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**Affective change in adult students in second chance  
mathematics courses: three different teaching approaches**

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## **Abstract**

A case study approach was used to explore second-chance mathematics through two larger courses and one individual study program. A different teaching approach, by committed experienced teachers, was used in each course. In evaluating their effectiveness, I focused on affective change in the students, relating this to their achievement. This study contributes to research on understanding good teaching of mathematics to adults.

Qualitative and quantitative data were collected over several years. Methods included: a questionnaire (including mathematics attitude and belief scales as well as demographic and open questions); interviews with students to gather more affective data and explore their reactions to the course approach; and the individual supervised study course was audio-taped for six months. Teachers of the larger courses were also interviewed about their goals for, and experiences with, the students. These multiple strands of evidence provide a complex overall picture of three, largely successful, teaching approaches. Each measure had its own contribution to make, and taken together they illuminated the ways in which affective change was related to achievement in the three contexts.

The higher achieving groups in each of the two larger courses entered the courses with more positive attitudes and beliefs than the lower achieving groups and subsequent affective changes reinforced these differences. The lower achieving groups completed the courses affectively worse off than when they started.

Students' reactions to these approaches were compared and found to reflect the nature of the approach. In addition to this finding, successful students' beliefs about mathematics changed in two of the courses. In the one-to-one course the teacher focused initially on understanding the students' fear of mathematics and early mathematical experiences. The student-focused teaching approach trusted and encouraged the growth of this student's mathematical thinking. Six months later the student felt empowered and had come to believe that mathematics was a creative and enjoyable process of discovering patterns. The second course focused on the mathematization of realistic situations. Successful students came to regard mathematics as useful, interesting, relating to real life. Successful students in the third course appreciated the carefully structured reintroduction to mathematics and were pleased they could finally do the mathematics they hadn't been able to understand at high school.



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# **1 Introduction**

Decades of experience teaching, mostly mature, students who have had a ‘rocky road’ in mathematics provided motivation for my thesis topic. The effectiveness of three courses developed for adults learning mathematics is the topic of my research study, which I hoped would contribute to the body of research on understanding good teaching of mathematics to adults. One course was a supervised study course for an individual, while each of the other two courses contained about 100 students. I chose to focus on affective change in the students, relating this change to their achievement in the course, using a case study approach. Qualitative and quantitative research methods are used. A questionnaire was developed to gather data, and it includes mathematics attitude and belief scales as well as demographic and open questions. Final marks in the two large courses were used as a measure of achievement. Further data came from interviews with a few students in similar demographic groups and with teachers of the courses, allowing an exploration of other variables which may have influenced students’ attitudes, beliefs, achievement level and their reactions to a particular course. This thesis is also the report of a journey of a quantitative researcher who became a qualitative researcher, replacing the analysis of existing data with the experience of gathering, and analyzing, both quantitative and qualitative data. I was inspired by qualitative data as it gave me a much more personal feel for each student’s experience learning mathematics.

My professional interests, research and teaching experience over the last 15-20 years have all contributed to my interest in this research project. My motivation for the topic of this research project is based on my previous experiences, both as a teacher of academically-able students who feared and avoided mathematics, and my involvement in equity initiatives particularly for girls/ women and mathematics.

Teaching mathematics to adults began for me in 1985 when I teamed up with a friend, who was both a university graduate but feared and avoided mathematics, to co-teach a ten week course called “Maths Anxiety” for adults at a local high school evening community class. We read material about

mathematics anxiety, common 'myths' about mathematics (Kogelman & Warren, 1978) and courses and resources developed for such students (Brown, 1984; Langbort & Thompson, 1985). We learned a lot from each other while planning and co-teaching this course. The following year I was asked to teach part-time in an academic assistance program, attached to the Counseling Centre, just beginning at the University of Auckland. A year or two later I was appointed to a permanent tutoring position in what was eventually known as the Student Learning Centre. I developed the mathematics and statistics program for the Centre and taught within it as well as tutoring part-time in the Departments of Mathematics and Computer Science. I also taught a mathematics content course, for several years, to primary and intermediate school teachers in the (post-experience) Diploma of Mathematics Education. In addition, in the post-graduate Diploma of Teaching, I taught mathematics to students with little of the background knowledge usually assumed to be required to learn how to teach mathematics at primary and intermediate schools. I have always felt privileged to have the opportunity to teach adults individually or in small groups and have formed many friendships over the years with mature students whom I have taught.

The students in all of these programs are largely 'mature' students, they are highly motivated, they have not seen (formal) mathematics or statistics for a long time and they are struggling with it in other courses, with varying degrees of anxiety and competence. They could be described as maths-avoidant (Buerk, 1982). Their first encounters with mathematics at school were usually not successful at some stage for a variety of reasons, although for some it was a subject they liked. Many do not realize how much mathematical knowledge and experience they have (Cockcroft, 1982; Fitzsimons, 1994; Harris, 2000; Noss & Hoyles, 1996). Most of these students are usually doing very well in their (non-mathematical) courses at university, have very high expectations of themselves and work to get high grades. They are usually juggling their study with parenting and/or work responsibilities. Some have supportive families, while others do not; some are sole parents. I have found these students very interesting and satisfying to teach and have discovered and developed many interesting resources which seem to be successful with such students. Many of my teaching methods have been informed by my involvement on the executive of EQUALS (NZ) since 1985, with a group of teachers who were concerned about the lack of participation of females and other under-represented groups in mathematics, and who organised in-service professional development. I gained much of my professional teaching support from the experienced teachers involved in this group. I needed this support because my

pedagogical/andragogical style was different, and needed to be in order to help the maths-avoidant, from the most common pedagogy employed at the university. My experiences, and vignettes of others' experiences in EQUALS (NZ), were included in a research study which explored the impact of this group over ten years on mathematics education in NZ. The study, written by Jill Ellis, Maxine Pfannkuch and myself (1998) was presented at the International Organisation for Women and Mathematics Education (IOWME) sessions at the 8<sup>th</sup> International Congress on Mathematics Education (ICME 8). My professional development in teaching continued during my attendance at a number of workshops in the USA in 1987 while on Research and Study Leave from the University of Auckland, supported by a Winston Churchill Fellowship to investigate programs which encouraged girls and women in mathematics and computing (Reilly, 1988).

A bridging mathematics course was developed in the early 1990's in the Department of Mathematics at the University of Auckland. This course did not seem to cater for the needs of the mature students who were returning to the study of mathematics with little background in mathematics as well as it met the needs of students who had studied mathematics, possibly not successfully, in senior high school more recently.

When the Mathematics Education Unit, in the Department of Mathematics at the University of Auckland, decided to pilot a new course (Mathematics 1) for students coming back to the study of mathematics in 1994, I was very interested to study its effectiveness. Typically these students may not have studied mathematics beyond the first few years of high school and may have failed mathematics examinations at that time.

I thought that the approach in Mathematics 1 might be successful with adults as it was based on teaching students how to mathematize realistic non-routine problems and therefore might utilize the prior experiences of adult students and change some of their attitudes about mathematics and beliefs about how mathematics might best be learned.

The Wellesley Program, a pre-degree full-time certificate program, had already been developed five years earlier to help students who wanted another chance to return to education, enabling them to access other tertiary courses of study in the future. Their level of background mathematics varied greatly, with very few having completed all five years of high school. Collecting similar data, to that collected in Mathematics 1, about the students in the compulsory mathematics course within this Program allowed me to explore the reactions of students to

the carefully structured and more traditional teaching approach used for this adult population.

I felt that evaluating these courses would help redevelop them to be more effective in the future. In fact many of my research results have already served this purpose. Such courses are important in addressing the needs of students who fear and avoid mathematics and who want to study in the pre-degree or degree programs at the university. If such students are ignored, they will incur personal and financial costs and it certainly will affect the retention of such students in our institutions. As a society we also need mathematically confident and competent biologists, psychologists, town planners and teachers, for example, to function more effectively in the workplace.

A third course was organized later, in 1995, when a 33 year-old businessman contacted me for assistance. I could see that supervised study with this individual who feared mathematics, carefully recording his process and progress, could become an interesting part of my project. He was happy to be taught by me as part of my research and was willing for our sessions to be audio-taped. I also gathered his responses to the questionnaire used in the other two courses.

In order to evaluate the first two courses I focused on the following questions. What effect does studying mathematics in this course have on students? What are the characteristics of students who are best suited to the style of teaching and assessment in each course? Did these courses achieve their broad goals, namely, to attract and help adult students who have not succeeded in previous mathematics study? Did characteristics of learners matter in the effectiveness of these courses? What are students' reactions to the particular approach?

I will now explain how I interpret some terms used in these research questions. What are the characteristics explored? Age, gender, mathematics background, ethnicity, first language, degree/option, and fluency in English were the demographic information gathered. Attitudes measured by attitudinal scales were enjoyment of mathematics and value of mathematics. Beliefs about themselves as learners of mathematics and beliefs about the learning of mathematics were also gathered.

What is meant by reaction? Reactions were gathered either by self-report, students' answers to open questions, by scores on scales, or by interview. During the interview students were asked to compare their learning experience in the current course with their experiences learning mathematics in the past using a projective technique.

What did I mean by best “suited”? A good level of achievement in the course based on their final grades; an increase in scores measuring enjoyment or value of mathematics; having beliefs change appropriately, ie about themselves as learners of mathematics and/or about the learning of mathematics.

The happenstance of the third course, and subsequent data analysis, has changed my focus. Themes which emerged from the data analysis of this third course, the supervised study course for the individual, have been used to structure the theoretical discussion of all three courses.



## 2 Literature Review

### 2.1 Introduction: History of research on the mathematics education of adults

The study of characteristics of successful mathematics education for adults is on the periphery of the field of mathematics education. Most attention in this field, in curriculum writing and in research, for example, has been focused on mathematics education in primary, intermediate and secondary schools. However, over the last decade or two this has begun to change. "Adults Learning Mathematics - a Research Forum" (a largely European organization) was established in 1994, holding an annual conference for practitioners and researchers since then. I presented a paper at their 1996 conference (Miller-Reilly, 1996), a later version of which appeared as a chapter in a book (Miller-Reilly, 2000). In Australasia, the more informal "Bridging Mathematics Network" was established by starting annual (or biennial) conferences in 1991. These networks, among others in North America, and the "recognition of the growing importance of a complex field which spanned all educational levels and which was likely to be linked to issues such as class, gender and race" (FitzSimons, 1996 p. 183), were the impetus for the higher profile of this area at the 8th International Congress on Mathematics Education (ICME8), held in 1996. A working group entitled "Adults Returning to Mathematical Education" worked towards proposing a set of recommendations related to mathematics education for the different populations of adults returning to the educational system. Two "major subgroups with distinctive orientations" emerged as the program evolved: "adults returning to, or embarking on, studies of basic mathematics; and those returning to study at the tertiary level" (FitzSimons, 1996 p. 183). I presented a paper to the latter group, analysing results from data gathered in one course at the University of Auckland (Mathematics 1) (Miller-Reilly, 1997). Recommendations of the working group emphasized taking into account the diverse needs, individual priorities and experiences of adults; using a cooperative approach to learning and doing mathematics in a way it is actually used in their lives; thus enabling "reflective and reflexive thinking" through mathematics, "critical citizenship" and "empowerment" of adults (FitzSimons, 1996 p. 186).

Mathematics educators in the field of adult numeracy and adults returning to study have much to contribute as a result of their experiences of working with students who have apparently been failed by their experiences of school mathematics and who have the maturity to reflect on these experiences.

A special interest group, adults learning mathematics, also began featuring at the annual Mathematics Education Group of Australasia (MERGA) conferences from mid-1990s.

Why does adult mathematics education matter? Coben (1996), in her 'agenda for adult learning in mathematics for the new millennium', lists several reasons: firstly, that "lifelong education is a fundamental human right, a hallmark of a civilized society"; secondly, there is the efficiency argument. Since mathematics is a core skill for employment and vocational training, adults need this skill. Thirdly, mathematics will become increasingly important. The importance of mathematics education for adults in our society motivates me in undertaking this research about the effectiveness of some second-chance mathematics courses at the University of Auckland, New Zealand.

My aims in this chapter are to place my study in the context of an appropriate body of work and to acknowledge the research that others have completed. It also enables me to establish whether my results are supported by other findings in the field or whether they differ. Literature on adult learning is the first research area I will discuss then, later in the chapter, literature on adults learning mathematics, with links to relevant theory in mathematics education.

First, in Section 2.2, I would like to explore what has been written about the concept of adulthood and adult education. The range of adult education providers is then discussed, as well as some typical characteristics of adult learners. The major topic which completes the first part of this chapter is adult learning theories, which suggest a range of appropriate methods for educating adults. I will then, starting in Section 2.3, explore the field of adults learning mathematics, with a particular focus on students returning to study at the tertiary level, i.e. bridging mathematics courses, in Australasia. The affective characteristics and learning experiences of bridging mathematics students are discussed in Section 2.4. Approaches and issues in teaching relevant to adults learning mathematics are in Section 2.5. Research findings in mathematics education in general will be interspersed as appropriate.

## 2.2 General Considerations in Adult Education

There are three characteristics which underpin any conception of adulthood according to Rogers (1986). Firstly, the idea of full development, elaborated by

the use of the word ‘maturity’, which describes both a state and an ideal. An adult is not only expected to have reached a stage of personal growth but also be pursuing further levels of it. Secondly, an adult possesses a greater sense of perspective, which allows them to make sound judgments about themselves and about others. Finally, the idea of autonomy i.e. that adults are responsible for themselves. Another idea is explored by Gustafsson and Mouwitz (2004 p. 4), that adulthood has two parts.

The first is a period dominated by forming a family and pursuing a career, while the second is characterized more by cultural and social activities of an experiential nature. In the first part of the adulthood phase studies tend to be narrow and instrumental, while studies in the second part are often driven by curiosity, pleasure and the desire for personal improvement.

Knowles (1984, cited in Knowles, Holton, & Swanson, 1998 p. 64) believes there are four viable definitions of an adult, namely, a biological, legal, social and psychological definition. These definitions are based on, respectively, the age when we can reproduce; the age when the law says we can vote, etc; when we start performing adult roles, such as full-time worker, partner/spouse and/or parent; when “we arrive at a self-concept of being responsible for our own lives, of being self-directing” (p. 64). Knowles states that “with regard to learning, it is the psychological definition that is most crucial” (p. 64). Self-direction will be discussed later, in the section on adult learning theories. In my study, age is the demographic information which will define the stage of adulthood of the participants. In addition, mathematical self-concept will also feature.

Adult education has not always been acknowledged as a legitimate field of study. Jarvis (1987 p. 166) believes that Houle is the scholar who has done most to “establish adult education as a legitimate field of university study and professional practice”. Adult education is also difficult to define because of inconsistent terminology. In New Zealand it is variously referred to as lifelong education, further education or continuing education (Lynch, 1998).

Adult education also takes place in a variety of settings. Some researchers have classified adult education in terms of settings. Cross (1981) suggests three categories: self-directed, organised instruction, or degree-credit. On the other hand, Brookfield (1986) classified settings by the formality of the setting. Informal settings included learning networks, community action, and self-directed learning. Formal settings included adult literacy and basic education, continuing professional, university or polytechnic education, training in industry and business. My research focused on three courses at the University of Auckland, two of which were formal organized courses, one for degree-credit

and one a compulsory course in a one-year full-time pre-degree certificate program. The third course in my study was less formal, not for degree-credit, partly self-directed and partly organized/structured by the teacher.

There are many definitions of adult education. The most well-known, according to Merriam and Brockett, is that proposed by Houle (1972, quoted in Merriam & Brockett, 1997 p. 7-8): "Houle argues it is a process involving planning by individuals or agencies by which adults alone, in groups, or in institutional settings ... improve themselves or their society". Brookfield (1986 p. 135) discusses some factors that make adult education distinctive: the importance of a certain emotional climate, the use of the learner's experience, the provision of plenty of evaluative information and the encouragement of collaboration and participation.

## **2.2.1 Adult Learning Theories**

### **2.2.1.1 Andragogy**

The andragagogical model defines teaching practices, concepts and methods, appropriate to adult learners. Knowles (1973, cited in Queensland University of Technology, 1995 p. 30) popularised this term and he initially saw andragogy as focusing on "self-directedness, previous experience as a resource, learning related to real life experiences, as well as increasing competence for life's tasks".

Criticisms of Knowles' Theory of Andragogy were based on the pedagogical/andragogical relationship, which Knowles assumed was dichotomous. Current thinking is that pedagogy and andragogy are at opposite ends of the same continuum and indeed, much current pedagogical thinking looks remarkably similar to these criteria for andragogy. Tenant (1988, cited in Queensland University of Technology, 1995 p. 30) criticised Knowles' assumption that "children are qualitatively rather than quantitatively different from adults".

Over the years, Knowles modified his ideas about andragogy, for example, acknowledging that the essence of the andragagogical model was the emphasis on process rather than content, assuming that the pedagogical model was based on content. Later, Knowles (1989, cited in Merriam & Brockett, 1997 p. 136) listed six assumptions that underlie the concept of andragogy.

The "need to know" is the first assumption of Knowles' "andragagogical model" i.e. "adults need to know why they need to learn something before undertaking to learn it". In some situations the educator may need to "help learners become aware of the 'need to know'" (p. 136).

The second andragogical assumption focuses on the adult "learners' self-concept". Because "adults have a self-concept of being responsible for their own decisions, for their own lives ... they resent and resist situations in which they feel others are imposing their wills on them" (p. 136).

The "role of the learners' experience" is the third assumption. Adults have "both a greater volume and different quality of experience from youths", so in any group of adults there will be a wider range of individual differences than is the case with a group of younger people. "Hence, greater emphasis in adult learning is placed on individualization of teaching and learning strategies" because the "richest resources for learning can reside in the adult learners themselves" (p. 136).

An important source of "readiness to learn", the fourth andragogical assumption, is the developmental tasks associated with moving from one developmental stage to the next. For further discussion of adult development see Section 2.2.1.3.1.

This andragogical model assumes that adults have a particular "orientation to learning", the fifth assumption. While children have often "subject-centred orientation" to learning (at school), adults are "life-centred (or task-centred or problem-centred)". They learn "new knowledge, understandings, skills, values and attitudes most effectively when they are presented in the context of application to real-life situations" (Knowles et al., 1998 p. 67).

Motivation is the topic of the final (sixth) assumption in this andragogical model. It is believed that the most potent motivators for adults are internal pressures (self-esteem, increased job satisfaction, quality of life). "All normal adults are motivated to keep growing and developing", but "this motivation is frequently blocked by such barriers as negative self-concept as a student (and) time restraints" (Tough 1979, cited in Knowles et al., 1998 p. 68). Wlodowski (1985, cited in Knowles et al., 1998 p. 149) believes that adult motivation to learn is the sum of four factors: adults need to be "successful learners"; adults need to "feel a sense of choice in their learning"; adults need to "learn something they value": they need to "experience the learning as pleasurable".

Merriam and Brockett acknowledge that concerns about andragogy raised by Pratt are important and insightful, that while andragogy has "contributed to our understanding of adults as learners it has done little to expand or clarify our understanding of the process of learning" (Pratt, 1993, quoted in Merriam & Brockett, 1997 p. 137).

### **2.2.1.2 Self-directed learning**

"As individuals mature, their need and capacity to be self-directing ... increases" (Knowles, 1984 p. 6). Self-direction includes the concepts of learner control over instruction, learner autonomy, the learner determining the adequacy of outcomes, learner and educators sharing control i.e. conventional power relationships are changed. Knowles et al (1998 p. 251) acknowledge that the concept of self-directedness is the most debated aspect of andragogy, and suggests there are two independent dimensions, self-teaching and personal autonomy. Brockett and Hiemstra (1991, quoted in Merriam & Brockett, 1997 p. 138-9) developed the Personal Responsibility Orientations model which holds there are two related dimensions. Firstly, the teacher's role and responses for self-directed learning are different from those needed in teacher directed learning. In self-directed learning teachers operate as consultants, referral and resource persons, producers of learning materials. They encourage self-diagnoses of needs, student formulation of objectives and student responsibility for design, implementation and evaluation of learning. Secondly, personal characteristics of the self-directed learner "predispose (them) toward taking primary responsibility" for learning.

The are several key trends in this body of knowledge which Merriam and Brockett (1997 p. 140) identify: "self-directed learning, however defined, is the most frequent way in which most adults choose to learn"; "there is a strong connection between self-directed learning and self-concept"; "several other personality and social characteristics seem to have some connection to self-directed learning". This relationship between self-directed learning and self-concept is related to connected teaching in mathematics which becomes apparent in my theoretical analysis in Chapter 8.

Mezirow (1983 p. 136) argued that the education of all adults should be "an organised and sustainable effort to assist (them) to learn in a way that enhances their capability to function as independent learners". In the same vein (Grow, 1991 p. 140) suggests that 'good teaching' is "situational" i.e. it is a match between "students' stage of self-direction" and "empowers the student to progress towards greater self-direction". Grow's (1991) Staged Self-Directed Model proposes that learners advance through stages of increasing self-direction and that teachers can help or hinder that development. Good teaching matches the learner's stage of self-direction and helps the learner advance toward greater self-direction. He proposed specific methods for teaching students at each stage, and explains several pedagogical difficulties as mismatches between teacher style and learner stage, especially the mismatch

between a student needing direction and a non-directive teacher. I will explore some matches and mismatches occurring in the courses in this study.

Brookfield (1999) suggests that the concept of self-direction is a form of practice that “dignifies people and respects their experiences” with the aim that “adults can gain increasing control over their lives”. It can include the ideas of human interdependence developed by Goldberger, Tarule, Clinchy, & Belenky (1996). Brookfield believes that at the “heart of self-direction are issues of power and control, particularly regarding the definition of acceptable and appropriate learning activities” (p. 4).

### **2.2.1.3 Transformation Theory**

While “experience is central to an understanding of the adult learner”, it is the “way in which individuals make meaning of their experiences, facilitating growth and learning” which forms the foundation of transformation theory (Merriam & Brockett, 1997 p. 140). Mezirow (1983 p.125) introduced the term “perspective transformation” to describe the changes in roles and relationships which he noticed in his national study of re-entry college women in the US. He defined this term as “the structure of psycho-cultural assumptions within which new experience is assimilated and transformed by one’s past experience”. Transformational learning (Clark, 1993, quoted in Merriam & Brockett, 1997 p. 142) “produces more far-reaching changes in the learner than does learning in general … it shapes people; they are different afterward in ways that they and others can recognize”. Cranton (1994, cited in Merriam & Brockett, 1997 p. 142) uses the strategies in transformative learning as a process for empowering adult learners. (Empowerment is discussed further in Section 2.2.1.5.) In response to claims that the theory ignored social change and social context, Mezirow (1989, cited in Merriam & Brockett, 1997 p. 142) indicates that “while social change may develop out of the transformation process, the impetus for such action must come from the learner; for the educator to initiate a specific political agenda would constitute indoctrination”.

#### **2.2.1.3.1 Adult Development**

There is clearly a link between adult education and adult development, as the previous section on transformation theory illustrates. There are many theories on adult development.

There are stage theories such as that proposed by Erik Erikson (1963) who divided the life span into eight stages, each of which centred on the resolution of a developmental crisis. Other stage theories have included Jane Loevinger's work on ego development (1976), William Perry's research on the stages of intellectual and ethical development

(1970), Lawrence Kohlberg's work on moral development (1969), Jane Fowler's perspectives on faith development (1981) and George Vailant's hierarchy of adaptive mechanisms (1977).

(Merriam & Brockett, 1997 p. 145)

Most of the above researchers view adult development as a series of stages or phases that adults go through, i.e. a sequential pattern of change. Another concept of adult development is based on life events, individual and cultural, such as childbirth or death of a spouse or partner, and a third key concept is based on transitions, "the natural process of disorientation and reorientation that marks the turning points on the path of growth" (Bridges, 1980, quoted in Merriam & Brockett, 1997 p. 145), which is connected to the research by Mezirow and others discussed in Section 2.2.1.3. Knowles (1998 p. 152) stated that an adult student's "life situation affects both (their) readiness to learn and readiness for andragogical-style learning experiences". The level of readiness of an adult is associated with one of the andragogical assumptions, the 'need to know' (Section 2.2.1.1). It is known that for many adult students "a major life change, transition, or developmental task is probably involved in the decision" to return to study (R. Smith, 1990 p. 50).

Criticisms of some of the theories of adult development above are that they largely describe white, middle-class and male perspectives. Gilligan's (1982) work has been influential in showing there are variations in the personality and moral development of the women and men. Research by Belenky et al (1987), based on Gilligan's work, suggests that women's ways of knowing also differ in some fundamental ways from how men know. They developed a theory of intellectual development with five stages of knowing: silent knowing, received knowing, subjective knowing, procedural knowing (which is sub-divided into separate and connected knowing) and constructed knowing. They indicate that separate and connected knowing are gender-related, the preferred learning styles of men and women respectively, that is, that women favour connected knowing as the way in which they come to know. These stages of knowing will be discussed in relation to knowing in mathematics in Section 2.5.2. Merriam and Brockett (1997 p. 153) believe that "some of the most exciting developments taking place in adult education today center on understanding women as learners (Tisdell, 1995)".

While essentialist issues have been raised regarding Belenky et al's (1987) theory, (e.g, Hare-Mustin & Maracek (1989) and Lewis (1989), both cited in Goldberger et al (1996 p. 28)), Goldberger (1996b) explicitly rejected this accusation, stating that they had not claimed any essential differences between women and men: "We did not claim that the five perspectives or ways of

knowing that we described were distinctively female" (p. 7). Rather, Goldberger (1996b) believes that their interviews in *Women's Ways of Knowing* uncovered salient themes, missing or de-emphasised in Perry's earlier theory; themes that "related to the experience of silencing and disempowerment, lack of voice, the importance of personal experience in knowing, connected strategies in knowing, and resistance to disimpassioned knowing" (p. 7).

A second criticism leveled at *Women's Ways of Knowing* was that it did not pay sufficient attention to race and culture. Schweickart (1996) discusses cultural difference in relation to Belenky et al's (1987) theory, and rejects silence as a passive position, drawing attention to the need for listening as an active position in communication. She believes that silent knowing and received knowing have different significance and different uses in different cultures, and in her culture, "among Filipinos, silence attends to wisdom" (Schweickart, 1996 p. 306). Subjugated knowledge was added by Hurtado (1996) to the five dimensions in *Women's Ways of Knowing*, knowledge that is "temporarily suspended or subjugated to resist structures of oppression and to create interstices of rebellion and potential revolution" (Hurtado, 1996 p. 386). She explained that it exists in the in-between spaces that multiple stigmatized identities create. Goldberger et al (1996) accepted these criticisms in *Knowledge, Difference and Power*, and Goldberger (1996a) noted increased concern since 1987 with the way in which social context and culture shape knowing which had changed the focus of the four authors of *Women's Ways of Knowing*. It was recognized that the low rating given to silence and received knowing needed to be revised, as these have different significances and uses in different cultures. The value assigned by people to these positions varies with the recognition of their importance to the listener.

How is Belenky et al's (1987) theory relevant to my study of adults who avoid mathematics? As Goldberger (1996b p. 7) noted, their theory uncovered themes common to many women's lives, in particular, those related to "silencing", "disempowerment" and "lack of voice". However, these experiences are also common amongst both men and women who fear and avoid mathematics (L. Taylor & Shea, 1996; Zaslavsky, 1994). Hence some of the perspectives, or ways of knowing, included in Belenky et al's (1987) theory of intellectual development are evident amongst such students of mathematics (Boaler & Greeno, 2000; Koch, 1996). In addition, Belenky et al's (1987) theory has been related to the teaching of mathematics by several researchers (Becker, 1995; Buerk, 1994; Morrow, 1996; Morrow & Morrow, 1995). I will use their resulting theory of 'connected teaching in mathematics' in Chapter 8 as the main framework for the theoretical discussion of my teaching approach with an adult

who feared and avoided mathematics. More discussion about aspects of connected teaching are in Section 2.5.

#### **2.2.1.4 Experiential Learning**

Taking cognizance of the learner's experience, as well as learning through experience, has always been an important part of adult education. "Experiential approaches to learning have become firmly rooted in adult learning practice" because (adults) "learn best when new information is presented in a real life context" (Knowles et al., 1998 p. 146). While experience is central to all the theories in adult learning previously discussed, Merriam and Brockett (1997 p. 153) state that "empirical research on experiential learning is less prevalent".

For Kolb (1984, cited in Knowles et al., 1998 p. 147), one of the main theorists in experiential learning, "learning was not so much the acquisition or transmission of content as the interaction between content and experience, whereby each transformed the other". As Knowles et al (1998 p. 149) continue, experiential learning means to "enhance transfer of learning into performance" and to "increase motivation to learn". It appears that experiential learning approaches "have the dual benefit of appealing to a learner's experience base as well as increasing the likelihood of performance change after training" (ibid). "Central to any experiential learning activity are student involvement, learner control, as well as the relationship of learning tasks to real life situations" (Boud & Pascoe 1978, cited in Queensland University of Technology, 1995 p. 33). Experiential learning focuses on process whereas traditional learning focuses on product (knowledge), the transmission model. The purpose of experiential learning may be personal, defining one's needs and pursuing them, and/or practical, application of learning, including empowerment (Section 2.2.1.5). "Experiential learning methods are designed to produce individuals who are motivated, assertive, adaptable, effective and communicative" (p. 33). Boud, Cohen and Walker (1993 p. 8-16) developed five propositions to help adult educators to develop effective experiential learning opportunities:

experience is the foundation of, and the stimulus for, learning;  
learners actively construct their experience; learning is a holistic process; learning is socially and culturally constructed; learning is influenced by the socio-emotional context in which it occurs.

In my research, teaching approaches in the courses studied contain aspects of an experiential learning approach.

#### **2.2.1.5 Major Concepts of Adult Education for Social Change**

Social-change education nearly always hinges on collaborative learning which is “locally initiated and locally controlled” (Merriam & Brockett, 1997 p. 251). However such learning conflicts with much adult education practice which is “more governed by competition, individual achievement and didactic modes of instruction (Belenky et al, 1986)” (p. 251).

The analysis of power in society leads to the notion of empowerment, which allows individuals or groups “to control certain aspects of their lives” (Hamilton, 1992, quoted in Merriam & Brockett, 1997 p. 253). This notion of empowerment has three dimensions: cognitive, understanding “their (mathematical) history, social status and conditions of social subordination”; psychological, meaning affective aspects of self-esteem and self-confidence; economic, development of “productive activity” to enable improved financial conditions.

To conclude this section on adult learning theories, I note that Knowles et al (1998 p. 122) suggest that the “best adult education practices allow maximum individual control and appeal directly to the needs most meaningful to the individual”. However, “facilitating learning in performance-oriented organisations often creates a tension between the assumptions underlying andragogical practice and the organisation’s performance requirements”. I believe that teachers of the two large courses in my study were working out a compromise between the performance requirements of a large university department and their aims in teaching. The individual study course did not have to deal with institutional requirements.

The notion of empowerment discussed above will, in my study, be based on all three dimensions, as they relate to mathematics; understanding their mathematical history and the social subordination experienced, e.g. shaming; the affective aspects of mathematical self-esteem and self-confidence; becoming more ‘numerate’ can offer options such as further education and career choice. I now focus on the field of adults learning mathematics.

### **2.3 Adults Learning Mathematics: the field of research and practice**

There is a great diversity of practitioners and researchers in adult education in mathematics. Adults are taught mathematics in a variety of settings, for example, in

colleges and universities, but also in community centres, voluntary organisations, in industry and commerce, and in government and non-government agencies of various kinds, including hospitals and prisons.

(Coben, 1996 p. 2)

There is diversity in the levels of mathematics taught and teachers/researchers have different foci, even though they share an interest in adults learning mathematics.

Their teaching or research may be mathematics as a subject in its own right or as a support subject. Their focus may be on the constitution of knowledge, including mathematical knowledge, or the context in which mathematics learning takes place, on the role of mathematics in social movements, or as part of critical citizenship, or, more generally, on the meaning of mathematics in adults' lives.

(Coben, 1996 p. 2)

In addition, "adults learning mathematics is an under-theorised domain which needs to draw upon as many relevant disciplines as possible in order to develop" (Benn, Maasz, & Wedege, 1999 p. 54). For example, research by Benn (1997 p. 182) considers the teaching and learning of emancipatory mathematics. She suggests that the process of learning must be active and problem-centred, not passive or learning from books; that it requires an andragogy that not only teaches competence in mathematical tasks but also "critical thinking, control over the discourse in mathematics and communication skills". Critical thinking means

identifying and challenging assumptions and imaging and exploring alternatives ... reflecting on a problem, testing new solutions, strategies or methods on the basis of that thinking, reflecting on the success of these actions in a context, further honing, refining and adapting.

Fitzsimons, Jungwirth, Maab and Schloeglmann (1996 p. 758) suggest that for a person to be "mathematically well-educated (or numerate)" requires three attributes; to have a "sound mathematical knowledge, that is, to know important concepts and methods and to be able to apply them appropriately; to have knowledge about their power and their limitations; to have an overview of, and a critical stance towards, the discipline".

Now I want to consider a small sub-group of the larger field of adults learning mathematics, namely, adults returning to study at the tertiary level of education with limited background in mathematics, because the courses in my research study are all in this category.

### **2.3.1 Adults returning to study at the tertiary level of education with limited background in mathematics**

I will consider the special needs of this group who require some type of 'bridging' course in mathematics. Bridging (or developmental) programs are provided by many institutions to support students who find themselves mathematically under-prepared for tertiary studies. For example,

mathematics is so pervasive that most university courses require, often implicitly, at least basic algebra and often more. ... Most students are literally shocked to find that a degree such as nursing or human resource management not only assumes pre-requisite mathematics but makes actual explicit demands in the course, including statistics. Students who may well have avoided mathematics in choosing their area of study are now forced to confront it.

(FitzSimons & Godden, 2000 p. 28)

Responses by institutions can vary greatly in the ways of assistance that are offered to students. For example, a small survey of bridging programs in some New Zealand universities (that I conducted and reported on (Miller-Reilly, Snyders, & Taylor, 2003)) indicated that each university has at least one foundation (enabling/preparatory) program, either full-time for one year or shorter, which was pre-degree. Some of these programs target overseas students, or mature returning students, or they target Maori<sup>1</sup> and Pasifica students and other under-represented groups. For students enrolled in a degree program, most NZ universities provide workshops, usually not for credit, for students without the assumed mathematics background for the subject they are studying. There are also courses for degree credit in most of these institutions, usually taken by students who did not take and/or pass mathematics in the last year of high school. These courses aim to help students meet quantitative requirements for their major, or bridge students into the standard calculus/algebra course, or are part of a pre-service primary teaching program. Most universities in New Zealand also have a learning centre which runs informal workshops, often timed when skills are needed in degree courses, and/or drop-in assistance.

Barnes (1989 p. 110-1) wrote about the development of the Mathematics Learning Centre at the University of Sydney in 1984, and wrote about the obstacles the students face. They include lack of prior knowledge, lack of effective approaches to studying and learning mathematics, lack of basic mathematics study skills, lack of understanding of mathematics symbols and a lack of problem solving skills – no repertoire of strategies. In addition, on the

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<sup>1</sup> Maori are the indigenous people of Aotearoa (New Zealand) and Pasifica refers to people with ancestors in the Pacific nations north of New Zealand.

affective side there is a lack of confidence in their ability to learn mathematics so there is a need to help them change their self-defeating talk - a small number have a severe anxiety about mathematics.

Administrative structures for bridging and developmental programs vary greatly. Many suffer from inadequate resources, including time and personnel. There is a high proportion of part-time staff, often at lower academic pay-scales, who are mostly female.

Lack of effective evaluation of such programs also results in very little publishable work. Bridging (and developmental) mathematics teachers face many challenges which make it difficult to do research. Staffing is a resource which is often constrained because of funding problems leading to heavy teaching loads. Schonberger (1985, quoted in Godden, 1994 p. 68) wrote

Heavy teaching loads make finding time for research a problem. Even those faculty who do have the necessary time, research skills, and interests seldom have graduate assistants or colleagues with whom to share research ideas and problems.

Research is a low priority for these bridging mathematics educators when time is at such a premium. Wepner states the

the relatively few available evaluations provide minimal or no statistical or longitudinal data to establish program effectiveness in terms of the primary goal of mathematics remediation.

Wepner (1986:6, quoted in FitzSimons & Godden, 2000 p. 30)

Galbraith (1990, cited in Godden, 1994 p. 68) discusses the state of research in the bridging mathematics area in Australasia and he

concluded that the small amount of research which has taken place has been segregated from teaching activities. Another major concern was that educators in this field have had minimal liaison with their peers, and thus have individually concentrated on similar levels of work. They have thus proceeded without the advantage of a more powerful, more comprehensive, joint research and development base.

Schonberger (1985, quoted in Godden, 1994) also raises the point that

Subjects of the investigations (the students) are less available than in investigations on other students due to external commitments (there are many mature age students) and due to attrition.

Godden (1994 p. 70) mentions several researchers (Galbraith, 1990; Kinsler & Robinson, 1990; Pegg, 1991) who point out, at the time I was beginning this research project, that this means there exists a unique opportunity in "bridging mathematics to do research, where so little has been undertaken". In addition,

the uniqueness of the teaching-learning situation in bridging and remedial mathematics deems that research conducted in the area should also have its own special features.

Smith and Schnuth (1990, quoted in Godden, 1994)

A review of research (FitzSimons & Godden, 2000 p. 30) also make this point when they refer to research by Godden and Pegg (1993) which

concluded that the strength of bridging mathematics programs, their great flexibility and student-centredness, was the very reason they were unable to be evaluated in the traditional manner of educational programs generally; they called for a new approach to evaluation in this important area.

In Australasia, what has helped bridging mathematics educators, and beginning researchers, has been the formation of the Bridging Mathematics Network in 1991, as I discussed earlier in this chapter. This network maintains an explicit focus on mathematics education for adults from Aboriginal, Maori and other indigenous cultures and others in the diverse population of adult learners who are under-prepared for post-secondary education in mathematics. It has fostered networking amongst previously isolated bridging mathematics educators. We have shared information about our programs and teaching experiences and, in addition, the Network has provided support and encouragement for many to begin, or continue, researching aspects of their programs. However, about 50% of the participants at the 2002 Australasian Bridging Mathematics Network (BMN) conference had not attended a BMN conference before. This illustrates the high turnover of educators in this area, probably due to the conditions of employment and status and support they receive.

In the US the development of a standards document in 1995 entitled *Crossroads in Mathematics* by the American Mathematical Association of Two-Year Colleges (1995) has been significant for developmental mathematics educators in that country. It is based on the learners' needs and interests, rather than on their mathematical deficiencies, "steering clear of a deficit model for adults' mathematics education" (O'Donoghue, 2000 p. 231). This practice is echoed in that of the educators involved in the Bridging Mathematics Network and in Adults Learning Mathematics. These practitioners and researchers, engaged in the mathematics education of adults, "favour learner-centred methods; contextualize mathematics; value learners' life experience; and appeal to learners' critical faculties" (p. 233).

### **2.3.2 Some theoretical frameworks in research on bridging mathematics**

Research in bridging mathematics is usually framed by socio-psychological theories such as theories of adult learning, theories of learning in higher education, affective aspects of learning or socio-cultural aspects of learning. Researchers are most likely to gather a mixture of quantitative and qualitative data, although now there is more emphasis on the qualitative.

Barnes (1994 p. 9) looked through a socio-constructivist lens when researching bridging mathematics and suggests that

a constructivist approach to the teaching and learning of mathematics suggests a change in the norms of the classroom. Interaction between teacher and student needs to take centre stage, and beliefs, including those of the teacher, need to be questioned. As teachers, we have to learn to play a different role.

Barnes advised bridging mathematics educators to do this, to deliberately seek to find out where their students' misconceptions are, in a non-threatening environment.

To try and understand our students' mathematics .... we need to listen and reflect on what we hear - and to 'de-center', that is, put ourselves in the student's position and try to see things from her or his perspective.

(Barnes, 1994 p. 9)

She suggests some strategies for bridging mathematics teachers:

- negotiate meaning,
- provide opportunities for reflection,
- choose problems which generate discussion.

Gordon (1994 p. 11) chose activity theory as her theoretical framework and suggests that activity theory might be a "helpful conceptual model for understanding the reluctant learner" because in this theory "experience, subjective perceptions and socio-cultural factors are interwoven". For educational activity to develop, according to Davydov and Markova (1983, quoted by (S. Gordon, 1994)

it is necessary to ascertain and create conditions that will enable activity to acquire personal meaning, to become the source of the person's self development

Gordon (1994 p. 16) asks what sort of self development could it be hoped adult students will achieve. She suggests three areas in which development could be

aimed for: "intellectual development, the development of students' conceptions of mathematics and approaches to learning it, and their personal growth":

- intellectual development: we would aim that our students have "improved capacity for abstract thinking, better methods of learning, and conscious control over the processes of learning. ... Educational activity in this context ... could also be directed at enhancing (students') ability to reflect, to understand the connections between mathematical concepts, to see ahead and to generalise" (p. 17)
- the development of students' conceptions of mathematics and approaches to learning it: she refers to research by her and her colleagues (Crawford, Gordon, Nicholas, & Prosser, 1993, 1994, 1998) remembering that "many students of mathematics enter university believing mathematics to be a fragmented collection of rules, formulae and algorithms, learned in much the same way as one would learn touch typing" (p. 17)
- their personal growth: we would aim to help students to "overcome their reluctance to tackle mathematics based courses, ... conquer long standing difficulties with mathematics or a severe lack of confidence in their abilities to do mathematics" and find that learning statistics is "empowering" (S. Gordon, 1994 p. 17).

Gordon (1994 p. 18) concluded by warning that teaching mathematics or statistics "without regard to students' personal concerns" will "perpetuate many students' reluctance to learn statistics at university". She continued

the task of educating these students is firstly to understand the effect of subjective goals, perceptions and evaluations of the learners ... our challenge is to find a way to communicate, so that these students may be encouraged to participate in ... high quality learning"

## 2.4 Affective characteristics and experiences of bridging mathematics students

### 2.4.1 Introduction

FitzSimons & Godden (2000 p. 17), in their survey of research on adults learning mathematics, state "clearly the affective domain is a critical influence on adult learners and their background experiences have helped to construct their attitudes and beliefs".

Students' beliefs about mathematics may influence their effective participation in, and reaction to, the courses in my research study. In addition, this approach

may change students' perceptions about the learning of mathematics. The belief systems which a problem-based curriculum engendered in participating students are discussed by Clarke (1992 p. 233) and compared with beliefs of students in a conventional mathematics class. Students in the former valued "interactive learning situations", writing and talking to assist their learning, whereas the latter valued "the teacher's explanations", "the textbook" and "drill and practice". McLeod (1994) asserts that research on student beliefs has made a substantial contribution to understanding the difficulties they have in solving non-routine problems and Schoenfeld (1989) exemplifies this by showing that problem solving performance can be undermined by beliefs. Student's experiences with mathematics give rise to a "mathematical world-view", or system of beliefs and expectations about mathematics, for example, thinking that only geniuses are capable of discovering or creating mathematics, expecting mathematical problems to be solved quickly, or failing to see connections between formal mathematics reasoning and discovery.

McLeod's (1992) analysis of the affective domain to include aspects of beliefs, attitudes and emotions is still a basis of much affective research today (Evans, Hannula, Philippou, & Zan, 2004). In the case of beliefs, McLeod discusses four categories that have been researched in relationship to the teaching and learning of mathematics:

- beliefs about mathematics, which includes beliefs that mathematics is a discipline consisting primarily of rules and procedures;
- beliefs about oneself as a learner of mathematics, self-confidence in learning mathematics, beliefs about the relationship of gender to interest or performance in mathematics;
- beliefs about mathematics teaching i.e. what a teacher should do to communicate mathematical ideas;
- beliefs about social context, which includes beliefs about what it means to be a student in a mathematics class, and beliefs about the way mathematics is seen in school as opposed to non-school settings.

A more recent framework of student's mathematical beliefs, developed by Op't Eynde, de Corte and Verschaffel (2002 p. 28), give three categories:

- "beliefs about mathematics education which include beliefs about mathematics as a subject, beliefs about mathematical learning and problem solving and beliefs about mathematics teaching in general;

- beliefs about the self which includes self-efficacy beliefs, control beliefs, task-value beliefs, and goal-oriented beliefs;
- beliefs about social context which include beliefs about social norms in their own class (the role and functioning of the teacher and the students) and beliefs about socio-mathematical norms in their own class.”

McLeod and McLeod (2002 p. 120) believe that this model is a “useful foundation” for future research. Stage and Kloosterman (1995 p. 294) claim that few studies examine learning in college classrooms from “the perspective of a specific campus or course setting”, that “few analyses exist of factors that relate to student success in remedial college-level mathematics courses” and that “research of beliefs of remedial college-level students is still limited although receiving some attention” (p. 295).

#### **2.4.2 Student’s mathematical beliefs and attitudes**

My belief is that one of the major problems for students is their perception of mathematics as something that's rote, something in which they have to shut off their own thinking and try to reproduce somebody else's thinking, without it having to make sense to them.

(Buerk 1994, cited in Jackson, 1995 p. 5)

It is not unusual to find that a large proportion of students in high school and university hold such beliefs. A study of secondary students (Southwell & Khamis, 1992 p. 505-6) found most believed that mathematics was “mainly facts and procedures to be memorised” and that “certain procedures must be followed in order to get the right answer”. Duranczyk and Caniglia (1998) studied developmental mathematics students enrolled in a pre-algebra, non-credit course. Students in this study believed “memorization of formulas, rules and procedures is a necessary ingredient to understanding” (p. 133), “they consider answers in maths as either right or wrong”. Categorizing students by gender and age gave helpful information. Nontraditional-age female students’ believe in the “importance of understanding and making connections in learning ... expressing interdependence and creativity in formulating solutions” (p. 134-5). Nontraditional-age male students’ believe that “working alone is more beneficial” and memorisation is important. Traditional-age students “espoused dualistic viewpoints” i.e. dependent on knowledge being transmitted (see Section 2.2.1.3.1). Traditional-age males felt that understanding was important but that mathematics must be shown, rather than created. The “belief that mathematics is procedural rather than conceptual is so implicit that students do not even realize it as a belief – to them, that *is* mathematics” (Oaks 1988, cited in Gourgey, 1992). Tertiary students who had studied mathematics

in the senior high school were the subject of another Australian study. This Sydney study (Crawford et al., 1994), seeking to identify conceptions of mathematics held by students beginning to study mathematics at university, found that the majority viewed mathematics as a necessary set of rules and procedures to be learnt by rote. Students with a strong fragmented view of mathematics, who believe mathematics to be numbers, rules and formulae with applications to problems, prefer learning mathematics by rote memorization and by doing examples for reproduction. Students with a strong cohesive view of mathematics, who believe mathematics is a way of thinking for complex problem solving and provides new insights for understanding the world, prefer learning mathematics by doing difficult problems to extend understanding and learning by looking for a broader context to extend understanding. The terms fragmented and cohesive are similar to the terms instrumental/procedural and relational/conceptual (Skemp, 1978), used above.

Mau (1991, cited in Stage & Kloosterman, 1995) conducted a qualitative study of a remedial college-level mathematics course which had created a high level of stress in both students and teachers. She found that the instructors wanted students to succeed but believed many students unwilling to put the time and effort required into what instructors perceived as an easy course. However, the students believed they were working hard and course expectations were unreasonable. When Mau investigated how students were spending their time she found that students "believed that simply memorizing formulas and algorithms was the best way to master course content" (p. 295), and those beliefs appeared to be the major cause of students' difficulty with the course. Mau's findings are an example of Stage and Kloosterman's (1995 p. 295) conclusion that

although a number of factors are responsible for (the high failure rate in these courses), beliefs about learning and doing mathematics seem to be a key to many students' inability to focus themselves enough to "survive" mathematics courses that they see as both emotionally and cognitively difficult.

Various researchers give some reasons for the development of such beliefs. FitzSimons & Godden (2000 p. 15), in their survey of research on adults learning mathematics, refer to a formal education tradition where learners have been encouraged to "disembed context and forget meaning in an attempt to universalize logical reasoning" which has, for many adults, lead to "low-level activity and rule-following". Fragmented curricula and some teaching-learning styles at school have tended to encourage memorizing techniques and rule-based learning (Barnes 1990). Students with fewer background courses in mathematics had "narrower views of mathematics" (Duranczyk & Caniglia 26

1998 p. 132). Hudson (1998 p. 79) interviewed and observed adults in a light engineering factory and found they were "using mathematical skills appropriately, effectively and with confidence" in the workplace but perceived themselves as failures in the subject in high school. He has serious questions about the mathematics curriculum that is implemented in schools.

Boaler's (1998) research supports Lave's view that

learners whose understanding is deeply circumscribed and diminished through processes of explicit and intense 'knowledge transmission' are likely to arrive at an understanding of themselves as 'not understanding' or as 'bad at what they are doing' even when they are not bad at it.

(Lave 1990:325 quoted in A. Watson, 1998 p. 6)

Buerk (1985 p. 61) found that her maths-avoidant students' "vision of mathematics (was) as an absolute ... at which one was very good or very bad". In stark contrast, her experience of mathematics was that mathematics was "creative, dynamic, evolving and in process" which "allowed for the expression of the personal, imaginative, and intuitive capabilities". Richardson (1994, cited in Justice & Dornan, 2001 p. 237) found that "older students were more likely to adopt a deeper, comprehension-focused approach to learning, whereas younger students tended to adopt a more surface-level, assessment-focused approach".

Such beliefs about mathematics affect students' beliefs about themselves as a learner of mathematics, their mathematical self-concept. "Developmental mathematics students, in addition to having poor mathematics skills, usually hold negative attitudes about math and about themselves that affect learning" (Gourgey, 1992 p. 10). It can also overwhelm students into the "coping state of inaction" which is the "hallmark of mathematics anxiety" (Hauk, 2005 p. 38). Duranczyk and Caniglia (1998) found that traditional-age females see mathematics as two separate disciplines, "mathematics that can be used in everyday life" and "mathematics that is taught in the classroom" (p. 135).

Ethnographic life history interviews were used as Lyn Taylor's (1995; 1996) research format. She attempts to be sensitive to the culture of the people she is interviewing and attempts to understand their life histories from an "emic" or "insider's" view of reality. Mathematics autobiographies reveal past mathematical experiences which are implicated in the beliefs and attitudes discussed above. A study by Caniglia and Duranczyk (1999 p. 2) revealed common themes "emerging from developmental mathematics students' autobiographies". Negative statements made up over 30% of 96 autobiographical introductions of students in a pre-algebra course, only 13% of the students wrote positively. Many students had negative experiences in

learning. Instructional practices that had negative effects were "teachers who 'made students feel stupid'" or "who graded unfairly", were "'boring', 'mean' and 'disliked their jobs and their students'" (p. 11). Some students disliked methods that involved competition or when "students took turns at answering basic fact drills" (p. 12). Students' writings included "failure identifiers, math anxiety, negative self-talk, and math avoidance". Factors that influenced their beliefs were external factors in 27% of autobiographies, for example, their poor performance, external exams, family member comments. Johnston (1995 p. 229) writes about her adult student Marie, whose memories of her school mathematical experiences at age 11 were negative, that of repeated shaming and repeated humiliation. Some students in the Caniglia and Duranczyk (1999 p. 10) study also had memories of good mathematics teachers: those who "taught clearly so that they could understand mathematics", those that "developed a caring environment", those that "spent time with them on a personal level", those who "made mathematics fun". At university, good instructors were enthusiastic, gave "clear and multiple explanations and spent time with students" (p. 11).

## **2.5 Approaches and issues in teaching relevant to adults learning mathematics**

### **2.5.1 Introduction**

Koehler and Grouws (1992 p. 123), in their survey article about the current status of research on mathematics teaching practices, explain that research perspectives are based on "different views of the acquisition or construction of knowledge" and that these views "lead to different views of teaching". Possibly elements of at least three of the five perspectives which they discuss are represented by the philosophies and practices of the teachers of the three courses in my research. One perspective, a "sociological or epistemological view", is that "students will only come to know mathematics if they learn mathematics in the way it is developed in the discipline. Teaching is viewed as helping students construct knowledge through problem posing and engaging students in mathematical discourse so that they might examine their own assumptions about mathematics" (p. 123). Another perspective, a "constructivist approach", "examines teaching behaviour from the viewpoint of how much it encourages or facilitates learner construction of knowledge" (p. 123). Teaching is viewed as a "continuum between negotiation and imposition as extremes and the teacher's role to "structure, monitor and adjust activities for students to engage in" (p. 119). A third perspective, a "mathematical content view", regards the teacher as an "agent of cognitive change for the learner". The

"goal is to design instruction sequences and develop instructional techniques (to) ... facilitate cognitive growth and change" (p. 123). The relationship between teaching approaches and teachers' conceptions of mathematics is complex but many differences can be explained by teachers' varying conceptions (Thompson 1992, cited in Carroll, 1995).

Lave (1990:309, quoted in A. Watson, 1998 p. 4) suggests that "it might be possible to learn math by doing what mathematicians do, by engaging in the structure-finding activities and mathematical argumentation typical of good mathematical practice". However, Watson (1998 p. 4) states that "doing what 'real' mathematicians are thought to do is not necessarily the same as learning how to use mathematics in adult life. ... School learning is more general in form and needs subsequent adaptation and application to use in other contexts".

Views of researchers in the field of adults learning mathematics vary. The need for "innovative approaches to teaching and learning" is one of the main points in Coben's (1996 p. 5) "agenda for adult learning in mathematics for the next millennium". Duranczyk and Caniglia (1998) state that implications of their study of developmental mathematics students enrolled in a pre-algebra, non-credit course are that "developmental mathematics courses should engage students in meaningful mathematics by addressing inadequate conceptual understanding, instead of focusing on drill and practice". Boaler (1998 p. 83) conducted ethnographic case studies of two schools with radically different approaches to the teaching of mathematics. One, Amber Hill, used "textbooks and whole-class teaching", the other, Phoenix Park, used open-ended projects. She found that students in Phoenix Park developed a more relational view of knowing, had developed a "'connected' way of working" (Boaler, 1997a p. 114) (see Section 2.5.6). These students "knew they only needed to remember an idea and move on from that" whereas Amber Hill students "tried to remember set pieces of knowledge and apply them" (Boaler, 1998 p. 88).

Fitzsimons and Godden (2000 p. 15), surveying results of research on adults learning mathematics, are more specific, stating:

methods of instruction which focus on technique and lack any overview, connectedness or historicity, together with an absence of community and manifestations of competitiveness rather than cooperation are discouraging to adult learners.

A report which reviews the literature on learning theories and frameworks applicable to the instruction of undergraduate college students (Stage, Muller, Kinzie, & Simmons, 1998) emphasizes the importance of diverse methods for

presenting academic material, activities to facilitate learning, and multiple ways for students to demonstrate their learning.

Koehler and Grouws (1992 p. 124) comment, “there is obviously considerable room for variation in the interpretation of effective teaching”.

Discussion about teaching approaches for adults learning mathematics will include many of the ideas in connected teaching. Connected teaching is based on certain theories of moral development (Gilligan, 1982) and of intellectual development (Belenky et al., 1987) which were introduced in Section 2.2.1.3.1. These theories broaden previous categorizations by including, or emphasizing previously ignored or devalued, ways which women develop intellectually and morally. Section 2.5.2 contains descriptions of the stages of intellectual development in *Women’s Ways of Knowing* and how they relate to knowing in mathematics.

### **2.5.2 The theory behind connected teaching**

In the “silent knowing” stage the learner’s “survival depends on blind obedience” (Erchick, 1996 p. 111), so she “accepts authority’s judgment of what is true” (Becker, 1996 p. 19). The learner has a “sense of feeling dumb and stupid” (Brew, 2001 p. 99), “does not speak and does not expect reasons to be given” (Erchick, 1996 p. 112). In this stage, “trying to know why is neither important nor possible” (p. 111). Brew (2001 p. 97) believes the silent perspective is “particularly pertinent for women returning to study mathematics due to the anxiety many have with the learning of mathematics as a consequence of poor experiences at school”.

The “received knowing” stage depends on knowledge being transmitted, authorities “hand down the truth (so) knowledge is dependent upon an external source” (Becker, 1995 p. 166). Hence the “receiver perspective most closely aligns with the emphasis on rule-based learning” (Brew, 2001 p.97), which is characterized by “students mechanically following the rules precisely” (Buerk, 1994 p. 46). However, there is a gender difference. “Men have a sense of identification with the authority and the knowledge, while women do not” (Erchick, 1996 p. 113). In this stage, Erchick continues,

no knowledge originates from the student, ... which can be particularly damaging in the study of mathematics as the student knows nothing about the creation of mathematical ideas, only of their transference.

This perspective aligns with the “absolute knower” of Baxter Magolda (1992), who is also a “dualistic thinker” and “confused by ambiguity”. Perry’s (1970) position of “dualism” is also pertinent here.

In the “subjective” perspective of knowing there is an awareness of an “inner resource for knowing and valuing” so that it is “no longer only the authority who knows” (Erchick, 1996 p. 113). This perspective “legitimizes women’s intuition” (Becker, 1995 p. 166). Erchick compares this perspective with the previous two voices.

For the silent and received knowers, the absolute truth that is known by the authorities is true and consistent for all; for the subjective knower, the absolute truth is true only for the individual.

(Erchick, 1996 p. 114)

Many women in the *Women’s Ways of Knowing* interviews “listened to outside voices much of their lives, only developing the subjectivist position perspective at the age of 45 or 50” (p. 113). Baxter Magolda’s (1992) “transitional” knowers are also struggling to “come to terms with knowledge being inherently uncertain and socially constructed”. Students at the ‘subjective’ or ‘transitional’ stage

begin to value their own experiences with mathematical concepts but still focus on getting ‘the right answer’. They can learn from the experience of their peers, for they come to realize that peers, as well as teachers and textbooks, have ideas worth considering. ... Writing opportunities and cooperative learning activities can furnish excellent environments for students to consider a variety of mathematical approaches and place their own ideas in context.

(Buerk, 1994 p. 46)

Writing their metaphors for mathematics is also another way that students “become aware of their own thoughts and the extent to which their teachers value this independent thinking” (p. 46) (see Section 8.2.3).

The procedural<sup>2</sup> knowing stage follows next in Belenky et al’s (1987) perspectives. It is here that, rather than “instantly accepting their own intuitive responses from their subjectivist self”, “procedural knowers become critical of their own thinking, it is here they begin reflective thought” (Erchick, 1996 p. 115). There are two parts to this stage: separate and connected knowing. “Separate knowers look to propositional logic to validate arguments, and are particularly suspicious of what ‘feels right’. Connected knowers, on the other hand, focus on the context and other people’s knowledge. Authority comes

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<sup>2</sup> The terminology can be confusing at this point as the meaning of the term ‘procedural’ in this context is rather different from the meaning attached to the same term in mathematics learning. Procedural knowing is about systematic reasoning but procedural learning in mathematics is about rule following.

from these shared experiences as opposed to some external power or status" (Becker, 1996 p. 20). However, "procedural knowers do not limit themselves to speaking in either the separated/knowledge voice or the connected/understanding voice, but, rather fluctuate between the two" (Erchick, 1996 p. 117). Becker (1995 p. 167) suggests that it is in the procedural knowing stage that

there is most conflict with the traditional way of knowing in mathematics. If the only knowledge accepted as valid is that which can be statistically demonstrated or is based on deductive logic, methods that are independent of the knower's actions, then that which is known through induction would be devalued. ... Knowledge developed through deduction is viewed as more valuable than knowledge gained through induction. But, how do we know what to set out to prove (separate knowing) if we do not first know things through inductive reasoning (connected knowing)!

Brew (2001 p. 97) suggests that the connected and separate knowing of Belenky et al's (1987) procedural perspective "appear to be precursors to Baxter Magolda's gender-related reasoning patterns ... of the 'independent knower'". She identified "interindividual" and "individual" knowers within this category, which were gender related.

The last stage in Belenky et al's (1987) model is "constructed" knowing, that "integration of thought, feeling and experience" (Erchick, 1996 p. 117). This is analogous to the Baxter Magolda's (1992) fourth and final category, "contextual knowing", in which the nature of knowledge is regarded as uncertain, but a system of judging the worth of evidence has developed.

Therefore the range is from "silence", which encompasses the feeling of being "mindless and voiceless and subject to the whims of authority" (Merriam and Caffarella 1991:192 quoted in Merriam & Brockett, 1997 p. 145) to "constructed knowledge", in which "women view themselves as creators of knowledge and they value both objective and subjective learning strategies" (p. 145).

Becker (1995 p. 168) suggests that connected teaching addresses issues important to women's learning of mathematics as identified in these models of intellectual development and "might help students to develop into constructed knowers of mathematics". Connected teaching and other teaching approaches which aim to bring "thought, feeling and experience" into the mathematics classroom are discussed in the following sections.

### **2.5.3 Recognising the need to focus on affective issues in teaching**

Typically, in curricula for courses (or books) developed for adults returning to study mathematics, the first topic is based on ways of acknowledging a student's mathematical attitudes, beliefs and experiences indicating the importance these authors attach to focusing on affective as well as cognitive domains (Brown, 1984; Buerk, 1982; Buxton, 1984; Langbort & Thompson, 1985; L. Taylor & Shea, 1996; Tobias, 1978) (see Section 8.5).

Many researchers (Becker, 1995; Buerk, 1982, 1985, 1996; Damarin, 1990; Goolsby, Dwinell, Higbee, & Bretscher, 1987; Morrow, 1996), point out that addressing affective issues early in (and during) any course of study is important. For example, Damarin (1990 p. 149) states that "to fail to recognize a student's anxiety, uncertainty, and concern about whether (they) are mathematically inferior is to deny an important part of the mathematical reality of the student". This view is supported by Stage, Muller, Kinzie and Simmons (1998 p. 1) who reviewed literature on learning theories and frameworks applicable to the instruction of undergraduate students as well as other relevant research. They comment that students in courses focusing on "developing self-efficacy as well as academic learning experienced dramatic successes" (see Section 8.2.3).

One way to focus on affective issues relevant to students is to allow students to safely express their feelings about mathematics. One such method is described by Gibson (1994, p. 7) who collected mathematics "metaphors" from her students at the start of the year, enabling them "to focus on their views of mathematics and of themselves as learners", and informing her about how they felt in previous experiences learning mathematics. Buerk (1996 p. 27) describes one value of gathering students' metaphors, that "the experience and ensuing discussion broadens mathematics to include language, imagery and reflection". Section 8.2.3 includes a broad discussion of the use of metaphors in teaching (Buerk, 1982, 1985, 1994).

Zaslavsky (1994) and Heckman & Weisman (1994) have discussed the fact, that there are many factors in our society that may have led to a student's difficulties with mathematics so that it is important that they do not blame themselves for their negative feelings about mathematics. Avoidance of mathematics by adults, particularly women, is well documented in the literature (Buerk, 1982, 1985, 1996; Fennema, 1995; L. Taylor & Shea, 1996; Tobias, 1978) (see Section 8.2.1).

Another important aspect in teaching mathematics to adults is the clarification of the students' expectations and needs and the teacher's expectations of them.

Barnes (1994 p. 3) says, we must "take into consideration the student's goals". Sells' (1979) pointed out that mathematics is a critical filter, affecting educational and career choices i.e. students need mathematics as a "means to achieve future change" (Coben, O'Donoghue, & FitzSimons, 2000 p. 19), a common theme among adults learning mathematics. This is a focus of several aspects of adult learning theory (see Sections 2.2.1.1 and 2.2.1.3.1).

Typical adult students have a "high level of motivation" which needs to be utilized so that they become "a full partner in the process" and this partnership is a "base on which to improve (their) learning" (Mullinix and Commings 1994, quoted in FitzSimons & Godden, 2000 p. 19). In connected teaching there is a similar emphasis, "the teacher and students engage in the process of thinking and discovering mathematics together" (Becker, 1995 p. 168) (see Section 8.4.2).

#### **2.5.4 Mathematics resources for adults**

A concern among adult mathematics educators is the resources available which encourage student enquiry and experimentation. Books have been especially developed with adults, particularly parents, in mind (Frankenstein, 1989; Johnston & Tout, 1995; Langbort & Thompson, 1985; Marr & Helme, 1987; Marr, Helme, & Tout, 1991; Stenmark, Thompson, & Cossey, 1986).

For example, there are books which allow students starting algebra to explore number patterns, using 'function machines' as a model (Britt & Hughes, 1985a; Brooker, Butel, & Carson, 1990; Langbort & Thompson, 1985), how to describe them algebraically, and algebraic shorthand. Using a spreadsheet for generating functions (Healy & Sutherland, 1991) can be useful for adults as spreadsheets are also commonly used in the workplace.

Adult mathematics educators and researchers write about the advantages of using concrete, visual materials. (American Mathematical Association of Two-Year Colleges, 1995; Dwinell & Highbee, 1989; McCoy, 1992; Zaslavsky, 1994). For instance, Burton (1987) suggests that a learning environment which encourages student enquiry and experimentation for mature students with a history of previous failure in mathematics capitalizes on their adult experience and their motivation. It "offers learning experiences which build confidence and a positive self-image" (Burton 1987 p. 306), establishing a basis for understanding mathematics. It helps the student to more effectively visualise mathematical structures and patterns and to learn about mathematics as a "pattern-searching discipline". Further discussion about this issue is developed in Section 8.5.

### **2.5.5 'Realistic mathematics' education**

"Realistic mathematics" education (de Lange, 1987; Streefland, 1991) was developed originally in The Netherlands. Essential aspects of this approach are "mathematization", the role of context and the "critical attitude" of students. A "real world" situation (either in the physical or social world, in the "inner" reality of mathematics or the real world of the student's imagination) is initially explored intuitively, to organise and structure the problem, trying to discover mathematical aspects of the problem. This process should lead to the "development, discovery or (re)invention of mathematical concepts" (de Lange, 1987 p. 37). This approach is a significant departure from the more traditional content based presentation of mathematics as there is an integration of mathematical process and content and an emphasis on the importance of letting students develop their own mathematical understandings. One of the three teaching approaches studied in my research is based on this instruction theory (see Chapter 6).

Van Groenestijn (1998) has developed a numeracy program for Adult (Basic) Education (ABE) based on the learning principles of constructivism (a learning theory) and Realistic Mathematics Education (RME) (an instruction theory). Students in ABE are used to managing their own lives i.e. their work, family and social activities, however in school situations they often present themselves as "completely dependent on teachers' decisions and activities". "They remember that working together (in school) was forbidden" (p. 228). "Creating learning situations in the way of RME and constructivism (encourages) students to learn to study more independently in a natural and more authentic way". Starting with "real life contexts or situations" where communication, interaction and reflection are important and where "students need to work cooperatively and constructively" (p. 225), "allows students to solve problems in their own manner" (p. 229).

Another goal of realistic mathematics education, which was mentioned earlier, is the development of critical attitude development (de Lange, 1987), in order that students are able to identify misuses of mathematics. Mellin-Olsen (1989) also reported some projects described in his research, developed by students, which led to action by students in their social environment. Skovsmose (1994) refers to this type of mathematical knowledge as "reflective knowing", an ability to evaluate the applications of the algorithms.

Two-stage tests as a form of assessment were introduced as part of "realistic mathematics education" in the Netherlands (de Lange, 1993), where

examination questions were developed which were more open to multiple strategies and results, to assess higher-order thinking skills, and to give students freedom to solve problems at their own ability level. Goals are that these tests should improve learning, and enable students to show what they know rather than what they don't know. After initial assessment of students' answers, the opportunity is given for students to rework and resubmit the tests. "Students can learn from their mistakes, making the testing process an interactive one that assists students in reaching their potentials" (Lajoie, 1995 p. 33).

Assumptions underlying National Council of Teachers of Mathematics (NCTM) (National Council of Teachers of Mathematics, 1989) standards include issues related to the nature and origin of mathematical knowledge (the epistemology) and what it means to learn and teach mathematics (the pedagogy). These assumptions call for a shift in epistemology in that mathematical knowledge is not simply a collection of skills and procedures. It is also the solving of non-routine problems and the building of mathematical arguments. The content cannot be separated from the ways in which students learn it, nor can the domain be separated from the pedagogy.

Many important principles are advocated by the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics, 1989), the National Statement on Mathematics for Australian Schools (1991) and emphasised in the New Zealand Mathematics Curriculum (Ministry of Education, 1992). The five general goals for all students that were set out in the Standards (National Council of Teachers of Mathematics, 1989 p. 5) were that they learn to value mathematics, become confident in their abilities to do mathematics, become mathematical problem solvers, and that they learn to communicate and reason mathematically. The NZ Mathematics Curriculum advocates a problem solving approach.

A balanced mathematical program includes concept learning, developing and maintaining skills and learning to tackle applications. These should be taught in such a way that students develop the ability to think mathematically. Students learn mathematical thinking most effectively through applying concepts and skills in interesting and realistic contexts which are personally meaningful to them.

(Ministry of Education, 1992)

## **2.5.6 The integrated development of process and content: Issues of context**

Contextual non-routine problems can address the "degree of discontinuity of performance" in mathematics across "school" and "everyday" situations (Boaler,

1993 p. 341) by "replicating real world demand" and enabling students to use "mathematics in different situations" (p. 371). Criteria for the selection of contexts which enable "authentic activity" to occur were developed by Mellin-Olsen (1987, cited in Heckman and Weissglass, 1994) and are discussed in Section 9.3.1. There is a need to include the "range and complexity of individual experience and interpretation" (Walkerdine, 1988). There needs to be an awareness that task contexts can be more personally meaningful to one group than another (Burton, 1987; Heckman & Weissglass, 1994; Lehmann, 1987; A. Rogers, 1986). Boaler (1993; 1997a) discusses how mathematics must connect with students' meaning and it must acknowledge the social environment. Cultural bias may also be implicated in the contexts used (Lajoie, 1995). Students from non-English-speaking backgrounds may also find the "demands of language or contextual detail constitute an excessive cognitive load" (Helme, 1994 p. 2). The use of contexts in two of the courses I am researching is discussed in more detail in Section 9.3.1.

Surveys of research on gender differences in achievement find that differences are only noticeable amongst those who achieve high grades (Leder, 1990; Linn & Hyde, 1989). Boaler (1997a p. 118) suggests that the "negative attitudes reported amongst bright girls and inequities present amongst the top 5 per cent of students may derive from some of the intrinsic features of 'top set' mathematics classrooms, rather than the personal inadequacies of girls". A number of other studies also indicate that the type of teaching and assessment can affect gender differences (Becker, 1995; Burton, 1994; Damarin, 1990;Forgasz, 1994; Linn & Hyde, 1989; Ocean, 1996; Vale, 1993a). Indeed, the results of Vale (1993b) and Burton (1994) seem to challenge research findings which indicate that gender differences occur in "complex problem solving" (Campbell, 1995) and on tasks that require "functioning at high cognitive levels" (Fennema, 1995). Damarin (1990 p. 148) suggests that we "need to explore how to help females deal with problem solving" because "numerous studies show boys outperform girls in tests of problem solving". She believes that "problem solving requires students to risk being wrong, to rely on internal intuitions". Boaler (1993 p. 371) suggests that this sort of flexibility of thinking occurs when students are dealing with the complexity of "real world demand". She (1997a p. 114) found that open-ended work which allowed students to "use their own ideas or think creatively" was preferred by girls more than by boys (see Section 2.5.1).

These research results seem to be referring to different types of complexity, complexity in abstract mathematics versus mathematical modeling. There is a different valuing of these two complexities. Johnston (1995 p. 231) comments

that mathematics is an “abstracted discourse that could refer to anything” and that “the abstract mathematics that most of us know ignores both its sources and its applications and consequences”. Walkerdine (1992, cited in Johnston, 1995 p. 232) argues, “abstract reasoning is not the ultimate pinnacle of intellectual achievement, but a massive forgetting which is intricately bound up with questions of power”. Heckman and Weissglass (1994) question the importance which is attached to certain kinds of abilities in Western society, in particular those measured by certain intelligence and aptitude tests. They point out that “intelligence and creativity are not limited to those who have certain abilities and ways of thinking”, and that “context and social circumstances emerge as important variables that interact with individual characteristics to promote learning and reasoning” (Heckman & Weissglass, 1994 p. 30).

One of the three teaching approaches which I studied involved one-to-one teaching (see Chapter 8). The use of students’ experiences, culture and interests as contexts make learning most meaningful and authentic to students and can be incorporated particularly successfully into one-to-one teaching (see Section 8.3.2). Boaler (1997a p. 114) also found that the open-ended work in the Phoenix Park school (see Section 2.5.1) allowed students to “use their own ideas or think creatively”, i.e. “allowed a ‘connected’ way of working (Becker, 1995; Gilligan, 1982)”. This open-ended work was preferred by girls more than by boys. Another of the teaching approaches which I studied incorporated open-ended work (Mathematics 1). Boaler (1993 p. 344) believes that “mathematical meaning” is possible when

mathematical activities emphasise the learner's involvement with mathematics rather than the teachers presentation of content, and when communication, negotiation and the resulting development of shared meanings are a part of the process.

