be removed from MT8 entirely without impacting on the content alignment of the bridging courses to the high school curricula.

# Chapter 5

# Recommendations

The importance of bridging courses as what is often student's first experience at university means that the bridging courses have a substantial opportunity to impact on both students well-being and retention. The fact that the demographic of students enrolling in these courses represents a slice of our population that is typically less advantaged than average also helps to emphasise the importance of these bridging courses from the perspective of social equity (Lee et al., 2008). The broader social issue of maths anxiety and maths-phobia is also beginning to impact on the Australian economy and will continue to do so at an increasing rate as our economy continues to rely more and more on industries that require maths-competence (King & Cattlin, 2015; Gordon & Nicholas, 2013b). The demographic of students enrolling in the bridging courses over-representing maths-anxiety means that such bridging courses are one of the key places to tackle the broad social issue of negative maths affect. Obviously addressing this issue at earlier stages is also critical, but a multi-pronged approach is important to capturing multiple generations of students simultaneously and hence sparking a broader perspective shift in the general population. In this final chapter, recommendations for improvements that could be made to the mathematics bridging courses run through the MLC at the UofA. Throughout this work, recommendations have been largely split into content alignment recommendations based on document analysis of the Australian high school curricula (see Section 4.3) and non-content recommendations based on the academic literature (see Section 3.6). More detailed discussion of the context and specifics of each recommendation is included in Sections 3.6 and 4.3, but in this chapter the most salient recommendations are summarised, and the ways in which these two avenues of research interact and the importance of considering both simultaneously is emphasised. Throughout the discussion of these recommendations, there are two things that should be kept in mind to contextualise the perspective from which these recommendations are made:

- First, allocation of resources (funding, human resources, etc.) is beyond the scope of this work. So although there are many recommendations that could be made that involve the allocation of additional resources to improvement of the bridging courses, influencing the allocation of resources is not a goal of this work, and so any recommendations that would require substantial additional resources to implement will be deemphasised, but not omitted entirely, from the discussion.

- As implied by the framework introduced in Section 1.2 and much of the literature discussed in Chapter 3, in order to achieve positive outcomes for the students it is critical to simultaneously consider both improvements to their mathematical ability/ performance/ skills (i.e. content knowledge), and their affect towards maths — i.e. directly addressing issues such as maths anxiety, poor maths self-efficacy, etc.

### Content Alignment to high school Curriculum

In line with the second point above, although content changes are listed in Section 4.3 that could be made to bring the bridging courses more closely into alignment with the high school curricula, it would be inadvisable to make all or even a substantial number of these changes at once. Instead, it is important to evaluate changes to content in terms of the learning experience associated to them. The interconnectedness of learning experience, as represented in Figure 1.2, cannot be understated, and is crucial to achieve positive outcomes. If students are having a negative affect reaction to a change in content this is critical to identify early and address either directly (by some direct intervention as discussed in Section 3.6, i.e. a reappraisal approach to adjusting students into a "productive struggle" mindset for example), or by changing the pedagogical approach to how the content is taught, by modifying the structure and order in which the content is taught, or even changing the content taught to result in a better affective reaction from students. One of the common points agreed upon broadly by the literature is that although students affect towards maths impacts on their performance and learning strongly, it is also possible to change students affect towards maths, and that doing so is critical to their success particularly in a bridging course context. Although the relationship between success and affect is not necessarily straightforward (Jansen et al., 2013), introducing content in a way that causes students to experience failure has the potential to be catastrophic if not handled carefully (i.e. in an environment which has already strongly re-enforced a "productive struggle mindset for example), and so adjustments to content should be made with care and with student's affect foremost in mind.

### Self-Paced and Feedback-Focused Assessment

The self-paced and feedback focused approach to assessment is certainly one of the highlights of MathsTrack as it is currently run. Although test anxiety has been delineated from maths anxiety (Kazelskis et al., 2000), they are none-the-less strongly correlated and the effect on reducing test anxiety of the self-pacing of MathsTrack should not be understated. That said, this feedback-focussed approach to assessment, which is widely considered best practice in the teaching profession, is also resource (time) expensive, relating to the first point discussed above (funding restrictions). It would be ideal to be able to expand on this feedback and support capacity in the bridging courses, but this would inevitably require additional resources.

### Encouraging Social Support Network Development

One recommendation coming from the literature that would not require substantial resources however, and would address two of the points raised in Section 3.6, is to incorporate a fixed day/time for bridging courses to come and get help at the MLC drop-in centre (or to book a seperate room for them). Providing tutors specifically would obviously be a question of resources, but would not necessarily even be entirely necessary. One of the primary advantages of having a fixed day/time for the bridging courses to come to a specific place, is that they would be more likely to meet each other, to make friends, to build social support networks that will help them through the bridging courses but also continuing beyond to their further studies. The importance of developing such a social support network, and the potential role of bridging courses in facilitating this, has been discussed widely in the literature (Trotter & Roberts, 2006; Peat et al., 2001; Leese, 2010; Gordon & Nicholas, 2013b) and is strongly supported by the "rite-of-passage" model of Clark and Lovric (2008). If this was organised to be part of the MLC drop-in services, then this would also be an opportunity to further expose the bridging course students to the "learning-to-learn" emphasis that is already an integral part of the MLC drop-in centre, and has been shown to be effective in the context of bridging courses (Zeegers & Martin, 2001).

### Differentiation of Content for Student Trajectory

Given the diversity of the student cohort entering the bridging courses as discussed in Section 1.1, and in particular the variety in their future directions differentiating the content of the bridging courses to tailor to these different trajectories could be of interest. Unfortunately, depending on how this differentiation is implemented, it could potentially be associated to a substantial requirement for additional resources, but with the flipped-learning model already incorporated into much of the bridging courses structure perhaps this cost could be mitigated somewhat. Specifically, the simplest partitioning of content to accommodate the different trajectories seems to be to split the content into three broad areas:

- **Foundational Concepts** such as rearranging equations, polynomials (quadratics mostly), index laws and fractions are important for establishing a good knowledge-base and also would be an ideal place to incorporate the "learning-to-learn" emphasis from the MLC drop-in centre as recommended by Zeegers and Martin (2001).

