
University Mathematics Bridging Courses: MathsStart, MathsTrack, A Review of Existing Approaches and Recommendations for Moving Forward.

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SCHOOL OF EDUCATION



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Abstract

In this thesis the University of Adelaide's mathematics bridging courses: MathsStart and MathsTrack are discussed in detail. Their context and purpose is reviewed, their content is aligned to national and state highschool curriculums, as well as appropriate entry-level university courses. The pedagogy and excellence in educational practice demonstrated through the University of Adelaide's maths learning center, which offers these bridging courses, is examined with an eye to learning from their exemplary approach to supporting what can be one of the most vulnerable cohorts of students attempting to enter into tertiary education. Ultimately, recommendations are made both in terms of exemplary practice that can be generalised to other such courses accross Australia, and also in terms of restructuring of the courses to better align them with national and state curriculums, while also tailoring their content to better suit the cohort of students enrolling in these courses.

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Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

I give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines.

Signed:

Date:

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My thanks also extend to Igusti Darmawan, my supervisor from inside the school of education, for his guidance but mostly support and his willingness to allow me to pursue a somewhat unorthodox project of my own construction, despite it being quite outside of his speciality area of educational research, none-the-less supporting me in my project.

- Lynda
- Linda
- Brendan, for doing his best,
- Ed
- Saman?
- Jason?
- Amanda?
- Trev?
- Students?
- ...

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Chapter 1

Introduction

University mathematics bridging courses serve an important stop-gap role in the Australian educational system, and other educational systems internationally.

This project can be thought of as consisting of two broad categories of work:

- A literature review of Australian mathematics bridging courses, a state of the field of research in this area, and commentary on the role, purpose, and approaches important to implementing effective and impactful mathematics bridging courses in Australia and internationally.
- Reflection on the mathematics bridging courses offered by the University of Adelaide: MathsStart and MathsTrack with a focus on existing strengths, and directions for potential improvement. This will be broken into two sub-sections of work, with one of the bigger contributions I offer being a curriculum mapping from the Australian Curriculum (AC) to South Australian Certificate of Education (SACE) through to MathsStart and MathsTrack. This curriculum mapping suggests potential areas for modification of the mathematics bridging courses to more closely align them with the AC and SACE. The second sub-section of work will be on the non-content aspects of the bridging courses (structure, assessment, timing, feedback, etc.), the strengths of their approaches in comparison to others, potential weaknesses, and recommendations moving forward both for MathsStart and MathsTrack, and for university mathematics bridging courses more broadly.

This thesis will be structured as follows:

- The remainder of this introductory chapter (Chapter 1), I will give a broad overview of the concepts, challenges, and setting for this project.
- In Chapter 2 I will provide an in-depth discussion of the existing literature, what is known, approaches attempted in the past both in Australia and internationally, and some deeper discussion on some of the particularly relevant related concepts, such as maths anxiety.
- One of the major contributions of this work is the curriculum mapping of the AC to SACE, to the content currently in MathsStart and MathsTrack, with commentary on how this mapping connects with typical first-year university mathematics courses. This mapping is discussed in Chapter 3, and will identify gaps and mis-alignment, discuss the tension between different perspectives on

the role of university mathematics bridging courses and how this impacts on content decisions, and potential modifications to the bridging courses content that would allow them to be more closely aligned with the AC should that be desirable.

- Finally, I will wrap up with commentary on what is being done well, recommendations for how to improve, and a summary of the work I have done outside of this thesis to generate resources and content that can be used to improve these programs moving forward in Chapter 4.

1.1 The Role of University Mathematics Bridging Courses in General

Students will usually enroll in university mathematics bridging courses because they are required to demonstrate a certain level of mathematical knowledge/ competence before commencing study at university, but either do not meet those requirements, or do but feel a lack of confidence in their abilities and feel like they need to refresh/ revise/ learn some mathematics prior to commencing their studies.

Reasons why these students do not either meet the entry requirements, or feel a lack of confidence in their abilities can be quite varied:

- A long period of time may have passed since they last studied mathematics (or studied at all). Adult-entry students are over-represented in bridging courses (REFERENCE?).
- They may have performed poorly in mathematics in highschool.
- They may have chosen not to study mathematics at a higher level in highschool.
- They may suffer from maths anxiety (which would make them likely to fit into the above two categories as well).

The role of mathematics bridging courses is to take these students, and:

- Bridge their content knowledge so they are prepared for university entry.
- Support the growth of their confidence and self-efficacy surrounding mathematics.
- Ultimately prepare them to be successful in a university context.

What content should be taught in a university bridging course is actually a question that has dramatically different answers from different perspectives on the role of such a course, even when restricting the question to purely knowledge-based content (and excluding the teaching of self-efficacy etc.):

- If you take the perspective that the role of such a course is to fill in the gaps in student's knowledge left from an incomplete or maths-light highschool education, then the content that should be taught should be up to and including the advanced year 12 Australian curriculum. This is particularly appropriate if you do not know the direction of the students, or if they are potentially just doing the bridging course with you and they are planning on studying a degree at a different university say, interstate.

- If you take the perspective that the role of such a course is to prepare students for entry into the particular courses they are about to commence studying, the content relevant to them will be dramatically different. The senior mathematics Australian curriculum is extremely generalist and contains many topics that would be completely irrelevant to any particular field of study.

In terms of choosing what content to teach in a university bridging course, the above two competing perspectives will often create tension between each other, making finding a happy compromise a difficult endeavour.

1.2 Context for the University of Adelaide Bridging Courses

The University of Adelaide offers two mathematics bridging courses, MathsStart and MathsTrack, through their maths learning center. According to the staff at the maths learning center, the vast majority of students enrolling in their bridging courses are aiming to end up in one of three places:

- Studying a tertiary degree at The University of Adelaide (approximately 60–70% of bridging courses at any one time),
- Studying a tertiary degree at James Cook University, or
- In the defence forces.

Only about a single student will not fit into any of the three categories above at any one time, so thinking of this as being the complete cohort of students is fairly close to being accurate. The distribution within these categories can also be broken down and the most common trends considered:

- Of the students aiming to enroll in a tertiary degree at the University of Adelaide, about 50% are aiming to study something in the Faculty of Engineering, Computer and Mathematical Sciences (ECMS) (i.e. Engineering, Mathematics, Computer Science, etc.), and about 10% are aiming to study something in the sciences, often veterinary science or oral health.
- Of the students aiming to enroll at James Cook University, most are aiming to enroll in medical degrees, with some interested in marine biology or veterinary science — broadly biological science in large.
- Of the students aiming to enlist in the defence forces, the majority of those enrolled in the bridging courses are doing so to meet their pre-requisite mathematics knowledge criteria for airforce pilot training.

To meet the needs of these cohort of students, it is clear there are two broad areas of content knowledge that different students will need:

- Calculus focused maths, differentiation, integration, and understanding of functions and graphs are fundamental to the students aiming to study in ECMS at the University of Adelaide, as most entry level mathematics and engineering subjects are very calculus-focused, and this calculus-emphasis is carried through both engineering and mathematics degrees.

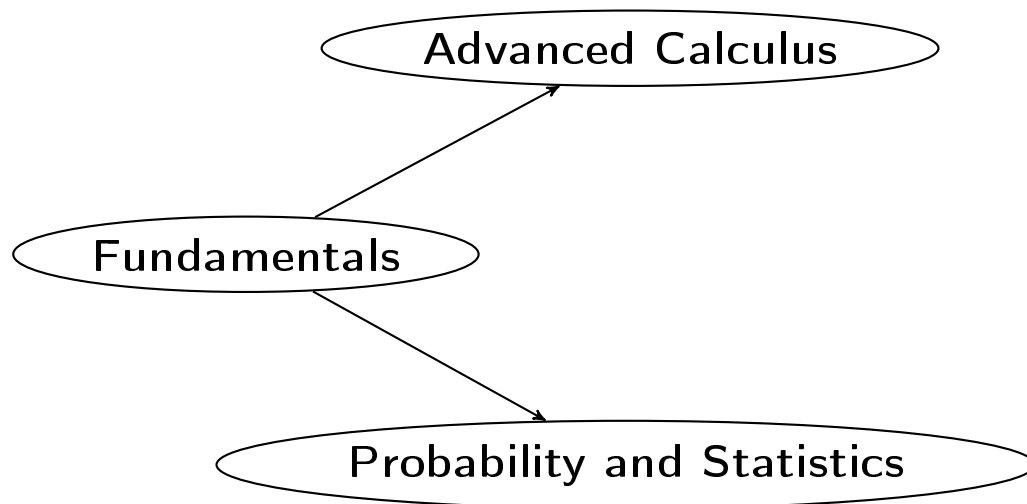


Figure 1.1: Ideal High-Level Content Structure for the University of Adelaide Bridging Courses

- Probability/ Statistics are an important focus of the science, with particularly biological science students having been found to struggle with the mathematical (largely statistical) requirements of their degrees (ADD REFERENCE HERE?).

Both of these focus-areas require a base of content knowledge of fundamentals, things like re-arranging equations, fractions, log-laws, etc. So it makes sense to structure the content of the bridging courses in a way aligned with this thinking, in order to best meet the needs of the students enrolled in the programs.

Chapter 2

Literature Review

2.1 Bridging Courses

TODO List (Papers to read in more detail, and incorporate into discussion):

- (Nicholas & Rylands, 2015) is very relevant but I can't find the actual paper, just references to it. Same with a number of Conference presentations by Nicholas, I got one by chance but it would be good to get more through more official channels.
- (Gordon & Nicholas, 2015) ... got pdf need to read
- (Nicholas, Poladian, Mack, & Wilson, 2015) ... got pdf need to read
- (Kajander* & Lovric, 2005) Multiple papers reference this study, I should read it and comment. According to (Gordon & Nicholas, 2013b) "(Kajander* & Lovric, 2005) found that time spent learning mathematics in the final years of high school was crucial for students' success in university calculus courses."
- (Clark & Lovric, 2008) Note the absence of a theoretical model and propose one.
- (Irwin, Baker, & Carter, 2018) Talks about the importance of alternate avenues to access education.
- It would be nice to get a copy of (Vygotsky, 1978).
- It would be nice to get a copy of (Wood, 2001).
- It would be nice to get a copy of (Bandura, 1997)
- Some of the books I found would be good to read more thoroughly too, (Volmink, 1994) for example, and/or (McGivney, 1996) for example.

2.1.1 "The Mathematics Problem"

"The Mathematics Problem" is a term originally coined by Howson et al. (1995) but that has continued to be relevant to the present day, receiving even greater attention and research in recent times. It refers to the trend of declining interest and participation of final year highschool students in mathematics, and the carry-over

effects this has on their success in tertiary education, and the impacts this has on the supply of highly technically skilled workers for industries which have increasing demand for graduates educated and skilled in either mathematical fields or other technical fields that in turn require competency in mathematics.

Barrington and Evans (2016) shows that in Australia, although the number of both advanced and intermediate mathematics year 12 students was increasing over the ten years from 2006 to 2015 (as the overall population of total year 12 students increased), the percentage participation in these subjects steadily declined. James (2019) updates the figures of Barrington and Evans (2016) with data up to 2017, showing a continuation of the same steady trend. These reports also highlight the significant gender gap that exists in mathematics participation in final year high-school students. The gender gap is more dramatic in advanced level mathematics than in the intermediate level, with 37.8% of advanced mathematics year 12 students identifying as female, especially when considering that 51.8% of year 12 students of that year were female. 2017 saw a significant jump in intermediate level mathematics participation by female students, with there being more female students than males for the first time in recorded history (James, 2019). The gender gap in mathematics education is a significant issue that needs to be taken into account when considering university mathematics entry, particularly as the gap is most pronounced in the advanced level subjects towards which are targeted at university entry. It is an issue recognised by the Australian Mathematical Sciences Institute (AMSI), who have committed significant resources towards programs intended to address this inequity. Perhaps the uptick in female student participation in intermediate level mathematics in 2017 could be partly attributed to some of these programs, such as the [CHOOSEMATHS](#) project. (Brown, 2009) gives a shocking wider-view picture of this overall trend, specifically that the proportion of year 12 students studying intermediate or advanced level mathematics has declined by 22% and 27% respectively from 1995 to 2007.

Observation, concern surrounding, and research of this decline in mathematics participation in senior highschools are not limited to Australia (Hourigan & O'Donoghue, 2007; Hoyles, Newman, & Noss, 2001). (Hoyles et al., 2001) as well as (Luk, 2005) further connect this trend to another: the apparent divergence of content (curriculum) between senior secondary and tertiary education. This divergence of curriculum is a point that will be explored extensively in Chapter ??.

QUOTE from (Gordon & Nicholas, 2013b):

(Thomas, Muchatuta, & Wood, 2009) argue that, while mathematics is used to solve problems in many industries, its contributions are often invisible, making it difficult for the community to see its value. They report that negative community attitudes towards mathematics and the perceived difficulty of its study make students less likely to select mathematics over other subjects at school. Failure to choose higher-level mathematics courses in high school can have serious consequences both for a student's success in university mathematics and on whether a student continues with his or her mathematical studies.