This is a characteristic of connected teaching. Morrow and Morrow (1995 p. 18) explain, if “students (are) encouraged to build on their entire knowledge base rather than leaving all personal experiences at the classroom door” this enables them to construct their own knowledge. Then this “weaving together of objective and subjective knowing in mathematics” (Becker, 1995 p. 170) can allow a course to become a “meaningful educational experience” (Morrow & Morrow, 1995 p. 18).

## **2.5.7 Acknowledging the prior rich learning experiences of adults**

Typically, adults can be quite blocked about mathematics in a formal setting while competent in everyday life because they do not realise the amount of mathematical experience they bring to a mathematics class (Cockcroft, 1982;

Fitzsimons, 1994; Harris, 2000; Noss & Hoyles, 1996). Explicit recognition and valuing of a student's prior knowledge and experience is an essential aspect of courses for adults learning mathematics. Helping "adults to recognise the mathematics that they can do and build on it" is one of the key points in Coben's (1996 p. 5) "agenda for adult learning in mathematics for the next millennium". Adult educators need to "draw attention to, and give a value to the often extensive learning that the adult has gained from non-formal and informal environments" (Gustafsson & Mouwitz, 2004 p. 3).

In connected teaching (Becker, 1995; Morrow & Morrow, 1995) the student is "in charge of the interaction" but the teacher's responses provide "a 'safety net' whereby the students' misconceptions are recognized and addressed" (Morrow & Morrow, 1995 p. 19). Barnes (1994) also advised bridging mathematics educators to deliberately seek to find out where their students' misconceptions are, in a non-threatening environment. FitzSimons and Godden (2000 p. 16), in their survey of research on adults learning mathematics, state that courses which "attempt to incorporate the learner's perspective ... can help to provide adults with self-confidence" (see Section 8.3.2 for further discussion on this topic.)

A related point in Coben's (1996 p. 5) "agenda for adult learning in mathematics for the next millennium" is that teaching "should proceed at a pace which suits the learners". Ramsden (1992) and Munn, MacDonald and Lowden (1992) also address this issue in research about teaching and learning in higher education. (See further discussion in Section 8.5.2).

### **2.5.8 Beliefs about mathematics**

The goal is "demystify the doing of mathematics", as Rogers (1995 p. 178-9) explains in her approach to teaching undergraduate mathematics students, by "calling (his) attention to mathematics as a creation of the human mind, making visible the means by which mathematical ideas come into being", engaging students in "purposeful, meaningful activity" (p. 179). Connected teaching echoes these ideas in advocating teaching mathematics "as a process, not a universal truth" (Becker, 1995 p. 168) so that the student can use his/her "intuition in an inductive process of discovery", experiencing creativity and enjoyment doing mathematics (Buerk, 1982, 1985; Burton, 1999b). The teacher can "value and nurture (students') intuitions" which is recognized as important in "making connections or links in the building of mathematical meaning" (Burton, 1999b p. 31). However, these processes are not always made visible to students (J. P. Smith & Hungwe, 1998).

The sense of agency which students can gain was expressed by Johnston (1995 p. 233). She “traveled with (her adult student) Marie, from a position as a victim... to the possibility of autonomy”. Marie talks about her journey, that she “came from being very damaged to increasingly feeling, ‘I am worthwhile’” (Johnston, 1995 p. 233). Examples of such a change are evident in a number of students in this research study. Section 8.4.2 contains further discussion of these issues.

### **2.5.9 Appropriate mathematical challenges: providing support but allowing student to take risks to develop their thinking**

Morrow and Morrow (1995 p. 19) describe one aspect of connected teaching as allowing the student to take on challenges with support because, “in order to grow intellectually, a student must take reasonable risks and be able to make mistakes”. Hence there is a need to create an environment where the student feels “no need to apologise for uncertainty”.

Buxton (1981 p. 62) emphasizes similarly that, “we must provide people with a high load of success or a level of failure they can tolerate”. However, Rogers (1995 p. 181) suggests that a “classroom climate characterised by safety and trust is essential for risk-taking to occur” because this requires courage and honesty. The types of risks include “risks that one’s assumptions are open to revision, risk that one’s insights are limited, and risk that one’s conclusions are inappropriate” (Ocean, 1996 p. 425). Rogers describes this approach as “student-sensitive pedagogy” because it is

grounded in the students’ own language, focused on process rather than content, and centres on the students’ individual questions and learning processes. Students who are ‘cared for’ in this way are set free to pursue their own legitimate projects (Noddings, 1984).

Rogers (1995 p. 178)

This sense of ‘care’ could be related to the positive aspect of one of the two moral perspectives identified by Gilligan (1982). Ocean’s research (1996; 1997) relates these moral perspectives to mathematics education. She examines Gilligan’s “Care perspective”, a perspective which emphasizes mutual dependence and connection to others, and the “Justice perspective”, which emphasizes individualism, independence, equality and fairness, relating these to the moral climate in the classroom. For example, teaching which includes values such as those of “co-operation, connection and communication” (Ocean, 1997 p. 8) illustrates the positive side of the Care perspective and “the negative side of Care morality in mathematics education is seen when Care slips into patronage” (Ocean, 1996 p. 427). When Rogers (1995) suggests that one must become a “caring teacher” in the specific sense of “caring” as in helping “the 40

other to grow and actualise himself”, she is talking about the positive aspect of the Care morality. Morrow and Morrow (1995 p. 19) expand on this idea and suggest that the connected teacher “must become skilled in active listening and asking questions that will allow the students to become more aware of her own thinking, as well as to decide which of her ideas to pursue further”. Connected teachers, as Becker (1995 p. 170) describes, “trust the students’ thinking and encourage them to expand upon it”, their role is as a facilitator and guide. “The teacher and students engage in the process of thinking and discovering mathematics together” (p. 168). As Mason (1992 p. 302) says, the “central task of mathematics teachers is to help pupils to discover and develop their inherent imaginative powers”. A fuller discussion of these issues is in Section 8.4.4.

It is important in connected teaching to “listen in a believing mode” (Elbow 1973, cited in Morrow, 1996 p. 7) as “there is an assumption that the speaker has a valid basis for his/her opinion”. The aim is to help the student to “elaborate, deepen and extend” the ideas they present. The alternative mode to the believing mode is the “doubting mode”, an “argumentative mode” where the presenter of an idea is “challenged to ‘prove’ the validity of the observation or claim”. The teacher asks the student for clarification or a more detailed explanation as part of the “belief-based-inquiry process” so that they become “used to justifying ideas”, not because the teacher disbelieves him/her but because they believe him/her and “believe in” him/her (Morrow, 1996 p. 7-8) (see Section 8.5.1).

Since my research studies three different approaches used in second chance mathematics courses, I will be referring to the research on teaching approaches covered above in this chapter.



## **3 Methodology**

### **3.1 Methodological Choices**

In investigating the effectiveness of three courses developed for adults learning mathematics, I chose to focus on affective change in the students, relating this change to their achievement in the course, using a case study approach. A case study is defined by Leedy, Newby & Ertmer (1997 p. 157) as an “in depth study of a phenomenon” in “its natural context” which usually “includes the point of view of the participants”. Romberg (1992 p. 57) suggests that, in a case study, the researcher is “writing a natural history of a particular situation”, collecting detailed information by using a variety of data collection procedures over a reasonable length of time. The phenomenon explored in this research study was teaching approaches in second chance mathematics courses. The three evaluative case studies have enabled me to “make comparisons, build theory, and propose generalisations” (Leedy et al., 1997 p. 157).

The types of data which I collected are described next in Section 3.2. My research started as a mainly quantitative study, using attitudinal scales (Section 3.2.2). However, the richness of replies to a few open questions (Section 3.2.2.2) encouraged me to expand the questionnaire to include more such questions. In addition, to understand the issues in more depth, I decided to interview (Section 3.2.2.3) some students, introducing a projective technique (Section 3.2.2.4) which was a powerful tool to facilitate and enrich responses to questions. The sequence of data analysis in the case studies is described in Section 3.3.

There has been an evolution of research methods in mathematics education, particularly over the last ten to fifteen years, from the use of predominantly statistical methods to a predominance of either qualitative methods or a mixture of both (Schoenfeld, 1994). Looking at the history of research on affect and mathematics learning since 1970, McLeod (1994, p. 643) says, “the vast majority of studies of affective issues have involved the use of questionnaires and quantitative methods”. While the quantitative research on attitudes in the 1970’s “showed some success in identifying important patterns of student responses to

mathematics, particularly in the area of gender-related differences", McLeod comments that

the results often ran contrary to expectations and complicated statistical analyses of questionable questionnaire data were not necessarily reflecting accurately what students were thinking and feeling.

(McLeod, 1994, p. 640)

McLeod suggests, therefore, the need to develop a wider variety of methods, including the "intelligent use of multiple research methods that fit the problems" (p. 591). For example, he says, clinical interviews or detailed observations could provide a "deeper understanding of the role of affective issues in mathematics learning and teaching" (p. 591). Romberg (1992 p. 60) also writes about the growing diversity in research methods and says that "although correlational studies and experiments are commonly conducted, many scholars are using different strategies and methods", adapted from other social sciences. Gordon and Langmaid (1988 p. 235) also comment on the development of "hybrid methodologies" in market research, designed to "maximise the strengths whilst overcoming the weaknesses" of qualitative and quantitative research approaches. The methodological choices I made in this study, collecting a substantial amount of qualitative as well as quantitative data for analysis, were therefore in tune with this movement towards the use of multiple research methods in research in mathematics education.

Koehler and Grouws (1992 p.125), in their survey article on research on mathematics teaching practices, suggest that some areas of research need more emphasis. In the area of student outcomes, more attention needs to be given "to non-cognitive outcomes and also to possible differential outcomes for particular groups of students". They also suggest that more research is needed on the postsecondary level of education. My study considers non-cognitive outcomes and differential outcomes for particular groups at the postsecondary level.

McLeod (1994, p. 641) suggested that it would be "more helpful if more studies that focus on affective issues would have stronger links to research on other topics related to the improvement of practice in mathematics education". He described one such study, an "investigation of the relation of confidence and other beliefs to the interactions of students and teaching in mathematics classrooms" and suggested that this work was "a useful example of how research on affect can be linked to research on teaching". The focus of my study addresses this issue because it links research on affect to another topic, namely, how students experience a teaching approach.

Stage, Muller, Kinzie & Simmons (1998) in their review of the literature on learning theories and frameworks related to the instruction of undergraduate students and its applicability in those classrooms state that "few authors have systematically tracked differences in learning across classes". My research study examines how students returning to the study of mathematics respond to three different teaching approaches.

## **3.2 Types of Data Gathered**

I gathered data using a variety of methods to "capture the complexity of issues" (McLeod, 1994, p. 644) in teaching adults returning to the study of mathematics. A questionnaire was developed to gather both quantitative and qualitative data, including scales, gathering demographic information, and asking open questions. Final marks in the two large courses were used as a measure of achievement. Further data came from my interviews with students in, and with teachers of, the courses.

Each of these types of data is discussed in turn, in Sections 3.2.1 and 3.2.2, considering the advantages and disadvantages of each.

### **3.2.1 Demographic information**

Gathering demographic data allowed me to analyse the data by subgroups, as overall results can mask what can be occurring in subgroups. Some demographic variables were chosen to coincide with information available from the departmental class rolls. This enabled me to determine how well the groups who completed the questionnaires represented the class. Other demographic variables were chosen because I thought, based on the literature, that the course could be more effective for some groups compared to others, namely, students of a particular age, gender, ethnicity, fluency level in English and background in mathematics. More detailed data was collected on many demographic variables than is reported in the data analysis which follows (see questionnaire in Appendix P) as, because of low numbers in some categories, the data is combined into broader groupings.

Achievement by age is given for two age groups, namely, 24 years or younger and 25 years or older, chosen to correspond as closely as possible to other studies (Trueman and Hartley 1996; Justice and Dornan 2001). In a comparative study of "mature" and "traditional entry" students at university in the UK, Trueman and Hartley (1996) divided students into three age groups: "traditional entry" students were < 21 years old; "borderline mature" students were 21-25 years old; "older mature" students >25 years old. In a study of differences

between "traditional-age" and "nontraditional-age" students in the USA (Justice & Dornan, 2001), the age groups used were 18-23 years for "traditional-age" students and 24-64 years for "nontraditional-age" students. In New Zealand, students who are 25 years or older are deemed to be independent adults, as eligibility for student allowances for these students does not include parents' income.

I gathered data about the students' first language but, after discussions with Sue Gordon in Australia, fluency in English was included because it is possibly a more appropriate variable than first language. A student's fluency level in English may be more likely to affect learning than a student's first language. Data on this variable, fluency in English, was gathered in October 1995 and March and October 1996. Students were requested to mark one of three categories: not fluent, fairly fluent, and very fluent. Students' responses were checked against their written answers to open questions and sometimes the entry in the fluency category was amended by me if the student's response seemed inappropriate. I am aware that the difference between oral and written fluency is not addressed. The teaching approach, or their peers, might also affect a student's response to this question.

I thought the overall program that the student was enrolled in would be of interest. Mathematics 1 students were asked if they were enrolled in a Science (BSc), Arts (BA) or 'other' degree program; Wellesley Program students were asked if they were in the Science or Arts option.

The ethnicity information was gathered in culturally appropriate categories for NZ. The first category was 'Maori', as the first nation people of NZ. 'Pakeha/European' was chosen as the second category, referring to New Zealanders whose ancestors came from Europe, largely from the British Isles. The 'Pacific Island' category referred to New Zealanders whose families came to NZ from Pacific nations north or northeast of NZ. The 'Asian' category refers to East Asian (usually Chinese) in NZ rather than South Asian (i.e. India/ Sri Lanka/ Pakistan). 'Indian' was the category which covered the latter ethnic group.

Definitions and measures of some mathematical attitudes and beliefs, as well as students' reactions to their course of study, are discussed next in Section 3.2.2.

### **3.2.2 Measures of Some Mathematical Attitudes and Beliefs as well as Reactions to the Courses**

First, I will introduce some definitions of mathematical attitudes and beliefs. Leder (1985 p. 18) states that "over the years many definitions of attitude have been proposed" and she discusses several. The most recent definition Leder mentions is by Fishbein and Ajzen (1975, quoted in Leder, 1985 p. 18): "attitude can be described as a learned predisposition to respond in a consistently favourable or unfavourable manner with respect to a given object". This definition assumes that "attitude is learned, it predisposes to action, the action towards the object is either favourable or unfavourable, and there is response consistency" (Leder, 1985 p. 18). Lalljee, Brown and Ginsburg (1984 cited in Leder, 1985 p. 18) have a different view of attitudes. They "construe attitude as communicative acts which imply favourable or unfavourable evaluations about a class of objects, persons or events". Leder (1985 p. 18-19) considers that "attempts to close the gap between definition and measurement of attitude continue to the present" and that "because of the difficulties matching the conceptualised components of attitude to their operational definitions and their quantification through measurable aspects of behaviour, such efforts are likely to continue".

McLeod (1992), in his survey paper on research on affect in mathematics education, finds that researchers think that formation of attitudes towards academic subject matter develops through a) the automisation of a repeated emotional reaction to the subject: or b) the transference of our existing attitudes to a new but related task. McLeod regards beliefs as distinct from attitudes or emotions in that they have the least affective involvement and are the most stable over time. In a more recent paper, Ruffell, Mason and Allen (1998 p. 2) view attitude as a "multi-dimensional construct with three interwoven components: cognitive, affective and conative" where the cognitive component refers to the "expressions of beliefs about an attitude object", the affective component refers to the "expression of feelings towards an attitude object" and the conative component as "expressions of behavioural intention".

Beliefs are defined by Cobb (1986, cited in Szydlik, 2000 p. 258) as an "individual's personal assumptions about the nature of reality ... which underlie goal-oriented activity". In particular, McLeod (1994, p. 641-2) makes the point that "research on beliefs has made substantial contributions to our understanding of the difficulties students have in solving non-routine problems" and that Schoenfeld had been especially influential in developing this line of research. Schoenfeld (1985 p. 45) states that "belief systems" are

“one’s mathematical world view, the perspective with which one approaches mathematics and mathematical tasks” which “shape cognition, even when one is not consciously aware of holding these beliefs” (p. 35).

A recent definition by Op’t Eynde, de Corte and Verschaffel (2002 p. 27) is based on their model that differentiates between students’ beliefs about mathematics education, about the self, and about social context.

Students’ mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about mathematics class context. These beliefs determine in close interaction with each other and with students’ prior knowledge their mathematical learning and problem solving in class.

So much for the definitions of mathematical attitudes and beliefs. Now a discussion of the measurement of such follows. Kulm (1980 p. 373), in his survey article on attitudes to mathematics, discusses how attitudes ought to be measured. He compares “the use of carefully normed, objective, reliable, easy-to-administer scales on the one hand and less structured, designed-for-the-situation, subjective, open-ended questions on the other” and suggests that “exploring factors of attitude must be done through the use of a variety of measurement approaches” (p. 381).

A variety of measurement approaches were used in my study to measure several mathematical attitudes and beliefs, namely scales (Section 3.2.2.1), open questions (Section 3.2.2.2) and interviews (Section 3.2.2.3).

### **3.2.2.1 Using scales to measure attitudes and beliefs**

Regarding measurement of attitude, Kulm (1980 p. 362) states that the “most widely used self-report procedure has been Likert’s summed-rating approach”, where “subjects are asked to respond to items by choosing the extent of their agreement on a five point scale”. A Likert scale contains a number of items indicating either a positive or negative attitude to the attitude to mathematics being measured. Questions have been raised about the value of the middle ‘undecided’ response category, but research suggests that “the middle category does provide information on those who are really undecided” (Bradburn & Sudman, 1991 p. 238). Some of the self-report measures of attitudes and beliefs in this study use Likert’s technique of summated rating scales.

Kulm (1980 p. 365) suggests several difficulties that we need to be aware of that may arise from the use of scales:

- using previously constructed scales may not be appropriate and it can be better to use a few well chosen items;
- it is possible to overlook important attributes of maths attitude; the characteristics measured may not be “meaningful and useful”;
- disparate items may be combined to produce a single score (hence overlooking data that are “more closely related to actual attitudes held by subjects”) i.e. “combining in meaningless ways characteristics that aught to be considered separately”; total mean scores may mask variations.

Many of these issues are addressed in Chapter 5, as are other issues which Kulm (1980) raises regarding research design. The dimensions of attitude, and types of beliefs, which were considered useful to measure with scales for this study are discussed. The appropriate scales to use for adults to measure these attitudes and beliefs are also discussed in Chapter 5.

### **3.2.2.2 Open questions**

Open questions were used in the questionnaire developed for this study in order to gather a variety of responses from students, that added to responses that have been elicited by other closed questions in the scales. As Kulm (1980 p. 362) states, asking open questions is one of the “simplest and most effective approaches” that can “help to make inferences about the effects of instructional approaches” and is a “valuable approach” to assessing attitude, both of which are relevant to this study. He finds it “interesting that very few studies employing innovative teaching strategies or curricular treatments have used attitude items that ask students to respond to teaching practices”. He thinks that such items would “yield far more information about the impact on students than items related to a general enjoyment of mathematics”. In addition, Kulm believes that “items related to teaching practices might reveal why student attitudes to mathematics were or were not changed by the treatment”. So he is suggesting that it may be better to measure attitude as it relates to the specific classroom situation (Kulm, 1980 p. 368). Many of the open questions in my study were chosen to give feedback about particular aspects of the teaching approaches and were carefully phrased so that they did not lead the students to a particular response.

However, the range of responses that are gathered by open questions can be hard to analyse. The method of analysis I used was to look for themes that emerged (i.e. the responses were content analysed and post coded).

### **3.2.2.3 Interviews**

Interviewing is a common means of collecting qualitative data (Merriam, 1998) and is “a conversation with a purpose” (Dexter 1970, cited in Merriam 1998 p. 71). Patton describes the purposes of interviews.

We interview people to find out from them those things we cannot directly observe... The purpose of interviewing is to allow us to enter into the other person's perspective.

(Patton 1990, cited in Merriam 1998 p. 72)

The interviews in this study were semi-structured, defined by Merriam (1998 p. 93) as “guided by a set of questions and issues to be explored, but neither the exact wording nor the order of the questions is predetermined”. Azazkis and Hazzan (1998 p. 429) suggest semi-structured interviews are “planned in advance but contingent upon the interviewee's response, allowing unplanned follow up questions, variations on planned questions and clarifying questions”. Merriam (1998 p. 87) suggests that “interviewer-respondent interaction is a complex phenomenon”. I took care to use open-ended questions or advanced tentative interpretations for the interviewee to respond to. My goal was, as Seidman (1991 p. 77) suggests, “to have enough distance to enable students to ask real questions and to explore, not to share, assumptions”.

### **3.2.2.4 A projective technique: Use of “metaphors”**

A metaphor is an alteration of a woordē from the proper and naturall meanyngē, to that which is not proper, and yet agreeth thereunto, by some lykenes that appeareth to be in it.

-Thomas Wilson 1525-1581  
(Wilson, 1982 p. 345)

This quotation was written over 400 years ago and is quoted in Madden (2001) from a recent edition of Wilson's work “The Arte of the Rhetorique”, edited by T. J. Derrick (Wilson, 1982). Encouraging students' responses in terms of analogies (similes or metaphors) was used in this research study as an interview technique and as a teaching technique. In this study, the term 'metaphor' will be used in the broad sense most commonly used in much of the research literature. Buerk (1996 p. 27) believes that Lakoff and Johnson (1980) and Burton (1980) all “use the word ‘metaphor’ in the broadest sense”, as she does, “to mean any comparison between two objects, ideas, concepts, or experiences”. The noun 'metaphor' is defined as 'application of the name of a descriptive term to an object to which it is not literally applicable' (Concise Oxford Dictionary). Presmeg (1997), surveying the theories of metaphor relevant to education, states that:

'metaphor' can be considered to be an implicit form of analogy, while 'simile' is an explicit form. ... A simile would specify 'domain A is *like* domain B while a metaphor would state, domain A *is* domain B... For both, the analogy refers to only some elements of the two domains: similar elements constitute the *ground* of the comparison; while the dissimilar elements constitute the *tension*. The ground and the tension are both essential elements of a metaphor (or simile). .... It is the recognition of simultaneous similarity and dissimilarity which gives metaphor (or simile) its special power to structure new experience in terms of old.

(Presmeg, 1997, p. 268-269)

In this research study I usually supplied the general subject area that the student would choose domain B from.

Pimm (1987) describes how metaphors distort, that they stress certain things at the expense of others and that this is part of their successful function. Knight (1992) talks about the nature of metaphor, that it is a statement which links objects, actions, processes and systems in a way which has the intention of "forcing the hearer to search among his associated ideas for possible connections" (Sutton 1978, cited in Knight, 1992 p. 130). One of the objects is chosen to emphasize or suppress certain properties of the other.

Much of the literature on metaphors, particularly in educational research, seems to fall into the following categories: to describe teachers' personal perspectives in both pre-service teacher education and in their teaching practice (Briscoe, 1991; Bullough, 1991; Chapman, 1997); to describe personal conceptions of mathematics (Buerk, 1982; Gibson, 1994; Jackson, 1995); as a tool in counselling (Bowman, 1995); to facilitate participants' evaluation of an educational program (Sims, 1981); to evaluate products in market research; to teach mathematical concepts (Knight, 1992; Mellin-Olsen, 1987; Roberts, 1998); and their use in the notation of maths (Pimm, 1987). In much of this literature the term metaphor is used in a broad sense, referring to similes or metaphors.

In both pre-service and in-service teacher education, teachers' personal perspectives have been clarified by the discussion of personal metaphors (Briscoe, 1991; Bullough, 1991; Chapman, 1997). Briscoe (1991) used a teacher's role metaphors to illustrate the process of teacher change in a case study of one teacher, who participated as a co-researcher. She commented that other studies of metaphor indicate that metaphoric language is an "extraordinarily powerful linguistic tool" to express meaning (Provenzo et al 1989, cited in Briscoe, 1991 p. 186).

Some research and articles describe the use of metaphors to illuminate students' personal conceptions of maths (Buerk, 1982; Gibson, 1994; Jackson, 1995). Buerk

(1996) reports how she first "became conscious of mathematical metaphors" when she realised how often she "heard metaphors" as her "math-avoidant teaching colleagues responded to mathematical situations" in her earlier research. Gibson and Buerk (1996 p. 26) devised a "protocol for the collection of metaphors" in 1988, a protocol which I used and adapted in this research study (see Appendix Z). Gibson (1994 p. 8-9) describes this protocol, asking her pre-calculus students to compare maths to specific objects, in order to discover her students' conceptions of, and attitudes to, mathematics. Responses could describe challenge and exhilaration, for example, "for me, maths is like a high mountain. It is terribly difficult going up, yet when you get there, you know you've accomplished something" (p. 8). On the other hand, responses could allude to frustration and powerlessness, for example, "for me maths is like a giant jigsaw puzzle, with all pieces the same colour", or, "maths at difficult times is like a hurricane - wrenches up the roots of your learning like the roots of a tree" (p. 9). Gibson reflected on some benefits of carefully using this technique: that students have a more accurate picture of themselves, and they appear to be more aware of their teacher's concern for them as learners. Buerk (1994) suggests that metaphors give clues about students' learning strategies and their conceptions of maths, for example, they may perceive that "maths is rote learning so that they shut off their own thinking and reproduce someone else's without expecting to make sense of it". This knowledge may help a teacher deal with the individual student in different ways (p. 5).

Bowman (1995 p. 208-9) experimented with the use of metaphors as a tool in counselling children and found that metaphors can enhance the affective realm by tapping into the "child within", and hence helps the counsellor to relate to the client. He found the use of metaphors can produce more "positive client ratings of empathy, regard, and expertness of the counsellor", can provide "alternative means of expressing facilitative responses", can provide "indirect feedback", and can "intensify positive effects of affirmations". Metaphors can also incorporate more playfulness and maybe more meaning into any message. A metaphor can provide a brief and "powerful message" that can "help us relate", using language that is "playful and flexible". "Metaphors can help clarify understanding of experiences that are not easily described literally" (Bowman, 1995 p. 206).

Sims (1981) used metaphor-generating sessions in order to facilitate participants' evaluation of an educational program, in order to organise participants' perceptions of patterns and resources. He used metaphors to take advantage of their projective properties, their synthesizing function, their generality and remoteness from specific problems. Knight (1992 p. 134)

advocated the use of metaphors in teaching mathematical concepts as it might encourage “development of metacognitive skills, aid memory and change attitudes to maths”.

Projective techniques in market research situations are discussed in (W. Gordon & Langmaid, 1988). They comment that these techniques were usually enjoyed, that they created new energy into group discussion and that they have an important role in qualitative research. While some see these techniques as unreliable, Gordon and Langmaid found them valuable when used in suitable situations. Students were usually asked to interpret their own metaphors - this was considered important by Gordon and Langmaid.

### 3.3 Sequence of Data Analysis in the Case Studies

The sequence of data analysis for the case studies for each of the large classes, Mathematics 1 and the Wellesley Mathematics Course (in Chapters 6 and 7), is presented in Table 3.1.

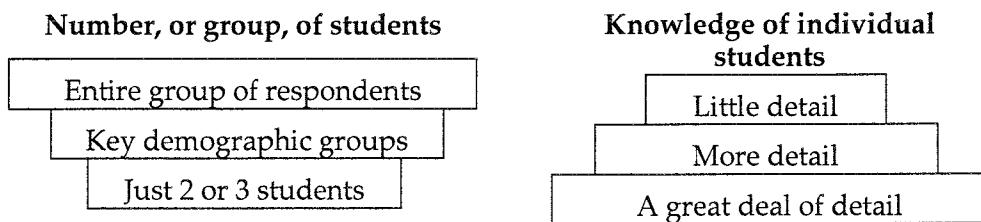
Table 3.1: The sequence of data analysis employed in two case studies: Mathematics 1 and the Wellesley mathematics course.

<b>Sequence of Data Analysis in Chapters 6 and 7</b>	<b>Sections of chapters</b>	<b>Numbers of students</b>	<b>Individual students included in analysis only if ...</b>
1. Quantitative analysis to determine key demographic groups with achievement differences	Maths 1 §6.1  Wellesley §7.1	Entire group of respondents	Demographic information known.
2. Quantitative and qualitative analysis of key groups in order to compare attitudes, beliefs and responses to open questions.	Maths 1 §6.2  Wellesley §7.2	Those in the key demographic groups	Students' responses to the open questions known also.
3. Detailed qualitative analysis of interviews of a few students.	Maths 1 §6.3  Wellesley §7.3	Just 2 or 3 students	In addition, mathematical background and current learning experiences are known.

This analysis could be pictured as a ‘sieve’, as each stage in the process ‘sifts’ out a smaller group of students to investigate further, as illustrated on the left in Figure 3.1 below. We begin with the entire group of respondents, then examine key demographic groups, and finally end each case study describing just the

few students who have been interviewed. On the other hand, the amount of detail known about the students increases at each stage of the 'sieve' just described, as pictured below on the right in Figure 3.1. Each succeeding stage in the data analysis reveals more information about the students. The most revealing data comes from the interviews of a few students in Mathematics 1 and in the Wellesley Mathematics course. This fact was one motivation for the third in-depth case study of an individual student.

Figure 3.1: A diagrammatic view of the data analysis viewed as a sieve, with fewer students involved at each succeeding stage, on the left. A similar view of the amount of information known about students at each succeeding stage of the data analysis is on the right.



## **4 Three Approaches to Second Chance**

### **Mathematics and Their Audiences**

#### **4.1 Introduction**

The three courses researched, all focusing on adults returning to the study of mathematics, are described in this chapter. These courses differed for a variety of reasons. For example, resource constraints meant that class sizes varied from teaching an individual, to teaching 20-25 students in each class in the Wellesley mathematics course, to over 100 students in each lecture for Mathematics 1. The following brief descriptions of each course provide a background against which chapters six, seven and eight can be read. Each of these chapters is a case study of one of the three courses introduced in this chapter.

Descriptions that follow are based on written materials, such as course outlines, annual reports and assessment tasks. Excerpts from interviews I conducted with Bill, Maxine and Moira, (three of the four teachers) are also included. The retrospective protocol for the semi-structured interviews with these teachers is included in Appendix A. Bill and Maxine developed and taught Mathematics 1<sup>3</sup> in 1994, Bill taught the course again in 1995 and part of the course in 1996. Moira had taught in the Wellesley mathematics courses<sup>4</sup> several years prior to, as well as during, my research study (1994-1996).

Section 4.2 contains a description of Mathematics 1, Section 4.3 describes the Wellesley mathematics course and Section 4.4 describes the supervised study course in mathematics for an individual. The key similarities and differences between the three approaches are discussed in Section 4.5 and the relevance of the four different populations in adult education categorised by Smith (1990) to the student populations in the three courses in this study is discussed in Section 4.6.

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<sup>3</sup> Mathematics 1 is officially listed as University of Auckland course MATHS 101.

<sup>4</sup> The compulsory Wellesley mathematics course is officially listed as University of Auckland course MATHS 091.

## 4.2 Mathematics 1 and Undergraduates

My goal is to work in partnership with the students in doing maths, so that you are learning maths together. ... It's about valuing people, that they are all at different starting points, and just working with them and letting them grow mathematically.

Maxine

### 4.2.1 Development of Mathematics 1

#### 4.2.1.1 Historical background

Traditionally the Department of Mathematics at the University of Auckland has catered for students who successfully passed school examinations in year 13 (Form 7, age 18 years), the Bursary examinations, and who wanted to major in the mathematical and physical sciences. However, within the last two decades, the percentage of students without the background knowledge for this level of study in mathematics has risen markedly. To meet the needs of these students a 'bridging' course<sup>5</sup>, which could be credited to a degree, was developed about 20 years ago (now known as MATHS 102 or Mathematics 2). However, the bimodal distribution of results each semester indicated that there were clearly two groups of students in this course: those who had been away from the formal study of mathematics for some time and those who were straight from high school with insufficient background knowledge for the standard calculus/algebra entry course. The former group did not appear to be catered for by the Mathematics 2 course. In his interview Bill said:

(Mathematics 2) attracted these other people who clearly wanted a maths course, there was nothing else: they were just lost, they got failed, they got disillusioned, they didn't perform. This group weren't even breaking the surface on that course.

The significant number of these students provided the motivation for the development of a new course (Mathematics 1), taught for the first time in 1994. Mathematics 2 was then able to be redesigned to better prepare students for entry into the next course (Mathematics 3), the entry course for students who have passed mathematics in the final year of high school. The new course was an opportunity to try an entirely different approach with students who were, typically, not very confident of their ability to learn mathematics and apprehensive about tackling it again.

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<sup>5</sup> Studying seven courses is a normal full time study load for a year of study for a degree.

The official listing of Mathematics 1, appearing in the Department of Mathematics booklet, describes the course in the following way:

MATHS 101                                  Mathematics 1

Restrictions: Only for students who have not studied Mathematics at Bursary level

This course is designed to build up confidence in approaching mathematical ideas, and helps to build up the mathematical skills required to assist students to take the course MATHS 102 (Mathematics 2) in their second semester or later. It is a suitable course for primary teachers in training. The course presents mathematics through thematic topics. You should enrol in MATHS 101 if you have little or no background in mathematics beyond 5th form level; otherwise you should enrol in one of the other Stage 1 courses in Mathematics.

(Mathematics Department, 1995)

#### **4.2.1.2 Selection of students**

Students who enrolled in the course in 1994 - 1996 were interviewed by one of the lecturers. Selection was based on the student's mathematics background and level of fluency in English. Many non-fluent students chose to do this mathematics course because they assumed the demands on their English would not be great, but as Bill said:

because the assessment had a lot of writing in it, second language students were penalised from the beginning and we actually tried to select them out.

Bill was "surprised" at

the number of people who came out of the woodwork, I think that is a good way of putting it, who suddenly turned up on the doorstep ... who had never been near a science faculty at all, or came for strange reasons. I don't know where they found out about it, although maths courses are listed in the Arts section of the calendar ...

In terms of ethnicities, Bill had no real expectations but his perception was that

there was a bigger range of ethnicity in Mathematics 1 than in any other maths course but, I mean, that shouldn't surprise us.

Section 4.2.2, which follows, explores the teachers' rationale for the approach used in Mathematics 1 and the main theoretical background that informed their development of the course.

#### **4.2.2 Teachers' Experience and Philosophy**

##### **4.2.2.1 Rationale for the approach: Students' previous failure requires a new approach**

The approach developed by Bill and Maxine, based on their teaching experiences and philosophies, was a significant departure from the more

traditional skills-based presentations of mathematics. Maxine explained that they did not want students to fail again as they had in the past, so there was a need for a new approach. She continued:

As far as I could see, it was students who didn't have a good grasp of the basic techniques in maths and didn't have a good grounding. The basic philosophy seemed to be, well, these students had failed in the past so it wasn't useful to give them a course that was full of the techniques and procedures of maths. That had been tried for 10-15 years and that course of action had actually failed so a new approach was needed to build up their confidence in maths. It wasn't any use just repeating the failures of the past.

Bill said that Maxine and he had often discussed "what did we really think (such) a course would be like?" Bill expressed their ideas in this way:

We suddenly thought well here's our chance to do all the things that we understood to be useful, with the ultimate aim to produce a course that, at the end of it, the students would still want to do maths. That's really what we were on about, you know, it was a course about turning people on. My memory of it was that was our dominating philosophy. ... We perceived that maths courses for students who are not so good were often totally skills oriented, we wanted it to be about maths, so that the students would understand what they were doing, using the idea of investigations and centring it in real experiences of the students now - the current emphasis in the new curriculum through secondary schools.

Maxine added:

They needed another type of course to develop, what I might call, a mathematical attitude or disposition, so an entirely different approach to maths.

Maxine talked of her experience of teaching mathematics in context.

I'd started that as a high school teacher and of course at College (in pre-service teacher education) it continued. You start with a problem and the problem generates the questions and then you'd try and solve these questions. There's a reason for actually doing the problem. You are taking a real situation and somehow you've got to get it over to a mathematical model. In that modelling you're going to make assumptions and so that in the end your solution or interpretation of the problem will depend on how you've modelled it. Instead of imposing the skills, the skills would come out of it. You'd need some practise at those skills and so those became the skills to practice. It wasn't 'we need to practise how to solve a particular equation (for example,  $1 + x^i = 5$ )', it was ... 'here's a problem and that came out'... 'well now how do we solve that? Well we could do it by trial and error but here's another way of solving it - maybe through logs or something like that', so the skills were used when it was necessary to use them.

The experiences and philosophies of Bill and Maxine enabled them to develop Mathematics 1 to be very different from the usual first year courses offered in the department. Section 4.2.2.2, which follows, looks at some of the research they based the course on.

#### **4.2.2.2 Theory underpinning the course: Realistic mathematics education**

The main theoretical background that had informed their development of Mathematics 1, mentioned by both teachers, was the instructional theory, called Realistic Mathematics Education, developed by the Freudenthal Institute in The Netherlands (de Lange, 1987). Bill:

We had a theoretical background for all that, it wasn't just 'oh this is a good idea'. The realistic theory of Freudenthal, which isn't just putting problems in context, it's saying you actually want situations and contexts that do the mathematical work that you want to do - just any context is not good enough. You have to research that and check it out. The realistic movement is much more complex I think than most people realise really.

Maxine expressed enthusiasm for this approach.

There are too many rote learners of maths who actually never had a feel for maths so it fitted those ideas. Of course I had been looking at de Lange and the need to start maths from a context and then actually start mathematizing problems. It fitted in with what I felt all students should have and so I was really keen to do it. I believe that all students were crippled because they weren't allowed to actually do maths. They'd just been following rules and they never saw the sense of maths, that it wasn't actually to solve problems, it wasn't to actually ask mathematical questions. I could see that you could develop a course where you're starting to get students to look at the world with mathematical eyes. They'd start to pose mathematical questions and out of that would start to come the maths problems, what maths was all about. It had to be set in a context - it wasn't an abstract thing.

Bill mentioned another practical "source of inspiration" being the materials from the curriculum division in Victoria, Australia (Lovitt, 1991a, 1991b).

We wanted mathematically rich contexts for the ideas that we thought were important and that's where our judgement came in. For example, exponential growth would often be regarded as a mathematical idea beyond (these students), but we felt it such an important one that it became part of the course.

In summary, the aim of Mathematics 1 was to build the mathematical confidence of students by providing this opportunity for them to engage in mathematical exploration and modelling in a variety of contexts, for students to extract the relevant mathematics, using many different strategies, encouraging them to become involved in mathematical activity rather than being passive listeners to mathematical facts.

#### **4.2.3 Curriculum and Assessment**

##### **4.2.3.1 Selection of social themes**

Each week the lectures and assignments were based on a mathematically rich social theme. Examples of these themes were environmental issues, maps, gambling, packaging and medicine. Maxine talked about how she and Bill chose and developed the themes.

You knew the skills that you wanted students to come out of the course with. ... Because of our background reading and research as lecturers, we were aware of the misconceptions that students had in maths and we were directly trying to confront them. For example, when we did the packaging topic, we knew that there were misconceptions about surface area and volume. We used that topic to actually confront them with these misconceptions, challenging them to think 'something's wrong here' by saying 'well what do you predict now – let's have a look'. In the chance topic the same thing happened. Knowing that people think a small sample is a representative sample we'd challenge them on that idea. In algebra, knowing that they've got no conception of a variable, so really focussing in on variables. Starting with spreadsheets and thinking of ways to get away from fruit salad algebra - making sure that there are links between context, table, graphs, and formula - that there are different ways of representing maths. Really bringing all your knowledge to bear in a course, so they'd get an inkling of the inner workings of maths and a conceptual understanding of maths.

#### **4.2.3.2 Course structure: Lectures, tutorials and assessment**

Mathematics 1 was taught through three one-hour lectures each week (for approximately 100 students) and a weekly collaborative tutorial, from March to October (the academic year). During each lecture hour, 5-10 minutes was set aside by Bill or Maxine for groups of students to talk about, or experiment with, an idea relevant to the lecture topic. Students worked in groups in the weekly collaborative tutorial to share the initial exploration of the task (assignment) related to the theme developed in the lectures that week.

Students were encouraged to work together on the open-ended tasks, investigations of mathematically rich situations, learning to communicate mathematically in order to increase their mathematical confidence and competence. Some examples of these tasks are in Appendix B. A start was made each week, in groups in tutorials, on an investigation related to the theme being studied in lectures. This enabled students to share the initial exploration of the task and build up some communal confidence. Tasks were completed in their own time and their folio of completed investigations was collected periodically for marking. Every 3 weeks the completed tasks were handed in and one was marked fully, the other two were given a little credit<sup>6</sup>. Some credit was also given for participating in these tutorials to emphasize the importance that was attached to this collaborative learning experience. Section 6.2.1.1 contains further discussion by the teachers and reactions of students to the tasks.

Two-stage tests were held twice during the year containing essay questions, short investigations and skills questions. (A sample test is contained in Appendix C.) This form of assessment was introduced as part of Realistic Mathematics Education in the Netherlands (de Lange, 1993), where examination

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<sup>6</sup>This practice has been adapted since 1996 because students felt there was little credit given for their work in the latter two tasks.

questions were developed which were more open to multiple strategies and results, to assess higher-order thinking skills, and to give students freedom to solve problems at their own ability level. Goals were that these tests should improve learning, and should enable students to show what they know rather than what they don't know. After an initial assessment of students' answers, the opportunity was given for students to rework and resubmit the tests. The marks for the two stages of the test were averaged to give the mark earned for the test. Section 6.2.1.2 contains further discussion by the teachers and reactions of students to the two-stage tests.

Bill commented that "formative assessment was a key idea that we played on", both in assignments (tasks) and in the two-stage testing. The final examination could not be two-stage but contained similar types of questions. An example of a final examination is contained in Appendix D.

Information given to students about the aims and objectives of the course, as well as the assessment regime, is in Appendix E. To summarise, the assessment was made up of:

Task participation	10%
Mathematical Assignments (6 in total)	20%
Two open book terms test (10% each)	20%
Final open book examination	50%

Coursework was therefore worth 50% and the final examination 50% of the final mark for the course.

#### **4.2.4 Overview Statements from Teachers**

Final overview statements from Maxine and Bill follow, summarising their values and their aims for this course.

Maxine:

The goal is to work in partnership with the students in doing mathematics, so that you are learning maths together. They will come up with different questions, you don't know the solutions so you'll do it together. Yes, it's about valuing people and that they are all at different starting points, just working with them and letting them grow mathematically. At some stage touching base with each one of them and somehow motivating them. I can remember in my tutorial seeing that growth (in various students). So this is the whole idea of starting in context - you start with something that's familiar to them and then gradually draw out maths - it's always back to something that they're familiar with.

You're developing a learning culture, a disposition towards liking mathematics, but you're liking it together. You're not up front with this

knowledge. They have got just as much to offer me, as you know, it's a two way thing. They can pose questions and I won't know the answer but together learning more about mathematics. It was to promote lateral thinking and creative thinking. The thinking is valued, ... not so much their powers with technical wizardry at maths but rather the thinking behind it - that posing (of) interesting questions.

You have a vision of what you want to do but what will happen in reality in the classroom is a lot different. I recognise that this is what I might think, but the student experience will be quite different. But I have to have those beliefs or that vision to know what I am working towards in the end. Of course this was just a first year and of course the first year is - you can only progress with that sort of thing afterwards.

Bill:

Well the word that comes to mind is open - open in the way people could respond to the course. I mean the actual course itself was open, as opposed to closed and determined in its outcomes. I think people got different things from the course and that was good. I don't think we got it but I know what we were trying to do was to make it quite multidisciplinary, wider in its vision than what you do when you're doing maths. That's the other sense of open, it's that doing maths is not just doing sums, that's really what we were trying to get at. I don't know how much we succeeded in it. That's the metaphor we have, of an expanse really like open fields rather than a closed city with particular pathways and things that you have got to go down. It's an open field that touches on different things and you can see it in different kinds of ways.

In a city buildings are all the same really, there's a set way of doing things which has been laid down and you don't question it, whereas when you go out in the countryside anything's possible really I think - that's exaggerating it but ... Some of the interesting things that have happened from this course are worth commenting on. We now have this question about what maths is going to be useful for primary teachers? Also what would a maths course for arts students be like? I don't think that MATHS 101 as it is currently conceived is quite right for either of these groups, but it's not a bad basis to start from. I think it's a good course and I think it has, overall, performed its function but, given the university environment, I think the potential student group is probably declining. Maybe MATHS 101 will actually decline and re-emerge in some different form – possibly maths for teachers or maths for arts.

### **4.3 The Wellesley Program for Educationally Disadvantaged Students**

Coming back as an adult is very hard - some of them are very brave.

Moira

#### **4.3.1 Development of the Wellesley Program**

##### **4.3.1.1 Historical background**

It is interesting to observe that New Zealand was almost certainly “the first country to establish a statutory body to coordinate adult education activities” (Dakin 1992:37, quoted in Lynch, 1998 p. 11) when the Education Amendment Act set up the Council of Adult Education in 1936. The political climate in the 1980s and the government publication “Learning for Life” emphasised

"removal of barriers to access for under-represented groups" in education and training (J. Burns & Little, 1995 p. 5). This led up to the 1987 Supplementary Funding Programme, when the government made available Contestable Equity funding with which the Wellesley Program was started. Many other initiatives were funded at this time. This funding was discontinued in 1993. The market-driven ideology now puts more emphasis on vocational training, rather than, what can be seen as, liberal education.

Concern about equal educational opportunity at the University of Auckland led to proposals for central (NZ) government equity funding in 1988 to develop, among other programs, a special course which prepared educationally disadvantaged<sup>7</sup> adults for tertiary study. Availability of funding in 1989 enabled the establishment of the Wellesley Program on the City Campus of the University of Auckland.

The mathematics part of the Program aimed to prepare students to enter Stage 1 mathematics at the bridging level (ie MATHS 101 or MATHS 102), to cover essential elements of the high school curriculum (including some work at year 13 level), to build students' confidence, to promote good study habits and to develop a sense of security in a university environment. This last aim was commented on as part of a talk Moira gave at the Australasian Bridging Mathematics Conference in 1998.

One of the features of our program which we value highly is our location. Each subject is taught within its own department, so mathematics for our students involves coming into the Mathematics Department for classes, and when they need help, finding their tutors' office in the heart of the mathematics building. The students become part of the university culture, they are not labelled as bridging students, or marginalised in any way. They are full members of the university community, indistinguishable from ordinary degree students. In this way they get exposure to a true university experience, but with certain important supports in place.

Other aims are discussed in Sections 4.3.2 - 4.3.4. Section 4.3.1.2 contains a description of the structure of the entire Wellesley Program.

#### **4.3.1.2 Overall Wellesley Program structure**

Two "options" were offered in the Wellesley Program, Arts option and Science option. A compulsory core for both options is English and Mathematics and

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<sup>7</sup> A quote from the Wellesley Program 1996 Annual Mathematics Report in Appendix M: "The interpretation of 'disadvantaged' is loose and students include those who: left school early because of financial difficulty in the family; attended a number of schools because of their family's circumstances; are redundant from the workforce and wish to change direction; were at-risk teenagers and dropped out of school; have been in prison; are parents and feel inadequate with their own lack of education; are single parents."

then students chose two other subjects. For the Arts option, Geography and History; for Science, two from Biology, Chemistry and Physics. Students receive five hours tuition per week in each of their four subjects. The mathematics test given at interview was used to determine the minimum level for acceptance into the Science option of the program; this affected a few students each year. Those who completed the year in all four subjects were awarded the Tertiary Education Foundation Certificate, which carried a grade for each subject. Wellesley Program courses were not credited towards a degree.

#### **4.3.1.3 Selection of students**

Places were made available for 100 students in 1995 and 1996 for this full time one-year program. Students were accepted only if they had a significant time break from their previous (high school) education<sup>8</sup>. Under-represented ethnic groups such as Maori and Pacific Island students, and women studying science, were target groups for the Wellesley Program.

Interviews were conducted with each of the applicants and the interview process included a small English writing task and some mathematics exercises. The aim was to choose students whom it was felt would have the best chance of success in the Program. Issues discussed included the student's financial and personal situation as these seem to have major impact on the retention and success of students in the Program. The principal mathematics tutor in the Wellesley Program, Moira, comments on the interview process as follows:

It's very difficult really, because we are trying to assess potential, not where people are at the moment. .... If we think that somebody is totally unsuitable, we will always direct them somewhere else, ... to a more basic program, offering them the chance to reapply the year after. When we do that, and they do re-apply, they are nearly always very successful students.

In selecting students in 1996, Moira writes in the 1996 Annual Report (in Appendix F),

we were able to direct to other courses those wishing to do Sciences but without sufficient mathematical background. Those with few numeracy skills were identified, and some were accepted into the Arts program because of other recommendations. These students were not so numerous this year because of the experiences of the past. It was decided to take fewer than the hundred students into the program, rather than overload the bottom stream (see Section 4.3.3.1) with students who struggled not only with mathematics, but with all subjects in their Arts course.

This selection process resulted in a higher pass rate (in MATHS 091) in 1996 (78%) than in 1995 (67%) and the same as in 1994 (80%). "It would seem to reflect the more stringent selection process, even with a smaller pool of

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<sup>8</sup>This requirement has been relaxed since 1999 (see Appendix D).

applicants to choose from", Moira continued to write in the 1996 Annual Report (in Appendix F), and also

the bottom stream was much more successful this year because it was not dominated by a group of students with no hope of passing. In fact seven students in this stream passed, and one gained a B- grade. Every stream this year had its share of good grades, showing that students can discover unsuspected talents, and that diagnostic maths tests are quite fallible.

There have always been concerns about the level of the dropout rate in the Wellesley Program and this issue is discussed in the next section.

#### **4.3.1.4 Non-completion rate**

The non-completion rate for the Wellesley Program has varied from 23% to 39% from 1992-1996, in particular, 33% in 1995 and 39% in 1996. This reasonably high rate may be an example of, as Smith (1990 p. 50) comments, the "under-educated" population in adult education being "four times more likely to drop out than other groups" (see Section 4.6).

Moira commented on the issue of the retention rate and how it is being addressed within the Program. She believes a common reason why students drop out is because of "personal difficulties".

I don't feel myself that it's often to do with the program itself or the work that they're asked to do. It's nearly always something in their own lives preventing them from finishing.

One of the things we tell them (at the interview) is that it is a one-off opportunity, and you want to choose the very best year to do it. Sometimes it's better to put it off. (For example,) if you're not in a good state, having come immediately out of a broken marriage, it's not a good time to be taking on the Wellesley Program. We've proved that! And women who have really little babies don't survive very well either because the demands are too high. The child needs to be a little bit older than a month or 6 weeks old before they start.

Another very common reason, which Moira believes causes many students to not complete the year, is "financial difficulties". She comments:

Establishing their financial position in an interview is, we find, really important. .... the lack of money for some of them is an enormous difficulty. ... We do a lot of financial counselling, trying to help them find ways around obvious difficulties.

One example was given in the 1996 Annual Report (Appendix F).

This hurdle directly affected at least one very able student who disappeared to find paid work in the last few weeks of the program, and consequently did less than justice to his final exams.

Students' motivation is also affected by the length of time since they attended high school. Moira commented:

We would prefer the students to have been out of high school for at least 2 years, because they're out of that school mentality. They have to be self-motivated, they have to want to do it.

Students who completed the Program were surveyed in 1996 for anecdotal evidence for why students dropped out (see Appendix F).

Such reasons are : death (1), ill-health (2), lack of money (4), pregnancy (2), feeling overqualified and already prepared for tertiary study (4), drugs and/or alcohol problems (2).

Significant problems were also "associated with lack of motivation after return from the five week inter-semestral break and placing the tests after this break". (In 1997, the year's program for Wellesley Program was re-structured and the inter-semester break of five weeks was reduced to three.)

Section 4.3.2, which follows, explores Moira's rationale for the approach used in the Wellesley Mathematics course.

#### **4.3.2 Teacher's Experience and Philosophy**

Moira wrote, in the transcript of her talk at the Australasian Bridging Mathematics Network Conference in 1998, about the importance of developing a "non-threatening", yet "challenging", atmosphere in the classroom.

In spite of widely different backgrounds and experiences, *all* students are making a new start in new circumstances, so in this sense at least they have something in common. The tutor needs to establish trust, and build confidence - the students who are prepared to question when they don't understand, who collaborate with one another, and seek help as it is needed, are the most likely to succeed. A climate is needed where there is friendly communication, where students feel secure, are treated as individuals, valued as members of the class, encouraged to contribute their own experiences, and quickly take responsibility for their own learning. Early success is an important feature in building confidence, so structuring the course to begin with something familiar works well.

Moira believed that another way to lessen fear within a maths lesson is to take particular care about the language that is used.

Students see mathematics from the context of their own culture and their own situation, so the language used needs to relate to these. Then students have a chance of making meaning and learning some mathematics. We need careful definition in appropriate terms for new vocabulary : we can't assume that we are all working with the same idea in mind. We need to be aware that many words, which we as mathematicians take for granted and use in the mathematics register, have different meanings in the English language. Words as simple as 'half' and 'share' are used more specifically or differently in mathematics. A process like 'addition' can be signalled in many different ways. For instance, I had a student last week ask what 'sum' meant in the context of a problem he was working on. The fact that he was able to ask, that no-one ridiculed him, that others were anxious to give him the answer, illustrates his security, and their interaction, but without knowledge of the language, ultimately he was stuck.

Moira's teaching experience had been in high school and in a "Social Welfare Home for disturbed adolescents". She explained at the interview that her experiences have made her

much more aware that, for the bulk of the Community, maths has very little relevance, and one of the most important things is to try and make it as relevant as possible for them. And to make them feel that it's not a difficult subject, that it is something they can all do. And that there's no need to be frightened of it and if you can bring the language down to the level that they can understand, they're often very agreeably surprised at how much they can do.

And you have to get them to the point that they're not frightened to ask, because you want it to come from them. I think we try to make it as user-friendly as possible. We take the trouble to meet them all, and to have talked with them for 5 or 10 minutes over the 2 months before the course starts. During the interview process I make a point of making some notes so I remember who they are and where they come from and I always try to know their names the first day.

Moira wrote in the transcript of her talk at the Australasian Bridging Mathematics Network Conference in 1998:

In the Wellesley Program, we have found that small groups of up to 25 students located within the Mathematics department of our university provide good working environments. The program we teach must be flexible enough to meet the needs of the students, yet at the same time structured enough to meet the aim of preparing the students for tertiary study.

Section 4.3.3, which follows, discusses the structure of the Wellesley Mathematics course, the curriculum which was specially planned to suit these students, and the assessment regime.

### **4.3.3 Structure, Curriculum and Assessment**

#### **4.3.3.1 Teaching in streams**

As mathematics is a compulsory part of the program, students come with a wide range of maths backgrounds. The mathematics test given at the initial interview had one section to assess basic numeracy, one section to assess problem solving skills, and a third to assess (algebraic) background (1996 Annual Report in Appendix F).

It was felt that a score of less than 10 (out of 30) indicated that a student would have significant difficulty with the mathematics course, and his or her other abilities needed to be looked at more critically. Most of those with very poor results in the preliminary mathematics test were advised to seek alternative courses. On the basis of other strengths, twelve students with a score of less than 10 were accepted (in 1996).

These 12 students' results were described in the 1996 Annual Report, and their different levels of achievement during the year indicated that "the mathematics test acts as an indicator, but should not stand alone in determining a student's

suitability for the program. It was most useful in determining the minimum level for acceptance into the Science option of the program". In the interview Moira explained that the test is held in order to "try to establish where their maths learning stopped" and she continued:

It's used as a means of seeing where we think they would fit in, because we have the four classes<sup>9</sup>. We try and see where the very best group is, and where the very worst group is, and keep those together, and then we just muddle up the rest!

We find that the people who need the most help are better in the same group because then they're not so embarrassed. And if everybody's starting off with the same disadvantage, you can bring them on more quickly.

These four classes, called streams, had about 20-25 students each initially and some movement of students between streams was permitted. The compulsory mathematics course was called MATHS 091. The top stream (stream 4) were expected to study an additional extension mathematics course (MATHS 092). Therefore students in stream 4 had to complete all the required assessment for these two courses, which included sitting two final examinations at the end of the year.

#### **4.3.3.2 Study skills**

There are general study problems that need to be addressed while teaching mathematics, as these students are returning to study after a break and may never have succeeded academically at school. The main issues mentioned by Moira were about time-management and helping the students to learn how to study effectively. There was a need to establish structure to encourage self-discipline, and also to motivate the students. Moira discussed how these issues were addressed within the maths course.

We try to give them some idea of what it means to study, that it is a time consuming occupation but that the rewards are there if you give it a try and do your best. They do then feel a lot of satisfaction. I try to communicate enthusiasm for what I'm doing and that does rub-off on them. But studying is a lot to do with organising your work and knowing how to flip through a unit of work to pick out the main points. To have a bit of practice of some of these skills, we give them a short 10-minute test every fortnight on the work covered in the preceding two weeks.

Moira mentioned that, in September, they give the students a study program for examinations which runs for the 6 weeks prior to these examinations. She mentioned that there is often a lack of structure in the students' lives, which causes problems.

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<sup>9</sup>These classes are called streams, a term which is similar to the term tracks used in the USA or the term sets used in the UK.

It's very difficult with the first (assignment), because they really don't understand a deadline, and you have to keep reminding them. ... We find it works better to be absolutely definite at the beginning, but also to offer as much help as we can to them so they're not entirely on their own doing it. They can come and ask for help at any time.

Students often have very little self-discipline, another thing. And it's a question of a lifestyle change, for a large number of them. ... They often have to make quite hard decisions.

#### 4.3.3.3 The mathematics curriculum

During Moira's talk at the 1998 Australasian Bridging Mathematics Network Conference she stated that "the mathematics course is skills-based because of the specific aim to prepare second-chance students for degree studies". During the interview Moira was asked if their teaching of mathematics followed the pattern of teaching skills and procedures before introducing problems in context. She indicated that they "did both".

Very often the students are much more comfortable if they have the skill first and they can see where it goes, though at the moment we're doing quadratic functions and we're doing that from a context point first. But teaching trig from the skills works extremely well. They get very good at the skills and they can normally apply that to problems. It seems to work well that way around. They get an enormous kick actually out of being able to use them. I had a young man say to me the other day: 'you know my brother - he's a builder - and he didn't know how long to make the timber on the rafters. He'd forgotten that and I know how to do trig and showed him!'

It is very much trial and error. If it doesn't work one way, we try it another way the next year and see if we have a bit more success.

Moira commented on recent curriculum development in 1996;

I think we've developed it a lot more into group activities. We've developed laboratory sessions where the students work on a problem together in a group. So we're trying to encourage a cooperative environment and we find that they are very good at helping one another. And that's a really good support for them because they do set up little study groups and they enjoy working on things together. So I think that's a huge development really from what we would have done in 1990. Yes.

And one of the reasons we did that was to try and mirror a little bit what goes on in Stage I so that when the students ... start Stage I maths they are used to cooperative group work. The course is meant to lead them on and so it's good to give them the same sort of experience that they will have in Stage I with Maths 1 or Maths 2.

At the end of every fortnight we give them a very short 10-minute test on the work in the last 2 weeks. ... We do that on the same day that we do the laboratory session, so that in an hour session, every fortnight, they will have this assessment.

The test is handed in, marked, and handed back. They get a small amount of credit for it. The group solutions to problems set in the laboratory session are also marked and returned to students and form part of the assessment for the course.

#### **4.3.3.4 Classes, tutorials and assessment regime**

Students receive five hours tuition per week in each of their four subjects so mathematics classes are held five hours a week, for 27 teaching weeks in a year. The classes included both teaching time and time for students to work on problems with help from their teachers.

The 1996 Study Guide (Appendix G) summarised the assessment regime for the course. Coursework included the following:

- ✓ 10 assignments, which count for 18% of the total mark. Assignments 1 and 9 in 1996 are in Appendix H.
- ✓ 8 short tests and 8 group laboratory sessions count for 10% of the total mark.
- ✓ 3 major tests (1 hour long) count for 12 %. Test 3 in 1996 is in Appendix I.
- ✓ 1 semester test (2 hours long) counts for 10%.

The final examination (3 hours long) counts for 50% of the total mark, although it can count for 100% if it gives a student a higher mark. The 1996 final examination is in Appendix J.

#### **4.3.4 Overview Statement**

As part of Moira's talk at the 1998 Australasian Bridging Mathematics Network Conference she stated:

Nearly all the students are anxious about 'doing' mathematics and they need to build up confidence in their own ability to tackle the subject again. They also need to realise that their added life experience since school can be a positive contribution, which may well outweigh the negative aspect of time away from formal learning.

To give these students the best chance of *learning* the skills they need for tertiary study, the role of the tutor has to go beyond teaching mathematical skills. We need to look at the climate in which the teaching takes place to give the best opportunity for learning.

The final discussion at Moira's interview centred on her main role and philosophy for the Wellesley mathematics course:

My role is to be approachable, to make what I do as non-threatening as possible. To take a personal interest up to a point, in the students: that's not an intrusive interest, that's being available if they want to talk to you. Not really wanting to interfere in their lives in any way, just to be able to listen, I think, not necessarily to offer advice. Be enthusiastic about the subject, and to try and lead as many people as possible into wanting to do more. Some of them are very talented, and it's a really exciting job if you can uncover a talent that somebody doesn't know they've got. I also think - never put them down. I think coming back as an adult is very hard - to be a student again.

You feel very diffident, and very exposed, and I just try and make them feel comfortable. Some of them are very brave. It's not easy to admit sometimes that you've messed up your life so far, but you do want to change it.

## **4.4 Individual Supervised Study in Mathematics**

### **4.4.1 Development of Academic Assistance for Individuals and Small Groups**

#### **4.4.1.1 Historical background**

In 1988, a central University of Auckland academic assistance unit was set up, initially attached to the counselling service. The assistance was available to students either as individual appointments with tutors or in small groups, and focussed on study skills, academic writing and mathematics. I developed, and taught in, the mathematics section of this unit from 1988 to 1999. Since 1999 some of the functions of this unit have been transferred to the Mathematics Education Unit in the Department of Mathematics, where I now teach.

#### **4.4.1.2 Selection of this individual**

While I was on Research and Study Leave in 1995, the head of the Department of Mathematics referred Charles on to me. Charles had contacted the Department to obtain help with mathematics. He was not currently studying at the University but had suspended his Bachelor of Commerce studies because of his difficulties with quantitative aspects of some courses. He felt that he wanted to tackle his phobia of mathematics. With his approval, I decided to teach Charles as part of my PhD research project and he allowed me to audiotape all teaching sessions. He also completed all the surveys and scales used to gather data for this study. We usually met once a week for 1-2 hours for a six-month period. I have had many years experience teaching mathematics to adult students who are usually very anxious and not confident about their mathematical ability and this student was one of the most math phobic students I have experienced teaching. I felt this would be quite a challenge, for us both.

### **4.4.2 My Beliefs and Philosophy about Teaching**

The approach used in this course was based on my strongly held belief that it is worth spending time working with one person and supporting them in facing their anxieties about mathematics while they start learning mathematics again. I have a genuine interest in this individual and also in discovering how their previous mathematical experiences may have contributed to their current beliefs and attitudes. This approach could be described as humanistic, a common approach in the practice of adult education. FitzSimons and Godden

(2000 p. 20), in their survey of research on adults learning mathematics, write that a humanistic approach provides an

atmosphere conducive to learning including a warm, supportive environment. This approach identifies learner's needs and interests, de-emphasizes or eliminates grading and tries to accommodate individual learning styles.

In such an atmosphere I believe in a person's ability to grow, both in the reduction of their avoidance, anxiety, or even phobia, about mathematics, and in their ability to increase their understanding of mathematics. I have a fascination, and an absorption, in their learning. My beliefs are echoed by Smith (1990 p. 49) who states "it has been said that a successful climate for adult learning is almost assured when people are truly treated as adults".

As the adult does mathematics, I am interested in responding to several aspects which I believe affect learning, for example, the emotions 'doing the mathematics' arouses in the student, their beliefs and attitudes about mathematics, as well as the mathematical processes they use. My aim is that the student will move from instrumental or rote learning to a more relational or conceptual focus on learning (Skemp, 1976) of mathematics.

The way these beliefs and aims were implemented in the course are described in Section 4.4.3, which follows. My methods are interspersed with quotes from each of Smith's (1990 p. 47) six "optimum conditions for learning for adults", as well as references to other relevant research.

#### **4.4.3 The Curriculum**

The curriculum is discussed in the next five sub-sections. The topic of my initial discussions with the student addresses one of Smith's (1990 p. 49) conditions for learning for adults, that an adult "learns in a climate that minimizes anxiety" and is explained in Section 4.4.3.1, which follows.

##### **4.4.3.1 Discussion of beliefs, attitudes and experiences**

While the aim of the course is to increase the adult student's confidence while doing mathematics, initially high priority is given to acknowledging and accepting the student's feelings about mathematics, their attitudes towards mathematics, their beliefs about their abilities in mathematics and their conceptions of mathematics. Most books for, and about, adults returning to the study of mathematics stress the importance of talking about the student's feelings and beliefs at the start of the course, often providing useful activities and ideas to use to do this (Brown, 1984; Buerk, 1982; Buxton, 1984;

Frankenstein, 1989; Kogelman & Warren, 1978; Langbort & Thompson, 1985; Ruedy & Nirenberg, 1990; Tobias, 1987; Zaslavsky, 1994).

The student's mathematical experiences in their family, or at school, which may have contributed to these beliefs, attitudes and anxieties, are also discussed. I think it is important to listen and accept their views about the impact of these experiences on their lives, and not blame the student. As Zaslavsky (1994) discusses, there are many factors in our society that may have led to the student's difficulties with mathematics so it is important that they do not blame themselves for their negative feelings about mathematics.

These initial discussions and acceptance of their beliefs and attitudes, as well as listening to their mathematical experiences, help to create an emotionally safe environment for the student to start learning mathematics.

As Barnes (1994 p. 3) says, we must "take into consideration the student's goals, values and beliefs about the nature and purpose of mathematics". Clarification of the student's goals and my expectations of the student are my next tasks (Section 4.4.3.2).

#### **4.4.3.2 Needs and expectations of the student and the teacher made explicit**

I discuss with the student their aims and goals for learning mathematics, for example, their personal and mathematical expectations of this supervised study course. I plan the course to match their needs as much as possible, and agree with Smith (1990 p. 47) that adults need to "have input into what, why and how they will learn". Mullinix and Commings' (1994, quoted in FitzSimons & Godden, 2000 p. 19) study of adult education concluded "that the high level of motivation in the students as full partners in the process should be used as a basis on which to improve their learning, grounded in their rich experiences".

In addition, I voice my expectations of the student, for example, that they will work outside the time spent with me. (I will usually give them some material to work on which enables them to continue learning more about the current topic out of class time.)

This discussion "helps the learner with ... autonomy-related aspects of (my) particular" course (R. Smith, 1990 p. 48). The "mode" of this course is in part "self-directed" (requiring the most autonomy of the student), in part "collaborative" (between the student and myself as we will learn from each other) and in part directed by me (the planning of an appropriate curriculum).

It is also important that the student's experiences and existing knowledge be recognised (Section 4.4.3.3).

#### **4.4.3.3 Adult experience and mathematical knowledge affirmed**

As I work with the student, I acknowledge their background mathematical knowledge. Barnes (1994) advised bridging mathematics educators to do this, to deliberately seek to find out where their students' misconceptions are, in a non-threatening environment.

To try and understand our students' mathematics ... we need to listen and reflect on what we hear - and to 'de-center', that is, put ourselves in the student's position and try to see things from her or his perspective.

(M. Barnes, 1994 p. 9)

Often this mathematical knowledge has come from their adult experience, and it is knowledge which they often do not realise they have. It is important that the "content and processes" of the course bear "a perceived and meaningful relationship to past experiences and that experience is effectively utilised as a resource for learning" (R. Smith, 1990 p. 47), taking into account that "their experience constitutes both a potential asset and a potential liability for learning". Adults often do not realise that they can be using quite complicated mathematical ideas and techniques (Cockcroft, 1982; Harris, 2000; Noss & Hoyles, 1996). This acknowledgement of their existing knowledge and ability builds confidence and encourages the student.

There is also an acceptance of their culture as the class context. This aligns with Friere's (1976, cited in FitzSimons & Godden, 2000 p. 20) view that educators need to "refrain from imposing their values on learners, and see themselves as co-learners, learning about the culture of the people among whom they are working, mutually responsible for growth and change". In this sense, the mathematical contexts I use in one-to-one teaching are the student's experiences and interests. Other aspects of teaching that I believe will enhance the learning experience of the student are encouragement of the student and care with questioning and pace (Section 4.4.3.4).

#### **4.4.3.4 Encouragement, pace and questioning**

I aim to give constant encouragement to the student. This is also a major theme in continuing education literature, as Nordstrom (1989, quoted in FitzSimons & Godden, 2000 p. 19) states, "adult learners are concerned with maintaining a positive self-concept and, although adult learners tend to be highly motivated to learn, they lack confidence in their ability to do so and require continuing

encouragement". Hallett (1983, quoted in Zaslavsky, 1994 p. 20) expresses a similar opinion, that students "learn far more from our faith that they can and will learn mathematics than from our most lucid explanations".

I plan to gradually expose the student to, the often 'feared', mathematics, and I deal with their anxieties as they do some mathematics. I am willing to support the student when their fear of mathematics surfaces again, for example, a panic attack when they can't see how to do a question. I am in agreement with Buxton (1981 p. 61) that "a violent reaction from the emotions completely ruins the reasoning process", and I talk to the student about this effect. It can be reassuring to them that it is their fear, rather than a lack of ability, that is affecting their thinking.

I try to be sensitive to the pace at which I introduce the mathematics so that the student is not discouraged. Buxton (1981 p. 62) also emphasizes that while teaching maths anxious adults "we must provide people with a high load of success or a level of failure they can tolerate". Buxton (1981 p. 103) continues, "the avoidance of emotional blockages does not lie in a totally laissez-faire approach; a basic premise is that some degree of pressure is appropriate".

There is a need to be careful how questions are asked. Often adult students have had teachers who have emphasized questions with a single answer which is right or wrong which, as Buxton (1981 p. 59) says, "enhances the sharpness of emotional response". Buxton (1981 p. 102) describes his experience teaching a group of maths anxious adults where, "despite the relaxed atmosphere which developed", an occasion when he did ask a question in a "fairly demanding way produced the characteristic fear and defensiveness which we need to notice and if possible avoid". It is not that we should not ask questions but it is the way they are asked and how we respond to the student's answer that is important. I agree with Buxton when he says:

Questions are easier to handle if we feel that the answers we give will be treated with respect. As a general guideline, all answers should be regarded as of interest. This is reasonably obvious since there are clearly more teaching points to be made with wrong answers than with right ones.

(Buxton, 1981 p. 103)

The type of mathematical content which I usually teach in a course for maths anxious adults is described next in Section 4.4.3.5.

#### **4.4.3.5 The mathematics content**

While it can be difficult to gauge the appropriate level of mathematics that the students can work comfortably at, to minimise their fear I use an investigative approach. This is usually a different approach than they have experienced in the past. I choose mathematical activities and problem solving tasks carefully to be more open-ended, often using manipulatives (hands-on materials) or diagrams to try and help students to more effectively visualise mathematical structures and patterns. My experience is, as research by McCoy (1992 p. 51) has found, that the careful "use of manipulative materials for mathematics instruction" will "reduce mathematics anxiety", in particular, for the remediation of adults who prefer a tactile-kinesthetic mode of learning. My aim is to engage the student in exploring topics in mathematics, "encouraging freedom (for the student) to experiment" (R. Smith, 1990 p. 48). Zaslavsky (1994 p. 181) writes about another adult mathematics educator who, like me, also "sees her role as that of a facilitator in helping adults to construct their own knowledge by studying and reflecting upon patterns, attributes and relationships" and who finds that concrete materials are "particularly appealing to people who learn best by touching and feeling".

The use of investigative ways of presenting mathematical topics also allows the adult, actually both of us, to have fun doing mathematics together. As I see the student becoming involved and enjoying doing mathematics, I agree with Boaler, Wiliam & Zevenbergen's (2000 p. 4) statement, in a more recent publication, that "students' enjoyment of mathematics was largely related to the extent to which they identified as a mathematics learner".

Materials I use come from courses and books written for (mathematics anxious) adults returning to the study of mathematics or for teachers of such students. For example, I have used materials that have been published in Australia (Marr & Helme, 1987), in Britain (Buxton, 1984) and in the USA (M. Burns, 1987, 1998; Langbort & Thompson, 1985; Ruedy & Nirenberg, 1990; Stenmark et al., 1986; Tobias, 1987; Zaslavsky, 1994). New Zealand textbooks were also used (Britt & Hughes, 1985a, 1985b; Brooker et al., 1990; Brooker, Butel, & Carson, 1991).