- **Calculus** focused maths, differentiation, integration, and understanding of functions and graphs are fundamental to the students aiming to study in ECMS at the University of Adelaide or more broadly for students aiming to continue into entry level mathematics and engineering courses as these are traditionally (and ubiquitously) very calculus-focused, and this calculus-emphasis is carried through both engineering and mathematics degrees.

- **Probability and Statistics** concepts on the other hand, which would need to be added to MathsTrack as discussed in Section 4.3, are an increasingly important focus in the sciences, particularly the biological and health sciences degrees in which students have historically been found to struggle with the mathematical (largely statistical) requirements of their degrees (Tariq, 2002).

It makes sense for the foundational concepts to be covered in MathsStart as this was always the intention of the course. MathsTrack in the meantime is entirely focussed on Calculus, so one approach might be to gradually introduce Probability and Statistics topics into MathsTrack, and potentially offer them as alternatives to some of the Calculus topics for students planning on studying degrees which do

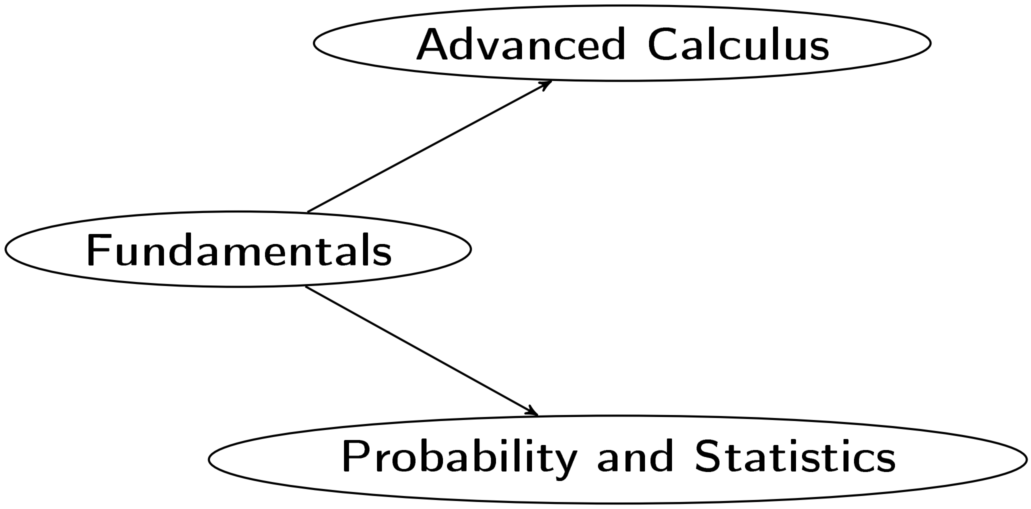


Figure 5.1: Ideal High-Level Content Structure for the University of Adelaide Bridging Courses

not need Calculus (see Figure 5.1), but have a statistics focus (such as health and biological science or medical degrees).

## 5.1 Further Research

During the course of this work, a number of additional research directions have been raised that have fallen beyond the scope of this work to pursue. The following are a list of these potential future research directions:

- A more comprehensive systematic review of the literature surrounding mathematics bridging and the secondary-tertiary transition.
- A review of all Australian universities, bridging courses they offer, and placing the UofA courses into that context. Some work on this was begun, and a list of universities and bridging courses was compiled but was not included in this document. Contact the author for details.
- Writing a probability and statistics topic booklet to be used as part of the MathsTrack course. Some work on harmonising the formatting of the MathsTrack booklets has been done but is not included in this document. Contact either the author, or the MLC at the UofA for details.
- Alignment of final year high school curriculum content across the other states of Australia in comparison to the AC, and internationally.
- Developing a measure of the "granularity" of a curriculum in terms of how much content is grouped into each topic, and investigating the effect of this granularity on learning.
- Relating to the "granularity" concept above, investigating which concepts are taught separately and which are taught together and comparing these groupings into content areas between curricula could lead to insights on impact on

learning. This could then potentially be extended and links be made to educational research into STEM where traditionally separate subjects are taught together.

- Extending the curriculum mapping further back to include more foundational knowledge and assumed skills by including the the year 10 curriculum and prior and linking key concept dependencies between year levels.
- One key difference in emphasis between the bridging courses and the AC/ SACE identified in Chapter 4 is the emphasis the bridging courses place on sketching graphs, and explicitly exploring the connection between how transformations (translations and dilations primarily) of a graph relate to algebraic changes to those functions. An interesting line of research would be to investigate the impact of this emphasis on learning.
- Review the old MT5, and map complex number concepts to the AC and SACE to bring it into alignment with the high school curricula in case it is re-introduced into MathsTrack.
- Long-term effects of maths anxiety interventions.

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# Appendices

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# Appendix A

# Key-Concept Level Description of Topics

Note, topics are identified using the code notation introduced in Table 4.1. The full **topic name is given in bold where applicable**, and then key concepts covered in that topic are listed.

| Code | **Name** and Key Concepts |
|---|---|
| MMu1t1 | **Functions and graphs**: Midpoint of a Line, $y = mx + c$, Quadratic Equations in Vertex and Factorised Forms, Inverse Proportions, Polynomials, Relations, Translations and Dilations |
| MMu1t2 | **Trigonometric functions**: Unit Circle, Radians, SOH CAH TOA, Sine Rule, Cosine Rule, Exact Values, Amplitude/ Period/ Phase, Length of Arc, Area of Sector |
| MMu1t3 | **Counting and probability**: Binomial Coefficients, Set Complement Intersection and Union, Probability, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, Conditional Probability, Independance |
| MMu2t1 | **Exponential functions**: Index Laws, Fractional Indices, Functions, Asymptotes, Graphs |
| MMu2t2 | **Arithmetic and geometric sequences and series**: Arithmetic and Geometric Sequences as Recurrence Relations, Limiting Behaviour, and Partial Sum Formulae, Growth and Decay |
| MMu2t3 | **Introduction to differential calculus** Average Rate of Change, First Principles, Leibniz Notation, Instantaneous Rate of Change, Slope of Tangent, Derivitive of Polynomials, Linearity of Differentiation, Stationary Points, Optimisation, Anti-Derivitives, Interpret Position-Time Graphs |
| MMu3t1 | **Further differentiation and applications**: Define $e$ as $a$ s.t. $\lim_{h\to 0} \frac{a^h - 1}{h} = 1$, Derivities of $e^x$ $\sin(x)$ and $\cos(x)$, Chain Product and Quotient Rules, Second Derivitives |
| MMu3t2 | **Integrals**: Integrate Polynomial Exponential and Trigonometric Functions, Linearity of Integration, Determine Displacement given Velocity, Definite Integrals, Fundamental Theorem of Calculus, (signed) Area Under a Curve |

