2 Unit Bridging Course – Day 4

The derivative of a function

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Derivative of a function

A derivative is concerned with how one quantity changes with respect to another quantity, in other words a rate of change.

The derivative of the function $y = f(x)$ with respect to x will show us how y changes as the value x changes. It gives us the slope, or gradient of the function.

The derivative of $y = f(x)$ with respect to x is represented by the following notations:

$$f'(x), \quad \frac{d}{dx}(f(x)), \quad \frac{df}{dx} \quad \text{or} \quad \frac{dy}{dx}.$$

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Differentiation of linear functions

Differentiating a linear function is to simply find the gradient or slope of that function. You learnt about gradients of linear functions in Day 2.

For example:

1. Differentiate $f(x) = 3x - 2$.
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2. Differentiate $f(x) = 9$.
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   The slope of y is -5, so $\dfrac{dy}{dx} = -5$.

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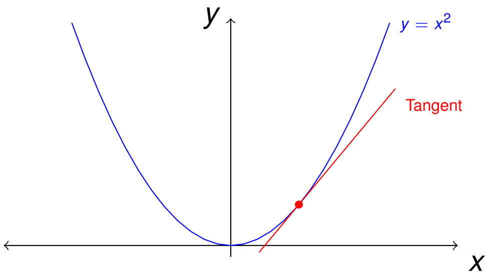
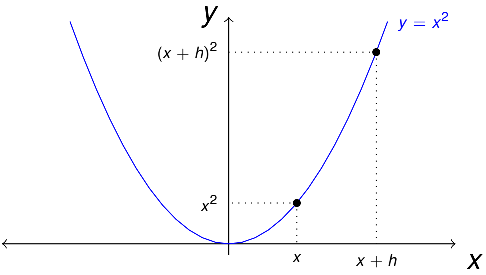


Differentiation of a quadratic

To find the gradient of the tangent to the curve $y = x^2$ we first take an arbitrary point (x, y) that is on the curve.

Then take another point on the curve with the x-coordinate $x + h$, where h is a small number. Its corresponding y-coordinate is $(x + h)^2$.



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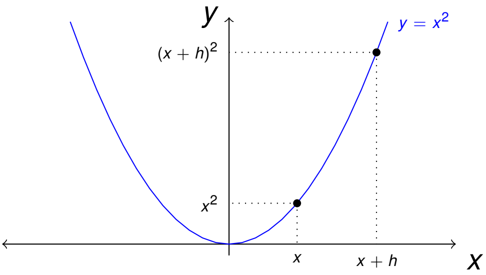
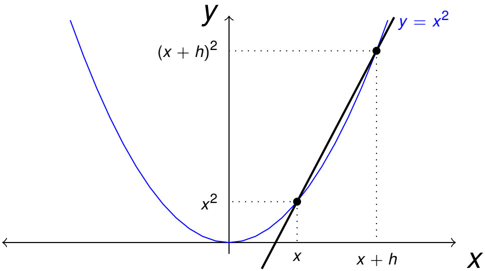


Differentiation of a quadratic

Draw a line through the 2 points.



The gradient of this line is:

$$\frac{\text{change in } y}{\text{change in } x} = \frac{(x+h)^2 - x^2}{(x+h) - x} = \frac{2xh + h^2}{h} = 2x + h.$$

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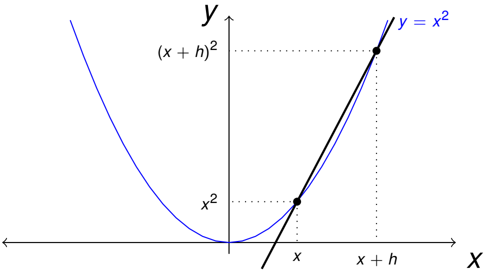

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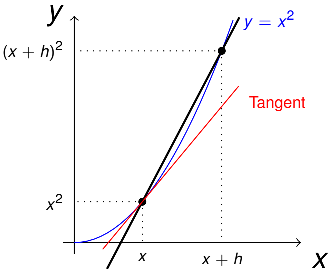

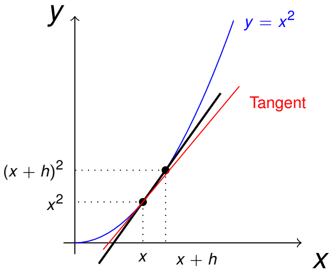

Differentiation of a quadratic

As the value of h approaches 0, the gradient of the line through the two points approaches the gradient of the tangent to the curve, i.e. $2x + h$ approaches $2x$.

Hence the gradient of the tangent to $y = x^2$ at point (x, y) is $2x$ or $\frac{dy}{dx} = 2x$.

This method is called the *Differentiation By First Principles*. In general, the derivative of a function f at x is given by:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if it exists.

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So, putting it all together, if $f(x) = x^2$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2}{h} \\
&= \lim_{h\to 0} 2x + h \\
&= 2x.
\end{aligned}
$$

Some differentiation rules

The following are some rules of differentiation:

- $\dfrac{d}{dx}(\text{constant}) = 0, \qquad \dfrac{d}{dx}(x) = 1, \qquad \dfrac{d}{dx}(x^2) = 2x.$
- $\dfrac{d}{dx}(kf(x)) = k\dfrac{d}{dx}(f(x))$, where k is a constant.
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Example

Differentiate $y = 5x^2 + 3x - 4$.

$$\frac{dy}{dx} = \frac{d}{dx}(5x^2 + 3x - 4) = \frac{d}{dx}(5x^2) + \frac{d}{dx}(3x) - \frac{d}{dx}(4).$$

Now

$$\frac{d}{dx}(5x^2) = 5\frac{d}{dx}(x^2) = 5 \times 2x = 10x,$$

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Hence $\frac{dy}{dx} = 10x + 3.$

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Differentiate $f(x) = 2 - 4x - x^2$.

$$f'(x) = \frac{d}{dx}(2) - \frac{d}{dx}(4x) - \frac{d}{dx}(x^2).$$

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$$\frac{d}{dx}(2) = 0,$$

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Practice Questions

Differentiate the following:

1. $f(x) = 2x - 5$
2. $y = 9 - 2x$
3. $f(x) = 3x^2 + 4x - 5$
4. $f(x) = x^2 - 4x - 6$
5. $y = x^2 - 5x$
6. $m = 2n^2 - 2n + 1$
7. $y = 7$
8. $q = p - 6p^2$
9. $f(a) = 4a^2 + 5a - 9$
10. $f(x) = 6x - 4x^2$.

Answer

Answers to practice questions:

1. $f'(x) = 2$
2. $\frac{dy}{dx} = -2$
3. $f'(x) = 6x + 4$
4. $f'(x) = 2x - 4$
5. $\frac{dy}{dx} = 2x - 5$
6. $\frac{dm}{dn} = 4n - 2$
7. $\frac{dy}{dx} = 0$
8. $\frac{dq}{dp} = 1 - 12p$
9. $f'(a) = 8a + 5$
10. $f'(x) = 6 - 8x$.