I believe that the instructional design described above "can help learners compensate if deficiencies are not too great for the learning purposes and task at hand" (R. Smith, 1990 p. 48), that is, it will usually optimise learning for adults with little background in, and much anxiety about, mathematics. This instructional design in a one-to-one teaching situation is flexible and can be modified to fit most "people's preferred styles of processing information and

preferred learning environments” (R. Smith, 1990 p. 49). This approach has many of the characteristics of connected mathematics teaching (Becker, 1995; Buerk, 1985; Morrow, 1996; L. Taylor & Shea, 1996), for example, “sharing the process of solving problems with students”, teaching mathematics “as a process, not a universal truth”, “engaging in the process of thinking and discovering mathematics together” with the student, “trusting the students’ thinking and encouraging them to expand upon it”, creating an environment where the student feels “no need to apologise for uncertainty” (Becker, 1995 p. 168-170). I aim for the student to become a relational learner of mathematics (Skemp, 1976), to broaden their strategies for knowing in mathematics (Belenky et al., 1987; Goldberger, 1996a), to broaden their view of the nature of mathematics (Buerk, 1994). Discussion of Belenky et al’s (1987) categories of intellectual development are in Chapter 2, and further discussion of the theory of connected teaching in mathematics is contained in Chapter 8.

I have now completed a description of each of the three courses and so now I summarise the key similarities and differences between the courses in this study in Section 4.5, which follows.

## **4.5 Key Similarities and Differences Amongst the Three Courses**

Key differences between the three courses are summarised in Table 4.1. Other key differences between the students in these different courses are highlighted in research by Smith (1990) which has categorised clientele groups in adult education (see Section 4.6 which follows). On the other hand, these three courses have, in common, students whose past experience was a rather ‘rocky road’ in mathematics. It is clear from the preceding sections of this chapter that all the teachers were committed, experienced, and cared about their students’ learning.

Table 4.1: A summary of the key differences between the three courses.

	<b>Mathematics 1</b>	<b>Wellesley Mathematics</b>	<b>Individual Supervised Study</b>
<b>Learning goal</b>	Students are returning to the study of mathematics. ie. Students are learning about mathematics.	Students are returning to formal study. ie. Students are learning about learning.	Student is returning to the study of mathematics. ie. Student is learning about mathematics.
<b>Optional or Compulsory</b>	Studying this mathematics course is optional, although some students indicate that they require this knowledge for other subjects.	Studying this mathematics course is compulsory in the Wellesley Program.	Studying this mathematics course is optional.
<b>Flexibility of Course Content</b>	Course content is predetermined but the open-ended nature of the tasks allow some variability in solutions presented by students.	Course content is predetermined and quite traditional.	Course content reflects the student's needs.
<b>Credit for Course</b>	This course is credited to a degree.	This course is credited to a pre-degree certificate.	This course was one for personal interest not credit.
<b>How the Course Dealt with Students' Anxiety about Maths</b>	The use of contexts, investigative work and two-stage tests aimed to provide motivation and build on existing knowledge, lessening students' anxiety.	Techniques to reduce anxiety used were: grouping students with similar ability, teaching in small classes, knowing students' names and circumstances.	A high priority was given to dealing directly with this student's anxieties about mathematics by talking about these feelings and about his early mathematical experiences.
<b>Size of Class(es)</b>	Lectures contained about 100 students. Tutorials: 25 students.	Classes contained 20-25 students.	One-to-one teaching.
<b>Aims of Curriculum</b>	A new approach, mathematizing realistic situations, to turn them on to mathematics.	Preparing students for tertiary study but students often present with difficult personal circumstances.	Math phobia was the main problem to work on so the student gained mathematical confidence and then more competence.

## 4.6 Different Populations in Adult Education

Four special populations are categorised by Smith (1990) in his research on clientelle groups in adult education. The first two of his categories seem relevant to the two large courses in this study. His first category is called "the under-educated" and seems to describe many of the students in the Wellesley Program. Many adults in this group are "prone to economic and health

problems, learning disabilities, low self-esteem and a sense of powerlessness" (R. Smith, 1990 p. 49). These students are "especially subject to anxiety and doubts about their learning ability" (R. Smith, 1990 p. 50). This "under-educated" population in adult education "are four times more likely to drop out than other groups". This may help to explain why there is a large group, about one-third, who drop out of the Wellesley Program each year (see Section 4.3.1.4).

The second of Smith's categories are those returning to university, to which many students in Mathematics 1, as well as some students in the Wellesley Program, will belong. He indicates that this group have fewer problems with health and resources, and are less prone to feelings of helplessness and doubts about the efficacy of education. They "recognise the need to learn ... but must adapt to the competitive arena" of the university, which could be an "environment likely to produce considerable anxiety". They are subject to heavy outside pressures, such as family and employment, but adult undergraduates show stronger capabilities for "conducting analytical enquiry" and for "independent study" than younger students (R. Smith, 1990 p. 50).

The individual who undertook the supervised study course in mathematics belonged to Smith's third category, the professional. Smith (1990 p. 51-2) describes "professionals" as having "a relatively high income, access to resources and a variety of life-styles", to be "heavily pressed for time" and that professionals' careers "tended to absorb a great deal of their time and energy". On the one hand "few professionals doubt their learning ability", however "their participation and learning are affected by the need to avoid revealing professional incompetence in public". This latter characteristic, as well as the fact the individual (Charles) had strong doubts about his learning ability in mathematics, means that the nature of one-to-one supervised study may have been an appropriate setting in which this individual could learn mathematics. The other category listed by Smith (1990), the older person (over 60), does not seem relevant to this study as very few students in the large courses were in this age group.



## 5 Method

Three different approaches to teaching adults returning to the study of mathematics are the focus of my research. This chapter describes, in Section 5.1, the demographic characteristics of the participants in these three second-chance mathematics courses. Materials used to gather data are listed in Section 5.2 and the procedure employed for data collection is described in Section 5.3. The pilot study and trial of the questionnaire are explained in Sections 5.4 and some decisions made about the general management of statistics are listed in Section 5.5.

### 5.1 Participants

Demographic characteristics of students in Mathematics 1 are described in Section 5.1.1, those of the Wellesley program in Section 5.1.2 and those of the individual undertaking supervised study in mathematics in Section 5.1.3. Comparison of these three demographic profiles is contained in Section 5.1.4.

#### 5.1.1 Demographic Characteristics of Mathematics 1 Students

Mathematics 1 was a full year course in 1995 and 1996, with lectures held from early March until mid-October. Chapter 4 contains a description of this course (Section 4.2). Below, in Table 5.1 and Table 5.2, the proportions of students are listed within each category of the two demographic variables available on the class roll, the students' gender and the degree in which they were enrolled. Initially, in Table 5.1, proportions are calculated for the entire class, which is defined to be the number of students who completed the course (65 students in 1995, 47 in 1996). These proportions will be used in two ways, to describe the demographic profile of the Mathematics 1 class in each year, and also to determine how well the groups who completed the questionnaire in 1995 and 1996 (Table 5.2) represent the class.

Hence, from the entire class (Table 5.1), we find that the ratio of BA students to BSc students are similar both years, 56:40 in 1995 and 51:43 in 1996, and the ratio of females to males is 57:43 in both 1995 and 1996.

Table 5.1: Numbers and percentages of students are listed for categories of the two demographic variables available on the class roll, gender and degree, for Mathematics 1 in 1995 and 1996.

Variable	Categories	Class Roll 1995		Class Roll 1996	
		n=65*		n= 47*	
		n	%	n	%
Degree	BSc <sup>#</sup>	25	40	20	43
	BA <sup>#</sup>	35	56	24	51
	Other	5	8	3	6
Gender	Female	37	57	27	57
	Male	28	43	20	43

\*The number of students who completed the Mathematics 1 course.

<sup>#</sup>Abbreviations: BSc, Bachelor of Science; BA, Bachelor of Arts.

A comparison can now be made between the percentages of students on the roll (i.e. the entire class) in categories of degree and gender (in Table 5.1) and the corresponding figures for the groups who answered the questionnaire in 1995 and 1996 (in italics in Table 5.2), to ascertain if the latter groups are reasonably representative of the whole class.

Table 5.2: Numbers of students and percentages in categories of the variables gender and degree for Mathematics 1 students who completed the questionnaire in 1995 and 1996<sup>#</sup>.

Variable	Categories	Mar*	1995	Oct*	1995	Oct	1996	
			n=50		n=42		n=36	
			n	%	n	%	n	%
Degree	BSc	21	42	14	33	14	39	
	BA	26	52	24	57	20	56	
	Other	3	6	4	10	2	6	
Gender	Female	32	64	27	64	21	58	
	Male	18	36	15	36	15	42	

<sup>#</sup>For 1996 only the October data is included because of the small sample size in March 1996, but March 1996 data is listed in Appendix L for completeness.

\*Abbreviations: Mar is March; Oct is October.

First, however, a summary is given of the numbers of students who answered the questionnaires in March and October, in 1995 and 1996, and then a comparison of these with the total number of students who enrolled in, and completed, the course, respectively.

In 1995, the questionnaire was answered near the beginning of the academic year, in March, by 59 of the 84 students enrolled in Mathematics 1 in March.

(Sixty five of these 84 students completed the course.) Since 9 students of this group of 59 did not complete the course, the proportions in the Table 5.2 are calculated for the 50 students who answered the questionnaire in March and who also completed the course. The questionnaire was also answered near the end of the academic year, in October, by 44<sup>10</sup> of the 65 students who completed the course; 37 students answered both questionnaires. (These figures are listed in Table 1 in Appendix K for reference.)

In 1996 a different situation arose. The questionnaire was completed in March by 46 of the 76 students enrolled in Mathematics 1 in March. However 22 of these 46 students did not complete the course and, in addition, two students gave no ID numbers so their achievement level could not be identified. The group of 22 completed questionnaires remaining was deemed too small to include in the following tables, because once these 22 students are divided into various demographic categories, the numbers in these categories become too small. The data analysis of this group is presented in Appendix L for completeness. In October 1996, 38<sup>11</sup> of the 47 students who completed the course answered the questionnaire; 22 students completed both questionnaires. Hence the only data included in the following two tables (Table 5.3 and Table 5.4) for 1996 is from the October group. (The numbers in each of the above groups are listed in Table 1 in Appendix K for reference.)

Table 5.2 lists, for Mathematics 1 students who completed the questionnaire in 1995 (March and October) and 1996 (October), the numbers of students and the percentages in categories of the two demographic variables, gender and degree. These variables were available on the class roll and were also collected on the questionnaire.

Although there were small demographic differences in the groups which answered the March and October questionnaires in 1995 and in October 1996, it can be seen, by comparing the proportions in each category of gender and degree, that the demographic characteristics of the three groups are similar (Table 5.2). It appears, from a comparison of corresponding proportions<sup>12</sup> in Table 5.1, that the groups of students who answered the questionnaire reflect, well enough, the demographic characteristics of the class as a whole in both

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<sup>10</sup> In October 1995, of the 44 students who answered the October questionnaire, 2 students gave no id so their achievement level cannot be identified, hence the data analysis is completed for 42 students.

<sup>11</sup> In October 1996, of the 38 students who answered the questionnaire, 1 student who completed the course work did not sit the final examination and 1 student gave no id, hence data analysis is completed for 36 students.

<sup>12</sup> Only proportions that differed by greater than 10%, compared to the corresponding figures on the class roll, were considered large enough to comment on.

1995 and 1996. (Note that the corresponding proportions for the March 1996 group, which are listed in Appendix L, are smaller for BSc students but similar for each gender group, compared to the other three groups in Table 5.2.)

The numbers of students and percentages in categories of the other demographic variables that were included in the questionnaire<sup>13</sup>, namely, ethnicity, age group, first language, fluency in English (included from October 1995) and mathematics background are listed in Table 5.3.

Proportions in age groups of students who answered the questionnaire are calculated for two categories, namely, students aged 24 years old or younger and those 25 years old or older. In the March (1995) and October (1995 and 1996) groups, the percentages of students in the younger group vary from 51 to 60% and those in the older group vary from 40 to 49%. The percentages of students who indicate that they have a background in mathematics at Form 5 level (or less) varies in the range 40-42%, and 58-60% have taken Form 6 mathematics or higher. Furthermore 60-63% indicate that they are fluent in English, while 36-39% indicate they have moderate or little fluency. Percentages in different ethnic groups are in the ranges 4-6% for Maori, 40-47% for Pakeha/European, 6-12% for Pacific Island, 26-33% for Asian and 8-14% are from other ethnic groups. These percentages (in ethnic groups) are largely reflected in the corresponding figures for first language, where 50-58% name English as their first language, 20-26% select Chinese, 3-10% a Pacific Island language, and 12-18% with a different first language. The majority of students in this last category specify Japanese. Since these demographic groups are so similar in terms of ethnicity and first language, no further data analysis will be done with the first language variable. (Note that the corresponding March 1996 percentages, which are listed in Appendix L, are similar for categories of ethnicity, first language and fluency, but differ for age and mathematics background. There are a higher proportion, than would be suggested by the other three groups in Table 5.3, of older students and of those with Form 5 or less mathematics background.)

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<sup>13</sup> The demographic variables selected for the questionnaire are described in Chapter 3, Section 3.2.1.

Table 5.3: Numbers of students and percentages in categories of five other demographic variables available only from the questionnaires completed by students in Mathematics 1 in March or in October in 1995 and 1996.

Variable	Categories	Mar	1995	Oct	1995	Oct	1996
			n n=50	%	n n=42	%	n n=36 <sup>#</sup>
Ethnicity	Maori	2	4	2	5	2	6
	Pakeha	23	46	17	40	17	47
	PI*	5	10	5	12	2	6
	Asian	13	26	14	33	12	33
	Other <sup>†</sup>	7	14	4	10	3	8
Age	≤ 24 years	29	58	25	60	18	51
Groups	≥ 25 years	21	42	17	40	17	49
First Language	English	26	52	21	50	21	58
Language	Maori	2	4	1	2	0	0
	PI*	3	6	4	10	1	3
	Chinese	10	20	11	26	9	25
	Other <sup>†</sup>	9	18	5	12	5	14
Fluency in English	Fluent	NA*		26	62	22	61
	Not Fluent			16	38	14	39
Maths	≤ Form 5 <sup>~</sup>	20	40	17	40	15	42
Background	≥ Form 6	30	60	25	60	21	58

<sup>#</sup> In October 1996, one student gave no age information.

\* Abbreviations: PI, Pacific Island; NA, data was not available.

<sup>†</sup> The majority of students in the 'other' category specified Japanese as their ethnicity and first language.

<sup>~</sup> Form 5 is the third year of high school at about age 15.

### 5.1.2 Demographic Characteristics of Wellesley Program Mathematics Students

A full description of the Wellesley program, which was taught in 1995 and 1996 for the full academic year, is given in Chapter 4 (Section 4.3). The tables below, Table 5.4 and Table 5.5, contain the proportions of students within each category for each demographic variable for the demographic variables available on the class roll, and those available only on the questionnaire, respectively, where data is available. Proportions are calculated in Table 5.4 for the whole class (defined to be the students who completed the course) for ethnicity, gender, options and streams, as these four variables were listed on the class roll. These proportions will be used in two ways, to describe the demographic profile of the Wellesley Mathematics class each year, and also to determine how

well the groups who completed the questionnaire in 1995 and 1996 represent the class.

It can be seen in Table 5.4 that proportions in categories of some demographic variables have varied from 1995 to 1996. The proportions of students in the Arts and Science Options are approximately 50% in 1995, but about 40% are in the Science Option in 1996. (The percentage of students in the Science option has varied from approximately 35% to 50% over the ten-year period 1989-1999 (see Appendix M).) In 1995, 57% indicate that they are Pakeha/European (compared to 43% in 1996), 28% are Maori (20% in 1996), 10% are Pacific Island (25% in 1996), and 5% indicate other ethnic groups (12% in 1996). The proportion of males to females from the class roll is approximately 2:1 in 1995, but in 1996 there are almost 50% in each group.

Table 5.4: Numbers and percentages of students are listed for categories of the four demographic variables available on the class roll for Wellesley Program 1995 and 1996.

Variable	Categories	Class Roll 1995		Class Roll 1996	
		n	%	n	%
Options	Science	31	53	21	41
	Arts	27	47	30	59
Gender	Female	19	33	27	53
	Male	39	67	24	47
Ethnicity	Maori	16	28	10	20
	Pakeha	33	57	22	43
	Pacific Island	6	10	13	25
	Others	3 <sup>†</sup>	5	6	12
Streams	1	11	19	14	27
	2	15	26	14	27
	3	14	24	11	22
	4	18	31	12	24

\*Although the roll contained 55 names, final marks were available for only 51 students in 1996. Four students completed the entire year's coursework but did not sit the final examination.

<sup>†</sup> For example, in 1995, other ethnic groups specified were 1 Asian (Chinese), 1 Somali and 1 Arab student.

A comparison can now be made between the percentages of students on the roll (i.e. in the entire class) in categories of option, gender, ethnicity and stream (in Table 5.4) and the corresponding figures for the groups who answered the questionnaire in 1995 and 1996 (in italics in Table 5.5) to ascertain if the groups

who answered the questionnaire were reasonably representative of the whole class.

Table 5.5: Numbers and percentages, for categories of demographic variables, of Wellesley Program students who answered the questionnaire in 1995 and 1996.

Variable	Categories	March 1995		October 1995		March 1996		October 1996	
		<i>n=46*</i>		<i>n=34*</i>		<i>n=47*</i>		<i>n=36*</i>	
		<i>n</i>	<i>%</i>	<i>n</i>	<i>%</i>	<i>n</i>	<i>%</i>	<i>n</i>	<i>%</i>
<i>Options</i>	<i>Science</i>	26	57	21	62	18	38	16	44
	<i>Arts</i>	20	43	13	38	29	62	20	56
<i>Gender</i>	<i>Female</i>	15	33	12	35	26	55	23	64
	<i>Male</i>	31	67	22	65	21	45	13	36
<i>Ethnicity</i>	<i>Maori</i>	11	24	7	21	8	17	7	19
	<i>Pakeha</i>	27	59	21	62	21	46	14	39
	<i>PI</i>	6	13	4	12	11	24	10	28
	<i>Others</i>	2	4	2	6	6	13	5	14
<i>Streams</i>	1	7	15	6	18	14	30	13	36
	2	13	28	5	15	12	26	8	22
	3	12	26	11	32	11	23	6	17
	4	14	30	12	35	10	21	9	25

\*March 1995, n=61 but 15 students withdrew; October 1995, n=38 but 4 students gave no id; March 1996, n=62 but 13 students withdrew and 2 gave no id; October 1996, n=44 but 4 students did not sit the final and 4 gave no id.

First, however, a summary is given of the numbers of students who answered the questionnaires in March and October, in 1995 and 1996, and then a comparison of these with the total numbers of students enrolled in, and who completed, the course, respectively.

The March questionnaire was answered by 61 of the 88 students enrolled in the Wellesley Program in 1995 (62 of the 87 students enrolled in 1996). Since 15 of these 61 students (and 13 of the 62 students in 1996) did not complete the course, the proportions in the table below are calculated for the 46 (47<sup>14</sup>) students who answered the questionnaire in March and who also completed the course. The October questionnaire was answered by 38<sup>15</sup> of the 58 students who

<sup>14</sup> In March 1996, of the 62 students who answered the questionnaire, 13 students withdrew and 2 students gave no id, hence data analysis is completed for 47 students.

<sup>15</sup> In October 1995, of the 38 students who answered the October questionnaire, 4 students gave no id, hence data analysis is completed for 34 students.

completed the course in 1995, and by 44<sup>16</sup> of 51 students who completed the year in 1996. Thirty students answered both questionnaires in 1995, 34 answered both in 1996. (The numbers in the groups above are listed in Table 2 in Appendix K for reference.)

Corresponding percentages in Table 5.4 and Table 5.5 can now be compared, for 1995 and 1996. Two differences<sup>17</sup> apparent by year are that fewer students in stream 2 answered the questionnaire in October 1995 and fewer males answered the questionnaire in October 1996. Otherwise it appears that, in terms of the proportions in each demographic category available both on the roll and in the questionnaire, the groups of students who answered the questionnaire in March or in October reflect, well enough, the demographic characteristics of the class as a whole in both 1995 and 1996 particularly when the comparison is made by year (as with Mathematics 1).

Table 5.6 contains the numbers and percentages of students in categories of four other demographic variables which were included in the questionnaire, namely, age group, first language, fluency in English (included from October 1995) and mathematics background.

In both the March and October groups in 1995 and March 1996, about 60% are in the younger age groups, and 40% in the older group. In October 1996 there are 50% in each age group. A high proportion of the students are fluent in English in all groups where data was available, varying from 83 to 97%. These percentages are reflected in the corresponding figures for first language, where 72-91% name English as their first language. Since these demographic groups are so similar, namely, fluency in English and first language, no further data analysis will be done with first language data. Percentages in categories of mathematics background in 1995 range from 70 to 79% with Form 5 or less mathematics, compared to the corresponding figures in 1996 which are 59 to 61%. The lower proportion in this group with the lower mathematics background in 1996 could reflect the change in selection policy that year, when fewer students with poor numeracy and literacy skills were selected for the Wellesley Program.

Demographics of the Wellesley Program over 10 years are contained in Part I of a report entitled "Wellesley Program 1989-1998: The First Ten Years" (Statham,

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<sup>16</sup> In October 1996, of the 44 students who answered the questionnaire, 4 students who completed the course work did not sit the final examination and 4 students gave no id, hence data analysis is completed for 36 students.

<sup>17</sup> Only proportions that differed by greater than 10%, compared to the corresponding figures on the class roll, were considered large enough to comment on.

1999), which includes the years in which data was collected for this study. This report is included for reference, with the permission of the author, as Appendix M.

Table 5.6 Numbers and percentages of students in categories of the demographic variables available only from the questionnaires completed by students in the Wellesley Program in March or in October, in 1995 and 1996.

Variable	Categories	Mar 1995		Oct 1995		Mar 1996		Oct 1996	
		n=46		n=34		n=47 <sup>#</sup>		n=36	
		n	%	n	%	n	%	n	%
Age	≤24 years	28	61	20	59	27	57	18	50
Groups	≥ 25 years	18	39	14	41	20	43	18	50
First language	English	38	83	31	91	36	77	26	72
	Other <sup>†</sup>	8	17	3	9	11	23	10	28
Fluency in English	Fluent	NA		33	97	40	85	30	83
	Not Fluent			1	3	7	15	6	17
Maths Backgr'nd	≤ Form 5	32	70	27	79	27	59	22	61
	≥ Form 6	14	30	7	21	19	41	14	39

<sup>#</sup>March 1996, 1 student gave no mathematics background information.

<sup>†</sup>Because of the small numbers in the four other categories, they are combined as 'Other'.

### 5.1.3 Individual Supervised Study in Mathematics

The independent study course was for an individual student who was male, pakeha, and aged 33 years. He had completed a degree in English literature and was part way through a commerce degree, but had suspended his study because of his inability to cope with the quantitative aspects of courses in this degree. He was taught for one to two hours, once a week, for 6 months. For this report he is referred to as Charles (not his real name).

### 5.1.4 Differences between the demographic profiles of the populations in the three courses

The demographic profiles of the populations in the three courses are summarized in Table 5.7, which lists the percentages in categories of demographic variables for all three courses.

The most pronounced differences between the demographic profile of Mathematics 1 and the Wellesley mathematics course are the proportion of Maori and Pacific Island students, the proportion of Asian students. There are a higher proportion of Maori and Pacific Island students in the Wellesley course

and a higher proportion of Asian students, and students who are not fluent in English, in Mathematics 1. Smaller differences are evident in mathematics background and gender: in Mathematics 1 there are a lower proportion of students with Form 5 or less and a higher proportion of females, compared to the Wellesley mathematics course. Similarities occur in the proportions in the two age groups in the Science and Arts degrees or options.

Table 5.7: Percentages of students in categories of demographic variables for the three courses in this study.

Mathematics 1	Wellesley Mathematics	Independent Study
Gender (from class roll): 57% Female* 43% Male	Gender (from class roll): 33 or 53% Female 67 or 47% Male	Male
Degree (from class roll): 40 or 43% BSc 51 or 56% BA	Pre-degree Options (from class roll): 53 or 41% Science 47 or 59% Arts	This student had completed a BA and was part way through a BCom.
Ethnicity (from questionnaire): 4 or 6% Maori 6 or 12% PI 40 or 47% Pakeha 26 or 33% Asian (Chinese)	Ethnicity (from class roll): 28 or 20% Maori 10 or 25% PI 57 or 43% Pakeha Few Asian (Chinese)	Pakeha
Age groups (from questionnaire): 51 or 60% $\leq 24$ years old 40 or 49% $\geq 25$ years old	Age groups (from questionnaire): 50 or 61% $\leq 24$ years old 39 or 50% $\geq 25$ years old	33 years old
Fluency (from questionnaire): 61 or 63% fluent in English	Fluency (from questionnaire): 83 or 97% fluent in English	Fluent in English

\* The 1995 and 1996 gender split for Mathematics 1 are identical.

Figure 5.1 gives bar graphs which summarise the demographic information listed in the tables above for Mathematics 1. Figure 5.2 gives bar graphs which summarise the demographic information listed in the tables above for the Wellesley Program. For any category, the average of the percentages, over years and over questionnaire groups (where appropriate), are used in the graphs. (The mean is used, despite differences in some proportions between 1995 and 1996, because the graphs in Figure 5.1 and Figure 5.2 are intended to show the

reader the main differences in the demographic profile of Mathematics 1 and the Wellesley Program which are summarized previously in Table 5.7.)

Figure 5.1: Bar graphs presenting average percentages of students in Mathematics 1 in categories of gender and degree from the class roll and, for categories of other demographic variables, from the groups who answered the questionnaire.

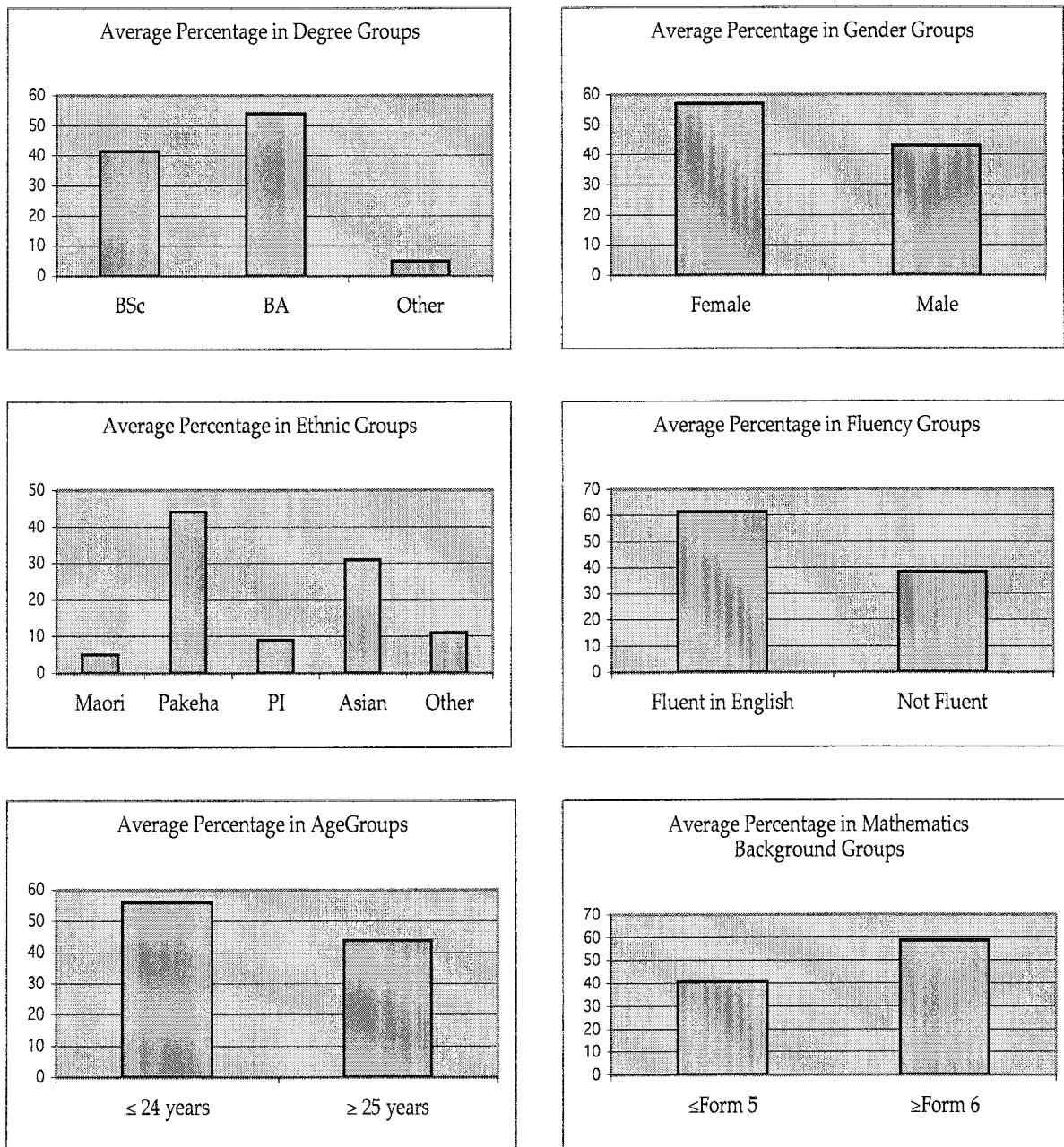
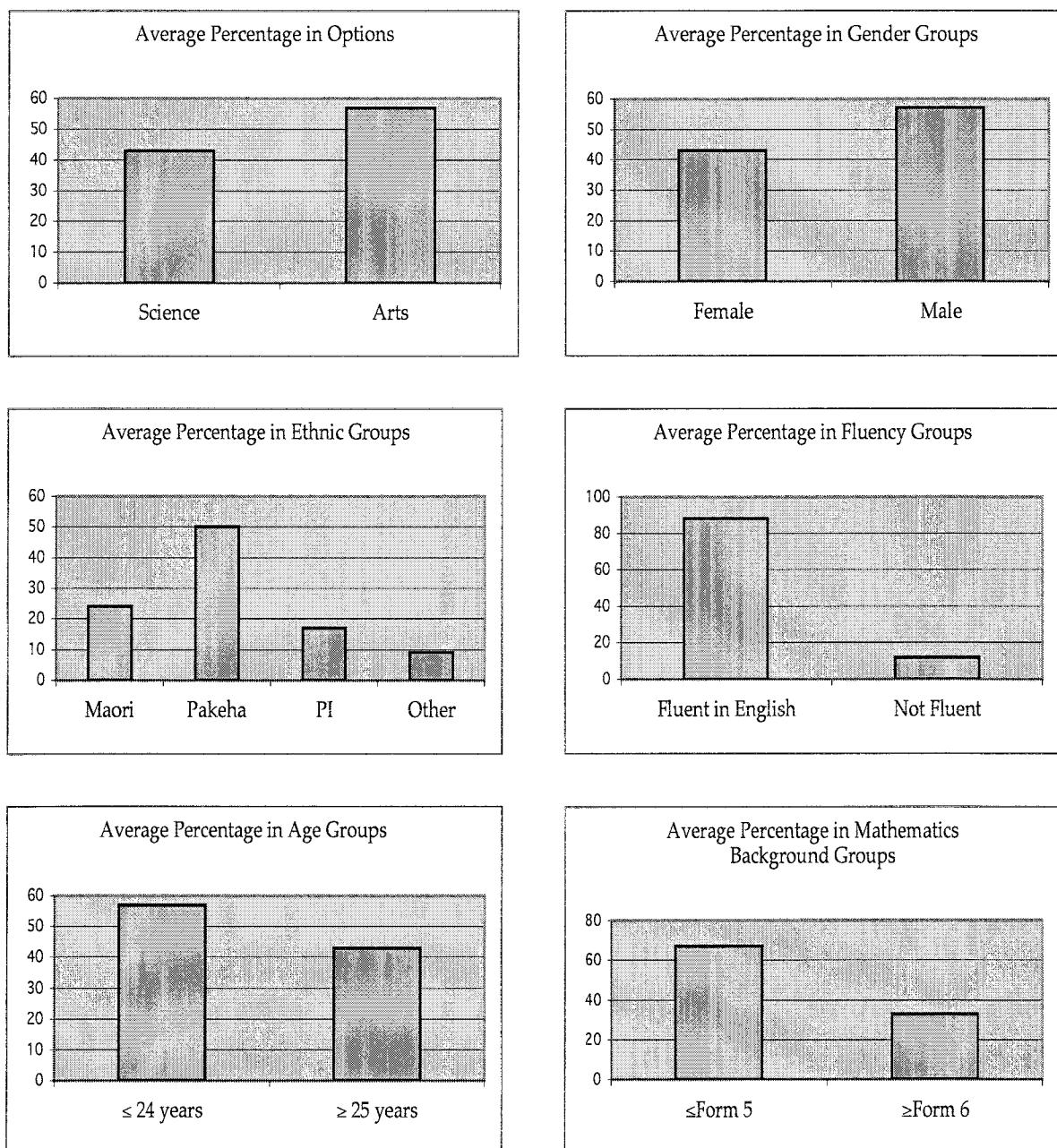


Figure 5.2: Bar graphs presenting average percentages of students in the Wellesley mathematics course in categories of gender, option and ethnicity from the class roll and, for categories of other demographic variables, from the groups who answered the questionnaire.



Section 5.2, which follows, contains a description of the materials used to gather data for this study.

## 5.2 Materials

A questionnaire was developed after a pilot study early in 1994 (Sections 5.4.1 and 5.4.2) and a trial of this questionnaire occurred later in 1994 (Sections 5.4.3, 5.4.4 and 5.4.5). Students completed the final questionnaire in 1995 and 1996, early in the academic year (March) and near the end of the academic year (October), in Mathematics 1 and the Wellesley Mathematics course. Charles, the individual undertaking supervised study in mathematics, completed the questionnaire at the beginning and the end of a 6-month period. Table 5.8 contains a list of the scales used in the questionnaire in 1995 and 1996.

Table 5.8: A summary of the scales used in the 1995 and 1996 questionnaires.

Attitudes and Beliefs Measured	Author of Scale and Number of Items
<i>Attitudes:</i>	
Enjoyment of Mathematics	Aiken (1974) - 11 items
Value of Mathematics	Aiken (1974) - 10 items
<i>Beliefs:</i>	
Beliefs about Mathematics Learning	Schoenfeld (1989) - 5 items
Mathematical Self-Concept *	Gourgey (1982) - 12 item subset °
Conceptions of Mathematics †	Crawford, Gordon, Nicholas & Prosser (1995a) – 19 items

\* Mathematical self-concept is viewed as a belief about oneself as a learner of mathematics, as suggested by McLeod (1992).

° This subset of the Gourgey (1982) scale was developed during the pilot study (Section 5.4.2).

† This scale was only included in the questionnaire in October 1996 (Section 5.4.2).

A full list of all items in these scales is in Appendix N. The procedure I employed to gather data is explained next, in Section 5.3.

## 5.3 Procedure

At one session of each lecture stream, or class, in March and again in October, lecturers in the two large courses gave the researcher time alone with the students to explain the purpose of the study before the questionnaire was handed out. It was emphasised that completing the questionnaire was optional, that their responses would be confidential to the researcher, and that the researcher was not involved in any marking for these courses. The length of

time between March and October may have avoided a problem raised by Kulm (1980 p. 370), in his survey of research into mathematics attitudes, that the initial questionnaire may sensitise students and "may cause subjects to examine their opinions and respond differently a second time regardless of intervening treatment".

Care was taken to choose a time to collect data that did not reflect general attitudes. The March questionnaire was given out after 2-3 weeks of class, thus avoiding the general attitude of excitement at the beginning of the year. The October questionnaire was timed to avoid the stressful time just before examinations as, "nearly concurrent with an examination, general attitude may be lower" (Kulm, 1980 p. 368). These data were collected for two years (1995 and 1996), for the main study, and the courses appeared to mainly exhibit similar characteristics each year.

Interviews were conducted with several students from each large course in both years. Students were mainly selected from those who indicated on the questionnaire that they were willing to be interviewed although, since selection was based on obtaining a range of students from different demographic groups, such as age, gender, ethnicity and level of fluency in English, some students were approached to ask if they would mind being interviewed.

These interviews covered their mathematical autobiography (their previous experiences learning mathematics and the attitudes of their family and teachers) and their experiences studying mathematics in the current course. Metaphors were used to help them describe their learning experiences. The semi-structured interview schedule for student interviews is in Appendix O.

#### **5.4 Development and Trial of Materials in 1994: Choice of Dimensions of Attitude, Types of Beliefs and Open Questions**

Attitudes, beliefs and reactions of students to the courses were measured by several self-report methods; scales, open questions, or interviews. The development of the materials early in 1994 and the trial of the materials<sup>18</sup> late in 1994 enabled me to choose dimensions of attitude, types of beliefs and the open questions to use in the final questionnaire. Appendix P contains a copy of the final questionnaire, which was used in the main study in 1995 and 1996.

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<sup>18</sup> A trial of the materials, which followed the development phase, with Mathematics 1 and the Wellesley mathematics course occurred later in 1994, in July and in October. The demographic profile of each of these courses was similar to the corresponding courses in the main study in 1995 and 1996.

#### **5.4.1 Attitude Scales Used in this Study**

One common way to measure attitudes to mathematics is to use scales, and most scales since the early-1970s “recognise explicitly different aspects of students’ attitudes to mathematics and report the effects of the different components separately” (Leder, 1985 p. 18), for example, the Aiken (1974) Scale has two dimensions, the Value of Mathematics and the Enjoyment of Mathematics.

The main purpose of the development of materials was to determine which dimensions of attitude to use for this study. Since one course in this research study emphasised the use of realistic contexts (Mathematics 1) and another course emphasised the student’s own context (supervised study for individual), I thought it was likely that the attitudinal dimension of value, or usefulness, of mathematics might be affected by a student’s exposure to these courses. Previous experiences of second-chance mathematics students result in, typically, a lack of confidence or enjoyment in mathematics, or anxiety about returning to the study of mathematics. All teachers of the courses in this study aimed to increase the mathematical confidence of students, so these aspects of attitude/belief were also of interest to me. The other issue when looking for scales, was to check they were relevant to adults, in particular, to second chance mathematics students in a tertiary setting.

Among the “more important and widely used scales” which Kulm (1980 p. 362) lists in his survey chapter on research on mathematics attitude are Fennema and Sherman (1976), developed for high school students, and Aiken (1974), developed for undergraduate students. There are 84-items in the Fennema and Sherman (1976) nine-scale package (confirmed as valid constructs by Broadbooks, Elmore, Pedersen & Bleyer (1981)<sup>19</sup>) and 21-items in the Aiken (1974) two-scale package (validated as bi-dimensional by Watson (1983)<sup>20</sup>). Both packages were constructed by Likert’s method of summated ratings. Both these mathematics attitude scales, and other Aiken scales, have been used in a number of research studies since they were developed (Hyde, Fennema, & Lamon, 1990; Nolen, Archambault, & Greene, 1976; Rech, Hartzell, & Stephens, 1993; Taylor, 1994; Watson, 1989; Yong, 1993).

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<sup>19</sup> These nine scales / domains were confirmed as valid constructs for a junior high school population by Broadbooks, Elmore, Pedersen & Bleyer (1981).

<sup>20</sup> The bi-dimensionality of the Aiken (1974) scale was validated for Australian first-year university students ( $n=287$ ) by Watson (1983 p. 1250), who had looked at the reliability and discriminant validity of these Enjoyment of Mathematics and Value of Mathematics scales and found that Aiken’s scale represented “two distinct aspects of attitude towards mathematics”. A three-factor structure was suggested by Nolen, Archambault & Greene (1976) based on an analysis of data from 76 elementary teachers, but Watson argues against these results.

Initially, I examined the Fennema-Sherman (1976) Scales to see if some of these scales would be suitable for this study. Fennema & Sherman's (1976) Usefulness of Mathematics Scale was considered but a number of statements in this scale made reference to mathematics being useful for work in the future, for example, "for my future work", "in my life's work", "help me earn a living", or "when I get out of school". Such statements are relevant to high school students or younger undergraduates, but not to the majority of students in the courses in my study. Janet Taylor (1994 p. 184), who was teaching mathematics to second-chance pre-degree adult students at an Australian university, also studied the relevance of the Fennema-Sherman (1976) Scales for adult learners and found that, in many of the scales, "references to parental influence and schooling practices may be irrelevant for many of our students".

The Aiken (1979) Scale had been administered for a number of years to adults returning to the study of mathematics in a pre-degree bridging program at the University of Southern Queensland, Australia (Taylor, 1995)<sup>21</sup>. A comparison was made, by Janet Taylor (1994), of the Fennema-Sherman (1976) Scales and the Aiken (1979) Scale, in terms of their suitability for use with adult students and she decided that Aiken's (1979) Scale was preferable to most of the Fennema-Sherman (1976) Scales. She (1994 p. 184) gave two reasons: the statements were clearer in the former scale and this "simplicity" was good for students who were new to tertiary study; and the Aiken Scale was shorter, another advantage, as a "long test would be intimidating to these students". However, the dimensionality of the Aiken (1979) Scale with this adult population was uncertain and being researched<sup>22</sup> (the results were published later (Taylor, 1997)), so my focus shifted to using the Aiken (1974) Scale which was of similar clarity and length to the Aiken (1979) Scale and therefore suitable for adults for the same reasons.

One dimension of the Aiken (1974) Scale, Value of Mathematics, made no reference to future work, which we have already observed was a disadvantage

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<sup>21</sup> Although the Aiken (1979) scale had been used in Australia for a number of years with adults returning to the study of mathematics in a pre-degree bridging program, Janet Taylor began to question the dimensionality of this scale with this student population. The Aiken (1979) scale, initially developed to survey 11-15 year old high school students in Iran, was "purported to measure four dimensions of attitude: Enjoyment, Motivation, Importance, and Freedom from Fear" (J. A. Taylor, 1997 p. 125-6). Janet Taylor found that no validation of this survey for adults students has been completed, so she researched the dimensionality of the scale with their students (n=430). The results of this research, which I discussed later in 1994 with her and which was published in 1997, found that the "Aiken (1979) Attitude to Mathematics Scale was two-dimensional when used with adult students" (p. 129).

<sup>22</sup> Although Aiken (1979) Scale was used in the trial of the questionnaire later in 1994, chosen then because of its use with second-chance adult students in Australia, the questioning of the dimensionality of this scale with these adult students, caused me to decide to use the Aiken (1974) Scale for the main study.

of Fennema & Sherman's (1976) Usefulness of Mathematics Scale, and items were also related to the extrinsic value of mathematics which may be more suitable for adults. The other dimension of this scale, Enjoyment of Mathematics, "encompassed not only a liking for mathematics problems but also a liking for mathematical terms, symbols and routine calculations" (Aiken, 1974 p. 67) and was also considered a useful dimension of attitude to measure. So for the above reasons, the Aiken (1974) Value of Mathematics Scale was considered more suitable to use in the main study in 1995 and 1996.

#### **5.4.2 Belief Scales Used in this Study**

Gourgey (1982) developed and validated a 27-item Likert-style scale, called the Mathematical Self-Concept Scale with undergraduate and graduate students studying a required basic statistics course at a university in the USA. Since this scale had been developed for a population similar to those in this research, it seemed appropriate to use it in this study. Gourgey defined mathematical self-concept as

beliefs, feelings or attitudes regarding one's ability to understand or perform in situations involving mathematics. The self as capable or incapable of learning or performing in mathematics, rather than the subject of mathematics, is the object of attitude.

(Gourgey, 1982 p. 3)

Mathematical self-concept, according to Reyes (1984, quoted in Relich, Way & Martin, 1994 p. 58) "relates to how confident a person is of being able to learn new topics in mathematics, perform well in class and do well on tests". McLeod (1994, p. 641) discusses how "confidence about learning mathematics ... has frequently been discussed as an attitude (Fennema & Sherman, 1976), but it may be useful to consider it as a belief about oneself". I decided, therefore, consider the measure of mathematical self-concept in this research as a measure of belief (about oneself as a learner of mathematics).

The Fennema-Sherman (1976) Mathematics Anxiety Scale did not have some of the disadvantages of other Scales in this package discussed earlier in this Section, and was included in the questionnaire for the pilot study. Richardson and Suinn (1972 p. 551) state that "mathematics anxiety involves feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations".

The Fennema and Sherman (1976) Mathematics Anxiety Scale and the Gourgey (1984) Self-Concept in Mathematics Scale were both included in the questionnaire for the pilot study, however, the correlation between the scores

on these two scales in the pilot study was very high (0.92). It was noted that Fennema and Sherman (1976) decided not to include their Mathematics Anxiety Scale in their combined package (which included their Confidence in Learning Mathematics Scale) administered to high school students, "since it correlated 0.89 with the Confidence scale" (Fennema & Sherman, 1976 p. 8). A high correlation between mathematics anxiety and confidence in learning mathematics was also found in a study of students in a developmental algebra course in a large southern state university in USA (Goolsby, Dwinell, Higbee, & Bretscher, 1987). Gourgey (1982 p. 9) also found a "moderately high correlation of the Mathematical Self-Concept Scale with the measure of mathematics anxiety" in her study which, she said, also lent "support to the validity" of her scale "as a measure of mathematical self-concept". Mathematical self-concept was more highly correlated to measures of achievement and "erroneous statements about mathematics" than was mathematics anxiety and Gourgey (1982 p. 9) suggested "mathematical self-concept may be a more powerful and more informative variable than mathematics anxiety". Relich et al (1994 p. 58), in their study of attitudes of pre-service teachers, believed that self-concept was a "better measure than mathematics anxiety of how people feel about themselves as teachers of mathematics and that self-concept in turn has an influence on formation of attitudes". Goolsby et al (1987), in their study of students in a developmental algebra course, also found that mathematics anxiety was not a very good predictor of achievement in mathematics. The research above suggests that mathematical self-concept may be a more informative variable than mathematics anxiety, hence I decided to include only a Mathematical Self-Concept Scale, considered to measure a "belief about oneself" as a learner of mathematics (McLeod, 1994 p. 641).

The length of the questionnaire became an issue at this stage. It was only possible to use approximately 20 minutes of lecture time for students to answer the questionnaire, and the Gourgey (1984) Mathematical Self-Concept Scale contained 27 items. A 12-item subset of the Mathematical Self-Concept Scale was selected, including six positive and six negative items. The high correlation (0.94) between the scores on this 12-item subset and the full length Mathematical Self-Concept Scale (Gourgey, 1984), used in the pilot study, indicated that the subset explained 82% of the variability of the longer original scale, i.e., if we know a student's score on one scale we can be 82% sure of predicting their score on the other. This shorter scale was used both in the trial of the questionnaire late in 1994 and also in the main study in 1995 and 1996.

Some of the teaching approaches in the three courses were considered to be innovative, as the teachers aimed to give these second-chance students a

different experience to that in their past. It was likely that their beliefs about the learning of mathematics might change over the period of time in the mathematics course. Gourgey's (1984) Beliefs about Mathematics Scale contained 17 items and was considered too long to include in the questionnaire. The Indiana Mathematics Beliefs Scales (Kloosterman & Stage, 1992; Stage & Kloosterman, 1991), although developed for remedial college mathematics students in the USA, were similarly rejected because of the length of these scales (30 items). A part of an instrument developed by Schoenfeld (1989), and modified for use with university students in Sydney by Wood and Smith (1993) was also considered. Six items explored students' beliefs about the learning of mathematics and I included them in the questionnaire for the trial of the questionnaire later in 1994 and in the main study in 1995 and 1996.

After some analysis of data collected in the main study in 1995, which identified changes in students' beliefs, it was decided to lengthen the questionnaire and include another belief measure in the questionnaire for October 1996, namely, the Conceptions of Mathematics Scale (Crawford et al., 1995a; Crawford, Gordon, Nicholas, & Prosser, 1995b). This scale is also listed in full, with the other scales used in the questionnaire, in Appendix N.

#### **5.4.3 Open Questions**

Open questions used in the trial of the questionnaire in July and October 1994 were examined and some changes made. A set of open questions, developed by the lecturers in Mathematics 1 to evaluate this new course in 1994, were answered anonymously by students at the end of that academic year and provided additional data. An analysis of the responses to these questions indicated how useful they were in obtaining students' reactions to specific aspects of the course, and hence would enrich any evaluation of the course (Miller-Reilly, 1995), so I adapted and included many of these questions in the 1995 instrument. I took care in the wording of the questions so that they were not leading the students to a particular response.

#### **5.4.4 Trial Interviews**

I conducted some trial interviews with a few students in Mathematics 1 and in the Wellesley Program in 1995. An opportunity was taken, during these interviews and also with a math phobic individual seeking assistance that year, to trial a projective technique. Metaphors were used in two different ways in the trial interviews. The metaphors were initially used to illuminate students' personal conceptions of mathematics, to learn how students think and feel about mathematics (Buerk, 1982; Gibson, 1994). They proved to be particularly

helpful in teaching Charles because he acknowledged how well they captured the intensity of his feelings about mathematics. Analysis of the interviews indicated that students were also using the metaphors to describe how they thought and felt about their mathematics learning experiences in the courses in which they were studying mathematics. As further interviews were held, I chose to use metaphors in this latter way because, as Kulm (1980 p. 360) also suggested, collecting students' responses to teaching practices might "reveal why students attitudes to mathematics were or were not changed by" new teaching strategies or curricula.

#### **5.4.5 Demographic Variables**

The trial of the questionnaire in 1994 involved students completing an initial questionnaire in July and a final questionnaire in October. The instrument used in October 1994 did not contain the demographic questions used in the initial data gathering in July 1994, but a number of students only completed the October questionnaire. The resulting lack of demographic data for these students limited the analysis I could do, so I decided that in future I would gather demographic information as part of both the initial (March) and the final (October) questionnaires for the main study in 1995 and 1996.

Appendix P contains a copy of a final questionnaire for the main study, in particular, the questionnaire used in October 1995 for Mathematics 1. In addition, the main parts of the Wellesley Program questionnaire which differ from the questionnaire given to Mathematics 1 students are also included in Appendix P.

### **5.5 Decisions on general management of the data analysis**

I made every effort to go to and fro between different parts of my data set so that all of it was inspected at some stage, so that my generalizations were applied to all the relevant data I gathered. I aimed to further strengthen the validity of my research by critically and comprehensively analysing the data. On different occasions I re-analysed some of the data in order to check the reliability of my categorizations of the data (Silverman, 2000).

As the data analysis progressed decisions were made, in particular, about the general management of the statistical analysis. These decision are listed:

- For missing values in responses to attitude scale items I decided to impute the mean for that item over the whole group. However if, for a particular student, the number of missing items for one scale was over

one third of the total number of items, then the score on the scale for this student was not included in the analysis.

- Scale items scored 0 to 4, as in Aiken (1974), where 0 was entered for Strongly Disagree, 1 for Disagree, up to 4 for Strongly Agree.
- The three scales are of slightly different lengths - 10, 11, or 12 items. The total scores for each student were scaled so that all scores would range from 0 (minimum) to 40 (maximum).
- Correlations were calculated only when 20 data points or more were available.
- For Mathematics 1, just 22 students who answered the questionnaire in March 1996 (Section 5.1.1) also answered the questionnaire in October that year (Sections 6.4 and 6.5). This group was considered to be too small to be broken up into demographic subgroups for analysis.



# **6 Mathematics 1**

Detailed analysis of data collected from students in 1995 and 1996<sup>23</sup> informs this case study of the Mathematics 1 course. Section 6.1 compares the final marks of students who completed the course, within categories of the demographic variables. This analysis determines the groups where achievement differences are greatest. The data from these particular groups is then examined in more detail. In Section 6.2 students' responses to open questions highlight how particular groups reacted to the teaching approach and these are compared to quantitative measures of some beliefs about, and attitudes to, mathematics. Interviews with a number of students provide additional qualitative data, which is analysed in Section 6.3.

## **6.1 Analysis of Achievement within Categories of Demographic Variables**

The first comparisons of achievement, in Section 6.1.1, are for the demographic variables available both on the class roll for the Mathematics 1 course and in the research questionnaire; gender, fluency in English and degree. Later in this section, a comparison is made between the class roll statistics and those of the groups of students who answered the questionnaire in March and October, to determine how representative these latter groups are. In Section 6.1.2, achievement statistics for demographic variables only available on the questionnaire, and not on the class roll, are analysed. A refinement of this analysis of achievement differences is contained in Section 6.1.3 to make a final determination of the demographic groups where these differences are greatest.

### **6.1.1 Demographics on the class roll: gender, degree and fluency in English**

Table 6.1 contains the medians of the final marks in the Mathematics 1 course calculated for categories of the demographic variables available on the class roll. This class roll contains only those students who completed the year's course and who sat the final examination. Achievement data was not available from a class roll in 1996.

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<sup>23</sup> Demographic characteristics of Mathematics 1 in 1995 and 1996 are discussed in Chapter 5, where proportions of students in categories of each demographic variable are listed.

Table 6.1: Medians and ranges of final marks for Mathematics 1 in 1995.  
 These achievement statistics are listed for the entire class for categories of the demographic variables available both on the class roll and in the research questionnaire.

Class Roll 1995 n=65*				
Variable	Categories	n	Final Marks Med <sup>†</sup>	Range <sup>†</sup>
Degree	BSc	26	71	71
	BA	33 <sup>†</sup>	62	88
	Other	6	68	43
Gender	Female	38	69	70
	Male	27 <sup>†</sup>	65	88
Fluency	Fluent	43	69	69
In English*	Not Fluent	19 <sup>†</sup>	55	83

\*In 1995, n=67, but 2 students gave no id; no fluency information was available for 3 students.

<sup>†</sup> Abbreviations: Med is Median.

<sup>†</sup> One student in 1995 achieved a final mark of 6%. This student was not fluent, male, and in a BA degree and hence the ranges for these demographic groups are particularly affected by this low final mark, which could be considered an outlier.

Largest differences in achievement in the 1995 class roll, for these three variables, are between the two groups based on fluency in English and between the two degree groups, BSc and BA. Medians of final marks differ by 14% between students who are fluent in English and students who have moderate or little fluency. For the BSc and BA groups, medians differ by 9%. Medians of final marks differ by 4% between female and male students. None of these differences are statistically significant.

I now examine how representative the groups of students who responded to the questionnaire in March and October 1995 are of the whole class by comparing their achievement statistics with those from the class roll (Table 6.1). Table 6.2 lists these corresponding achievement statistics for 1995 and 1996 in categories of the three demographic variables available both on the class roll and in the questionnaire.

Medians of the final marks, and differences in achievement, by gender, degree and by fluency in English in the roll in 1995 (in Table 6.1) are similar to corresponding statistics for the groups of students who responded to the questionnaire in March and/or October 1995 (in the first and second column of medians in Table 6.2).

Table 6.2: Achievement statistics<sup>#</sup> for Mathematics 1 students who answered the questionnaire, in March and October 1995 and October 1996, in categories of the three demographic variables also available on the class roll.

Variable	Categories	Mar <sup>†</sup>	1995	Oct <sup>†</sup>	1995	Oct	1996
		n=50		n=42*		n=36*	
		n	Med	n	Med	n	Med
Degree	BSc	21	72	14	71	14	86
	BA	26	64	24	61	20	74
	Other <sup>#</sup>	3	-	4	68	2	-
Gender	Female	32	69	27	69	21	79
	Male	18	69	15	63	15	72
Fluency in English	Fluent	NA <sup>a</sup>		26	70	22	80
	Not Fluent			16	60	14	75

\*October 1995, n=44 but 2 students gave no id; October 1996, n=38 but 1 student did not sit the final and 1 gave no id.

<sup>†</sup>Abbreviations: NA, data was not available; Mar is March; Oct is October.

<sup>#</sup>For sample sizes of 3 or less, medians were not calculated because they are not meaningful.

<sup>a</sup>Fluency in English was gathered only from October 1995.

It is also noted that, although no comparison with a class roll in 1996 was possible<sup>24</sup>, the direction of the differences by achievement in each demographic group in 1995 are echoed in the statistics for the group of students who answered the questionnaire in October 1996<sup>25</sup> (the last column of Table 6.2). It appears, from this comparison of corresponding medians in these two tables, that the groups of students who answered the questionnaire reflect, well enough, the achievement characteristics of the class as a whole in 1995. These characteristics are echoed in the October 1996 group also.

Achievement statistics for the demographic information available on the class roll are now completed so I will analyse additional demographic information gathered only in the questionnaires in Section 6.1.2.

### 6.1.2 Demographics available only on the questionnaire: ethnicity, age and mathematics background knowledge

Medians of the final marks for the Mathematics 1 course in 1995 and 1996 are listed in Table 6.3 for categories of the demographic variables available only

<sup>24</sup> Achievement data was not available from a class roll in 1996.

<sup>25</sup> The number of students who answered the questionnaire in March 1996 and who also completed the course was considered too small to include in Table 6.2 (see Chapter 5, Section 5.1.1, for more details) but these achievement statistics are included in Appendix R for completeness.

from the questionnaires in March 1995, October 1995 and October 1996.  
 (Students who did not complete the course are excluded.)

Table 6.3: The numbers of students and the medians of final marks for Mathematics 1 students in 1995 and 1996 in categories of the demographic variables available only from the questionnaires.

Variable	Categories	Mar	1995	Oct	1995	Oct	1996	
			n=50 <sup>†</sup>		n=42		n=36*	
			n	Med	n	Med	n	Med
Ethnicity	Maori	2	-	2	-	2	-	
	Pakeha	23	72	17	69	17	82	
	PI	5	50	5	50	2	-	
	Asian	13	62	14	66	12	74	
	Others	7	69	4	72	3	-	
Age	≤ 24 years	29	68	25	64	18	76	
Groups	≥ 25 years	21	70	17	69	17	77	
Maths	≤ Form 5	20	56	17	53	15	79	
Background	≥ Form 6	30	71	25	72	21	77	

\*October 1995, 1 student gave no age information.

<sup>†</sup>Consideration of one final mark as an outlier, 6%, affects medians for Asian students (becomes 66) and for the group with Form 6 or more mathematics background (70), only in March 1995.

Achievement differences between the two largest ethnic groups in the class, Pakeha/European and (East) Asian, largely reflect the differences in achievement by fluency in English, hence the demographic variable ethnicity will not be analysed separately further. The large difference in achievement between the two mathematics background categories in 1995 is not repeated in 1996, where there is little difference in achievement between these two groups. It appears that the group with Form 5 or less mathematics background in 1996 is composed of higher achieving students than were in this category in 1995. Because of this inconsistency no further analysis of the mathematics background variable for Mathematics 1 is included in this study. There is a consistent small difference in achievement by age.

Hence, achievement differences between categories of the demographic variables that are reasonably consistent over 1995 and 1996, although sometimes small, are those for fluency in English, degree, gender and age. However, none of these differences are statistically significant.

Larger differences in achievement might be revealed by a two-factor breakdown of final marks for demographic groups because differences can be masked by

insufficiently fine analysis or breakdown of variables. Section 6.1.3 contains this refinement of the analysis of achievement differences.

### **6.1.3 Analysis of achievement differences in the two-way interactions of demographic variables**

A refinement of the analysis of achievement differences was completed by investigating all the two-way interactions between the demographic variables which show consistent achievement differences over 1995 and 1996. (See Appendix Q for the full list.) This two-factor breakdown of the final marks showed larger achievement differences than those in Section 6.1.1 and 6.1.2, for all four demographic variables where achievement differences were found to be consistent.

Table 6.4, a subset of the table in Appendix Q, lists the two-way analyses which show the largest achievement differences in both years<sup>26</sup> for particular demographic combinations<sup>27</sup>.

The achievement difference between students who are fluent in English and those who have moderate or little fluency is evident in two of the two-way results. Fluent students in the younger age group achieve better than non-fluent younger students (an 18% and 11% difference in achievement in 1995 and 1996 respectively), and a similar pattern occurs for female students (a 14% and 7% difference). Both the differences in achievement by fluency are statistically significant in 1995 ( $p=0.01$ ,  $p=0.03$  respectively).

The achievement difference between the older and younger age groups ( $\geq 25$  years and  $\leq 24$  years respectively), in favour of the older age group, largely disappears for fluent students. However, for students with moderate or little fluency in English there is a difference in levels of achievement. For non-fluent students, the older age group achieve better than younger students (an 8% and 16% difference). In addition, BA students who are older achieve better than BA students in the younger age group (a 10% difference both years), however the majority of younger BA students are not fluent in English, so my data suggests that this difference is mostly a fluency effect, and hence it will not be investigated further. Only the non-fluent students will be analysed by age.

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<sup>26</sup> Table 6.4 includes the two-way interactions where the achievement differences in the median scores are more than 10% in at least one year.

<sup>27</sup> Another comparison can be made between the achievement statistics on the class roll and analogous figures for the questionnaire group in October 1995 (Table 6.4), for the two-way interaction of gender by fluency in English, which again indicates that we have a representative group in October 1995. On the 1995 class roll fluent female students achieved a median (and range) of 71 (54), fluent males 66 (69) and non-fluent females 58 (57), respectively, very similar to the corresponding statistics in Table 6.4.

Table 6.4 Achievement of students in categories of two-way interactions of the demographic variables for Mathematics 1 students in October 1995 and 1996. The figures listed are for categories where the largest achievement differences occurred.

Achievement differences are occurring for categories of this variable	Categories of two-way interactions		Oct n	1995 n=42*		Oct n	1996 n=36*	
				Med	Range		Med	Range
FLUENCY	≤ 24 years	Fluent	14	71	63	9	80	26
	≤ 24 years	Not Fluent	11	53	64	9	69	48
FLUENCY	Female	Fluent	13	72	43	14	81	44
	Female	Not Fluent	14	58	64	7	74	48
AGE	≤ 24 years	Not Fluent	11	53	64	9	69	48
	≥ 25 years	Not Fluent	5	69	51	5	77	20
AGE	≤ 24 years	BA	14	60	64	10	67	48
	≥ 25 years	BA	10	70	56	10	77	47
GENDER	Female	Fluent	13	72	43	14	81	44
	Male	Fluent	13	63	69	8	70	31
DEGREE <sup>†</sup>	≤ 24 years	BSc	9	74	45	7	82	24
	≤ 24 years	BA	14	60	64	10	67	48
DEGREE <sup>†</sup>	Male	BSc	5	74	32	7	86	31
	Male	BA	10	59	69	8	71	42

\*October 1995, n=44 but 2 students gave no id; October 1996, n=38 but 1 student withdrew and 1 student gave no id.

<sup>†</sup> The few students enrolled in "other" degrees were omitted from this analysis (Oct 1995, n=5; Oct 1996, n=2).

The difference in achievement between BSc students and BA students in Mathematics 1, to a large extent, reflects the difference in achievement between students with different levels of fluency in English, as there are very few students who have moderate or little fluency in English enrolled in a BSc. However, fluent BSc students do achieve at a slightly higher level than fluent BA students (8 and 4% difference). BSc students in the younger age group (≤24 years old) achieve at a higher level than young BA students (a 14% and 15% difference), however most younger BA students are not fluent in English so my data suggests that this is a fluency effect. We also find that male BSc students achieve at a higher level than male BA students (a 15% difference both years), however, the size of one group reduces to only 4 students for the analysis (for

students who completed both March and October questionnaires) in the next section. Hence no further analysis will be done by degree.

A gender difference in achievement in favour of females occurs for fluent students, that is, females who are fluent in English achieve higher than males who are fluent (a 9% & 11% difference in achievement). There seems to be no difference in achievement between females and males who have moderate or little fluency in English. (This latter result is only available in 1996 as data was gathered from only 1 non-fluent male student in 1995.) Another indication of a gender difference in achievement is highlighted by analysis of the pass rates and retention rates by gender for the whole class in 1995. Although 51% of students who enrolled in Mathematics 1 were female, 57% of those who sat the final examination were female, which indicates a higher dropout rate for males. Forty-two percent of females passed, compared to 31% of males, and 9% of female students achieved A passes, compared to 6% of male students.

A demographic profile of the reasonably large number of students who completed the March 1996 questionnaire but who later withdrew from the course ( $n=22$ ) shows that two-thirds were not fluent in English, over 80% were younger ( $\leq 24$  years old) and over half were male. It would appear that the students who withdrew were all from at least one of the demographic groups which did not achieve as well as others, that is, they were either not fluent in English, younger or male.

In the rest of this chapter, data will be examined in more detail for the particular demographic groups where achievement differences are greatest, that is, by levels of fluency in English (particularly for the younger age group or for female students), by age groups (for non-fluent students) and by gender (for fluent students). This analysis not only considers whether students' reactions to the course help us understand their levels of achievement, but also if their achievement is related to quantitative measures of their mathematical attitudes and beliefs. Some data was gathered using open questions and interviews and other data was gathered using Likert-style statements.

## **6.2 Students' Reactions, Beliefs and Attitudes: Analysis of Questionnaire Data**

I consider the relationship between some quantitative measures of students' mathematical attitudes and beliefs and students' achievement in the course in Section 6.2.1, which focuses on the class as a whole. Section 6.2.2 explores how students react to the realistic investigative approach used in Mathematics 1 by examining their responses to the open questions.

### **6.2.1 The relationship between achievement in the course and some quantitative measures of students' beliefs and attitudes**

A discussion of the relationship between achievement in the course and some quantitative measures of students' beliefs and attitudes, in particular, students' beliefs about themselves as learners of mathematics and students' attitudes to mathematics, follows.

The relationship between achievement in the course and students' beliefs about themselves as learners of mathematics is discussed first. A subset of Gourgey's (1982) Mathematical Self-Concept Scale was used in the questionnaire to measure this belief. (Chapter 5 contains a discussion of the trial of this scale.) Correlations are calculated between this scale score and the students' final marks in the course, a measure of achievement. Table 6.5 lists the correlations of the Mathematical Self-Concept Scale scores with final marks, in March and in October 1995.

Table 6.5: Correlations of a subset of Gourgey's Mathematical Self-Concept Scale (1982) with final Marks in the Mathematics 1 course, in March and in October 1995, are listed.

Correlation	Year	n <sup>28</sup>	Mathematical Self-Concept Scores in March*	n	Mathematical Self-Concept Scores in October
Final marks	1995	37	0.27: p=0.11	37	0.35: p=0.03

\* One student's score is excluded because it was regarded as an outlier.

It appears that students' mathematical self-concept in October (just prior to sitting their final examination) is more highly correlated with their achievement in the course than are their attitudes (i.e. enjoyment and value of mathematics) in March, and is statistically significant. This parallels Gourgey's (1984 p. 17) result that "achievement and mathematics self-concept are significantly related" and agrees somewhat with other studies which have "found a consistently high positive relationship between self-concept and mathematics achievement" (Relich, Way, & Martin, 1994 p. 7).

The relationships between achievement in the course and students' attitudes to mathematics are discussed next. Aiken's (1974) Mathematics Attitude Scale was included in the questionnaire to measure students' attitudes to mathematics.

<sup>28</sup> In order to compare correlations for the same group of students in March and October, correlations were calculated for the 37 students in 1995 who completed both questionnaires (in March and in October) that year, a slight reduction in the size of the groups analysed earlier: 42 students answered the October questionnaire. Because of the small number of students in 1996 who answered both questionnaires (n=22), 1996 data is not included.

Two attitudinal dimensions were measured by this scale; Enjoyment of Mathematics and Value of Mathematics. Correlations are calculated between these scale scores, as a measure of attitude, and the students' final marks in the course, as a measure of achievement. Table 6.6 lists the correlations of the Enjoyment of Mathematics Scale scores, and the Value of Mathematics Scale scores, with final marks, in March and in October, 1995.

Table 6.6: Correlations of Aiken's (1974) Enjoyment of Mathematics Scale scores, and Aiken's (1974) Value of Mathematics Scale scores, with Final Marks in the Mathematics 1 Course, in March and October (1995), are listed.

Correlation	Year	n	Enjoyment of Maths Scores in March	n	Enjoyment of Maths Scores in October
Final marks	1995	37	-0.05: p=0.76	37	0.22: p=0.20
Correlation			Value of Maths Scores in March		Value of Maths Scores in October
Final marks	1995	30	0.28: p=0.09	30	0.4: p=0.01

Students' attitude scores for both scales in October are more highly correlated with their achievement in the course than are their attitudes in March. (This situation also occurs in the Wellesley mathematics course (Section 7.2.2.2).) Students' attitude to the (extrinsic) value of mathematics correlates more highly with their achievement in the course than does their enjoyment of mathematics. (The reverse situation occurs in the Wellesley mathematics course where students' enjoyment of mathematics correlated more highly with their achievement in the course than did their attitude to the (extrinsic) value of mathematics (Section 7.2.2.2).) One correlation in Table 6.6, the Value of Mathematics scale scores in October with achievement, is statistically significant ( $p = 0.01$ ). That this correlation is significant could be because students with a more positive attitude to the (extrinsic) value of mathematics (by the end of the course) may well achieve better in Mathematics 1, a course which uses contexts to emphasize how mathematics can model situations. The converse could be expected also, that lower scores on the Value of Mathematics scale could indicate students who do not believe in the extrinsic value of mathematics, so the use of contexts may not interest, and could even confuse, such students thus lowering their achievement. It appears that students' attitude to the (extrinsic) value of mathematics correlates a little more highly with achievement in the course than students' mathematical self-concept. Appendix S contains the medians of the scores for the three scales discussed above. The results in this

section give some evidence of Gourgey's (1984 p. 17) statement that "attitudes and performance are completely intertwined".

The rather low correlations above (Table 6.5 and Table 6.6) are not unusual according to Kulm (1980). In his survey article about attitudes to mathematics he states "the attitude-achievement relationship is not as strong as common sense might expect" which "seems paradoxical" (p. 367), that there is "generally low correlation between attitude and achievement" (p. 373) and that "most experimental treatments" are "ineffective" in "producing significant improvements in mathematics attitude" (p. 375). Although Relich et al (1994 p. 5) report on a number of studies which reveal "a low but significant correlation" between "attitude and achievement".

I have now completed a discussion of some quantitative analyses which looked at the class as a whole, namely, relationships of some of the quantitative belief and attitudinal measures to students' achievement in the course. Section 6.2.2, which follows, explores how students react to the realistic investigative approach used in Mathematics 1, by examining their responses to the open questions. These responses were content-analysed and post-coded. The qualitative analysis is sometimes compared to relevant quantitative results. The analysis of data also focuses on demographic groups which have the largest achievement differences in the course.

### **6.2.2 Responses related to special features of the approach in Mathematics 1**

The analysis of the students' responses to the open questions have allowed me to focus on their experiences in the course and how the teachers' goals have, or have not, been realised. This analysis is interspersed with relevant comments from Bill and Maxine (who developed and taught Mathematics 1). Special features of the course which are explored in this section include the use of realistic contexts and non-routine problems (Section 6.2.2.1), the use of two-stage tests (Section 6.2.2.2), aiming to improve the mathematical confidence of students (Section 6.2.2.3), and the use of collaborative groups on tutorials (Section 6.2.2.4).

In Section 6.2.2.1, I will first determine the major themes which emerge from students' responses to all the open questions, i.e. overall reactions of students to the use of realistic contexts and non-routine problems in Mathematics 1. Then I will focus on responses to three particularly relevant questions. The main themes of all responses to the question *Has this course affected your ability to investigate mathematical situations?* are listed. Responses to two other questions

are analysed by the demographic groups in which achievement varied most. Responses to the question *What do you think about the maths you have done in this course?* are analysed by fluency in English (Section 6.2.2.1.1) and by gender (Section 6.2.2.1.2). Responses to the question *What did you think of the tasks which were given out in tutorials?* was analysed by age (Section 6.2.2.1.3). Relevant quantitative results are compared with the qualitative analysis throughout this section.

### **6.2.2.1 The use of realistic contexts and non-routine tasks**

Examining the students' responses to all of the open questions in October 1995 and October 1996, we find that 50% and 80% of the students, respectively, mentioned the contextualised nature of the course, a noticeable theme running through the responses of the whole class. Students said the course "introduced maths in a very practical and living way", there were "interesting topics relating to real life", "I liked best that I can see relationships between mathematics and actual situations", and

The best thing that I like about this course is we can relate all the mathematics functions that we learned to the real world, or real life.

I like Mathematics 1 because it is useful to every day life. It is with us all the time, whether one is aware of this or not.

I liked Mathematics 1, as rather than give us mathematical equations and tell us how to do them, it gave us some idea as to how the problem evolved and therefore its applications.

I feel more confident about how mathematics play an important role in every day living.

It has been applied to many "real" "useable" situations rather than just equations on a course.

It is interesting to note that, in 1995, students who liked the contextualised nature of the course all belonged to at least one of the categories of demographic variables that achieved at a higher level in this course, that is, they were either fluent in English, older, and/or female.