| Code | **Name** and Key Concepts |
|---|---|
| MMu3t3 | **Discrete random variables**: Frequencies, General Properties, Expected Value, Variance, Standard Deviation, Bernoulli and Binomial Distribtions |
| MMu4t1 | **The logarithmic function**: Logs as Inverse of Exponentials, Log-Scales, Log Laws, Log Function Graphs, Natural Log, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, $\int \frac{1}{x}dx = \ln(x) + c$ for $x > 0$ |
| MMu4t2 | **Continuous random variables and the normal distribution**: Probability Density Function, Cumulative Distribution Function, Probabilites Expected Value, Variance and Standard Deviation as Integrals, Linear Transformation of Random Variables, Normal Distribution using Technology |
| MMu4t3 | **Interval estimates for proportions** Simple Random Sampling, Bias, Sample Proportion, Normal Approximation to the Binomial Proportion, Wald Confidence Interval, Trade-Off Between Width and Level of Confidence |
| | |
| SMu1t1 | **Combinatorics** Multiplication of Possibilities, Factorial Notation, Permutations with and without Repeated Objects, Union of Three Sets, Pigeon-Hole Principle, Combinations, Pascals Triangle |
| SMu1t2 | **Vectors in the plane**: Magnetude and Direction, Scalar Multiplication, Addition and Substraction as a Triangle, Vector Notation, $a\mathbf{i} + b\mathbf{j}$ Notation, Scalar Dot Product, Projection, Parallel and Perpendicular Vectors |
| SMu1t3 | **Geometry**: Notation for Implication ($\Rightarrow$) and Equivalence ($\Leftrightarrow$), Converse ($B \Rightarrow A$) Negation ($\neg A \Rightarrow \neg B$) and Contrapositive ($\neg B \Rightarrow \neg A$), Proof by Contradiction, $\forall$ and $\exists$ Notation, Counter-Examples, Circle Theorems, Quadrilateral Proofs in $\mathbb{R}^2$ |
| SMu2t1 | **Trigonometry**: Graph and Solve Trig Functions, Prove Various Trig Indentities, Reciprocal Trig Functions |
| SMu2t2 | **Matrices**: Notation, Addition and Scalar Multiplication of Matrices, Multiplicative Identity and Inverse, Determinant, Matrices as Transformations |
| SMu2t3 | **Real and complex numbers**: Rationality and Irrationality, Induction, $i = \sqrt{-1}$, Complex Numbers $a + bi$ and Arithmetic ($+$, $-$, $\times$, $\div$), Complex Conjugates, Complex Plane, Complex Conjugate Roots of Polynomials |
| SMu3t1 | **Complex numbers**: Modulus and Argument, Arithmetic ($\times$, $\div$, and $z^n$) in Polar Form, Convert between Polar and Cartesian Form, De Moivre's Theorem, Roots of Complex Numbers, Factorising Polynomials |
| SMu3t2 | **Functions and sketching graphs**: Composition of Functions, One-to-One, Inverse Functions, Absolute Value Function, Rational Functions |
| SMu3t3 | **Vectors in three dimensions**: $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ Notation, Equation for Spheres, Parameterised Vector Equations, Equations of Lines, the Cross Product, Equation for a Plane, Systems of Linear Equation (Elimination Method) and Geometric Interpretation of Solutions, Kinematics via Differentiation of Vector Equations, Projectile and Circular Motion |

| Code | **Name** and Key Concepts |
|---|---|
| SMu4t1 | **Integration and applications of integration** Substitution, $\int \frac{1}{x}dx = \ln\|x\|+c$ for $x \neq 0$, Inverse Trig Functions and their Derivitives, Integrate $\frac{\pm 1}{\sqrt{a^2-x^2}}$ and $\frac{a}{a^2+x^2}$, Partial Fractions, Integration by Parts, Area Between Two Curves, Volume of Solids of Revolution, Numerical Integration using Technology |
| SMu4t2 | **Rates of change and differential equations**: Implicit Differentiation, First-Order Seperable Differential Equations, The Logistic Equation, Kinematics (Rates of Change) |
| SMu4t3 | **Statistical inference**: Central Limit Theorem and the Resulting Confidence Interval for a Mean |
| | |
| S1M1 | **Functions and graphs**: Equations for a Line, Slope, y-intercept, Intersection of Lines, Reciprocal Function, Asymptotes, Functions vs Relations, Domain, Range, Function Notation |
| S1M2 | **Polynomials**: Quadratic Equations in Vertex and Factorised Forms, Quadratic Formula, Completing the Square, The Leading Coefficient and Degree of a Polynomials, Cubics, Quartics |
| S1M3 | **Trigonometry**: Pythagoras, SOH CAH TOA, Cosine Rule, Sine Rule, Unit Circle, Exact Values, Sine and Cosine Functions, Radians, Length of Arc, Area of Sector, Amplitude, Period, Phase, $\tan(x) = \frac{\sin(x)}{\cos(x)}$ |
| S1M4 | **Counting and statistics**: Factorial, Permutations, Multiplication Principle, Combinations, Discrete vs Continuous Random Variables, Mean, Median, Mode, Range, Interquartile Range, Standard Deviation, Normal Distribution, |
| S1M5 | **Growth and decay**: Index and Logarithm Laws, Exponential Functions and their Graphs |
| S1M6 | **Introduction to differential calculus**: Average Rate of Change, First Principles, Notation $f'(x) = \frac{df}{dx}$, $\frac{d}{dx}x^n = nx^{n-1}$, Linearity of Differentiation, Slope of Tangent, Increasing vs Decreasing, Local and Global Maxima and Minima, Stationary Points, Sign Diagram |
| S1M7 | **Arithmetic and geometric sequences and series**: Arithmetic and Geometric Series as Recurrance Relations and Explicit Expressions, Partial Sums, Limiting Behaviour |
| S1M8 | **Geometry**: Circle Properties, Proofs (Direct, Contradiction, and Contrapositive) |
| S1M9 | **Vectors in the plane**: Component (column) vs $ai + bj$ Notation, Length and Direction, Linear Combinations of Vectors, Scalar Dot Product, Projection, Angle Between Two Vectors and Parallel/ Perpendicular, Geometric Proof |
| S1M10 | **Further Trigonometry**: Sketch Trigonometric Functions with Translations and Dilations, Solve for Angles, Trigonometric Identities, Reciprocal Trigonometric Functions |
| S1M11 | **Matrices**: Linear Combinations of Matrices, Matrix Multiplication, The Identity, Inverse Matrices, The $2\times 2$ Inverse, The $2\times 2$ Determinant, Linear Transformations (including rotations, reflections and composition) |

