2 Unit Bridging Course - Day 11

Logarithms

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Definition

Recall from Days 7-8 that given the expression a^x:

- x is called the *index*, *power*, or *exponent* of a;
- a is called the *base*.

However, an index/power/exponent is also called a **logarithm.**

In particular, given

$$y = a^x,$$

x is called the "**logarithm of y to the base a**", written as:

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Examples

Here are some examples of logarithms to the base 2:

- $\log_2 4 = 2$, since $4 = 2^2$.
- $\log_2 8 = 3$, since $8 = 2^3$.
- $\log_2 \frac{1}{8} = -3$, since $\frac{1}{8} = 2^{-3}$.

Here are some examples of logs to the base 10:

- $\log_{10} 100 = 2$, since $100 = 10^2$.
- $\log_{10} 1000 = 3$, since $1000 = 10^3$.
- $\log_{10} 0.001 = -3$, since $0.001 = 10^{-3}$.

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$\boxed{\textbf{log}}$ and $\boxed{\textbf{ln}}$ on your Calculator

The $\boxed{\textbf{log}}$ button on your calculator calculates logarithms to the base 10.

E.g. $\boxed{\textbf{log}}$ $\boxed{\textbf{1000}}$ will display 3, since $1000 = 10^3$.

Of particular interest to us is the $\boxed{\textbf{ln}}$ button, which provides logarithms to the base e.

E.g. $\boxed{\textbf{ln}}$ $\boxed{\textbf{5}}$ will display roughly 1.61, since $5 \approx e^{1.61}$.

Logarithms to the base e are called **natural logarithms**.

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Some Properties of Logarithms

Note that $\boxed{\textbf{log}}$ $\boxed{\textbf{1}}$ and $\boxed{\textbf{ln}}$ $\boxed{\textbf{1}}$ both give 0 on your calculator. In fact,

$$\log_a 1 = 0$$

for any base $a > 0$, since we know from Day 7 that $a^0 = 1$.

Moreover, recall from Day 8 that all exponential functions of the form $y = a^x$ (where $a > 0$) output strictly positive numbers for all x. Therefore it does not make sense to take the log of zero or any negative number.

Indeed, trying to $\boxed{\textbf{log}}$ or $\boxed{\textbf{ln}}$ any number ≤ 0 will result in an 'Err' error message being displayed on your calculator!

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The Logarithm Laws

Here are some useful ‘laws’ for manipulating log expressions:

The Logarithm Laws

The following is true for the log to *any* base:

1) $\log(AB) = \log A + \log B, \quad \text{for } A > 0 \text{ and } B > 0.$

2) $\log\left(\dfrac{A}{B}\right) = \log A - \log B, \quad \text{for } A > 0 \text{ and } B > 0.$

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The Logarithm Laws (cont.)

Example

Simplify $2\ln(x+1) + \ln(2x)$ to a single logarithm.

$$2\ln(x+1) + \ln(2x)$$

$$= \ln(x+1)^2 + \ln(2x) \qquad \longleftarrow \qquad \text{using log law \#3}$$

$$= \ln\left[(x+1)^2 \times (2x)\right] \qquad \longleftarrow \qquad \text{using log law \#1}$$

$$= \ln\left[2x(x+1)^2\right] \qquad \longleftarrow \qquad \text{upon simplifying the inside}$$

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Practice Questions

- Simplify $\frac{1}{2}\ln\left(x^2\right) - \ln(x^3)$ into a single logarithm.
- Write $\ln\left(x^2\sqrt{3x+1}\right)$ as sums and multiples of simpler logarithms.

Answers

- $\ln\left(\frac{1}{x^2}\right)$.
- $2\ln x + \frac{1}{2}\ln(3x+1)$.

Summary

- $y = a^x \iff x = \log_a y$, where $\log_a y$ is spoken as the "*logarithm of y to the base a.*"
- The $\boxed{\textbf{log}}$ and $\boxed{\textbf{ln}}$ buttons on your calculator provide logarithms to the base 10 and e, respectively.
- $\log(ab) = \log a + \log b, \quad \text{for } a > 0 \text{ and } b > 0.$
- $\log\left(\dfrac{a}{b}\right) = \log a - \log b, \quad \text{for } a > 0 \text{ and } b > 0.$
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