Responses to the question *Has this course affected your ability to investigate mathematical situations? If so, in what way?* were mainly positive and indicated that, for some students, the course promoted lateral thinking and creative thinking. Students commented that they "developed a more critical approach", "now look for different approaches and to think laterally", or that Mathematics 1 "assisted me in problem solving, e.g., brainstorming and lateral thinking", or "makes you think about the situation fully". Additional responses to this

question show how other students have reacted to the investigative approach, for example, "it's given me confidence to tackle some situations", also

Yes, the idea of not to accept every mathematical problem on face value, to investigate a problem & maybe even come up with new answers.

The course has a good approach; it gives guidance to know how to solve something, how to tackle a problem - that's beneficial!

Maths 1 has helped me to answer a written question and to turn it into a mathematical form answer.

Yes, it makes you think about the situation fully.

Yes. Because the problems and tasks we have to deal with are applied to real life situations rather than simply a whole bunch of numbers, it makes maths a whole lot easier to understand and therefore easier to investigate.

Yes, material is presented in such a way as you become more confident at doing more complex investigations.

A reasonable proportion of students have been able to transfer the mathematical understanding gained in this course to real world situations, starting to "look at the world with mathematical eyes" (Maxine):

Yes, I might be more likely to look for a mathematical model or formula and apply it.

Yes, it has created a lot of ideas for me to investigate other mathematical situations.

Yes, I can now see how to approach these situations and how to work it out logically.

I tend to think more mathematical way in every day life.

A few students did not give positive responses to *Has this course affected your ability to investigate mathematical situations?*, for example "I'm not sure", "no, not at all" and "no".

Clearly, proportions of students' answers belonging to different response categories vary by demographic variable so it was important to compare the responses of students in the higher and lower achieving demographic groups. Proportions varied, for example, by fluency and by gender in answer to the question, *What do you think about the maths you have done in this course?* Table 6.7 lists the percentage of students' answers within each of the response categories for three demographic groups.

Table 6.7: Proportions of students answering in response categories for the question *What do you think about the maths you have done in this course?* for fluent males and females and non-fluent females in Mathematics 1\*.

		Very good. Learned more. Thinking more mathematically	Relevant. Interesting. Practical.	Negative response	No answer
Female	Fluent in English	31%	46%	15%	8%
Female	Not Fluent in English	50%	8%	21%	21%
Male	Fluent in English	60%	26%	7%	7%

\* The data for non-fluent males is not included in this table because there were only two students in this category.

Comparison of the proportions of responses for fluent females with non-fluent females, or for fluent females and fluent males, allowed us to examine the effect of fluency or gender, respectively. The analysis of proportions of responses to this question is also compared to quantitative data about students' beliefs about the learning of mathematics in the next section.

#### **6.2.2.1 Analysis by fluency of the question “What do you think about the maths you have done in this course”: compared to students’ beliefs about the learning of mathematics**

Comparison of the first and second rows of Table 6.7 indicates almost half the fluent females mentioned the relevant and interesting nature of the course, while less than a tenth of non-fluent females commented on this aspect. Half of the non-fluent females, compared to 31% of fluent females, felt they were learning more or that the course was very good, the largest response category for the non-fluent group. Nearly half (42%) of the non-fluent females either did not respond or gave negative responses compared to 23% of fluent females.

A couple of the negative responses from the non-fluent group were:

(Mathematics 1) is too hard in some way. It's too many writing words. Do some more real maths.

They teach the easy things but when I do it feel very hard. Use the easy way to teach, is better.

I now consider the quantitative data on students' beliefs about the learning of mathematics, for non-fluent and fluent female groups, and this analysis of proportions of responses of these groups to the question above.

Students' beliefs about the learning of mathematics were examined by analysing their responses to five statements (Schoenfeld, 1989) listed in Appendix N. Non-fluent and fluent female groups entered the course, in March 1995, with conceptual beliefs about the learning of mathematics, although the non-fluent group are undecided in some responses. By October, there are clear differences between the response patterns of the higher and lower achieving groups by fluency. The fluent group have stayed strongly conceptual but the non-fluent group have changed their views by October and are focusing on algorithmic rather than conceptual ways of learning mathematics, although a good proportion are undecided in their views, indicating some uncertainty. This uncertainty could possibly be due to difficulty understanding the statements in the questionnaire or it could be due to the use of contextual non-routine problems in Mathematics 1. Diagrams illustrating the response pattern for female students who are fluent in English and for females who have moderate or little fluency in English for the five belief statements, and some commentary, are in Appendix U (Figure 1 to Figure 5).

Bill commented on his awareness of how students who were not very fluent in English were coping with the course.

There was the problem with the second language all along that was the biggest problem ... Because the assessment had a lot of writing in it second language students were penalised from the beginning. We actually tried to select them out. They were clearly more suited to Maths 2. So when we identified people in trouble we'd suggest they change. We missed people anyway but then there were a few Asian students who really absolutely got into it. One of our classic cases was a Japanese student who has now gone on to major in maths and is pulling A's. There was no way she could have gone into Maths 2 at that stage. She really needed that initial step, there's no question. She's talked to me about it since. So you know there was the isolated case of the fact she was second language but was ok. Yes, and she got huge support from tutors. She was an older student too so mature in her attitudes and she really benefited - well that's her reporting of it. I have a very clear sense that she would not have succeeded in Maths 2 - she might have succeeded because of her support but she wasn't ready for it in terms of background. I think the second language thing is still a problem but the other thing I would say about that is what it's done is make second language a problem in mathematics the same as it's a problem in geography or English. You know it's not any worse it's just actually I would see it as not facing up to the reality that in mathematics you need to communicate. If you're not in the language of instruction that actually it should be harder. If the mathematics course is such that your ability in English isn't a problem then I think you're missing some of the maths. That's how I'd say it and I think I'd be in argument with people here but ...

Comparison of the proportions of responses for fluent females and fluent males now allow us to examine the effect of gender.

#### **6.2.2.1.2 Analysis by gender of the question “What do you think about the maths you have done in this course”: compared to students’ beliefs about the learning of mathematics**

Comparison of the first and third rows of Table 6.7 shows almost half of the fluent females (46%) mention the relevant, interesting and practical nature of Mathematics 1, however, only 26% of fluent males comment on this aspect. The largest response category for fluent males (60%) was that the course was very good or that they had learned more, while 31% of fluent females responded in this category. Negative responses were given by 15% of the fluent females and by 7% of fluent males.

Another examination of students' beliefs about the learning of mathematics, by analysing their responses to five statements (Schoenfeld, 1989) (listed in Appendix N), show little difference between the higher and lower achieving groups by gender in March (1995). By October, however, fluent females indicate a more relational or conceptual approach to the learning of mathematics and fewer of this higher achieving group are undecided in their views, compared to the slightly lower achieving fluent males. Diagrams illustrating the response pattern for fluent female students compared to fluent male students for the five belief statements, and some commentary, are in Appendix U (Figure 11 to Figure 15).

I will now examine the students' reactions to another special feature of this course, the open-ended (or non-routine) assignments (or tasks). To obtain an idea of how different demographic groups have reacted to the non-routine tasks assigned in the course another open question is analysed, a question about the tasks (assignments). Categories of responses to the question *What did you think of the tasks which were given out in tutorials?* indicates that a significant proportion of students found the investigations difficult. In October in both 1995 and 1996 about 40% of all students expressed some reservations about the open-ended tasks. Several reasons were given. Reasons given by 25% of the students were:

- i) they were hard (some younger non-fluent and some older fluent students),
- ii) they found the tasks took too long to do (reasons given by some younger and older fluent students), or
- iii) the marks allocated to them did not reflect the time taken to complete them (some older fluent students).

### **6.2.2.1.3 Analysis by age of the question “What did you think of the tasks which were given out in tutorials”: compared to students’ beliefs about the learning of mathematics**

Achievement differences by age are only noticeable among students who were not fluent in English, where younger students did not achieve as well as older students. Comparing responses of non-fluent students by age (in 1996) to the question *What did you think of the tasks which were given out in tutorials* we find that one-third of the older non-fluent group comment on a lack of clarity of wording in some of the tasks, and two-thirds of this group was positive. On the other hand, for younger non-fluent students, about two-thirds thought the tasks were hard but one-third were positive about them. Representative comments from this younger non-fluent group in 1996 are:

They were good. We could revise the topics then we studied more because of tasks.

I thought they were really good because it had real life relevant with mathematics.

It is quite useful materials for learning.

The tasks were quite interesting, but some of it was quite hard to achieve.

Sometimes is easy but sometimes is really hard.

Some representative comments from the higher achieving older non-fluent students were:

That was good.

I think of the tasks which were given out in tutorials were a good practice and interest.

Some of the question is not so specific. It is better to express what answer is needed.

I had difficulty in some, especially when I was doubt of how it was supposed to be calculated or worked out. Otherwise they were OK. It tests how you understood the lecture.

So it seems that the older non-fluent students were more able to cope with the demands of the language used in the tasks than the younger non-fluent group.

Students' beliefs about the learning of mathematics were also examined by analysing their responses to five statements (Schoenfeld, 1989) listed in Appendix N. In comparison to the analysis by fluency or gender, there are few patterns discernable to differentiate beliefs about the learning of mathematics by age when these five statements are examined. This may largely be due to the smaller size of the groups. Diagrams illustrating the response pattern for non-fluent students who are older ( $\geq 25$  years old) with those who are younger ( $\leq 24$  years old).

years old), for the five belief statements, and some commentary, are in Appendix U (Figure 6 to Figure 10).

Bill had the feeling that “the older students did better” in the course. A good example of this is a quote from an older fluent male student, asked to comment on the tasks in an interview (see Section 6.3.1), said the tasks were

all fairly open-ended.... You're given the initial exercise but then you've got to frame something around it, and more or less design a maths problem for it. You're given the initial direction and you sort of take it from there really. So it does open it up a bit. And I suppose by writing, then you're starting communicating it, aren't you. I feel that by developing those sort of skills you could just about go into geography and geology, and these other places where we .... have written lab work to do, (and) start doing those a lot easier I think. Because you start moseying and putting things in the proper perspective, on track. I've developed that through Maths 1, I think. It's an area probably people miss, that side, as opposed to just manipulating figures.

Maxine comments on how she adapted to the demands of assigning open-ended problems to students.

The only thing I realised was that, when we gave students open-ended problems to work on, they actually needed some guidance. So I developed a marking scheme - it was an open marking scheme - there would be so many marks for restating the problem, for example - I can't remember the details - so much for the task or for where they'd drawn a diagram that was peculiar to that problem. It gave them a little bit of guidance as to how to get through the problem. If you look at an open-ended problem, ninety percent of them will do it a particular way, but you've got to allow there's the other ten percent who will do it a completely new way and that was encouraged. This wasn't the only way this was just a marking scheme that may help them. They were encouraged to think of another way to do it. Then I'd just adapt the marking scheme to the way they'd done it. Because students had never done it before, they weren't sure what was expected of them. It seemed that they needed that guidance and that the marking schedule actually helped them through that process, so I think that was necessary. Actually the markers were having problems as to how to mark it and so we'd realised both the marker and the students actually needed some guidance.

Well I think you learn as you go - the first assignment I set I gave them an option it was too much for them - so you sort of learnt on the go.

Bill thought that

the tasks had worked pretty well right from the beginning. We got better at judging what was a good task, yes, we just got better at that. My experience with Maths 2 tells me as well that, if you've got a task that's moderately ok, you'll get good feedback. Because the thing about collaborative tutorials is the interaction between the students, because it's open, and because the dynamic depends on the student, anything that'll get them talking will probably be productive.

The answers to another question *What have you liked least about this course?* also indicates that some of the students found tasks difficult. About 25% of the whole class in October 1995 mentioned task difficulty in answer to this question, whereas only about 10% of the 1996 students responded this way. It

seems that the teachers were getting better at judging what would work well as a task.

In contrast, right from the first year the course ran, students were overwhelmingly positive about another innovative aspect of Mathematics 1, the use of two-stage tests during the year (Section 6.2.2.2).

### 6.2.2.2 The use of two-stage tests

Two-stage tests were held twice during the year, containing essay questions, short investigations and skills questions. This form of assessment was introduced as part of "realistic mathematics education" in the Netherlands (de Lange, 1993), where examination questions were developed which were more open to multiple strategies and results. The aim was to assess higher-order thinking skills, to give students freedom to solve problems at their own ability level, and to test processes rather than product. These tests aimed to improve learning, and enabled students to show what they knew rather than what they didn't know. The tests, components of coursework assessment, were done in two parts. The first part was an open book test, sat in a two hour scheduled period under little time pressure. After these answers were marked, the opportunity was given for students to rework and resubmit the tests. Each student was interviewed briefly when they re-submitted the test to check their understanding. The re-submitted test was then re-marked and the two scores were averaged.

Maxine discussed the two-stage test in her interview.

We used De Lange's method of assessment where the students were given their test and that was given an initial marking and then they had to go away and complete it. For the first test, it was recognised that they would not be able to do it in a test situation. It had to be set so that there were higher level thinking skills involved and that they weren't going to get one hundred percent - it was impossible. I thought it was very successful because they found out what they couldn't do, then there was a two week gap where there was a frenetic activity as they found out how to do it. I remember them coming into your office and really learning where their gaps were and I thought that was a very good learning experience. It wasn't enough to get fifty percent in the test and say 'I've passed', they actually had to get a hundred percent. They had to know the work really well. So I thought that this test was the strength of the course. Then when we interviewed them about what they'd done I thought that was really good because they seemed to really enjoy telling me what they'd done and how they'd done the problem. Part of the procedure was to check that it was their own work. Again I came up against misconceptions. I remember one student who didn't think it was possible to have a negative percentage. Through that discussion I learnt more and they learnt more. I felt they really enjoyed talking to someone about how much they knew or what they knew. You'd give them a curly question and they'd just say 'well you've got me there' and it was very open and honest. It was though we really cared about their learning and were trying to get them to learn more. It was a partnership process.

Bill also comments on the use of two-stage tests.

The assessment we'd chosen, that double marking, was time consuming but we didn't change it because we just felt it was so valuable - it was such a good idea.

He comments on the on-going development of Mathematics 1.

I think most of our focus (in 1995) was on getting the tests and exams right. I think we were too open about it (in 1994). We didn't direct the students enough - we didn't give them enough models. So it was really trying to be fairer to the students on the assessment issues as to what we were expecting - so a little more direction in the test and exam questions.

If we examine students' reactions to the two-stage tests, we find that they largely agree with Bill and Maxine's responses. About 90% of the students in both years gave positive responses to the question *What do you think about being able to redo term tests?* Reasons given in their responses included the opportunity to "really learn" from their mistakes, or that it was reassuring to have this chance to improve your marks. At the affective level, students mentioned that it took the pressure off the exam either because they had a fear of exams, or because they were speakers of other languages.

Most students seem to appreciate the value of testing in two stages in order to learn during the testing process.

Yes, it was good not only to gain extra marks but also to get students to redo, and rethink problems rather than discarding them.

Yes. It not only lets one a chance to earn more marks but really learn from the test.

Excellent! Test are normally completed and forgotten. A chance to revisit and complete them (fix mistakes) very reassuring.

Several students mentioned that the two-stage test addressed some of the negative affective aspects of tests.

Yes it is good for people who are always nervous in tests and examinations.

Very good. I did very bad in the exam due to the 'fear' of exams and time limitation. I will have a chance to improve my marks again.

Excellent. I learnt so much doing them again, and without the pressure which I didn't cope with well.

Another question asked *What comment do you have about the style of the term test questions?* Although a good proportion of the students were positive about the style of the questions, about 20% in 1995 and 1996 had reservations about having essay questions in the test. Some more negative comments were:

I thought the essay-type questions were a bit too general but then again it gave me more ways of doing or answering the question.

Fine. I don't like it when the mathematical information is all jumbled up in the stem question. It is better for me when it is listed.

Too many essays and not enough straight mathematics.

It seems that, overall, the use of two-stage tests was received very positively. An aim of the course, to increase the mathematical confidence of students, is discussed next (Section 6.2.2.3) by analysing responses to one relevant question, *Has this course affected your mathematical confidence?* by fluency in English (Section 6.2.2.3.1) and gender (Section 6.2.2.3.2), demographic groups in which achievement differences were greatest.

### 6.2.2.3 Aiming to improve the mathematical confidence of students

Students' responses to the question *Has this course affected your mathematical confidence?* indicate that this aim has been achieved for many students, although there is a mixed reaction from students in lower achieving groups. Of the students who responded at the end of the year, in 1995 over 60% of 44 respondents, and in 1996 over 80% of 36 respondents, indicated that their confidence had increased. I now investigate whether these responses differ by fluency or gender. Table 6.8 below lists, for 1995, the percentage of fluent females and males and non-fluent females that indicate a definite increase, some increase, or no increase in mathematical confidence.

Comparing fluent and non-fluent females allows consideration of the effect of levels of fluency in English, and comparing fluent females and males allows us to look at differences by gender.

Table 6.8: Response categories to the open question *Has this course affected your mathematical confidence?* for fluent female and male students and for females who had limited fluency in English\*.

		Definite increase in confidence.	Some increase in confidence	Already confident or too easy.	No increase in confidence	No answer
Female	Fluent	62%	0%	23%	15%	0%
	Not Fluent	21%	14%	14%	14%	36%
Male	Fluent	47%	20%	13%	7%	13%

\* The data for non-fluent males is not included in this table because there were only two students in this category.

### **6.2.2.3.1 Analysis by fluency of the question Has this course affected your mathematical confidence?**

Comparison of the first and second rows of Table 6.8 indicates one third of the females (35%) who are not fluent in English indicate increased mathematical confidence, compared to about two thirds of females (62%) who are fluent in English. A high proportion (62%) of females who are fluent in English indicate a definite increase in mathematical confidence, compared to 21% of females not fluent in English. Over one third of the non-fluent females (36%) did not respond to this question, compared to none of the fluent female group.

Comments from females who are fluent in English:

Dramatically. I am going to be a teacher and I took this paper specifically to gain more confidence. I'm not so scared now about finding ideas for teaching.

Absolutely - it has greatly improved my attitude towards maths - i.e. I am not so afraid of maths - I could face another paper.

Absolutely - I know I do have the ability to understand maths.

Yes it has shown me I am capable of doing maths problems.

No I don't think so. I was fairly confident before.

I found I prefer manual calculations instead of in-depth understanding.

Comments from females who are not fluent in English:

Maybe I could look at things in different points of view.

It makes me realise how important to learn maths and it is a must for everyone to know.

I don't think so. I quite enjoy it even though it's been a long time ago I learned these.

Not really, it's kind of affected my grammar while doing the task.

### **6.2.2.3.2 Analysis by gender of the question Has this course affected your mathematical confidence?: compared to students' beliefs about themselves as learners of mathematics**

Comparison of the first and second rows of Table 6.8 indicates a high proportion of fluent females (62%) indicate a definite increase in mathematical confidence compared to the fluent males (47%), although 20% more of the fluent male group indicated some increase in confidence. A larger proportion of fluent females (15%), compared to fluent males (7%), indicated no increase in confidence. Some of the fluent males (13%) did not respond to this question, compared to none in the fluent female group.

Comments below from males who are fluent in English can be compared to those from fluent females above.

Yes, I feel more confident about how mathematics plays an important role in every day living.

Yes I have a much better idea of how maths is useful as a tool type thing. Given me a better idea of what kind of maths to apply to a certain situation.

It has probably made me less confident, but it has shown me that others prefer different learning styles which is important for me as a teacher.

Not really. It was probably too easy for me. The maths was very interesting and probably quite useful in the 'working' world.

In addition to this data from students' responses to an open question, students' beliefs about themselves as learners of mathematics were measured with a subset of Gourgey's (1982) Mathematical Self-Concept Scale both in March and October. This data provides an interesting comparison with the above analysis of responses to the question *Has this course affected your mathematical confidence?* Median scores for the scale are listed in Appendix T for comparison of groups in categories of demographic variables where achievement differences are greatest, that is, by levels of fluency in English (particularly for the younger age group or for female students), by age groups (for non-fluent students) and by gender (for fluent students)<sup>29</sup>.

The higher achieving groups by fluency and age enter the Mathematics 1 course with a more positive mathematical self-concept than the lower achieving groups. However the opposite pattern occurs for the comparison by gender where, as was the case for mathematical attitudes, fluent males enter the course with a more positive mathematical self-concept than fluent females, although fluent females are the higher achieving group. This result is "consistent with all current literature, (that) males rate themselves more highly than females on this construct" (Relich et al., 1994 p. 16). By October, for higher achieving groups (fluent, older or female students) there is either a gain, or no change, in the medians of their Mathematical Self-Concept Scale scores. It is noted that for fluent females there is a reasonably high increase in Mathematical Self-Concept Scale scores (11%), the highest of any group analysed. In contrast, all comparisons indicate that the lower achieving groups (non-fluent, younger, male) have decreased their scores in this scale from March to October, so they

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<sup>29</sup> This analysis is completed using data from the group of students who have completed both the March and the October questionnaires in order to obtain a reliable measure of any change in scale scores from March to October. Therefore there is a slight reduction in the size of the groups analysed: in 1995 there are 37 students in this group, compared to 42 students who answered the October questionnaire. Because of the small number of students in 1996 who answered both questionnaires (n=22), 1996 data is not included in comparisons of scores.

have finished the course feeling less self-confidence as learners of mathematics than when they began the year.

Another particular emphasis in Mathematics 1 was actively encouraging students to interact mathematically in groups in tutorials.

#### **6.2.2.4 Working in groups in tutorials**

Maxine was positive about how the tutorials worked for most students.

I think the tutorials really went well. They got into their own small groups and then they had problems which they had to solve as a group. You could really interact with the students and challenge them. You could get to know the students more and start to look at their thinking, start to see where there were problems and then bring those out into the open. All of them were getting something out of it. They seemed to work well together. There were of course some shy ones and some not so good but I felt that they all took part and, because it was a small group, they got to know one another and respect one another. I found that with some people who felt they were maths failures actually weren't. They were really good at this type of thinking. They were good problem solvers and they were delighted. You know I remember those tutorials as enjoyable, that there was lots of interaction. The students always came and seemed to enjoy them even though it was five o'clock at night.

When asked if there were students or groups which did not benefit from the course or who found it difficult, Maxine said that she couldn't

remember any in my tutorial but there were probably some who never could get away from the fact that they needed a formula so that they could never open their mind that this was a different way of doing maths and that they needed to get involved in it. There are ones who really want to learn and get involved and like to participate and then there's ones who are cynical and sit on the sideline. It's just human nature and we just had the whole gamut there.

One open question addressed the issue of working in groups, *What are your reactions to working with a group on the task in the tutorials?* Reactions to this question varied by age (for non-fluent students) (Section 6.2.2.4.1).

##### **6.2.2.4.1 Analysis by age of the question What are your reactions to working with a group on the task in the tutorials?**

Almost two-thirds of the younger (non-fluent) students (in October 1995) gave unqualified positive responses to this question, compared to about one-third of the older students. The other two-thirds of the older (non-fluent) students gave positive comments which were qualified in some way, as did 20% of the younger group, suggesting that the groups were not always effective. Comments made about working with a group in tutorials include the following:

Older non-fluent students:

It's good to work with others, sharing ideas.

(Working in a group in tutorials was) very excellent. Without this I don't know if I can cope because that's where most of help comes from. I have to make sure I understand.

I love (working in a group in tutorials) only if people are willing to contribute or share.

Sometimes I said my opinion (in the tutorial groups), but not always.

Younger non-fluent students:

(In tutorials) it's good, because we can get more ideas when working with a group.

I think it's good but I myself am not good at this sort of way of working, so it was pretty hard to me.

Section 6.2.3 contains a richer, more detailed, description of a few students in each of the demographic groups which showed the largest achievement differences.

### **6.2.3 Reactions from Selected Students in Key Demographic Groups**

The following four sub-sections allow some comparison of responses to the open questions, as well as some interview data and scale scores<sup>30</sup>, by fluency, by age and by gender. Students were chosen from a range of levels of achievement in the course. Non-fluent students were chosen from a range of levels of fluency as well as a range of achievement levels.

Reasons why levels of fluency have affected achievement can be explored by comparing the reactions of younger fluent students in Section 6.2.3.1 (Rachel, Samantha, Louisa and Sarah) with reactions of younger non-fluent students in Section 6.2.3.2 (Akiko, Fumico, Zamani and Sak Kuen).

Reasons why age has affected achievement can be examined by comparing reactions of this latter group, younger non-fluent students, with reactions of older non-fluent students in Section 6.2.3.3 (Kwan, Lee and Hii).

The effect of gender on levels of achievement can be explored by comparing responses of fluent female students in Section 6.2.3.1 with those of fluent males in Section 6.2.3.4 (Warren) and Section 6.3.1 (John's interview).

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<sup>30</sup> The Mathematics Self-Concept scores are listed when the student has completed both the March and October questionnaires.

Section 6.2.3.1, which follows, lists some responses by younger fluent female students.

### **6.2.3.1 Responses from younger female students who are fluent in English**

Rachel and Samantha, the first two young fluent female students discussed in this section, were positive about Mathematics 1 and both passed. Rachel's final mark was 88% and Samantha scored 64%. The third student, Louisa, achieved 72% but was quite negative about the course. Sarah, the final student, failed the course with final marks 45% and she had mixed reactions to the course. Comments of these younger fluent students can be compared to those of younger non-fluent students in Section 6.2.3.2, which follows.

Rachel was a fluent student, in the 20-24 age group, female, who had a Form 6 mathematics background. Her final mark in the course was 88%. Her Mathematics Self-Concept scores were 23/40 in March and similar, 24/40, in October. She indicated that her mathematical confidence had not been changed by her experiences in Mathematics 1 as she "was fairly confident before".

(Maths 1) has been very practical, interesting and relevant in many ways. The course was clearly set out and the design of the course was such that one could learn, through interaction, examples and experiments rather than solely from the student's hard work and study away from class.

She sometimes found the tutorials a "bit boring" and that some tasks "were too repetitive from lectures". Her reaction to working in a group in the tutorials was "sometimes this could be difficult due to establishing one group as members come and go". Regarding the two-stage tests, she said, "This is one of the best things about the course. This is excellent learning and provides a good basis for it. It is good especially for those who make stupid mistakes under pressure." She concluded: "The course has a good approach. It gives guidance to know how to solve something, how to tackle a problem - that's beneficial! I think this course has helped me to look for different approaches and to think laterally."

Samantha was a fluent student, in the 20-24 age group, female, who had a Form 6 mathematics background. Her final mark in the course was 64%. Her Mathematics Self-Concept scores were 25/40 in March and decreased to 22/40 in October.

I liked Maths 1 because for the first time in years it actually felt like I was learning maths not just memorising it. I liked its relationship to the real world, interest points and layout. It has been very practical and interesting.

In answer to the question about whether the course had affected her mathematical confidence she said

Dramatically. I am going to be a teacher and I took this course specifically to gain more confidence. I'm not so scared now about finding ideas for teaching. One of my biggest problems in maths was discovering where to begin on a problem, now I can do this a bit easier.

She thought there were too many assignments as "they took a lot of time to do", however she thought "they were necessary but the work load was quite heavy". She found the group work "good" and continued: "It relies on everyone's strengths, weaknesses and contributions for you to learn. You also get different points of view on how to do things." The two stage test was "fantastic - it took the worst thing out of a test, the stress. You could relax and not panic." Altogether she found the approach "refreshing; it gets away from all the university hoop-la and lets you learn in a relaxed and friendly environment".

Louisa was a fluent student in the 20-24 year age group, female, who had previously studied mathematics to age 16 (Form 6). Her final mark in the course were 72%. She said "I liked the course because the work was easy but I disliked the way the course was presented. (I liked best) the support from other students." In answer to the question about whether the course had affected her mathematical confidence she said "I found out I prefer to do manual calculations instead of in-depth understanding". She found the tasks "time-consuming" and although "some tasks were good and interesting, others were manual and boring". Working in groups was "okay sometimes - it depends on the sort of mood I'm in". She thought that the two stage test was a "good idea but should it be averaged?" To the question regarding the maths that was done in the course, she said: "I could understand the maths if it wasn't covered in too much theory".

Sarah was a fluent student, in the 20-24 years age group, female, whose mathematics background was Form 4 (age 14). Her final mark in the course was 45%. Her Mathematics Self-Concept scores were 14/40 in March and 15/40 in October, comparatively low. She "liked the course" as it "moved at a good pace". The course "slightly increased my confidence, but I still freak out with maths. I sat the terms test and could hardly answer anything." Some of the mathematics covered she has "found very useful", for example, "when studying environment in relation to my degree in biology". She said that some of the tasks "seemed pretty pointless" to her but she "enjoyed working in a group situation". She found the two-stage test was "an excellent idea - I totally lose it in a maths exam situation".

Comments from young non-fluent students are included in Section 6.2.3.2, which follows, from students who have achieved well in the course and some who have not done as well. Their reactions can be compared to those above

(Section 6.2.3.1) in order to gain some insight into the situation of comparative levels of achievement by levels of fluency.

### **6.2.3.2 Comments from younger female students with moderate or little fluency in English**

Students were selected to illustrate both moderate and little fluency in English and a range of levels of achievement (43% to 88%). The first two students, Akiko and Fumico, had moderate fluency in English and both did well. The second pair of students, Zamani and Suk Kuen, had little fluency and both did not pass.

Akiko was a female student with moderate fluency in English. She had studied mathematics to Form 6 level (age 16) and was in the 20-24 years age group. Her final mark in Mathematics 1 was 82%. Her Mathematics Self-Concept scores were 10/40 in March and unchanged in October, comparatively low. When asked what she liked best about Mathematics 1, she said she could "see relationships between mathematics and actual situations". When asked what she disliked she said: "I did not dislike the course, I suppose because I did enjoy the classes quite often". Questioned about the tasks, she said that "some were hard but most of them are ok". Her reaction to working in a group was "I think it's good but I myself am not good at this kind of way of working, so it was pretty hard to me". Regarding the two stage test, she says: "It is really good. One good thing is that we can make sure that parts we were not sure, and another thing is that we have 2 chance which helped me a lot!" However the style of the questions was difficult: "essay style was pretty hard"; "explaining maths clearly was a hard task". In answer to the question about whether the course had affected her mathematical confidence she said: "The skills in this course was all familiar to me so I did not find something new about skills themselves." When questioned about the mathematics done in the course, she said: "it was interesting; all maths topics were related to the real world in some way, which was interesting to know".

Fumico is an 18-year-old female student with moderate fluency in English who had studied mathematics to age 16 (Form 6). Her final mark was 70%. She said that she had always "been bad at maths" but she liked best "the ways of approaching" in this course. What she liked least were the "tests and assignments" as they "require a lot of English". The tasks "needs a lot of time & consideration and I often had to go to the library to get some information". She says "for me it's good" to redo the terms test, "to compare with the other tests I have less pressure." However, regarding the term test questions she says "I know that this course is not normal math course but I'm not sure if we really need to write essays on math course." Her mathematical confidence "may be"

affected, as she "could look at things in different points of view". Regarding the mathematics done she says, "I got mathematical idea" and now she "tends to think more mathematical way in every day life". The course approach was "quite interesting". She "really enjoyed the lectures".

Zamani was a female student, with little fluency in English, in the 20-24 age group, who studied mathematics to age 15. Her final mark in Mathematics 1 was 43%. Her Mathematics Self-Concept scores were 24/40 in March and decreased to 21/40 in October. She liked the course "because it was interesting, but it wasn't the thing I expect before". She said that "it was very good to work in group - I like it". She liked the two stage test and wrote "First is good and you got chance to get good and better mark than first, then you can learned more when you review that". Her comment about the mathematics done in the course was "I thought is going to be calculates but it wasn't, it's more about thinking of mathematical, but is good".

Suk Kuen is a female student, in the 20-24 age group, with little fluency in English. She studied mathematics to age 17 (Form 7). Her final mark was 44%. Her Mathematics Self-Concept scores were 23/40 in March and decreased to 19/40 in October. Her first comment was "I like this course and easy to understand maths." She continues "the lecture is good" but "the assignment is too difficult". "It can improve to make sure the answer" is her reaction to the chance to redo the tests. When asked about the approach, she says, "some of the topics is hard to understand". She did not answer many of the open questions.

Responses in the discussion above (Section 6.2.3.2) can be compared to those that follow (in Section 6.2.3.3), from older students who are not fluent. Older non-fluent students achieved better in the course than non-fluent younger students.

### **6.2.3.3 Comments from older students with moderate or little fluency in English**

One of the largest achievement differences by age was for students who were non-fluent in English. This sub-section lists comments by a sample of students who are older but have moderate or little fluency in English. These students are selected to illustrate a range of levels of fluency and a range of achievement levels.

P.W. Kwan was an older male student who had limited fluency in English whom I interviewed. His final mark was 93%. He had worked for the same

company in Hong Kong for 22 years but came to NZ six years ago. He doesn't have to work now and can "come to school".

You know one thing I regret very much is that I don't have very good education before. ... I begin to work at age 12, life is very hard at that time - my parents can't provide my pocket money because life is so hard.

He liked the course and thought it was "interesting" and "easy to understand". I asked him to think of his experience in this mathematics course and, if it were weather, what would it be like. He replied:

I think it is a fine day, the sun is shining high, I feel really delighted, and the birds are singing, the flowers are blooming and all the grass is so green and I step on it and I just feel very delighted in walking over there, just jumping ... things like that. That's how I feel.

He didn't like "the homework question which is too open" and thought that "formula should be explained more detailly". His comments about the tasks were: "some of the question is not so specific. It is better to express what answer is needed." He thought the groups were "excellent" and that "everyone participate well in the group". Asked if he liked the chance to re-do the terms test he said; "yes, it helps to understand the question in the test, but explanation for each question should be given". He thought that the style of the test questions was "acceptable". He thought the course had affected his mathematical confidence and said "it increase my knowledge and confidence in mathematics!" The mathematics covered was "good, very practical" and the course "had created a lot of idea for me to investigate other mathematics situations" but that "more example should be given". When asked to think of a kitchen utensil that reminded him of Mathematics 1 he replied the "main thing", the "thing you use for stir fry", it's a "thing you must use" because "no matter what question you solve in the future, you must use this". As a tool, Mathematics 1 reminded him of a "plough" which will "break soil up" and that he will "use this maths to break ground in the future".

Su Jung Lee was an older female student with limited fluency in English. She scored 77% in the course. She liked Mathematics 1 "very much because it's made me think mathematically". The tasks were "good". Her reactions to working with a group were "sometimes it's no need but I think we should have talk with a group". When asked what she thought about being able to re-do term tests she said: "that's good idea; if you don't do that (re-do) I don't know that I write wrong way." The style of the test questions were "fine". She did not respond to several questions but did say she "liked" the mathematics done in the course and that the approach was "good".

Buo Chong Hii was an older male student who had limited fluency in English. His final mark in the course was 75%. He liked Mathematics 1 as it “let me recall back long time ago, what I have learned”. What he liked best was “all the new idea what I haven’t learned before”. He thought the course could be improved for him by “only explain more clearly”. The tasks “were a good practice and interest”, and when working in groups he “learned and got new idea, learned what (he) didn’t know”. Regarding the chance to re-do the terms tests he says “that let me know where are my mistake then learned it again”. The test questions were “standard term test questions”. He thinks that the course did affect his ability to investigate mathematical situations, but does not comment on how. He thinks that the course affected his mathematical confidence and “that slowly make more interest maths, better basics of maths”.

These examples give us some idea of how older students who had moderate or little fluency in English reacted to Mathematics 1. We can compare these responses to those of younger non-fluent students in the previous sub-section to gain some insight into the different achievement levels between the older and younger non-fluent students. To explore the effect of gender, responses in Section 6.2.3.1 from fluent female students can be compared to reactions of a fluent male student in Section 6.2.3.4 (which follows) and in Section 6.3.1 (John’s interview).

#### **6.2.3.4 Comments from a fluent male student**

Warren was a younger male student who was fluent in English. He achieved 64% in the course. He liked Mathematics 1 as “it explored the practical applications of maths” but “the essays that were required for the tests/exams” were what he did not like. “This is supposed to be maths not arts”, he wrote. He thought some of the tasks were “very practical” but “some are very time consuming”. Working in a group was “good” as it “makes it enjoyable and the opportunity to share ideas”. The term test questions included “too many essays and not enough straight maths” but he thought it was “a good idea” to be able to re-do the test as “it enables you to see where you have gone wrong”. He thinks that the maths was “useful” and that his mathematical confidence has “increased as understanding has grown”. He does not think that the course affected his ability to investigate mathematical situations – “no not at all”.

Reactions of another fluent male student are described in Section 6.3.1, in the first of the interviews in the following section.

## **6.3 Interviews analysed in detail**

Interviews with a number of students provided additional qualitative data. A presentation and discussion of parts of in-depth interviews with three students follows. It looks at some of their early mathematical experiences and also their reaction to aspects of the Mathematics 1 course. Gibson's (1994) protocol for using metaphors (the Mathematics Metaphor Questionnaire) was modified and used during these interviews to help students to describe mathematics and their learning experiences in mathematics. The students were usually asked to interpret their own metaphors, considered important by Gordon and Langmaid (1988) in their discussion of the use of projective techniques in qualitative market research. The first student is an older male student, and the other two interviews are with younger non-fluent female students.

### **6.3.1 “Digging up as opposed to raking over”: John’s story**

At the time of the interview John was 44 years old. He studied mathematics up to age 15, in a small rural school. “We didn’t get a good grounding in maths”, it was “rote-learning”, “not much application”. He had 20 years work experience in the electrical and electronics trades, learning some specialised mathematics for these jobs. He was in his fourth year as a part-time student at the University of Auckland, having studied biology, geography, chemistry and geology. He had seen the need to strengthen his background mathematics and so enrolled in Mathematics 1.

He was asked to imagine if Mathematics 1 was weather, what would it be like, and he replied “It would be a nice sunny day”. He said that he was “not so hesitant about getting into a maths problem, trying to sort through it” whereas before he would have “walked away”. Mathematics 1 “does help you build a lot of confidence in what you’re doing in maths” which he attributed to the “continued learning process” encouraged in Mathematics 1. “With (the labs in Maths 1), where you’ve got to go away and finish them and develop them more, you tend to remember. That’s one big advantage.”

When John was asked to think of a kitchen utensil that Mathematics 1 was like, he replied “I don’t know. Probably like an opener, can open up things”. This metaphor succinctly symbolises his comments, such as, “I can see it opening up a lot of areas”, “developing different angles”, the “sorts of skills” you need in “geography labs”, for example. He commented that the assignments are “open-ended” and you “more or less design a maths problem for it so it does open (the assignment) up quite a bit”.

If Mathematics 1 were a tool, he described it as “digging up as opposed to raking over”. This metaphor seems to refer to the depth of understanding he has achieved in Mathematics 1, as opposed to superficial knowledge. This is illustrated by his comparative comments that most people miss out on learning these skills and just learn to “manipulate figures” and that there is “more time to think about it, and also thinking from different angles as well, as opposed to a rote-learning type situation”. You learn to “put things in the proper perspective, and think about what’s happening. I’ve developed that through Maths 1.”

John passed Mathematics 1 with a B grade and completed his BSc three years later, majoring in geography.

The following two interviews, in Sections 6.3.2 and 6.3.3, are with younger non-fluent females. Typically this demographic group had difficulty with the investigative nature of Mathematics 1, however, these two students have had very different experiences in this course.

### **6.3.2 “Frypan - I use it often”: Yuko’s Story**

At the time of the interview Yuko was 21 years old and it was her first year of study (full-time) at the University of Auckland. She arrived in NZ from Japan two years prior, attending language schools in Auckland for about 18 months. Her parents lived in Japan so Yuko said “once I decided (the courses I wished to study) I told (my parents) I’m going to take these subjects and they said that’s ok.” She was hoping to major in languages. She studied mathematics right through high school.

I found it hard to understand and follow the meaning of much of what Yuko was saying during the interview. When I introduced the idea of using metaphors to describe her experiences, she became more animated and seemed to enjoy this method. It helped to clarify my understanding of experiences that she did not easily describe literally (Bowman, 1995) because of the level of her oral fluency in English. It became a powerful tool for her to express her meaning, particularly as she interpreted most of the metaphors she used.

To the question “What if high school maths were an animal?” Yuko replied that her experience learning maths in Japan was like a lion. “Lion is cute animal but” it always “try to catch animal”, kills them and eats some meat then kills another animal and “eats half of” that one. There is lots of “left-over meat”. She “studied lots of maths (and) forgot lots of maths”. It was “thrown away”. Her experience in Mathematics 1 reminded her of “the cow - eating all the time”. “Cows don’t

eat a lot because" they move food from "stomach to mouth" and eat it again. "I study mathematics and whenever I need some skills I remind of the cow and then chew again."

She was asked to think about the utensils she would use in a kitchen and decide what kind of utensil would be like mathematics in high school. She replied: "Dishes and tin opener - put away - forget where you put away. So many kinds of dishes and I don't know which ones to use, but I cook something. Tin opener? I don't use tin opener so I forgot where I put it and how to use it, but I know what it is." Yuko interpreted this metaphor as "It's useful but forgotten how to use it, like the mathematics. There are so many skills around. Which skill I can use?" Mathematics 1 was like "knife, fork and spoon, fry-pan, I use them often. I know where it is, I know how to use them. I use them so many times so I remember where I put it. I know the formula, I know when I have to use it." She liked the approach in Mathematics 1 better, "because we can use this mathematics. I know when I think about things in a mathematics way and I like this thinking in mathematical way. When I study this course, ... you have to use mathematics to solve these (assignment) problems - (they) are likely to happen. I want to study mathematics more because I feel it's useful now. I'd like to study mathematics next year as well."

Yuko achieved an A grade in Mathematics 1 and a B+ in Mathematics 2, in the second semester. After successfully completing all the courses she enrolled in for the year, she left the University of Auckland.

### **6.3.3 “A narrow and bumpy street”: Keiko’s story**

Keiko was 21 years old and moved to NZ five years earlier with her family. She completed all five years of high school, taking mathematics until age 15. At the time of the interview it was her first year at the University of Auckland and she was studying Mathematics 1, Korean and Chinese language and culture. She said "my parents ask me to do maths or science. ... If I have to choose between maths and language would choose language. This year I told my parents I can't do maths very well - I've forgotten everything." They replied that she should try, as "maths is quite important in our life". But she said "actually, I thought myself as well I should study maths - a little bit - thought (this course) easy one for you".

When I asked "If your early high school experience were a way to travel, what would that way be?" Keiko replied "a straight road ... nothing there, just straight, a nice wide road". Her interpretation of this metaphor was "because easy to understand and easy to solve the problems. If hard, then teacher or

friend always help you." If high school maths were a kitchen utensil, Keiko said it "must be a knife because useful, and everyday have to use ... easy to cut ... (to solve problem) although sometimes need to sharpen it, so need to think a bit first sometimes."

For her experience in Mathematics 1, the 'way to travel' would be a "very difficult way ... going to the wrong way and back again ... a narrow and bumpy street ... lots of hills". In Mathematics 1, the 'kitchen utensil' is "an electric element (not oven) ... electric one is so inconvenient ... if busy and want hot water, it's not like gas, takes a long time to heat!" These metaphors seemed to indicate that she found Mathematics 1 difficult, which is illustrated by her comparison "I think question not hard if I understand well but because in high school just question and answer, question and answer, so easy. But here you have to write why. Sometimes I just don't know why!" Mathematics 1 has been "Uncomfortable! Very!"

The most important thing she has learned about mathematics in Mathematics 1 is that the "questions always about daily life so I think it's quite useful but at moment I don't need to use it". Keiko achieved a B pass in Mathematics 1 and completed her BA in three years of full-time study, majoring in Chinese and Korean. She studied Mathematics 2 in the final year of her degree and achieved a marginal pass.

#### **6.3.4 Discussion of interviews**

The first interview in this section enabled me to look in more detail at the experience of a mature male student's experience in Mathematics 1, a positive experience which dovetailed well with geography, his main area of study.

The latter two interviews allowed me to focus on two non-fluent students, and hence find some of the most telling data in the study of this course. Both Yuko and Keiko belonged to the same demographic group, non-fluent younger students, which have not achieved as well as other groups in Mathematics 1. But a deeper look at both students through these interviews has made me realise that Mathematics 1 has been a very different experience for each of the two young Japanese women. I examine what may have made the difference.

Yuko seems to be less dependent on her parent's approval for the subjects she studied than Keiko, who still lived at home with her parents. Some of the quantitative measures for Yuko and Keiko reflect their stories above. Their scores on the Mathematical Self-Concept Scale, a measure of belief in oneself as a learner of mathematics, are different. Keiko's score decreased markedly from

March to October (18/40 to 8/40), indicating a loss of mathematical self-confidence, whereas Yuko's scores are 23 and 21 respectively. Yuko entered the course with more mathematical self-confidence than Keiko and her confidence level was unchanged in October. Keiko had less high school mathematics background than Yuko, which may have affected her ability to handle the content of Mathematics 1.

Keiko believed that she learned mathematics effectively by "memoris(ing) everything, all the formula and so on ... quite easy to" and this is reflected in her responses to statements in the questionnaire which were measuring beliefs about the learning of mathematics. Keiko agreed with the statements *The best way to do well in maths is to memorise all the formulas* and *To solve maths problems you have to be taught the right procedure, or you cannot do anything* in March and also in October, at the end of the course. By contrast, Yuko was undecided about the former belief statement in March and disagreed with it in October, and she disagreed with the latter statement in March and in October. They were both undecided about the statement *Everything important about maths is already known by mathematicians* in March but, by October, Yuko disagreed and Keiko agreed with this statement. Their responses to these three statements probably indicate that Yuko is a more confident relational learner of mathematics whereas Keiko is a less confident instrumental learner of mathematics. However, in two other statements, *Maths problems can be done correctly in only one way* and *In maths you can be creative and discover things by yourself*, they both responded in a manner which indicated a more relational approach (disagreeing and agreeing, respectively).

It is perhaps unfortunate that Keiko could not have been given the choice of studying the Wellesley mathematics course. (The Wellesley Program is not available for students in a degree program.) It may have suited her needs better than the approach in Mathematics 1.



## 7 Wellesley Program Mathematics

Chapter 7 contains a detailed analysis of data collected from students in the Wellesley mathematics course in 1995 and 1996<sup>31</sup>. Section 7.1 contains an analysis, within categories of the demographic variables, of the final marks of students who completed this course. This analysis will be used to determine the demographic groups where achievement varies most. Further analysis of both qualitative and quantitative data in this chapter focuses on a comparison of these groups. Section 7.2 explores how students react to the teaching approach in the Wellesley mathematics course, by analysing their responses to open questions, comparing these with some quantitative measures of beliefs. Interviews with two students provided additional qualitative data, which is analysed in Section 7.3.

### 7.1 Analysis of Achievement within Categories of Demographic Variable

The first comparisons of achievement, in Section 7.1.1, are for the demographic variables available on the class roll for the Wellesley mathematics course; options, streams, gender and ethnicity. In Section 7.1.2 demographic information gathered by the questionnaire and not available on the class roll is analysed.

#### 7.1.1 Demographics on the Class Roll: Options, Streams, Gender and Ethnicity

Table 7.1 contains the medians of the final marks in the Wellesley mathematics course, within categories of those demographic variables, which were available on the class roll. The class roll used contains only those students who completed the year's course and who sat the final examination.

The largest differences in achievement, for these four variables, are between the two option groups and the four stream groups. Medians of final marks differ between students in the Arts option and those in Science by 49% in 1995 and

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<sup>31</sup> Demographic characteristics for the Wellesley mathematics course in 1995 and 1996 are discussed in Chapter 5, where proportions of students in categories of each demographic variable are listed.

22% in 1996 ( $p<0.001^{32}$ ). For the streams, medians differ by 67% in 1995, and 38% in 1996, between students in stream 1 and those in stream 4. The differences were statistically significant ( $p<0.001$ ) in both 1995 and 1996 between streams 1 and 3, streams 1 and 4 as well as streams 2 and 4; between streams 3 and 4 only in 1996. Differences in 1995 were larger than the corresponding differences in 1996, probably because of a change in the admission policy in 1996; fewer students were admitted with poor numeracy and literacy backgrounds.

Table 7.1: Medians and ranges of final marks for Wellesley mathematics course 1995 and 1996. These achievement statistics are listed for the entire class for categories of the demographic variables available on the class roll. An additional column in 1996 (the second) lists medians of marks on the entry mathematics test.

Variable	Categories	Class Roll 1995 n=58			Class Roll 1996 n=51		
		n	Final Marks		Test   Final Marks	n	Med
			Med	Range			
Options	Science	31	82	65	21	63	73
	Arts*	27	33	82	30	57	51
Gender	Female	19	74	67	27	50	58
	Male*	39	53	95	24	57	61
Ethnicity	Maori*	16	46	89	10	33	51
	Pakeha	33	71	94	22	60	71
	Pacific Island	6	39	73	13	37	57
	Others	3	-		6	57	67
Streams	1	11	20	50	14	30	46
	2	15	49	75	14	47	59
	3	14	68	61	11	60	60
	4	18	87	49	12	80	25

\*Six students in 1995 achieved final marks which ranged from 3% to 11%. They were all male, all in the Arts option and three were Maori, hence these medians and ranges are affected by six low final marks. If these final marks were considered as outliers then the median (range) for Arts = 49 (65), for males = 66 (68) and for Maori = 53 (72).

The additional (second) column in the 1996 figures above is the median of scores on the entry mathematics test given at the interview. The differences between the medians of scores in the entry mathematics test are not always

<sup>32</sup> For example, in 1996 the 95% confidence interval indicates the mean mark for students in the Science option is somewhere between 8 and 28 marks higher than the mean mark for Arts students.

reflected in the corresponding difference in the medians of the final scores for categories of some demographic variables. For example, for Arts and Science option groups, there is a small difference between the medians of the scores on the entry test, but a larger difference between the medians of the final scores. In contrast, larger differences occur within both sets of test and final median scores in comparisons between different streams. A smaller difference in achievement is evident from the class roll figures between ethnic groups as Pakeha students score approximately 25% more than Maori students in 1995, 20% more in 1996 ( $p=0.01$ ). The differences in achievement between Pakeha and Pacific Island students, or between male and female students, were not statistically significant.

The reasonably large size of the ranges, generally smaller in 1996 than in 1995, indicates a wide spread in the final marks for students in categories of demographic variables. Figure 7.1 contains graphs which illustrate the spread of the final marks in each stream, compared to test marks, for 1996.

The graph on the left shows the small spread in the marks on the mathematics test at interview for each stream. The graph on the right shows the overlapping range of final marks between streams for the whole class in 1996, although the medians increase from stream 1 to 4. There is a similar pattern in 1995 and a similar overlapping range of marks between options.

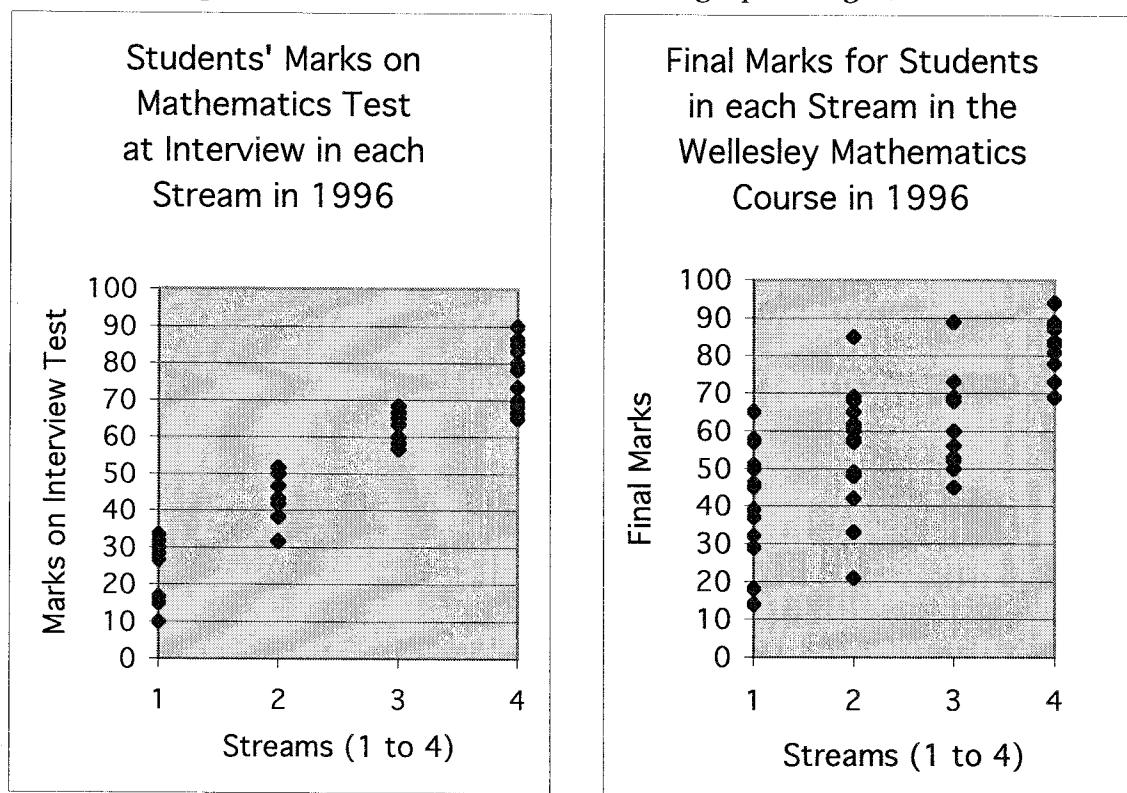
Now we examine how representative the groups of students who responded to the questionnaire in March or October are of the whole class (in 1995 and 1996). Appendix V lists the achievement statistics for the students who answered the questionnaire in March and/or in October (1995 and 1996) for the four demographic variables also available on the class roll, so that comparisons can be made with the statistics in Table 7.1<sup>33</sup>. Medians of the final marks are fairly similar in March 1995 and March 1996 compared with the corresponding figures from the class roll. However, in October 1995, there are fewer lower scoring Arts, male and Pacific Island students who responded to the questionnaire, because medians in those demographic categories are 12-20% higher in October than medians for the whole class. In October 1996, fewer lower achieving Science, Pakeha and stream 3 students have responded to the questionnaire, because the medians in those demographic categories are 11% higher in October than those for the whole class. (It is perhaps not surprising that lower achieving students were attending less regularly than others and were therefore probably not present when the questionnaire was handed out in

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<sup>33</sup> A decision was made to only comment on medians that differed by greater than 10% compared to the corresponding figures on the class roll.

October.) Otherwise, it appears, from this comparison of corresponding medians, that the groups of students who answered the questionnaire reflect, well enough, the achievement characteristics of the class as a whole in both 1995 and 1996.

Figure 7.1: A comparison of the range of marks by stream in 1996 for the test at interview (graph on left) and for the final marks (graph at right).



Achievement statistics for the demographic information available on the class roll are now completed, so we turn our attention to demographic information gathered only in the March and October questionnaires, which are discussed next, in Section 7.1.2.

### **7.1.2 Demographics Available Only on the Questionnaire: Age, Mathematics Background Knowledge and Fluency in English**

Medians of the final marks for the Wellesley mathematics course in 1995 and 1996 are listed in Table 7.2 for categories of the demographic variables available only from the questionnaires. (Students who did not complete the course are excluded.) Fluency in English was not gathered in March 1995.

The higher achievement of students in the younger age group ( $\leq 24$  years old) in 1995 is not evident in 1996, i.e. either the achievement level of the older group ( $\geq 25$  years old) is higher (in October) or both age groups achieve at a similar level (in March). Because of this inconsistency in the data, no further analysis

will be done by age. There is little difference in achievement in 1996 between students who are fluent in English and those who have moderate or low fluency, therefore a decision was made to do no further analysis by fluency.

Table 7.2: Medians of final marks for the Wellesley mathematics course in 1995 and 1996 for categories of the demographic variables available only from the questionnaires (completed in March or October).

Variable	Categories	March 1995		October 1995		March 1996		October 1996	
		n=46*		n=34*		n=47*		n=36*	
		n	Median	n	Median	n	Median	n	Median
Age	≤24 years	28	68	20	70	27	58	18	59
Groups	≥ 25 years	18	53	14	53	20	61	18	69
Fluency in English	Fluent	NA		33	68	40	59	30	65
	Not Fluent			1	-	7	57	6	63
Maths	≤ Form 5	32	58	27	57	27	50	22	54
Backgr'nd	≥ Form 6	14	67	7	85	19	73	14	84

\*March 1995, n=61 but 15 students withdrew; October 1995, n=38 but 4 students gave no id; March 1996, n=62 but 13 students withdrew and 2 gave no id; October 1996, n=44 but 4 students did not sit the final and 4 gave no id.

A large difference in achievement is generally found for different levels of background in high school mathematics, a difference also evident in the levels of achievement in different streams, for example, students' median marks in stream 1 and in stream 4 vary by 38% (1996) or 67% (1995). Streaming was determined by students' results in the test given at the time of their interviews, and it is another measure of the students' mathematics background. Since the differences in median marks between streams are bigger than those between the different mathematics backgrounds, a decision was made to use the stream variable to explore the effect of mathematics background knowledge, the major effect on achievement in the Wellesley mathematics course.

Demographic and further achievement information about each stream are discussed next, in Section 7.1.3.

### 7.1.3 The profile of each stream

Table 7.3 lists, for each stream, the numbers of students on the class roll (who completed the course), the percentage who passed and the range (and median) of marks they achieved in the test given to them at their interview.

Table 7.3: Achievement of students in each stream in 1995 and *in 1996* (*in italics*) in the mathematics test given at the interview and the pass rate.

Stream	Number of students in 1995 (1996); n=58 (n=55)	Range of Marks (%) on the Mathematics Test at Interview in 1995 (1996)	Median Mark	Pass rate in 1995 (1996)
1	n=11 (n=16)	10 - 27 (10 - 33)	23 (30)	9% (36%)
2	n=15 (n=15)	30 - 48 (37 - 53)	43 (47)	47% (64%)
3	n=14 (n=12)	50 - 60 (57 - 70)	53 (60)	79% (91%)
4	n=18 (n=12)	60 - 97 (63 - 90)	70 (80)	94% (100%)

The difference between 1995 and 1996 in the pass rate in each stream is possibly due to the change in admission policy in 1996: fewer students with poor numeracy skills were accepted into the Program in 1996.

Table 7.4 shows the demographic profile of each stream. Data included from the class roll are the numbers of students from each option, gender and ethnic group. Numbers in each age group are from the respondents of the March or October questionnaires.

Data in Table 7.4, from stream 1 to stream 4, indicate that there are progressively fewer students from the Arts option and more from the Science option, although streams 2 and 3 have similar proportions. There are also progressively fewer female or younger students from stream 1 to stream 4 in 1996 however the reverse pattern occurs in 1995. There is no clear pattern for males or for older students, although there are progressively fewer older students in 1995 from stream 1 to stream 4. There are also progressively fewer Maori and Pacific Island students from stream 1 to stream 4 in 1996, but there is no clear pattern in 1995, although there are fewer Maori students in streams 3 and 4 than in stream 1 and 2. Most of the Pacific Island students are in stream 3 in 1995. The numbers of Pakeha students generally increase from stream 1 to stream 4 in both years. It is interesting to also note that in 1996 in stream 1, 38% indicate that they enjoyed and/or did well in mathematics at high school: the corresponding figure for stream 4 is 64%.

Table 7.4: Demographic profile of each stream in 1995 and *in* 1996 (*in italics*).

Stream/ Year	n	Arts or Science Options		Age		Gender		Ethnicity#		
		Arts	Science	≤ 24	≥ 25	Female	Male	Maori	Pakeha	PI
1    1995	11	10	1	1	7	3	8	4	4	3
	<i>1996</i> <i>16*</i>	<i>16</i>	<i>0</i>	<i>10</i>	<i>6</i>	<i>12</i>	<i>4</i>	<i>4</i>	<i>1</i>	<i>7</i>
2    1995	15	7	8	9	4	5	10	6	7	2
	<i>1996</i> <i>15*</i>	<i>8</i>	<i>7</i>	<i>7</i>	<i>8</i>	<i>7</i>	<i>8</i>	<i>3</i>	<i>6</i>	<i>5</i>
3    1995	14	7	7	8	5	4	10	2	0	12
	<i>1996</i> <i>12*</i>	<i>5</i>	<i>7</i>	<i>6</i>	<i>6</i>	<i>7</i>	<i>5</i>	<i>2</i>	<i>6</i>	<i>2</i>
4    1995	18	3	15	13	3	8	10	4	10	1
	<i>1996</i> <i>12</i>	<i>4</i>	<i>8</i>	<i>5</i>	<i>7</i>	<i>3</i>	<i>9</i>	<i>0</i>	<i>9</i>	<i>1</i>

\* Numbers in these rows include four students who dropped out late, in October 1996 (Stream 1, n=2; Stream 2, n=1; Stream 3, n=1).

# Counts of Asian and Other categories are not included (n=0-2).

It seems that in both years, any analysis which compares stream 1 and stream 4 - the demographic groups where achievement differences were greatest - will also largely reflect the effects of option and ethnicity, as well as their previous experience learning mathematics (i.e. whether, in the past, they enjoyed learning mathematics or did well).

The rest of this chapter discusses students' reactions to the teaching approach in the Wellesley mathematics course and attempts to determine whether this approach moderates student's attitudes and beliefs. Comparisons will be made between the demographic groups where achievement differences were greatest, that is, focusing only on stream 1 and stream 4.

Open questions were included in the October questionnaires (see Appendix P) to ascertain students' reactions to the course while other data was gathered using Likert-style statements. Analysis of questionnaire data is contained next, in Section 7.2. Detailed analysis of two interviews is in Section 7.3.

## 7.2 Students' Reactions, Beliefs and Attitudes: Analysis of Questionnaire Data

The first two sub-sections (Sections 7.2.1 and 7.2.2) look at the class as a whole. Section 7.2.1 contains an analysis of all responses to the open questions, and Section 7.2.2 the relationship between some quantitative measures and students' achievement in the course. Sections 7.2.3 - 7.2.5 contain detailed analyses of

three open questions asked in the questionnaire in October 1995 and 1996. These data are all analysed by the students' mathematics background knowledge i.e. streams, because student achievement differences were greatest between those students with the greatest amount of background knowledge (stream 4) and those with least (stream 1). In Section 7.2.4, the qualitative analysis is compared to some relevant quantitative results.

### **7.2.1 Students' Responses to Some Special Features of the Wellesley Mathematics Course**

Sections 7.2.1.1 and 7.2.1.2 contain an analysis of all responses to the open questions, drawing out the major themes.

#### **7.2.1.1 Returning to study mathematics**

While covering essential elements of the high school mathematics curriculum in the Wellesley mathematics course, Moira said that she aimed "to make students feel that it's not a difficult subject but it is something they can all do". This aim has been achieved for many students. The main theme which emerged from students' responses to all the open questions, mentioned by approximately 80%<sup>34</sup> of the students who answered the questionnaire in October 1995 or October 1996, was that they were pleased with the amount of mathematics they had learned in the course and had found the pace reasonable.

Students' excitement at being able to do mathematics, when they had not expected to be successful, is echoed in the following statements:

For me it has been challenging and a realisation that I can do it.

This was the first time in my life that I realise I can do it!

(I liked it) because I found that I'm capable of maths which I didn't before.

I think the Wellesley Program is an awesome program. I feel better about myself and I'm learning a lot of things in maths that I never knew.

(I liked it) as it is fast moving, interesting and informative. I have learned all the stuff I had forgotten or didn't know to begin with. A good basic grounding.

A number of students compared the Wellesley mathematics course with their previous school experience. Their comments included:

I learned more than I did at school and quicker.

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<sup>34</sup> This percentage may be a little higher than for the full class because the medians of the final marks of the October group are higher than for the overall class, indicating that a smaller percentage of lower achieving students answered the October questionnaire.

It has helped me to understand the subject more clearly than when I was at school.

(I liked it) because I learned much more than I did at school.

I found this course valuable because it has enabled me to catch up on some very large gaps in my mathematical education

Several students were pleased to have the opportunity to try to learn mathematics again after a long break away from study. Some comments included:

I like Wellesley Program maths because, being away from school so long, I was able to start all over again.

I like it a lot because it brings you up slowly back into the education system.

I enjoyed it. Being an older student and out of education for many years it was good to be given a basic grounding again before it got complicated.

The pace that the material was presented suited many students. Samples of students' comments included:

The tutors go at a good pace for people.

Everything was logically laid out and taken very slowly right from the beginning.

I think the course was ok at approaching a mathematical subject step by step.

We have covered a lot of work at about the right pace.

I liked the Wellesley Program maths because you start from the basics and work your way up.

I have enjoyed the Wellesley Program maths because it helps me understand more and is very thorough.

Straight forward and full on. I think that the way in which the course is structured and taught has helped my understanding.

It was good as it worked into an area rather than being thrown in the deep end.

However, some students (about 20%) found the pace too fast for them.

Not enough time to consolidate on things learnt.

I disliked the pace at which we go through the topics. Too quick on some of the areas I find difficult.

Tutor was too fast for me. I can't keep up. I won't let her down but just myself.

Too much to cram in one year and needs to go slower for people who have trouble – maybe introduce tutorials.

I have found it hard – as to get through the work the tutor has to go fast.

A bit slower would be nice - I did not like trying to remember all the formulas.

Another aim of the Wellesley course was to prepare students to enter Stage 1 mathematics at the bridging level and to develop a sense of security in a university environment. Comments related to this aim included the following:

The program gives me a chance to give tertiary studies another go.

Excellent all round basis for a smooth transition into university next year.

I liked it from the point of view that it gave me a chance to enter university and to have a shot at the degree I wanted. Not wanting to return to school for personal reasons, it was a mature way of studying.

However, for some it did not achieve this goal, for example, the comment:

My maths is not advanced enough to sit a course next year.

This main theme emerging from the responses of the Wellesley mathematics students (that they were pleased with the amount of mathematics they had learned in the course and found the pace reasonable) is different from the main theme which emerged from an analysis of the responses of Mathematics 1 students, namely, the relevance and usefulness of mathematics. This latter theme was a minor theme to emerge from the responses of Wellesley mathematics students.

### **7.2.1.2 Reactions to other aspects of this course**

In 1996 collaborative tutorials were introduced, as Moira explained:

We've developed laboratory sessions where the students work on a problem together in a group. So we're trying to encourage a cooperative environment and we find that they are very good at helping one another. And that's a really good support for them because they do set up little study groups and they enjoy working on things together.

No questions were asked for students to specifically comment on the collaborative tutorials in 1996 but several students commented on them, both positively and negatively.

(I liked best) the exciting lab exercises.

(I liked best) the collaborative labs. It gives us an opportunity to work with different members of the class and the opportunity to work through examples in a verbal environment.

(I liked best) the collaborative learning and the regular short tests.

I still fail to see purpose of collaborative laboratory / assignments.

The collaborative assignments are unfair assessment and often require more time than what is allocated.

The tutors had handed out the resource material in small booklets (three booklets in 1996) in order not to overwhelm the students with too much material at once. A few students commented on these and reactions varied:

(I liked best) the chapter books because they were easy to understand and you were able to work at your own pace.

(I liked best) the clarity, the explanations and the resources provide i.e. worksheets, revision exercises and topic books.

The thing that I liked least about this paper is that the resources or the book that has given to us, because there are not clear explanation. I think the tutor or co-ordinator of this program should provide the right book resources which is one book rather than piece by piece.

Moira mentioned that, in September, they gave the students a study program which ran for the 6 weeks prior to the examinations, one example of the help given to improve study skills. Helping the students to study effectively was important for many Wellesley students because many had never succeeded academically at school and there was often a lack of structure in their lives. Moira discussed how some of these issues were addressed:

We try to give them some idea of what it means to study, that it is a time-consuming occupation but that the rewards are there if you give it a try and do your best. Studying is a lot to do with organising your work and knowing how to flip through a unit of work to pick out the main points. To have a bit of practice of some of these skills, at the end of every fortnight we give them a very short 10-minute test on the work in the last 2 weeks.

No specific questions were asked about the testing regime but some students commented, positively and negatively, on the tests:

I like the regular short tests.

(I liked least) the tests and assignments. I think more revision is needed before a maths test.

(To improve the course the teachers need to) realise that although some people may not have performed well in the maths competency tests, this may be due to lack of confidence or practice- not lack of ability. Therefore, sometimes it's a little frustrating when you want to move on to the next subject.

(I liked least the) tests - I'm hopeless at remembering formula and how to use them.

To continue the focus on the whole class I consider the relationship between some of the quantitative measures and students' achievement in the course in Section 7.2.2.

## **7.2.2 The relationship between achievement in the course and some quantitative measures of students' beliefs and attitudes**

A discussion of the relationship between achievement in the course and some quantitative measures of students' beliefs and attitudes, in particular, students' beliefs about themselves as learners of mathematics and students' attitudes to mathematics, follows.

Students' beliefs about themselves as learners of mathematics and the relationship of this belief to achievement in the course are discussed now. A subset of Gourgey's (1982) Mathematical Self-Concept Scale was used in the questionnaire to measure this belief. (Chapter 5 contains a discussion of the trial of this scale.) Correlations are calculated between this scale score and the students' final marks in the course, a measure of achievement. Table 7.5 lists the correlations of the Mathematical Self-Concept Scale scores with final marks, in March and in October, 1995 and 1996. It appears that, in 1995, students' mathematical self-concept in October (just prior to sitting their final examination) is more highly correlated with their achievement in the course than are their beliefs in March. In 1996, correlation coefficients were similar in March and October. All four correlations are statistically significant. As for Mathematics 1, this parallels Gourgey's (1984 p. 17) result that "achievement and mathematics self-concept are significantly related" and agrees with other studies which have "found a consistently high positive relationship between self-concept and mathematics achievement" (Relich et al., 1994 p. 7).

Table 7.5: Correlations of a subset of Gourgey's Mathematical Self-Concept Scale scores with Final Marks in the Wellesley Mathematics Course, in March and in October, 1995 and 1996 (*in italics*) are listed.

Correlation	Year	n <sup>35</sup>	Mathematical Self-Concept Scores in March	n	Mathematical Self-Concept Scores in October
Final marks	1995	30	0.39: p=0.03	30	0.64: p=0.00
	1996	34	<i>0.66: p=0.00</i>	34	<i>0.60: p=0.00</i>

<sup>35</sup> In order to compare correlations for the same group of students in March and October, correlations were calculated for the 30 students in 1995, and 34 in 1996, who completed both questionnaires (in March and in October) that year, a slight reduction in the size of the groups analysed. In 1995, we have 40% of the students in Stream 1 and 70% of stream 4, down from 42% and 72% respectively in October 1995. For 1996, we have 74% of the students in Stream 1 and 63% of stream 4, down from 77% and 67% respectively in October 1996.

Students' mathematical self-concept correlates more highly with achievement in the course than did the measure of students' enjoyment of mathematics and their attitude to the value of mathematics which are discussed next. Aiken's (1974) Enjoyment of Mathematics and Value of Mathematics sub-scales were used in the questionnaire to measure these attitudes to mathematics. Correlations between these scale scores and the students' final marks in the course, one measure of achievement, are calculated for the Wellesley mathematics course in 1995 and 1996. Table 7.6 lists these correlations in March and in October, 1995 and 1996.

Table 7.6: Correlations of Aiken's (1974) Enjoyment of Mathematics Scale, and Value of Mathematics Scale, scores with Final Marks in the Wellesley Mathematics Course, in March and in October, 1995 and 1996 (*in italics*) are listed.

Correlation	Year	n	Enjoyment of Maths Scores in March	n	Enjoyment of Maths Scores in October
Final marks	1995	30	0.38: p=0.05	30	0.53: p=0.00
	1996	34	<i>0.44: p=0.01</i>	34	<i>0.55: p=0.00</i>
Correlation			Value of Maths Scores in March		Value of Maths Scores in October
Final marks	1995	30	0.27: p=0.15	30	0.50: p=0.01
	1996	34	<i>0.18: p=0.31</i>	34	<i>0.20: p=0.25</i>

In most cases students' attitudes in October are more highly correlated with their achievement in the course than are their attitudes in March, which also occurs in the Mathematics 1 course (Section 6.2.2.2). It appears that, in both years, enjoyment of mathematics usually correlates more highly with achievement in the course than the students' attitude to the (extrinsic) value of mathematics did. (The reverse occurred in the Mathematics 1 course, where students' attitude to the (extrinsic) value of mathematics correlated more highly with their achievement in the course than did their enjoyment of mathematics (Section 6.2.2.2).) All four correlations of enjoyment of mathematics with achievement are statistically significant which lends support to the number of different research studies which Boaler (1997b p. 173) found which "linked mathematical enjoyment with mathematical ability or competence". Only one correlation of the Value of Mathematics scale scores in October with achievement, is statistically significant ( $p = 0.01$ ). The results in this section give stronger evidence than was the case in Mathematics 1 of Gourgey's (1984 p. 17) statement that "attitudes and performance are completely intertwined".