2.1.2 Effects of Conditions on Entry to University Education

TODO (Papers to review on the topic and include into discussion):

- (King & Cattlin, 2015) On the effects of relaxing tertiary entry conditions in Australia.

QUOTES:

In Australia, the situation concerning the mathematical readiness of undergraduate students is exacerbated by university entry structures. Many universities, including the university featured in this study, do not have subject prerequisites for entry into their degree programmes. Rather, the 'assumed knowledge' for each programme is published, including the mathematics subjects that students are 'assumed' to have studied at school. However, students may be accepted into a degree programme, such as engineering, science or economics, even if they do not have that assumed knowledge. Consequently, the mathematical underpreparedness of commencing undergraduate students is an important issue for university teachers not only in mathematics itself but for many other disciplines. (Gordon & Nicholas, 2013b)

2.1.3 Models and Frameworks

Self-Efficacy, Maths Anxiety, and Bandura's Social Cognitive Theory

Quote from (Galligan & Taylor, 2008):

In a quantitative approach, Klinger (2006) investigated attitudes, self-efficacy beliefs, and mathematics anxiety of a diverse group of pre-tertiary adult learners using a self-designed survey and pre- and post-testing. He found that although students initially shared the negative views reported in the wider population, these views changed significantly by the conclusion of the topic(s). He concluded that adults' perceptions of, and capacity to engage with, mathematical content was strongly influenced by early learning experiences, but like authors before, confirmed that these could be changed. In a later study, Klinger (2008) repeated the analysis with commencing arts, humanities, and science students and found the arts/humanities students to have significantly lower mathematical self-efficacy and poorer attitudes than the science students. The results were strongly linked with gender, with females scoring lower than males. He concluded that these kinds of results indicated that undergraduate support programs would have to address such poor attitudes to enable students to succeed and develop mathematical literacy necessary for their discipline.

In two similar quantitative studies underpinned by Bandura's social cognitive theory (C. S. Carmichael, Dunn, & Taylor, 2006; C. Carmichael & Taylor, 2005), motivation (particularly self-efficacy) and other factors such as prior knowledge were examined as measures of prediction of success in a university preparatory mathematics course. Results suggested

that students' assessment of their confidence was based, in part, on their current levels of knowledge and skills, and that this assessment influenced their current performance. Females and those who had been away from study for a long period of time had lower entering levels of confidence and self-efficacy than males and recent school leavers, but final performance in the course was not significantly different.

(Taylor & Galligan, 2006) used conversation theory framework to design a multimedia package to simultaneously develop students' mathematical skills while improving mathematics confidence. The evaluation of this program indicated the effectiveness of this approach. It is apparent through the work discussed above that although attitudes and beliefs about mathematics are important for students enrolled in bridging programs, the programs can change students' attitudes and beliefs about mathematics as well as their achievement. An alternative to the above is to design programs that can assist in self-selection. Egea (2004) has designed and evaluated an online self assessment tool that assists students to choose the appropriate level of bridging mathematics program. It has been specifically designed with mathematics anxious students in mind, and participating students indicated that the test was 'enjoyable', 'fun', 'easy', and even 'relaxing', confirming that its design had addressed both the cognitive and affective domains.

A new, specific, framework

Clark and Lovric (2008) suggest thinking of the secondary-tertiary mathematics education transition as a 'rite of passage' and to leverage the extensive knowledge of such events from fields such as anthropology and culture studies (relating concepts such as culture shock), to allow us to better understand this transition and design interventions that will be more effective.

2.1.4 Temporary Section: Quotations from Selected Relevant Papers

Quote from (Galligan & Taylor, 2008):

Miller-Reilly (2006), in a significant long term analysis of two large bridging courses and one individual bridging program in New Zealand, used quantitative and qualitative measures to compare students' reactions. These multiple strands of evidence provided a complex overall picture of three largely successful teaching approaches. A one-to-one supervised course focused on understanding fear of mathematics and early mathematics experiences. The course empowered the student who came to believe that mathematics was a creative and enjoyable process. A second course (100 students) focused on the mathematization of realistic situations. Here, students came to regard mathematics as useful, interesting, and relevant to real life. The third course (100 students) was a carefully structured re-introduction of mathematics. The students appreciated the course and were pleased that they could now do mathematics that they could not do in school. Students in all programs were highly motivated,

mature, and had not seen formal mathematics for some years. One surprising result from the study was that if students were unsuccessful they were, in fact, worse off than before, and often confused. A significant component of the study was the focus placed on dealing with students' mathematics anxiety or fear. Quantitative measures and qualitative descriptors indicated a decrease in mathematics anxiety throughout the duration of the three programs, and in the larger courses this correlated with achievement. Beliefs about mathematics in general, however, did not necessarily change, although students in the larger courses did see the practical nature of mathematics.

Quotes from (Gordon & Nicholas, 2013b):

- A number of universities offer mathematics bridging courses – short preparatory courses available before students commence their degree programme – as one of the ways to provide students with a way forward with their chosen degree programme, and to ameliorate students' difficulties with mathematics (Croft, Harrison, & Robinson, 2009; MacGillivray, 2009). Recent reviews (Galligan & Taylor, 2008) of the limited research into bridging mathematics in the Australasian region have indicated consistent areas of investigation, including evaluation of specific courses, diagnostic tests and other ways of determining students' needs and overcoming mathematics anxiety.
- The extent to which self-motivation and independent learning are required at university can be particularly problematic for students coming to university for the first time (Murtagh, 2010) and has been shown to be a source of concern to many incoming students, particularly in view of the widening participation of 'new' demographic groups in higher education (Leese, 2010).
- A further area of research concerns students' well-being as they enter higher education, with levels of strain shown to be generally highest during the first semester of university life (Bewick, Koutsopoulou, Miles, Slaa, & Barkham, 2010).
- Various factors have been identified that ease the transition of students into university, enhance the early student experience and appear to contribute to improved rates of retention. These include activities that help students find their feet, make friends and get to know other students on their programme (Trotter & Roberts, 2006); learning-to-learn programmes (Zeegers & Martin, 2001) and workshops facilitating the early formation of social networks and peer groups (Peat, Dalziel, & Grant, 2001).
- On a different plane, and perhaps less predictably, many students reported seeking or realising the value of their bridging courses that went beyond the published aims of the courses.
- The interviews show the value students place on interaction with peers and teachers during the bridging courses. As reported in our introduction, social and interactive aspects of learning in early university education are formative in students' adjustment to and retention in higher education (Peat et al., 2001; Trotter & Roberts, 2006). This could be particularly critical for students where

family or friends are unfamiliar with the discourse and ways of learning in the university context (Leese, 2010).

- Students' collective construction of mathematical knowledge – what (Vygotsky, 1978) calls co-knowing could enhance students' confidence and enjoyment of learning mathematics and increase students' persistence in tackling complex problems, an outlook shown to stand them in good stead in ongoing mathematical study (Carlson, 1999).

Quotes from (Nicholas & Rylands, 2015):

- Gha it would be great to get this paper.

Quotes from (Poladian & Nicholas, 2013):

- In Australia, university entry criteria may also inadvertently exacerbate the problem of low uptake of the higher levels of mathematics in senior secondary school (Varsavsky, 2010).
- This creates a 'tension' between a student gaining access to a certain degree and being adequately prepared mathematically for that degree (Gordon & Nicholas, 2013a). Students, who have elected not to study intermediate or advanced mathematics at senior secondary school, have a difficult choice; either they accept that they have closed their access to quantitative disciplines for the immediate future or they enrol in a degree program for which they are mathematically under-prepared.
- In an Australian study, (Rylands & Coady, 2009) analysed data on the performance of first year university students in four mathematics and mathematics-related subjects. They concluded that a student's secondary school mathematics background, and not their ATAR, has a dramatic effect on pass rates: 77% of students with only elementary mathematics failed a basic mathematics subject. NOTE: This supports results of (Kajander* & Lovric, 2005) around this 'tension'.
- In Australasia, mathematics bridging courses have been part of the tertiary preparation scene for many years but there has been little research on their effectiveness (Rylands & Coady, 2009; MacGillivray, 2009; Galligan & Taylor, 2008). In 2006, (Galligan & Taylor, 2008) posed two (of four) unanswered questions within bridging mathematics as: How is success defined in bridging mathematics activities? Are successful bridging mathematics students successful university students?
 - For the first question, there are inherent difficulties in defining and measuring success in bridging courses. Godden & Pegg [12] suggest that formal evaluation of bridging mathematics programs may be contrary to the aims of the programs, and undermine their major strengths of flexibility and student-centred approach. They argue that traditional evaluative techniques are 'just not possible' and 'risk losing the essence of the support and assistance so necessary for these students'. NOTE: This is from MERGA 1993 conference proceedings, tough to find.

- For the second question, internationally, bridging mathematics programs have been shown to be highly effective at resolving skill deficiencies for some students (Kajander* & Lovric, 2005; Bahr, 2008). In a large US study, (Bahr, 2008, p.442) found that 'remediation has the capacity to fully resolve the academic disadvantage of math skill deficiency' for the quarter of students who 'remediated successfully', but the likelihood of successful remediation declined sharply as the 'depth of remedial need' increased. The latter finding echoes (Wood, 2001)'s remark that bridging programs do not work for very mathematically weak students.

•

Quotes from (Johnson & O'Keeffe, 2016) (a representative of a seemingly deep literature on the topic originating from Ireland):

- (Hardin, 2008) highlights that in recent years, the 'face of higher education' has changed, with a more diverse range of learners now entering third-level education. (Hardin, 2008) notes that in 1987, the number of adult learners 2 in College or University in the U.S. had increased to 4.9 million and the 2010 projections were set at 6.8 million. In the Irish context, the National Adult Learning Organisation (Aontas) identified that in 2012, adult learners accounted for 15% of all third-level full-time students and 96% of all part-time students. According to Aontas (2012), these percentages equate to circa 6000 full-time and 1500 part-time adult learners each year. (Murtaugh, Burns, & Schuster, 1999) point out that increases in the number of adult learners can cause additional retention worries for university policy-makers, as research shows that attrition rates have been found to increase with age. Further to this statistic, studies such as those conducted by (House, 2000) and (Tsui, 2007) indicate that significant numbers of students dropout of STEM degree programmes within the first two years, which highlights the importance of addressing this retention issue as early as possible in a student's career. One approach that has proven effective in addressing this issue is to encourage students to engage with mathematics learner support provisions. (Lee, Harrison, Pell, & Robinson, 2008) advocate that appropriate engagement with mathematics learner support can have a positive impact on student retention and progression.
- In response to the changing profile of students, the Mathematics Learning Centre (MLC) at the University of Limerick has adapted its services to meet the altered needs of the students. Such changes are outlined by O'Keeffe, O'Donoghue, and Gill (2011) and include a move away from 'fire-fighting' measures to provide more front end, pre-ventive support measures. Such measures include the provision of a mathematics bridging programme entitled 'Head Start Maths' in conjunction with diagnostic testing of first-year science and technology mathematics students.
- According to Postle, Clarke, and Bull (1995), bridging programmes usually come in two forms; longer programmes designed as pre-university programmes or shorter programmes tailored specifically to meet the needs of a particular group. While Cobbin and Gotstlelow (1993) note that the number and type of such programmes are varied, (Benn & Burton, 1994) believe that mathematics

should be an essential element of all such programmes. The nature of a short bridging programme is elaborated on by Taylor and Galligan (2005) who explain that specific tailored bridging programmes can also be broken down into two categories, those which are pre-degree 'stand-alone' courses or those which offer on-going support.

