# 2 Unit Bridging Course – Day 1

## Introduction to Functions

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In higher-level mathematics, functions represent fundamental objects of study, the understanding of which are therefore crucial to your mathematical success at university.

The purpose of functions is to describe **relationships** between different **variables** — for instance, between:

- a person and their birthday.
- a historical year and the world population at that time.
- a person's age versus the size of their vocabulary.

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So what exactly are functions? Intuitively, a function can be thought of as a "machine" that takes in **inputs** and spits out a **unique outputs**, often according to some **rule**.

## Example

As a basic example, the oven machine can be regarded as a function with the rule 'to cook', which takes in uncooked food as input and outputs cooked food.
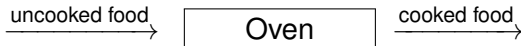
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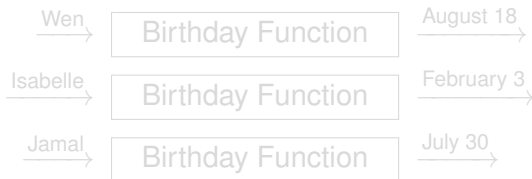
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As another example, consider a 'birthday function' that takes in people, and as a rule, spits out their birthdays as output:

| Wen → | Birthday Function | August 18 → |
| Isabelle → | Birthday Function | February 3 → |
| Jamal → | Birthday Function | July 30 → |

Notice that outputs are indeed unique, since no person can have two birthdays!

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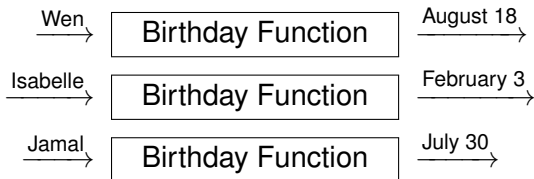$$\xrightarrow{\text{Wen}} \boxed{\text{Birthday Function}} \xrightarrow{\text{August 18}}$$

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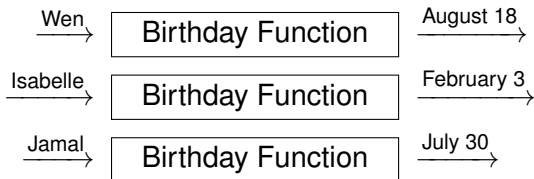
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Of course, a birthday depends on the person we are referring to. That is, changing the person will likely change the birthday.

Thus, outputs are known as **dependent variables**, whose values are dependent upon our choice of inputs, known as **independent variables**.

If we name the independent variable $p$ (for $p$erson) and the dependent variable $b$ (for $b$irthday), then $b$ depends on $p$.

Terminology-wise, we say that $b$ is a **function** of $p$, written as $b = f(p)$ and colloquially spoken as "**$f$ of $p$**" (or sometimes as the "**function value at $p$**" or the "**function evaluated at $p$**").

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In this formal notation, we can therefore write:

- $f(\text{Wen}) = \text{August 18}$;
- $f(\text{Isabelle}) = \text{February 3}$;
- $f(\text{Jamal}) = \text{July 30}$.

When reading this notation, simply keep in mind that whatever is inside the brackets – such as Wen – is the input, and $f(\text{Wen})$ is then the resulting output (with the function's rule now applied):

$$\xrightarrow{p} \boxed{f} \xrightarrow{f(p)}$$

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In your university studies, you will most often be working with functions defined by **formulas**.

## Example

Consider a formula that describes the relationship between the dosage $d$ (in ml) of threadworm medication required for a person of weight $w$ (in kg):

$$d = \frac{w}{5}.$$

So a man with a body weight of 70kg would require a dose of 14ml of medication, while a child of 20kg would require a dose of 4ml.

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Because *d* depends on *w* and each value of *w* produces exactly one value of *d*, *d* is a function of *w*. That is, $d = f(w)$ in the aforementioned notation.

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## Example

For our drug function, we can draw up a table of values by evaluating $f(0), f(20), f(40), f(60), f(80)$ and $f(100)$:

| weight $w$ | 0 | 20 | 40 | 60 | 80 | 100 |
| --- | --- | --- | --- | --- | --- | --- |
| dosage $f(w)$ | 0 | 4 | 8 | 12 | 16 | 20 |

Note that a table consists of only a *finite* number of values and thus cannot provide as much information as a formula. Had our drug function been given through this table alone, it would not be clear what dosage an adult of say, 70kg would require. But according to the formula, we know that this would be $\frac{70}{5} = 14$ml.

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However, a table does allow us to represent functions in yet another form - it's **graph** (sometimes called a *curve*).

This is done by treating each $(w, f(w))$ pair in the table as $(x, y)$ coordinates in the $xy$-plane.

It is for this reason that function inputs are sometimes referred to as **x-values**, and function values as **y-values**.

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For our drug function, this entails first plotting the coordinates

$(0, 0), (20, 4), (40, 8), (60, 12), (80, 16), (100, 20)$

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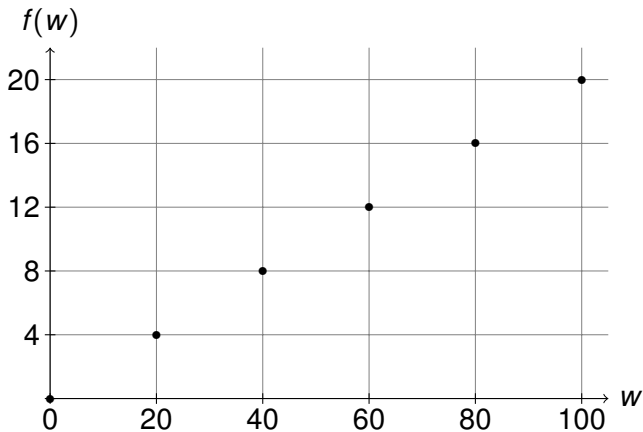
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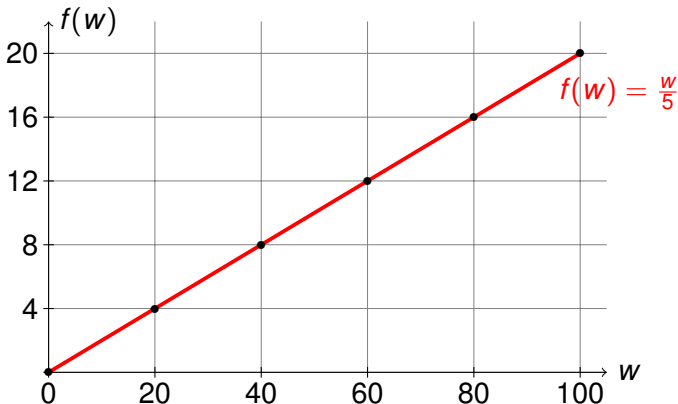
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$(0, 0), (20, 4), (40, 8), (60, 12), (80, 16), (100, 20)$

provided by the table.

The graph is then constructed by connecting all of these points.



The graph is **linear** in shape.

- Intuitively, functions are 'machines' that take in inputs and eject unique outputs often according to some rule.

- Functions are commonly defined by *formulas*, but may also be expressed as *tables* or *graphs*.

- The $f(x)$ notation is used to describe formulas for functions, where $x$ represents the input and $f(x)$ represents the output.

- A graph of a function can be drawn by obtaining a collection of different $(x, f(x))$ pairs from its formula and plotting them as coordinates on the *xy*-plane.