As mentioned in the case study of Mathematics 1 (Chapter 6, Section 6.2.2.2), Kulm's (1980) opinion, based on his survey article was that "the attitude-achievement relationship is not as strong as common sense might expect" (p. 367), that there is "generally low correlation between attitude and achievement" (p. 373) and that "most experimental treatments" are "ineffective" in "producing significant improvements in mathematics attitude" (p. 375). While this is the case for students' attitude to the extrinsic value of mathematics, it is not for students' enjoyment of mathematics. Relich et al (1994 p. 5) found a number of studies which reveal "a low but significant correlation" between "attitude and achievement" which is definitely true for students' enjoyment of mathematics for the Wellesley mathematics course.

I have now completed a discussion of some qualitative and quantitative analyses which looked at the class as a whole. Hence, some major themes which have emerged from the analysis of responses from the class to all the open questions have now been discussed. Relationships of some of the quantitative belief and attitudinal measures to students' achievement in the course have also been covered. The analysis of data now focuses on stream 1 (students with the least mathematics background) and stream 4 (students with the most mathematics background), since mathematics background knowledge had the largest effect on achievement in the course.

The following three sections (Section 7.2.3 – 7.2.5) contain a detailed analysis of the responses to three open questions asked in October (1995 and 1996). Responses to one of these questions are also compared to a quantitative measure of students' beliefs about themselves as learners of mathematics.

The demographic profile of the streams indicated that 10 of the 13 students in stream 1 (October 1996) were Maori or Pacific Island students, and all were from the Arts option, whereas 1 of the 8 students in stream 4 was Pacific Island, and 6 were from the Science option. Hence the effect of the stream variable also could be related to the effects of option and ethnicity (particularly Pakeha and Maori). This demographic information<sup>36</sup> is given alongside a student's response, as well as their gender and final marks for the course.

### **7.2.3 Students' Responses to the Question: *What do you think about the maths you have done in this course?***

The responses to this question in October 1995 were content analysed and post coded and the categories are listed as follows:

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<sup>36</sup> Abbreviations used to present this information are: M, male; F, female; Sci, Science option; PI, Pacific Island ethnicity.

A. Students indicate that they have gained much more mathematical confidence (n=8, 21%).

B. Students indicate more mathematical confidence (n=21, 55%).

In group A and B the two most common reasons students gave were either that they could now do more high school mathematics or particular mathematics skills or that they could now understand mathematical concepts more, or think more logically.

C. Students indicate a little more mathematical confidence gained (n=3, 8%)

D. Students indicate no confidence gained (n=5, 14%)

E. No response (n=1, 3%)

This analysis indicates that most students in the October group (83%<sup>37</sup>) have gained confidence by doing this mathematics paper. In category A and B, over 50% (16 of 29) indicate that they can now do more high school mathematics or do particular mathematics questions that they were unable to attempt before.

Almost all students who responded in category A were in stream 4. For students whose responses were in categories C, D, or E, over half were from stream 1 and they failed the course. The other three responses in categories C, D, or E were from streams 3 and 4. They had achieved very well in the course so it is possible to interpret these students' responses as stating that they already felt confident and hence did not feel they had gained more.

Analysis of students' responses to a third question by the variable stream (i.e. background knowledge in mathematics) is discussed next, in Section 7.2.3. Sample responses from stream 1 and stream 4 are listed, in Section 7.2.3.1 and 7.2.3.2, respectively.

### **7.2.3.1 Stream 1 responses**

Responses of stream 1 students, in 1995 and 1996, varied in answer to the question *What do you think of the mathematics you have done in this paper?* Their final marks ranges were 10-53% and 14-65%, respectively.

Comments of students in 1995 or 1996 in stream 1 will be separated into the groups who did not sit, who did not pass and those who passed.

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<sup>37</sup> This percentage may be a little high compared to that for the full class because the medians of the final marks of the October group are higher than for the overall class, indicating that a smaller percentage of lower achieving students answered the October questionnaire.

Two students who did not sit the final examination, although they completed all the course work, commented:

There's a lot of rules and numbers and formulas to remember but that's maths.	Did not sit the final, Arts, PI, F
No idea. I'm always slow on maths.	Did not sit the final, Arts, PI, M

Responses of some of the students whose final marks were 29-46% were:

I suppose it is meant to be designed to be practical, but in practice I would find it hard to apply.	Final mark 10%, Arts, Pakeha, M
Because I have to.	Final mark 20%, Arts, Maori, M
Pretty easy at the start, harder as this year goes on.	Final mark 29%, Arts, PI, M
I have found it hard as, to get through the work, the tutor has to go fast.	Final mark 30%, Sci, Pakeha, F
Although I pick up things slowly, I can still understand.	Final mark 37%, Arts, Asian, F
For me it has been challenging and a realisation that I can do it.	Final mark 39%, Arts, Pakeha, F
Better than what I had done at school, even though it ain't too great.	Final mark 46%, Arts, Maori, F
Simple if you really apply yourself.	Final mark 46%, Arts, PI, F

Responses from those who scored 50% or more:

It can be hard but also challenging at the same time.	Final mark 50%, Arts, Maori, F
Quite reasonable but slowly getting harder towards the end of the year.	Final mark 51%, Arts, Maori, F
Most of it has refreshed my memories of secondary experience, but has also shed new light.	Final mark 53%, Arts, PI, M
I love it, it is just the matter of taking time to learn formulas in different changes.	Final mark 58%, Arts, PI, F

Responses from students in stream 4, with the highest scores on the mathematics test sat on entry to the Wellesley Program, follow in Section 7.2.3.2. They are noticeably different from responses in stream 1.

### 7.2.3.2 Stream 4 responses

Stream 4 students in 1995 and 1996, whose final scores ranges were 49-98% and 73-94%, respectively, gave positive responses to the question *What do you think of the mathematics you have done in this paper?*, although some indicated that it had been easy at times for them.

A lot simpler than first perceived.	Final mark 49%, Sci, Pakeha, M
What I have done could have been better if I had not had a year which had been rife with personal problems but, considering this, I am satisfied with the work I have done.	Final mark 69%, Sci, Pakeha, F
I feel, to some extent, that the maths I have learnt has little application outside university at this stage in my life.	Final mark 82%, Sci, Pakeha, M
More advanced than what I have done before.	Final mark 83%, Sci, Pakeha, M
It will help to bridge the gap between the past few years and next year.	Final mark 84%, Arts, Asian, F
What was old hat sucked. What was new was a good challenge.	Final mark 84%, Sci, Pakeha, M
Relatively easy, though helpful in many areas.	Final mark 89%, Sci, Pakeha, M
Most of it is relevant. Practical applications were shown to us for many parts of the syllabus studied, which is great.	Final mark 90%, Sci, Pakeha, M
Very important – it has built strong foundations about which I feel very confident in continuing in maths.	Final mark 98%, Sci, Pakeha, M

Analysis of students' responses to a third question by the variable stream (i.e. background knowledge in mathematics) is discussed next, in Section 7.2.4. Sample responses from stream 1 and stream 4 are listed, in Section 7.2.4.1 and Section 7.2.4.2, respectively.

### 7.2.4 Students' Responses to the Question: *Has this course affected your mathematical confidence?*

Teachers of the Wellesley mathematics course aimed to increase the mathematical confidence of students. Moira explained that she wanted to convince students that they need not "be frightened of maths", and said:

if you can bring the language down to the level that they can understand, they're often very agreeably surprised at how much they can do.

This aim has been achieved for the majority of students. In answer to the question *Has this course affected your mathematical confidence? Explain please*, of the students who responded at the end of the year, about 80%<sup>38</sup> in 1995 and about 70% in 1996 indicated that their confidence had increased. Students' responses to this question are now analysed for stream 1 and 4. In addition, a comparison will be made with a measure of students' beliefs about themselves as learners of mathematics. These beliefs have been measured with a scale, a subset of Gourgey's (1982) Mathematical Self-Concept Scale (listed in Appendix N). After each student's comment, their scores (out of 40) on this scale in March and October are listed (in italics). The mean scores on this scale in October 1995 were 19.2 (for stream 1) and 30.3 (for stream 4), and in October 1996 they were 21.8 (stream 1) and 28.4 (stream 4). The means for all scales and a commentary are in Appendix W.

#### **7.2.4.1 Stream 1 responses**

Stream 1 students' responses varied in answer to the question *Has this paper affected your mathematical confidence? Explain please*. The comments of students in 1995 or 1996 in stream 1 will be grouped. Comments by students who did not pass are listed first, those who did not sit are listed second and finally comments from those who passed the mathematics course.

For those students who failed, some comments were negative;

No - this paper has made me realise my maths is abysmal.	SC(17, 19)	Final mark 14%, Arts, Maori, F
Sometimes of so many formula to remember.	SC(28, 23)	Final mark 29%, Arts, PI, F
By getting low scores in tests my self-esteem has gone down.	SC(11, 7)	Final mark 30%, Sci, Pakeha, F

Other responses were tentatively positive;

Yes a little more confident to try and attempt to work other maths out.	SC(26, 23)	Final mark 25%, Arts, Pakeha, F
Well, it's given me the opportunity to try.	SC(15, 11)	Final mark 29%, Arts PI, M
Learning the basics has given me confidence.	SC(17, 17)	Final mark 32%, Arts PI, M

<sup>38</sup> These percentages may be a little high compared to that for the full class because the medians of the final marks of the October group are higher than for the overall class, in 1995 and 1996, indicating that a smaller percentage of lower achieving students answered the October questionnaire.

Yes, I can do some equations that I haven't done before, but I hate maths.	SC(21, 18)	Final mark 37%, Arts, Asian, F
Yes, prior to this course I had very little confidence in my own ability.	SC(16, 15)	Final mark 39%, Arts, Pakeha, F
It has built my confidence in maths in some aspects.	SC(23, 18)	Final mark 46%, Arts, Maori, F
In terms of mind boggling yes because it makes me feel that I could 'probably' do extension maths - 'I hope'.	SC(22, 28)	Final mark 46%, Arts, PI, F

The following comment indicates this student had little appreciation of her level of mathematics knowledge.

Yes tremendously. I enjoy working alongside my two teenage daughters with their maths, they are 4th and 6th form. (I might add I'm a bit more advanced at this stage).	SC(18, 22)	Final mark 18%, Arts, Maori, F
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One student was very positive, although dropped the course just before the final examination:

No! I see maths in a new light e.g. I love it.	SC(31, 29)	Did not sit the final, Arts, PI, F
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Stream 1 students who passed the mathematics course gave positive responses:

Yes, because I know a lot more on maths. I feel I can do anything.	SC(20, 23)	Final mark 50%, Arts, Maori, F
It has boosted my confidence a lot.	SC(22, 36)	Final mark 51%, Arts, Maori, F
In a good way, I'm more confident with numbers and figures now than 5 years ago.	SC(23, 29)	Final mark 58%, Arts, PI, F

The responses from students in stream 4, with the most mathematics background knowledge, follow in Section 7.2.4.2.

#### 7.2.4.2 Stream 4 responses

Stream 4 students gave the most positive responses to the question *Has this paper affected your mathematical confidence? Explain please.* Responses, and scores on the subset of Gourgey's (1982) Mathematical Self-Concept Scale, included:

It has boosted my mathematical confidence immensely, encouraging me to go further and deeper than before.	SC(30, 29)	Final mark 53%, Sci, Other, M
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Yes. In the fifth form I gave up because I couldn't do it. Here I try until I can do it. SC(25, 27)	Final mark 69%, Sci, Pakeha, F
Yes. I have learned all the stuff I had forgotten or didn't know to begin with. SC(32, 32)	Final mark 81%, Arts, Pakeha, M
By doing harder problems and getting them right, I've become more confident, arrogant and cocky in maths than I already was!!! SC(29, 33)	Final mark 84%, Sci, Pakeha, M
Yes, I feel a lot more confident. I understand figures I see and hear on the radio, newspaper. I understand how economic things work better and things are constructed - I am more aware. SC(28, 23)	Final mark 84%, Sci, Pakeha, F
Yes it has given my ego an incredible boost. I was not overly confident mathematically at the beginning of the year, but now I know I can do it. SC(28, 29)	Final mark 90%, Sci, Pakeha, M
Not very much – I've always been confident about maths, but it has increased with the learning of new concepts. SC(37, 38)	Final mark 92%, Sci, Pakeha, F
Yes it has greatly improved. I no longer feel intimidated and have really consolidated my understanding of the concepts. SC(NA)	Final mark 98%, Sci, Pakeha, M

Two students indicated they were confident already and one said:

Not really, I have always been confident of my mathematical ability. SC(28, 31)	Final mark 89%, Sci, Pakeha, M
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While there is some variation in the Mathematics Self-Concept Scale scores in each stream, those in stream 1 are generally lower than scores in stream 4 i.e. the mathematical self-concept of students seems to depend, to some extent, on their mathematics background knowledge. Their level of background knowledge could be related to their mathematical experiences in the past, and these will have influenced their beliefs about themselves as learners of mathematics.

Changes in scores from March to October vary. For students in stream 1 who did not pass the course the changes are largely negative indicating that these beliefs are influenced negatively by their experience in the Wellesley mathematics course. However, for those students in stream 1 who passed the course the changes are positive indicating an increased mathematical self-concept.

Stream 4 students have started the course with higher scores, so it might be expected that there would be little change, however they have generally

increased their scores i.e. they have been influenced positively by their experience in the Wellesley mathematics course. While Mathematical Self-Concept Scale scores improved for a majority of students in stream 4, for a few students this was not the case. In 1995 the only two students in stream 4 whose Mathematical Self-Concept Scale scores decreased (percentage differences of -31% and -6%) were male with mathematics background of Form 5 or less and their final marks were the lowest in stream 4 (49% and 53% respectively). Both of these students may have stayed in an inappropriate stream. One of these students may have wanted to move as he commented on the streaming in answer to the question *Suggest ways the Program could be improved*; he said "a better test at the beginning of the year to sort out which stream people fit into". Improved streaming in 1996 seems to have decreased this problem. Two students in 1996 whose scores decreased by a reasonably large amount (by -29% and -18%) were both female. They commented that the pace of the course was too fast, although they both achieved 84% in the course. In answer to the question *What have you liked least about this course?* these two students said: "The pace at which we go through the topics is too quick on some of the areas I find difficult", and "I found the amount of new content difficult even though I'm working hard". That two of the best prepared female students in the Wellesley mathematics course who were placed in the top stream, have commented negatively on the pressure and pace of the work, may indicate some disaffection. Boaler (1997b) discovered that many of the high ability secondary school students, particularly girls, who were grouped and taught in a top stream became disaffected and underachieved. She observed that the lessons in the top stream were "taught with an air of urgency" and that students felt that the pace of the lessons were too fast (p. 172). In the top Wellesley stream in 1995 three-quarters of the students below the median of the final marks were female. This may indicate that the pace of the material in this stream has caused female students to underachieve.

In summary, comparisons indicate that the higher achieving group (stream 4) entered the course with a more positive mathematical self-concept than the lower achieving group (stream 1). For the higher achieving group there was usually an increase in their scores over the year. The pressure of work in stream 4 impacted on some female students. Some other students in stream 4 may have stayed in an inappropriate stream. Most of the stream 1 students who did not pass the course have decreased their scores in this scale from March to October, indicating many of them have finished the course feeling less self-confidence as learners of mathematics than when they began the year. However, for the

students in stream 1 who did pass the course there is evidence of an increase in their mathematical self-concept.

Analysis of students' responses to a third question for stream 1 and stream 4 (i.e. by background knowledge in mathematics) is discussed next, in Section 7.2.5. Sample responses from each of these two streams are listed, in Section 7.2.5.1 and Section 7.2.5.2, respectively.

### **7.2.5 Students' Responses to the Question: *What do you think of the paper's approach?***

An analysis of the 1995 responses found that about 80%<sup>39</sup> of students gave positive responses. All the negative responses came from Stream 1 students.

#### **7.2.5.1 Stream 1 responses**

Stream 1 responses to the question *What did you think of the paper's approach?* The comments of students in 1995 or 1996 in stream 1 will be separated into the groups who did not sit, did not pass and those who passed the mathematics course.

The two students who did not sit the final (having completed all the coursework) said "ok" and "very different."

Those who sat the final examination but did not pass said "good" and "excellent" and

I think in the early context of the year it is rushed with lots of assignments which makes it difficult when you have been away from school for a long time.

Final mark 10%,  
Arts, Pakeha, M

Depending on which area we are doing. Some you relaxed at the approach and others you were incline to worry or panic.

Final mark 18%,  
Arts, Maori, F

Too much to cram in one year and needs to go slower for people who have trouble - maybe introduce tutorials.

Final mark 30%,  
Sci, Pakeha, F

It's OK, we started from the beginning so we build up and understand things clearly.

Final mark 37%,  
Arts, Asian, F

Great.

Final mark 39%,  
Arts, Pakeha, F

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<sup>39</sup> This percentage may be a little high compared to that for the full class because the medians of the final marks of the October group are higher than for the overall class, indicating that a smaller percentage of lower achieving students answered the October questionnaire.

Students who passed, as well as making the comments "great" and "very good", said:

Careful and a good pace.	Final mark 50%, Arts, Maori, F
Problems: linear graphs, area and fractions. Taught well but I need one-on-one tuition to understand more thoroughly.	Final mark 51%, Arts, Maori, F
Very straightforward and very easy to understand and acknowledge.	Final mark 53%, Arts, PI, M

The responses from students in stream 4, with the most mathematics background knowledge, follow in Section 7.2.5.2.

### 7.2.5.2 Stream 4 responses

Stream 4 students' responses to the question *What did you think of the paper's approach?* included:

Good – logical order of subjects studied and clear explanations of concepts, making everything easy to understand.	Final mark 69%, Sci, Pakeha, F
Good. Diverse. Thorough.	Final mark 81%, Arts, Pakeha, M
Good, but I think the people in the higher streams (such as myself) should be spared the simple stuff that is quite frankly a waste of our time, effort, ink and paper (the material things matter when on a student budget you know!!)	Final mark 84%, Sci, Pakeha, M
I like the variation of topics and range. I like the collaborative learning and the regular short tests.	Final mark 84%, Arts, Asian, F
Excellent, though I felt rushed. I would have been better to do a short 5th or 6th form maths paper before this.	Final mark 84%, Sci, Pakeha, F
I think the paper's approach is good, it covers a wide range of useful mathematical concepts.	Final mark 89%, Sci, Pakeha, M
Good – logical order of subjects studied and clear explanations of concepts making everything easy to understand.	Final mark 92%, Sci, Pakeha, F
Excellent all round basis for a smooth transition into university next year.	Final mark 98%, Sci, Pakeha, M

These responses above, by students in streams 1 and 4, to the question *What did you think of the paper's approach?* could be compared to the students' beliefs about

the learning of mathematics. Data gathered relating to these beliefs are analysed next, in Section 7.2.6.

### **7.2.6 Students' beliefs about the learning of mathematics**

Beliefs about the learning of mathematics were examined by analysing students' responses to five statements (Schoenfeld, 1989) (listed in Appendix N). Differences between response patterns for stream 1 and stream 4 occur in all five statements. More lower achieving stream 1 students exhibit beliefs in an algorithmic or instrumental (Skemp, 1978) approach to learning mathematics than in the higher achieving stream 4 group. Stream 1 students indicate, in three statements, no change in this focus after the year's course. Another statement reveals more students believing in an algorithmic approach in October than did in March. However, in one statement there is an indication that, by October, more in this group feel that mathematics is more accessible to them. A high proportion of stream 4 students indicate they are conceptual or relational (Skemp, 1978) learners of mathematics in March and more of the students in this group believe in a conceptual approach to learning mathematics by October. (Diagrams illustrating the response pattern for students in stream 1 and stream 4 for the five belief statements, together with some commentary, are in Appendix X).

The Conceptions of Mathematics Scale (Crawford et al., 1994), included in the questionnaire only in October 1996, was developed as a result of an investigation to identify conceptions of mathematics held by beginning university students and their approaches to study of mathematics. The majority of students in their study viewed mathematics as a necessary set of rules and procedures to be learnt by rote.

This 19-item scale, a five-point Likert scale, yields two sub-scales. Eleven items formed the Fragmented sub-scale, 8 items formed the Cohesive sub-scale. A fragmented view of mathematics means students' conceptions of mathematics are as numbers, rules and formulae with applications to problems. Students with a strong fragmented view of mathematics prefer learning mathematics by rote memorization and by doing examples for reproduction. Scores on the Fragmented sub-scale collate the range of fragmented views about mathematics and the associated learning preferences. A cohesive view of mathematics means that students' conceptions of mathematics are as a way of thinking for complex problem solving and that mathematics provides new insights for understanding the world. Students with a strong cohesive view of mathematics prefer learning mathematics by doing difficult problems and by looking for a broader context

to extend understanding. Scores on the Cohesive sub-scale collate the range of cohesive views about mathematics and the associated learning preferences. The terms fragmented and cohesive are similar to Skemp's (1978) terms instrumental and relational, used above.

Table 7.7 lists the medians and standard deviations of students' scores, by stream, for the Conceptions of Mathematics Scale (Crawford et al., 1994).

Table 7.7: Medians and Ranges of Crawford et al's (1995a) Conceptions of Mathematics Scale scores in October, 1996, for Streams 1 and 4.

Stream	n	Fragmented view of mathematics		Cohesive view of mathematics	
		Median	Range	Median	Range
1	13	41.0	18	28.0	26
4	8	34.5	32	32.0	12

These results show a reasonable difference between the scores for stream 1 and stream 4. Students in stream 1 hold a more fragmented view, and a less cohesive view, of mathematics learning than those in stream 4.

Scores on the Fragmented sub-scale, which collated a range of fragmented views about mathematics and associated learning preferences, ranged from 18, the least fragmented view, to 53. The median of these scores was 41, lower quartile 39 and upper quartile 43. Of the 6 students who scored from 44 to 53, those who held the most fragmented views of mathematics in the class, 5 were from stream 1 and 1 from stream 2. Of the 4 students who scored from 18 to 31, those who held the least fragmented view of mathematics, 3 were from stream 4 and 1 from stream 3. Unexpectedly, I found that three other students with low scores (33, 35, 36) were from streams 1 and 2 and that these students were the highest achievers from streams 1 and 2 (final marks 69, 50, 65 respectively).

Scores on the Cohesive sub-scale, which collated a range of cohesive views about mathematics and associated learning preferences, ranged from 14, the least cohesive view, to 40. The median of these scores was 32, lower quartile 28 and upper quartile 32. Of the 6 students who scored from 14 to 26, those who held the least cohesive views of mathematics in the class, 5 were from stream 1 and 2 from stream 2. Of the 8 students who scored from 32 to 40, those who held the most cohesive view of mathematics, 7 were from streams 3 and 4. One student with a high score (36) was from stream 1 and, maybe not surprisingly, this student was among the highest achievers in stream 1 (final mark 51).

Recall that Stream 1 students are predominantly from the Arts option, and over half are either Maori or Pacific Island students. On the other hand, stream 4 students are predominantly from the Science option and Pakeha. Hence the above analysis of stream 1 and stream 4 (in Sections 7.2.3 - 7.2.6), the demographic groups where achievement differences were greatest, also largely reflects the effects of option and ethnicity.

To enable us to examine, in more detail, the variables that affect the achievement of students, several students were interviewed. Two interviews are presented next, in Section 7.3.

## 7.3 Interviews

Interviews with a number of students provided additional qualitative data. A presentation and discussion of parts of in-depth, semi-structured interviews with two students follow. Some of their early mathematical experiences and also their reactions to aspects of the Wellesley mathematics course are considered. Gibson's (1994) protocol for using metaphors (the Mathematics Metaphor Questionnaire) was modified and used during these interviews to help students to describe their learning experiences in mathematics. The students were usually asked to interpret their own metaphors, considered important by Gordon and Langmaid (1988) in their discussion of the use of projective techniques in qualitative market research. The two interviews are with female students, one Maori and one Pakeha, who had similar scores on the mathematics test at the start of the year i.e. both were placed in stream 3. Tania is one of few Maori students in the upper streams. (Streams 3 and 4 contained students who scored above the median on the entry test in mathematics.)

### 7.3.1 Introduction to Mary and Tania

Interviews of Mary and Tania were conducted, for Mary, just after she had completed the Wellesley Program and, for Tania, about 2 years after she had completed. Both Mary and Tania had a child not long after leaving high school. Both did well enough in the entry mathematics test to be put into Stream 3, although Mary moved to Stream 1 during the year.

Mary's final mark in the Wellesley Program mathematics course was 30%, a failing grade. Tania's final mark in the Wellesley Program mathematics course was 68%, a passing grade. Why did Mary do poorly and Tania do well? Details of their interviews follow, illustrating some of the reasons why this happened.

### 7.3.2 Mary

At the interview Mary summed up how she did in her year's study as "not too good - not as well as I expected in maths". She also didn't do well in biology and chemistry, saying she "had a real struggle in chemistry". She said it was "completely different" from high school and that all these subjects "tied in together".

Mary remembered that she was "average at maths" all through primary and intermediate schools and in the third and fourth forms<sup>40</sup>. A good experience at that time was her "times-tables", because they "still stick in her mind" and she "uses them regularly". Now she can help her son "through the same thing". As soon as she "hit fifth form" it was algebra that was the problem. Her father was a draftsman so "he understood maths" and Mary said "I got freaked out because he couldn't understand why I couldn't understand and he used to get annoyed". Her father got her a tutor for maths and she still didn't understand. Mary continued, "then I just convinced myself that I just couldn't do algebra - that I'd never be able to. It was probably like a phobia - when I see algebra I freak out." Her mother "wasn't very good at maths either and so for her it wasn't important". Mary did not sit School Certificate Mathematics but sat the NZ Certificate in Mathematics<sup>41</sup>, passing this subject. She left school after Form 5, giving birth to a son soon after. A few years later she returned to high school as an adult day student in 1990, passing English and geography and she passed the sixth form year. As she says "it's only the sciences and maths that I've got a problem with, and that's what I want to end up working in".

Her goal on entering the Wellesley Program was to go on to study for a BSc, but, as she says:

not long into the first term I realised I was just taking off more than I could chew. As the year went on I realised I couldn't do it at the moment. I learned a lot through doing this, what not to do more than what to do.

She has since amended this goal. She plans to study for a Certificate in Applied Science at a Technical Institute. Full-time study was too difficult this year in the Wellesley Program, as the sole parent of her 9-year-old son, so she plans to study for the Certificate part-time over 4 years. She'll avoid maths for her first year, to allow her to "get the feel of things and maybe tackle it next year". She

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<sup>40</sup> The third and fourth forms are the first two years of high school, when students are aged about 13-14 years.

<sup>41</sup> Most high school students were able choose from two alternate mathematics courses offered for national examinations in the fifth form (age 15 years): School Certificate Mathematics and the less abstract, more applied, NZ Certificate in Mathematics.

always has "tried to avoid doing maths" and the "thought of doing anything with maths is pretty scary", even now.

She said she didn't want to take mathematics, however, in the Wellesley Program mathematics was compulsory. Mary explained:

If there has been any way around it, I certainly wouldn't have taken it. During the first term I coped reasonably well, then it went from bad to worse. I decided to move from stream 3 to stream 1, the easiest stream. In stream 3, when it came to tests, I didn't know what I was doing. It was quite bad. I felt much better going into stream 1 as, even though they did the same work, it was just at a much slower pace.

When asked about what her expectations were, starting to study mathematics again, she said:

I was scared because I knew what it was going to be. Actually it was worse than I thought, because I hadn't realised that about half of it was algebra. I think most of terms 2 and 3 were connecting with algebra. Because I didn't master the basics that we did, I was lost for the rest of the year. Actually it was worse than I thought it would be but I learned some maths - I actually did finally master some algebra, just beginning to understand some by the end of the year, but we were way passed those topics by that stage. I used to get quite excited when I'd understand something, the basic stuff - I was quite pleased. This course did give me some hope that, given the right sort of time and input from a tutor, I will eventually understand it.

After high school she had felt that she "would never be able to do maths" but "having been made to do it" and then seeing that she did understand some of it "was good".

Did she think maths would be useful? "No. The only useful thing would be that I would be able to do it at stage one level". She said that she only did it "because I had to, mind you, there were some calculations in biology and chemistry where it would be useful, but that's about it".

When Mary was asked how she studied maths she said:

I just wrote everything down, everything that (the tutor) said, any method to do things. I wrote and I wrote. I wouldn't really know what I was writing. When it came to studying it, I'd sit down and then look at it, I'd refer to examples in the book and then I would just get confused. I couldn't decide which process to use for which equation. Then I'd get annoyed because I'd think that I had it, then I'd do an example and then something would go wrong. It used to take a few goes before the answer came out right. It was really stressful. I'd persevere a fair length of time but then I did less and less of it because I dreaded it. Then it would be the last thing that I'd do. I knew I'd be there for hours. I'd sit there on one problem for ages, probably a few hours and quite often I'd just get confused. I wasn't achieving anything, just stressing myself out. Then I'd avoid it if I could.

When asked what she liked least in the mathematics course, she said:

there's lots to choose from. I didn't like how everybody else understood it and used to almost gloat about the marks they got. That was one of the

reasons I moved streams. In stream 1 we were all the same, they were all on my level.

I asked her to think of ways she thought the course could be improved and she said "better streaming". She also wondered "if some of the people that were there should have even been there". She continued:

I was quite bad but some of them couldn't even add or subtract properly, and I wonder what that did for their self-esteem. There were a few who couldn't do any of the work. Lots of people didn't turn up and I wonder if it was just because they were just completely out of it, they didn't know what to do. It's not going to make them feel better about themselves, cos I know what I felt like.

She said that they were encouraged to come and get help but there wasn't enough time available, especially as she got further behind. Also the "speed" - the class "moved too fast - way too fast. It was too fast for all of us in stream 1. Some people in stream 3 found it too fast too. It was too much crammed into one year - too intense."

I asked Mary to respond to the questions on the Mathematics Metaphor Questionnaire in order to more fully understand her attitudes to and beliefs about mathematics. Table 7.8 lists the questions and her responses.

Table 7.8: Mary's responses to the Mathematics Metaphor Questionnaire.

<i>The questions</i>	<i>Mary's responses</i>
<i>Describe maths to someone</i>	Scary, confusing
<i>What does it feel like to do maths?</i>	Scary. It's like you've got this problem and then you've got these steps, and then there's an answer. It's all the steps in the middle, that's the scary bit. It's not like other things. There's an answer and that's it - it can't be this or that - it's not creative for me. I have seen that for others it could be interesting and quite exciting.
<i>If maths were weather, what weather would it be?</i>	Gloomy and grey - it's negative - when I think about maths I think yucky thoughts.
<i>If maths were food, what food would it be?</i>	Something disgusting - I hate meat - like a disgusting beef stew.
<i>If maths were a hobby, what hobby would it be?</i>	A stressful one - but hobbies are enjoyable!
<i>If maths were a way to travel, what way to travel would it be?</i>	Fast - like a fast train - it's going so fast and I'm trying to understand what it's doing - too fast!
<i>If maths were a colour, what colour would it be?</i>	Black because it's something dark and gloomy.

<i>The questions (continued)</i>	<i>Mary's responses</i>
<i>If maths were a way to communicate, what way to communicate would it be?</i>	I suppose there would be formulae for everything and x's everywhere. I wouldn't survive it. I wouldn't be able to communicate. It would be a completely foreign language.
<i>If maths were an animal, what animal would it be?</i>	A lion, big strong, powerful and scary - a big male lion with a mane. Even though they are beautiful animals, they're pretty scary to be near. Maths feels powerful to me because it's bigger than me and I don't understand it.
<i>If maths were a building, what building would it be?</i>	A concrete skyscraper - a huge one. It's big, looks powerful and can take over you - like a nightmare.
<i>If maths were a plant, what plant would it be?</i>	A prickly weed, prickly gorse say, because it is yucky, not a nice pretty little plant. You stay away from it because it hurts.
<i>If maths were music, what music would it be?</i>	Probably something loud, heavy metal probably because I don't like it. It's aggro, everything is black and dark.
<i>If maths were a kitchen utensil, what kitchen utensil would it be?</i>	A sharp knife, like a stabbing - it gets you where it hurts. A very sharp knife - quite often you cut your finger on them - not just an ordinary knife - it's really dangerous and they hurt - maths is pain.
<i>If maths were a garden tool, what garden tool would it be?</i>	A pitchfork - you can stab things, poke things - it's not a nice tool either - not gentle.

Mary's comments about the Mathematics Metaphor Questionnaire, after answering the questions, were:

Just sitting here going through that is almost like therapy, because I'm sitting here thinking I've got real problems with maths. It's almost a phobia-type thing, that's the closest thing I can think of. I've connected maths with feelings using these questions. It's very good. I was answering them and thinking 'oh my goodness'. Could I have a copy of the questions? Maybe it would be a good thing to connect with other things that you have problems with, things that seem too big and not possible to overcome. I didn't realise how much of a problem it was until I just answered these. I think my problem with maths is worse than I had realised. When you associate all the things you don't like, I guess maths would be in that category, definitely. That's neat. I'm glad I've come in.

She summed up her experience in this mathematics course as follows:

My attitude hasn't changed much during the year because I'm still trying to avoid it but there's a bit of hope. I'm getting to the point where I'm going to do it because I want to beat it. I feel now that maths has beat me but I would be so happy if I would be able to do stage one and pass it, then I would have conquered it. That would be one of my ambitions, to understand it and conquer it in the end.

The second interview presented in this chapter (with Tania) is in Section 7.3.3.

### 7.3.3 Tania

At the interview Tania talked first about her family background.

I finished in the sixth form, not successfully, but I hung in there till sixth form. I think I only passed Maori. And School C<sup>42</sup> I passed in Home Economics, Maori, barely English and barely Geography. Dropped maths. Just did maths when it was compulsory! I think I did some maths in fifth form but not School C maths.

My mum was keen on education really, she's actually a teacher. But my father, I think he said he'd finished school at about 8. There was a bit of a split with my mum and dad with that because my dad wanted me to go to work when I was 15 I think, and he didn't really see any sense in it. My mum thought that education was the key to improving my life. She also had the belief of the Maori culture being probably more important than the education was.

It's funny though because my dad could do things that I suppose would take quite a lot of learning. He could build an extension onto the house, and he only had an education up to 8. He'd be able to rattle off, just adding up things, just practical stuff - not schooled.

I was born in Whangarei, but I've lived most of my life in Auckland. My mum was born in a small town, but she was sent to boarding school, the family was got together and they sponsored her to go to boarding school in Church College in Hamilton. She was probably privileged in that way. I think from that experience she's always used her education to benefit Maori people. She's now working at a Maori school.

I asked her about the attitudes of her teachers at school

Maths was ok but I remember actually one teacher which just really threw me off maths. I remember his name was Mr X and he'd just rattle it all off and I just felt like I was getting left behind. When I said I couldn't do it, he'd just rattle it off again. And I wasn't able to work through it properly and I just lost interest. I just distinctly remember him. I think that was my School C year.

She talked about after she left school and deciding to apply for the Wellesley Program.

After I left school I just did some factory work and then a delivery job, and I had a reception job with a hearing-aid company, and that's about it. The job that I was working burnt down and we got redundant and then I got pregnant. He was born in October 1993. He's four now - in his second year I joined up with the Wellesley Program.

Why? I suppose the small voice of my mother in my head and, being a mother myself, I thought I wanted to give him a few more opportunities. I've probably realised that I needed to do something to better myself, because I've become a single mum. I was on the DPB<sup>43</sup>, and I didn't want to stay on it. It was probably that, plus working when I was pregnant, I got real bad back pains. So I needed some other occupation. I just applied for the university prospectus. I read in there and whatever they had put in there I knew that fitted me. I wasn't too sure about tackling stage I papers, straight on, without doing, I hadn't done anything since I left school.

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<sup>42</sup> School C is an abbreviation of School Certificate, a national examination held at the end of the fifth form (age 15 years).

<sup>43</sup> DPB is an abbreviation for the Domestic Purposes Benefit, an allowance paid by the NZ government to mothers who were solo parents.

I was secretly applying, didn't let anybody know, because I wasn't too sure what I was in for. Then just sat the small maths test they gave us, and went to the interview. I was given approval at the time of the interview, so that was great! I was rapt! And it gave me that time to make plans for childcare. I used the Kohanga Reo<sup>44</sup>. It was convenient for me. It was so close you could just pop in. I found it hard just sort of leaving him there but he seemed to have enjoyed it, and being an only child.

I asked Tania if she would have taken mathematics in the Wellesley Program if it had been optional. She said "I probably wouldn't have taken maths if it wasn't compulsory".

We started off with the basics which was quite good. I still thought that something was going to pop up and I was going to lose it. I know how to add and subtract but, like who wants to know what  $x$  is, who the heck is  $x$ !

Her initial experiences attending the Wellesley Program classes at the university were quite daunting.

I actually found when I first came to University I couldn't even talk to people, or look at them really. I'd only stick to the Wellesley Program because I knew they were all on the same level, then somebody told me this little story and from that day it changed. There was 3 people and I whisper to this person this certain number and say to you 'guess what number I'm thinking of' and you'd say the wrong number and you'd say 'what was the number?' You see the only difference was that this person has been told. It doesn't make anybody special.

She talked about how attending the Wellesley Program has changed her social life and how she has studied with others in the Program.

Well I've got two sets of friends now. And I know when I was doing the Wellesley Program maths at home, doing homework and that, my friends and family were just - whoa! - like backed right off and start thinking that you must be getting clever or something like that, but I would just say - anybody can do it. If I can do it, anybody can do it. But then you've got a different group from around University, others, more valuable and confident to tackle it. There were a few of us, we'd get together when there was assignments, and we'd talk about it. Some were still lost as to how to do it but most of us could discuss it. We'd come up with different answers then we'd say "how did you come up with that" then they'd go through it - sort of get a sense of which was the most logical.

I asked her what was the most important thing that she learned about maths that year.

Probably that it just takes practice. I mean, being brainy or not brainy, it's not that. I always thought it was something that you were clever and if you couldn't do maths well you haven't got that sort of brain. I think it was probably the main thing, that it just took practice, repetition. Anybody could really do it, it's just a matter of trying yourself.

I asked how she now felt about her ability to learn maths. She said there had been a "big shift".

I had not thought of doing anything mathematical or whatever from 5th form, and now I've got the book, 'Statistics Without Tears'. so I know there's

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<sup>44</sup> Kohanga Reo is the name given to Maori immersion pre-schools.

been a big shift. With the Wellesley Program – (the tutor) was such a good teacher and if you didn't get it she'd just go over and over and over until the light switched on. All you had to do was ask. And probably with the others in the class, I felt we were at the same level. At school I suppose there were some that weren't, so it was hard to find the median pace. After doing the Wellesley Program I felt confident and I know if I don't figure something out, I'll go and get a book or something and try and come back. There are seven of us in the family. I'm second youngest. I'm probably the only one that's gone on to tertiary education. But they're all quite curious now! And with the Wellesley graduation, most of them came. And I got a few prizes, so that was good. I got third overall for arts students and second for geography.

I asked Tania to respond to the questions on the Mathematics Metaphor Questionnaire to describe her experiences learning mathematics, both in high school and in the Wellesley mathematics course. Her responses follow, in Table 7.9.

Table 7.9: Tania's responses to the Mathematics Metaphor Questionnaire.

Tania's High School Experiences	Tania's Experience In The Wellesley Mathematics Course
<b>Weather<sup>45</sup>:</b>  Whirlwinds. There were times when I probably was angry too at times. I probably played up a lot in class too, I did actually, to be honest. I remember going to his classes sometimes knowing that I wasn't going to do any work, I was just going to disrupt - only in maths.	At the beginning it was cold. I was quite happy to just stick to myself, not HAPPY, but I just didn't want to engage in any conversation when I didn't know what the hell was going on. Probably about two thirds of the way through I saw the sunshine because, by the time exams came, I was confident. If I thought about the sun, it would be like it was bright and happy.
<b>Food:</b>  Probably more hunger! Can't think of any food. When you're hungry, and anxious and there were things that you probably wouldn't normally do, forget food, I suppose.	Food.. Rice. It's not my favourite food, but I like it now and then, and a little bit fills me up.

<sup>45</sup> A typical question under each subject category would be: *If your experience learning maths at high school were weather, what kind of weather would it have been?*

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**Tania's High School Experiences**  
**(continued)**

**Tania's Experience In The Wellesley  
Mathematics Course**

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**Way To Travel:**

Bucking Bronco! You're going to fall off easily. I've been to watch it. You know how it starts off slow, you try to hold on and it just gets faster and faster - you eventually just fall off. It starts off slowly, it's not something that you're used to but you're able to hold in and just eventually fall off. Once I fell off I started to disrupt the class.

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I've got this picture of a snail in my head when you said that. It's not fast, it went slowly and you didn't have to be scared of anything.

**Kitchen Utensil:**

Potato masher! Just really separating a whole lot of things that I probably could have done, and separating that from some other things... it was all mushy!!! In my brain it just didn't work out. It was just all these words, figures and everything not making any sense.

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Probably a spoon, a medium sized dessertspoon, yes. A spoon that reminds me of dessert, nice things. I love dessert and it was a good experience.

**Garden Tool:**

Those sprinklers, you know, probably because I was the one who wasn't getting any water, but it was something you needed. It wasn't like a hose, not everybody got some, just a sprinkling of people.

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A hand mower, a push one, probably because I felt more in control of it. And it did a lot, it mowed down a lot of the reservations I had about maths.

As she made these comparisons she commented:

It's neat that I can compare something; it's probably a lot more light-hearted now that I can actually do maths, probably can compare it to something. If it were prior to the Wellesley Program, I wouldn't have known any better - you know what I mean? I've been shown two different ways of learning maths.

I asked her which one of these images she liked best and she chose the travelling metaphor.

Tania, at the time of the interview in 1998, was studying stage II, the second year, of the Bachelor of Planning, a four-year course of study.

Last year I just didn't get my bearings until the end of the second semester. Actually, I knew it was an area that I wanted to get involved with, just from reading the outline of the course, but I suppose we had to do all the basic techniques, and the history of everything, and you didn't actually do anything meaty until the second semester.

But now I look back and think 'well it was all relevant', and 'we needed it'. Oh, I'm enjoying it this year. Probably I've got more direction with the whole thing. I suppose just the papers are better and more interesting, because I really want to specialise in area resource management.

She noticed differences between the Wellesley Program and a university degree course:

I found that in stage 1 courses, people tend to be more competitive. They're younger too. We had to do population projection, and I just find that people are less forthcoming, they just say 'it's easy, just do this'. It was not as competitive in the Wellesley Program. I only have one friend that I can study with, the only one who's about the same age as me. So it could be that is the difference. There are probably half and half male and female. Maori? Just the two of us. I think there's about 35 in the course altogether.

I asked her whether, having studied some mathematics in the Wellesley Program, this had affected her study for the planning degree. She said that she felt

a lot more confident. Because I have gained so much out of the Wellesley program just being able to do it, firstly. I think I graduated with a B in maths and I think if I had just gone straight onto this course I wouldn't have got the basics. I had a mind block about them. So once I had finished the Wellesley program, I was a lot more confident to even try them.

My son is four now and is going to Kura Kaupapa<sup>46</sup> for half days, he's got a program for going from Kohanga to Kura, as long as there is somebody there to supervise him from the family. I could go in, but usually my sister goes in with him because she's trying to learn Maori as well. So she just takes him in and looks after him.

I asked whether the Wellesley mathematics course had been useful in her current course and she replied

Last semester we had a course on fundamental skills, it's actually quite a big bit of the overall degree really. So the research side of it, like getting statistics and being able to present them in different ways, I'm still learning.

She talked about how her first year had gone:

I managed to pass all my papers last year! Even from the first semester to the second semester my marks have improved. Although I've passed everything, I think I must have got about two C's in the first semester and ended up in the second semester with two B's and two B plusses. Just really want to keep improving those marks!

I replied "it's really nice when you can see yourself improving isn't it, very encouraging!" Tania responded:

Yes! Yes, and I think I'm going hopefully to get, not a paid job but, work experience with Waitakere City Council as a Maori Issues officer. So I'm hoping that'll help me too. Still a bit to do but I've changed from before the Wellesley Program I suppose.

Tania graduated with her degree in Planning in 2001 with honours. She had several job offers and chose one in a small city, working in the City Council in Maori planning. Tania's success in the first year of the Planning degree

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<sup>46</sup> Kura Kaupapa Maori is a Maori immersion primary school (students aged 5-12 years).

encouraged staff there to select another Wellesley Program student the following year into that degree, which has limited places.

#### **7.3.4 Comparison between Mary and Tania**

These interviews have allowed us to focus on these two students, and hence find some of the most telling data in the study of this course. Both Mary and Tania start the year as students in stream 3, a stream where students have, typically, achieved successfully in the Wellesley mathematics course. But a deeper look at both students' experience through these interviews has made us realise that this course has been a very different experience for each of the two women. What has made the difference?

Mary studied alone and spent hours not getting anywhere with mathematics problems. Tania organised a study support group to help her. Tania had a strong role model in her mother, who valued education and was a teacher herself. Mary's mother didn't expect her to do well in maths because she herself hadn't. There seemed to be more whanau (family) support in Tania's case, so maybe this enabled her to cope with the demands of a full time study program while parenting a young child compared to Mary, who found it very difficult. Moira said that Tania "gathered other students around her". Neither Pam nor Moira, the two teachers in the Wellesley mathematics course, remember Mary.

If there had been more explicit encouragement and help to form study support groups in the Wellesley Program, the experience may have been very different for Mary. What if Mary had found an opportunity to study mathematics in a one-to-one supervised study course as Chris did (see Chapter 8)? Such a course could have been a much more positive experience for her.

Mary and Tania responded, specifically and differently, to the use of metaphors in the interviews. Their comments were related to the different ways the metaphors were used.

For Mary they were used to illuminate her personal conceptions of mathematics, how she thought and felt about mathematics. Her response to their use was positive and, in fact, she found them useful as they had made clear to her the depth of her negative feelings about mathematics, of which she was unaware, which she then described as a "phobia". She became aware of the need to address these feelings in the future.

For Tania the metaphors were used to enable her to describe and compare her mathematics learning experiences, in high school and in the Wellesley

mathematics course. She commented that it would have been impossible for her to answer these questions in the past. She was pleased that learning mathematics again in the Wellesley mathematics course meant she was able to create these comparative analogies.

Some of the quantitative measures for Mary and Tania reflect their stories above. Mary's scores on the Mathematics Self-Concept Scale, a measure of belief in herself as a learner of mathematics, changed from an initial score of 11/40, the lowest in Stream 3, to a final score of 7/40, indicating her low mathematical self-concept decreased during the course. On the other hand, Tania's scores changed from an initial score of 13/40, similar to Mary's initial score, to a final score of 27/40, indicating that her mathematical self-concept had improved significantly during the course.

Mary and Tania's responses to the statements concerning their beliefs about learning mathematics sometimes differed. (Responses were requested on a Likert 5 point scale.) Both Mary and Tania 'disagreed' with the two statements *Maths problems can be done correctly in only one way* in March and also in October, and both 'agreed' with the statement *The best way to do well in maths is to memorise all the formulas* in March and also in October. However, while Mary and Tania both 'strongly agreed' with the statement *To solve maths problems you have to be taught the right procedure, or you cannot do anything* in March, Mary did not change her belief and Tania changed, 'disagreeing' with the statement in October.

This information indicates that Mary's approach to learning mathematics was instrumental and did not change during the course and that she lost confidence in her mathematical ability. Tania's responses indicate that her approach to learning mathematics has changed during the course, and she has gained confidence.



## 8 Individual Supervised Study and Discussion

*I've gone from a foggy mess to sense that a door has been opened and there is light. What we've done is gone back to the smallest door, but in opening the smallest door we've generated the greatest light.*

Charles, October 1995

One student's journey from viewing mathematics as "the most disgusting, unappealing building" to one "with form, balance and symmetry" is the focus of this chapter. This journey was supervised study undertaken by an intellectually able 33 year-old man, Charles, who feared and avoided mathematics.

### 8.1 Introduction

Charles contacted me by phone, early in 1995, after an appointment with a clinical psychologist. He had already contacted the Head of the Department of Mathematics who suggested he call me, as I had years of experience teaching students who had had a 'rocky road' in mathematics. Charles had seen an article recently about mathematics anxiety. His mathematical ability was "atrocious", he said. He had failed School Certificate Mathematics, a national examination at age 15 years. Mathematics was hard for him even in Primary School. He felt like he was "running a race with one leg", and was at a turning point in his career. He was in business management in the city and was a share broker before that. In 2002, he was still involved in the business sector, as a partner in a property development company, which he had established in 1997.

At this time I was working on my PhD research and could see that teaching an individual, and carefully recording the process and progress, could become an interesting part of my project. He was happy to be taught by me as part of my research and was willing for our sessions to be audio-taped. Large affective and cognitive changes in Charles, described by him as well as creatively and effectively captured using metaphors<sup>47</sup> (Table 8.3), indicate that the course was successful for him and hence very satisfying for me. In fact, results from the analysis of this data have become the most interesting and exciting part of my research. It is clear that, by the end of the course, Charles has "changed his

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<sup>47</sup> Using the Mathematics Metaphor Questionnaire (Gibson, 1994)

relationship to mathematics and broadened (his) view of what mathematics was and how (he) could learn it" (Buerk, 1994 p. 46).

A summary of my teaching approach<sup>48</sup> follows:

- I plan to work alongside the student, as we are in the learning enterprise together, learning from each other. I am trying to learn about the student's "values and beliefs about the nature and purpose of mathematics", "to understand (their) mathematics" and to "see things from their perspective" (M. Barnes, 1994 p. 9). At the same time I plan for them to experience that they are capable of learning mathematics;
- I acknowledge the student's autonomy as an independent adult by being willing to consider changing direction at any time, responding to requests from them, letting them take the initiative when they have the confidence or need to do this. I am willing to work together with the student on any mathematics from their workplace or home environment, encouraging them to bring such material to class. Before we meet each week, I usually prepare several related topics in mathematics so that I can change mathematical direction if it seems appropriate;
- I am willing to accept that I could make mistakes as a teacher. If I don't know the answer to some question, I am willing to find out and discuss it later.

In order for Charles to gain confidence and learn some mathematics appropriate to his needs I aimed to introduce the mathematical topics very carefully in the following ways:

- to choose topics that would inform me of his existing mathematical knowledge and strengths, to recognise this existing knowledge and be willing to learn from him, in particular, ways he uses mathematics already;
- to choose topics that would encourage and challenge but not overwhelm him, acknowledging any negative feelings this study brought up;
- to choose a variety of ways to explore these topics in order to engage Charles, so that he would start to enjoy exploring mathematical concepts and 'discover' some mathematical ideas, i.e. to encourage 'aha' moments. I wanted to impart the creative, intuitive aspect of doing mathematics.

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<sup>48</sup> A more detailed description of my beliefs, attitudes and my goals as a teacher are in Chapter 4, Section 4.4.

As I wrote the summary above, I became aware of how closely my teaching approach resembles that of “connected mathematics teaching” (Becker, 1995 p. 168). Categories of intellectual development were proposed in *Women’s Ways of Knowing* (WWK) by Belenky, Clinchy, Goldberg and Tarule (1987). These are explained more fully in Chapter 2, Sections 2.2.1.4 and 2.5.2. Brew (2001), Boaler and Greeno (2000), Erchick (1996) and Koch (1996) and have written about these categories in relation to perspectives in learning mathematics. In addition, Becker (1995), Buerk (1985) and Morrow (1996) have developed implications of these categories of intellectual development for the teaching of mathematics, connected mathematics teaching. For example, Becker (1995 p. 169) discusses connected teaching in mathematics in relation to the issues of “voice”, “first-hand experience”, “confirmation of self as knower”, “problem-posing”, “believing versus doubting”, “support versus challenge” and “structure versus reality”. Since my teaching approach seemed similar to that of connected teaching, it seemed appropriate to theorise the results of the analysis of Charles’ data through this ‘lens’. For this reason much of the theoretical discussion is included in this case study, alternating with the data analysis. Results will also be interpreted through the literature on adult learning and adults learning mathematics. This theoretical analysis of Charles’ data will be used to cast further light on elements of Mathematics 1 and the Wellesley mathematics course. The theoretical discussion of these two latter courses will appear in Chapter 9, Sections 9.2 and 9.3.

My initial teaching goal was to create a safe environment for Charles to re-commence his study of mathematics (Section 8.2) hence I encouraged Charles to talk about his experiences of, and feelings about, mathematics. I acknowledged and accepted his account of his previous mathematical experiences (Section 8.2.1), and his attitudes and beliefs about mathematics (Section 8.2.2). Mutual planning of the course, described in Section 8.2.4, was negotiated by discovering Charles’ mathematical expectations and needs, followed by discussions of my expectations of him. I interpret these results (in Sections 8.2.3 and 8.2.5) from the theory of connected teaching, adults learning mathematics and adult learning, as well as theory related to the use of projective techniques in the classroom.

Section 8.3 gives an overview of the mathematics taught in the course. Firstly, Section 8.3.1 contains an overall brief description of the topics covered in the course, from May to October 1995, in order to describe the curriculum followed and to show the development of his confidence and knowledge over the six-month period. I did not introduce new topics until Charles had shown me sufficient understanding of, or interest in, the current topic. Interwoven with these topics were discussions of his mathematical experiences, his culture,

interests and workplace mathematics. I interpret this “weaving together (of) multiple aspects of (his life)” (Morrow & Morrow, 1995 p. 18), in Section 8.3.2, from the theory of connected teaching and adults learning mathematics.

Section 8.4 gives some outcomes of this course, in particular, describing affective changes in Charles. Teaching one mathematical topic is described in detail, in Section 8.5, linking with other ideas in the theory of connected teaching and in adults learning mathematics in Sections 8.5.1 and 8.5.2. Section 8.6 discusses later contacts and developments with Charles.

## **8.2 Acknowledging Negative Experiences, Feelings and Reactions to Mathematics**

My initial aim was to acknowledge and accept Charles' account of his mathematical experiences and his attitudes to, and beliefs about, mathematics. I did this to create an emotionally safe environment for Charles to approach the study of mathematics. This supportive atmosphere is necessary in connected teaching because the student is encouraged to “take reasonable risks and be able to make mistakes” to, as Morrow and Morrow (1995 p. 20) describe, “gain a sense of their own voice in mathematics”. Charles acknowledged the success of this aspect of my approach (Section 8.4.3, Table 8.4:F). Typically, in curricula for courses (or books) developed for adults returning to study mathematics, the first topic is based on ways of acknowledging a student's mathematical attitudes, beliefs and experiences indicating the importance these authors attach to focusing on affective as well as cognitive domains (Brown, 1984; Buerk, 1982; Buxton, 1984; Langbort & Thompson, 1985; L. Taylor & Shea, 1996; Tobias, 1978).

To begin this process of acknowledgement of his experiences I asked him to write, and bring with him to our first session, his mathematics autobiography, a story of his mathematical experiences up until now, especially the support, or otherwise, of his teachers and parents. He was happy to read it to me during our first class. Some details of his mathematical experiences up until the present were also gathered using a Mathematical Autobiography Questionnaire (University of Central Queensland, 1992), in Appendix Y, which contained open questions.

In order to gather information about his mathematical self-concept and attitudes about mathematics he completed two questionnaires, the Maths Attitude questionnaire which was also answered by students in Mathematics 1 and the Wellesley Program, as well as a questionnaire which elicited his views of mathematics using analogies.

### **8.2.1 Acknowledgement of Charles' mathematical experiences in school, family and employment**

During the first four sessions I found that we spent a *significant* proportion of the time talking about his previous mathematical experiences and his attitudes to mathematics. While some discussion was initiated by me, as detailed above, Charles often talked about his experiences and resulting feelings, to make sure I understood. Interspersed with these discussions I carefully started to do some mathematics with him, even during the first session, using hands-on materials in a visual, geometric, investigation of multiplication. (Teaching this topic is explained in detail in Section 8.5.1 where some transcripts of our interactions are included.)

The summary of his background experiences by age, listed in Table 8.1, are a synthesis of his written responses to autobiographical questions with other comments he made during the first four classes.

Table 8.1: Charles' mathematics autobiography

AGE	Experiences
5-8 years	I was bored, disinterested, eventually resulting in a lack of real understanding of the concepts. I remember numbers and not really understanding their conceptual role but being reasonably comfortable with their practical value. I was experimenting with words to communicate and pictures at this stage. I thought visually and hence numbers were somewhat out of my realm of thought and interest. From this point I believe my understanding was weak. I recognised in myself that I didn't understand maths and I certainly didn't enjoy failing. My hearing was tested and found to be A1, yet the teachers merely assumed I wasn't applying myself, when in reality and on reflection I was disinterested and didn't understand.
10 years	I was failing tests (unless simple mechanisms such as multiplication tables as an example were learned by repetition). I remember studying for a multiplication test early one morning, memorising simply by repetition. The concept was entirely foreign and I was starting to feel inadequate. Now I understand these simple concepts. [He is referring to the mathematics we have started doing and that he is understanding it.]
13 years	IQ test for streaming, no problems whatsoever with English and comprehension, knew I was hopeless at maths, and therefore ended up in a lower stream than that which really I should have been in, which pulled me back enormously. On all other subjects I was bored because I could do them. I wasn't being extended. I didn't realise that because I was poor at maths, though good at English, that I'd been held back - this affected my self-confidence.
15 years	I was working to apply myself but failed School Certificate Maths. I did Chemistry for School Certificate which I failed largely because of the maths component. Maths homework was a complete nightmare. Despite listening attentively and taking the necessary notes, I could <u>never*</u> apply the concepts to a different set of numbers. It was totally frustrating! Despite assistance from Dad, I never really improved.

Table 8.1 (continued)

16, 17 years	The same applied as per age 15, failed UE and Bursary maths and nobody seemed to understand. I just didn't comprehend it. <u>I failed every maths exam throughout my secondary school career.</u> Therefore my association with and attitude towards maths was completely negative. <u>Application was not the issue, understanding was!</u>
17 years	Shop work - dealing with money, a little scary to start with, but in reality extremely simple. I recall thinking a lot about, what if I make a mistake, the well known feeling of numbers beating me, but they didn't. It was in reality simple and practical, and I actually enjoyed overcoming this fear.
20 years	After the completion of my 1 <sup>st</sup> degree, majoring in English literature, I was unable to complete my 2 <sup>nd</sup> degree (Commerce) because of the mathematical component.
21 years	I had to leave a fabulous job, sharebroking, trading and providing advice for clients on equities, fixed interest. I was offered an opportunity to become a partner at 21, but I had this total fear of maths and recall breaking out into a sweat at the prospect of undertaking this work. I was surrounded by numbers, and I just thought, I can't do this, this is ridiculous. But, in reality, they could see skills that were good for them in me, which I can recognise now but ....
30 years	Continuing fear of maths. There were experiences where I just completely freaked with maths! How could I possibly use a computer under pressure? By listening, observing, taking notes in my own language and being determined, I largely overcame this fear to become competent but it was an enormous effort. It took an unbelievable amount of energy. I can hardly explain it. I can never relax when dealing with those things. I have to be on my guard always. It's so tiring.
33 years	An experience I had in the past month: sensitivity analysis, and a feasibility study on a large block of land owned by the company whom I work for in Upper Queen Street. I recall thinking 'how can I do this?' When I'd complete any part of the exercise I'd review it and think 'how did I get this result? Is it correct? I'm not sure.' Then I'd review it again and again... In other words, complete insecurity in my own ability. In general my analysis was correct, however it took me forever to reach a result and the task seemed quite daunting.  Maths from a practical perspective, eg., profit or loss, shares, etc, isn't a problem. Conceptual maths is like an entirely bizarre language with which I have <u>no rapport whatsoever</u> . Continued failure with maths is frustrating, hurtful and demeaning. I wonder whether I have the maths version of dyslexia.

\* Any underlining of text in this table is that done by Charles.

Some extracts from the initial sessions with Charles follow, in order to give some idea of how I was responding to his comments, trying to acknowledge and accept his accounts of his background experiences, so that he would feel comfortable working with me on mathematics. For example, after Charles has finished reading aloud his autobiography during the first session, he said

- Charles: That's really true, it is just horrible. You imagine what it's like  
-  
Barbara: For so many years too.  
Charles: Oh, and it's not application.  
Barbara: Yes - and knowing you've tried very hard.  
Charles: I really have.  
Barbara: Yes, yes.

Later in Session #3 he referred to his background again,

- Charles: It's such a shame no-one - I had a chat to Mum and Dad about it actually, and they feel so bad because they're very caring, intelligent, communicative, aware, sensitive people. They just didn't understand.
- Barbara: Understand that you were having such a struggle?
- Charles: No-one really understood.
- Barbara: Mmm. Yes.
- Charles: You're the first person. Probably because you're studying the learning difficulties of adults in your research, how it occurs and how it can be remedied. Consequently you're sensitive to people's feelings, I would say, in respect of the subject.

His experience seems similar to that of the trainee teacher in a study by Carroll (1994) who attributed the cause of her failure in mathematics to teachers who identified the problem but did nothing to help her. Later, remembering how hard he had tried to learn mathematics at school, Charles adds,

- Charles: The application I put into my maths at school, I put into my other subjects. I would come first or second or third in these, and in maths I'd still be failing.
- Barbara: Yes, hitting your head against a brick wall.
- Charles: Yes.
- Barbara: Mmm. Well you really did try hard didn't you?
- Charles: Oh incredibly!
- Barbara: And you're willing to do it again?
- Charles: Yes.
- Barbara: I won't let you hit your head against a brick wall, OK?
- Charles: OK.
- Barbara: Alright.
- Charles: Deal.
- Barbara: OK, deal, yes (laughs), alright.

In all three extracts above I am attempting to understand and accept Charles' account of his experiences in mathematics. He acknowledges (in the second extract above) that I am the first person whom he feels has understood how "frustrating, hurtful and demeaning" his experiences have been. Johnston (1995) writes about Marie, whose memories of her school mathematical experiences at age 11 were similar, that of repeated shaming and repeated humiliation.

I took care to not blame Charles when he described these past experiences. I believed it was important to listen and accept his views about the impact of these experiences on his life. The results of not addressing his lack of understanding in the past was clear, his avoidance and fear of mathematics. He was hurt and puzzled by his past experiences, assuming that he was to blame, that he did not have the ability to learn mathematics. Zaslavsky (1994) also discusses this issue, that there are many factors in our society that may have led to a student's difficulties with mathematics so that it is important that they do not blame themselves for their negative feelings about mathematics. L. Taylor &

Shea (1996, p. 60) mention the anger felt at "being robbed of understanding" which led to Shea's avoidance of mathematics as an adult. Such avoidance of mathematics by adults, particularly women, is well documented in the literature (Buerk, 1982, 1985, 1996; Fennema, 1995; Tobias, 1978). Heckman and Weisman (1994 p. 32) also emphasize this point when they say:

The teacher ... recognizes that difficulties in learning are a result of past experiences of educational failure, distressing experiences such as ridicule or criticism that have disempowered the learner, lack of experience in the situated environment or lack of support in ... a school.

Alongside this acknowledgement of past mathematical experiences is the acknowledgement of the strength of his feelings, his beliefs about the learning of mathematics and his conceptions of mathematics (Section 8.2.2).

### **8.2.2 Acknowledgment of Charles' attitudes to, and beliefs about, mathematics**

Feedback on his attitudes to, and beliefs about, mathematics were explored on 29<sup>th</sup> May (Session #3) when he brought the Mathematics Metaphor Questionnaire (Buerk, 1996; Gibson, 1994)<sup>49</sup> which he had completed at home. This questionnaire is in Appendix Z. I went through the questionnaire with him. He gave graphic, profoundly negative, responses (see Table 8.2).

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<sup>49</sup> This protocol for the collection of metaphors was developed by Buerk and Gibson in 1988 and is used regularly by them in the classroom setting (Buerk, 1996).

Table 8.2: Charles' responses to the Mathematics Metaphor Questionnaire

The questions	Charles's responses in May
Imagine yourself doing or using maths, what does doing or using maths feel like? List all the words and phrases.	Using maths from a practical point of view, i.e., profit and loss is great. Maths for the sake of it, all that garbage: cosine, co-efficient, derivative, square root are just nonsensical, non-stimulating, academic gain. I still sort of look at it and I don't even know what it is really. I think it's a self-confidence thing, a lot of it.  On a mountain - skiing on blue ice, with no edges, blindfolded. That's what it feels like! [Barbara: That's frightening! Charles: It is frightening!]
Think about the things that maths is like.	A house with no doors or windows, you can't get out of it. A mad scientist - humourless, colourless.
If maths were weather, what kind of weather would it be?	Humidity which you can't escape, you shower and freshen up, five minutes later you're sticky, dirty, and uncomfortable. There's a bit of a theme coming through there - can't escape it.
If maths were food what kind of food would it be?	Tripe, the only food I can't eat, it's disgusting.
If maths were food how would you eat it?	I wouldn't, I loathe it.
If maths were a hobby, what kind of hobby would it be?	Counting grains of sand on a beach. Looking for an emerging pattern or formula because it's a futile exercise for me.
If maths were a way to travel what means of travel would it be?	In a leaky boat, because you'll never reach your destination.
If maths were a colour, what colour would it be?	It would be the absence of colour. Why? Because it offers no life.
If maths were a way to communicate, what way would it be?	A forgotten dialect, making communication, two-way communication, impossible.
If maths were an animal, what kind of animal would it be?	A hyena. Why? Because it scavenges my self-confidence.
If maths were a building what kind of building would it be?	The most disgusting, unappealing structure in history. Maybe a prison, white, grey and ugly.
If maths were a plant, what kind of a plant would it be?	Miles of indoor house plants, everywhere. I just can't stand that.
If maths were music what kind of music would it be?	Head banging, non-rhythical rubbish.
Write a paragraph using a word that best describes what maths as a subject is like for you.	For me maths is most like a scavenger-predator, rearing its head when I least want it. Always succeeding in removing my self-confidence and sense of self. It brings me down, holds me back, confuses, and upsets me. I hate it.

An extract of the dialogue following his powerful negative images about mathematics<sup>50</sup> (Buxton, 1984) is given below to show how Charles and I completed this discussion. This extract also illustrates the two-way nature of our dialogue, because he knew that this information might be useful for, and used in, my research.

- Barbara: Mmm, yes. Well, we're going to have to work on that, aren't we? We will work on it.
- Charles: OK. You can see now why I want to know whether it is a black hole - they're my genuine feelings. 99 percent of people, when they say they're not good at whatever, they maybe can be taught - but these feelings are pretty strong.
- Barbara: Very strong, yes, I agree. Well, we'll just go slowly because there are some strong feelings and it will probably be difficult for you.
- Charles: Well I'm enjoying it so far! Primarily for two reasons: First, I've actually learned something, and secondly, you understand. You're the first person who I've come across who can genuinely understand. It's a huge relief. Is this useful stuff for you?
- Barbara: Oh yes, thank you very much. I do appreciate it.
- Charles: OK, you're welcome. I just hope it helps you and me. It obviously helps you understand me and maybe it will help you understand, if ever you come across somebody else in a similar situation.
- Barbara: Sure. Thank you.

In this extract Charles is acknowledging that he is beginning to enjoy and understand mathematics and is also recognizing my acceptance and understanding of his feelings. Wlodowski (1985, cited in Knowles et al., 1998 p. 149) believes that one of the four factors that increase adult motivation to learn is enjoyment, that they need to "experience the learning as pleasurable".

To obtain a quantitative measure of Charles' attitudes to, and beliefs about, mathematics, during the fourth session Charles completed the Mathematics Attitude Survey which was part of the questionnaire completed by Mathematics 1 and Wellesley mathematics students (see Appendix P). His initial scores (June, 1995) on the attitude scales (out of 40), and the mean (and standard deviation) of these scores for Mathematics 1 students in March 1995, were:

	Charles	Mathematics 1
Enjoyment of Mathematics	14	24 (6)
Value of Mathematics	27	30 (5)
Mathematical Self Concept	6	21 (6)

These initial scores for Charles showed that he had little enjoyment of mathematics but a positive view of the extrinsic value of mathematics. He has

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<sup>50</sup> The powerful negative responses given by Charles are similar to those experienced by Buxton (1984) when he was teaching mathematics to an adult student who was anxious about mathematics. He makes the response, "you want me to know how bad you are, and then accept you?" The student replies, "Yes, yes that's it exactly".

little belief in his own mathematical ability. These scores are well below the means for Mathematics 1<sup>51</sup> students except for the measure of the extrinsic value of mathematics, which is similar. The range of scores on this test when given in March to Mathematics 1 students is listed in Section 8.4.2.

### **8.2.3 Understanding where the student is affectively to create a supportive environment: the theory**

Taking plenty of time during the classes in the first few weeks to acknowledge Charles' current feelings about mathematics and past experiences dealing with mathematics, enabled me to reach my goal of developing a rapport with him. An emotionally safe environment was created for him to start learning mathematics. He acknowledges this in his analysis of the six-month course, entitled "What Has Made The Changes" (in Section 8.4.3, Table 8.4:A) saying

Barbara approached my problem, which was very real, in a thoughtful, gentle and completely encouraging manner. She was able to empathise with me and fully understand what had been an on-going and seemingly never ending horrible experience.

He said, several times in the first few weeks, that I was the first person who had "understood" these feelings and experiences. His relief at being understood was palpable, which supported my belief, similar to others (Becker, 1995; Buerk, 1982, 1985, 1996; Damarin, 1990; Goolsby et al., 1987; Morrow, 1996), that addressing affective issues early in (and during) any course of study is important. As Damarin (1990 p. 149) states, "to fail to recognize a student's anxiety, uncertainty, and concern about whether (they) are mathematically inferior is to deny an important part of the mathematical reality of the student". Similarly, research by Goolsby et al (1987 p. 11) on high risk university students studying mathematics leads them to emphasize that mathematics instructors and counselors "need to focus on strategies which build confidence as well as those which reduce anxiety for high risk mathematics students", that "it is imperative that instructors focus attention on both the affective and cognitive domains". This view is supported by Stage et al (1998 p. 1) who reviewed literature on learning theories and frameworks applicable to the instruction of undergraduate students as well as other relevant research. They comment:

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<sup>51</sup> I have chosen to compare his scores with those of students in Mathematics 1 as I believe he is similar to this population, rather than the Wellesley student population who were let down by their education generally, rather than just mathematics.

researchers have demonstrated that constructs related to self-efficacy are positively related to achievement, and, in several instances, classes designed for low-achieving students that focused on developing self-efficacy as well as academic learning experienced dramatic successes.

Evidence of the dramatic success experienced by Charles is in Sections 8.4.2 and 8.4.3. Charles' understandable fear of mathematics was expressed very clearly and particularly vividly when he used metaphors<sup>52</sup>. His metaphors for mathematics were collected using a protocol developed by Gibson (1994) and Buerk (1996). Buerk has informed my thinking on the use of metaphors in mathematics teaching and in educational research (Buerk, 1982, 1985, 1996). She describes one value of gathering students' metaphors is that "the experience and ensuing discussion broadens mathematics to include language, imagery and reflection" (Buerk, 1996 p. 27).

Many of Charles' metaphors illustrated extreme fear, several containing images of death and destruction, for example: *What does using or doing maths feel like?* "Skiing on blue ice, with no edges, blindfolded", or, *Think about the things that maths is like:* "A house with no doors or windows. You can't get out of it." As with many other (adult) students, the metaphors were about "intense feelings of anger, despair and frustration", viewing mathematics as "something beyond (his) control" (Gibson, 1994 p. 8). Learning mathematics had been a "demoralizing" experience and he had felt "manipulated by mathematics", an "object of humiliation" (Buerk, 1996 p. 28).

Charles expressed satisfaction with the way the metaphors had captured the intensity of his feelings, as did Buerk's (1996) students, and he emphasized to me how genuine these feelings were. This illustrated how metaphoric language is an "extraordinarily powerful linguistic tool to express meaning" (Provenzo et al 1989, cited in Briscoe, 1991 p. 186). The use of metaphors "takes advantage of the metaphor's projective properties, its synthesizing function, its generality i.e. its remoteness from specific problems" (Sims, 1981 p. 402). It allowed me to understand the intensity of his feelings which were not as easy to describe literally (Bowman, 1995). I believe, as do Buerk (1996) and Gibson (1994), that it is important to accept as genuine, and not belittle, the feelings that these images express. After I had read his metaphors Charles said "you're the first person who I've come across who can genuinely understand – it's a huge relief". So, as Buerk (1996 p. 27) found with her students, the experience was therapeutic for Charles, a "catharsis", a "relief", "somehow freeing to get the negative feelings out" and to have them accepted. (Further discussion about the relief

<sup>52</sup> I will use the word 'metaphor' in the "broadest sense", as Buerk (1996 p. 27) does, to mean "any comparison between two objects, ideas, concepts, or experiences".

experienced by maths-avoidant students is in Section 8.4.1 (Buxton, 1981)). By using metaphors he also became much more aware of my concern for him as a learner (Gibson, 1994) and my empathy and regard for him. It changed the way we “viewed each other” (Gibson, 1994 p. 10). I realised Charles was extremely fearful of mathematics and therefore of the need to be extremely careful introducing mathematics to him again. I was able to more appropriately meet his needs because, as Buerk found, metaphors can give clues about the students’ “learning strategies” and their “conceptions of mathematics” (Buerk 1994, cited in Jackson, 1995 p. 5). Charles showed no understanding of the creative, human, or conceptual aspects of mathematics. While “practical maths, e.g. profit and loss, shares, is not a problem”, Charles wrote at the start of the course, he had “no rapport whatsoever” with “conceptual maths”.