- O'Donoghue (2004) summarised the key issues of the 'Mathematics Problem' in the Irish context and found that foremost among these issues were students' mathematical shortcomings and deficiencies. Prior to commencing third-level education, some students may not have had adequate opportunity to develop the pre-requisite skills needed and so it is up to the institutions to provide additional assistance to help these students survive, progress and succeed. The additional assistance needed as a consequence of the students' lack of preparedness places added financial costs on the institutions as well as affecting student self-efficacy, retention and progression rates within the institutes. This problem though is not restricted solely to Ireland with Cuthbert and MacGillivray (2003) discussing the lack of mathematical confidence among first-year engineering students and (Rylands & Coady, 2009) highlighting the lack of appropriate mathematical background among students at their respective institutes in Australia. In Canada, (Kajander* & Lovric, 2005) highlighted what they termed the 'transition gap' between secondary- and tertiary-level mathematics and in the UK the decline in numeracy skills among first-year biosciences undergraduate students has been highlighted by (Tariq, 2002).
- Astin and Oseguera (2005) and (Croft et al., 2009) agree that the mathematics skill level at entry of students undertaking STEM degrees is one of the primary factors which impacts on student retention. (Robinson, 2003) suggests that more advanced mathematics and science programmes in second-level education will minimise such attrition. Further to this, (Kitchen, 1999) previously highlighted that some mathematics departments within universities have already felt a need to introduce remedial mathematics into the first-year teaching programmes. (Moses et al., 2011) suggest advanced and targeted preparatory programmes (outside of the normal university preparation) better prepare students for third level and they suggest that those without such preparation may be more likely to dropout.
- In the Irish context, this changing profile of students studying mathematics at the University of Limerick is documented by (Faulkner, Hannigan, & Gill, 2010) who noted that between 1998 and 2008, there has been a 20–25% reduction in students attending their first service 4 mathematics lecture, a 12–16% reduction in the number of students entering service mathematics modules with higher level 5 mathematics and an 8–12% increase in the number of non-standard students. 6 Such changes place additional pressure on support services like MLCs whose primary function is to provide the necessary and appropriate support to all university students.
- Focusing on adult learners, who constitute the largest cohort of non-standard students at the University of Limerick, (Burton, 1987) and Klinger (2006) as well as (Klinger, 2011) indicate that negative preconceptions are of major concern, both preconceptions of mathematics, in general, and also of their own

abilities. (Bandura, 1997, pg391) defines self-efficacy as ‘people’s judgement of their capabilities to organize and execute courses of action required to attain designated types of performance’. Self-efficacy is vital among all students but particularly among adult learners as an individual’s beliefs of self-capability has been shown to affect motivation, performance, achievement, effort, willingness to persist with a task, as well as the anxiety they experience (Bandura, 1997; Pajares & Miller, 1994; Pajares, 1996; Pajares & Miller, 1997; Pajares & Graham, 1999). Woodley (1987) (cited in (McGivney, 1996)) noted that the main personal factors that contribute to dropout are: self-perception, being disorganised, not having sufficient study skills and lacking in self-confidence. This suggests that an individual’s self-efficacy plays a role in their decision with regard to dropping out.

- (Hackett & Betz, 1989) and (Pajares & Miller, 1994) and Pajares and Miller (1995) also found that self-efficacy can have an impact on career choice. In these studies, it was found that mathematical self-efficacy is a stronger predictor of students’ mathematical interest and choice of degree programmes than either prior mathematical achievement or mathematical outcome expectations. Self-efficacy also influences how often mathematics is used, as well as an individual’s willingness to pursue advanced work in mathematics, and even the choice of prospective occupations (Dutton and Dutton 1991). Engineers Ireland (2010) highlight that this avoidance of mathematics, and mathematics-related courses, at university will eventually prove detrimental when attempting to build a knowledge economy. This point was also stressed decades before by (Hembree, 1990, pg34) when he stated that ‘when otherwise capable students avoid the study of mathematics, their options regarding careers are reduced, eroding the country’s resource base in science and technology’.
- Further highlighting the importance of mathematics, (Volmink, 1994) and Noyes (2007) both stressed its essential role with Noyes (2007, 1) stating that ‘[mathematics]’ sacred position as the gatekeeper to many education, employment and life opportunities is now firmly established’. This gatekeeping function was also emphasised by Russel (2005) who stated that a student’s leaving certificate mathematics grade was a key determinant in that student successfully progressing through an engineering programme. Therefore, in the case of adult learners, who can enter degree programmes through non-standard application avenues and bypass the leaving certificate examinations, it is essential that appropriate mathematical support provisions be put in place to lessen the difficulties associated with commencing third-level education.

2.2 Maths Anxiety

Why is Maths Anxiety Important?

Maths anxiety is hugely prevalent, the 2012 Programme for International Student Assessment (PISA) report states that across Organisation for Economic Co-operation and Development (OECD) countries, over 30% of 15 year old students “get very nervous doing mathematics problems”, and over 60% of students “worry about getting poor grades in mathematics” (OECD, 2013). As teachers our foremost concern

should be for the wellbeing of our students. It has been shown that students with a high level of maths anxiety often literally experience the anticipation of a maths task as visceral pain (Lyons & Beilock, 2012b). There is a clear and overwhelming moral imperative (and ethical duty of care) on us to do everything in our power to protect students in our care from maths anxiety.

Even if the wellbeing issue was not enough, there is also a clear maths anxiety-performance connection, and all the stakeholders in a student's academic success in maths. One example of this is highlighted by Foley et al. (2017) who juxtaposes the internationally rising demand for Science, Technology, Engineering and Mathematics (STEM) professionals with the negative correlation between maths anxiety and performance shown in the 2012 PISA report (OECD, 2013) to highlight the relevance of addressing maths anxiety in filling this demand. The relationship between maths anxiety and maths-qualified professionals in the workforce is supported throughout the literature: when a student has low self-concept (correlated with high maths anxiety), they will tend not to enroll in maths beyond the minimum requirements for graduation (Ashcraft, Krause, & Hopko, 2007), and students' affect towards maths can predict their university major (LeFevre, Kulak, & Heymans, 1992). Beyond this example, the list of stakeholders in a student's academic success in maths goes on and on: parents; the student's themselves; schools (which are often funded based on the results of standardised testing such as National Assessment Program — Literacy and Numeracy (NAPLAN)), and teachers amongst them.

Maths Anxiety as Distinct from General Anxiety

The existence of maths anxiety as “emotional disturbances in the presence of mathematics” has been noted as early as the 1950's, Dreger and Aiken Jr (1957) even postulated that what he tentatively designated “Number Anxiety” and later became to be known as Maths Anxiety could be a distinct syndrome from general anxiety. Later the landmark meta-study of Hembree (1990) supported this hypothesis, showing a correlation of only 0.38 between maths anxiety and general anxiety. In more recent times, this hypothesis has also been confirmed by Young, Wu, and Menon (2012) using functional magnetic resonance imaging (fMRI) to show that the brain activity in a person experiencing maths anxiety is measurably distinct from that in a person suffering general anxiety. These later studies, as well as the work of Kazelskis et al. (2000) and more, have also delineated maths anxiety from test anxiety, and these different anxieties existing as meaningfully distinct constructs is now quite well accepted. For more on the history of maths anxiety, Suárez-Pellicioni, Núñez-Peña, and Colomé (2016) offers a more detailed review.

Frameworks for Understanding Maths Anxiety

Only a few studies focus on maths anxiety itself (primarily fMRI studies such as those of Young et al. (2012) or Lyons and Beilock (2012b)). Instead the bulk of the literature is focused on the maths anxiety-performance link. Specifically, there seem to be two distinct theories being pursued and I will adopt the terminology of Ramirez, Shaw, and Maloney (2018) to describe them: the “Disruption Account” and the “Reduced Competency Account”. Ramirez et al. (2018) go on to make a convincing argument that although these two theories might seem to compete, they

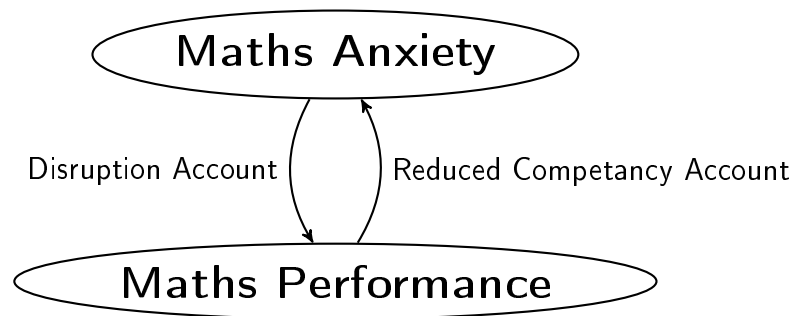


Figure 2.1: The Interpretation Account of Ramirez et al. (2018) for the maths anxiety-performance link showing how the Disruption Account and the Reduced Competency Account can be compatible.

are not actually mutually exclusive and instead quite compatible with each other. Ramirez et al. (2018) suggests a third “Interpretation Account” which encapsulates observations made by both lines of research, see Figure 2.1.

First, a little more detail on the existing theories. The “Disruption Account”, spearheaded by the work of Ashcraft et al., is centered around the concept of working memory (Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007). Specifically that anxiety about maths takes up students working memory, which prevents them from using that working memory to complete maths tasks and thereby impacts their performance. The “Reduced Competency Account” on the other hand proposes the opposite causality: that lower ability in maths leads to negative experiences associated to maths, which in turn cause maths anxiety to develop. There is also a significant body of work to support this hypothesis, including the milestone meta-analysis of Hembree (1990) and the longitudinal study of Ma and Xu (2004) which found that although past maths anxiety was correlated with future maths performance it was a small effect, while past maths performance had a strong effect on future maths anxiety.

Complexities in Finding Effective Interventions

These theoretical views are of course broad oversimplifications of what is an incredibly complex and interconnected topic. They also imply very different approaches for intervention. The “Reduced Competency Account” would imply interventions to boost maths performance and hence allow students to experience success in math should also help to reduce maths anxiety. The results of Supekar, Luculano, Chen, and Menon (2015) seem to support this hypothesis as when students are given an intensive 8-week tutoring program to boost their maths skills, this is associated to a reduction in maths anxiety. The earlier work by Faust (1996) further supports this by demonstrating an anxiety-complexity effect in which low and high maths anxiety groups performed similarly on low complexity problems, but in high complexity problems the high anxiety groups performance was impacted. On the other hand, Jansen et al. (2013) showed that it is not necessarily that simple, by showing that when students experience more success they attempt more problems and perform better. However their improved performance is almost completely predicted by the number of problems they attempted, not their experience of success, and their level

of maths anxiety was not affected in a significant way which raises a lot of interesting but unanswered questions about this approach.

On the other side of attempted interventions are those in line with the “Disruption Account”, in which the maths anxiety itself is addressed in the hopes that will free up extra working memory and hence boost students performance. Park, Ramirez, and Beilock (2014) demonstrate a direct and successful attempt at this in which they used expressive writing exercises to help guide students self-perceived narratives about their maths experiences and thereby reduce their maths anxiety. Notably the approach of Park et al. (2014) is in line with successful treatments for clinical anxiety disorders (see McNally (2007); Becker, Darius, and Schaumberg (2007); Foa et al. (2005)). Another approach that has shown success in this vein does not attempt to directly reduce the anxiety experienced, but rather reappraise it’s symptoms (Jamieson, Peters, Greenwood, & Altose, 2016). This is another technique from clinical psychology in which stress is reconceptualised as a coping tool, an evolutionary method for heightening performance in response to a challenge to be overcome, instead of a symptom of exposure to something to be feared and avoided. This change in the perspective of stress is also very much in line with the “Interpretation Account” of Ramirez et al. (2018).

The work of Wang et al. (2015) showed the role that intrinsic motivation has mediating the relationship between maths anxiety and performance, and suggested the importance of a mindset centred on viewing the process of learning maths as one of “productive struggle”. This reconceptualisation to a ‘productive struggle’ model is supported by other literature as well, Lin-Siegler, Ahn, Chen, Fang, and Luna-Lucero (2016) exposes students in a classroom to struggles experienced by famous scientists in order to help normalise the concept of productive struggle, and Hiebert and Grouws (2007) discuss the importance of this same concept in a maths context.

One of the implications of the “Interpretation Account” is that if an intervention targets only one of these two possible links in the cycle (see Figure 2.1), the cycle may re-establish itself after the intervention is over and negate any potential longterm effects. However there is only a very limited amount of research out there on such longterm effects, and several authors have discussed the need for further research into this (Suárez-Pellicioni et al., 2016; Chang & Beilock, 2016). My hypothesis is that a multi-faceted approach targetting both directions simultaneously could disrupt the cycle shown in Figure 2.1 and result in significant longterm effects.

Instruments for Measuring Maths Anxiety

In order to track the effectiveness of these interventions, we will be collating assessment results as a measure of performance, but will also want to measure maths anxiety and maths affect/ self-concept. Significant work has been done over the years to develop psychometrics to measure maths anxiety, almost exclusively consisting of self-reporting surveys (with the exception of some more modern fMRI work, such as that of Lyons and Beilock (2012a)). We will use a recently developed scale: the Maths Anxiety Scale — Revised (MAS-R) of Bai, Wang, Pan, and Frey (2009), which has been shown to be remarkably self consistent by incorporating both positive and negative affect items (Bai, 2011). It is short, easy to implement, and cheap in comparison to fMRI methods. In order to measure maths self-concept, Jansen et al. (2013) modified the Perceived Competence Scale for Children of Harter (1982) to measure “Math Competance”. The methodological process employed by Jansen et al.

(2013) was quite rigorous and so we will use their instrument, or a minor modification thereof (we will do it in English), to measure maths self-concept.

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Chapter 3

Curriculum Mapping

One of the important roles of university mathematics bridging courses (such as MathsStart and MathsTrack) is to fill the content knowledge gap for students who did not complete mathematics to a sufficiently high level in highschool, or completed it long enough ago that they need to re-learn the material, but wish to commence study at a university level in subjects that have a high level of required knowledge in mathematics.

