2 Unit Bridging Course – Day 12

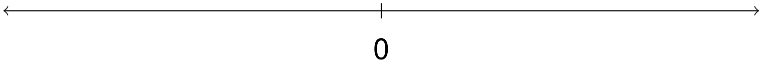
Absolute values

Clinton Boys

THE UNIVERSITY OF SYDNEY

The number line

The number line is a convenient way to represent all numbers:



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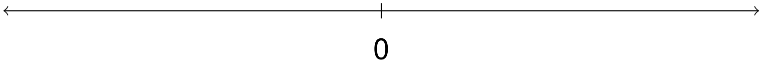

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The position of a number on the number line is completely specified by two pieces of information:

(i) its magnitude or absolute value, that is, the absolute distance of the number from zero, and

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If $x \geq 0$, the absolute value of x is just x itself since its sign is positive:

$$|x| = x \quad \text{if } x \geq 0.$$

However if $x < 0$, the absolute value of x is the positive number which sits on the other side of zero on the number line. This number is exactly the negative of the negative number x:

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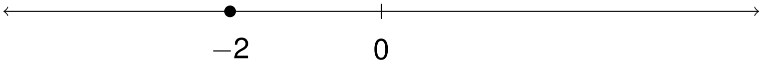

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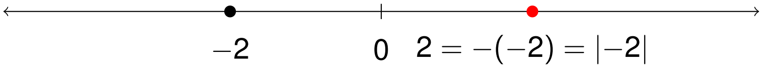

Example

(i) $|3| = 3$

(ii) $|-4| = 4$.

Equations involving absolute values

Suppose we are given the equation

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and asked to solve it for x. Since we now understand absolute values, we know there are exactly two possibilities:

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Equations involving absolute values

Practice questions

Find the values of x which satisfy the following equations:

(i) $|x-2|=4$

(ii) $|3x+1|=2$

(iii) $|2+x|=1.$

Equations involving absolute values

Answers

(i) $x = 6$ or $x = -2$

(ii) $x = 1/3$ or $x = -1$

(iii) $x = -1$ or $x = -3$.

Graphing absolute values

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This function will look the same as the function $y = x$ when x is positive, but when x is negative it will be the *negative* of $y = x$.

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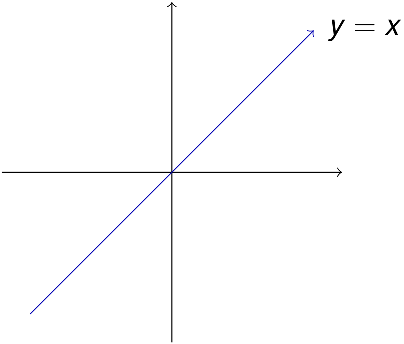

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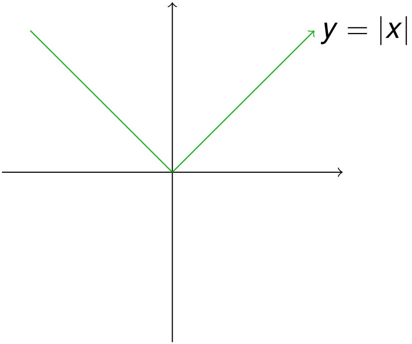

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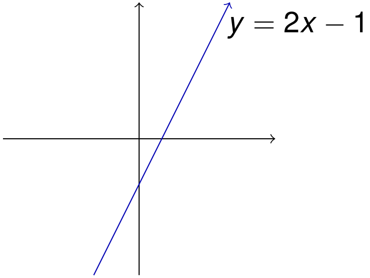

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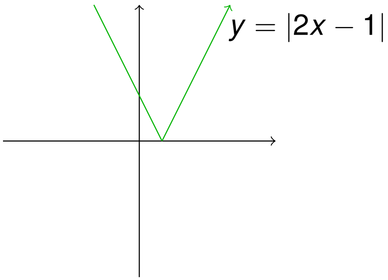

Graphing absolute values

Practice questions

Sketch the following curves:

(i) $y = |3x - 4|$

(ii) $y = \left|x^2 - 1\right|$

(iii) $y = |\sin x|$

Answers

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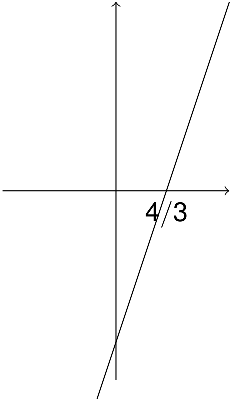

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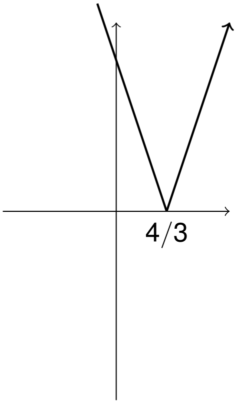

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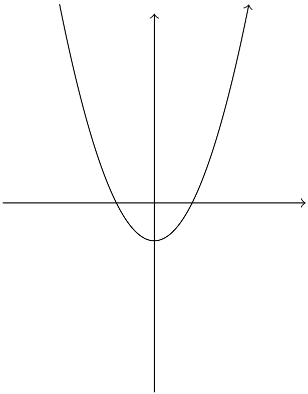

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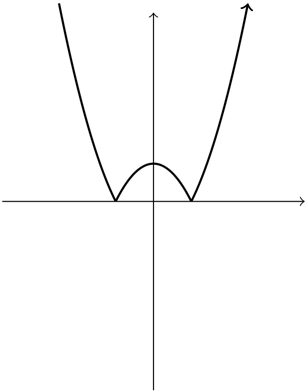

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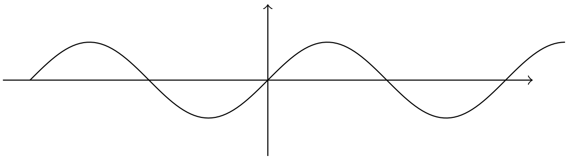

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