| Code | **Name** and Key Concepts |
|---|---|
| S1M12 | **Real and complex numbers**: Rationals, Irrationals, Interval Notation, Induction, $i = \sqrt{-1}$, Real and Imaginary Components, Complex Conjugates and Arithmetic, Argand Diagram, Modulus, Complex Roots of Polynomals |
| S2MM1 | **Further differentiation and applications**: S1M6, Chain Product and Quotient Rules, $e = 2.718\ldots$, $\frac{d}{dx}e^x = e^x$, $\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$, Second Derivatives, Concavity and Points of Inflection |
| S2MM2 | **Discrete random variables**: Random Variables, Discrete vs Continuous, Probability Functions and Distributions, Properties of Probabilities, Frequency, Expected Value $E[X] = \sum xp(x) = \mu_X$, Standard Deviation $\sigma_X = \sqrt{\sum(x-\mu_X)^2p(x)}$, Uniform Bernoulli and Binomial Distributions |
| S2MM3 | **Integral calculus**: Anti-differentiation, Reversing Chain Rule for $\int f(ax+b)dx$, Linearity of Integration, Finding the Constant of Integration, Area Under the Curve as Upper and Lower Sum Approximations, Definite Integral, Area Between Two Functions and Between a Negative Function and the x-axis, Fundamental Theorem of Calculus, |
| S2MM4 | **Logarithmic functions**: Logs as Inverse of Exponentials, Log-Scales, Log Laws, Sketching $y = a\ln(b(x-c))$, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, For $x > 0$ $\int \frac{1}{x}dx = \ln(x) + c$ |
| S2MM5 | **Continuous random variables and the normal distribution**: $P(X = x) = 0$, Probability Density Function, $\mu_X = \int_{-\infty}^{\infty} xf(x)dx$, $\sigma_X = \int_{-\infty}^{\infty}(x-\mu_X)^2f(x)dx$, $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, Standard Normal $Z = \frac{X-\mu}{\sigma}$, Simple Random Sampling, For $X \sim (\mu, \sigma)$ and $X_i \sim iidX$ Sampling Distributions of $S_n = \Sigma_{i=1}^n X_i$ $(n\mu, \sigma\sqrt{n})$ and $\bar{X}_n = \frac{S_n}{n}$ $(\mu, \frac{\sigma}{\sqrt{n}})$, If $X$ is Normally Distributed, then so are $S_n$ and $\bar{X}_n$, Central Limit Theorem (CLT) |
| S2MM6 | **Sampling and confidence intervals**: Confidence Interval for a Mean using CLT $\left(\bar{x} - z^*\frac{s}{\sqrt{n}}\right) \leq \mu \leq \left(\bar{x} + z^*\frac{s}{\sqrt{n}}\right)$, Wald Interval for a Proportion |
| S2SM1 | **Mathematical induction**: Initial Case and Induction Step |
| S2SM2 | **Complex numbers**: Cartesian vs Polar Form, Real and Imaginary Components, Modulus and Argument, Arithmetic in both Cartesian and Polar Forms, de Moivre's Theorem including Negative and Fractional Powers, Geometric Properties of the Argand Plane, Complex Arithmetic as Transformations, $n^{\text{th}}$ Roots of a Complex Number, Factorising Polynomials with Complex Roots |
| S2SM3 | **Functions and sketching graphs**: Function Composition, Informal Intro to Domain and Range, One-to-One, Inverse Functions, Absolute Value Function, Graphing Rational Functions |

| Code | **Name** and Key Concepts |
|---|---|
| S2SM4 | **Vectors in three dimensions**: Notation, Equations of a Line in $\mathbb{R}^3$, Scalar Dot Product, Vector Cross Product, $\|\mathbf{a} \times \mathbf{b}\|$ is the Area of their Parallelogram, Equation for a Plane in $\mathbb{R}^3$, Systems of Linear Equations, Geometric Interpretation of No/Unique/Infinite Solutions to a System of Linear Equations in $\mathbb{R}^3$ |
| S2SM5 | **Integration techniques and applications**: Integration by Substitution, Using Trigonometric Identities for Integration, Derivatives of Inverse Trigonometric Functions (so $\int \frac{\pm 1}{\sqrt{a^2-x^2}}dx$ and $\int \frac{a}{a^2+x^2}dx$, Integration by Parts, Partial Fractions for Integrating Rational Functions, Area Between two Curves, Volume of Solids of Revolution |
| S2SM6 | **Rates of change and differential equations**: Implicit Differentiation, First-Order Seperable Differential Equations, The Logistic Differential Equation, Parameterised Curves, Example: if $\mathbf{v} = \frac{d}{dt}(x(t), y(t))$ is Velocity, $\|\mathbf{v}\|$ is Speed, and so the Arc Length along the Parameterised Curve is $\int_a^b \sqrt{\mathbf{v} \bullet \mathbf{v}}dt$, Trigonometric Parameterisations (unit circle, and non-circular parameterisations) |
| MS1 | **Numbers & Functions**: Natural Numbers, Integers, Rational Numbers, Real Numbers, Functions, Intervals |
| MS2 | **Linear Functions**: Equation for Linear Functions, Simultaneous Linear Equations, Sketching Linear Inequalities |
| MS3 | **Quadratic Functions**: Sketching a Parabola, General Form of a Quadratic, Translations and Dilations |
| MS4 | **Rational Functions**: Sketching Reciprocal Functions (Hyperbola), Lines of Symmetry, Limits and Asymptotes |
| MS5 | **Trigonometry I**: Pythagoras, Similar Triangles, SOH CAH TOA, Trigonometric and Inverse Trigonometric Functions using Technology, Exact Values |
| MS6 | **Trigonometry II**: Unit Circle, Sketching Trigonometric Functions, Finding all Solutions to Trigonometric Equations, The Sine Rule, The Cosine Rule, Introductory Trigonometric Identities, Radians |
| MS7 | **Exponential Functions**: Index Laws, Sketching Exponential Functions, $e = 2.718\ldots$, Growth and Decay |
| MS8 | **Logarithms**: Natural Logarithm, Logarithm Laws, Using Logarithm to Fit Growth/Decay Functions, Half-Life/ Doubling Time |
| MT1 | **Polynomials**: Polynomial Division and "Remainder Theorem", Factor Theorem Linking Zeros to Factors, Continuous vs Discontinuous Functions, Smoothness, Sketching Factorised Form of Polynomials, Factorising Polynomials, The Quadratic Formula |
| MT2 | **Matrices**: Order, Notation, Linear Combinations of Matrices, Matrix Multiplication (Associative but not Commutative, Distributes across Linear Combinations), The Identity Matrix, Powers of Square Matrices, Matrix Transpose, Systems of Linear Equations, Matrix Inverse, $2 \times 2$ determinant, The $2 \times 2$ Inverse, $n \times n$ Inverses, Elementary Row Operations, |