There are two angles from which this required knowledge can be seen: the knowledge required for the university study intended, and knowledge expected from highschool graduates. As we will come to see, these two angles or perspectives can be quite dramatically different. From the perspective of knowledge expected from highschool graduates, the AC serves as a good guide, but even so the exact content knowledge expected of students having completed highschool in Australia varies for a number of reasons:

- To begin with, the AC specifies four levels of mathematics: essential mathematics, general mathematics, mathematical methods, and specialist mathematics. Our focus will be on the higher two of these: mathematical methods and specialist mathematics, as these are the ones often associated to university entry into mathematics-intensive courses.
- Different states within Australia teach different curriculums, with varying degrees of alignment to the AC. In South Australia the primary curriculum taught in senior secondary school is SACE, and so we will focus on that.

The other perspective is of course the knowledge required for entry level university mathematics courses. This will vary hugely from course to course: a entry level calculus course will require very different knowledge than an entry level statistics course, for example. Even within one discipline of mathematics, different universities will have very different expectations of entry level students: in particular, South Australian universities will often structure their entry level mathematics courses to align with SACE even though not all their students have completed SACE, because of the majority who have it is still useful for them to do so. For example, the University of Adelaide re-structured it's first year mathematics courses in 2018 to match changes in SACE. Similarly, universities interstate will often structure their entry level courses to align with their local senior highschool curriculum.

This places a difficult tension on mathematics bridging courses as to what content to teach. Although many of the students enrolling in the mathematics bridging

courses at the university of adelaide do so with the intention to begin study at the University of Adelaide (and hence might benefit from SACE structured content), many do not. Even amongst those that do, some may end up going to a different university interstate or even overseas — plans change. So it is important to try and maintain some connection to a broader set of knowledge expected in general and not necessarily remain laser focussed on the requirements of the particular university courses most students are going to be attempting. This is one of the reasons why the AC is a useful construct as even though some states do not align to the AC as well as others, it still forms a guiding structure at a national level and individually considering the curriculum taught in each state is... beyond the scope of this work. Tailoring the content of the bridging courses more narrowly to target entry into particular disciplines (say calculus/ matrix algebra/ statistics for example) could potentially still be of interest down the line, but is likely to be unrealistic with the current resources available to the maths learning center.

This chapter will examine the alignment of the content of MathsStart and MathsTrack (the mathematics bridging courses offered at the university of adelaide) with the AC and SACE. First, in Section 3.1, some notation will be introduced and the content of each of the three curriculums will be reviewed:

- The AC senior mathematics subjects mathematical methods and specialist mathematics,
- The SACE curriculum stage 1 mathematics, stage 2 mathematical methods, and stage 2 specialist mathematics,
- The University of Adelaide's bridging courses: MathsStart, and MathsTrack.

Then, these will be mapped to each other in Section 3.2 (see Figure 3.1), and alignments/ misalignments discussed. Finally, the discussion throughout around alignment and gaps between the content of these curriculums and courses will be summarised, explanations and reasons for these discrepancies discussed, and potential modifications to content suggested.

Beyond that, this chapter will also briefly discuss the alignment of these bridging courses to first year university mathematics courses and bridging courses offered by other universities in Australia, and discuss the relationship between the gaps in alignment between the AC/SACE and the bridging courses and the requirements of these first year university courses.

3.1 Content

3.1.1 Notation

Each of the senior highschool curriculums, as well as the university bridging courses, being considered here is broken down into topics, with each topic containing a number of key concepts. In Section 3.2, the alignment between these curriculums and bridging courses will be considered thoroughly at both a topic-level, and to the finer detail of particular key concepts. In order to abstract away some of the complexity of considering the topic-level alignment, and be able to present the topic-level alignment in a meaningful way abbreviated codes will be used to identify each topic. These abbreviated codes are presented in Table 3.1 and will be used for the remainder of this chapter.

Table 3.1: Abbreviated codes for topics within the AC and SACE senior mathematics subjects: Mathematical Methods and Specialist Mathematics, as well as the University of Adelaide's bridging courses: MathsStart and MathsTrack. Square brackets ([]) are used to indicate numeric values that can vary.

Code	Meaning
MMu[#1]t[#2]	AC Senior Mathematical Methods Unit [#1], Topic [#2]
MMu[#1]t[#2]	AC Senior Specialist Mathematics Unit [#1], Topic [#2]
S1M[#]	SACE Stage 1 Mathematics, Topic [#]
S2MM[#]	SACE Stage 2 Mathematical Methods, Topic [#]
S2SM[#]	SACE Stage 2 Specialist Mathematics, Topic [#]
MS[#]	Maths Start, Topic (Booklet) [#]
MT[#]	Maths Track, Topic (Booklet) [#]

3.1.2 Within-Topic Key Concepts

Description of each topic in the AC Mathematical Methods and Specialist Mathematics Topics, SACE stage 1 mathematics, stage 2 mathematical methods and stage 2 specialist mathematics, and the University of Adelaide's MathsStart and MathsTrack programs. For brevity a code is used to identify each topic, see Table 3.1, and then for each topic its name is given in bold followed by a list of the key concepts covered in that topic. These are discussed at length below, and this table is intended to be used as reference material for that discussion.

Some notes on the way the key concepts are summarised:

- This key concept summary is intended for a reader deeply familiar with the content, and as such it is heavily condensed and uses notation and terminology without the usually appropriate rigorous definitions.
- Concepts relating to "interpretation" and application in a general sense are omitted. The assumption is that to the intended readers, these should go without saying. For example, in S1M2 the key concept "Quadratic Equations in Vertex and Factorised Form" is included, but this implies a variety of auxiliary knowledge which is not explicitly included in the key concept summary: the interpretation of roots and vertices, deducing vertices and roots from the equation of a quadratic, or deducing the equation of a quadratic given these bits of information, etc. It is intended that an experienced maths educator should be able to deduce such surrounding concepts from the key concepts that are listed.

Producing this curriculum mapping was a delicate balance between being broad and vague in order to be able to present the entire curriculum mapping within a single frame of view, and yet still be granular enough so that specific content is clear and explicit and useful actionable recommendations can be made. This balance was achieved by presenting these curriculums at two levels of detail:

- At a topic level (see Figure 3.1). This is intended to give the broad strokes, and show the entire mapping in a single frame of view (a page, in this case).

It is also intended to be reference material for the following more detailed discussion, to aid the reader in structuring the information contained in the more detailed discussion and place each piece of information into where it belongs in the bigger picture.

- At a key concept level, this is what will be presented for the remainder of this section, and intended to be the granular level at which content is presented specifically enough that recommended actions can be understood explicitly and implemented easily. Note that although the key concept level is much more granular than the topic level discussion, it is still intended as a summary and does not include every single detail of the content, as discussed above.

Note: This is a huge table. I could maybe put it in an appendix?

Code	Name and Key Concepts
MMu1t1	Functions and graphs: Midpoint of a Line, $y = mx + c$, Quadratic Equations in Vertex and Factorised Forms, Inverse Proportions, Polynomials, Relations, Translations and Dilations
MMu1t2	Trigonometric functions: Unit Circle, Radians, SOH CAH TOA, Sine Rule, Cosine Rule, Exact Values, Amplitude/ Period/ Phase, Length of Arc, Area of Sector
MMu1t3	Counting and probability: Binomial Coefficients, Set Complement Intersection and Union, Probability, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, Conditional Probability, Independence
MMu2t1	Exponential functions: Index Laws, Fractional Indices, Functions, Asymptotes, Graphs
MMu2t2	Arithmetic and geometric sequences and series: Arithmetic and Geometric Sequences as Recurrence Relations, Limiting Behaviour, and Partial Sum Formulae, Growth and Decay
MMu2t3	Introduction to differential calculus Average Rate of Change, First Principles, Leibniz Notation, Instantaneous Rate of Change, Slope of Tangent, Derivative of Polynomials, Linearity of Differentiation, Stationary Points, Optimisation, Anti-Derivatives, Interpret Position-Time Graphs
MMu3t1	Further differentiation and applications: Define e as a s.t. $\lim_{h \rightarrow 0} \frac{a^h - 1}{h} = 1$, Derivatives of e^x $\sin(x)$ and $\cos(x)$, Chain Product and Quotient Rules, Second Derivatives
MMu3t2	Integrals: Integrate Polynomial Exponential and Trigonometric Functions, Linearity of Integration, Determine Displacement given Velocity, Definite Integrals, Fundamental Theorem of Calculus, (signed) Area Under a Curve
MMu3t3	Discrete random variables: Frequencies, General Properties, Expected Value, Variance, Standard Deviation, Bernoulli and Binomial Distributions
MMu4t1	The logarithmic function: Logs as Inverse of Exponentials, Log-Scales, Log Laws, Log Function Graphs, Natural Log, $\frac{d}{dx} \ln(x) = \frac{1}{x}$, $\int \frac{1}{x} dx = \ln(x) + c$ for $x > 0$

Code	Name and Key Concepts
MMu4t2	Continuous random variables and the normal distribution: Probability Density Function, Cumulative Distribution Function, Probabilities Expected Value, Variance and Standard Deviation as Integrals, Linear Transformation of Random Variables, Normal Distribution using Technology
MMu4t3	Interval estimates for proportions Simple Random Sampling, Bias, Sample Proportion, Normal Approximation to the Binomial Proportion, Wald Confidence Interval, Trade-Off Between Width and Level of Confidence
SMu1t1	Combinatorics Multiplication of Possibilities, Factorial Notation, Permutations with and without Repeated Objects, Union of Three Sets, Pigeon-Hole Principle, Combinations, Pascals Triangle
SMu1t2	Vectors in the plane: Magnetude and Direction, Scalar Multiplication, Addition and Substraction as a Triangle, Vector Notation, $a\mathbf{i} + b\mathbf{j}$ Notation, Scalar Dot Product, Projection, Parallel and Perpendicular Vectors
SMu1t3	Geometry: Notation for Implication (\Rightarrow) and Equivalence (\Leftrightarrow), Converse ($B \Rightarrow A$) Negation ($\neg A \Rightarrow \neg B$) and Contrapositive ($\neg B \Rightarrow \neg A$), Proof by Contradiction, \forall and \exists Notation, Counter-Examples, Circle Theorems, Quadrilateral Proofs in \mathbb{R}^2
SMu2t1	Trigonometry: Graph and Solve Trig Functions, Prove Various Trig Identities, Reciprocal Trig Functions
SMu2t2	Matrices: Notation, Addition and Scalar Multiplication of Matrices, Multiplicative Identity and Inverse, Determinant, Matrices as Transformations
SMu2t3	Real and complex numbers: Rationality and Irrationality, Induction, $i = \sqrt{-1}$, Complex Numbers $a + bi$ and Arithmetic ($+$, $-$, \times , \div), Complex Conjugates, Complex Plane, Complex Conjugate Roots of Polynomials
SMu3t1	Complex numbers: Modulus and Argument, Arithmetic (\times , \div , and z^n) in Polar Form, Convert between Polar and Cartesian Form, De Moivre's Theorem, Roots of Complex Numbers, Factorising Polynomials
SMu3t2	Functions and sketching graphs: Composition of Functions, One-to-One, Inverse Functions, Absolute Value Function, Rational Functions
SMu3t3	Vectors in three dimensions: $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ Notation, Equation for Spheres, Parameterised Vector Equations, Equations of Lines, the Cross Product, Equation for a Plane, Systems of Linear Equation (Elimination Method) and Geometric Interpretation of Solutions, Kinematics via Differentiation of Vector Equations, Projectile and Circular Motion
SMu4t1	Integration and applications of integration Substitution, $\int \frac{1}{x} dx = \ln x + c$ for $x \neq 0$, Inverse Trig Functions and their Derivitives, Integrate $\frac{\pm 1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$, Partial Fractions, Integration by Parts, Area Between Two Curves, Volume of Solids of Revolution, Numerical Integration using Technology