Acknowledging students’ feelings about mathematics and past experiences dealing with mathematics was, in some ways, easy to achieve with Charles because the one-to-one teaching situation was so suitable for him. On the other hand, it was also hard because of the extreme fear of mathematics which he expressed. It is usually much more difficult to develop such a rapport with students in a larger group.

The next theme in the data analysis, covered in Section 8.2.4, is linked to another of my aims, discovering Charles’ reasons for taking this course and his mathematical expectations and needs. I also needed to clearly impart my expectations of him.

#### **8.2.4 Clarification of Charles’ expectations and needs and my expectations of him**

In the following extracts from early classes it is clear that his decision to seek help to try removing the block he had to mathematics was extremely important to him. His early mathematical experiences had damaged his self-esteem. For Charles, mathematics was most like a hyena,

a scavenger-predator, rearing its head when I least want it; always succeeding in removing my self-confidence and sense of self. It brings me down, holds me back, confuses and upsets me. I hate it.

One of the “most potent motivators for adults” undertaking study is “internal pressures”. An important reason for him to study mathematics was the desire for increased “self-esteem” i.e. it had personal value (Wlodowski 1985, cited in Knowles et al., 1998 p. 149). Other motivators for adults will be discussed throughout this chapter. Motivation is one of the six current key assumptions underlying andragogy (see Chapter 2, Section 2.2.1.1).

During our first appointment on May 15 he mentioned some of his strengths and discussed other reasons why he wanted to study mathematics. Charles said he "loved language, economics, creativity, colour, form, business", "especially colour and form and trading, they're instinctive". He continued:

If somebody threw a group of colours at me, I know what goes well with it, just instinctively. On the one hand there's this business trading thing and on the other is this creative side. To really succeed on the business side, I think that I really need to understand maths. Now I could run with my creative side, and I may still do so, however if I do it now I'm doing so without a choice. I have no choice at the moment.

The second time we met (23<sup>rd</sup> May) he discussed the "enormous amount of change" that he was going through in his personal life then. A serious relationship had broken up, he was selling a house in Wellington, he was going to resign his job and take a holiday because he "needed" it. He was "coping very well" and had "found a lot of strength which (he) didn't think was there". He again talked about the lack of choice of career options because his maths was "abysmally weak" and continued:

In order to make a valid career choice I have to either say 'OK, it is, for want of a better description, that horrible black hole, or it's not'. But if it is that horrible big black hole, then I have to say 'it's there Charles, it's a reality, you've got to live with it, you've got to work with it'. I've reached an age (32-33) that I have to start making some pretty serious choices. I have to find out whether there is a choice because, if there's not a choice, OK, I do have other skills which are quite strong and are undeveloped. Can you see the importance of this? I mean it's actually quite a big life decision.

Later, during this second appointment, he came back to check that I realised how significant this chance was for his career. I also emphasized that it would take a lot of effort on his part.

- Charles: Barbara, you do understand the significance of what I'm doing? I don't mean to labour it but it is really quite significant.  
Barbara: Yes, I understand that. And I think we can get a long way together, particularly if you're really determined (Charles: I am) and you're willing to put time in, because it could be hard, you know. [I am making explicit my expectations of Charles.]

During the third appointment (29<sup>th</sup> May) Charles again emphasized the importance of this course to him and said:

I'm making a major career decision - if you've average skills in maths, for example, and strong in others you will succeed. But you can't have the A and the E. You can have an A and a C. It's just soul destroying. You're understanding more now I think. I can see the benefit of you getting me to do this stuff. It took quite a while to think about it, but it's good you know.

When asked about some of his concerns about re-commencing the study of mathematics, Charles replied:

- Charles: How long will it take before we can adequately assess whether it is merely learning gaps or a black hole. I want to get on with my career. I want to make a choice and feel comfortable with it. I don't want to really spend a year trying to work out if it is this big black hole.
- Barbara: I would doubt very much that it's a black hole, but also I don't want to minimize how much time it might take to get you moving again but, as you said last time, already you felt that you'd learned something.
- Charles: I have. [Charles is acknowledging that he is starting to learn mathematics.]
- Barbara: So we have to take it gradually. I have to get a feel for your needs in mathematics too, and we'll be able to just look at those topics.
- Charles: It'll be you who I'll be looking for guidance from, in terms of what we need to do, because you'll recognise it.
- Barbara: That's fine. We need a bit of time to work, before I can - we can - see the rate of progress, alright?
- Charles: I just want to obtain an average level of mathematical ability which will make me feel better about myself, which will enable me to progress, I believe, on the business side.
- Barbara: Sure, sure. Yes that's alright. It just does take time, and it does take effort. But you're prepared to do that? [I am making explicit my expectations of Charles.]
- Charles: I'm fully prepared. I wouldn't take on anything else for the year if I'm doing this. I'll just study.
- Barbara: I'm pleased about that, it means we can work hard together. In the future, I'll give you even more homework, as you can cope. [I am making explicit my expectations of Charles.]

To summarize, firstly, I am aware of the significance of this course to him and that this course is a means to achieve future change. Secondly, negotiations between Charles and myself have clarified what we each require to reach our goals. He has stated his goals and that he is prepared to put a lot of effort into studying mathematics, acknowledging that he is starting to learn mathematics. I have also discussed my expectations, the effort I expect of him outside of class time while doing this course.

The theory on adult learning is the main theoretical support for this aim to understand where an adult student is developmentally and to negotiate with them about plans for their study and is discussed next, in Section 8.2.5.

### **8.2.5 Students' goals and mutual planning: the theory**

Adults may look to education to help them to understand self and others somewhat better and to revise a personal narrative as part of the process of rebuilding and constantly reshaping a life.

(West, 1995 p. 154)

Charles has immediately made it clear that he urgently needed to know whether he had any ability to do mathematics, bracing himself to find out if he had "a black hole" where understanding mathematics was concerned, because of the continuing negative impact this had on his career. That Charles' mathematical needs and goals were discussed early in the course is in

agreement with Barnes (1994 p. 3) who says, we must "take into consideration the student's goals". Charles was at a turning point in his career, that he felt like he was "running a race with one leg", supporting Sells' (1979) view that mathematics is a critical filter, affecting educational and career choices. It is clear that other typical "potent motivators" for adult learners are present for Charles; "increased job satisfaction and quality of life" (Knowles et al., 1998 p. 149). According to Houle's (1988, cited in Merriam & Brockett, 1997 p. 132) categories of reasons for participation in education, Charles is "goal-oriented learner" i.e. he is "participating to meet specific objectives". L. Taylor & Shea (1996) document the experience of an adult student (Shea) whose lack of competence and confidence in mathematics was standing in the way of her career goal, to become a primary school teacher. Charles wanted choice in his life, to not feel forced into making decisions because of his avoidance of anything mathematical. As is common among adults returning to study, he has investigated the benefits that he will gain from the study, if successful, and the negative consequences of not doing it (Tough, 1979, cited in Knowles et al., 1998 p. 64). This course was a "means to achieve future change" (Coben et al., 2000 p. 19), a common theme among adults learning mathematics. It is known that for many adult students "a major life change, transition, or developmental task is probably involved in the decision" to return to study (R. Smith, 1990 p. 50). This is a rich source of their "readiness to learn", another of the six key assumptions underlying andragogy (Merriam & Brockett, 1997). Charles was at a time of transition in his life.

Negotiations between Charles and myself early in the study course clarified what we each require of each other to reach our goals. He has asked for guidance from me about what he needs to study and I suggested I need more feedback about his mathematical needs and will require him to devote time to study outside class time. As is typical with adult students, he has a "high level of motivation" which I have utilized so that he is "a full partner in the process" and this partnership is a "base on which to improve (his) learning" (Mullinix and Commings 1994, quoted in FitzSimons & Godden, 2000 p. 19). In connected teaching there is a similar emphasis, "the teacher and students engage in the process of thinking and discovering mathematics together" (Becker, 1995 p. 168). "Mutual planning" relates to another of the six key assumptions underlying andragogy, (Knowles et al., 1998 p. 135) that adults "need to know". Knowles et al (1998 p. 133) lists three dimensions to the "need to know"; "*how* the learning will be conducted, *what* learning will occur and *why* this learning is important", all issues discussed by Charles and I.

Andragogy's core adult learning principles take the learner seriously. They go beyond basic respect for the learner and view the adult learner as a primary source of data for making sound decisions regarding the learning process.

(Knowles et al., 1998 p. 183)

I have engaged Charles "as a collaborative partner for learning" (Knowles et al., 1998 p. 133) meeting the andragogical principle of the "need to know". Another of the six aspects of Knowles' et al (1998 p. 64) andragogical model was the "learners' self-concept". Since "adults have a self-concept of being responsible for their own decisions, for their own lives ... they resent and resist situations in which they feel others are imposing their wills on them" (Knowles et al., 1998 p. 65). This also addresses issues of adult motivation. Wlodowski (1985, cited in Knowles et al., 1998 p. 149) believes that adult motivation to learn is the sum of four factors of which one is "to feel sense of choice in their learning". This mutual planning could also be regarded as an "autonomy-related" aspect of a course (R. Smith, 1990 p. 48). The "mode" of this independent study course was in part "self-directed" (requiring the most autonomy of the student), in part "collaborative" (between the student and myself as we will learn from each other) and in part directed by me (the facilitator of an appropriate curriculum). The topic of self-direction is a major topic in adult learning theory.

The next section of this chapter (Section 8.3) describes the mathematics content that was covered in this course, held in the second half of 1995. The pace of the course was dictated by the time it took for Charles to understand topics. Topics were also suggested by Charles, for example, he requested to learn how to do long division (of a multi-digit number by a single digit number), which is not necessarily a topic I would have planned to spend time on.

### **8.3 Teaching the Mathematics: An Overview**

The mathematical content of the course is outlined in Section 8.3.1, which follows, including key comments Charles made which illustrate his affective changes and growing understanding. (Guidelines I typically follow to introduce mathematics to a mathematics avoidant adult student such as Charles are listed earlier, in Section 8.1.) Later, in Section 8.5.1, are details of the interactions between myself and Charles while exploring the first topic, a geometrical investigation of patterns in the number system, to illustrate my teaching approach and how Charles responded.

Notation used in Section 8.3.1:

Italics type: *Teaching techniques and the atmosphere engendered.*

Underlined type: Charles' growing confidence and his first enjoyment of mathematics.

Bold type: **The mathematical understanding Charles is gaining.**

Bold and Italics: ***'Aha' moments of discovery by Charles.***

### 8.3.1 The mathematics content of the course

Topics covered from May to October 1995 in this independent study course are now briefly described. Each weekly session (of 1-2 hours) began with talking about homework that had been set the previous time, discussing successes and difficulties.

In choosing mathematical topics, I took into account Charles' needs and interests. I was aware that the main use he made of mathematics in his work was arithmetic. He said "maths from a practical perspective, eg. profit or loss, shares, etc, isn't a problem", so I decided to start by looking at patterns in our number system, using an investigative approach. As he was possibly unfamiliar with these patterns, exploring them would help him in his work, and it would enable me to assess his background mathematical knowledge. I thought it might interest him to look at some of the history of number systems, which was the topic I introduced next, which would also increase his knowledge of the decimal number system. Following this we worked on some decimals and fractions. I also realized that he was very fearful of algebra, as he said "conceptual maths is like an entirely bizarre language with which I have no rapport whatsoever". I thought it would greatly increase his mathematical confidence to become familiar with the beginnings of algebra, to generate, understand and use simple formulae. I had a lot of experience to draw on in making this decision as I have been very interested and satisfied to see many adults gain confidence and enjoy being able to do some algebra, a topic they usually have never understood.

As described previously in this chapter, during the first four appointments we discussed his experiences, beliefs and attitudes, his and my expectations of this course of study, as well as beginning to do some mathematics.

During the first appointment on 15 May, we spent time reading and discussing his maths autobiography (see Section 8.2) but also started some number theory - an investigation of multiplication (M. Burns, 1987; Stenmark et al., 1986)

through a geometric investigation of rectangular arrays (see Section 8.5 for details).

During the second session on 23 May Charles discussed, at some length, how this course was part of a very important decision to do with his choice of career. We discussed his mathematical needs and the amount of work this would entail (see Section 8.3). We continued the investigation of multiplication, making a start on forming the multiplication chart from rectangles we had cut out.

On 29<sup>th</sup> May, our third meeting, we read and discussed his responses to the Mathematics Metaphors and Mathematics Autobiography questionnaires (see Section 8.2). We completed more of the multiplication chart started last week and homework continued this topic by looking at multiplication designs in this chart.

At the fourth session, 6<sup>th</sup> June, he completed the Mathematics Attitude scales (the same as those used for the other two courses in this study). We reviewed factors, multiples, primes, even and odd numbers, patterns in multiples of 10, 9 and 3: he asked to do some division of larger numbers, so we tried division by 10, then by 3.

He completed marking multiples of 9 on a multiplication chart before our fifth session on June 12 (see Section 8.5); we practiced division again, on request. Some topics on the history of number systems were introduced; Babylonian, Chinese and Roman systems (Joseph, 1991; Marr & Helme, 1987); we discussed powers of 10.

Some weeks elapsed before we met again on 17<sup>th</sup> August, as we both had been away for a month, so topics covered previously were reviewed. This review led to a discussion of Scientific Notation using a scientific calculator; the decimal number system (grouping by 10's, 100's); multiplication and division by multiples of 10.

At our seventh class, on 22<sup>nd</sup> August, we discussed the meaning of whole numbers (Frankenstein, 1989), at one stage we used base 10 blocks. Charles commented that already he has noticed that when he has mathematics to do he says to himself 'I will be able to do it', rather than 'I don't know if I can do it', an indication of his growing confidence.

On 1<sup>st</sup> September we discussed whole numbers again. Charles had brought an example of the mathematics he was using at work and explained it to me. He was going to have a style of outdoor chairs made and was going through the

process of pricing them. The cost was \$355.00 and sale price was \$536.05 (+ GST<sup>53</sup>). He calculated the “margin” by dividing the “markup” by the sale price. For his chair the “margin” was 33.77% (which he immediately expressed as 0.3376). He knew that a 50% “markup” yielded a 30% “margin”. His confident analysis of this situation seemed to be an example of “situated abstraction”, where the context which an adult is very familiar with “provides a well-defined and crucial anchor for the mathematical model” (Hoyles & Noss, 1998 p. 76).

We then started algebra by exploring number patterns, using ‘function machines’ as a model (Britt & Hughes, 1985a; Brooker et al., 1990; Langbort & Thompson, 1985), initially patterns from word to word (for example, a machine which ‘outputs’ the last letter of the word which in ‘input’), then word to number, and finally number to number function machines. **He was very excited by this work, commenting that another word for algebra could be “associations” or “connections”, which indicates his growth in mathematical understanding and confidence.**

At the beginning of the next session on 4<sup>th</sup> September he told me one of the homework problems had “stumped him”. I found that it was impossible to do! I was very apologetic as it had affected him greatly and he had felt very discouraged. I encouraged him in the future to just leave such problems and bring them in for me to see at the next class. We then discussed some problems using decimals, fractions but returned to algebra on 8<sup>th</sup> September when I introduced tile patterns (Britt & Hughes, 1985a), how to describe them algebraically, and algebraic shorthand e.g.  $a \times 2 = 2a$ . He commented that he has “gone a long way – especially on the fear side – I think I’m going to enjoy algebra” i.e. affective change has occurred. We looked again at the pricing of his chair - markup and margin calculations. We discussed decimal representation of tenths and hundredths, and he completed exercises on this topic for homework.

During our eleventh session on the 14<sup>th</sup> September, we worked through problems on decimal fractions, tenths, hundredths, and thousandths. Introductory exercises using graphs were worked on together early next class.

We returned to algebra again for the session on 18<sup>th</sup> September, spending the remaining four teaching sessions of the year exploring patterns and how to describe them in different ways: in words and algebraically.

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<sup>53</sup> Goods and Services Tax in New Zealand, currently at 12.5%.

A start was made on using a spreadsheet for generating functions. There was an educational reason for my use of spreadsheets (Healy & Sutherland, 1991) but they are also commonly used in his workplace. After I showed him how to enter some formulae, he entered a formula for me to guess. *Then Charles took the initiative, thinking of the idea of entering one function followed by another.* We entered some into the spreadsheet, which led to the need to introduce the convention for the order of operations (using the mnemonic BEMA). We investigated  $R = (I + 3) \times 2$  and  $R = I \times 2 + 3 \times 2$ . He was very excited by this topic.

During our thirteenth meeting on 27<sup>th</sup> September we worked on the spreadsheet again, practicing entering a formula and 'filling down' to generate a sequence of numbers. He said "Is this algebra?" and when I said that it was, he wanted to know where it was used and then said "I am enjoying this – this is the best thing that has happened to me this year!" For homework: more practice at writing down the algebraic representation of one function machine followed by another.

We discussed again, on 2<sup>nd</sup> October, how algebraic formulae incorporated the convention for the order of operations. *He commented that when he gets "stuck" now, he just leaves it to discuss with me. He doesn't feel as "devastated" as he used to be when this happened – affective change has occurred.* He feels like "we are covering a lot", and we are. For homework: finding patterns in Pascal's triangle and substituting values in simple linear formulae.

We discussed the 'row sum' in the Pascal's Triangle initially on 9<sup>th</sup> October, our fifteenth class, used the spreadsheet, explored the recurrence formula for this pattern, but this quadratic pattern was too difficult for him. I decided to return to the previous examples of function machines and we worked on these and he could write clearly, in words, what each formula meant. Homework: finding two types of formulae (universal and recurrence) to describe patterns done previously.

He made an extra appointment for 13<sup>th</sup> October as he was confused about recurrence formulae. We looked at the function machines explored previously, the universal formulae for these, eg  $r = 2i + 6$ , etc and then explored the recurrence formulae for these examples, eg  $r_1 = 6, r_n = r_{n-1} + 2$ . We explored both formulae on a spreadsheet also and expressed the formulae in words. He planned to go through an early handout on function machines and develop recurrence formulae for them. Plan for Monday - our last session - we reviewed all we had covered, his suggestion, and explored metaphors for describing his feelings about mathematics, my suggestion which he immediately responded

positively to saying, they "would be very different", the course had been "rewarding - very" and he had a "great feeling of accomplishment".

The final appointment for the year was on 16<sup>th</sup> October. It was our seventeenth session. We discussed how this course had affected him and he says "I felt a cog go in my brain for the first time". He now "knows he can (do maths), given time – before it felt impossible". He completed the Mathematics Metaphor Questionnaire, the Mathematics Attitude Scale and the Mathematical Self-Concept Scale again. For homework he has agreed to write two pages explaining what aspects of the course he thinks have helped him make the changes in his beliefs about, and attitudes to, mathematics, enabling him to learn mathematics (see Section 8.4.3).

To summarise, by the end of the 6-month course, Charles has now understood patterns in our number system, some history of number systems, and more about the decimal number system. He has also understood the beginnings of algebra, to generate, describe and use simple formulae.

A thread running through all the preceding four themes is that Charles is the context of this supervised study course in mathematics. I considered that his previous experience and current experiences were an important aspect of the course and interpret this goal from the theory of connected teaching, adults learning mathematics and adult learning next, in Section 8.3.2.

### **8.3.2 Contexts used in the mathematics course and acknowledging the prior rich learning experiences of adults**

Charles' experiences, culture and interests are the contexts that I use in one-to-one teaching as well as real problem solving examples from his work experience. I encouraged Charles to bring examples of mathematics used by him in his workplace. He brought examples of his financial planning, costing/pricing etc. I believe that "weaving together objective and subjective knowing in mathematics" (Becker, 1995 p. 170) allowed the course to become a "meaningful educational experience" (Morrow & Morrow, 1995 p. 18) for Charles. This is a characteristic of connected teaching. Morrow and Morrow also explain, if "students (are) encouraged to build on their entire knowledge base rather than leaving all personal experiences at the classroom door" (Morrow & Morrow, 1995 p. 18) this enables them to construct their own knowledge. There was an acceptance of his culture as a young man in business as the class context. This aligns with Friere's (1976, cited in FitzSimons & Godden, 2000 p. 20) view that educators need to "refrain from imposing their values on learners, and see themselves as co-learners, learning about the culture of the people among

whom they are working, mutually responsible for growth and change". I thought it was important to consider Charles' "coping mechanisms", "the way that (he) managed situations in life", which D'Ambrosio (1985, cited in Boaler, 1993 p. 345) emphasizes in his cultural perspective of mathematics. I believe that "mathematical meaning" for Charles was possible because he was making "connections between (his) knowledge and present knowledge" (Bishop 1985, cited in Boaler, 1993 p. 344). This connection is possible, Boaler (1993 p. 344) believes, when "mathematical activities emphasise the learner's involvement with mathematics rather than the teachers presentation of content and when communication, negotiation and the resulting development of shared meanings are a part of the process".

Explicit recognition and valuing of his prior knowledge and experience was an essential aspect of my teaching approach i.e. the course was student-centred. Charles states that an important aspect of this course was "encouraging my memory and actually saying on many occasions, 'you'll probably find you realise more than you first thought'" (Section 8.4.3, Table 8.4:D). Other researchers emphasize the need to acknowledge adult experience. For example, helping "adults to recognise the mathematics that they can do and build on it" is one of the key points in Coben's (1996 p. 5) "agenda for adult learning in mathematics for the next millennium". This recognition is important as

self-assessments made by mathematics anxious students are not necessarily realistic indicators of ability yet those students perpetuate their low self-concepts by failing to recognize their mathematical accomplishments.

(Goolsby et al., 1987 p. 5)

Gourgey (1984 p. 15) also makes this point when she writes that students' "self evaluation and anxiety level are not realistic assessments of their ability". Charles was an example of such a person. Typically, adults can be quite blocked about mathematics in a formal setting while competent in everyday life because they do not realise the amount of mathematical experience they bring to a mathematics class (Cockcroft, 1982; Fitzsimons, 1994; Harris, 2000; Noss & Hoyles, 1996). FitzSimons and Godden (2000 p. 16), in their survey of research on adults learning mathematics, write that "from the perspective of mathematicians, adults are using complicated mathematical ideas and techniques" in the workplace and in everyday life, but are "seemingly unaware of their significance in mathematical terms".

Knowles et al (1998 p. 65-6) include, as one of the six parts of Knowles' andragagogical model, "the role of the learners' experience" because adults have "both a greater volume and different quality of experience from youths" and the

"richest resources for learning can reside in the adult learners themselves". However, in any group of adults there will be a "wider range of individual differences than is the case with a group of younger people ... hence, greater emphasis in adult education is placed on individualization of teaching and learning strategies" (p. 66). Hence a one-to-one course such as that with Charles has a great chance of success. However, there are potential negative effects of adults' prior experiences as adult minds can be closed to "new ideas, fresh perceptions, and alternative ways of thinking". The most effective "adult educators try to discover ways to help adults examine their habits and biases and open their minds to new approaches" (p. 66). Schon, (cited in Knowles et al., 1998 p. 140) as a key researcher in this area, refers to this as "double-loop learning" where learners are required "to change their mental schema in a fundamental way" because the learning doesn't "fit the learners' prior experiences or schema".

I aimed to help Charles become aware of his own thinking. I also found it very interesting to gain an insight into his thinking because this knowledge helped me to encourage him in mathematically productive ways. All these aims are characteristics of connected teaching (Becker, 1995; Morrow & Morrow, 1995). He was "in charge of the interaction" but my responses provided "a 'safety net' whereby (his) misconceptions (were) recognized and addressed" (Morrow & Morrow, 1995 p. 19). Barnes (1994) also advised bridging mathematics educators to deliberately seek to find out where their students' misconceptions are, in a non-threatening environment.

To try and understand our students' mathematics ... we need to listen and reflect on what we hear - and to 'de-center', that is, put ourselves in the student's position and try to see things from her or his perspective.

(M. Barnes, 1994 p. 9)

FitzSimons and Godden (2000 p. 16), in their survey of research on adults learning mathematics, state that courses which "attempt to incorporate the learner's perspective ... can help to provide adults with self-confidence". Charles benefited in this way. The marked increase in his self-confidence is clear from many of his statements throughout the course (see Section 8.3.1, in particular).

Descriptions of the final results of this individual study course, which was completed in October 1995, are in Section 8.4. Charles' affective and cognitive changes are marked.

## 8.4 Results: Analysis of Final Affective Data

During Charles' final session with me in October 1995, we discussed how this course had affected him. He said now "I know I can, given time", whereas "before it felt impossible". A transcript of part of our discussion in his final session is included in Section 8.4.1. The Mathematics Metaphor Questionnaire was completed again, and his responses in May and in October are compared in Section 8.4.2. He completed the Mathematics Attitude survey again at this time, and these scores are listed in Section 8.4.2. At my request he agreed to write a statement describing what he thought were the key aspects of the course that had enabled him to make the changes he has described. This statement is included and discussed in Section 8.4.3. and theorized in Section 8.4.4.

### 8.4.1 Charles' last class in October 1995: looking back over the last six months of study

During the final session in October 1995, Charles talked about what this course of supervised study in mathematics had meant to him.

Charles: I've gone from foggy mess to sense that a door has been opened and there is light. You can see some light, and you know there's a mile of other doors to be opened. It's not that every time you open the door it's still dark.  
Barbara: Yes I see. That's nice imagery too. So it used to be dark?  
Charles: It was always dark, and I just probably expected it to be. What we've done is gone back to the smallest door, but in opening the smallest door we've generated the greatest light. And although each door subsequent to that which we open is larger, the light sort of remains the same. It's just the expanse maybe becomes greater.  
Barbara: That's really interesting. Thank you, that's great.  
Charles: You're welcome. Major changes there. Huge.  
Barbara: Very satisfying for me and for you.  
Charles: Oh it's been incredibly satisfying for me Barbara.  
Barbara: And for me.  
Charles: Has it?  
Barbara: Yes.  
Charles: Oh great.  
Barbara: Oh it's lovely to see what you've been able to achieve.  
Charles: But I'm enjoying it.  
Barbara: And you're enjoying it.  
Charles: I'm enjoying it. I'm finding it rewarding. It's so rewarding.  
Barbara: And your attitude has changed a lot. You're now not panicking when something doesn't quite gel. You're coming back to me, or leaving it till next time.  
Charles: Yes.  
Barbara: And that's really a big change isn't it.  
Charles: Oh yes.  
Barbara: Because earlier on, to start with, you would have just felt all desperate again really.  
Charles: Well yes because I was never getting ahead, whereas now I've got a bit of foundation and I'm comfortable with it.  
Barbara: And how is it affecting your day-to-day work?  
Charles: Oh just general confidence when it comes to using maths. When it comes to business matters, it's not a problem, but when it's something complex, some spreadsheet which I

- haven't done, it doesn't concern me if at first I don't see what's happening, because I know if I analyse it, slowly I will. So my attitude, that's what has changed.
- Barbara: Yes, whereas before you would have looked at it and thought "help, I'll never understand this"?
- Charles: Yes, "just show me the result", whereas now "ah, I can see how we got this one". So it's an attitude thing, it's a confidence thing. You look at it more objectively, feel more at ease.
- Barbara: Ok. Well that would have made a big difference.
- Charles: Huge! Enormous!
- Barbara: That's great. That's great to hear.
- Charles: I'm just so grateful.
- Barbara: That's good, I'm very pleased.

We both, understandably, were very pleased with the results of the course. Since Charles had been one of the most maths phobic students I had taught, I was initially afraid that I would not be able to break through that strong fear to achieve changes in his attitudes and feelings towards mathematics, as well as help him learn any more mathematics. As I developed an understanding of his experiences and fears (which he confirmed), a rapport developed, which enabled affective changes and learning to take place for Charles. During our third class he talked about "enjoying it" because he had "learned something" and because I understood, the first person he had "come across who has genuinely understood" his situation (see Section 8.2.2). He continued "it's a huge relief". Buxton discusses the relief commonly experienced by maths-avoidant students.

The problem in changing the attitudes of people who have developed a great distaste for mathematics is that the best we can achieve initially through success is *relief* at avoiding an anti-goal.

(Buxton, 1981 p. 106)

He defines the general distinction between a goal and an anti-goal is "between the existence in the environment of desired end or of something from which we wish to escape" (p. 105). He continues to discuss the feeling of relief:

Relief, although in some ways resembling pleasure, has one important difference – it is not an experience that we seek to repeat, since it obviously involves suffering anxiety first.

(Buxton, 1981 p. 106)

Buxton continues to talk about the aim, in teaching mathematics anxious adults, of moving the student beyond this "*relief* at avoiding an anti-goal".

It is only through regular experience of elation that one can want to continue. The initial, and very demanding task, is to shift people from the anti-goal avoidance course to the goal seeking course.

(Buxton, 1981 p. 106)

Charles previously had felt that mathematics was "impossible", that "every time (he) opened the door (to it) it was still dark - it was always dark and I just expected it to be – a foggy mess". Now he has the "sense that a door has been opened and there is light". Charles previously would have said "just show me the result" but now he says "I know if I analyse it slowly I will" be able to say "ah I see how we got this one". He believes that his attitude and confidence have changed. I think that Charles has "shifted from the anti-goal avoidance course to the goal seeking course" (p. 106).

Buxton then talks about the ever-present danger of despair, even after people shift to the goal seeking course.

There is a danger ... that someone newly on this (goal seeking) route will experience emotions in an enhanced form, and, if disappointment is later experienced it is very bitter.

(Buxton, 1981 p. 106)

He describes an adult student who had "been gaining greatly in confidence throughout" his teaching sessions but who, after giving the wrong answer to a question, experienced "feelings of bitter disappointment and deep despair, of an intensity totally out of keeping with the event" (p. 106).

I had also noticed, as I gradually exposed Charles to mathematics and dealing with his anxieties as he did some mathematics, Charles' panic could return easily. Since Charles was so fearful of mathematics, I took a great deal of care in how I asked questions of Charles and how I responded to his answers. Often adult students have had teachers who have emphasized questions with a single answer which is right or wrong which, as Buxton (1981 p. 59) says, "enhances the sharpness of emotional response". I also experienced, as Buxton (1981 p. 102) did, when he describes his experience teaching a group of maths-avoidant adults, "despite the relaxed atmosphere which developed", an occasion when you inadvertently ask a question in a "fairly demanding way produces the characteristic fear and defensiveness which we need to notice and if possible avoid". I supported him when his fear of mathematics surfaced again, for example, his panic when he couldn't see how to do a question. I am in agreement with Buxton (1981 p. 61) that "a violent reaction from the emotions completely ruins the reasoning process", and I talked to Charles about this effect. Hauk (2005 p. 38) describes this effect also by stating that

the dissonance between cognitive and affective demands may overwhelm her into the coping state of inaction that is the hallmark of mathematics anxiety.

I think it can be reassuring to a student that it is their fear, rather than a lack of ability, that is affecting their thinking.

Since I had supported Charles as he experienced despair many times, particularly in our earlier sessions, it was particularly pleasing to hear him say in October 1995 that now “it doesn’t concern (him) at first (if he) doesn’t see what is happening” and that he knows, given time, he will be able to work it out. This “general confidence when it comes to using maths” which Charles now feels is reflected in the metaphors he uses for mathematics when he completes the Mathematics Metaphors Questionnaire again in October 1995. His responses are discussed in Section 8.4.2, which follows.

#### **8.4.2 Charles’ final mathematics metaphors indicate new beliefs**

According to Thompson (1992 p. 132) conceptions of mathematics can be viewed as “conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics”. There are a number of different conceptions of mathematics which are commonly believed (Ernest, 1995). Three such views are:

- a Platonist view, that mathematics is a static, unified body of knowledge which is discovered rather than created consisting of interconnecting structures connected by logic;
- a problem solving view of mathematics as a field of human creation and invention which is dynamic, continually expanding and open to review where mathematics is seen as a process of enquiry not a finished product;
- an instrumental view, that mathematics is a set of accumulated facts, rules and skills to be used by trained persons.

These differing views of mathematics can be seen as being located on a continuum. Romberg (1992) found that most people see mathematics as a fixed body of knowledge, set in final form, involving manipulation of numbers and geometric deduction. Mathematics is thought of as a cold and austere discipline, without scope for creativity. Contrasted with this is the view that mathematics is a dynamic and changing discipline. In the past two decades new technologies are providing scope for the development of new mathematics and making other aspects of mathematics obsolete. The following discussion of Charles’ metaphors will reveal a significant change in his conceptions of mathematics, from the instrumental view to the human creation view.

Table 8.3 lists Charles' responses to the Mathematics Metaphor Questionnaire, in May 1995, when he started the supervised study in mathematics, then in October 1995. The change in the metaphors he has used highlights the changes he had made during that time, espousing new beliefs about the learning of mathematics and new conceptions of mathematics.

Table 8.3: Charles' responses, in May and October 1995, to the Mathematics Metaphor Questionnaire

The Questions	Responses in May	Responses in October
Imagine yourself doing or using maths, what does doing or using maths feel like? List all the words and phrases.	<p>Using maths from a practical point of view, i.e., profit and loss is great. Maths for the sake of it, all that garbage: cosine, coefficient, derivative, square root, are just nonsensical. Non-stimulating, academic gain. I still sort of look at it and I don't even know what it is really. I think it's a self confidence thing, a lot of it.</p> <p>On a mountain - skiing on blue ice, with no edges, blindfolded. That's what it feels like!</p>	<p>Enjoyable, investigative, pattern-forming, rewarding, satisfying.</p> <p>If you had to describe maths to someone, what would be the words or phrases you would think of to use?</p> <p>Patterns, solutions, stimulating, rewarding. Intellectually stimulating, engaging your brain. Brain food, but different brain food to the arts.</p>
Think about the things that maths is like.	A house with no doors or windows, you can't get out of it. A mad scientist - humourless, colourless.	I'd say form, structure, balance, patterns, problem, solution, principles. I know there's more - I'm just not getting the right words for it.
If maths were weather, what kind of weather would it be.	Humidity which you can't escape, you shower and freshen up, five minutes later you're sticky, dirty, and uncomfortable. There's a bit of a theme coming through there - can't escape it.	<p>Clear, clean. Well, it's funny though, as soon as I said clear I also thought, it's almost four seasons in a day. You can have a problem, which maybe a storm, but if you apply what you've learned, it becomes very fine.</p> <p>[Barbara: It seems to end up fine? Charles: Yes, Yes definitely.]</p>
If maths were food what kind of food would it be	Tripe, the only food I can't eat, it's disgusting.	<p>I see really good quality - like a really good quality cut of meat on a plate with a very simple herb sauce, with really beautiful vegetables but simple. Do you know what I mean? It doesn't need to be dressed.</p> <p>Why? I just think it has its own path, it's quite clear, clean and straight forward. I don't mean straight forward in terms of simple, but it's uncluttered. This could be my interpretation, going from the cluttered to understanding something, the clutter is starting to dwindle.</p>
If maths were food how would you eat it?	I wouldn't, I loathe it.	I want to enjoy eating it.

Table 8.3 (continued)

The Questions	Responses in May	Responses in October
If maths were a hobby, what kind of hobby would it be?	Counting grains of sand on a beach. Looking for an emerging pattern or formula because it's a futile exercise for me.	Quite a stimulating and rewarding hobby, something which is good for the mind. It engages cogs which previously haven't turned.  I imagine that at some stage it may be a little bit like painting, if you're very good at it, because I think that at times you would go through periods of incredible productivity and creativity. At other times you'd find that something wasn't quite right. You just couldn't create the view that you would like to. With painting more so but, I think of the previous session where part of the circle, part of the pattern, was incomplete in my mind. But as soon as it was complete, I was away again. Once you understand you can develop - remember, I'm coming from a negative position to a positive position.
If maths were a way to travel what means of travel would it be?	In a leaky boat, because you'll never reach your destination.	Concorde, because its sophisticated, and you have to have a full understanding of everything that is necessary to fly that piece of sophisticated machinery correctly in order to get it in top performance. You can fly it incorrectly, it would still fly, but you wouldn't be getting peak performance. And the more you know, the greater its ability to fly efficiently. In a way its a bit like skiing as well because you can ski on day one, but until you understand and learn and apply all the techniques, you don't ski well. If you are into skiing you're always learning and you're always actually striving to improve. It's quite an unusual sport. You have to understand in your head what should be going on before you can apply it to your body.
If maths were a colour, what colour would it be?	It would be the absence of colour. Why? Because it offers no life.	Not green yet. Not yet. What would it be? Previously I would have said it's no colour. So now it's definitely a colour. It's almost like two colours, blue with yellow. The depth and the solution, the dark and the light, the dark being the problem, the light being the solution. Yellow's quite a cheerful colour, and blue is quite a strong colour. Yes.
If maths were a way to communicate, what way would it be?	A forgotten dialect, making two-way communication impossible.	Direct. There may be two ways of expressing the same situation but either way would be direct and clear.
If maths were an animal, what kind of animal would it be?	A hyena. Why? Because it scavenges my self-confidence.	Antelope. Why? Quite definitely in action, when it moves, it moves quickly.

Table 8.3 (continued)

The Questions	Responses in May	Responses in October
If maths were a building what kind of building would it be?	The most disgusting, unappealing structure in history. Maybe a prison, white, grey and ugly.	It would have a lot of form, it would have balance, it would have symmetry, quite classical. There would definitely be a pattern. I would never have said that before. I can now see how a mathematician and an artist can be one and the same. Well when you look at a classical design from Roman times, which is often used in churches, you've got five or seven divisions down the side, and three in the middle. They look superb. The Greeks, I think, used a similar ratio. Symmetry is very pleasing to the eye - it creates balance. I think maths is a little bit like that because it starts with rubble and you've got to find a pattern which creates a structure from the rubble. Once you've got the pattern, you're away. The development is quite clear, quite straightforward, quite direct, quite cheerful.
If maths were a plant, what kind of a plant would it be?	Miles of indoor house plants, everywhere. I just can't stand that.	Probably a Parisian street tree - great form - you see them a lot in Paris. You see them a bit here. When they're planted correctly in large avenues or groups, they look fantastic. They're again form, structure. Well, within that mass, there's a lot going on.
If maths were music what kind of music would it be?	Head banging, non-rhythmic rubbish.	A concerto, starts fairly simply and develops, and ends fairly simply. But it has a definite beginning and a definite end, and quite powerful. So a small problem can become extremely large, but once resolved, it's small. You go through the process, you develop and every path is related to the previous path.
Read through your list of words that describe maths saying maths is, write a paragraph describing a way that maths as a subject is like for you.	For me maths is most like a scavenger-predator, rearing its head when I least want it. Always succeeding in removing my self-confidence and sense of self. It brings me down, holds me back, confuses, and upsets me. I hate it.	NA. This question was not answered in October 1995.
What about if maths is a kitchen utensil? What would it be?	NA. This question was not answered in May 1995.	I think tongs. Why? Because of multi-use: you can pick something up with them irrespective of its shape; you can flip something with them; you can pierce the surface of something with them.
What if maths was like a tool?	NA. This question was not answered in May 1995.	A wheel, because you can move with it, and it continues to move, and the pace with which it moves, is dependent on the input. The greater the input, the greater the movement. But it will move, even if you don't. It's ongoing, it never stops. You can move it faster, or slow it down, depending on what you're trying to do with it. So it's useful, it's a useful tool. It's a tool which doesn't have a limited or specific use? - it's ongoing the whole time.

The metaphors are a graphic description of the changes that Charles has made over 6 months study in the independent course. His view changed from one of extreme negativity (in May 1995) to an emotionally positive and intellectually engaged attitude to mathematics (Ocean & Miller-Reilly, 1997) (in October 1995).

A vivid picture emerges from the creative images Charles uses for his metaphors about mathematics which indicate new beliefs about the learning of mathematics and new conceptions of mathematics. I believe that Charles' metaphors and summary comments (see Section 8.4.3) indicate that I have "demystified the doing of mathematics", as Rogers (1995 p. 178-9) explains in her approach to teaching undergraduate mathematics students, by "calling (his) attention to mathematics as a creation of the human mind, making visible the means by which mathematical ideas come into being", engaging him in "purposeful, meaningful activity" (p. 179). Connected teaching echoes these ideas in advocating teaching mathematics "as a process, not a universal truth" (Becker, 1995 p. 168) so that the student can use his/her "intuition in an inductive process of discovery". Charles' metaphors indicate that he experiences doing mathematics as a creative process and "without intuition, there is no creativity in mathematics" (Wilder, 1984:43, quoted in Burton, 1999b p. 27).

A mathematician Halmos (1968 p. 389) described mathematics as a "creative art" because "mathematicians create beautiful object", because "mathematicians live, act, and think like artists". He stated that mathematicians make "vague guesses, visualize broad generalizations and jump to unwarranted conclusions", involved in a "cyclical process of guessing, visualizing, and conclusion-jumping" (Halmos, 1968 p. 380-1). Buerk (1985 p. 62) describes her experience of mathematics as a "process" that is "subjective, intuitive, inductive – playfulness of ideas". I believe that I have "valued and nurtured (Charles') intuitions" and "also recognized the importance of making connections or links in the building of mathematical meaning" (Burton, 1999b p. 31). These processes are not always made visible to students. As Smith and Hungwe (1998 p. 46) say,

if guessing and the resulting cycle of inquiry does not become visible to students, they are left with only the public mathematics – the carefully crafted propositions and polished arguments they see in their texts.

Charles' metaphors indicate that his experience with mathematics has become similar to that of Buerk's (1985 p. 61) and that of other mathematicians, as he

now believes that “maths is creative, dynamic, evolving and in process” which “allows for the expression of the personal, imaginative, and intuitive capabilities”. Buerk compares this view with that of her maths-avoidant students, so similar to Charles’ view at the start of the course: the “vision of mathematics was as an absolute … at which one is very good or very bad” (p. 61).

Let’s analyse some of the imagery that Charles uses. In May he indicated extreme fear of doing mathematics, comparing it to “skiing blindfolded on blue ice” or being unable to escape from a house because it had “no doors and windows”, whereas in October he described mathematics as “enjoyable” and compares it to “yellow”, where “yellow is a cheerful colour”. Mathematics has become enjoyable to Charles. Instead of an example of “disgusting food (tripe)” which he “loathes”, mathematics has become, by October, “really good quality” food i.e. Charles now believes that mathematics has value, has worth. Mathematics has “scavenged” his “self-confidence” in the past but now he is confident enough to feel that he can direct the pace and movement of it like a “wheel”. He no longer sees mathematics as “dualistic”, where he has no “agency” and where “right answers are to be memorized by hard work” (Buerk, 1982 p. 23). He now has “agency”, he sees doing mathematics as a process, a path, an evolving story.

Charles said that mathematics (other than arithmetic) was being taught at school just “for the sake of it”, but he now believes that mathematics is “useful”, and its use is not “limited” or “specific” but rather it is “multi-useful” like kitchen “tongs”. Mathematics was “non-stimulating” and has become “intellectually stimulating” and “good for the mind”. Charles believes that mathematics is playing with ideas (Halmos, 1968). He compared it to “blue”, where “blue is a strong colour”. For mathematicians, Burton (1999a p. 29) finds in her study of their views and practices, “far from understanding being something which is only driven by knowledge, there is both a need to know and an associated pleasure in knowing which is its own reward.” This “indivisibility of the cognitive and the affective” (p. 29) I also see expressed in Charles’ metaphors. Futility is a theme in many images he used in May, for example, “it’s a futile exercise - like counting grains of sand on a beach”, “a leaky boat – you’ll never reach your destination”. In contrast, by October, he talked about mathematics as “rewarding”, “satisfying”, as “Concorde, because it’s sophisticated – the more you know, the greater its ability to fly efficiently”. Charles has experienced both the cognitive and affective aspects of mathematics. Mathematics as “non-rhythmic music”, “nonsensical” and “a forgotten dialect where two-way communication is “impossible” has become

"direct" and "clear". Instead of the "most disgusting unappealing structure", by October, his building would have "form", "balance" and be quite "classical". He can now see "how a mathematician and an artist can be one and the same" i.e. he now believes that mathematics is creative and imaginative. Hersh (1998:61, quoted in Burton, 1999a p. 27) points out that "intuition is an essential part of mathematics". Charles talked, in October, about mathematics as "pattern forming", "investigative" and where "every path is related to the previous path" i.e. he now believes that mathematics is inductive and evolving – a process. Charles' metaphors in October show how "he now had the idea that mathematics is creative; that it involves a search for patterns, for possible routes to solutions, for relatedness amongst ideas and concepts" (Ocean & Miller-Reilly, 1997 p. 18). I have traveled with Charles, as Johnston (1995 p. 233) "traveled with Marie, from a position as a victim... to the possibility of autonomy".

Changes in his beliefs about the learning of mathematics, gathered using his responses to the five statements which follow (Schoenfeld, 1989), also reflect some of the changes evident in the metaphors Charles used in June and October. They indicate a less algorithmic and more conceptual focus on learning mathematics, for example, he 'agrees' with the statement *The best way to do well in maths is to memorise all the formulas* in May but, by October, he 'disagrees' with it. He also initially is 'undecided' about the statement *In maths you can be creative and discover things by yourself* but, by October, he 'agrees' with it. However he is 'undecided' about *To solve maths problems you have to be taught the right procedure, or you cannot do anything* in both May and October. His responses to the other two statements were quite confident in May and remain unchanged: he 'disagrees' with the two statements *Maths problems can be done correctly in only one way* and *Everything important about maths is already known by mathematicians*.

Some changes in Charles' attitudes and beliefs are also reflected in the quantitative data. Charles' beliefs about himself as a learner of mathematics, as measured by a scale, have changed markedly over the six-month course i.e. they indicate a much more positive mathematical self-concept at the end of the course. At the beginning of the course his score on the subset of Gourgey's (1982) Mathematical Self-Concept Scale was 6/40 but six months later it was 22/40, an increase of 16. As a comparison, the range of differences in individual scores, from March to October 1995, in Mathematics 1 was -8 to +11, the mean score in March 21.1/40 and the mean gain -0.2. I have chosen to compare his scores with those of students in Mathematics 1 as I believe he is similar to this population, rather than the Wellesley student population who were let down by their education generally, rather than just mathematics.

Charles' scores on the Mathematics Attitudinal Scales (Aiken, 1974) have also changed over the six-month course. Scores on the Enjoyment of Mathematics scale increase from 14/40 to 27/40, an increase of 13, indicating much more enjoyment of mathematics. As a comparison, the range of differences in individual scores, from March to October 1995, in Mathematics 1 was -8 to +11, the mean score in March 24.4/40 and mean gain -0.7. While his score for the Value of Mathematics scale was quite high at the beginning (27/40), it increased during the course (32/40), an increase of 5, i.e. his attitude regarding the extrinsic value of mathematics remained high. As a comparison, the range of differences in individual scores, from March to October 1995, in Mathematics 1 was -8 to +6, the mean score 29.2/40 and the mean gain 0.6.

I asked him to describe, from his point of view, what it was about the course that had created these positive changes. Section 8.4.3, which follows, includes his statement and also compares the points he makes with my teaching goals.

#### **8.4.3 Charles' summary statements, his letter to the University, and my teaching goals**

I requested this statement, as part of my research, with the intention of including it as part of the case study. Charles was very happy to write it, describing what he thought has caused these changes in his mathematical self-concept and his conceptions of mathematics.

##### **What Has Made The Changes**

In the very first instance, actually facing up to the problem and seeking advice from a psychologist. Had the said psychologist told me in the very first session that I didn't have a problem with mathematics I would have thought him inept. However, by way of a process he was able to illustrate to me that in all likelihood I had the inherent capacity to cope with maths. Indeed as he pointed out reasonably active people often do, which, was quite the reverse of what I originally thought.

Stage two was making contact with Barbara Reilly, who has been nothing short of fabulous.

Barbara approached my problem, which was very real, in a thoughtful, gentle and encouraging manner. Having experienced students of a similar nature, she was able to empathise with me and fully understand what had been an on-going and seemingly never ending horrible experience.

What exactly has Barbara done which has been so enlightening? I need to list the points and ideas which have made the change as there have been so many.

1. Identifying the gaps.

2. Really starting with the basics. Letting me know it was fine not to understand some rudimentary concepts with certainty. Previously I had been greatly embarrassed by my lack of certainty.

3. Encouraging my memory and actually saying on many occasions: "you'll probably find you realise more than you first thought".
4. Approaching problems in bite-sized pieces.
5. Patterns. These were a major as they illustrated a concept (an alien one at that) in a visual fashion which I could relate to.
6. Building on knowledge and the basics as I learnt engendered great confidence.
7. Being completely encouraging and motivating. What an amazing teacher.
8. Letting me know it was OK if I didn't get it first hit.
9. Once the logic emerged from the patterns it was fun. Now I thoroughly enjoy maths and would like to spend more time discovering it. In some instances it's like reading a fabulous book, interesting and expanding your perceptions. At other times it's like feeling a cog turn in your brain for the very first time.

Charles' summarised aspects of the course he thinks worked for him, allowing him to work through the phobia he had about mathematics and feel confident to learn mathematics again.

I taught Charles for a period of time in 1996. Later in 1997 he called me to talk about how he was responding to mathematics in his work environment now and to ask to do more study with me. We did not continue with mathematics classes again as I was busy. He mentioned that he had written a letter, to indicate his appreciation of the study course he completed with me, to the Registrar of the University of Auckland in 1997, unbeknown to me. I had heard nothing about it but he gave me a copy, which is now included (with his permission).

7 June 1997

Dear Mr Nicoll

I write to you to commend Mrs. Barbara Reilly of the Mathematics Department, Auckland University.

My path to Barbara's door is somewhat interesting but worthy of note given what has been achieved. As a school student although being in an upper / middle stream I had a consistently poor record in mathematics. Despite increased efforts this situation prevailed and I continued to fail examinations whilst passing all other courses. Interestingly this consistent pattern of failure in mathematics was never addressed from an academic perspective. Unfortunately whilst at University the same inability with mathematics prevailed precluding me from concluding my second degree. This for me was a very real problem and one which was hugely frustrating.

Having graduated I've pursued a successful career in property and sharebroking, both with a marketing bias. Business mathematics has never been a problem, however, pure mathematics had rendered my own self confidence a real blow. In many respects this was like walking an internal tight rope.

I mentioned this situation in passing to my G.P. who suggested I discuss this with a Psychologist. Although sceptical (as I viewed my mathematical abilities as a black hole or the numbers version of dyslexia) I pursued the suggested course of action and within two sessions was at Barbara Reilly's door.

Having explained this background to Barbara an interesting and very fulfilling course was embarked upon. Barbara addressed my situation from a principles and patterns perspective as opposed to repetition and rules. Looking back I believe it was the latter which turned me off as a child. I switched off and the rest was history.

Barbara tapped into a method of learning which has enabled me to gain some of the basics and genuinely enjoy the subject. Over the course of twelve to sixteen months Barbara Reilly has provided an invaluable platform from which I am able to develop skills and a genuine confidence that I can undertake such tasks without fear or misgivings.

Barbara's methods were clear, considered and extremely encouraging. I am most grateful to Barbara and would count this as a highlight of the last two years.

I would like to thank the University and Barbara for this opportunity.

Yours sincerely,

Charles

I now compare Charles' comments in October 1995 and in his letter in June 1997 with my teaching methods and aims in Table 8.4.

Since Charles is a graduate in English literature, this last item in the table is a particular significant analogy! Charles' comments are like Marie's, who says that she "came from being very damaged to increasingly feeling, 'I am worthwhile'" (Johnston, 1995 p. 233).

Table 8.4: Comparing Charles' statements in October 1995 with my teaching methods and aims.

My Teaching Methods or Aims		Charles' Statements
A	I aimed to create an emotionally safe environment for Charles to start learning mathematics again by, initially, listening to his mathematical experiences, his beliefs and feelings about mathematics, and not blaming him for these negative experiences and feelings (Section 8.2).	Barbara approached my problem, which was very real, in a thoughtful, gentle and encouraging manner. ... she was able to empathise with me and fully understand what had been an on-going and seemingly never ending horrible experience. Barbara's methods were clear, considered and extremely encouraging.
B	I attached no blame to the level of Charles' background mathematical knowledge (Section 8.2.1). I had a fascination and absorption in his learning.	Really starting with the basics. Letting me know it was fine not to understand some rudimentary concepts with certainty. Previously I had been greatly embarrassed by my lack of certainty.
C	I aimed to find out Charles' misconceptions in a non-threatening environment.	Identifying the gaps.
D	Knowing that Charles would be using some mathematics in his adult life without realizing it, I aimed to acknowledge and use this knowledge (Section 8.3.2).	Encouraging my memory and actually saying on many occasions: "you'll probably find you realise more than you first thought".
E	I aimed to move at a pace which suited Charles, treating him as an equal, and allowing him to have input into what and how he learned.	Approaching problems in bite-sized pieces. Building on knowledge and the basics as I learnt engendered great confidence.
F	I aimed to support Charles facing his fear as he tried to learn mathematics again.	Letting me know that it was OK if I didn't get it first hit.
G	I aimed to give Charles constant encouragement (Section 8.5.1).	Being completely encouraging and motivating. What an amazing teacher.
H	I aimed to help Charles to effectively visualize mathematical structures and patterns (Section 8.5).	Patterns. These were a major as they illustrated a concept (an alien one at that) in a visual fashion which I could relate to. Barbara addressed my situation from a principles and patterns perspective as opposed to repetition and rules.
I	I aimed to encourage experimentation, discovery of patterns and enjoyment of mathematics (Section 8.5).	Barbara tapped into a method of learning which has enabled me to gain some of the basics and genuinely enjoy the subject - an interesting and very fulfilling course Now I thoroughly enjoy maths and would like to spend more time discovering it. In some instances it's like reading a fabulous book, interesting and expanding your perceptions.

#### 8.4.4 Appropriate mathematical challenges: providing support but allowing student to take risks to develop their thinking

Charles acknowledges that an important aspect of my teaching approach was that I let him know "it was fine not to understand" some concepts "with certainty". He had previously been "greatly embarrassed" by his "lack of

certainty" in his understanding of "some rudimentary concepts" (Table 8.4:B). Because I aimed to find a balance between providing success while at the same time challenging Charles, I thought that it was particularly important to avoid blaming him. Morrow and Morrow (1995 p. 19) describe this as an aspect of connected teaching, creating an environment where the student feels "no need to apologise for uncertainty". Heckman and Weisman (1994 p. 32), in their study of contextualized teaching, also state "the teacher does not blame the student for inadequate knowledge".

Another integral part of my approach is echoed by Buxton (1981 p. 62), when he describes his method of teaching maths-avoidant adults. He emphasizes that, while "we must provide people with a high load of success or a level of failure they can tolerate", "the avoidance of emotional blockages does not lie in a totally laissez-faire approach; a basic premise is that some degree of pressure is appropriate" (p. 103). This is similar to an aspect of connected teaching which Morrow and Morrow (1995 p. 19) describe as allowing the student to take on challenges with support because, "in order to grow intellectually, a student must take reasonable risks and be able to make mistakes". Rogers (1995) relates this issue to her undergraduate mathematics classes and says

A classroom climate characterised by safety and trust is essential for risk-taking to occur and for students to be willing to test their ideas and thoughts and develop fluency in mathematical language.

Rogers (1995 p. 181)

This freedom to take risks is an essential part of the so-called "Moore Method" of teaching, developed by the eminent mathematician R. L. Moore at the University of Texas, Austin, from 1920 (Parker, 2005). He used it especially at graduate level with outstanding results. His 50 PhD graduates include several of the leading mathematicians in the USA. It is interesting to observe that successful teaching at the high graduate levels has similar features as that described above, which has been successful with Charles, an adult student returning to mathematics.

The types of risks include "risks that one's assumptions are open to revision, risk that one's insights are limited, and risk that one's conclusions are inappropriate" (Ocean, 1996 p. 425). In addition, Lampert (1990) suggests taking risks also requires courage and honesty. Rogers describes this approach as "student-sensitive pedagogy" because it is

grounded in the students' own language, focused on process rather than content, and centres on the students' individual questions and learning processes. Students who are 'cared for' in this way are set free to pursue their own legitimate projects (Noddings, 1984).

Rogers (1995 p. 178)

This sense of 'care' could be related to the positive aspect of one of the two moral perspectives identified by Gilligan (1982). Ocean's research (1996; 1997) relates moral perspectives to mathematics education. She examines Gilligan's "Care perspective", a perspective which emphasizes mutual dependence and connection to others, and the "Justice perspective", which emphasizes individualism, independence, equality and fairness, relating these to the moral climate in the classroom. For example, teaching which includes values such as those of "co-operation, connection and communication" (Ocean, 1997 p. 8) illustrate the positive side of the Care perspective and "the negative side of Care morality in mathematics education is seen when Care slips into patronage" (Ocean, 1996 p. 427). This negative side of Care was highlighted in research by Walkerdine (1989, cited by (Ocean, 1996) where UK high school teachers entered girls into a less difficult examination to "protect" the girls not "push" them. When Rogers (1995) suggests that one must become a "caring teacher" in the specific sense of "caring" as in helping "the other to grow and actualise himself", she is talking about the positive aspect of the Care morality. She suggests that

caring teachers are able to focus primarily, or at least initially, upon the students rather than upon the subject matter, with the idea that the route to the subject is not imposed from without, but rather illuminated from within. They do no work upon their students, but with their students, looking at the subject matter from their perspective and at their level.

Mayeroff (1971 quoted in P. Rogers, 1995 p. 178)

This environment fosters the intellectual growth needed for students to broaden their strategies for knowing in mathematics. I focused on Charles' knowledge and perspective so that this would inform the course design and curriculum. Morrow and Morrow (1995 p. 19) expands on this idea and suggest that the connected teacher "must become skilled in active listening and asking questions that will allow the students to become more aware of her own thinking, as well as decide which of her ideas to pursue further". I trusted and valued Charles' thinking, believing that my role was as a facilitator and guide. Connected teachers, as Becker (1995 p. 170) describes, "trust the students' thinking and encourage them to expand upon it". Hallett (1983, quoted in Zaslavsky, 1994 p. 20) expresses a similar opinion, that students "learn far more from our faith that they can and will learn mathematics than from our most lucid explanations".

Becker uses a metaphor to describe connected teaching and sees the teacher "as a midwife",

nurturing the student's newborn thought in a supportive environment. The first thought of this midwife/teacher is to preserve the student's fragile, newborn thought. The midwife/teacher then

supports the evolution or growth of this thought. The goal is not to replace it with a different, teacher-generated ‘better’ thought. Rather it is to help the student’s thought grow, mature and develop. Both the teacher and the student engage in this process.

Becker (1995 p. 170)

With support and encouragement from me, Charles could cope with struggling for understanding, because I was “shar(ing) the process of solving problems with (him)” (p. 168). In connected teaching, “questions can be posed and potential answers explored together” (p. 170) i.e. “the teacher and students engage in the process of thinking and discovering mathematics together” (p. 168).

## **8.5 Teaching the Mathematics: Detailed Description Teaching One Topic**

Section 8.5.1 gives details of the interactions between Charles and I while teaching and understanding the first topic, as an example, a geometric investigation of patterns in the number system.

Carrying out this investigation of these whole numbers allowed Charles to better understand the multiplication tables he would have been exposed to at school. I agree with Buxton when, in a book he wrote for adults to use to study mathematics again, he says:

We need to be able to break down (i.e. factorise) all the numbers up to 100 which can be broken down, and to know which ones cannot be broken (i.e. factorised). This is a more extensive aim than just the tables and it makes more sense.

(Buxton, 1984 p. 9)

He suggests that such an investigation allows the students to “absorb the tables”, “to have a good acquaintance with the properties and relationships between the numbers up to 100” and that “this ‘good acquaintance’ is part of numeracy” (Buxton, 1984 p. 16).

Section 8.5.1, which follows, contains a detailed analysis of the teaching of one topic using hands-on material. Charles enjoyed the process of discovering mathematical patterns in visually explorative ways (Section 8.4.3, Table 8.4:H & I). He acknowledges this in his analysis of the six-month course, entitled “What Has Made The Changes”, writing

Once the logic emerged from the patterns it was fun. Patterns were a major as they illustrated a concept (an alien one at that) in a visual fashion which I could relate to. Now I thoroughly enjoy maths and would like to spend more time discovering it.

I have found that using manipulatives (hands-on materials), models or diagrams can help the student to more effectively visualize mathematical structures and patterns and can suit their style of learning, as it did for Charles. His new enjoyment learning mathematics is also reflected in his scores for the Enjoyment of Mathematics scale (Aiken, 1976) which increased markedly over six months (Section 8.4.2). Very similar pedagogy occurs in connected teaching where the teacher "emphasize(s) visual representation" to develop "first hand experience" (Morrow, 1996 p. 15). My aim was for Charles, as Morrow and Morrow (1995 p. 20) describe, to "gain a sense of (his) own voice in mathematics". His belief in himself as a learner of mathematics improved and is reflected in his scores for the Mathematics Self-Concept Scale (Gourgey, 1982), which increased markedly over the six months of the course (Section 8.4.2). These quite dramatic changes in Charles, seen also in his metaphors, may be because he was learning mathematics as "an activity" and "knowledge acquired in this way has a power which is out of all proportion to its quantity" (Wilder, 1984:44 quoted in Burton, 1999b p. 31)

Other adult mathematics educators and researchers write about similar approaches. McCoy (1992 p. 51) found that the careful "use of manipulative materials for mathematics instruction" reduces the anxiety often associated with doing mathematics. Zaslavsky (1994 p. 181) writes about a mathematics educator who saw "her role as that of a facilitator in helping adults to construct their own knowledge by studying and reflecting upon patterns, attributes and relationships" and who finds that concrete materials are "particularly appealing to people who learn best by touching and feeling". The American Mathematics Association of Two-Year Colleges (1995) standards document, "Crossroads in Mathematics", also points to "the need for models, manipulatives and visual representation of concepts in developmental programs that lead students to abstract symbolic forms" (Kennedy, 2000 p. 2). Dwinell & Highbee (1989), examining academic success among high-risk first-year university students, found that students were likely to prefer a hands-on learning style and learning through interaction and visual stimuli rather than through lecture and text.

My aim was to allow Charles the "freedom to experiment" (R. Smith, 1990 p. 48), which is one of Smith's (1990 p. 47) six "optimum conditions for learning for adults". A learning environment which encourages student enquiry and experimentation is also suggested by Burton (1987), based on her experience teaching and researching a teacher education course for mature students with a history of previous failure in mathematics. Such an approach capitalizes on their adult experience and their motivation, "offering learning experiences which build confidence and a positive self-image" (Burton 1987 p. 306),  
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establishing a basis for understanding mathematics. It helps the student to more effectively visualise mathematical structures and patterns and to learn about mathematics as a “pattern-searching discipline”.

I was aiming to address Charles’ “inadequate conceptual understanding”, which Duranczyk and Caniglia (1998 p. 136) recommend for developmental mathematics education, so that he could experience “the interconnectedness and rich relationships in mathematics”. Some relationships were discovered by Charles when he explored the topic which is explained in detail next in Section 8.5.1. I took care to acknowledge his discoveries to emphasize my belief in him (Morrow, 1996) and my trust in his thinking (Becker, 1995).

Notation used in Section 8.5.1:

Italics type: *Teaching techniques and the atmosphere engendered.*

Underlined type: Charles' growing confidence and his first enjoyment of mathematics.

Bold type: **The mathematical understanding Charles is gaining.**

Bold and Italics: ***'Aha' moments of discovery by Charles.***

### 8.5.1 Investigating multiplication through a geometric investigation of rectangular arrays

*For heaven's sake! Look at the patterns! Amazing!*

Charles

I began teaching some mathematics during the first session (15<sup>th</sup> May), very carefully watching his reactions because I realised his extreme fear of mathematics. This first topic, investigating multiplication through a geometric investigation of rectangular arrays (M. Burns, 1987), used material I had used successfully many times with adults. Students usually get very involved in this investigation, find it interesting and different to their expectations of a mathematics class. They usually discover number patterns, are excited to have learned some mathematics, surprised that they can ‘do maths’ and usually I am able to start assessing their background mathematical knowledge.

In some earlier sections in this chapter (Section 8.2.2 and 8.3.1) I have mentioned some of his responses to this material as he started learning mathematics again. I now give details of how this topic is taught, to further illustrate how some of my teaching aims were met.

During the initial phases of this investigation I became aware of his lack of knowledge of basic patterns in the number system. I noted that he was even a little unclear of which sequence, either 1,3,5,7,... or 2,4,6,8,..., were odds or evens. He usually reminded himself by thinking about a dinner party, as it would be "odd" with an odd number. Later I pointed out that some numbers can only be represented by one rectangle and that these were called prime numbers. He thought all odd numbers should be prime numbers and was reluctant to omit 9 from the list of primes.

As he arrived at his second appointment (23<sup>rd</sup> May), his reactions to the first session were:

Charles: I actually learnt something.  
Barbara: That's good - I thought you did.  
Charles: I actually felt quite good about that.

He then discussed how he had been thinking about the mathematics we had done last session, how he understood what prime numbers were now and visualised the rectangles for prime numbers, that they could only be represented by one long rectangle. *The use of rectangles seemed to draw on his strengths as a visual learner.* This geometric investigation has utilized one of his "preferred learning styles of processing information", which Smith (1990 p. 47) names as an important condition for adult learning. We can see in the following transcript that the investigation *has increased his understanding.*

Charles: The primes, odds and evens, you know, now I can think in my head of primes in terms of the multiplication factor of one and then when I'm thinking about that, I visualise in terms of rectangles. Like, for primes, one by three, five, seven, not nine, 11, not 12, 13, not 15, and so on. Odds I know, and evens, I'm getting to terms with each. I mean, that is so basic, but I understand that now.  
Barbara: Yes, yes. So you have a feel for that now.  
Charles: Oh yes, absolutely!

We started cutting out the rectangles that we had outlined on a grid last session. The following excerpts illustrate how we were involved in the activity together, and how he was now feeling confident enough to sometimes help organise and initiate a new part of the investigation. I had achieved my aim to "involve the learner", which Smith (1990 p. 47) suggests is an important condition for adult learning. The excerpts below are examples of how the use of manipulatives also gave Charles the "freedom to experiment" (R. Smith, 1990 p. 49), and the chance to "construct his own knowledge by ... reflecting upon patterns and ... relationships" (Zaslavsky, 1994 p. 181). He is relaxed and enjoyed the investigation most of the time, which supports McCoy's (1992) finding that the use of manipulative materials reduces mathematics anxiety. The importance of

reducing anxiety is listed as another important condition for adult learning by Smith (1990). I also shared in the hands-on nature of the investigation, helping him cut out the rectangles, as part of the effort to put myself in the "student's position and try to see things from (their) perspective" (M. Barnes, 1994 p. 9). The shared experience became enjoyable, which Charles would not have expected.

Charles: (*Charles is initiating the next stage.*) Shall we do even numbers?  
Barbara: Alright, OK, that's fine.  
...  
Charles: (*We were involved in the activity together, both cutting out rectangles.*) We're doing four. OK, we'll do six.  
Barbara: Oh good, now you're away. We've got to remember all the different rectangles we can make for each number I guess too.  
Charles: Oh, do you want that as well?  
Barbara: Yes.  
Charles: Oh. Sorry, OK.  
Barbara: That's alright.  
Charles: That's cool. (*Charles is showing growing confidence and enjoyment.*) Here's six, we'll get another - shall we get other fours and sixes? (*Charles was now feeling confident enough to sometimes help organise and initiate a new part of the investigation.*)  
Barbara: Yes, that'll be good.  
..  
Charles: Here's another eight.  
Barbara: Another eight coming up! (*laughs*) Here we are. What do you want to do next?  
Charles: I'm just thinking if there are any other sets of eight. (*He has a clearer understanding of the activity here.*)  
Barbara: Are there any more rectangles with eight squares? Yes.  
...  
Charles: And twelve (*pause*) - oh hell.  
Barbara: Oh well, not to worry. We know that's there.  
Charles: We know what we mean. (*A sense of working together and sharing ideas on this investigation has developed.*)  
Barbara: We know! (*laughs*) OK.  
..  
Charles: This is another 16. I'll do another (*pause*). There's another 16. So how many sets of 16 have we got. Four-fours are 16, eight-twos are 16, 16 (*pause*), I haven't got- oh yes - I have two-eights.  
Barbara: Happy?  
Charles: Yes. Yes. (*He is enjoying and involved in this activity*)

At the start of the third session (29<sup>th</sup> May), he indicated the effort he is putting into this course outside of the time spent with me. He showed me a folder and said, "See I've even got a folder for all of this - I've rewritten all our notes."

He had completed more of the multiplication chart by laying the rectangles we had cut out on the top left-hand corner, writing the products in the chart under the bottom right hand corner of each rectangle. Three entries for 16 in the chart are illustrated in Figure 8.1, using the 8 by 2 rectangle and the 4 by 4 rectangle.

Figure 8.1: An illustration of how two rectangles (with 16 squares) were used to form part of the multiplication chart.

1	2	3	4	5	6	7	8	9	10
2							16		
3									
4			16						
5									
6									
7									
8	16								
9									
10									

As he continued to form the multiplication chart he said:

Well I can see six sets of five, and what that results in, (Barbara: Yes) or 10 sets of eight and what that results in. And I can see how each axis relates to the other, in terms of the result. One axis to the other will give you a result. I knew nine-nines were 81 by repetition, but I didn't realise there were nine sets of nine.

The materials have led to a deeper understanding of multiplication patterns. I suggested we look at a particular row to see if we could find any patterns. He suggested the '10' row and noticed:

Well it increases by the base number - adding ten in each instance. In essence this is, for example, using the other axis, for 2 at the top, is just two sets of your principle 10, so that becomes 20. If you want to utilise the other axis for the seven, it's seven lots of 10.

I suggested he try to use the word "multiple" and he said "120 is a multiple of 10". I asked if there is another way we could say this, using the word "factor"?

- |          |  |
|----------|--|
| Charles: | Each number within the row is a factor of 10.  |
| Barbara: | No, it's not a factor of 10, 10 is a factor of each number. ( <i>My response has provided a 'safety net' to recognize and address his misconceptions (Morrow &amp; Morrow, 1995) but no blame is attached – see Section 8.2.1.</i> ) |
| Charles: | Each number in that row has 10 as a factor.  |
| Barbara: | Yes. So 90 is a multiple of 10 (Charles: Yes) and 10 is a factor of 90.  |
| Charles: | Can I write that down?   |
| Barbara: | Yes certainly.   |

As he wrote, he said "90 is a multiple of 10, 10 is a factor of 90. Let me read that. Yes. That makes sense." (I am introducing mathematical language.) He continued,

- Charles: How would I therefore define multiple? Multiple of 10. I know that (pause). OK, yes, I sort of understand it. (*He is still struggling with this concept.*)
- Barbara: We'll talk about it some more. So we've been talking about this row as being the multiples of 10, right?
- Charles: Yes.
- Barbara: So why don't you write down 'the multiples of 10 are ...' (*I am asking for clarification and justification of his ideas because I 'believe in' him (Morrow, 1996).*)
- Charles: OK. 10, 20, 30, 40, 50.

He then asked about the factors of 10. We reviewed what factors are, and he said "10 is a factor of 90". I asked "why?" and he said "ten-nines are 90". I talked about 10 dividing exactly into 90 meant that 10 is a factor of 90. He then said

- Charles: **Oh, what divides exactly into 10 are factors of 10.**
- Barbara: Yes. Good. That's a description of a factor of 10, right. So what are the factors of 10? 10 is a factor because it divides exactly into 10.
- Charles: 20.
- Barbara: Does 20 divide exactly into 10?
- Charles: Oh no. (*He recovers from his mistake without feeling devastated as he did in the past.*)
- Barbara: No. Right. These are both complementary ideas, multiples and factors.
- Charles: 10 divides exactly into 10.
- Barbara: Right - what else divides exactly into 10?
- Charles: 5.
- Barbara: Five. Is 5 a factor of 10?
- Charles: **Yes - and 2 is a factor, and 1.**
- Barbara: Yes. Are there any other numbers that divide into 10, or have we all the factors of 10?
- Charles: **All the factors are there.**
- Barbara: We've got them all, haven't we?
- Charles: **I've got it now - so factors divide exactly into a number.**

The extracts above illustrate my "listening in a believing mode" (Elbow 1973, cited in Morrow, 1996 p. 7) as "there is an assumption that the speaker has a valid basis for his/her opinion". My responses aimed to help him "elaborate, deepen and extend" the ideas he presents. The alternative mode to the believing mode is the "doubting mode", an "argumentative mode" where the presenter of an idea is "challenged to 'prove' the validity of the observation or claim", a very common approach employed in traditional mathematics classrooms. In the extracts above and below I ask for clarification or a more detailed explanation as part of my "belief-based-inquiry process" so that he becomes "used to justifying ideas", not because I disbelieve him but because I believe him and "believe in" him (Morrow, 1996 p. 7-8).