| Code | **Name** and Key Concepts |
|---|---|
| MT3 | **Vectors and Applications**: Directed Line Segment Notation for Vectors, Magnetude/ Length and Direction, Linear Combinations of Vectors, Component and $a\mathbf{i} + b\mathbf{j}$ Notation, Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$, Scalar Dot Product, Equation for a Plane in $\mathbb{R}^3$ |
| MT4 | **Systems of Linear Equations**: Augmented Matrix for Systems of Linear Equations, Elementary Row Operations, Row-Echelon Form, Solutions to Systems of Linear Equations and Geometric Interpretations in $\mathbb{R}^2$ and $\mathbb{R}^3$, Matrix Inverses by Gauss-Jordan Elimination |
| MT6 | **Differentiation**: Rates of Change, Gradient, First Principles, Limit Notation, Derivative Notation, $\frac{d}{dx}x^n = nx^{n-1}$ (including $n = 0$ and $n = 1$), Linearity of Differentiation, Product Rule, Quotient Rule, Chain Rule, Implicit Differentiation, Normal to a Curve |
| MT7 | **Applications of Differentiation**: Sketching Polynomials and Rational Functions (Intercepts and Asymptotes), Continuity, Sign Diagrams, Increasing and Decreasing, Stationary Points, Points of Inflection, Concavity, Optimisation, |
| MT8 | **Exponential and Logarithm Functions**: Sketching Exponential Functions, $e = 2.718\ldots$, $\frac{d}{dx}e^x = e^x$, Natural Logarithm, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, Growth and Decay, Surge Models, Logistic Models |
| MT9 | **Integration**: Area Under a Curve, Lower and Upper Sums, Definite Integrals, Definite Integrals of Negative Functions, Linearity of Integration, Properties of Definite Integrals, Fundamental Theorem of Calculus, Antiderivatives, Indefinite Integrals, Integrating by Reversing the Chain Rule, Integration by Substitution, Area Between two Curves, Summation Notation (Appendix) |

# References


Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of experimental psychology: General*, *130*(2), 224.

Ashcraft, M. H., & Krause, J. A. (2007, Apr 01). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review*, *14*(2), 243–248. Retrieved from https://doi.org/10.3758/BF03194059 doi: 10.3758/BF03194059

Ashcraft, M. H., Krause, J. A., & Hopko, D. R. (2007). Is math anxiety a mathematical learning disability? In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (p. 329-348). Baltimore, MD, US: Paul H Brookes Publishing.

Bahr, P. R. (2008). Does mathematics remediation work?: A comparative analysis of academic attainment among community college students. *Research in Higher Education*, *49*(5), 420–450. Retrieved from https://doi.org/10.1007/s11162-008-9089-4

Bandura, A. (1997). *Self-efficacy: The exercise of control*. Macmillan.

Barrington, F., & Evans, M. (2016). *Year 12 mathematics participation in australia-the last ten years* (Tech. Rep.). Australian Mathematical Sciences Institute. Retrieved from https://amsi.org.au/publications/participation-in-year-12-mathematics-2006-2016/

Becker, C. B., Darius, E., & Schaumberg, K. (2007). An analog study of patient preferences for exposure versus alternative treatments for posttraumatic stress disorder. *Behaviour Research and Therapy*, *45*(12), 2861 - 2873. Retrieved from http://www.sciencedirect.com/science/article/pii/S0005796707001118 doi: https://doi.org/10.1016/j.brat.2007.05.006

Brown, G. (2009). *Review of education in mathematics, data science and quantitative disciplines: Report to the group of eight universities* (Tech. Rep.). Group of Eight (NJ1). Retrieved from https://eric.ed.gov/?id=ED539393

Burton, L. (1987, Aug 01). From failure to success: Changing the experience of adult learners of mathematics. *Educational Studies in Mathematics*, *18*(3), 305–316. Retrieved from https://doi.org/10.1007/BF00386200

Carlson, M. P. (1999). The mathematical behavior of six successful mathematics graduate students: Influences leading to mathematical success. *Educational Studies in Mathematics*, *40*(3), 237–258. Retrieved from https://doi.org/10.1023/A:1003819513961

Carmichael, C., & Taylor, J. A. (2005). Analysis of student beliefs in a tertiary preparatory mathematics course. *International Journal of Mathematical Education in Science and Technology*, *36*(7), 713–719. Retrieved from https://doi.org/10.1080/00207390500271065

Carmichael, C. S., Dunn, P. K., & Taylor, J. A. (2006). Identifying at-risk students: is it possible in a tertiary preparation course for adults?.