Code	Name and Key Concepts
SMu4t2	Rates of change and differential equations: Implicit Differentiation, First-Order Seperable Differential Equations, The Logistic Equation, Kinematics (Rates of Change)
SMu4t3	Statistical inference: Central Limit Theorem and the Resulting Confidence Interval for a Mean
S1M1	Functions and graphs: Equations for a Line, Slope, y-intercept, Intersection of Lines, Reciprocal Function, Asymptotes, Functions vs Relations, Domain, Range, Function Notation
S1M2	Polynomials: Quadratic Equations in Vertex and Factorised Forms, Quadratic Formula, Completing the Square, The Leading Coefficient and Degree of a Polynomials, Cubics, Quartics
S1M3	Trigonometry: Pythagoras, SOH CAH TOA, Cosine Rule, Sine Rule, Unit Circle, Exact Values, Sine and Cosine Functions, Radians, Length of Arc, Area of Sector, Amplitude, Period, Phase, $\tan(x) = \frac{\sin(x)}{\cos(x)}$
S1M4	Counting and statistics: Factorial, Permutations, Multiplication Principle, Combinations, Discrete vs Continuous Random Variables, Mean, Median, Mode, Range, Interquartile Range, Standard Deviation, Normal Distribution,
S1M5	Growth and decay: Index and Logarithm Laws, Exponential Functions and their Graphs
S1M6	Introduction to differential calculus: Average Rate of Change, First Principles, Notation $f'(x) = \frac{df}{dx}$, $\frac{d}{dx}x^n = nx^{n-1}$, Linearity of Differentiation, Slope of Tangent, Increasing vs Decreasing, Local and Global Maxima and Minima, Stationary Points, Sign Diagram
S1M7	Arithmetic and geometric sequences and series: Arithmetic and Geometric Series as Recurrence Relations and Explicit Expressions, Partial Sums, Limiting Behaviour
S1M8	Geometry: Circle Properties , Proofs (Direct, Contradiction, and Contrapositive)
S1M9	Vectors in the plane: Component (column) vs $ai + bj$ Notation, Length and Direction, Linear Combinations of Vectors, Scalar Dot Product, Projection, Angle Between Two Vectors and Parallel/ Perpendicular, Geometric Proof
S1M10	Further Trigonometry: Sketch Trigonometric Functions with Translations and Dilations, Solve for Angles, Trigonometric Identities, Reciprocal Trigonometric Functions
S1M11	Matrices: Linear Combinations of Matrices, Matrix Multiplication, The Identity, Inverse Matrices, The 2×2 Inverse, The 2×2 Determinant, Linear Transformations (including rotations, reflections and composition)
S1M12	Real and complex numbers: Rationals, Irrationals, Interval Notation, Induction, $i = \sqrt{-1}$, Real and Imaginary Components, Complex Conjugates and Arithmetic, Argand Diagram, Modulus, Complex Roots of Polynomials

Code	Name and Key Concepts
S2MM1	Further differentiation and applications: S1M6, Chain Product and Quotient Rules, $e = 2.718\dots$, $\frac{d}{dx}e^x = e^x$, $\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$, Second Derivatives, Concavity and Points of Inflection
S2MM2	Discrete random variables: Random Variables, Discrete vs Continuous, Probability Functions and Distributions, Properties of Probabilities, Frequency, Expected Value $E[X] = \sum xp(x) = \mu_X$, Standard Deviation $\sigma_X = \sqrt{\sum (x - \mu_X)^2 p(x)}$, Uniform Bernoulli and Binomial Distributions
S2MM3	Integral calculus: Anti-differentiation, Reversing Chain Rule for $\int f(ax + b)dx$, Linearity of Integration, Finding the Constant of Integration, Area Under the Curve as Upper and Lower Sum Approximations, Definite Integral, Area Between Two Functions and Between a Negative Function and the x-axis, Fundamental Theorem of Calculus,
S2MM4	Logarithmic functions: Logs as Inverse of Exponentials, Log-Scales, Log Laws, Sketching $y = a \ln(b(x - c))$, $\frac{d}{dx} \ln(x) = \frac{1}{x}$, For $x > 0$ $\int \frac{1}{x} dx = \ln(x) + c$
S2MM5	Continuous random variables and the normal distribution: $P(X = x) = 0$, Probability Density Function, $\mu_X = \int_{-\infty}^{\infty} xf(x)dx$, $\sigma_X = \int_{-\infty}^{\infty} (x - \mu_X)^2 f(x)dx$, $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, Standard Normal $Z = \frac{X-\mu}{\sigma}$, Simple Random Sampling, For $X \sim (\mu, \sigma)$ and $X_i \sim iid X$ Sampling Distributions of $S_n = \sum_{i=1}^n X_i$ ($n\mu, \sigma\sqrt{n}$) and $\bar{X}_n = \frac{S_n}{n}$ ($\mu, \frac{\sigma}{\sqrt{n}}$), If X is Normally Distributed, then so are S_n and \bar{X}_n , Central Limit Theorem (CLT)
S2MM6	Sampling and confidence intervals: Confidence Interval for a Mean using CLT $\left(\bar{x} - z^* \frac{s}{\sqrt{n}}\right) \leq \mu \leq \left(\bar{x} + z^* \frac{s}{\sqrt{n}}\right)$, Wald Interval for a Proportion
S2SM1	Mathematical induction: Initial Case and Induction Step
S2SM2	Complex numbers: Cartesian vs Polar Form, Real and Imaginary Components, Modulus and Argument, Arithmetic in both Cartesian and Polar Forms, de Moivre's Theorem including Negative and Fractional Powers, Geometric Properties of the Argand Plane, Complex Arithmetic as Transformations, n^{th} Roots of a Complex Number, Factorising Polynomials with Complex Roots
S2SM3	Functions and sketching graphs: Function Composition, Informal Intro to Domain and Range, One-to-One, Inverse Functions, Absolute Value Function, Graphing Rational Functions
S2SM4	Vectors in three dimensions: Notation, Equations of a Line in \mathbb{R}^3 , Scalar Dot Product, Vector Cross Product, $ \mathbf{a} \times \mathbf{b} $ is the Area of their Parallelogram, Equation for a Plane in \mathbb{R}^3 , Systems of Linear Equations, Geometric Interpretation of No/Unique/Infinite Solutions to a System of Linear Equations in \mathbb{R}^3

Code	Name and Key Concepts
S2SM5	Integration techniques and applications: Integration by Substitution, Using Trigonometric Identities for Integration, Derivatives of Inverse Trigonometric Functions (so $\int \frac{\pm 1}{\sqrt{a^2 - x^2}} dx$ and $\int \frac{a}{a^2 + x^2} dx$, Integration by Parts, Partial Fractions for Integrating Rational Functions, Area Between two Curves, Volume of Solids of Revolution
S2SM6	Rates of change and differential equations: Implicit Differentiation, First-Order Separable Differential Equations, The Logistic Differential Equation, Parameterised Curves, Example: if $\mathbf{v} = \frac{d}{dt}(x(t), y(t))$ is Velocity, $ \mathbf{v} $ is Speed, and so the Arc Length along the Parameterised Curve is $\int_a^b \sqrt{\mathbf{v} \bullet \mathbf{v}} dt$, Trigonometric Parameterisations (unit circle, and non-circular parameterisations)
MS1	Numbers & Functions: Natural Numbers, Integers, Rational Numbers, Real Numbers, Functions, Intervals
MS2	Linear Functions: Equation for Linear Functions, Simultaneous Linear Equations, Sketching Linear Inequalities
MS3	Quadratic Functions: Sketching a Parabola, General Form of a Quadratic, Translations and Dilations
MS4	Rational Functions: Sketching Reciprocal Functions (Hyperbola), Lines of Symmetry, Limits and Asymptotes
MS5	Trigonometry I: Pythagoras, Similar Triangles, SOH CAH TOA, Trigonometric and Inverse Trigonometric Functions using Technology, Exact Values
MS6	Trigonometry II: Unit Circle, Sketching Trigonometric Functions, Finding all Solutions to Trigonometric Equations, The Sine Rule, The Cosine Rule, Introductory Trigonometric Identities, Radians
MS7	Exponential Functions: Index Laws, Sketching Exponential Functions, $e = 2.718\dots$, Growth and Decay
MS8	Logarithms: Natural Logarithm, Logarithm Laws, Using Logarithm to Fit Growth/Decay Functions, Half-Life/ Doubling Time
MT1	Polynomials: Polynomial Division and “Remainder Theorem”, Factor Theorem Linking Zeros to Factors, Continuous vs Discontinuous Functions, Smoothness, Sketching Factorised Form of Polynomials, Factorising Polynomials, The Quadratic Formula
MT2	Matrices: Order, Notation, Linear Combinations of Matrices, Matrix Multiplication (Associative but not Commutative, Distributes across Linear Combinations), The Identity Matrix, Powers of Square Matrices, Matrix Transpose, Systems of Linear Equations, Matrix Inverse, 2×2 determinant, The 2×2 Inverse, $n \times n$ Inverses, Elementary Row Operations,
MT3	Vectors and Applications: Directed Line Segment Notation for Vectors, Magnitude/ Length and Direction, Linear Combinations of Vectors, Component and $a\mathbf{i} + b\mathbf{j}$ Notation, Vectors in \mathbb{R}^2 and \mathbb{R}^3 , Scalar Dot Product, Equation for a Plane in \mathbb{R}^3

Code	Name and Key Concepts
MT4	Systems of Linear Equations: Augmented Matrix for Systems of Linear Equations, Elementary Row Operations, Row-Echelon Form, Solutions to Systems of Linear Equations and Geometric Interpretations in \mathbb{R}^2 and \mathbb{R}^3 , Matrix Inverses by Gauss-Jordan Elimination
MT6	Differentiation: Rates of Change, Gradient, First Principles, Limit Notation, Derivative Notation, $\frac{d}{dx}x^n = nx^{n-1}$ (including $n = 0$ and $n = 1$), Linearity of Differentiation, Product Rule, Quotient Rule, Chain Rule, Implicit Differentiation, Normal to a Curve
MT7	Applications of Differentiation: Sketching Polynomials and Rational Functions (Intercepts and Asymptotes), Continuity, Sign Diagrams, Increasing and Decreasing, Stationary Points, Points of Inflection, Concavity, Optimisation,
MT8	Exponential and Logarithm Functions: Sketching Exponential Functions, $e = 2.718\dots$, $\frac{d}{dx}e^x = e^x$, Natural Logarithm, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, Growth and Decay, Surge Models, Logistic Models
MT9	Integration: Area Under a Curve, Lower and Upper Sums, Definite Integrals, Definite Integrals of Negative Functions, Linearity of Integration, Properties of Definite Integrals, Fundamental Theorem of Calculus, Antiderivatives, Indefinite Integrals, Integrating by Reversing the Chain Rule, Integration by Substitution, Area Between two Curves, Summation Notation (Appendix)

3.1.3 AC Mathematical Methods and Specialist Mathematics

AC has four levels of mathematics, including also essential and general mathematics. Mathematical Methods and Specialist Mathematics are the two highest level mathematics, intended (partly) for preparation to university entry. Broadly speaking, the content of these two subjects can be grouped into several areas:

- Functions and Graphs, which is broken up primarily into families of functions, with some extra concepts thrown into some generalist topics:
 - Polynomials and Rational Functions (MMu1t1, SMu3t2),
 - Exponentials and Logarithms (MMu2t1, MMu2t2, MMu4t1)
 - Trigonometric Functions (MMu1t2, SMu2t1)
- Calculus, which is largely structured similarly to the Functions and Graphs: breaking it up by the type of function you are doing calculus on, splitting up differentiation from integration, and throwing in rules of differentiation and approaches to integration with a few extra concepts along the way (MMu2t3, MMu3t1, MMu3t2, SMu4t1, SM4t2).
- Geometry and Linear Algebra: Mostly vectors, with some matrices, systems of equations, and even circle theorems. This is the topic in which the concept of proof is primarily attempted to be introduced, and perhaps that is the reason why it is entirely contained within the Specialist Mathematics curriculum (SMu1t2, SMu1t3, SMu2t2, SMu3t3),

- Complex Numbers, as well as rational/ irrational numbers, etc. (SMu2t3, SMu3t1), and finally
- Probability and Statistics, with some combinatorics thrown in for good measure (MMu1t3, MMu3t3, MMu4t2, MMu4t3, SMu1t1, SMu4t3)

Although there are a couple of topics (MMu2t2 for example) which although they relate to these broader areas, also contain a substantial amount of other content.

More detailed discussion of the specific key concepts of interest will follow in Section 3.2 as a natural part of comparisons between curriculums, as that is where the specifics will be important. These sections are intended to give a broader overview of the structure of the content in these curriculums

3.1.4 SACE Stage 1 Mathematics, Stage 2 Mathematical Methods and Specialist Mathematics

SACE follows the AC fairly well at a broad level (although there are differences in the details, which will be discussed in Section 3.2. Broadly though the topics in the three SACE subjects here, although split into three subjects instead of two, can be broadly grouped into areas much the same as the AC:

- Functions and Graphs, which is broken roughly as:
 - General Concepts (S1M1, S2SM3),
 - Polynomials and Rational Functions (S1M2, S2SM3),
 - Exponentials and Logarithms (S1M5, S2MM4, S1M7)
 - Trigonometric Functions (S1M3, S1M10)
- Calculus, which similarly split up by the type of function you are doing calculus on, differentiation vs integration, and throwing in rules of differentiation and approaches to integration with a few extra concepts along the way (S1M6, S2MM1, S2MM3, S2MM4, S2SM5, S2SM6).
- Geometry and Linear Algebra: Mostly vectors, with some matrices, systems of equations, and even circle theorems. This is the topic in which the concept of proof is primarily attempted to be introduced, and perhaps that is the reason why it is entirely contained within the Specialist Mathematics curriculum (S1M8, S1M9, S1M11, S2SM4),
- Complex Numbers, as well as rational/ irrational numbers, etc. (S1M12, S2SM2), and finally
- Probability and Statistics, with some combinatorics thrown in for good measure is included almost exclusively in Stage 2 Mathematical Methods (S1M4, S2MM2, S2MM5, S2MM6)

Although there are a couple of topics (S1M7, for example — very similarly to MMu2t2) which although they relate to these broader areas, also contain a substantial amount of other content. This grouping also leaves one topic out in the SACE context: S2SM1 which is essentially just the concept of mathematical induction.