We looked at the '10 row' again and he seemed to understand that every number is a multiple of 10. I talked about viewing the number patterns from two angles. He suggested that we were looking at them from "different perspectives". He then noticed that they all had a zero in common. (I found it quite amazing that he did not know this pattern and could still cope well with calculations of profit and loss etc.)

Barbara: Right, they end with a zero. Write it down, another discovery! (*I am acknowledging his mathematical power indicating I believe in him (Morrow, 1996).*)  
Charles: *All multiples of 10 end in a zero.*

I decided to extend this idea and asked him about larger numbers.

Barbara: OK. Is 250 a multiple of 10?  
Charles: Yes it is.  
Barbara: Why?  
Charles: 10 by 25.  
Barbara: 10 by 25. And how can you quickly recognise-  
Charles: **Zero at the end as well.**

Buxton (1984 p. 5) commented on a similar investigation of whole numbers which he wrote for adults and said "the activity of understanding and seeing connections has the effect of processing the facts to make them more digestible".

We looked at a few more similar examples then Charles made a discovery as he looked at the row containing multiples of nine.

Charles: *God, I've just seen something I've never seen before as well!*  
Barbara: Oh tell me, tell me!  
Charles: Well nine - the second digit, every time you multiply it the second digit decreases. *I never noticed that. Never!*  
Barbara: Excellent! Let's look at that row.  
Charles: I shouldn't really look at nine though, that was the only maths test I ever passed! (*He jokes.*)  
Barbara: (*laughs*) That's OK! Let's write them down. What can we say about that row that starts with 9. 'All those numbers are ...' (*I am asking for clarification and justification of his ideas because I believe in' him (Morrow, 1996).*)  
Charles: **Well, multiples of 9.**  
Barbara: Multiples of nine. Excellent.  
Charles: **9, 18, 27, 36, 45, 64, all multiples of nine.** What are factors of nine?  
Barbara: We could look at that.  
Charles: Just for my own benefit. **Factors of 9 are 1, 3, and 9 itself.**  
Barbara: Excellent! And now going back to the 9 row, what's a sentence we can write about 36 and nine using the word factor?  
Charles: 36 is a factor of 9? **9 is a factor of 36 - yes - 36 is being divided by 9.**  
Barbara: Yes, 9 divides exactly into 36. Right, so you've understood that 36 is a multiple of 9, and 9 is a factor of 36.  
Charles: **Yes, 36 is a multiple of 9.**  
Barbara: Happy about that?  
Charles: **Yes. That amazes me though.**

- Barbara: Now, let's look at this amazing thing that we've just seen - what did you discover? (*In the use of 'you' I am acknowledging his mathematical power indicating I believe in him* (Morrow, 1996).)
- Charles: Well, every time you multiply 9 by 1 or any other greater number in sequence the result shows the last digit decreasing by one.
- Barbara: Yes. What's happening to the first digit too?
- Charles: *Oh god, it's doing it as well!*
- Barbara: What is it doing? (*I am asking for clarification and justification of his ideas.*)
- Charles: *It's increasing by one!*
- Barbara: Yes, you've got two patterns going on? (*In the use of 'you' I am acknowledging his mathematical power indicating I believe in him* (Morrow, 1996).)
- Charles: *Yes, I've never noticed that - never! The first digit increases by one every time multiplication factor increases by one with a limit to nine.*
- Barbara: What's the relationship between the number you're multiplying nine by and that first digit now?
- Charles: For heaven's sake! It's one less! Look at the patterns! Amazing!

The extracts above illustrate my constant encouragement of Charles, for example, the use of "excellent" several times. In addition, I acknowledged his mathematical power in using the word "you" several times, for example, when I said "you have got two patterns going on?" Charles confirms my encouragement as one successful aspect of my approach when he says that my approach was "completely encouraging and motivating" (Table 8.4:G). This is a major theme in continuing education literature, as Nordstrom (1989, quoted in FitzSimons & Godden, 2000 p. 19) states, "adult learners are concerned with maintaining a positive self-concept and, although adult learners tend to be highly motivated to learn, they lack confidence in their ability to do so and require continuing encouragement". I think it is important to clearly affirm him and give him credit for his discoveries and I agree with Mason (1992 p. 302) when he says that the "central task of mathematics teachers is to help pupils to discover and develop their inherent imaginative powers". This is also consistent with Lyn Taylor's (1995 p. 699) view, that students often attribute their successes to luck or to their excellent teachers but said it was "important to help students internalize and take credit for their own successes". Being "successful learners" is one of the four factors which Wlodowski (1985, cited in Knowles et al., 1998 p. 149) believes increases adult motivation to learn. The other three factors have already been discussed in Sections 8.2.2, 8.2.4 and 8.2.5, so theoretical confirmation of Charles' comment, in his summary statement, that I was "completely encouraging and motivating" (Section 8.4.3).

Charles then discussed architecture and garden design. He talked about odd numbers of objects being more pleasing to the eye, for example, planting odd numbers of plants of the same variety, odd numbers of columns on ancient Greek buildings. After this discussion, I suggested we add the digits on a

multiple of 9. The sum was 9 and he said "heaven's sake!". We tried adding digits on other multiples of 9. He said "Amazing! Is nine quite unusual in that regard?" I mentioned that we could look at multiples of 3. He then tried adding the digits in multiples of 3 and we found that the sum was always a multiple of 3.

He asked me if I had "ever come across anyone who has been as poor at maths as I have been". I described another student whom I had taught who had not known about odd and even numbers, so she had difficulties finding a house when given the address. She had found it just amazing to learn about this pattern and others.

Charles: That's amazing, good on her!  
Barbara: Yes, I know I felt that too. Because it took a lot ...  
Charles: It takes quite a lot of courage.  
Barbara: It does take a lot of courage.  
Charles: Not many people would do that.  
Barbara: No, I think so. So I do admire you for doing it, facing it, you know - it's hard work.  
Charles: Never, ever, did I think I would!

We decided to move on, looking at other patterns made by highlighting, in colour, all the multiples of a particular number on a multiplication chart (Stenmark et al., 1986). He seemed to find it very satisfying to see patterns emerging and said "it looks good, all very even and balanced". He coloured multiples of 10 and of 5 and said "It does create a fantastic pattern when you look at it like that!" As he wrote down a summary of what he had learned in this session, I also explained factor trees, as a way of finding all the prime factors of a number. We talked about even numbers always having a factor of 2. He said "Great for parties, bad for gardens!" As he left he said "That was great, Barbara - again!" His anxiety had receded markedly and he was enjoying learning mathematics, two teaching aims valued by me had been met.

He asked to do some division at the next session, so we tried division by 10 then completed some examples of division by 3. He had problems with 0's in the middle of the answer and this was clarified, eg  $\overline{4004} \overline{3)12012}$ . At the start of the fifth appointment, 12<sup>th</sup> June, we practiced long division again on request, then moved onto the next topic. (In Section 8.4.1 I have given some idea of what else was covered in the course, his reaction to these topics, and also an indication of how his knowledge and confidence were developing.)

The detailed analysis in this section of the first topic Charles studied has illustrated the changes in his feelings and his new understanding of this topic. It

has also illustrated how many of my teaching aims have been met. In particular, the aim of engaging Charles at his level of mathematical understanding.

### **8.5.2 Engaging at the Students' Level of Mathematical Understanding**

Of the eight aspects of the teaching methods in the independent study course which Charles identified as the most important reasons why his course was so successful for him, four of them focused on the fact that I engaged with him at an appropriate mathematical level and pace. In his analysis of the six-month course, entitled "What Has Made The Changes" (Section 8.4.3), he points out that he thinks the course was successful for him because I "identified the gaps, really started with the basics, approaching problems in bite-sized pieces". One of my aims had been achieved, to be sensitive to the level and pace at which I re-introduced Charles to mathematics.

A key point in Coben's (1996 p. 5) "agenda for adult learning in mathematics for the next millennium" is that teaching "should proceed at a pace which suits the learners". Ramsden (1992), addressing issues about teaching and learning in higher education, said that good teachers must have the facility to engage students at their level of understanding. This issue was also a key finding in a research study in Scotland. How teachers dealt with students' differing amounts of background subject specific knowledge was the most common problem mentioned by students in science, mathematics, or engineering courses in higher education or further education in Scotland, investigated by Munn et al (1992). Students stressed the advantages of having the appropriate background knowledge and that they felt staff often wrongly assumed that they possessed this knowledge. The proportion of students reporting this concern was higher in advancing degree courses than in access courses. Staff who used "student-centred rather than teacher-centred methods" in the pacing of the content dealt most effectively with students' differing backgrounds. They planned a "gradual increase in difficulty" in the material, "encouraging student success at the beginning of the course so that confidence was developed", and "allowing the first few weeks of the course to serve as a foundation so that staff and students could see where they were" (Munn et al., 1992 p. 18). The individual study course was student-centred which did lead to early success, clearly a result of focusing the course on Charles' cognitive and affective needs in mathematics.

McLeod (1994), in his survey of research into affect and mathematics learning, discusses research which explores the relationship of mathematics anxiety to variation in instructional patterns and gives the result of Clute's study, that "anxious students do less well in discovery lessons than with expository

teaching" (Clute 1984, cited in McLeod, 1994 p. 640). Charles' experience suggests otherwise, so that maybe a sufficiently supportive environment allows for non-expository teaching to be effective with anxious students.

## 8.6 Later Developments

In 2000, Charles contacted me again and described to me the impact that our course of study was still continuing to have in his working life, how good he continued to feel about his new attitudes and knowledge in mathematics. Charles experience supports Parker's (1997, cited in Safford-Ramus, 2004 p. 57) conclusion, that "overcoming mathematics anxiety during adulthood" is a "transition of major magnitude", an important "life event". It also confirms Morrow & Morrow's (1995 p. 20) statement that "gaining a sense of their own voice in mathematics", as a result of connected teaching, is a "very powerful experience" for a student. He enquired about continuing to study mathematics with me, which we did for a few months. At my request he wrote a statement about his current attitude to mathematics and the usefulness of his new knowledge in his workplace now.

It is now approximately four years since I commenced tuition with Barbara Reilly of the Mathematics Department, Auckland University. My weekly tuition of approximately one hour spanned a period of nine to twelve months.

Prior to commencing tuition with Barbara Reilly I would have described my pure mathematics skills as severely limited. This limitation had, from my own perspective, been a significant impediment. Despite my abilities with basic and business mathematics I was acutely aware of my shortfall in what I referred to generally as "maths". This shortfall manifested itself as a lack of self-confidence when approaching most situations involving mathematics. This mindset was frustrated by an innate knowledge that this should not be the case, as my other skills were well developed. In a way fear and frustration reigned.

I am delighted to say that I now approach mathematics with a degree of pleasure and relative confidence.

Business mathematics seldom becomes extremely complex, other than for say a business analyst, as such some principles provide a guiding hand and the whole process is manageable. An appropriate example of the great value of this tuition is my ability to understand, use and enjoy algebra. A common method of assessing the value of shares within a company is the discounted cash flow model or DCF, which employs algebra, and in particular BEMA<sup>54</sup>. For example in assessing the required rate of return, and placing a value on K, the following equation is adopted:

$$K = R_f + B (R_m - R_f)$$

Applying BEMA, together with other information, make assessment of the above possible.

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<sup>54</sup> BEMA was the mnemonic used to remember the order in which arithmetic operations are completed.

The knowledge gained from expert tuition has enabled me to tackle such problems without fear and in a reasonably timely fashion. The method of teaching adopted by Barbara has opened up doors within my brain which were firmly closed and enabled me to progress with a greater sense of confidence.

I would rate the knowledge gained and consequent confidence as the highlight of the last four years.

In 2003, Charles contacted me again because he wanted me to look at some details of a mathematical model he had developed for his business. It seems that he has now "become excited about possibilities of posing his own problems and inventing new knowledge" by trying "to provide insights into mathematical connections with other areas" – he is becoming a "constructor of knowledge" in mathematics (Morrow & Morrow, 1995 p. 20), an overall aim of the connected teaching approach. This is related to the constructive perspective of knowing of *Women's Ways of Knowing* (Belenky et al., 1987). I gave him (positive) feedback about his mathematical model and in return he agreed to read this case study. I wanted to check, as Lyn Taylor (1995) did with the subjects of her ethnographic life histories, if I had reflected his views accurately in describing his learning experiences in 1995. The few editing changes he suggested have been incorporated into the manuscript. He wanted a copy of this chapter, his 'journey', so that he could share it with a colleague whom he thought avoided mathematics. Charles was now providing support for another mathematics anxious adult, which Parker (1997, cited in Safford-Ramus, 2004 p. 57) believes is the final part of a six-stage process. She identified this process in a number of adults, interviewed in her research, who were mathematics anxiety success stories. The other five stages are clearly visible in Charles' case study: his "perception of a need", his "commitment to address the problem" by taking "specific actions to become more comfortable with mathematics", his "recognition of a turning point having been reached", which has resulted in a change in his "mathematical perspectives".

Charles' letters and his continuing contacts with me illustrate long-lasting change. I notice that when he works with me now, he looks for patterns in the mathematics, which he calls "principles", and is then very satisfied with his learning. Davis (1992 p. 731) states that "for serious long term learning, one does not learn facts, one acquires a culture". It seems that Charles has "acquired a culture", for example, he knows that mathematics is about recognizing and describing patterns. As Damarin (1990 p. 150) writes, the "major objective of mathematics instruction must be that all students learn that they can learn mathematics". I believe, from the evidence in this chapter, that Charles has "learned that he can learn" mathematics. Connected teaching of mathematics seems to be a very suitable approach for adults who avoid mathematics. Charles

has realized that he has developed the power to do mathematics. Information or knowledge is one thing, power or ability is quite another. Boaler (1998 p. 87) has pointed this out in her study of two schools with completely different approaches to the teaching of mathematics. In the traditional, examination-oriented, school “teachers tried to give the students knowledge” while at the open, project-based, school the students “learned how to do things” (quote from a student). Much earlier, Moore (1948, quoted in Parker, 2005 p. 246) put it quite succinctly: “What does information amount to compared to power?”.

## **9 Final Discussion**

In this final chapter I aim to pull together the three strands of this study. I will initially present an overview of the findings of the data analysis for the three courses, all focusing on adults returning to the study of mathematics (Section 9.1). I have already theorized the in-depth data analysis for the individual supervised study course in Chapter 8. This discussion provides insights which are now related to the findings for the two larger courses, Mathematics 1 and the Wellesley mathematics course, in Sections 9.2 and 9.3. The use of metaphors in the interviews proved to be very useful and is discussed in Section 9.4. Overall conclusions and recommendations follow in Sections 9.5 and 9.6.

### **9.1 Overview of Findings**

During the six-month supervised study course in mathematics, the individual student (Charles) changed his view from one of extreme negativity to an emotionally positive and intellectually engaged attitude to mathematics. Such a course, working sensitively with a committed adult, without the demands of a fixed curriculum and assessment system, can be an ideal environment, as it was for this student and teacher. He had a strong desire to find out if he had a “black hole” in mathematics because of the negative impact his extreme mathematics avoidance has had on his career. In this course for the first time he felt his mathematical experiences and feelings were understood. After the course, Charles has new beliefs about the learning of mathematics and new conceptions of mathematics. He believes that mathematics is creative, rewarding, satisfying; that it is a process which involves a search for patterns, for possible routes to solutions, for relatedness amongst ideas and concepts. He now has agency, tools to use when he encounters mathematics again. He reported more mathematical confidence, particularly at work, and more enjoyment of mathematics, both of which were reflected in marked increases on some attitudinal scale scores and a much more positive belief in himself as a learner of mathematics.

Interviews with teacher-developers of the two larger courses enabled me to gain some insight into the goals of these experienced and highly committed teachers. The main themes emerging from the reactions of successful students

to the particular approach used in their course reflected the particular nature of each course. In Mathematics 1, successful students were positive about its contextualised approach, enjoying learning that mathematics was practical, useful, interesting and related to actual situations in real life. There were both positive and negative reactions to the conflict with their expectations of a mathematics class. In the Wellesley mathematics class, successful students were pleased with the amount of mathematics they had learned in the course, much of which they had not understood in high school and, generally, found the pace suited them even though for some it was reasonably challenging and hard. However, many lower-achieving students in Mathematics 1 found its approach confusing with too much writing required and, in the Wellesley course, many lower-achieving students found the pace too fast.

In both of the larger courses, most of the higher achieving groups entered the course with more confident mathematical beliefs and reported feeling more mathematically confident by the end of the course than the corresponding lower achieving groups. These differences were also reflected in the quantitative measures of attitudinal scales and of both their beliefs in themselves as learners of mathematics and beliefs about the learning of mathematics. For example, the latter changed during the course to a less algorithmic and more conceptual focus on learning mathematics. On the other hand, most of the lower achieving groups in the larger courses not only entered the course indicating less confidence in, and enjoyment of, mathematics than the corresponding higher achieving groups, but also reported less confidence by the end of the course than they had at the beginning.

Further data came from interviews with a few students in similar demographic groups in each of the larger courses. Interviews allowed more detailed exploration of the variables described above, as well as discovering other factors which seem to have influenced these students' attitudes, beliefs, achievement level and reactions to a particular course. The effect of the different cognitive perspectives and different mathematical beliefs and attitudes of these students impacted on their affective and cognitive changes during the courses. Use of a projective technique enabled these students to vividly portray their current experiences learning mathematics and to compare these with their previous experiences.

One of the benefits of this study design, in which three approaches were investigated and compared, is that data relating to one course can cast further light on the other courses. In particular, the very rich in-depth data relating to my one-to-one work with Charles provided insights that were not possible in

the studies of the two larger courses, but these insights can nonetheless be used to extend the conclusions and elucidate further the findings from these two studies. So, in the following section (Section 9.2), I will use the same analytic lens through which I have explored the one-to-one study to present an extension of this theoretical discussion related to the findings of the two larger courses, Mathematics 1 and the Wellesley mathematics course.

Table 9.1 presents a summary of the key findings and themes as they relate to the approaches in the three courses.

Table 9.1: A summary of the key findings and themes as they relate to the three approaches.

	Individual Supervised Study	Mathematics 1	Wellesley Mathematics
<b>Aims of curriculum were largely achieved.</b>	Mathematics avoidance and fear needed to be understood and changed as the student experiences	The approach, mathematizing realistic situations, aimed to 'turn them on' to mathematics. All students are in a degree program.	The approach aims to prepare students for tertiary study (not necessarily in mathematics) but students have wide range of background mathematics and often present with difficult personal circumstances and few study skills.
<b>Reflected in successful students' reactions which were related to each course's approach.</b>	mathematics as an enjoyable, creative, pattern-searching discipline, with connections to his context.	RESULT: Successful are turned on to the mathematics as useful, interesting, relating to real life situations.	RESULT: Successful students are very pleased they can finally do the mathematics they couldn't do at school. Unsuccessful students are affectively worse off by the end of the course.
<b>Different for unsuccessful students ...</b>	RESULT: He now believes mathematics is creative, satisfying, enjoyable. He expresses confidence and a sense of agency.	Unsuccessful students are confused and finish the course affectively worse off.	
<b>How the Course Dealt with Students' Fear of Maths</b>	A high priority was given to dealing directly with this student's anxieties about mathematics by talking about these feelings and about his early mathematical experiences.	The use of contexts, investigative work and two-stage tests aimed to provide motivation and build on existing knowledge and so lower students' anxiety	Techniques to reduce anxiety used were, grouping students with similar ability, teaching in small classes, knowing students' names and circumstances.
<b>Flexibility of Course Content</b>	Course content reflects the student's needs, both for a different experience with mathematics than in the past, and in relation to his life and work.	Course content is predetermined but the open-ended nature of the tasks allows some choice for students in solutions presented.	Course content is predetermined and quite traditional. Topics carefully introduced and structured.

<b>Is the material presented at the right pace?</b>	Tailored to his mathematical needs and he appreciated this aspect of the course very much. He is now confident with the mathematics at work.	The contextual nature of the problems was appreciated by the fluent, female and older non-fluent students.  For many non-fluent it was confusing & hard to understand.	Many appreciated the carefully structured and paced re-introduction to mathematics.  For students in stream 1 who failed it was too rushed.
<b>Is the course flexible enough?</b>			
<b>Is the course sufficiently supportive?</b>	Large increase in score on mathematical self-concept in scale.	The fluent and older non-fluent students enter the course with more positive beliefs in themselves as learners of mathematics than other groups and these beliefs are more positive by the end of the course.	Students in streams 1 and 4 enter course with different beliefs – stream 4 more positive.
<b>Students' beliefs in themselves as learners of mathematics</b>	Big change in confidence reported.	For the younger non-fluent students, these beliefs are less positive than they were at the start of the course.	Students in stream 1 who fail course lose confidence. Students in stream 1 who pass gain confidence.  Students in stream 4 usually gain confidence.
<b>Attitudes: Enjoyment &amp; (extrinsic) Value of Mathematics</b>	Enjoyment score increased a lot. Value score stayed high and increased.	Value scores at the end of the course correlated significantly with achievement.	Enjoyment scores at the end of the course correlated significantly with achievement.
<b>Students' beliefs about the learning of mathematics.</b>	He becomes a conceptual learner of mathematics. There is evidence of 'constructed knowing' in mathematics.	Example: Fluent and non-fluent students enter course with similar beliefs. Fluent stay conceptual learners. Non-fluent become less conceptual learners by end of course than they were at the start of the course.	Streams 1 and 4 enter course with different beliefs – stream 4 are the more conceptual learners – although a subset of stream 1 are conceptual learners and they pass.  Students in stream 1 who fail largely stay rote learners of mathematics.  Students in stream 4, and those in stream 1 who pass, largely stay conceptual learners.
<b>Beliefs about mathematics</b>	Mathematics is, by the end of the course, creative, a process, satisfying	Mathematics, by the end of the course, can be used to mathematize 'realistic' situations. It is practical.	Beliefs about mathematics largely unchanged, although some see its practical nature.

## **9.2 Acknowledging Negative Experiences, Feelings and Reactions to Mathematics**

### **9.2.1 Understanding the affective background of students in order to create a supportive environment**

It is widely acknowledged in the literature (Becker, 1995; Buerk, 1982, 1985, 1996; Damarin, 1990; Goolsby et al., 1987; Morrow, 1996) that addressing the affective, in addition to the cognitive, domain is a crucial matter. Section 8.2 discusses how the affective needs of Charles were addressed. This current section shows how this was important in both the larger courses also. Maxine, one of the teachers in Mathematics 1, mentioned that the course was “about valuing people”. Moira, one of the teachers in the Wellesley mathematics course, mentioned trying to know the students’ names the first day. She had met them all briefly because all students were interviewed so she was also aware of some of their past experiences in school (see Section 8.2.5). She aimed to make the course “user-friendly”, to be “approachable”, “to never put the students down” i.e. no blame (Zaslavsky, 1994), and to “try and make them feel comfortable”. These teachers were aware that many of the students were fearful of, and had avoided, mathematics (Buerk, 1982, 1985, 1996; Fennema, 1995; Tobias, 1978). A reasonably high proportion of students in both of the larger courses (40-60%) referred positively to the teaching or to an atmosphere that was conducive to their learning, even though there was not a specific question in the questionnaire about this. These students appreciated the care of the teachers, and the atmosphere created, enough to comment especially on it.

Although, in Mathematics 1, the classes were held in large lecture theatres, during most lectures an attempt was made to have all students involved in groups doing an activity during the hour. One hour per week was also spent in smaller tutorial groups where collaborative group work was encouraged. Further discussion on groups is in Section 9.3.3.

The Wellesley mathematics course was taught in small classes of less than 25 students, possibly allowing a supportive atmosphere to develop more easily than in a large lecture. The support felt by students varied. For example, Tania, interviewed 18 months after successfully completing the Wellesley mathematics course, acknowledged how helpful her teacher had been to her and that she “just needed to ask”. However, it took time to feel successful in this class. She reports feeling more comfortable about two-thirds of the way through, and not until after she had taken the initiative to form a study support group around her. In contrast, Mary, who was unsuccessful in the course, just ‘wrote everything down’ as a “passive receiver of knowledge” (Boaler & Greeno, 2000

p. 181) and tried to understand it later, with little success. Mary's way on knowing in mathematics could be described as a received knower, one of Belenky et al's (1987) stages of knowing. Koch (1996 p. 174) also described Ann as a received knower as she "learned by listening to others, particularly those in authority", and "had little confidence in her own mathematical ability". Maybe Mary did not ask for enough help or think about organizing peer support. She painted a rather gloomy picture of others in her low stream who were struggling more than her. Her beliefs about herself as a learner of mathematics were affected negatively. See the discussion of streaming in Section 9.3.2 and affective changes in Section 9.3.3. She experienced a powerful realization about how 'maths-phobic' she was as a result of my use of metaphors in her interview, similar to Gibson's (1994) experience that the technique gives students a more accurate picture of themselves.

### **9.2.2 Students' goals and mutual planning**

A 'time of transition', similar to that which Charles was experiencing (Section 8.2.4), is also seen, to a greater or lesser degree, in the larger courses (R. Smith, 1990). Students applying for the Wellesley Program were usually struggling "to rebuild and move beyond the fragments of a life" (West, 1995 p. 154) which would have provided them with motivation for the course i.e. they would have been ready to learn (Merriam & Brockett, 1997). Their previous educational experiences had often been unsuccessful. The interview given to each student before acceptance in the Wellesley Program was an important opportunity to hear of each student's needs and goals. Teachers assessed these and advised each student whether the Program, or the timing of it, was appropriate, offering alternative suggestions if not. For all accepted students, this Program was definitely a chance to create more options for their future (Coben et al., 2000). For Mathematics 1 students, the reasons they were studying this course were either as an opportunity to try learning mathematics again, often after some time away from it, or because they needed mathematics for other subjects they were studying (Knowles et al., 1998). For younger Asian students in Mathematics 1 another type of transition was evident. This course was a cultural and linguistic 'transition' for them, with mixed results (see Sections 8.3.2 and 9.3.3).

A discussion of students' goals, and mutual planning of the course, was not possible to accomplish in the other two courses in the detail that was possible with Charles (Section 8.2.4). Some "mutual planning" (Knowles et al., 1998 p. 135) and discussion of "*how* the learning will be conducted, *what* learning will occur and *why* this learning is important" (p. 133) occurred in the interviews of

students in the Wellesley Program. While students in the Program may feel that it would meet their needs in improving the quality of their lives by opening up more choices for future opportunities, the fact that the mathematics course was compulsory will have negatively affected the motivation of some students since adults usually need “to feel a sense of choice in their learning” (Wlodowski 1985, cited in Knowles et al., 1998 p. 149). However, the teachers may have tried to “make an intellectual case for the value of learning” mathematics as one way to counter a lack of motivation (Knowles et al., 1998 p. 65). One dimension of motivation, “why the learning is important” (p. 135), has been addressed in Mathematics 1 by the use of the realistic situations, which students investigated and modeled mathematically. These have provided motivation for many students but not for all. See Sections 8.3.2 and 9.3.1 (which follows) for discussions about contexts with respect to fluency and gender.

## 9.3 Teaching the Mathematics

### 9.3.1 The use of contexts in the mathematics course

Issues about the contexts used in the curriculum materials in the larger courses could also be raised here. It was of course not possible to base these courses on any one student’s culture or interests, as was done for Charles. But contexts used in non-routine problems in Mathematics 1, in particular, allowed for some recognition of prior knowledge and the rich experience of many students (Coben, 1996; FitzSimons & Godden, 2000; Knowles et al., 1998). The nature of the realistic non-routine tasks in Mathematics 1 has required student involvement and some learner control, encouraging “the interaction between content and experience, whereby each (can) transform the other” (Knowles et al., 1998 p. 147), particularly when students took advantage of the teachers’ encouragement to students to adapt the tasks to their interests. These are characteristics of experiential learning. There has been some blending of “objective and subjective knowing in mathematics” (Becker, 1995 p. 170), allowing the course to become a “meaningful educational experience” (Morrow & Morrow, 1995 p. 18) for many students. This is a characteristic of connected teaching where students are encouraged to learn by including their personal experiences of mathematics also. One half to three quarters of the students in Mathematics 1 reacted positively to the contextualised nature of the course, using terms such as “applicable”, related to “real life”, “practical” (Chapter 6, Section 6.2.2.1). It seems that many students, particularly fluent, older or female students, have been able to recognize the possibility of transferring the mathematical understanding gained in this course to real world situations. This agrees with Boaler’s (1993 p. 345) findings which suggest that “problems and

investigations which are open can link with a student's meaning by allowing the attainment of personal goals". However, non-fluent younger females in the course typically did not react as positively as other groups to the contextualised nature of the course. On the other hand, one such student, Yuko, was successful. In her interview, reported in Section 6.3.2, she commented a number of times that she had not realized before that mathematics was useful. Her imagery of the lion versus the cow illustrated the difference between her experiences learning mathematics in Mathematics 1 compared to high school.

So it seems that the approach in Mathematics 1 has addressed, for many students, the "degree of discontinuity of performance" in mathematics across "school" and "everyday" situations (Boaler, 1993 p. 341). The open-ended investigations may have "replicated real world demand" where problems are complicated in terms of "the relative influence of subjectivity, experience, communication, process and content" (p. 371). Maybe the tasks in Mathematics 1 "reflect(ed) this complexity" and required enough "application and negotiation" which allowed students to "develop an understanding of meaning, derivation and applicability" (p. 371). The positive reactions of many students to the contextualized nature of this course suggest that they have learned "mathematics through the integrated development of process and content" and have begun "to understand mathematics and to be capable of using this mathematics in different situations" (p. 371). This is particularly clearly reflected in the comment from one student who said "I liked this paper because it helped me to understand mathematics and use it in my day to day experiences". (See other similar quotes in Section 6.2.2.1). For these students, the contexts used seem to have included enough of the "range and complexity of (their) individual experience and interpretation" (Walkerdine 1988, quoted in Boaler, 1993 p. 345).

Some criteria for selecting contexts have been developed by Mellin-Olsen (1987, quoted in Heckman & Weissglass, 1994 p. 32) to encourage "authentic activity", which Heckman and Weissglass (1994 p. 30) believe is "paramount for learning". Mathematics 1 has implemented some of these criteria. Mellin-Olsen suggested that learning mathematics must occur "in the context of a wider project of interest to the learner" (p. 32). The teachers in Mathematics 1 chose contexts which they hoped would interest students, but it was not possible to consult with students. However, the teachers did permit students to adapt the tasks to their interests. It seems that contexts chosen were of interest to a reasonable proportion of the students, but others found them difficult to understand. Another criteria was that the learning occur within a "context of cooperation" (p. 32). Encouraging students to work collaboratively in groups in

the tutorials, met this criteria to some extent. Mathematics 1 also tried to encourage students to "see knowledge of the discipline as part of a complex web of values and activities that affected the environment and society" (p. 32), another criteria. The large proportion of students who commented on the course's usefulness and relevance indicate this criteria was met for many.

Making a gender comparison, not only do fluent females achieve a little better than fluent males overall in Mathematics 1, there is a higher proportion of this female group in the top grades. Usually the opposite situation occurs, as Leder (1990 p. 13) reports "differences in the proportions of males and females who score exceptionally well are particularly striking". Linn & Hyde's (1989 p. 26-7) meta-analysis found that differences between males and females remain for high scores on the SAT-M test in the USA, with males achieving higher, otherwise these differences were very small. The high achievement of fluent females in this paper seems to confirm the results of research done by Vale (1993a p. 566), who found that females "performed better than males on investigative projects and non-routine problem solving", and the conclusion ofForgasz' (1994 p. 49-50) study, that "the form of the assessment task is implicated in the direction of the gender difference found in mathematical performance". In addition, Burton (1994 p. 17), in a comparative study of assessment in mathematics in ten European countries by gender, says "it is evident that changes in a relativistic direction in learning and assessing styles lead to improvements in the performance of females". Mathematics 1 has this characteristic. Linn & Hyde (1989 p. 26-7), in a meta-analysis of research on gender and mathematics, gave examples of situations that minimised gender differences. They were: classes where all students were provided with "feedback on the use of problem-solving strategies rather than memorised algorithms"; or environments that "encouraged expression of ideas by all"; or curricula that emphasized the relevance of mathematics. All of the above characteristics of Mathematics 1 could have contributed to the more conceptual approach to the learning of mathematics, the higher pass rate, better retention rate and higher proportion of A grades for (fluent) females compared to (fluent) males.

As far as fluency in English is concerned, students who were not fluent in English gave negative responses to the question about "the maths" in Mathematics 1 as well as to the question about whether Mathematics 1 affected their mathematical confidence. As discussed previously, task contexts can be more personally meaningful to one group than another (Burton, 1987; Heckman & Weissglass, 1994; Lehmann, 1987; A. Rogers, 1986). Boaler (1993) discusses how mathematics must connect with students' meaning and it must

acknowledge the social environment. Maybe the contexts used had little meaning for, particularly younger, non-fluent students. Cultural bias may be implicated in the contexts used (Lajoie, 1995). Students from non-English-speaking backgrounds may also find the "demands of language or contextual detail" in Mathematics 1 "constitute an excessive cognitive load" (Helme, 1994 p. 2). One comment from such a student seemed to indicate this load when she says the maths is "easy but hard" (see Chapter 6, Section 6.2.2.1.1).

One reason why this approach was not successful for certain groups of students could be because the selection of contexts was not usually "negotiated between teacher and learner", another criteria mentioned by Mellin-Olsen (1987, cited in Heckman and Weissglass 1994 p. 32). Although students were encouraged to adapt the tasks, maybe the teachers in Mathematics 1 had not considered "how their present values affect their choices" of contexts (Heckman & Weissglass, 1994 p. 30). The teachers in Mathematics 1 were fluent in English, older and largely female, hence the contexts may have reflected the experiences, interests and values of these demographic groups. Maybe it is no accident that these were the demographic groups that achieved well in the course.

### **9.3.2 Engaging at the Students' Level of Mathematical Understanding**

Mathematics 1 and Wellesley mathematics teachers used many student-centred rather than teacher-centred methods and early success was achieved by many students in these two larger courses. In Mathematics 1, Maxine acknowledged that since "they are all at different starting points (her aim was) just working with them and letting them grow mathematically". Care taken to introduce topics to encourage early success (Munn et al., 1992), and the contextual non-routine problems allowed for some variation in students' mathematics background. This approach was successful for a large proportion of the students. However, as I discussed previously in Section 8.3.2, the contexts used in this course material suited some students more than others and there have been difficulties in meeting all students' needs because of differing English language backgrounds and different expectations about learning mathematics.

The teachers in the Wellesley mathematics course aimed to make the course manageable so that students would feel that they could do it. They made efforts to carefully structure the course to suit the particular population of students who usually entered the course. An explicit recognition of the formal background mathematics knowledge of Wellesley students was gained through testing at the interview and then grouping students into streams. This organization of students aimed to engage them at their level of mathematical

understanding. In Wellesley the main goal was the gradual introduction and revision of school mathematics to prepare them for degree study. These principles have been implemented to good effect for a large proportion of students in the Wellesley mathematics course, with many students commenting favourably on the appropriate pace of this course (Chapter 7, Section 7.2.1.1). The main theme which emerged from Wellesley students' responses to all the open questions was that they were pleased with, and excited about, the amount of mathematics they had learned in the course - they felt empowered in school mathematics. A high proportion of the students reported a gain in mathematical confidence.

However, it has been more difficult to meet the needs of students with little mathematics background. Because of the structure and resourcing of the Program, all students were expected to complete the same mathematics course and many in stream 1, largely lower-achieving students, found the pace too fast. Some students in the highest stream, where an additional mathematics course was studied, also found the pace too fast (see discussion in Chapter 7, Section 7.2.4.2).

A summary of research results on the long-term effects of the homogeneous grouping of mathematics students in high school, (Davenport, 1993 p. 1), indicates that the practice "fails to increase learning and seems to widen gaps between students deemed to be more or less able". In addition, it was found that female and minority students were "more often placed in the lower tracks", a trend noticeable in the Wellesley Mathematics course in 1995 and 1996 for Maori and Pacific Island students, also in 1996 for female students. There are generally fewer of these students in the higher streams. I agree with Davenport (1993 p. 1) who comments, this is a "troubling result considering the low representation of female and minority students in science and mathematics areas".

Many students in the Wellesley Program belong to the "under-educated" category in Smith's (1990 p. 50) clientele groups amongst adults. Smith suggests that students in this category are "especially subject to anxiety and doubts about their learning ability" and that "establishing a climate for minimising anxiety" is the "most crucial condition for learning success in this population". The inappropriate level of the existing mathematics course for many students in stream 1 is unlikely to have generated a "climate for minimising anxiety" for these students.

### **9.3.3 Appropriate mathematical challenges and affective change**

Although a number of factors are responsible for this, beliefs about learning and doing mathematics seem to be a key to many students' inability to focus themselves enough to "survive" mathematics courses that they see as both emotionally and cognitively difficult

(Stage & Kloosterman, 1995 p. 295)

A higher proportion of students in stream 1, students with the least background in mathematics, entered the Wellesley mathematics course with procedural beliefs about the learning of mathematics than in stream 4, those with the most background mathematics. This is consistent with Duranczyk & Caniglia's (1998 p. 132) finding, that students with fewer background courses in mathematics had "narrower views of mathematics". Many of these students had negative beliefs about themselves as learners of mathematics also. It may not be surprising to find such a high proportion of students with such beliefs because an Australian study (Crawford et al., 1994), seeking to identify conceptions of mathematics held by beginning university students, found that the majority viewed mathematics as a necessary set of rules and procedures to be learnt by rote.

The approach in the Wellesley mathematics course has had little effect on the many students with instrumental beliefs in stream 1, the lowest-achieving group, particularly those in stream 1 who failed the course. These students had the least cohesive and most fragmented views about the learning of mathematics in the class. Mau (1991, cited in Stage & Kloosterman, 1995) conducted a qualitative study of a similar situation in a remedial college-level mathematics course. She found that the instructors wanted students to succeed but believed many students unwilling to put the time and effort required into what instructors perceived as an easy course. However, the students believed they were working hard and course expectations were unreasonable. The course created a high level of stress. When Mau investigated how students were spending their time she found that students "believed that simply memorizing formulas and algorithms was the best way to master course content" (p. 295), and those beliefs appeared to be the major cause of students' difficulty with the course. The fact that this mathematics course was a compulsory part of the Wellesley Program may mean that some of the students in the lowest stream also had little motivation, which could have contributed to this result. Their already negative beliefs about themselves as learners of mathematics were more negative by the end of the course.

It is very different for students in stream 1 who passed the course. They entered with more positive beliefs about themselves as learners of mathematics than

others in stream 1 and often felt more positive about themselves as learners of mathematics by the end of the course. These, largely female, students also had some of the most cohesive and least fragmented views about the learning of mathematics in the class (Crawford et al., 1994). This is consistent with Stage and Kloosterman's (1995 p. 308) finding that, in a tertiary 'remedial' mathematics course, "women who had more positive beliefs about the nature of mathematics and about their own ability were more likely to succeed". It seems higher achievement is not necessarily a question of prior mathematical knowledge, as was found by Hartnell's (2000) study of students in a bridging mathematics course at Waikato University in New Zealand.

There is not such a marked difference in these beliefs between higher and lower achieving groups, by fluency in English, in Mathematics 1. Both groups have conceptual beliefs about the learning of mathematics although some of the non-fluent group are uncertain and less conceptual in their responses. The approach in Mathematics 1 has had a detrimental effect on beliefs of the non-fluent group who, by October, are focusing on algorithmic rather than conceptual ways of learning mathematics, although a good proportion are undecided in their views, indicating some uncertainty. This uncertainty could possibly be due to the use of contextual non-routine problems in Mathematics 1 (see Section 8.3.2). McLeod (1994) points out that non-routine problems, which should lead to more enjoyment and more interest in mathematics, may not because of a conflict with students' beliefs. Schoenfeld (1989) also showed that problem solving performance can be undermined by beliefs. Smith (1991 p. 29) found that "students who adopted a reproducing orientation" achieved lower results in a tertiary preparation course or dropped out of the course. Crawford et al (1993), in their study of first year undergraduate mathematics students, also found evidence of similar relationships between conceptions of mathematics, with related approaches to learning, and course results.

For the higher achieving groups in both courses, it was a different story. They held stronger conceptual beliefs about the learning of mathematics and reported feeling more mathematically confident by the end of each course. For these groups the small classes (in Wellesley) and tutorial groups (in Mathematics 1), with supportive teachers, have provided the "moral climate" necessary for students to "nurture each other's thoughts to maturity" (Becker, 1995 p. 170). Values such as those of "co-operation, connection and communication" were likely to be present so that many students felt "cared for" and not patronized (Ocean, 1997 p. 8). Many students have felt well enough supported to take the risks (and make the mistakes) necessary to grow

mathematically (Morrow & Morrow, 1995) and to broaden their strategies for knowing in mathematics (Goldberger, 1996a).

The contextual non-routine problems set each week were intended to show students that mathematics was a constantly changing product of human invention, reflecting the belief that mathematics should make sense: make sense to students and make sense of the world around us. For the successful students in Mathematics 1, meeting this challenge of these contextual non-routine problems has changed their conceptions of mathematics. Now mathematics is useful and relevant. It has “demystified the doing of mathematics … calling attention to mathematics as a creation of the human mind, making visible the means by which mathematical ideas come into being … and engaged students *within* the classroom in purposeful, meaningful activity” Rogers (1995 p. 178-9).

Although not in the original objectives for this research I unexpectedly found that using a projective technique in the interviews was helpful.

#### **9.4 An interview technique**

Using metaphors in the interviews with Mathematics 1 and Wellesley Program students gave some particularly interesting, useful and rich results. These provided a trigger for conceptualisation, and comparison, of the students' experiences learning mathematics (Briscoe, 1991; Sims, 1981). My experience conducting these interviews reflects that of Gordon & Langmaid (1988, p. 89) who report that the use of metaphors “created new energy in the interviews”. Chapman (1997 p. 202) states that “metaphors are used in framing the meaning one assigns to events – a way of understanding our perspective”. Taking advantage of this property of metaphors, I also extended the use of this technique to find out how students would describe and compare their mathematics learning experiences, both in their current course and in school. Each student's metaphors enabled me to understand more clearly how their current and their previous experience learning mathematics had affected them (Gibson, 1992). The students were usually asked to interpret their own metaphors which was deemed important by Gordon and Langmaid (1988) in their discussion of the use of this projective technique in qualitative market research. The metaphors used by these students illustrate the “synthesizing function” of a metaphor, “a metaphor's ability to compress a great deal of peripheral, intuitive and emotional content into one symbol”, and their “generality and remoteness from specific problems” (Sims, 1981, p. 402). The use of metaphors seemed to tap into students' feelings about mathematics (Buerk, 1996) and feelings about the learning of mathematics. This use of the

technique appears to be a novel and powerful approach in this area of research in mathematics education.

For students with limited fluency in English, it helped to clarify my understanding of experiences that these students could not easily describe literally (Bowman, 1995) because of the level of their oral fluency in English. For example, for Yuko and Keiko, it became a powerful tool for them to express their meanings, particularly as they interpreted most of the metaphors they used.

## **9.5 Conclusions - my contribution to the literature and limitations of the study**

This study contributes to research on understanding good teaching of mathematics to adults. Research in this area is important because mathematics is a prerequisite (explicit or implicit) for many professions and occupations and these courses act as gatekeepers, filtering students out of careers they might otherwise pursue.

The multiple strands of evidence have been woven together to complete an complex tapestry of these three, largely successful, teaching approaches in second chance mathematics courses. It is a multifaceted story with different teachers, different approaches, and the different journeys of the students. The same research tool has been used and all the data gathered come together to create a rich overall story. Each measure has its own contribution to make. Nothing stands alone but together these measures provide ample evidence of the relationships between affective change and achievement.

One of the strengths of this study is that it is possible to compare reactions of students to three different methods of instruction, all taught by highly committed experienced teachers. These teachers carefully planned a learning experience that they felt would be most beneficial to the students in each course and for most students it has been successful. In analyzing students' responses to particular teaching practices I have discovered that these responses are influenced by the nature of the course (probably also a sign that the courses have been well taught).

In the individual study, Charles' previous experience of silencing, disempowerment and lack of voice in mathematics, resulting in mathematics avoidance and fear, needed to be understood. The student needed support as he experienced a different mathematics which emphasized the importance of personal experience in knowing mathematics i.e. connections to his life and

work, as well as connected strategies in learning mathematics, focusing on his thinking and moving on from there. Six months later he believes mathematics is creative, satisfying, enjoyable and a process of discovering patterns. He expresses confidence and a sense of agency. The approach in Mathematics 1, that of mathematizing realistic situations, aimed to 'turn them on' to mathematics. Successful students reactions indicate they now realise that mathematics is useful, interesting, relating to real life situations. Successful students in the Wellesley mathematics course are very pleased they can finally do the mathematics they couldn't do at high school. Many appreciated the carefully structured and paced re- introduction to mathematics. Stage et al (1998) believe that few authors have systematically tracked differences in learning across classes. My study contributes to this area.

The inclusion of the individual study course as a third teaching approach in this study has changed the focus of, and enriched, this study markedly. Rather than the original design, the evaluation and comparison of two courses, where my only involvement was as a researcher, I became intimately involved in this third approach. I have needed to develop the knowledge to reflectively analyse my own teaching. Moreover, although I knew that I had helped many students in the past to become more mathematically confident, I had never gathered data to illustrate the changes. Using metaphors to discover Charles' feelings about mathematics was an amazing experience for me, as it was such a powerful way for him to express his feelings about mathematics at the start of the course. Also the use of the metaphors six months later gave information about the change in his beliefs about mathematics and his beliefs about the learning of mathematics that I believe I could not have gathered any other way. Analysing this data was particularly energizing and absorbing to me, and became the focus of the discussion of the whole study. This illustrates how the in-depth analysis and discussion of one part of this study, the one-to-one course, has provided insights not possible with the studies of the larger courses, and these insights have been used to extend the conclusions and elucidate further the findings from these two courses.

The examination of these different approaches reveal different reasons why students beliefs and attitudes have changed. Negative affective change in part resulted from the combination of poor background knowledge, negative attitudes and beliefs, and the inappropriate pace and level of the material taught in the Wellesley mathematics course. For Mathematics 1 a combination of less fluency in English and younger age together with the complexity of the situations the students had to mathematize led to negative affective change. There are other issues which could impact on the motivation of such students,

such as the compulsory nature of the Wellesley mathematics course, or that non-fluent students in Mathematics 1 could be enrolled in this course because of parental pressure. One of the important results of this study is that failed students leave a course affectively worse off than when they started. Students' beliefs about themselves and the nature of mathematics, the level of enjoyment and valuing of mathematics, are clearly factors in the success or failure of students in the courses I have studied.

Metaphors about mathematics were used initially to acknowledge, as a teacher, the individual student's fear of mathematics. Then, as a researcher, they were used to monitor changes in his beliefs about mathematics and beliefs about himself as a learner of mathematics, both of which changed markedly during the six month course. Metaphors prove to be a powerful teaching and research tool. The nuances in the imagery of the metaphors were a powerful way to show how Charles' beliefs had changed. I then decided to use metaphors in the interviews of selected students from the large classes. Hence, some responses to questions about their experiences learning mathematics, both in the past and in the current course, were facilitated and enriched through the use of metaphors.

The use of metaphors also seemed to give students a more accurate picture of themselves (Gibson, 1994), for example, Mary and Charles, which may link with recent research.

The ways we teach students to be aware of their beliefs may have more implications for changing those beliefs and improving students learning that anything else we can do. Certainly we need more research on strategies to help students monitor the way that beliefs and affect may influence their performance in mathematics classes.

(McLeod & McLeod, 2002 p. 122)

Charles' awareness of his beliefs has almost certainly influenced his motivation for, and his performance in, the course. Mary's new found awareness of the intensity of her negative beliefs about mathematics seemed to make her more determined to conquer her fear in the future (Section 7.3.2).

The strengths of this study have associated limitations. For example, the amount of data in the individual study course permitted a deeper analysis of the issues in a way that was not possible in the other courses. As usual, there was a tension of breadth versus depth in relation to the amount of data collected.

One feature of the study was that the teachers I interviewed who taught the larger course were colleagues and friends of mine. The positive aspect of this

was that they were very willing to be involved and were always willing to be accommodating when I needed to gather data etc. I also understood more about these courses through continuing conversations in our workplace. The limitation of close relationships with the teachers of these courses was an awareness of the need to be sensitive to issues which are largely unspoken in a workplace.

## **9.6 Recommendations**

I would recommend the continuance of the two large courses as they have met the needs of most of the students enrolled. More attention, however, needs to be paid to some groups whose needs have clearly not been met. I realize that this issue can be related to a lack of resources available to meet such a variety of needs, as well as the motivation of such students.

In the Wellesley mathematics course, the large proportion of stream 1 students who fail may indicate a need for an appropriate curriculum for these students to learn in the time frame available. If it is not possible to provide this then it may be necessary to put in place extra help. An alternative is to consider not making mathematics compulsory. Otherwise an option is to not accept students with a very limited background knowledge of mathematics into the Program. (In 2002 this course has been semesterised and only the first semester course in mathematics is compulsory. This solution will still not meet the needs of students who need the material presented at a more gradual pace.) Mary's experience in the Wellesley mathematics course indicates that the teachers might need to actively help the students to form study groups.

In Mathematics 1 there were no resources available to offer additional help to students for whom English was limited. Any contextual investigative course needs to provide extra assistance to students who are not fluent in English. Teachers who develop a contextual investigative course need to be aware that contexts must be chosen with the student population in mind and must reflect their interests or, ideally, be chosen by them. There is generally a positive reaction to the tutorial groups, supporting research indicating the effectiveness of learning cooperatively in small groups, although some responses suggest that more attention needs to be paid to the quality of the group process.

Charles was extremely fortunate in that the resources were available for his individual course of study when he approached the Department for help. In very few institutions are such resources available for such a purpose. The very significant affective changes in Charles show that in some cases such allocation of resources is timely and efficient.

## References

- Aiken, L. R. (1974). Two Scales of Attitude Toward Mathematics. *Journal for Research in Mathematics Education*, 5, 67-71.
- Aiken, L. R. (1979). Attitudes Towards Mathematics and Science in Iranian Middle Schools. *School Science and Mathematics*, 79, 229-234.
- American Mathematical Association of Two-Year Colleges. (1995). *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*. Memphis: American Mathematical Association of Two-Year Colleges.
- Azazkis, R., & Hazzan, O. (1998). Interviewing in Mathematics Education Research: Choosing the Questions. *The Journal of Mathematical Behaviour*, 17(4), 429-439.
- Barnes, M. (1994). Interaction Between Theory and Practice in Bridging Mathematics. In J. Nicholas & S. Gordon (Eds.), *Proceedings of the Fourth Australian Bridging Mathematics Network Conference* (Vol. 1, pp. 1-10). Sydney, Australia: Australian Bridging Mathematics Network.
- Barnes, M. S. (1989). A Rescue Operation. In D. C. Blane & G. C. Leder (Eds.), *Educating Girls: Practice and Research* (pp. 105-117). Sydney: Allen & Unwin.
- Baxter Magolda, M. (1992). *Knowing and Reasoning in College: Gender-related Patterns in Students' Intellectual Development*. San Francisco: Jossey-Bass.
- Becker, J. R. (1995). Women's Ways of Knowing in Mathematics. In P. Rogers & G. Kaiser (Eds.), *Equity in Mathematics Education: Influences of Feminism and Culture* (pp. 163-174). London: The Falmer Press.
- Becker, J. R. (1996). Research on Gender & Mathematics: One Feminist Perspective. *Focus on Learning Problems in Mathematics*, 18(1, 2 & 3), 19-25.
- Belenky, M. F., Clinchy, B. M., Goldberg, N. R., & Tarule, J. M. (1987). *Women's Ways of Knowing: The Development of Self, Voice and Mind* (Tenth Anniversary ed.). New York: Basic Books.
- Benn, R. (1997). *Adults Count Too: Mathematics for Empowerment*. Leicester: The National Organisation for Adult Learning (NIACE).
- Benn, R., Maasz, J., & Wedege, T. (1999). 'Adults Learning Mathematics' as a Community of Practice and Research. In M. v. Groenestijn & J. O'Donoghue (Eds.), *Mathematics as Part of Lifelong Learning: Proceedings of Adults Learning Mathematics - A Research Forum (ALM-5)*, Utrecht, 1998 (pp. 54-63). London: Goldsmiths College, University of London.

- Boaler, J. (1993). Encouraging the Transfer of 'School' Mathematics to the 'Real World' Through the Integration of Process and Content, Context and Culture. *Educational Studies in Mathematics*, 25, 341-373.
- Boaler, J. (1997a). *Experiencing School Mathematics: Teaching Styles, Sex and Setting*. Buckingham, UK: Open University Press.
- Boaler, J. (1997b). When Even the Winners are Losers: Evaluating the Experiences of 'Top Set' Students. *Journal of Curriculum Studies*, 29(2), 165-182.
- Boaler, J. (1998). A Tale of Two Schools: Alternative Teaching Approaches and Situated Learning. In A. Watson (Ed.), *Situated Cognition and the Learning of Mathematics* (pp. 83-91). Oxford: Centre for Mathematics Education Research, University of Oxford.
- Boaler, J., & Greeno, J. G. (2000). Identity, Agency, and Knowing in Mathematics. In J. Boaler (Ed.), *Multiple Perspectives in Mathematics Teaching and Learning* (pp. 171-200). London: Ablex Publishing.
- Boaler, J., Wiliam, D., & Zevenbergen, R. (2000). *The Construction of Identity in Secondary Mathematics Education*. Paper presented at the 2nd International Mathematics Education Conference (MES2), Portugal.
- Boud, D., Cohen, R., & Walker, D. (Eds.). (1993). *Using Experience for Learning*. Buckingham: Society for Research into Higher Education and Open University Press.
- Bowman, R. P. (1995). Using Metaphors as Tools for Counseling Children. *Elementary School Guidance and Counseling*, 29, 206-216.
- Bradburn, N. M., & Sudman, S. (1991). The Current Status of Questionnaire Research. In Biemer, Groves, Lyberg, Mathiowetz & Sudman (Eds.), *Measurement Errors in Surveys*: John Wiley & Sons.
- Brew, C. R. (2001). Tracking Ways of Knowing with the Role of Teachers and Peers: an Adult Mathematics Classroom. In M. J. Schmitt & K. Safford-Ramus (Eds.), *A Conversation Between Researchers and Practitioners: Proceedings of the 7th International Conference of Adults Learning Mathematics - A Research Forum (ALM7)*, 2000 (pp. 97-103). Cambridge, MA: NCSALL, Harvard University.
- Briscoe, C. (1991). The Dynamic Interactions Among Beliefs, Role Metaphors, and Teaching Practices: A Case Study of Teacher Change. *Science Education*, 75(2), 185-199.
- Britt, M., & Hughes, P. (1985a). *Making Sense with Mathematics 1A*. Auckland, NZ: Heinemann Education.
- Britt, M., & Hughes, P. (1985b). *Making Sense with Mathematics 1B*. Auckland, NZ: Heinemann Education.
- Broadbooks, W. J., Elmore, P. B., Pedersen, K., & Bleyer, D. R. (1981). A Construct Validation Study of the Fennema-Sherman Mathematics Attitudes Scales. *Educational and Psychological Measurement*, 41, 551-557.

- Brooker, B., Butel, C., & Carson, P. (1990). *Maths Matters 2*. Christchurch, NZ: Caxton.
- Brooker, B., Butel, C., & Carson, P. (1991). *Maths Matters 1*. Christchurch, NZ: Caxton.
- Brookfield, S. D. (1986). *Understanding and Facilitating Adult Learning*. San Francisco: Jossey-Bass.
- Brookfield, S. D. (1999). *Reclaiming and Problematizing Self-Directed Learning as a Space for Critical Adult Education*. Paper presented at the 29th Annual SCUTREA Conference, Warwick, 1999.
- Brown, S. E. (1984). *Who's Afraid of Numbers?: A Math Anxiety Workshop Offered at University of Houston - Clear Lake*. Paper presented at the Fourth International Congress on Mathematics Education (ICME4), Adelaide, Australia.
- Buerk, D. (1982). An Experience with Some Able Women Who Avoid Mathematics. *For the Learning of Mathematics*, 3(2), 19-24.
- Buerk, D. (1985). The Voices of Women Making Meaning in Mathematics. *Journal of Education*, 167(3), 59-70.
- Buerk, D. (1994). Broadening Students' Conceptions of Mathematics: Theory into Action. In D. Buerk (Ed.), *Empowering Students by Promoting Active Learning in Mathematics* (pp. 45-48). Reston, Virginia: The National Council of Teachers of Mathematics, Inc.
- Buerk, D. (1996). Our Open Ears Can Open Minds: Listening To Women's Metaphors For Mathematics. *Focus on Learning Problems in Mathematics*, 18(1, 2 & 3), 26-32.
- Bullough, R. V. J. (1991). Exploring Personal Teaching Metaphors in Preservice Teacher Education. *Journal of Teacher Education*, 42(1), 43-51.
- Burns, J., & Little, M. B. (1995). *Tertiary Bridging into Science and Technology: An Evaluation of the Effectiveness of the Certech Learning Centre, Massey University*. Palmerston North, New Zealand: Educational Research and Development Centre, Massey University.
- Burns, M. (1987). *A Collection of Math Lessons from Grade 3 through 6*. New York: The Math Solutions Publications.
- Burns, M. (1998). *Math: Facing an American Phobia*. Sausalito, CA: Math Solutions Publications.
- Burton, L. (1987). From Failure to Success: Changing the Experience of Adult Learners of Mathematics. *Educational Studies in Mathematics*, 18, 305-316.
- Burton, L. (1999a). Communications: Exploring and Reporting upon the Content and Diversity of Mathematicians' Views and Practices. *For the Learning of Mathematics*, 19(2), 36-38.

- Burton, L. (1999b). Why is Intuition so Important to Mathematicians but Missing from Mathematics Education? *For the Learning of Mathematics*, 19(3), 27-32.
- Burton, L. (Ed.). (1994). *Who Counts? Assessing Mathematics in Europe*. Stoke-on-Trent: Trentham Books Limited.
- Buxton, L. (1981). *Do You Panic About Maths? Coping with Maths Anxiety*. London: Heinemann Educational Books Ltd.
- Buxton, L. (1984). *Mathematics for Everyone*. New York: Schocken Books.
- Campbell, P. B. (1995). Redefining the "Girl Problem in Mathematics". In W. G. Secada, E. Fennema & L. B. Adajian (Eds.), *New Directions for Equity in Mathematics Education* (pp. 225-241). Cambridge: Cambridge University Press.
- Caniglia, J., & Duranczyk, I. M. (1999). *Understanding Mathematics Backwards: A Case Study Analysis of Students' Mathematical Beliefs Through Autobiographies*. Paper presented at the National Association of Developmental Education (NADE) Conference.
- Carroll, J. (1994). What Makes a Person Mathsphobic? A Case Study Investigating Affective, Cognitive and Social Aspects of a Trainee Teacher's Mathematical Understanding and Thinking. *Mathematics Education Research Journal*, 6(2), 131-143.
- Carroll, J. (1995). Primary Teachers' conceptions of mathematics. In B. Atweh & S. Flavel (Eds.), *Eighteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)* (pp. 151-155). Darwin, Australia: Mathematics Education Research Group of Australasia.
- Chapman, O. (1997). Metaphors in the Teaching of Mathematical Problem Solving. *Educational Studies in Mathematics*, 32, 201-228.
- Clarke, D., Wallbridge, M., & Fraser, S. (1992). The Other Consequences of a Problem-Based Mathematics Curriculum. In B. Southwell, B. Perry & K. Owens (Eds.), *Proceedings of the Fifteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)* (pp. 222-235). NSW, Australia: The Mathematics Education Research Group of Australasia.
- Coben, D. (1996, 9-11 October 1996). *Putting Adults into the Equation: an Agenda for Adult Learning in Mathematics for the New Millennium*, from <http://www.leeds.ac.uk/educol/documents/000000044.htm>
- Coben, D., O'Donoghue, J., & FitzSimons, G. E. (Eds.). (2000). *Perspectives on Adults Learning Mathematics: Research and Practice*. Dordrecht: Kluwer Academic Publishers.
- Cockcroft, W. H. (Ed.). (1982). *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*. London: Her Majesty's Stationery Office.
- Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1993). Learning Mathematics at University Level: Initial Conceptions of Mathematics. In

- B. Atweh, C. Kanes, M. Carss & G. Booker (Eds.), *Proceedings of the Sixteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)* (pp. 209-214). Brisbane, QLD: The Mathematics Education Research Group of Australasia.
- Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1994). Conceptions of Mathematics and How it is Learned: The Perspectives of Students Entering University. *Learning and Instruction*, 4, 331-345.
- Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1995a). *The Conceptions of Mathematics Questionnaire*. Sydney, Australia: University of Sydney.
- Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1995b). Patterns of Meaning of Students' Mathematical Experiences at University. In B. Atweh & S. Flavel (Eds.), *Eighteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)*. Darwin, Australia: Mathematics Education Research Group of Australasia.
- Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1998). Qualitatively Different Experiences of Learning Mathematics at University. *Learning and Instruction*, 8(5), 455-468.
- Cross, K. P. (1981). *Adults as Learners*. San Francisco: Jossey-Bass.
- Damarin, S. K. (1990). Teaching Mathematics: A Feminist Perspective. In C. R. H. Thomas J. Cooney (Ed.), *Teaching and Learning Mathematics in the 1990s* (pp. 144-151). Reston, Virginia 22091: National Council of Teachers of Mathematics.
- Davenport, L. R. (1993). *The Effects of Homogeneous Groupings in Mathematics* (No. ED359065). Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Davis, R. B. (1992). Reflections on Where Mathematics Education Now Stands and on Where It May Be Going. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 724-734). New York: Macmillan Publishing Company.
- de Lange, J. (1987). *Mathematics, Insight and Meaning: Teaching, Learning and Testing Mathematics for the Life and Social Sciences*: Vakgroep Onderzoek Wiskundeonderwijs en Onderwijscomputercentrum, Rijksuniversiteit Utrecht.
- de Lange, J. (1993). Assessment in Problem-oriented Curricula. In N. L. Webb & A. F. Coxford (Eds.), *Assessment in the Mathematics Classroom: 1993 Yearbook* (pp. 197-208). Reston, Virginia: National Council of Teachers of Mathematics.
- Dwinell, P. L., & Highbee, J. L. (1989, March). *The Relationship of Affective Variables to Student Performance: Research Findings*. Paper presented at the 13th Annual Conference of the National Association of Developmental Education, Cincinnati, OH.
- Ellis, J., Miller-Reilly, B., & Pfannkuch, M. (1998). Ways in Which Some Women Came to Influence Mathematics Education in New Zealand Through the Equals Network 1985-1995. In C. Keitel (Ed.), *Social Justice and*

*Mathematics Education: Gender, Class, Ethnicity and the Politics of Schooling* (pp. 220-237). Berlin: International Organisation of Women and Mathematics Education & Freie Universität Berlin.

Erchick, D. B. (1996). Women's Voices and the Experience of Mathematics. *Focus on Learning Problems in Mathematics*, 18(1, 2 & 3), 105-122.

Ernest, P. (1995). Values, Gender and Images of Mathematics: A Philosophical Perspective. *International Journal of Mathematics, Science and Technology Education*, 26(3), 449-462.

Evans, J., Hannula, M., Philippou, G., & Zan, R. (2004). *Affect and Mathematical Thinking: Thematic Working Group 2 at the Third Conference of the European Society for Research in Mathematics Education*, from <http://www.dm.unipi.it/~didattica/CERME3/proceedings/>

Fennema, E. (1995). Mathematics, Gender and Research. In B. Grevholm & G. Hanna (Eds.), *Gender and Mathematics Education: an ICMI Study in Stiftsgården Akersberg, Hoor, Sweden 1993* (pp. 21-38). Lund: Lund University Press.

Fennema, E., & Sherman, J. (1976). Fennema-Sherman Mathematics Attitudinal Scales: Instruments Designed to Measure Attitudes Toward the Learning of Mathematics by Females and Males. *JSAS Catalog of Selected Documents in Psychology*, 6(31), (Ms. No. 1225).

Fitzsimons, G. (1994). *Teaching Mathematics to Adults Returning to Study*. Melbourne: Deakin University.

FitzSimons, G. E. (1996). Working Group 18: Adults Returning to Mathematical Education. In C. Alsina, J. M. Alvarez, M. Niss, A. Perez, L. Rico & A. Sfard (Eds.), *Proceedings of the 8th International Congress on Mathematical Education* (pp. 183-187). Sevilla: S.A.E.M. 'THALES'.

FitzSimons, G. E., & Godden, G. L. (2000). Review of Research on Adults Learning Mathematics. In D. Coben, J. O'Donoghue & G. E. FitzSimons (Eds.), *Perspectives on Adults Learning Mathematics: Research and Practice* (pp. 13-45). Dordrecht: Kluwer Academic Publishers.

FitzSimons, G. E., Jungwirth, H., MaaB, J., & Schloeglmann, W. (1996). Adults and Mathematics (Adult Numeracy). In A. J. Bishop, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International Handbook of Mathematics Education* (Vol. 2, pp. 755-784). London: Kluwer Academic Press.

Forgasz, H. (1994). *Society and Gender Equity in Mathematics Education*. Victoria, Australia: Deakin University.

Frankenstein, M. (1989). *Relearning Mathematics: A Different Third R – Radical Maths*. London: Free Association Books.

Gibson, H. (1994). "Math Is Like a Used Car": Metaphors Reveal Attitudes toward Mathematics. In D. Buerk (Ed.), *Empowering Students by Promoting Active Learning in Mathematics* (pp. 7-12). Reston, Virginia: The National Council of Teachers of Mathematics, Inc.

- Gilligan, C. (1982). *In a Different Voice : Psychological Theory and Women's Development*. Cambridge, Mass.: Harvard University Press.
- Godden, G. (1994). A Guide to Tertiary Bridging Mathematics Research to Mid-1992: Including an Annotated Bibliography. In *Proceedings of the Fourth Australian Bridging Mathematics Network Conference* (Vol. 1, pp. 67-91). Sydney, Australia: University of Sydney.
- Goldberger, N. R. (1996a). Cultural Imperatives and Diversity in Ways of Knowing. In N. R. Goldberger, J. M. Tarule, B. M. Clinchy & M. F. Belenky (Eds.), *Knowledge, Difference and Power: essays inspired by Women's Ways of Knowing* (pp. 335-371). New York: BasicBooks.
- Goldberger, N. R. (1996b). Introduction: Looking Backward, Looking Forward. In N. R. Goldberger, J. M. Tarule, B. M. Clinchy & M. F. Belenky (Eds.), *Knowledge, Difference and Power: essays inspired by Women's Ways of Knowing* (pp. 1-21). New York: BasicBooks.
- Goldberger, N. R., Tarule, J. M., Clinchy, B. M., & Belenky, M. F. (Eds.). (1996). *Knowledge, Difference and Power: essays inspired by Women's Ways of Knowing*. New York: BasicBooks.
- Goolsby, C. B., Dwinell, P. L., Higbee, J. L., & Bretscher, A. S. (1987). *Factors Affecting Mathematics Achievement in High Risk College Students*. Paper presented at the Annual meeting of the American Educational Research Association.
- Gordon, S. (1994). Understanding the Reluctant Learner of Statistics: Some Examples from My Practice. In J. Nicholas & S. Gordon (Eds.), *Proceedings of the Fourth Australasian Bridging Mathematics Network Conference* (Vol. 1, pp. 11-18). Sydney, Australia: Australian Bridging Mathematics Network.
- Gordon, W., & Langmaid, R. (1988). *Qualitative Market Research*. Aldershot: Gower.
- Gourgey, A. F. (1982). *Development of a Scale for the Measurement of Self-Concept in Mathematics*. Unpublished manuscript.
- Gourgey, A. F. (1984). *The Relationship of Misconceptions About Math and Mathematical Self-concept to Math Anxiety and Statistics Performance*. Paper presented at the American Educational Research Association, New Orleans, LA, USA.
- Gourgey, A. F. (1992). Tutoring Developmental Mathematics: Overcoming Anxiety and Fostering Independent Learning. *Journal of Developmental Education*, 15(3), 10-12.
- Grow, G. O. (1991). Teaching Learners to be Self-Directed. *Adult Education Quarterly*, 41(3), 125-149.
- Gustafsson, L., & Mouwitz, L. (2004). *Adults and Mathematics - A Vital Subject* (NCM Report 2002:3). Goteburg, Sweden: National Center for Mathematics Education (NCM).

- Halmos, P. R. (1968). Mathematics as a Creative Art. *American Scientist*, 56(4), 375-389.
- Harris, M. (2000). Women, Mathematics and Work. In D. Coben, J. O'Donoghue & G. E. FitzSimons (Eds.), *Perspectives on Adults Learning Mathematics: Research and Practice* (pp. 171-190). Dordrecht: Kluwer Academic Publishers.
- Hartnell, D. (2000). The Differences Between Successful and Unsuccessful Students on a Bridging Mathematics Course. In J. Bradley & M. Kemp (Eds.), *Proceedings of the Bridging Mathematics Network Conference, Perth* (pp. 9-16). Perth, Western Australia: Murdoch University.
- Hauk, S. (2005). Mathematics Autobiography Among College Learners in the United States. *Adults Learning Mathematics International Journal*, 1(1), 36-56.
- Healy, L., & Sutherland, R. (1991). *Exploring Mathematics with Spreadsheets*. Oxford: Basil Blackwell Limited.
- Heckman, P. E., & Weissglass, J. (1994). Contextualized Mathematics Instruction: Moving Beyond Recent Proposals. *For the Learning of Mathematics*, 14(1), 29-33.
- Heckman?, P. E., & Weissglass, J. (1994). Contextualized Mathematics Instruction: Moving Beyond Recent Proposals. *For the Learning of Mathematics*, 14(1), 29-33.
- Helme, S. (1994). *Mathematics Embedded in Context: The Role of Task Context in Performance, Task Perceptions and the Solution Methods of Adult Women Students*. Unpublished Masters of Education Thesis, Australian Catholic University.
- Hoyles, C., & Noss, R. (1998). Anchoring Mathematical Meanings in Practice. In K. Gravemeijer (Ed.), *Introductory Texts: International Conference on Symbolizing and Modeling in Mathematics Education*. Utrecht: Freudenthal Institute, Utrecht University.
- Hudson, B. (1998). Learning Mathematically as Social Practice in a Workplace Setting. In A. Watson (Ed.), *Situated Cognition and the Learning of Mathematics* (pp. 71-80). Oxford: Centre for Mathematics Education Research, University of Oxford.
- Hurtado, A. (1996). Strategic Suspensions: Feminists of Color Theorize the Production of Knowledge. In N. R. Goldberger, J. M. Tarule, B. M. Clinchy & M. F. Belenky (Eds.), *Knowledge, Difference and Power: essays inspired by Women's Ways of Knowing* (pp. 372-392). New York: BasicBooks.
- Hyde, J. S., Fennema, E., & Lamon, S. J. (1990). Gender Differences in Mathematics Performance: A Meta-Analysis. *Psychological Bulletin*, 107(2), 139-155.
- Jackson, A. (1995). Voices from the Reform Movement. *Humanistic Mathematics Network Journal*, 12, 1-6.

- Jarvis, P. (1987). *Adult Learning in the Social Context*. London: Croom Helm.
- Johnston, B. (1995). Mathematics: An Abstracted Discourse. In P. Rogers & G. Kaiser (Eds.), *Equity in Mathematics Education: Influences of Feminism and Culture* (pp. 226-234). London: The Falmer Press.
- Johnston, B., & Tout, D. (1995). *Adult Numeracy Teaching - Making Meaning in Maths*. Melbourne: Community and Further Education Board.
- Joseph, G. G. (1991). *The Crest of the Peacock: Non-European Roots of Mathematics*. London: I. B. Tauris.
- Justice, E. M., & Dorman, T. M. (2001). Metacognitive Differences Between Traditional-Age and Nontraditional-Age College Students. *Adult Education Quarterly*, 51(3), 236.
- Kennedy, P. A. (2000). Concrete Representations and Number Line Models: Connecting and Extending. *Journal of Developmental Education*, 24(2).
- Kloosterman, P., & Stage, F. K. (1992). Measuring Beliefs About Mathematical Problem Solving. *School Science and Mathematics*, 92(3), 109-115.
- Knight, G. (1992). 'A Formula is a Leather Armchair': Metaphor and Mathematics Education. *SAME Papers*, 130-139.
- Knowles, M. S. (Ed.). (1984). *Andragogy in Action*. London: Jossey-Bass Publishers.
- Knowles, M. S., Holton, E. F., & Swanson, R. A. (1998). *The Adult Learner* (Fifth ed.). Houston, Texas: Gulf Publishing Company.
- Koch, L. C. (1996). Development of Voice in the Mathematics Classroom. *Focus on Learning Problems in Mathematics*, 18(1, 2 & 3), 164-175.
- Koehler, M. S., & Grouws, D. A. (1992). Mathematics Teaching Practices and Their Effects. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 115-126): Macmillan Publishing Company.
- Kogelman, S., & Warren, J. (1978). *Mind Over Math – Put yourself on the road to success by freeing yourself from math anxiety*. New York: McGraw-Hill.
- Kulm, G. (1980). Research on Mathematics Attitude. In R. J. Shumway (Ed.), *Research in Mathematics Education* (pp. 356-387). Reston, VA, USA: National Council of Teachers of Mathematics.
- Lajoie, S. P. (1995). A Framework for Authentic Assessment in Mathematics. In T. A. Romberg (Ed.), *Reform in School Mathematics and Authentic Assessment* (pp. 19-37). Albany, NY, USA: State University of New York.
- Lampert, M. (1990). When the Problem Is Not the Question and the Solution Is Not the Answer: Mathematical Knowing and Teaching. *American Educational Research Journal*, 27(1), 29-63.
- Langbort, C., & Thompson, V. (1985). *Building Success in Math*. California: Wadsworth Publishing Company.