Chang, H., & Beilock, S. L. (2016). The math anxiety-math performance link and its relation to individual and environmental factors: a review of current behavioral and psychophysiological research. *Current Opinion in Behavioral Sciences*, *10*, 33 - 38. Retrieved from http://www.sciencedirect.com/science/article/pii/S2352154616300882 (Neuroscience of education) doi: https://doi.org/10.1016/j.cobeha.2016.04.011

Chubb, I. W., Findlay, C., Du, L., Burmester, B., & Kusa, L. (2012). *Mathematics, engineering & science in the national interest* (Tech. Rep.). Australian Office of the Chief Scientist. Retrieved from http://www.chiefscientist.gov.au/2012/05/mes-report/

Clark, M., & Lovric, M. (2008, Sep 1). Suggestion for a theoretical model for secondary-tertiary transition in mathematics. *Mathematics Education Research Journal*, *20*(2), 25–37. Retrieved from https://doi.org/10.1007/BF03217475

Croft, A. C., Harrison, M. C., & Robinson, C. L. (2009). Recruitment and retention of students–an integrated and holistic vision of mathematics support. *International Journal of Mathematical Education in Science and Technology*, *40*(1), 109–125. Retrieved from https://doi.org/10.1080/00207390802542395

Dreger, R. M., & Aiken Jr, L. R. (1957). The identification of number anxiety in a college population. *Journal of Educational Psychology*, *48*(6), 344.

Faulkner, F., Hannigan, A., & Gill, O. (2010, Jun). Trends in the mathematical competency of university entrants in ireland by leaving certificate mathematics grade. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *29*(2), 76–93. Retrieved from https://doi.org/10.1093/teamat/hrq002

Faust, M. W. (1996). Mathematics anxiety effects in simple and complex addition. *Mathematical Cognition*, *2*(1), 25–62.

Foa, E. B., Hembreee, E. A., Cahill, S. P., Rauch, S. A. M., Riggs, D. S., Feeny, N. C., & Yadin, E. (2005). Randomized trial of prolonged exposure for posttraumatic stress disorder with and without cognitive restructuring: outcome at academic and community clinics. *Journal of consulting and clinical psychology.*, *73*(5).

Foley, A. E., Herts, J. B., Borgonovi, F., Guerriero, S., Levine, S. C., & Beilock, S. L. (2017). The math anxiety-performance link: A global phenomenon. *Current Directions in Psychological Science*, *26*(1), 52-58. Retrieved from https://doi.org/10.1177/0963721416672463 doi: 10.1177/0963721416672463

Galligan, L., & Taylor, J. (2008). Adults returning to study mathematics. In H. Forgasz et al. (Eds.), *Research in mathematics education in australasia 2004-2007* (pp. 99–118). Rotterdam: Sense Publishers.

Godden, G., & Pegg, J. (1993). Identified problems impeding effective evaluation of tertiary bridging mathematics programs. In *Proceedings of the sixteenth annual conference of the mathematics education research group of australasia (merga)* (pp. 297–302).

Gordon, S., & Nicholas, J. (2013a). Prior decisions and experiences about mathematics of students in bridging courses. *International Journal of Mathematical Education in Science and Technology*, *44*(7), 1081–1091. Retrieved from https://doi.org/10.1080/0020739X.2013.823249

Gordon, S., & Nicholas, J. (2013b). Students' conceptions of mathematics bridging courses. *Journal of Further and Higher Education*, *37*(1), 109-125. Retrieved from https://doi.org/10.1080/0309877X.2011.644779 doi: 10.1080/0309877X.2011.644779

Gordon, S., & Nicholas, J. (2015). What do bridging students understand by 'assumed knowledge'in mathematics? *International Journal of Innovation in Science and Mathematics Education (formerly CAL-laborate International)*, *23*(1).

Hackett, G., & Betz, N. E. (1989). An exploration of the mathematics self-efficacy/mathematics performance correspondence. *Journal for Research in Mathematics Education*, *20*(3), 261–273. Retrieved from http://www.jstor.org/stable/749515

Hardin, C. J. (2008). Adult students in higher education: A portrait of transitions. *New Directions for Higher Education*, *2008*(144), 49–57.

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, *21*(1), 33–46. Retrieved from http://www.jstor.org/stable/749455

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning*, *1*, 371–404.

Hourigan, M., & O'Donoghue, J. (2007). Mathematical under-preparedness: the influence of the pre-tertiary mathematics experience on students' ability to make a successful transition to tertiary level mathematics courses in ireland. *International journal of mathematical education in science and technology.*, *38*(4). Retrieved from https://doi.org/10.1080/00207390601129279

House, J. D. (2000). Academic background and self-beliefs as predictors of student grade performance in science, engineering and mathematics. *International Journal of Instructional Media*, *27*(2), 207. Retrieved from https://search.proquest.com/docview/204274271

Howson, A. G., Barnard, D. G., Crighton, N., Davies, A. D., Gardiner, J. M., Jagger, D., ... Steele, N. C. (1995). *Tackling the mathematics problem* (Tech. Rep.). London Mathematical Society, The Institute of Mathematics and its Applications and The Royal Statistical Society.

Hoyles, C., Newman, K., & Noss, R. (2001). Changing patterns of transition from school to university mathematics. *International Journal of Mathematical Education in Science and Technology*, *32*(6), 829–845. Retrieved from https://doi.org/10.1080/00207390110067635

Irwin, E. L., Baker, S., & Carter, B. (2018). What 'counts' as numeracy preparation in enabling education programs? results of a national audit. *Journal of Academic Language and Learning*, *12*(1), A141–A155. Retrieved from http://www.journal.aall.org.au/index.php/jall/article/viewArticle/520

James, S. (2019). *Year 12 mathematics participation in australia 2008 – 2017* (Tech. Rep.). Australian Mathematical Sciences Institute. Retrieved from https://amsi.org.au/preview-year-12-mathematics-participation-in-australia-2008-2017/

Jamieson, J. P., Peters, B. J., Greenwood, E. J., & Altose, A. J. (2016). Reappraising stress arousal improves performance and reduces evaluation anxiety in classroom exam situations. *Social Psychological and Personality Science*, *7*(6), 579-587. Retrieved from https://doi.org/10.1177/1948550616644656 doi: 10.1177/1948550616644656