3.1.5 MathsStart and MathsTrack

MathsStart can be seen as essentially an introduction to functions, MathsTrack then takes this, and extends it primarily to calculus, taking a little detour along the way to cover some vector and matrix geometry/ systems of linear equations concepts. So broadly grouping the topics in a way analogous to above:

- Functions and Graphs, roughly broken up into:
 - General Concepts (MS1, MS2),
 - Polynomials and Rational Functions (MS3, MS4, MT1),
 - Exponentials and Logarithms (MS7, MS8), and
 - Trigonometry (MS5, MS6)
- Calculus, similarly first introducing differentiation on polynomials with various other general concepts (MT6, MT7) and then exponentials and logarithms (MT8) and finally also integration (MT9).
- Geometry and Linear Algebra (MT2, MT3, MT4).

Note the missing topic 5 in MathsTrack, this used to be part of the course some time ago but is currently no longer included in the content of the course, and so is not included here.

3.2 Curriculum Mapping

Figure 3.1 shows the topic-level alignment between the AC (senior mathematical methods and specialist mathematics), SACE (stage 1 mathematics, stage 2 mathematical methods and stage 2 specialist mathematics), MathsStart and MathsTrack. This is the broad, eagle-eye, view of the alignment between the content in these topics.

Although key concept level alignment between topics connected with lines in Figure 3.1 is not always perfect, it is fairly strong in most cases. Individual cases with any misalignment will be discussed in the remainder of this chapter.

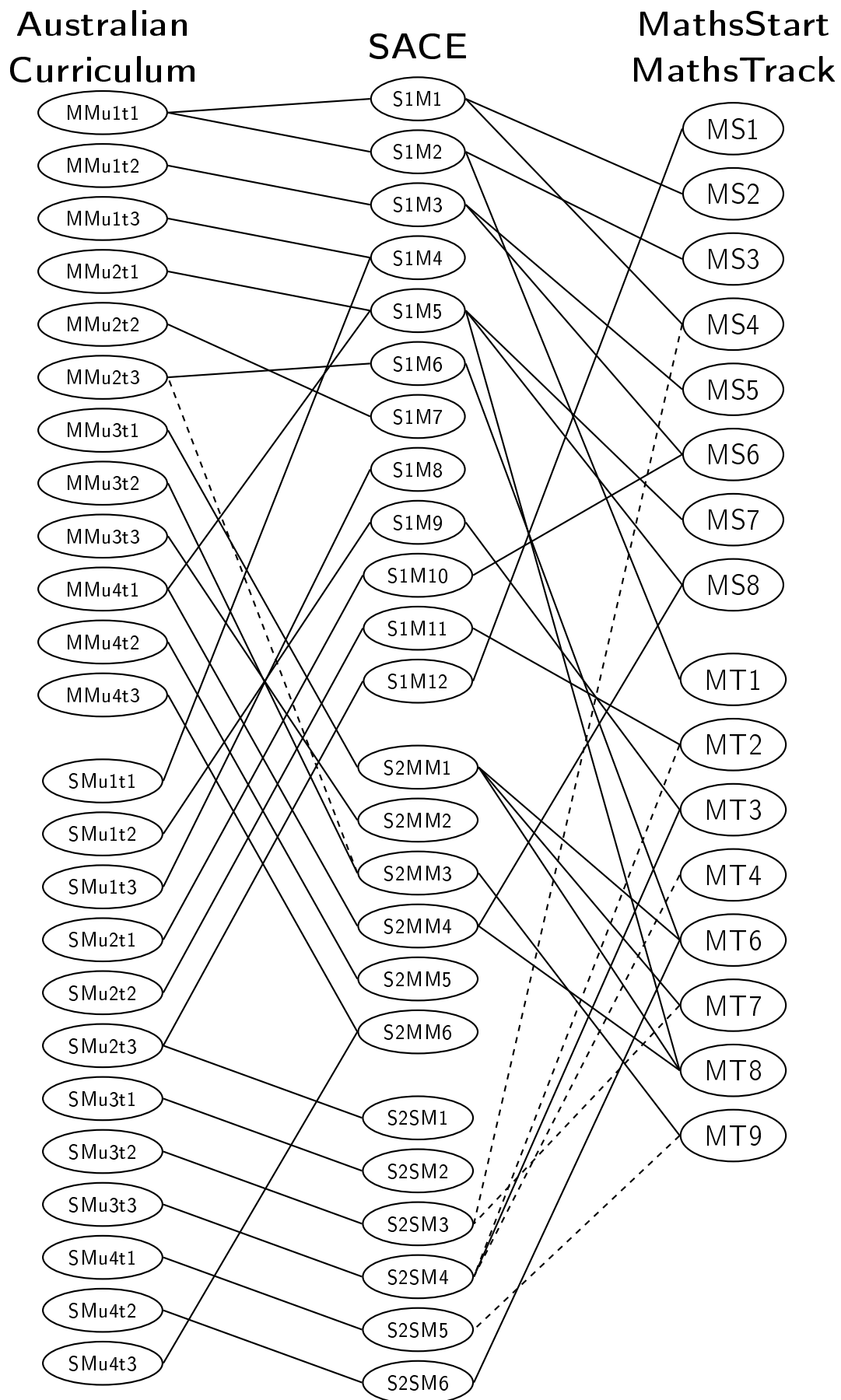


Figure 3.1: Curriculum Mapping

3.2.1 AC to SACE

At a glance, there appears to be a very good one-to-one alignment at the topic level between the AC and SACE. Broadly speaking the biggest difference between these two curriculums is their structure. The AC content is structured into two subjects (mathematical methods and specialist mathematics) which span "senior highschool", which most commonly would equate to both years 11 and 12 in Australia. Each of these two subjects contains 12 topics. The SACE content on the other hand is split into stage 1 (commonly year 11 in Australia) and stage 2 (commonly year 12), stage 1 consists of a single subject "mathematics" with 12 topics, and stage 2 is split into two (mathematical methods and specialist mathematics) each with 6 topics. So the total number of topics is actually the same across the board between the AC and SACE, and they seem to match almost exactly with a pattern in which the first 6 topics of both the AC mathematical methods and specialist mathematics constitute SACE stage 1 mathematics and the remaining 6 topics in each of the AC subjects align to the corresponding SACE stage 2 subject.

Note: The content is almost identical between these, the structure is simply different. Given how we don't really care about the structure, but rather care about the content maybe it would be more useful to structure this discussion into content areas rather than base it on the curriculum structure.

As usual however, the devil is in the details:

- In terms of Functions and Graphs:
 - General Concepts, Polynomials and Rational Functions: In both the AC and SACE this area is split into two: basic introduction and advanced concepts. The basic introduction topics align well (MMu1t1 to S1M1 and S1M2), with only slight differences in terminology (AC refers to inverse proportion while SACE refers to reciprocal for example) and focus (SACE puts much more of an emphasis on polynomials, separating it into its own topic (S1M2) and breaking it down into much more granular concepts). The advanced concepts are covered in SMu3t2 and S2SM3 are essentially identical.
 - Exponentials and Logarithms: There is essentially perfect alignment between the concepts for logarithms between MMu4t1 and S2MM4. Concepts around exponentials however are a little less straightforward. Similarly, MMu2t2 is almost exactly the same as S1M7, they are both centered on the introduction of recurrence relations, partial sums, and linking this back to exponential functions. I include these topics under exponentials as they link to those concepts, but really they focus on concepts around sequences and series, it's just they don't connect to anything else better than they connect with exponentials, so here they go, really they are a fairly distinct set of concepts though. However it is in the alignment between MMu2t1 and S15 that there is a difference: S15 includes Log-Laws, while MM2t1 does not, focusing only on Index Laws. This is not actually a difference in content between the AC and SACE as the log laws are covered in the AC in MMu4t1. They are actually repeated in the SACE curriculum, covered both in S1M5 and then again in S2MM4.
 - Trigonometry: MMu1t2 matches almost identically to S1M3, with the biggest difference being that in the AC the unit circle interpretations/

definitions of $\sin(x)$, $\cos(x)$, and in particular $\tan(x)$ are emphasised, where in SACE $\tan(x)$ in particular is introduced instead as $\frac{\sin(x)}{\cos(x)}$. That being the biggest difference between the two should emphasise how similar they are in terms of content. Similarly, SMu2t1 and S1M10 align just about perfectly.

- Calculus:

- SMu4t1 aligns perfectly with S2SM5, both covering integration by parts, by substitution, inverse trig substitutions in integration problems, volume of solids of revolution, partial fractions and area between two curves.
- SMu4t2 aligns well to S2SM6, both covering implicit differentiation, solving first-order separable differential equations, and the logistic equation. However there are some differences in that the AC goes on to focus on rates of change, while SACE instead decides to focus on parameterised curves, trigonometric parameterisations, etc.
- MMu2t3 and S1M6 both introduce differentiation by leading in with the concept of average rate of change, first principles and lead into linearity of differentiation, derivatives of polynomials, slope of the tangent and optimisation but in SACE S1M6 introduces the terms “increasing” and “decreasing” and sign diagrams, which are not mentioned in MMu2t3 (or AC?), while MMu2t3 introduces the concept of an antiderivative.
- MMu3t1 and S2MM1 align perfectly introducing the chain, product, and quotient rule. Introducing $e = 2.718\dots$ in the same way (using first principles to explore $\frac{d}{dx}a^x$ for different a , derivatives of $\sin(x)$ and $\cos(x)$, and second derivatives.
- MMu3t2 and S2MM3 are very closely aligned, both introducing both definite and indefinite integrals of polynomials, exponentials, and trigonometric functions, linearity of integration and the fundamental theorem of calculus, they have diverge slightly in their approach to definite integrals. In particular, SACE S2MM3 introduces the concepts of upper and lower sums and the definite integral as the unique number between the two as the size of the rectangles approaches zero, while in the AC MMu3t2 this is not discussed. Also, S2MM3 introduces anti-differentiation, a concept introduced in the AC MMu2t3 but not introduced in SACE S1M6, instead being covered here in S2MM3.

Note how although most derivatives are introduced in differentiation specific topics, $\frac{d}{dx} \ln(x)$ is introduced in a separate topic entirely about logarithm functions in both the AC (MMu4t1) and SACE (S2MM4), and I categorise these topics under ‘Functions and Graphs’ above because I see these topics as an introduction to logarithms, but they do also contain concepts around calculus (of logarithm functions).

- Geometry and Linear Algebra

- Vectors in the Plane are covered in SMu1t2 and S1M9, with the content being very well aligned and the only really notable difference being the inclusion of geometric vector proofs in SACE S1M9 which is not included in SMu1t2, instead being introduced but restricted to other topics... i.e.

- Proof and Circle Theorems which are covered in SMu1t3 to S1M8. Both these cover the same "content" in the sense of theorems: circle theorems, but they also both attempt to broach the difficult topic of proof, methods of proof, and some of the language around proof, and they take quite different approaches to this. The AC SMu1t2 is quite explicit specifying the introduction of language around formal logic: implication, equivalence, converse, negative, contrapositive, contradiction, 'for all' and 'there exists', counter-examples. On the other hand, SACE S1M8 simply specifies proof to be investigated as "justification of properties of circles", and only briefly mentions specifics of language and methods as suggestions not specifying them as being required components of the curriculum and instead leaving the approach and specific content chosen to be used to introduce the concept of proof much more open to interpretation by the teacher.
- Matrices, covered in SMu2t2 and S1M11 are essentially identical in content covering matrix notation, linear combinations of matrices, matrix multiplication, matrix identity and inverses (and determinants), and the perspective of matrices as linear transformations.
- Vectors in 3D in SMu3t3 and S2SM4 are also introduced very similarly in terms of content: cross product, equations for lines and planes, systems of equations and geometric interpretation of their solutions. One of the main differences however is in how they apply these concepts, the AC SMu3t3 includes a focus on parameterised vector equations, the equation for a sphere, and in particular kinematics: projectile and circular motion in 3D, which are not covered in SACE S2SM4, which instead remains more abstract with these concepts, and on the other hand the examples required are less complex to interpret.
- Complex Numbers are introduced in two topics, a basic and an advanced topic, in both curriculums. The basic topics, SMu2t3 in the AC and S1M12 in SACE are quite similar in their base content: rational/irrational numbers, i , complex arithmetic, conjugates, and complex roots of polynomials. However there are a couple of key differences between the two: first, induction is introduced in the AC SMu2t3 while in SACE it is separated into its own separate topic: S2SM1. The second key difference is that interval notation is explicitly introduced in SACE S1M12, while in the AC interval notation seems to be neglected. The advanced topics SMu3t1 and S2SM2 on the other hand align almost perfectly in content.
- Probability and Statistics is the topic area in which the alignment between the AC and SACE is at its loosest, and the most substantial differences in content exist between the two.
 - Combinatorics: MMu1t3, SMu1t1, and S1M4. The overlap between the AC and SACE for these topics is essentially concepts around permutations, factorial (and the 'multiplication principle'), combinations. Although it is notable that the AC MMu1t3 extends the concept of combinations to binomial coefficients and Pascal's triangle while SACE does not. Beyond

- these common concepts, both curriculums have some introductory probability content, but they take very different approaches to this. The AC does this via set theoretic concepts, union intersection and complement of sets, the pigeonhole principle, and then probability notation ($P(A)$) for set complement, intersection and union and introduces basic probability concepts from this angle (for example, $0 \leq P(A) \leq 1$), including conditional probabilities ($P(A|B)$). On the other hand, SACE S1M4 has introductory statistics concepts (as opposed to introductory probability concepts). Specifically, S1M4 reviews mean median and mode, interquartile range, standard deviation, and introduces the basic concepts around the normal distribution. S1M4 also introduces the distinction between discrete and continuous random data/ variables, not quite introducing the concept of a 'random' variable yet, but still. Very big difference in approach between these topics.
- Introduction to Distributions/ Random Variables: Discrete (MMu3t3 and S2MM2), and Continuous (MMu4t2 and S2MM5). There is quite good alignment between these topics actually. For both discrete and continuous general definitions of expected value and variance are given. For discrete the uniform, examples of arbitrary non-uniform, the bernoulli, and binomial distributions are introduced. For continuous the uniform, restricted domain polynomial, and normal distributions are considered, and transformations of normal distributions (in particular to get the standard normal) are considered. The one key difference is that in SACE the central limit theorem is explicitly explored, while its significance is much less explicit in the AC.
 - Confidence Intervals: The confidence intervals introduced are the same across both curricula, specifically the normal approximation to the binomial confidence interval for a proportion (Wald interval, MMu4t3) and the standard normal distribution confidence interval for the mean of a continuous variable (SMu4t3) are both introduced in SACE S2MM6. However the approach taken to justifying these confidence intervals is a little different, in SACE the justification is very central limit theorem centric, relying on the introduction to that concept in S2MM5, while in the AC instead many of these concepts (including the central limit theorem itself) are simply stated and students are encouraged to test them by simulation. Although SACE also takes this simulation approach to justification it is emphasised less, and the introduction of the concepts around the central limit theorem are much more explicit.

Note: Cumulative Distribution Function not mentioned in SACE

The only substantial difference in content is the concept of proof by induction, which is in the SACE curriculum but not the AC. This is represented in Figure 3.1 by S2SM1 which is an entire topic on induction with no link to the AC, although induction is also briefly introduced earlier in SACE in S1M12.

TODO: Review above dot points, trim/edit down, condense, and write a paragraph here beginning "In summary, ..." or "To summarise, ..."

If we rearrange the topics in Figure 3.1 into their five broad topic areas: Functions and Graphs, Calculus, Geometry and Linear Algebra, Complex Numbers, and Probability and Statistics, we get a much clearer picture, as shown in Figure 3.2.

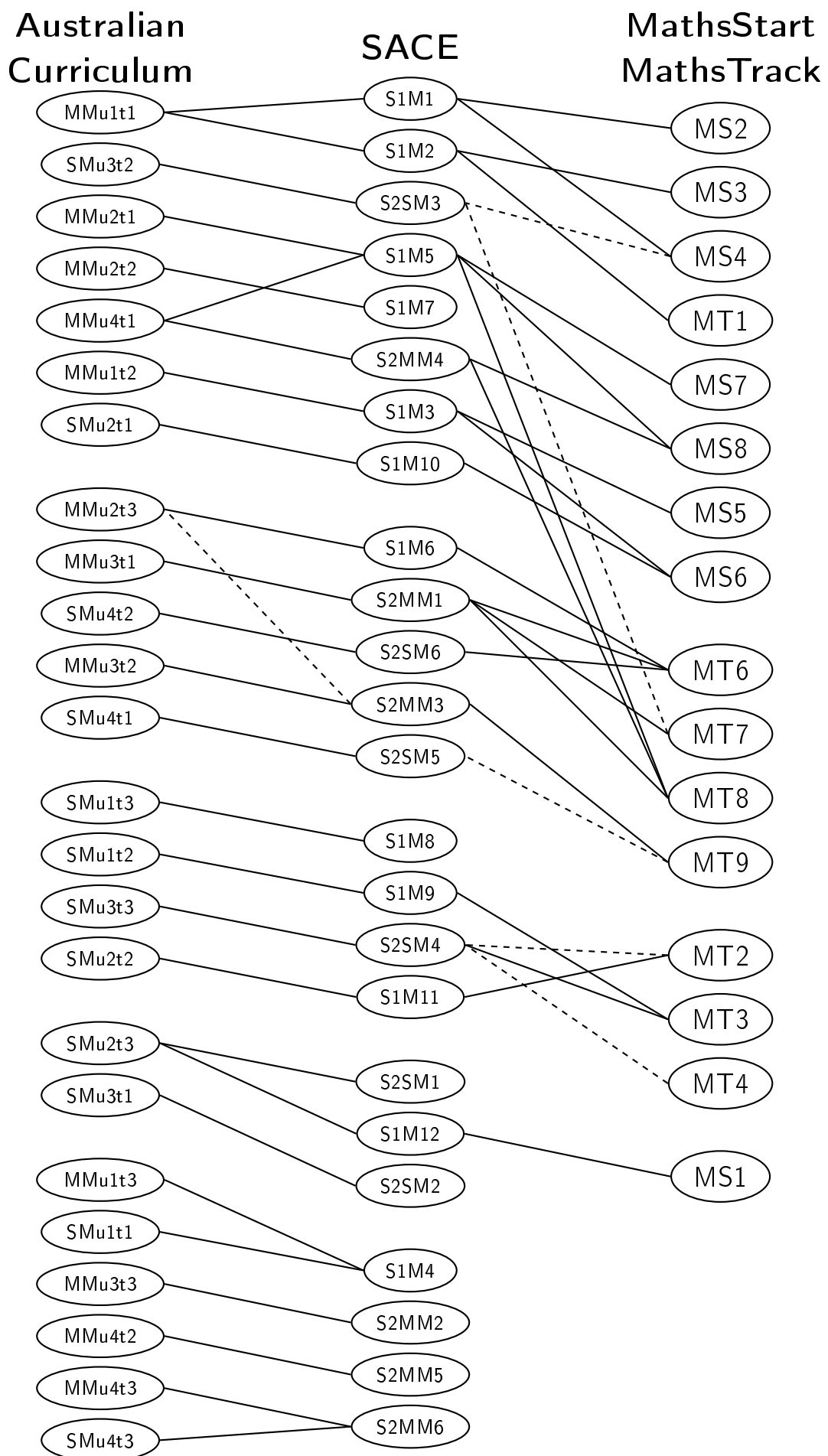


Figure 3.2: Curriculum Mapping

3.2.2 AC and SACE to MathsStart and MathsTrack

In the broad sense of areas of mathematics the topics can be grouped into naturally, as discussed above in Section 3.1, the topics that are covered in the AC and SACE but not in MathsStart or MathsTrack are complex numbers, and Probability/ Statistics/ Combinatorics. Note that the 'missing' MT5 is a topic on complex numbers that is currently being omitted from the bridging course. So given that those areas are not covered in MathsStart or MathsTrack currently, let's take a look in more detail (at a key concept level) at the alignment of the topics that are covered in MathsStart and MathsTrack.

- Functions and Graphs. This topic essentially covers the entire of MathsStart, and the first topic of MathsTrack MT1, and it makes sense to split it into the usual three sub-topics:
 - General Concepts, Polynomials and Rational Functions.
 - * Polynomials and Rational Functions have an interesting binary tree structure, with MMu1t1 splitting into both S1M1 and S1M2 in SACE, which each split into MS2, MS4 and MS3, MT1 respectively in the bridging courses. S1M1 covers mainly linear equations, but also reciprocal functions and asymptotes, while in MathsStart these are split into MS2 and MS4 respectively. Similarly S1M2 covers polynomials, including quadratics and related concepts as well as higher order polynomials, while in the bridging course these are separated into an entire topic just on quadratics (MS3) in MathsStart which is still less in-depth than the introduction to quadratics in S1M2, which introduces rearrangements of quadratics into vertex and factorised form, and then later a more in-depth topic on polynomials in MathsTrack (MT1), which covers some of the more detailed concepts on quadratics missed in MS3 (such as the quadratic formula), and goes on to higher order polynomials.
 - * S2SM3 introduces advanced general concepts on functions: domain and range, function composition, one-to-one, inverse functions, graphing more general rational functions (not just reciprocal functions), and the absolute value function. These concepts are not covered in the bridging courses, and could be useful, but on the other hand, are part of the specialist mathematics curriculum, so aren't the highest priority to include.
 - Exponentials and Logarithms. Both the AC and SACE introduce the concept of exponential functions via recurrence relations describing geometric sequences. Although this is certainly not the only (or even necessarily the best) way to introduce and understand exponential functions, it is the way prescribed by the AC and SACE curriculums, so it might be valuable. On the other hand, the way the number e is introduced is actually identical across AC, SACE, and the bridging course — which is remarkable given how many different ways this could be done. Interestingly, exponent laws and logarithm laws (as well as basic properties of exponential and logarithmic functions) are introduced in S1M5 of SACE, and are split into

the two topics MS7 and MS8, it seems that the granularity of the MathsStart program is roughly a factor of two more granular than the SACE curriculum, which is pretty interesting. Could be a direction for future research on some measure of “degree of granularity” of a course/ program. It is also interesting to note the location of S2MM4 in Figure 3.2 in the Functions and Graphs section, rather than the differentiation section. This is a choice because of how concepts of exponents and logarithms are very distinctly separated in the AC into MMu2t1 and MMu4t1, while in SACE introductory concepts for both are introduced in S1M5 and S1M7, but while advanced concepts around logarithms (including calculus) have their own topic in SACE — S2MM4, advanced concepts (such as calculus) around exponential functions do not, and are instead lumped into more general calculus topics (S2MM1 in SACE and MMu3t1 in AC). I’ll include a more indepth discussion of this under calculus, as this section ought to focus on the function and graph properties, but there is significant overlap between the two in the way the content is arranged (perhaps rightfully so).

- Trigonometry There is actually fairly good alignment in the trigonometry sections, although they are organised differently (S1M5 and S1M10 in SACE and MS5 and MS6 in MathsStart), the content is fairly well aligned. SACE covers some graphing slightly more comprehensively, talking about translations and dilations for example (a concept from MS3 that could be “translated” here effectively, linking the concepts and chaining them from topic to topic a little more strongly).
- Calculus, it makes sense to organise this under the MathsTrack topics (maybe I should do that for functions and graphs above as well actually).
 - MT6: Differential calculus is introduced very similarly, through first principles etc. in both MT6 and S1M6, MT6 actually goes beyond the content of S1M6, introducing also the product, chain and quotient rules (which are covered in S2MM1) and implicit differentiation (which is covered in S2SM6). On the other hand, MT6 does not cover increasing and decreasing (which is in S1M6), which is instead covered in MT7. Also, the concept of a normal to a curve is introduced in MT6, but it is not really covered anywhere in SACE (apart from sort of in S2SM4 using cross products).
 - MT7: Covers a few concepts from S1M6 that were skimmed over in MT6, as well as some of the more advanced function and graph concepts such as sketching rational functions which is only covered in S2CM3.
 - MT8: Similarly, taking the more advanced concepts from the functions and graphs topics of SACE, and combining them with the introductory calculus concepts MT8 introduced differentiation of exponents (from S2MM1) and logarithms (S2MM4), at the same time as re-hashing concepts from MS7 and MS8 and revising them (such as introducing exponential and logarithm functions/ sketching them, etc.). Notably, surge models and logistic models are introduced as well. Surge models are not covered anywhere in the AC or SACE as far as I can tell, and logistic models are only introduced in S2SM6.