Jansen, B. R., Louwerse, J., Straatemeier, M., der Ven, S. H. V., Klinkenberg, S., & der Maas, H. L. V. (2013). The influence of experiencing success in math on math anxiety, perceived math competence, and math performance. *Learning and Individual Differences*, *24*, 190 - 197. Retrieved from http://www.sciencedirect.com/science/article/pii/S1041608012001951 doi: https://doi.org/10.1016/j.lindif.2012.12.014

Johnson, P., & O'Keeffe, L. (2016). The effect of a pre-university mathematics bridging course on adult learners' self-efficacy and retention rates in stem subjects. *Irish Educational Studies*, *35*(3), 233-248. Retrieved from https://doi.org/10.1080/03323315.2016.1192481 doi: 10.1080/03323315.2016.1192481

Kajander, A., & Lovric, M. (2005). Transition from secondary to tertiary mathematics: Mcmaster university experience. *International Journal of Mathematical Education in Science and Technology*, *36*(2-3), 149–160. Retrieved from https://doi.org/10.1080/00207340412317040

Kazelskis, R., Reeves, C., Kersh, M. E., Bailey, G., Cole, K., Larmon, M., ... Holliday, D. C. (2000). Mathematics anxiety and test anxiety: Separate constructs? *The Journal of Experimental Education*, *68*(2), 137-146. Retrieved from https://doi.org/10.1080/00220970009598499 doi: 10.1080/00220970009598499

King, D., & Cattlin, J. (2015). The impact of assumed knowledge entry standards on undergraduate mathematics teaching in australia. *International Journal of Mathematical Education in Science and Technology*, *46*(7), 1032–1045. Retrieved from https://doi.org/10.1080/0020739X.2015.1070440

Kitchen, A. (1999). The changing profile of entrants to mathematics at a level and to mathematical subjects in higher education. *British Educational Research Journal*, *25*(1), 57-74. Retrieved from https://onlinelibrary.wiley.com/doi/abs/10.1080/0141192990250105

Klinger, C. M. (2006). Challenging negative attitudes, low self-efficacy beliefs, and math-anxiety in pre-tertiary adult learners. *Connecting voices: Practitioners, researchers & learners in adult mathematics and numeracy*, 164–171.

Klinger, C. M. (2008). On mathematics attitudes, self-efficacy beliefs, and math-anxiety in commencing undergraduate students. In *Crossing borders-research, reflection and practice in adults learning mathematics. proceedings of the 13th international conference on adults learning mathematics* (pp. 88–97).

Klinger, C. M. (2011, Feb). 'connectivism': A new paradigm for the mathematics anxiety challenge? *Adult Learning Mathematics: An International Journal*, *6*(1), 7–19. Retrieved from https://eric.ed.gov/?id=EJ1068259

Lee, S., Harrison, M. C., Pell, G., & Robinson, C. L. (2008). Predicting performance of first year engineering students and the importance of assessment tools therein. *engineering education*, *3*(1), 44–51. Retrieved from https://doi.org/10.11120/ened.2008.03010044

Leese, M. (2010). Bridging the gap: supporting student transitions into higher education. *Journal of further and Higher Education*, *34*(2), 239–251. Retrieved from https://doi.org/10.1080/03098771003695494

LeFevre, J.-A., Kulak, A. G., & Heymans, S. L. (1992). Factors influencing the selection of university majors varying in mathematical content. *Canadian journal of behavioural science*, *24*(3).

Lin-Siegler, X., Ahn, J. N., Chen, J., Fang, F.-F. A., & Luna-Lucero, M. (2016).

Even einstein struggled: Effects of learning about great scientists' struggles on high school students' motivation to learn science. *Journal of Educational Psychology*, *108*(3), 314.

Luk, H. S. (2005). The gap between secondary school and university mathematics. *International Journal of Mathematical Education in Science and Technology*, *36*(2-3), 161–174. Retrieved from https://doi.org/10.1080/00207390412331316988

Lyons, I. M., & Beilock, S. L. (2012). When math hurts: math anxiety predicts pain network activation in anticipation of doing math. *PloS one*, *7*(10), e48076.

Ma, X., & Xu, J. (2004). The causal ordering of mathematics anxiety and mathematics achievement: a longitudinal panel analysis. *Journal of Adolescence*, *27*(2), 165 - 179. Retrieved from http://www.sciencedirect.com/science/article/pii/S0140197103001064 doi: https://doi.org/10.1016/j.adolescence.2003.11.003

MacGillivray, H. (2009). Learning support and students studying mathematics and statistics. *International Journal of Mathematical Education in Science and Technology*, *40*(4), 455–472. Retrieved from https://doi.org/10.1080/00207390802632980

McGivney, V. (1996). *Staying or leaving the course: Non-completion and retention of mature students in further and higher education*. National Institute of Adult Continuing Education.

McNally, R. J. (2007). Mechanisms of exposure therapy: How neuroscience can improve psychological treatments for anxiety disorders. *Clinical Psychology Review*, *27*(6), 750 - 759. Retrieved from http://www.sciencedirect.com/science/article/pii/S0272735807000074 (New Approaches to the Study of Change in Cognitive Behavioral Therapies) doi: https://doi.org/10.1016/j.cpr.2007.01.003

Mohandas, R., Wei, M. H., & Keeves, J. P. (2003). Evaluation and accountability in asian and pacific countries. In *International handbook of educational research in the asia-pacific region* (pp. 107–121). Springer.

Moses, L., Hall, C., Wuensch, K., Urquidi, K. D., Kauffmann, P., Swart, W., ... Dixon, G. (2011). Are math readiness and personality predictive of first-year retention in engineering? *The Journal of Psychology*, *145*(3), 229-245. Retrieved from https://doi.org/10.1080/00223980.2011.557749

Murtagh, L. (2010). They give us homework! transition to higher education: the case of initial teacher training. *Journal of Further and Higher education*, *34*(3), 405–418. Retrieved from https://doi.org/10.1080/0309877X.2010.484057

Murtaugh, P. A., Burns, L. D., & Schuster, J. (1999). Predicting the retention of university students. *Research in higher education*, *40*(3), 355–371. Retrieved from https://link.springer.com/article/10.1023/A:1018755201899

Nicholas, J., Poladian, L., Mack, J., & Wilson, R. (2015). Mathematics preparation for university: entry, pathways and impact on performance in first year science and mathematics subjects. *International Journal of Innovation in Science and Mathematics Education (formerly CAL-laborate International)*, *23*(1).