- MT9: All the integration is fit into this single topic in MathsTrack, which students inevitably find challenging. This covers essentially all of S2MM3, and then goes a little further with the notable addition being integration by substitution, which in SACE is only covered in S2SM5. Notably summation notation is also introduced (in an appendix) in MT9, an important bit of notation that students often struggle with in first year university, but that isn't anywhere in the SACE curriculum as far as I can tell.
- Geometry and Linear Algebra is where SACE and the bridging courses begin to really diverge in earnest.
 - MT2 aligns well to S1M11, although it goes a little further and also introduced row operations, a concept not introduced in S1M11 although it is introduced in S2SM4 in a quite different context, not so much to do with matrices in the pure sense, but instead focussing on the connection to solving 3×3 systems of linear equations. This aspect, of solving systems of equations, is introduced in MT4 and actually gone into in great depth, while the concept seems tacked on and is not gone into in detail at all in S2SM4.
 - MT3 Introduces vectors and vector concepts in both \mathbb{R}^2 (concepts covered in S1M9) and \mathbb{R}^3 (concepts covered in S2SM4). The overlap between S1M9 and MT3 is substantial, with MT3 covering most of the concepts in S1M9, although S1M9 goes into a little more detail on scalar dot products (a concept covered in both), and also introduces the concept of orthogonal projection, and even throws in a dash of geometric styled proof. S2SM4 also goes significantly further, most notably introducing the concept of the vector cross product which is not covered in MT3, although both introduce equations for planes in \mathbb{R}^3 .
 - MT4 focuses on systems of linear equations, and although this concept is introduced in S2SM4, it is covered in MT4 in a much more detailed, granular way. Also, MT4 introduces Gauss-Jordan Elimination, an algorithm not explicitly introduced in SACE as far as I can tell (although it is implied).

MS4: Note the link to S2SM3

- MS1: Maybe Introduce Interval Notation along with Intervals?

3.3 Notes on interpreting

Figure 3.2, the dashed lines represent tenuous connections, usually a single concept shared:

- the one between AC and SACE essentially represents the concept of anti-differentiation,
- the one between S2SM3 and MT7, as well as the one between S2SM3 and MS4 essentially represents sketching rational functions, although in MS4 only reciprocal functions and transformations thereof are considered, this concept of

sketching these graphs and the properties of these graphs is heavily emphasised as a way to explore the concepts connected to these functions.

- the one between S2SM5 and MT9 essentially represents integration by substitution.
- the one between S2SM4 and MT2 essentially represents row operations, in MT2 introduced on matrices, but in S2SM4 it is introduced explicitly in the context of solving 3×3 systems of linear equations. Similarly the dashed line between S2SM4 and MT4 represents essentially the same concept in S2SM4, but in MT4 the system of equations perspective is explored, which is not really done as much in MT2.

3.4 Conclusions/ Summary

Broadly the content for functions and graphs, as well as calculus is well aligned between the bridging courses and the AC/ SACE, although the bridging courses tend to mix the two more, merging the more advanced concepts from functions and graphs into the calculus topics. This makes perfect sense to do, especially considering how inter-connected the areas are in the first place. Bigger differences between the content of the bridging courses and the AC and SACE exist in the other area though, with big differences in linear algebra and geometry, complex numbers almost not being covered in the bridging courses (although they used to be covered more, see the discussion relating to MT5), and the biggest difference being that statistics and probability are not covered in the bridging courses at all.

Broadly, a big difference in emphasis between the bridging courses and SACE is the emphasis the bridging courses place on sketching graphs, and explicitly exploring the connection between transformation (translations and dilations primarily) of a graph relate to algebraic changes to functions. Although this is covered in SACE to some degree, it is largely implicit and left to reading between the lines, while it is quite explicit in the bridging courses.

My recommendations for MathsStart would be to include some work on fractions, index laws, and more emphasise on re-arranging equations as in my experience these are the topics and concepts that students need the most from middle school mathematics (up to year 10) and form foundations for building other concepts with in senior highschool. This is already the implicit purpose of MathsStart in a sense, and this is good. Aside from that, concepts that could be added to the bridging courses to bring them more in alignment with the AC and SACE are:

- I'm not sure if I missed it somewhere, but a formal introduction of the definition of a function (and hence the alternative — a relation) would be good as the concept of a relation is in the AC and SACE, although it is not used much. It is a little useful later when doing implicit differentiation, for example.
- S2SM3 introduces advanced general concepts on functions: domain and range, function composition, one-to-one, inverse functions, graphing more general rational functions (not just reciprocal functions), and the absolute value function. This concepts are not covered in the bridging courses, and could be useful, but on the other hand, are part of the specialist mathematics curriculum, so aren't the highest priority to include.

- I like the way quadratics are introduced in MS3, particularly as the concepts used to introduce them (dilations, translation) are very applicable in the AC and SACE in many places. On the other hand there are a couple of concepts here that are missed in the gap between MS3 and MT1, specifically rearranging quadratics algebraically to get them in vertex form and factored form, although these forms are introduced implicitly in MS3 and interpreted, algebraic rearrangement of them could be emphasised more, especially given how this is one of the key concepts we want students to be picking up as they go through MathsStart.
- Both the AC and SACE introduce the concept of exponential functions via recurrence relations describing geometric sequences. Although this is certainly not the only (or even necessarily the best) way to introduce and understand exponential functions, it is the way prescribed by the AC and SACE curriculums, so it might be valuable.
- Overall having the concepts link from topic to topic more, for example in MS3 dilations and translations of quadratic functions are considered. It would be useful if these concepts were re-visited in MS6 for example when looking at graphing/ sketching trigonometric functions, as this is covered in SACE explicitly but also because it connects the concept to multiple different topics and applications (different kinds of functions).
- Scalar dot product of vectors is introduced in MT3, but SACE goes a little further, also introducing the concept of orthogonal projection in \mathbb{R}^2 in S1M9.
- I think it would be a good idea to introduce interval notation along with the concept of intervals when that concept is introduced in MS1.

while concepts currently in the bridging courses that could be removed as they are not part of the AC or SACE include:

- Implicit differentiation is technically only in Stage 2 Specialist Mathematics, so could potentially be removed from MT6.
- Normal to a curve is a concept introduced in MT6 but not used anywhere in the SACE curriculum (as far as I can tell), apart from briefly in S2SM4 when it is used in the context of vector cross products and equations for a plane — a very different context.
- Surge models are not anywhere in the AC or SACE, so could potentially be removed from MT8 entirely.
- Logistic models are only included in Stage 2 Specialist Mathematics, so could potentially be removed from MT8. Alternatively if it was kept, it could be introduced with concepts around differential equations, which is how it is introduced in Stage 2 Specialist Mathematics. At the moment, it is introduced as a model (an equation), not a solution to a differential equation, so even as it is it is very different content.
- Integration by Substitution is only covered in Stage 2 Specialist Mathematics, so could potentially be removed but is also very useful in first year university

calculus-based courses (such as Maths IA etc.) so I'm not sure. Depends, as most of these do, on the mindset: it would be useful content for the students to be exposed too early, or you could look at it from the perspective of they are not expected to know it going into a course like Maths IM.

- Concepts covered in S2SM4 on geometry in \mathbb{R}^3 are technically only covered in Specialist Mathematics, so could be cut in principal if we are operating under the assumption bridging course students will be doing Maths IM. So concepts like vector cross product, and the equation for a plane, to be specific.
- Technically, the Gauss-Jordan Elimination method introduced in MT4 is not anywhere in SACE, but that said it is somewhat implied a little in the topic that discusses the other concepts in MT4 (around systems of linear equations) — S2SM4. That said, this entire topic and broad concept of systems of linear equations is entirely contained in Stage 2 Specialist Mathematics, so it could be argued to not be included for that reasoning.

Other recommendations:

- Maybe spread out the introduction of integration? Potentially introduce both differentiation and integration, and then introduce them together on each individual type of function? Not sure.

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Chapter 4

Moving Forward: Improvements

4.1 Current Strengths of MathsStart and MathsTrack

Moving forward is a two-part process:

- Recognise what is being done well, encourage and recognise it, and continue to support its ongoing excellence.
- Recognise what can be improved on, gaps that may exist, and address them with specific actionable changes.

4.1.1 SQWIGLES

SQWIGLES is an abbreviation (see Figure ??) developed at the University of Adelaide maths learning centre with several purposes in mind:

- As a way to guide tutors working at the maths learning center on how to best help the students.
- To scaffold self-reflection in teaching and education when working one-on-one with students by providing specific actions one can focus on paying attention to and reflecting on.
- As a tool to focus efforts on self-improvement, by choosing one action to improve on at a time it can provide an avenue towards improvement that feels achievable, and can provide concrete progression.

SQWIGLES also had some ancillary and unintended benefits, as once an educator is engaged actively in self-reflection, even if it is prompted by paying attention to one specific action, they notice other things perhaps even things unrelated to SQWIGLES itself. David Butler, the current maths learning center coordinator at the University of Adelaide, wrote [a blog post about SQWIGLES](#) which is quite informative and has more detail, but here I will provide a brief overview as it is a very beneficial tool that generalises well beyond mathematics education.

As teachers we are constantly encouraged to reflect on our practice and continually aim to improve and develop our skills, and rightfully so. Teaching is a process of continual improvement. Often however, we can reflect on our practice and either be

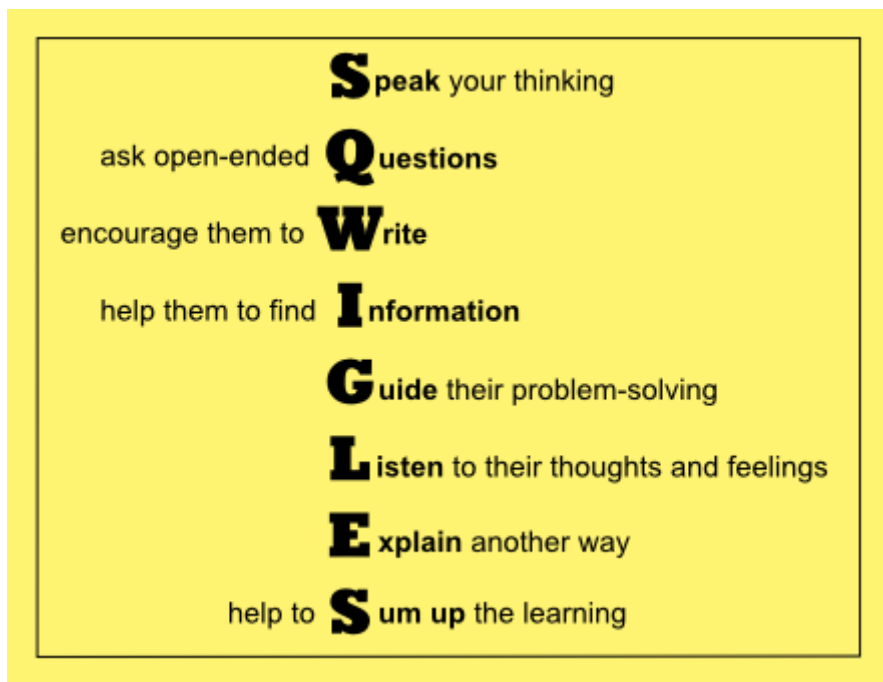


Figure 4.1: SQWIGLES

overwhelmed by the amount of observations we make and not know where to start, or be unsure what to reflect on, exactly — what aspects of our practice to spend the time to critically analyse. Teaching is an incredibly broad and diverse profession with many aspects, and improvement is always possible in every aspect of it. However pursuing improvement takes time (a resource we as teachers are very low on), and attention and energy, and so it is completely untenable to pursue improvement in all aspects of teaching simultaneously. In the face of this, we often suffer from either decision paralysis, or choosing relatively insignificant aspects of our teaching to focus on. SQWIGLES can serve as a guide for both where to focus our self-reflective attention, and as a list of suggestions for individual aspects of our teaching practice to focus on improving one at a time.

Speak Your Thinking

- Mathematics as language, speaking to translate.
- Mathematics as the process of reasoning, speaking to illustrate that process.

Ask Open-Ended Questions

Encourage Them to Write

Help Them Find Information

Guide Their Problem-Solving

Listen to Their Thoughts and Feelings

Explain Another Way

Help to Sum Up the Learning

4.1.2 Staff Culture at the Maths Learning Center

4.1.3 Self-Paced Assessment and Content Speed (!!!)

Link to Maths Anxiety literature review.

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Conclusions and Recommendations

With respect to the bridging courses run through the university of adelaide's maths learning centre: MathsStart and MathsTrack,

- The self-paced and feedback focused approach to assessment is certainly the highlight of the programs, should be continued, encouraged, potentially further resourced, expanded, and recommended to other bridging course facilitators.
- The role of bridging courses as what is often student's first experience at university implies that potentially students wellbeing and retention could be improved by structuring the programs to provide more opportunities for students to meet each other and work together: either in the maths learning center drop-in area, or a seperate area, but potentially assigning a certain time on a certain day perhaps weekly or fortnightly during which students are encouraged to come and work together, could allow them to make freinds, build social networks, and better aclimitise them to the university environment in order to better prepare them for success in their studies.
- The smallest but perhaps easiest to implement improvement could be to better align the course content with curriculum, both the highschool curriculum (AC/SACE) in the case of students doing the bridging course to then comence study interstate or overseas, or with specific first year entry level courses, to better match the potential gaps in knowledge students may encounter.

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