Nicholas, J., & Rylands, L. J. (2015). Hsc mathematics choices and consequences for students coming to university without adequate mathematics preparation. *Reflections: Journal of the Mathematical Association of New South Wales*, *40*(1), 2–7. Retrieved from https://researchdirect.westernsydney.edu.au/islandora/object/uws:29593

Organisation for Economic Co-operation and Development (OECD). (2013). *Programme for International Student Assessment (PISA) 2012 results: ready to learn: students' engagement, drive and self-beliefs (volume iii): preliminary version*. PISA, OECD, Paris, France. Retrieved from http://www.oecd.org/pisa/keyfindings/pisa-2012-results-volume-iii.htm (viewed 4 Feb 2019)

Pajares, F. (1996). Self-efficacy beliefs in academic settings. *Review of Educational Research*, *66*(4), 543–578. Retrieved from https://doi.org/10.3102/00346543066004543

Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary Educational Psychology*, *24*(2), 124 – 139. Retrieved from https://doi.org/10.1006/ceps.1998.0991

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *American Psychological Association*, *86*(2), 193–203. Retrieved from http://dx.doi.org/10.1037/0022-0663.86.2.193

Pajares, F., & Miller, M. D. (1997). Mathematics self-efficacy and mathematical problem solving: Implications of using different forms of assessment. *The Journal of Experimental Education*, *65*(3), 213-228. Retrieved from https://doi.org/10.1080/00220973.1997.9943455

Park, D., Ramirez, G., & Beilock, S. L. (2014). The role of expressive writing in math anxiety. *Journal of experimental psychology. Applied*, *20 2*, 103-11.

Peat, M., Dalziel, J., & Grant, A. M. (2001). Enhancing the first year student experience by facilitating the development of peer networks through a one-day workshop. *Higher Education Research & Development*, *20*(2), 199–215. Retrieved from https://doi.org/10.1080/07294360123888

Poladian, L., & Nicholas, J. (2013). Mathematics bridging courses and success in first year calculus. In *Proceedings of the 9th delta conference on the teaching and learning of undergraduate mathematics and statistics* (pp. 150–159).

Ramirez, G., Shaw, S. T., & Maloney, E. A. (2018). Math anxiety: Past research, promising interventions, and a new interpretation framework. *Educational Psychologist*, 1–20.

Robinson, M. (2003). Student enrollment in high school ap sciences and calculus: How does it correlate with stem careers? *Bulletin of Science, Technology & Society*, *23*(4), 265-273. Retrieved from https://doi.org/10.1177/0270467603256090

Rylands, L. J., & Coady, C. (2009). Performance of students with weak mathematics in first-year mathematics and science. *International Journal of Mathematical Education in Science and Technology*, *40*(6), 741–753. Retrieved from https://doi.org/10.1080/00207390902914130

Suárez-Pellicioni, M., Núñez-Peña, M. I., & Colomé, À. (2016, Feb 01). Math anxiety: A review of its cognitive consequences, psychophysiological correlates, and brain bases. *Cognitive, Affective, & Behavioral Neuroscience*, *16*(1), 3–22. Retrieved from https://doi.org/10.3758/s13415-015-0370-7 doi: 10.3758/s13415-015-0370-7

Supekar, K., Iuculano, T., Chen, L., & Menon, V. (2015). Remediation of childhood math anxiety and associated neural circuits through cognitive tutoring. *Journal of Neuroscience*, *35*(36), 12574–12583. Retrieved from http://

www.jneurosci.org/content/35/36/12574 doi: 10.1523/JNEUROSCI.0786-15.2015

Tariq, V. N. (2002). A decline in numeracy skills among bioscienceunder graduates. *Journal of Biological Education*, *36*(2), 76-83. Retrieved from https://doi.org/10.1080/00219266.2002.9655805 doi: 10.1080/00219266.2002.9655805

Taylor, J. A., & Galligan, L. (2006). Mathematics for maths anxious tertiary students: Integrating the cognitive and affective domains using interactive multimedia. *Literacy and numeracy studies*, *15*(1), 23. Retrieved from https://epubs.scu.edu.au/tlc_pubs/126/

Thomas, J., Muchatuta, M., & Wood, L. (2009). Mathematical sciences in australia. *International Journal of Mathematical Education in Science and Technology*, *40*(1), 17–26. Retrieved from https://doi.org/10.1080/00207390802597654

Trotter, E., & Roberts, C. A. (2006). Enhancing the early student experience. *Higher Education Research & Development*, *25*(4), 371–386. Retrieved from https://doi.org/10.1080/07294360600947368

Tsui, L. (2007). Effective strategies to increase diversity in stem fields: A review of the research literature. *The Journal of Negro Education*, 555–581. Retrieved from www.jstor.org/stable/40037228

Tyler, R. W. (1949). *Basic principles of curriculum and instruction.* University of Chicago Press.

Varsavsky, C. (2010). Chances of success in and engagement with mathematics for students who enter university with a weak mathematics background. *International Journal of Mathematical Education in Science and Technology*, *41*(8), 1037–1049. Retrieved from https://doi.org/10.1080/0020739X.2010.493238

Wang, Z., Lukowski, S. L., Hart, S. A., Lyons, I. M., Thompson, L. A., Kovas, Y., . . . Petrill, S. A. (2015). Is math anxiety always bad for math learning? the role of math motivation. *Psychological Science*, *26*(12), 1863-1876. Retrieved from https://doi.org/10.1177/0956797615602471 (PMID: 26518438) doi: 10.1177/0956797615602471

Wood, L. (2001). The secondary-tertiary interface. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (Vol. 7, pp. 87–98). Kluwer, Netherlands.

Young, C. B., Wu, S. S., & Menon, V. (2012). The neurodevelopmental basis of math anxiety. *Psychological Science*, *23*(5), 492-501. Retrieved from https://doi.org/10.1177/0956797611429134 (PMID: 22434239) doi: 10.1177/0956797611429134

Zeegers, P., & Martin, L. (2001). A learning-to-learn program in a first-year chemistry class. *Higher Education Research & Development*, *20*(1), 35–52. Retrieved from https://doi.org/10.1080/07924360120043630