Leder, G. C. (1985). Measurement of Attitude to Mathematics. *For the Learning of Mathematics*, 5(3), 18-21, 34.

Leder, G. C. (1990). Gender Differences in Mathematics: An Overview. In E. Fennema & G. Leder (Eds.), *Mathematics and Gender* (pp. 10-26): Teachers College, Columbia University.

Leedy, P., Newby, T. J., & Ertmer, P. A. (1997). *Practical Research: Planning and Design* (6th ed.). Upper Saddle River, N.J.: Merrill.

Lehmann, C. H. (1987). *The Adult Mathematics Learner: Attitudes, Expectations, Attributions*. Paper presented at the Annual Meeting of the American Educational Research Association, Washington DC.

Linn, M. C., & Hyde, J. S. (1989). Gender, Mathematics, and Science. *Educational Researcher*, 18(8), 17-19, 22-27.

Lovitt, C. (1991a). *Maths Problem Solving and Modelling for Year 11*. Melbourne: Thomas Nelson Australia.

Lovitt, C. (1991b). *Maths Problem Solving and Modelling for Year 12*. Melbourne: Thomas Nelson Australia.

Lynch, A. (1998). *An Exploration of Historical and Theoretical Perspectives the Inform the Use of Collaboration in Teaching Mathematics to Adults*. Unpublished Dissertation for DipMathEd, University of Auckland, Auckland, New Zealand.

Madden, J. (2001). Book Review. *Notices of the American Mathematical Society*, 48(10), 1182-1188.

Marr, B., & Helme, S. (1987). *Mathematics: A New Beginning*. Melbourne: Northern Metropolitan College of TAFE.

Marr, B., Helme, S., & Tout, D. (1991). *Breaking the Maths Barrier*. Canberra: Department of Employment Education and Training.

Mason, J. (1992). Imagery, imagination and mathematics classrooms. In D. Pimm (Ed.), *Mathematics, Teachers and Children: A Reader* (pp. 297-302). London: Hodder & Stoughton.

McCoy, L. P. (1992). Correlates of Mathematics Anxiety. *Focus on Learning Problems in Mathematics*, 14(2), 51-57.

McLeod, D. B. (1992). Research on Affect in Mathematics Education: A Reconceptualization. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596). New York: Macmillan Publishing Company.

McLeod, D. B. (1994). Research on Affect and Mathematics Learning in the JRME: 1970 to the Present. *Journal for Research in Mathematics Education*, 25(6), 637-647.

McLeod, D. B., & McLeod, S. H. (2002). Synthesis - Beliefs and Mathematics Education: Implications for Learning, Teaching, and Research. In G. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A Hidden Variable in*

- Mathematics Education?* (pp. 115-123). London: Kluwer Academic Publishers.
- Mellin-Olsen, S. (1987). *The Politics of Mathematics Education* (Vol. 4). Dordrecht, Holland: D.Reidel Publishing Company.
- Mellin-Olsen, S. (1989). Creative Uses of Mathematics in Social Contexts. In C. Keitel, P. Damerow, A. Bishop & P. Gerdes (Eds.), *Mathematics, Education and Society, Science and Technology Education* (Vol. Document Series No 35): UNESCO.
- Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Education*. San Francisco, CA: Jossey-Bass.
- Merriam, S. B., & Brockett, R. G. (1997). *The Profession and Practice of Adult Education*. San Francisco: Jossey-Bass Publishers.
- Mezirow, J. D. (1983). A Critical Theory of Adult Learning and Education. In M. Tight (Ed.), *Adult Learning and Education* (Vol. 1, pp. 124-138). London: Croon Helm Ltd.
- Miller-Reilly, B. (1995). A New Approach for Students Returning to Mathematics. In *Readings from the Mathematics Education Unit 1995* (pp. 81-94). Auckland, New Zealand: Mathematics Education Unit, School of Mathematical and Information Science, University of Auckland.
- Miller-Reilly, B. (1996). Exploration And Modelling In A University Mathematics Paper: Perceptions Of Adult Students, *Adults Learning Mathematics: A Research Forum*. Brighton, U.K.
- Miller-Reilly, B. (1997). Reactions Of Adults To An Investigative Mathematics Course. In G. Fitzsimons (Ed.), *Adults Returning to Study Mathematics, Papers from Working Group 18, 8th International Congress on Mathematical Education (ICME8)* (pp. 101-118): The Australian Association of Mathematics Teachers Inc.
- Miller-Reilly, B. (2000). Exploration and Modelling in a University Mathematics Course: Perceptions of Adult Students. In D. Coben, J. O'Donoghue & G. E. FitzSimons (Eds.), *Perspectives on Adults Learning Mathematics: Research and Practice*. Dordrecht: Kluwer Academic Publishers.
- Miller-Reilly, B., Snyders, M., & Taylor, J. (2003). *Bridging Mathematics Courses and Initiatives*. Paper presented at the Remarkable Delta03: Fourth Southern Hemisphere Symposium on Undergraduate Mathematics Teaching, Queenstown, New Zealand on 23-27 November 2003.
- Ministry of Education. (1992). *Mathematics in the New Zealand Curriculum*. Wellington, NZ: Author.
- Morrow, C. M. (1996). Women and Mathematics: Avenues of Connection. *Focus on Learning Problems in Mathematics*, 18(1, 2 & 3), 4-18.
- Morrow, C. M., & Morrow, J. (1995). Connecting Women with Mathematics. In P. Rogers & G. Kaiser (Eds.), *Equity in Mathematics Education: Influences of Feminism and Culture* (pp. 13-26). London: The Falmer Press.

- Munn, P., MacDonald, C., & Lowden, K. (1992). *Helping Adult Students Cope. Mature Students on Science, Mathematics and Engineering Courses* (Reports - Research/Technical No. SCRE -RR -39). Edinburgh: Scottish Council for Research in Education.
- National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author.
- Nolen, W. F., Archambault, F. X., & Greene, J. F. (1976). *Exploration of Mathematics Attitude: An Empirical Investigation of the Aiken Scale*. Paper presented at the Annual Meeting of the New England Educational Research Association.
- Noss, R., & Hoyles, C. (1996). *Windows on Mathematical Meanings: Learning Cultures and Computers*. Dordrecht: Kluwer Academic Publishers.
- O'Donoghue, J. (2000). Perspectives on Teaching Adults Mathematics. In D. Coben, J. O'Donoghue & G. E. FitzSimons (Eds.), *Perspectives on Adults Learning Mathematics: Research and Practice* (pp. 229-234). Dordrecht: Kluwer Academic Publishers.
- Ocean, J. (1996). Ethics, Morality, and Mathematics Education. In P. C. Clarkson (Ed.), *Proceedings of the Nineteenth Annual Mathematics Education Research Group of Australasia (MERGA) Conference* (pp. 422-429). Melbourne, Australia: Deakin University Press.
- Ocean, J. (1997). Is there both Care and Justice in the Mathematics Classroom? *The Weaver: A Forum for New Ideas in Education*.
- Ocean, J., & Miller-Reilly, B. (1997). Black Holes and Beginning Teachers: A Connected Approach to Teaching Mathematics. *The Australian Mathematics Teacher*, 53(4), 17-20.
- Op 't Eynde, P., de Corte, E., & Verschaffel, L. (2002). Framing Students' Mathematics-Related Beliefs. In G. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 13-37). London: Kluwer Academic Publishers.
- Parker, J. (2005). *R. L. Moore: Mathematician and Teacher*. The Mathematical Association of America.
- Pimm, D. (1987). *Speaking Mathematically: Communication in Mathematics Classrooms*. London: Routledge and Kegan Paul.
- Presmeg, N. C. (1997). Reasoning with Metaphors and Metonymies. In L. D. English (Ed.), *Mathematical Reasoning: Analogies, Metaphors, and Images* (pp. 267-279). London: Lawrence Erlbaum Associates.
- Queensland University of Technology. (1995). *Adult Education in the Workplace and Community: Study Guide*. Brisbane: QUT Open Learning Unit.
- Ramsden, P. (1992). *Learning to Teach in Higher Education*. London: Routledge.
- Rech, J., Hartzell, J., & Stephens, L. (1993). Comparison of Mathematics Competencies and Attitudes of Elementary Education Majors with

- Established Norma of a General College Population. *School Science and Mathematics*, 93(3), 141-144.
- Reilly, B. J. (1988). *Winston Churchill Fellowship Report*: University of Auckland.
- Relich, J., Way, J., & Martin, A. (1994). Attitudes to Teaching Mathematics: Further Development of a Measurement Instrument. *Mathematics Education Research Journal*, 6(1), 56-69.
- Richardson, F. C., & Suinn, R. M. (1972). The Mathematics Anxiety Rating Scale: Psychometric Data. *Journal of Counselling Psychology*, 19, 551-554.
- Roberts, T. (1998). Mathematical Registers in Aboriginal Languages. *For the Learning of Mathematics*, 18(1), 10-16.
- Rogers, A. (1986). *Teaching Adults*. Milton Keynes, England: Open University Press.
- Rogers, P. (1995). Putting Theory into Practice. In P. Rogers & G. Kaiser (Eds.), *Equity in Mathematics Education: Influences of Feminism and Culture* (pp. 175-185). London: The Falmer Press.
- Romberg, T. A. (1992). Perspectives on Scholarship and Research Methods. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 49-64). New York: Macmillan Publishing Company.
- Ruedy, E., & Nirenberg, S. (1990). *Where Do I Put the Decimal Point? How to Conquer Math Anxiety and Increase your Facility with Numbers*. New York: Barnes & Noble Books.
- Ruffell, M., Mason, J., & Allen, B. (1998). Studying Attitude to Mathematics. *Educational Studies in Mathematics*, 35, 1-18.
- Safford-Ramus, K. (2004). A Report on Selected Dissertation Research. In J. Maasz & W. Schloeglmann (Eds.), *Learning Mathematics to Love and Work in our World: Proceedings of 10th International Conference of Adults Learning Mathematics (ALM10) - A Research Forum, Strobl (Austria)*, 2003. Linz, Austria: Univeritatsverlag Rudolf Trauner.
- Schoenfeld, A. H. (1985). *Mathematical Problem Solving*. New York: Academic Press, Inc.
- Schoenfeld, A. H. (1989). Explorations of Students' Mathematical Beliefs and Behavior. *Journal for Research in Mathematics Education*, 20(4), 338-355.
- Schoenfeld, A. H. (1994). A Discourse on Methods. *Journal for Research in Mathematics Education*, 25(6), 697-710.
- Schweickart, P. P. (1996). Speech is Silver, Silence is Gold: The Asymmetrical Intersubjectivity of Communicative Action. In N. R. Goldberger, J. M. Tarule, B. M. Clinchy & M. F. Belenky (Eds.), *Knowledge, Difference and Power: essays inspired by Women's Ways of Knowing* (pp. 305-331). New York: BasicBooks.
- Seidman, I. E. (1991). *Interviewing as Qualitative Research: a Guide for Researchers in Education and the Social Sciences*. New York: Teachers College Press.

- Sells, L. (1979). The Mathematics Filter and the Education of Women and Minorities. In L. H. Fox, L. Brody & D. Tobin (Eds.), *Women and the Mathematical Mystique* (pp. 66-75). Baltimore: Johns Hopkins University Press.
- Silverman, D. (2000). *Doing Qualitative Research: A Practical Handbook*. London: SAGE Publications.
- Sims, D. C. (1981). *Context-Sensitive Evaluation Technology in Bilingual Education*: Eastern Michigan University, Ypsilanti, MI (National Institute of Education (ED), Washington, DC.
- Skemp, R. R. (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teaching*, 77, 20-26.
- Skemp, R. R. (1978). Relational Understanding and Instrumental Understanding. *The Arithmetic Teacher*, 3, 9-15.
- Skovsmose, O. (1994). Towards a Critical Mathematics Education. *Educational Studies in Mathematics*, 27, 35-57.
- Smith, J. P., & Hungwe, K. (1998). Conjecture and Verification in Research and Teaching: Conversations with Young Mathematicians. *For the Learning of Mathematics*, 18(3), 40-46.
- Smith, R. (1990). *Learning How to Learn: Applied Theory for Adults*. Milton Keynes: Open University Press.
- Smith, R. (1991). *Learning Styles of Mature Age Students in Tertiary Preparation Courses: Their Influence on Course Selection and Student Success*. Unpublished manuscript, Toowoomba.
- Southwell, B., & Khamis, M. (1992). In B. Southwell, B. Perry & K. Owens (Eds.), *Proceedings of the Fifteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)* (pp. 497-509). NSW, Australia: The Mathematics Education Research Group of Australasia.
- Stage, F. K., & Kloosterman, P. (1991). Relationships between Ability, Belief and Achievement in Remedial College Mathematics Classrooms. *Research and Teaching in Developmental Education*, 8(1), 27-36.
- Stage, F. K., & Kloosterman, P. (1995). Gender, Beliefs, and Achievement in Remedial College-Level Mathematics. *Journal of Higher Education*, 66(3), 294-311.
- Stage, F. K., Muller, P., A. , Kinzie, J., & Simmons, A. (1998). *Creating Learning Centered Classrooms. What Does Learning Theory Have To Say?* (No. 26(4)). Washington DC: Association for the Study of Higher Education. [BBB14212], ERIC Clearinghouse on Higher Education, Washington, DC. [BBB15669], George Washington Univ., Washington, DC. Graduate School of Education and Human Development. [BBB32577].
- Statham, M. (1999). *Wellesley Programme 1989 to 1998: The First Ten Years*. Unpublished report, University of Auckland, Auckland, New Zealand.

- Stenmark, J. K., Thompson, V., & Cossey, R. (1986). *Family Math*: The Regents, University of California.
- Streetland, L. (1991). *Fractions in Realistic Mathematics Education: A Paradigm of Developmental Research* (Vol. 8). London: Kluwer Academic Publishers.
- Szydlik, J. E. (2000). Mathematical Beliefs and Conceptual Understanding of the Limit of a Function. *Journal for Research in Mathematics Education*, 31(3), 258-276.
- Taylor, J. A. (1994). Attitude to Mathematics of Adult Students, *Bridging Mathematics Network Conference* (Vol. 1, pp. 182-187). Sydney, Australia.
- Taylor, J. A. (1995). *Attitude to Mathematics of Adults Returning to Tertiary Study in the Distance Mode*. Paper presented at the Regional Collaboration in Mathematics Education - an ICMI Regional Conference, Melbourne, Australia.
- Taylor, J. A. (1997). Factorial validity of the Aiken Attitude to Mathematics Scales for adult pre-tertiary students. *Educational and Psychological Measurement*, 57(1), 125-130.
- Taylor, L. (1995). Documenting and Exploring Ethnographic Mathematical Life Histories. In R. P. Hunting, G. E. Fitzsimons, P. C. Clarkson & A. J. Bishop (Eds.), *Proceedings of the ICMI Regional Collaboration in Mathematics Education Conference* (pp. 695-701). Melbourne, Australia: Bowater Reding.
- Taylor, L., & Shea, M. (1996). An Elementary Teacher's Mathematical Life History. *Focus on Learning Problems in Mathematics*, 18(1, 2 & 3), 52-87.
- Thompson, A. G. (1992). Teachers' Beliefs and Conceptions: A Synthesis of the Research. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 127-146). New York: Macmillan Publishing Company.
- Tobias, S. (1978). *Overcoming Math Anxiety*. New York: W.W. Norton.
- Tobias, S. (1987). *Succeed with Math: Every Student's Guide to Conquering Math Anxiety*. New York: The College Board.
- Trueman, M., & Hartley, J. (1996). A Comparison Between the Time-Management Skills and Academic Performance of Mature and Traditional-Entry University Students. *Higher Education*, 32(2), 199-215.
- Vale, C. M. (1993a). Sex Difference in Achievement in VCE Mathematics, *Mathematics Education Research Group of Australasia (MERGA)* (pp. 563-567).
- Vale, C. M. (1993b). *Sex Difference in Achievement in Year 12 Mathematics and Non-routine Problem Solving*. Unpublished Master of Education, University of Melbourne.
- van Groenestijn, M. (1998). Constructive Numeracy Teaching as a Gateway to Independent Learning. In D. Coben & J. O'Donoghue (Eds.), *Proceedings*

*of Adults Learning Mathematics - A Research Forum (ALM-4), Limerick, 1997* (pp. 224-231). London: Goldsmiths College, University of London.

Walkerdine, V. (1988). *The Mastery of Reason*. London: Routledge.

Watson, A. (1998). Why Situated Cognition is an Issue for Mathematics Education. In A. Watson (Ed.), *Situated Cognition and the Learning of Mathematics* (pp. 1-9). Oxford: Centre for Mathematics Education Research, University of Oxford.

Watson, J. M. (1983). The Aiken Attitude to Maths Scales: Psychometric Data on Reliability and Discriminant Validity. *Educational and Psychological Measurement*, 43(4), 1247-1253.

Watson, J. M. (1989). An Exploration of Gender Differences of Tertiary Mathematics. *Mathematics Education Research Journal*, 1(1), 1-10.

West, L. (1995). Beyond Fragments: Adults, Motivation and Higher Education. *Studies in the Education of Adults*, 27(2), 133-156.

Wilson, T. (1982). *Arte of rhetorique*. New York: Garland Publishing Inc.

Wood, L. N., & Smith, N. F. (1993). Students' Ideas About Mathematics. In B. Atweh, C. Kanes, M. Carss & G. Booker (Eds.), *Sixteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)* (pp. 593-597). Brisbane, QLD: The Mathematics Education Research of Australasia.

Yong, F. (1993). Attitudes Towards Mathematics and Science of African-American, Mexican-American, and Chinese-American Middle Grade Students Identified as Gifted. *Focus on Learning Problems in Mathematics*, 15(1), 52-61.

Zaslavsky, C. (1994). *Fear of Math: How to Get Over It and Get On with Your Life*. New Brunswick, New Jersey: Rutgers University Press.

## **Appendices**



## Appendices

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## **Appendix A:**

### **Retrospective Protocol For Teacher Interviews**

This is the retrospective protocol for teacher interviews for Mathematics 1, in particular, for Bill and Maxine. (I adapted this schedule for the teacher interview (Moira) for the Wellesley Program).

Initial discussion aimed to help the teacher recall the process and development of this course and the philosophy behind it. This course was taught first in 1994 so the developmental process will have mainly occurred during 1993 or earlier. What do you remember about this process?

How long had you been teaching in the Department of Mathematics in the University at this stage? How different was this job from your other recent teaching positions? (For example, autonomy, ...)

What were your (and others in the Mathematics Education Unit) main concerns that influenced the way you developed this new course?

Do you remember discussions in meetings at that time? Who was involved? (ie specific events/discussions/collaborative development). Who took responsibility for what?

What were your personal reactions to developing this new course? (For example, reluctant, excited, ...)

Who were you designing this course for? What were the characteristics of students you had in mind when planning the course? (For example, age, gender, ethnicity, maths background, ...)

Which of your past experiences, either personally or professionally, do you feel contributed to your philosophy and goals for this course? What reading(s) contributed to your philosophy for this course?

Who do you think it worked for best? When you think back are there any specific instances?

How do you think it worked that first year (1994)? What worked really well? What did you think could be improved? Did you make changes in aspects of the 1995 course? What were they?

Do you have anything that you want to add?



## **Appendix B:**

### **Three sample tasks set in Mathematics 1 (MATHS 101) in 1995**

#### **PATTERNS TASK**

##### **AT THE SUPERMARKET**

The diagram opposite shows one way (a triangular pyramid) that supermarkets use to display cans of beans.

Your team of three has been asked to provide FOODMARKET management with some options for stacking cans. For example, it may be possible to stack cans in pyramids with square-bases, pentagonal bases and so one. There may also be some sensible options other than pyramid stacks.

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture

#### **TEAM TASK**

Consider at least three options for can-stacking and plan a report in which you provide as much information as possible about each option. For example, you might consider the number of cans that can be placed in each layer, the number of cans required for your stack and the overall size of the stack (height of stack and floor space needed).

#### **INDIVIDUAL TASK**

Complete the task begun by your team and write a report giving details of your options for presentation to FOODMARKET management. In addition to your report you should also write a paragraph reflecting on your learning experiences in doing this task: where did you get help, what was easy or difficult, what you have learnt as a result of completing this task.

The following questions may help you organise your report.

- What is my objective?
- What stacking options did I consider?
- What steps did I take?
- What difficulties arose and how did I overcome them?
- What mathematical content have I used?
- What mathematical generalisations did I make?

#### **ASSESSMENT CRITERIA**

The following criteria will be used to mark your report:

Extent of consideration of practicable stacking options (4).

Extent and accuracy of mathematical aspects for the options considered (8).

Organisation and presentation of report. (4)

Reflective statement on your experiences in doing this task. (2)

## MEDICINE TASK

### Triage: Disastrous Decisions

In the few hours after a disaster there are often more injured people than there are facilities to care for them. The time immediately after the injury occurs is the most critical for the survival of the patient. This means that the medics on the scene must make a lot of difficult decisions about who gets medical care and who does not. Their aim in making these decisions will be to maximise the number of survivors.

Imagine that a disaster has occurred and that an initial assessment of the injured has been done. An estimate of their chances of survival is made on past experience.

#### Probability of survival

No. of People	If treated Immediately	After 1hr	After 2 hrs	After 3 hrs
Critical: 25	0.40	0.15	0.05	0.00
Serious: 60	0.80	0.60	0.40	0.10
Injured: 110	1.00	0.90	0.80	0.50

It is known that more injured are expected to arrive. There are facilities for 40 people to be treated immediately.

- Decide who gets treated and how many survive the first hour in each category.

After one hour more injured have arrived as follows: 15 critical, 30 serious and 30 injured. Medical facilities are now available for 80 people.

- Decide who gets treated and how many survive the second hour in each category.

After the second hour the final group of injured have arrived: 10 critical and 20 serious. Medical facilities are available for 150.

- Decide who gets treated and how many survive the third hour in each category.
- All survivors at that time get treatment in the following hour. How many survive in total out of the 300 who came to your medical centre? Could you improve this figure.

## MAPS TASK

### Finding your Bearings

#### Situation

You and your friend have gone out in a small boat into Auckland harbour to go fishing. You have tried several places without success. Finally, you find a spot where you catch a lot of large schnappers. You would like to be able to return to this exact spot tomorrow and catch some more.

You are situated somewhere in the channel between Rangitoto Island and the North Shore beaches (see accompanying map). How will you locate yourself so that you can return to exactly the same spot the next day?

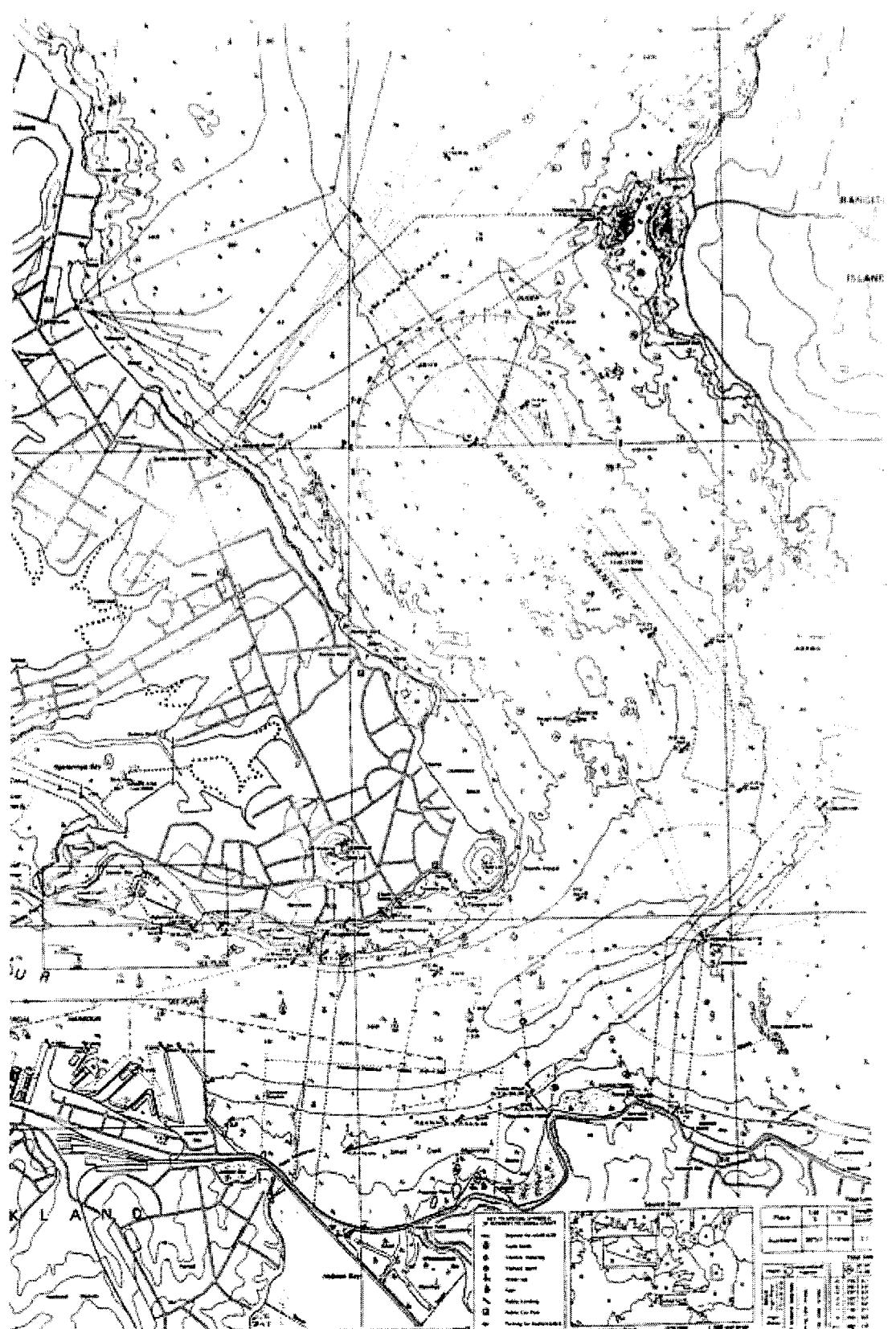
- a) Assume that you have no maps or navigation equipment on board your boat. Think of several ways to locate your position. Explain how each method works (draw diagrams or use the map if necessary), and discuss which method is liable to be the most accurate.
- b) Assume that you have a compass and a map on board your boat. What improvements could be made to your methods?

#### TEAM TASK

Discuss these situations and note as many methods as possible. Explain the methods to each other so that everyone in the group fully understands them, using diagrams if necessary. (Spare copies of the map are available). Discuss their accuracy.

#### INDIVIDUAL TASK

Write up a report analysing several methods for situation a) and situation b). Include diagrams or maps.



## **Appendix C:**

### **An example of a two-stage test given in 1995 in Mathematics 1 (MATHS 101)**

#### **TERMS TEST ONE IN 1995**

##### **INSTRUCTIONS:**

*There are three sections to this test. They are worth **EQUAL** marks. You must complete all sections. You should spend about the same amount of time on each section (about 30 mins). You may do the sections in any order.*

*After you have handed in this test, it will be marked and returned to you. You then have **TWO WEEKS** in which to improve your answers. At the end of that time, you may hand it in again for re marking. You may be required to answer questions about the extra work you have done. You will be given the **AVERAGE** of your two marks.*

*Please keep this test paper to help with re-doing your test.*

Answer ONLY ONE of the following questions:

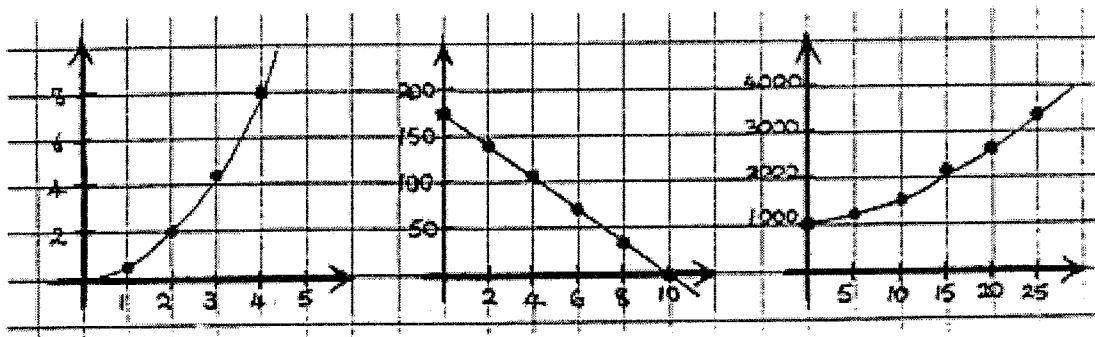
Question 1: Packaging

A new type of canned tomatoes is being marketed. The manager wishes to use a can which contains the same volume ( $400\text{ml} = 400\text{cm}^3$ ) but which has different dimensions than the usual cans. He favours one which is tall and slim. Write a report explaining the implications of his idea, and making a recommendation. (Note: The design must be a can, i.e. cylinder shaped).

OR

Question 2: Patterns

The following graphs illustrate different types of relationships: linear, quadratic and exponential (NOT in that order). Use the graphs to explain these three types of relationships. Give examples from the real world.



OR

Question 3: Environment

Write an essay about the following newspaper article. Explain how the figures in the article could have been calculated.

**NZ OLDER AND BIGGER**

The New Zealand population increased by more than 50,000 in the year to March 31, Statistics New Zealand said yesterday. The population at that date was 3,592,000 said the Deputy Government Statistician, Mr Dennis Trewin.

That was an increase of 50,400, or 1.4 per cent on the previous year and continued the trend of the past four years. The population grew faster during that period than it had in the late 1980's. The population also appears to be growing older.

At the end of March, 11.7 per cent of the population was aged 65 and over, up from 10.3 per cent a decade earlier. Children under 15 made up 23.2 per cent of the population, down from 24.6 per cent in 1985. The average age of New Zealanders was 34.5 years, compared with 33 years a decade earlier.

-NZPA

## SECTION B. MATHEMATICAL SITUATIONS

Investigate ONLY ONE of the following situations:

### Question 1: Packaging

Miriama wishes to make a box to contain some chocolates she has made for a dinner party. She has a rectangular piece of card which measures 40 cm by 24 cm. Explore some of the possibilities for boxes she could make.

OR

### Question 2: Patterns.

An Insurance Company offers several different pay-out options on a Retirement Savings Scheme. All options start at age 65. Explore which option you think is the best.

- Option A: \$1000 in the first month, \$1020 in the second month, \$1040 in the third month, and so on increasing by \$20 each month until death.
- Option B: \$4000 in the first month, \$3080 in the second month, \$3060 in the third month, and so on decreasing by \$20 each month until death.
- Option C: 5 cents in the first month, 10 cents in the second month, 20 cents in the third month, and so on, doubling in value every month until death.

OR

### Question 3: Environment

The Department of Conservation is trying to maintain the stocks of a particular fish which is very popular with local fishermen. Their research has shown that, if there is **NO** fishing, then the population will grow at 5% per year. At present there are estimated to be 5000 fish per square km. They wish to ensure that there are always at least 2000 fish per square kilometre. The effect of fishing regulations are given in the table. Investigate and recommend some alternative policies for the allowable catch over the next ten years. A policy may change from year to year.

Allowable Catch (daily quota per person)	Resulting Annual Growth
0	5%
5	3%
10	1%
15	-1%
20	-3%
25	-5%

## SECTION C: SKILLS

Answer ALL questions. Each question is worth FIVE marks (30)

**Question 1.** The volume of a cube is  $2000 \text{ cm}^3$ . Find the surface area of this cube.

**Question 2.** Pythagoras' Theorem states that, for a right-angled triangle with sides  $a$ ,  $b$ , and  $c$ :

$$a^2 = b^2 + c^2$$

Make  $c$  the subject of the formula, and find  $c$  if  $a = 13.5$  and  $b = 8.7$ .

**Question 3.** Write a formula and sketch a graph of the following sequence:

3, 6, 12, 24, 48, 96, .....

What type of sequence is this?

**Question 4.** The 8<sup>th</sup> term of the sequence 8, 15, 22, ..... has the value 57.

Devise another sequence for which the eighth term is also 57, and give a formula for it.

**Question 5.** Find the annual growth rate of a population which increases from 348000 to 592000 in 15 years. Give this rate as a percentage.

**Question 6.** A vehicle travels 2245km on a trip and uses 236 litres of petrol.

Find the petrol consumption in km per litre.

(a) Find the petrol consumption in litres per 100 km.

(b) Find the petrol consumption in litres per 100 km.

## **Appendix D:**

### **An example of a final examination test set in 1996 in Mathematics 1 (MATHS 101)**

**UNIVERSITY OF AUCKLAND**

**445.101**

**445.101E**

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**EXAMINATION FOR BSC BCOM BA BED DIPMATHSED 1996**

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### **MATHEMATICS**

#### **Mathematics One**

**(Time allowed: THREE hours)**

**NOTE:**      Answer All the Questions from **Section A**                            **(40marks)**  
                        Answer One Question from **Section B**                            **(30 marks)**  
                        Answer One Question from **Section C.**                            **(30 marks)**

#### **SECTION A: Skills Questions**

**ANSWER ALL QUESTIONS FROM THIS SECTION**

1.      A sequence has the following terms:

8, 12, 18, 27, 40.5, .....

(a)      Find a formula for this sequence.

(b)      What is the 8th term?

**(5)**

2.       $540 = 780 (1 - r)^6$

(a)      Solve this equation by finding the value of  $r$ .

(b)      Express  $r$  as a percentage.

**(5)**

3. The largest bonfire ever constructed in the world was lit on 19 April 1987 in Espel in the Netherlands. In the shape of a cone, it stood 28 m high and had a base circumference of 84 m. Find the radius of the base and the volume of the bonfire, before it burnt up.

(5)

4. On the way home Barry passes through three sets of traffic lights. The probability that the first set is green when he reaches them is  $\frac{3}{5}$ , the probability that the second set is green is  $\frac{2}{3}$ , and the probability that the third set is green is  $\frac{3}{4}$ . Draw a probability tree and use it to find the probability that he has to stop at:

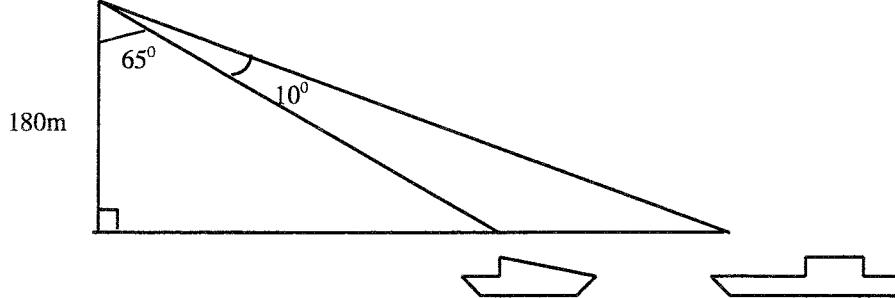
- (a) just one set of lights  
(b) at least one set of lights

(5)

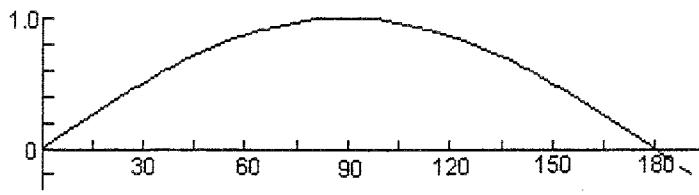
5. (a) Express 9 (base ten number) as a base two number.  
(b) Express 10101 (base two number) as a base ten number.  
(c) Change  $341_5$  to base 10.

(5)

6. Sightings are taken to ships from the top of a light-house, which is 180 m above sea level.
- (a) Find the distance of each ship from the bottom of the lighthouse.



- (b) Here is part of the curve for  $y = \sin x$ . What is an approximate value for  $\sin 135^\circ$ .



(5)

CONTINUED

7. The formula for the surface area in square metres of a human body may be given by  $S = 0.007184 \times H^{0.725} \times W^{0.425}$  where  $W$  is the weight in kg and  $H$  is the height in cm.
- (a) Find the surface area of a person who is 170 cm tall and weighs 85 kg.
- (b) (i) If this person who is 170 cm tall increases their weight from 85 kg to 90 kg, what is their new surface area?
- (ii) What is the percentage increase in weight for this person?

(5)

8. In a magic square, the sum of the numbers in each row, each column and each diagonal is the same. This sum is called the magic constant.

- (a) Complete this magic square.

		6
3	5	
4		2

- (b) If you add 4 to all the numbers in the magic square, what is the magic constant now?
- (c) If the magic constant of a  $3 \times 3$  square is  $M$  and you add  $k$  to all the numbers in the square, what is the new magic constant in terms of  $M$  and  $k$ ?

(5)

**CONTINUED**

## SECTION B

## Mathematical Situations

ANSWER ONE QUESTION FROM THIS SECTION.

There are four questions to choose from, each is worth 30 marks.

1. The Fibonacci sequence is the pattern 1, 1, 2, 3, 5, 8, 13, 21, .....where each term is found by adding the previous two terms together.
  - (a) Consider three consecutive Fibonacci terms. Find the difference between the square of the middle number and the product of the first and third terms. Repeat this at least three times. What have you found?
  - (b) Consider four consecutive Fibonacci terms. Find the product of the first and the last of these numbers and the product of the middle two terms. Repeat this at least three times. What have you found?

We can generate the sequence using algebra. Each term is the sum of the two previous terms.

$x, y, x+y, x+2y, 2x+3y, \dots$

- (c) Continue this pattern for the next 9 terms.
- (d) Find the relationship between the sum of the first 5 terms and the seventh term.
- (e) Find the relationship between the sum of the first 6 terms and the eighth term.
- (f) Find a rule for the sum of the first  $n$  terms and the  $(n+2)$ th term.
- (g) Find the sum of the first 10 terms without adding them all up. Show your working.

### 2. Magic Numbers

#### Part A

- (1) (i) Calculate  $2^1, 2^2, 2^3, 2^4, 2^5$ .  
(ii) Is there a pattern in the last digit of each number?  
(iii) What will be the last digit of  $2^{100}$ ?
- (2) (i) Repeat your investigation for  $3^n$ .  
(ii) Predict the last digit in the number raised to the power of 100.  
(iii) Are there any digits which give the same last digit when they are raised to any power?

(20)

#### Part B

The sum of the proper divisors of a number, A, is calculated by adding all those numbers which divide exactly into A, (including 1 but not including A itself).

For example, the sum of the divisors of 24 is

$$1 + 2 + 3 + 4 + 6 + 8 + 12 = 36$$

A pair of natural numbers, M and N, are called AMICABLE if each is equal to the sum of the proper divisors of the other. The smallest pair of amicable numbers is 220 and 284. Check that this pair of numbers is amicable.

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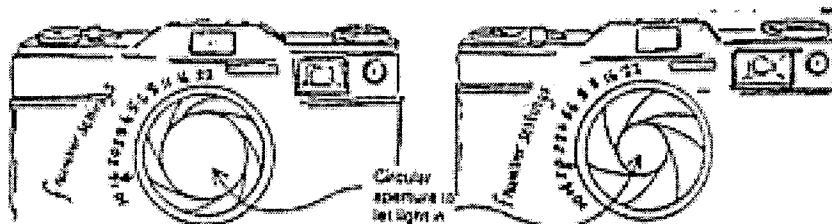
### 3. Patterns

#### Throwing Light on Cameras

The f-number of a camera tells you about the size of the opening which allows light through to the film. The standard f-number sequence is:

1.00    1.41, 2.00, 2.83, 4.00, 5.66, 8.00, 11.31, 16.00, .....

Reducing the f-number by one setting, for example from 11.31 to 8.00, doubles the light allowed through the opening by doubling the area. So the smaller the f-number, the larger the opening.



The setting on this camera is f/1.4.

The setting on this camera is f/8.

(a) The opening at  $f = 1.41$  is a circle with radius 1.6 cm. Calculate

(i) the area of the opening at

$$(\alpha) \quad f = 1.41$$

$$(\beta) \quad f = 2.00 \quad (\text{half the area at } f = 1.41)$$

(ii) the radius of the opening at  $f = 2.00$ .

(b) Copy and complete this table for the first eight f - numbers.

Setting (s)	1.00	1.41	2.00	2.83	4.00	5.66	8.00	11.31
Area ( $\text{cm}^2$ )	16.08	8.04			1.005			
Radius( $r$ cm)	2.26	1.6			0.57			0.2

(c) Draw a graph of  $r$  against  $s$ . Put  $s$  on the x-axis and  $r$  on the y-axis. Describe the relationship between  $r$  and  $s$ .

(d) The f-numbers form a sequence. Can you find a formula connecting consecutive terms?

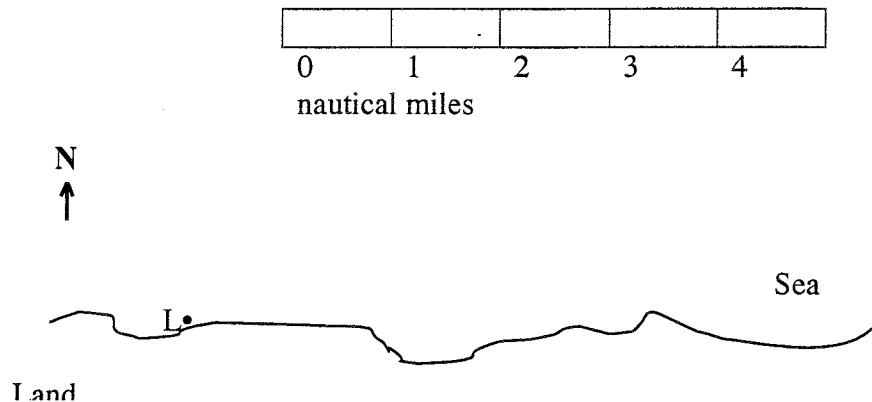
(e) Using your formula, find the next four f - numbers after 11.31.

(30)

CONTINUED

#### (4) Maps

The navigator on a boat knows she is close to shore, but because of fog, can only see the lighthouse at L.



- (a) The diagram above shows part of a map of the area. Copy the map.
- (b) At 1350 the navigator takes a bearing from the boat to the lighthouse of  $148^{\circ}$ . What will be the back-bearing from the lighthouse to the boat? Use this back-bearing to draw a line from L to the boat.
- (c) The boat is initially somewhere along this line, but the navigator does not know where. She does know the boat is travelling due East at 6 knots. (1 knot is 1 nautical mile per hour). Mark any point X on the back-bearing line and draw the boat's course due East from this point.
- (d) At 1420 the navigator sights the lighthouse at L again and takes the bearing to the lighthouse as  $206^{\circ}$ . Calculate the back-bearing from L to the boat and draw this line on your map from L.
- (e) Calculate how far the boat has sailed (in nautical miles) in the time between sightings of the lighthouse.
- (f) Mark the boat's position after travelling this distance along its course from X, as Y.
- (g) X and Y should both be on the back-bearing lines. Slide the line XY up or down on the map, keeping it parallel, until you have the correct distance. The navigator now knows exactly where the boat is at 1420. Measure the distance LY.

(30)

**CONTINUED**

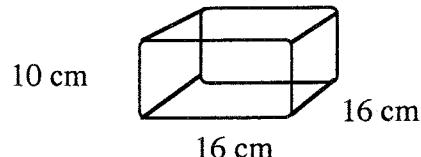
## **SECTION C: Essays**

**ANSWER ONE QUESTION FROM THIS SECTION.**

There are four questions to choose from, each is worth 30 marks.

### **1. Packaging**

Ice cream often comes in cuboid shaped containers.



An industrial designer suggests that the same volume of ice-cream could be packaged more cheaply in a container the shape of a cylinder, with height equal to the diameter of the base. Explain why a cylinder might be preferred to a cuboid and in particular, discuss the quantity of plastic required in both cases.

(20)

Another designer suggests the shape of a cube would be best. Which container would allow the ice-cream to freeze more quickly? Why do you think the manufacturer prefers the original cuboidal shape?

(10)

### **2. Medicine**

Genetic diseases may be recessive or dominant.

(a) Cystic fibrosis is a recessive genetic disease. It is the genetic disease that affects children most often. A child with cystic fibrosis has two recessive genes, one from each parent, so if a child has cystic fibrosis, both parents must be carriers of the disease. Being a carrier means that they have one gene in their cells, but because it is recessive it does not cause them to have the disease.

Complete a table for the possible combinations of genes from two parents who both carry the recessive gene. Discuss the various probabilities for children having the disease, being a carrier, or being free of the disease.

(20)

(b) Some other diseases are called dominant because only one gene carrying the disease is necessary to have the disease. Discuss the situation for people who have a dominant disease.

(10)

**3. Environment** Population

The population of a city is 25 437 at the beginning of the year 1980. The average birth rate is 4.6% and the death rate is 3.2%, and net migration from the city has been about 1670 per year. Complete a table to investigate the growth, or decline of the city over the next 5 years.

(15)

Illustrate your calculations with a sketch graph. Extrapolate your graph to predict the population for the year 2000. Explain why your predictions from the graph may be wrong.

(15)

**4. Magic** Prime Numbers

**Part A**

All counting numbers can be described as being either composite or prime.

Use examples to show what we mean by the words "composite" and "prime".

(12)

**Do Part B or Part C**

**Part B**

Many people in history, such as Eratosthenes, Euler, Fermat and Mersenne, have tried to find reliable methods for finding prime numbers. Describe some of the methods used and comment on how successful they have been.

(18)

**OR Part C**

In each of the following diagrams 2 out of 5 squares are shaded.



- (a) If there are altogether 10 ways to show  $\frac{2}{5}$ , explain why there must also be ten ways to show  $\frac{3}{5}$ .
- (b) Show that counting the different ways of representing fractions in the manner above can lead to the pattern of numbers that form Pascal's Triangle. You should count the different ways to show halves ( $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}$ ) and then thirds ( $\dots, \dots, \dots$ ) and so on.
- (c) Describe at least 4 patterns of numbers in Pascal's Triangle and describe how each pattern can be generated.

(18)

## **Appendix E:**

### **Mathematics 1 (MATHS 101) Resource Folder 1996 (part)**

#### **Course Information and Assessment**

## **COURSE INFORMATION**

### **1. Course Description**

Mathematics One is a 2-point Foundation Course in Mathematics for students who have very little background in the subject. It is especially aimed at students who have not studied mathematics for some time, those who lack confidence in mathematics, and those who require mathematics as part of non-science courses. It provides a preparation for further courses in mathematics and statistics.

#### Duration and Timetable

Maths One will be a full year course.

Maths One will be run in one stream on:

Monday	10 am
Tuesday	10 am
Wednesday	10 am
Thursday	One laboratory at either 9am., 10am., 12am., or 2 pm. First semester: B25 Maths/Physics Building.

#### Lecturers

Bill, Pam and Moira (with contact information)

#### Entrance Requirements

Entry to Maths One will be for:

- All those who have not done F6 Maths or equivalent, or
- If Form 6 mathematics is the highest Maths qualification, students will be interviewed to determine whether they should enrol in Mathematics One or Mathematics Two. Notwithstanding the above, special cases may be enrolled after an interview.

Maths One may not be taken concurrently with any other Maths or Statistics courses except STATS 10X.

Students may be permitted to enrol in Maths Two (MATHS102) concurrently in the second semester.

#### Resources

The main resource is this loose leaf folder. A variety of pages are added to this as the course proceeds. These will include: historical background, notes, illustrative examples or applications, work sheets, exercises, interactive lecture notes and up-to-date statistics or data.

#### Aims of this Course

- To build the confidence of students to do mathematics. MATHS 101 will provide both an opportunity for many different types of students to get a foothold on mathematics and the inspiration to continue to study and to use mathematical ideas.

2. To learn to construct simple mathematical models in order to solve real life problems. The idea of a mathematical model is an important basis of this course. Students should develop their mathematical knowledge through consideration of a variety of contexts and real-life situations.
3. To develop the ability to initiate mathematics learning. Students should become willing to use their own experience, printed and electronic materials, technology, and communication with others in order to develop mathematical understanding and to solve problems mathematically.
4. To learn when mathematical skills might be applied. This involves learning some new skills and their role in problem solving, knowing when other techniques must be found, and knowing what other (unlearnt) techniques are available.

### Objectives

It is hoped that this course, by involving students in real life material, will encourage them to think mathematically about things around them. The course will be conducted in a co-operative mode and wherever possible students will work together on open-ended tasks. Students will be involved in mathematical activity rather than listening passively or memorising facts. Although this course is not skill based, students will be expected to use scientific calculators and their use will be necessary in tests and examinations. The use of other technology, in particular computers, will be introduced by lecturers but will not be part of the examinable skills of the students. Students will have the opportunity to work on problems in the computer laboratories, in particular using spreadsheets.

In particular, over this course, students will gain confidence and practice in:

- (a) expressing quantities in numerical form, and calculating with and manipulating these numbers.
- (b) expressing relationships and quantities in symbolic form, and manipulating these symbols.
- (c) expressing relationships in graphical form, and interpreting these.
- (d) handling data in a variety of ways in order to answer questions.
- (e) developing a critical attitude to data and mathematical ideas as they apply to the real world.
- (f) recognising and using some standard mathematical relationships such as growth relationships, linear, quadratic and circular relationships.
- (g) developing ideas which lead into further mathematics, eg. rates, infinitesimals and limits.

### Curriculum

A set of themes will be developed in lectures with relevant background ideas and resources. Within these sessions, and in laboratories students will be expected to explore certain tasks. The curriculum is drawn from the following themes, each taking two to three weeks.

- Aspects of Design For Packaging
- Plans and Maps
- The Magic of Numbers
- Describing and Generating Patterns and Sequences
- Environmental Calculations
- Drugs and Medicine
- Chance, Gambling and Simulations
- Swings and Rotations

A feature of the course will be the laboratory approach in which students, working in groups and individually, will undertake a variety of tasks related to each theme.

## 2. Year's Timetable 1996

Week Starting	Topic	Laboratory	Deadlines
26th Feb	Packaging	29th	
4th March	Packaging	7th	
11th March	Packaging	14th	
18th March	Patterns	21st	Assignment 1 due Thursday
25th March	Patterns	28th	
1st April	Patterns	4th	

\*Easter Holiday\*

17th April	Environment	18th	Assignment 2 due Thurs
22nd April	Environment	no lab.	
29th April	Environment	2nd	
6th May	Plans and Maps	9th	Assignment 3 due Thurs
13th May	Plans and Maps	16th	Term's Test 8/5 Wed 6-8pm
20th May	Plans and Maps	23rd	Test Rewrites due 23rd Thurs
27th May	Chance and Gambling	30th	Assignment 4 due Thurs

\*Mid-Year Break\*

15th July	Chance and Gambling	18th	
22nd July	Chance and Gambling	25th	
29th July	Drugs and Medicine	1st	Assignment 5 due Thurs
5th August	Drugs and Medicine	8th	
12th August	Drugs and Medicine	15th	
19th August	The Magic of Numbers	22nd	Assignment 6 due Thurs

\*Mid-Semester Break\*

9th Sept	The Magic of Numbers	12th	Term's Test 11th Wed 6-8pm
16th Sept	The Magic of Numbers	19th	
23rd Sept	Swings and Rotations	26th	
30th Sept	Swings and Rotations	3rd	Test Rewrites due 3rd Thurs
7th Oct	Swings and Rotations	10th	
14th Oct	Revision		

21st Oct - -

\*Examinations\*

### **3. Lectures and Laboratories**

#### **Lectures**

In keeping with the active nature of this course, you can expect to be active in lectures. Lecture time is not a time to sit back and listen and take the odd note. You will be expected to DO things.

For example you may be given out notes which are incomplete - you will need to listen to the lectures and write on them as it proceeds. You will also be asked to turn to your neighbour to discuss some point, or to work with them on a small task. You will be asked to work on problems during the lecture, and to offer your working to the class.

The lecturers and tutors expect that you will have many questions, comments and contributions to make. These can be discussed in laboratory time, but it is hoped that you will speak out in lectures: ask questions, challenge what is being presented, ask for clarification or offer another method for the task being presented.

#### **Laboratories**

The Thursday session is a compulsory practical session in smaller classes of about 15-20 students. Groups will be available at several times during the day. You will stay in the same laboratory all year. There are no lectures on that day.

Each laboratory group will have one tutor who will stay with the group for the whole year, and who will be the marker of your assignments. It is expected that the tutor will get to know you well, and will help with any problems you have with the course. The tutors will be available at certain other times during the week. At any time during the year you may contact your tutor, or the course convenor (Bill).

During the laboratories, you will be expected to work in groups on cooperative tasks. Marks will be given for effective participation (see Assessment). These tasks are an important part of the course, and regular attendance at the laboratories is required.

### **4. Mathematics Beyond MATHS 101**

This course is primarily aimed at helping you to do mathematics in all aspects of your life. Mathematical ideas are often presented in the newspaper, and we all engage in mathematical thinking in many ways. For example in financial transactions like loans, mortgages or investments; in arranging times and schedules; when gardening or cooking; when building fences or sewing; when analysing statistics of all kinds. You often do mathematics in your daily life. We hope that this course will improve your ability to handle these mathematical situations.

In addition, many students may wish to extend their formal studies in mathematics after this course. MATHS 101 partly prepares you to take the courses MATHS 102 (Maths Two) and STATS 1xx (Level One Statistics). It is possible to enrol in MATHS 102B (Maths Two) in the second half of the year, provided that your progress in MATHS 101 in the first half year is satisfactory.

## ASSESSMENT

### 1. An Overall View

This course is about **doing** mathematics, and so the assessment is designed to reflect:

- a) your participation in the course;
- b) all the work you do during the year;
- c) the new mathematical skills you have achieved;
- d) the way you communicate in mathematics, written and oral;
- e) the new mathematical understandings you have achieved;
- f) your ideas about mathematics;

We are not so interested in what you can learn by rote, nor what you can achieve in short periods of time working under pressure. Thus there is less emphasis on the final exam than usual, and this exam will not be a set of problems to be solved within a given time limit.

The assessment is made up of:

Task participation	10%
Mathematical Assignments (6 in total)	20%
Two open book terms test (10% each)	20%
Final open book examination	50%

### 2. Task Participation

During laboratories you will be given tasks to do, sometimes in groups, sometimes individually. The tasks will generally be open-ended mathematical investigations rather than closed problems for which you must find a solution. During the year you will be shown how to pose your own questions, and how to communicate and record your results.

You are expected to attend all your laboratories and to participate in the tasks set. For each laboratory you attend and make progress on the task presented you will be given  $\frac{1}{2}\%$  towards your final mark (to a maximum of 10% after 20 laboratories). The work completed in laboratories should be kept, and handed in as part of the assignments.

Attendance means arriving within the first ten minutes, and staying until at least ten minutes from the end. Participation means contributing to group discussions and showing evidence of progress on the set tasks.

### **3. Assignments**

In each three-week period on a topic you will have worked on three tasks in your laboratories. One of these tasks will become the assignment for that period and is marked in detail. However, the other two tasks for that topic will also earn extra marks if they are handed in. You will be expected to do some extra work on your own on the designated task after working on it in a group.

The material should be organised so that it is easy to understand, and should be written up so that the marker can follow the train of thought.

Assignments are to be handed in on the following dates:

1.	Thursday	21 March	4pm	(Packaging Topic)
2.	Thursday	18 April	4pm	(Pattern Topic)
3.	Thursday	9 May	4pm	(Environment Topic)
4.	Thursday	30 May	4pm	(Mapping Topic)
5.	Thursday	1 August	4pm	(Chance Topic)
6.	Thursday	22 August	4pm	(Medicine Topic)

On City Campus, assignments should be placed in the boxes provided in the Basement of the Maths/Science building.

Assignments should be stapled or in a large envelope, and should have the standard cover sheet attached, with name and I.D. number filled in, and the time of your laboratory. These cover sheets are available from the SRC or your lecturer.

In place of the medicine topic, Assignment 6, you may do a topic of your own choosing.

You are encouraged to work together on these investigations, but you must present your own work in the assignment, and be prepared to discuss your work with the tutors.

For each task a list of marking criteria will be available.

### **4. Terms Tests**

There will be two terms tests, each worth 10% of the final mark. Each test will be conducted in two stages.

The first stage will be a written test taken under test conditions. It will be a one and a half hour test, although you may take up to two hours if you wish. It is not intended that you should be working under time pressure. You may take into the test any calculators, books, notes, dictionaries or other reference material.

These tests will be marked and returned with comments and an indication of where further work could be done. The second stage consists of you doing further work on your test at home over the next two weeks. At the end of that time the test will be

marked again. It is expected that you will get help from other people for this second stage of the test. However you may be asked to discuss your answers with the marker, so it will be necessary to fully understand the work which is handed in. The mark awarded will be the average of the marks after the first stage and the marks after the second stage.

Each term's test will generally cover the three themes taught prior to the test. Sample terms tests are included in this folder. There will be three types of questions, given about equal weighting:

- a) exercises and context problems requiring calculations and "answers".
- b) an essay-type question which will give you an opportunity to discuss your views on some aspect of mathematics;
- c) questions which will give you the opportunity to describe the kind of investigations you have been doing in lectures and tutorials, with a small, new investigation to spend time on.

The dates of the tests are:

Terms Test 1:	First stage:	Wed 8th May 6-8pm
	Papers Returned:	Wed 15th May
	Second stage:	Revised papers due Thurs 23rd May
Terms Test 2:	First stage:	Wed 11th Sept 6-8pm
	Papers Returned:	Wed 25 Sept
	Second stage:	Revised papers due Thurs 30th Sept

## 5. Final Examination

The final examination will cover all the material in the course and it will be three hours long.

Like the Terms Tests it will contain three types of questions in approximately equal proportions.

It will also be set so that it should take less than 3 hours, but you may take the full 3 hours if you wish. It will be open book (ie. you may take in whatever written material, notes, dictionaries you wish). You may also take in any calculators.

A sample Final Examination is included in this folder.



## **Appendix F:**

### **Wellesley Program 1996 Annual Mathematics Report**

#### **INTRODUCTION**

Interviews for the one hundred places available on the 1996 Wellesley Program began at the end of November 1995 and continued until the middle of February 1996. There was a reasonable demand for application forms, although it was noticeably less than in the previous year. Since there was no advertising of the program prior to Christmas, this was not unexpected. It was also apparent that many who requested application forms did not return them. As students were being asked to complete the standard university ATE form, most of which was irrelevant to Wellesley Program applicants, it was resolved to design a simple pre-application form to serve as the initial contact for the 1997 intake. Ninety-six students were offered places and the 1996 program began with eighty seven students enrolled.

Over a hundred prospective students were interviewed, and applicants were asked to do a mathematics test at their interview. The test had one section to assess basic numeracy, one section to assess problem solving skills, and a third to assess background. It was felt that a score of less than 10 (out of 30) indicated that a student would have significant difficulty with the mathematics course, and his or her other abilities needed to be looked at more critically. Most of those with very poor results in the preliminary mathematics test were advised to seek alternative courses. On the basis of other strengths, twelve students with a score of less than 10 were accepted. Of these, three dropped out of the program during the year, but five of the remaining nine passed mathematics. Furthermore, of the nine students, six achieved 'success' by passing three of their four papers in total, even if they failed mathematics. The poorest student overall in the Wellesley Program this year was the student who had the lowest score (2) in the mathematics test. However the student with the next lowest score (4.5) was successful in passing three papers (not mathematics), although with low grades. The mathematics test acts as an indicator, but should not stand alone in determining a student's suitability for the program. It was most useful in determining the minimum level for acceptance into the Science option of the program.

The test and interview were used as criteria to divide the students into four streams for mathematics and to identify a maximum of twenty-five students to be offered places in the extension paper MATHS 092. As in previous years, all students were taught the Basic Course MATHS 091 and the top stream of students was offered an additional Extension Course MATHS 092. Each stream had five hours teaching per week and in addition the tutors were available to help students individually as required.

Teaching began on Monday February 26th, the same time as the normal university year. Although full year papers were retained, the semester breaks were followed with one exception: in the inter-semester period, teaching continued for an extra week with lectures finishing on Friday June 7th. As the students did not have examinations at this time, they had an effective five week break which proved to be a major disruption, and the cause of much loss of motivation.

## **STATISTICS**

### **ENROLMENT STATISTICS**

#### **Gender**

87 Students enrolled	43 male	44 female
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46 Arts students	23 male	23 female
41 Science students	20 male	21 female

#### **Ethnicity**

87 students enrolled	: 37 Pakeha	16 Maori	21 Pacific Is	13 Other
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46 Arts students:	16 Pakeha	10 Maori	13 Pacific Is	7 Other
41 Science students:	21 Pakeha	6 Maori	8 Pacific Is	6 Other

### **DROP-OUT STATISTICS**

34 students did not complete the year. This represents 39% of our students and compares with 1995 (33%), 1994 (36%), 1993 (32%), 1992 (23%), and 1991(23%).

In the Arts option	15 did not complete			
	9 male	6 female		
	7 Pakeha	2 Maori	4 Pacific Is	2 Other

In the Science option	19 did not complete			
	9 male	10 female		
	8 Pakeha	3 Maori	3 Pacific Is	5 Other

#### **Comparison with 1992, 1993, 1994, 1995:**

<b>Drop-Out Rate</b>	<b>1996</b>	<b>1995</b>	<b>1994</b>	<b>1993</b>	<b>1992</b>
Male	42%	37%	46%	28%	25%
Female	36%	25%	24%	36%	20%
Pakeha	41%	36%	34%	24%	28%
Maori	31%	18%	41%	41%	26%
Pacific Island	33%	50%	50%	40%	6%
Other	54%	25%	20%	33%	21%
Arts	33%				
Science	46%				

It is noticeable that again female students showed more determination to complete the program than did male students. The retention of Pacific Island students was very encouraging when looked at in comparison to previous years, but overall the loss of students is a cause for concern. (A further analysis of the statistics is attached.)

To gain some information on possible reasons for the increased loss of students in 1996, a survey was designed (as attached). Significant problems seem to be associated with lack of motivation after return from the five week inter-semester break, placing of tests after this break, and the usual lack of money. Since the survey was answered by those who completed the program, only anecdotal evidence is available to provide some reasons for students leaving. Such reasons are : death (1), ill-health (2), lack of money (4), pregnancy (2), feeling overqualified and already prepared for tertiary study (4), drugs and/or alcohol problems (2).

### 9.6.1.1.1 THE BASIC COURSE MATHS 091

51 students sat the basic mathematics paper MATHS 091 at the end of the year.

**Results:**  $\frac{40}{51}$  or 78% of students "passed" gaining a grade of C or better.

Overall results were: 12 A (24%) 10 B (20%) 18 C (35%) 11D (22%)

In the Arts option:

$\frac{20}{30}$ (67%) passed	8 male	12 female
	7 Pakeha	5 Maori 5 Pacific Is 3 Other

In the Science option:

$\frac{20}{21}$ (95%) passed	10 male	10 female
	11 Pakeha	3 Maori 5 Pacific Is 1 Other

The difference between Arts and Science students reflects the selection procedure, where applicants for places in the Science option were required to have reasonable mathematical background.

Statistics which follow are the percentages of those who completed the course (including 1991, 1992, 1993, 1994, 1995 comparison).

Gender		1996	1995	1994	1993	1992	1991
<b>Male Pass Rates :</b>	Arts	57%	33%	64%	41%	56%	57%
	Science	100%	100%	94%	93%	83%	67%
	<b>Overall</b>	<b>75%</b>	<b>63%</b>	<b>81%</b>	<b>65%</b>	<b>72%</b>	<b>63%</b>
<b>Female Pass Rates</b>	Arts	75%	67%	68%	44%	45%	43%
	Science	91%	79%	93%	100%	54%	88%
	<b>Overall</b>	<b>81%</b>	<b>75%</b>	<b>79%</b>	<b>67%</b>	<b>48%</b>	<b>62%</b>
<b>Ethnicity</b>		<b>1996</b>	<b>1995</b>	<b>1994</b>	<b>1993</b>	<b>1992</b>	<b>1991</b>
<b>Pakeha Pass Rates:</b>	Arts	78%	46%	69%	50%	62%	50%
	Science	92%	95%	95%	94%	75%	95%
	<b>Overall</b>	<b>86%</b>	<b>75%</b>	<b>84%</b>	<b>75%</b>	<b>69%</b>	<b>70%</b>
<b>Maori Pass Rates</b>	Arts	63%	14%	70%	11%	50%	100%
	Science	100%	80%	100%	100%	50%	83%
	<b>Overall</b>	<b>73%</b>	<b>53%</b>	<b>79%</b>	<b>38%</b>	<b>50%</b>	<b>86%</b>
<b>Pacific Is Pass Rates</b>	Arts	63%	40%	25%	38%	30%	17%
	Science	100%	100%	100%	100%	80%	25%
	<b>Overall</b>	<b>77%</b>	<b>50%</b>	<b>50%</b>	<b>44%</b>	<b>47%</b>	<b>21%</b>
<b>Other Pass Rates</b>	Arts	60%	100%	100%	100%	60%	100%
	Science	100%	100%	80%	100%	83%	50%
	<b>Overall</b>	<b>67%</b>	<b>100</b>	<b>88%</b>	<b>100</b>	<b>73%</b>	<b>75%</b>

Female students continue to perform well, and Pakeha students also. There was a major improvement in the performance of Pacific Island students this year, not only did they complete the program, but they had better academic success in mathematics.

## THE EXTENSION COURSE MATHS 092

Twenty-one students began the year in the top stream, thirteen completed the year and twelve sat the extension paper MATHS 092 at the end of the year.

### Results

$\frac{11}{12}$  (92%) students gained a 'pass', being a grade of C or better.

11 students passed:	8 male	3 female
	9 Pakeha	0 Maori    1 Pacific Is    1 Other

The overall results were:

2A (17%), 4B (33%), 5C (42%), 1D (8%)

Statistics which follow (including 1992, 1993, 1994, 1995 comparison) are percentages of those who completed the course.

Gender	1996	1995	1994	1993	1992
Male Pass Rates:	Overall 89%	80%	79%	55%	40%
Female Pass Rates:	Overall 100	71%	75%	67%	86%

### Ethnicity

Pakeha Pass Rates:	Overall 90%	89%	60%	55%	67%
Maori Pass Rates:	Overall N/A	50%	100%	33%	33%
Pacific Is Pass Rates:	Overall	100%	100%	N/A	100% 33%
Other Pass Rates:	Overall 100	67%	100	100	80%

One outstanding student gained an A+ in both MATHS 091 and MATHS 092. A second student gained A passes in both MATHS 091 and MATHS 092.

## TUTORS

This year the mathematics course was taught by 2 tutors

Pamela	Stream 1 and Stream 3
Moira	Stream 2 and Stream 4 (the extension stream)

Marking was done by a student marker Ann, herself a mature student with an appreciation of the difficulties of returning to education as an adult.

The Resource Centre was used for collection of assignments but we continued to keep our own records for entering marks because our assessment program included fortnightly tests and laboratories as well as assignments. We processed all our own marks and generated our own result sheets.

## **GENERAL**

### **1. MATHS 091**

The more comprehensive pre-selection testing introduced first in 1993 was helpful in allowing us to select our students for 1996. We were able to direct to other courses those wishing to do Sciences but without sufficient mathematical background. Those with few numeracy skills were identified, and some were accepted into the Arts program because of other recommendations. These students were not so numerous this year because of the experiences of the past. It was decided to take less than the hundred students into the program, rather than overload the bottom stream with students who struggled not only with mathematics, but with all subjects in their Arts course. The greater the number of applicants, the better the opportunity to select students with a reasonable chance of success on the program.

In Paper MATHS 091, 78% of students who completed the program 'passed' by reaching a C grade or better. This is comparable with the 1994 program (80%), and better than 1995 (67%). It would seem to reflect the more stringent selection process, even with a smaller pool of applicants to choose from. The bottom stream was much more successful this year because it was not dominated by a group of students with no hope of passing. In fact seven students in this stream passed, and one gained a B- grade. Every stream this year had its share of good grades, showing that students can discover unsuspected talents, and that diagnostic maths tests are quite fallible.

As always, those students who attended regularly and persevered in taking responsibility for their own learning, were able to make the most of their ability and to achieve as much as possible. For some students this may not have been quite a pass, but still represented an enormous achievement.

Although the number of men and women in the program has been about equal, again women have better records in academic success. The two top students in MATHS 091 are women. Two of the top three Arts students are women, and two of the top three Science students are women.

### **2. MATHS 092**

Students selected for the extension paper MATHS 092 were enrolled in this as a separate paper for which they paid an extra fee. The identification of potentially more able students, who would like the opportunity to extend their mathematics, is still a difficulty. The extra work involved in taking on MATHS 092 can be a burden to some students, while others not offered the chance feel that they could have benefited.

### **3. Evaluation**

In the second semester the tutors requested an evaluation of the course and its teaching. The results were positive and encouraging.

Wellesley Program students took part in a research study on attitudes towards mathematics being conducted by Barbara Miller-Reilly. This involved the students completing two questionnaires over the year, one at the beginning of the course and one at the end.

Late in the second semester, as a response to the high drop-out rate of students, a survey seeking to establish some cause for this was conducted. As a result, it has been decided to re-structure next year's program for Wellesley Program students. The inter-semester break of five weeks will be reduced to three, by adding one week of teaching to the end of the first semester, giving the students a week's study break, and then offering a substantial one or two hour test in each subject over the following week. This will bring the semester to an end for Wellesley program students at the same time as for all other students. They will then have the same three week's break and begin the second semester at the same time as other students. It is hoped that this will cause less disruption to established study patterns, and also give the students a sense of accomplishment and completion at the end of the first semester.

### **4. Assistance**

From previous experience, we know that financial hardship plays a major part in students' ability to complete the course. This year we have again been fortunate to gain the support of the Alumni Association with the provision of some funds for emergency relief. The administration of the fund has been transferred to the university system, which makes it rather more daunting for the students to access. This hurdle directly affected at least one very able student who disappeared to find paid work in the last few weeks of the program, and consequently did less than justice to his final exams.

### **5. Collaborative Laboratories**

In order to prepare students for papers MATHS 101 and MATHS 102, a fortnightly collaborative laboratory was set up for all Wellesley Program students in 1996. This involved the students working in small groups on a problem to be solved within the class time and presenting a written solution. As such exercises are an important part of the assessment procedure at Stage 1 level, it was felt that an introduction to the process, with a small credit to be gained, was a positive step to take.

### **6. Graduation Ceremony**

As always the successes of those students who have done their best is a great reward. The presentation of the 1996 certificates took place on December 6th at a graduation ceremony conducted by the Chancellor of the University. Professor Ivan Reilly, who had welcomed the students in February on behalf of the university, was a guest speaker.

## SUCCESS OF EX-STUDENTS

### (a) Students enrolled in Mathematics papers in 1996

	No. of Students	No. of Papers	No. of Passes	No. of A or B Passes
Stage 1	25	39	21	10
Stage 2	5	10	8	5
Stage 3	2	2	1	1

The top mathematics students from 1994 and 1995 are both pursuing studies in mathematics with great success.

### (b) Students enrolled in Statistics papers in 1996

	No of Students	No of Papers	No of Passes	No of A or B Passes
Stage 1	19	19	12	5
Stage 2	8	9	9	6
Stage 3	1	2	2	2
Masters	1	2	2	1

### (c) Graduates

Several ex-Wellesley Program students were capped in graduation ceremonies this year. Nine graduated from the University of Auckland, including one MSc in Biological Sciences, and one BSc in Statistics. One student graduated from Massey University and another from Otago University.

Moira, Mathematics Tutor    Deputy Co-ordinator Wellesley Program  
Pam, Mathematics Tutor



## **Appendix G:**

### **Wellesley Program Mathematics Course**

#### **MATHS 091 Study Guide 1996**

##### **Description**

This is a one-year course designed for students who at present lack the necessary background for tertiary courses in mathematics.

##### **Course Outline**

	<b>Chapter No</b>	<b>Approx No of Classes</b>
<b>Semester 1</b>	1      Arithmetic	10
	2      Proportion and Percentage	10
	3      Area and Volume	10
	4      Statistics and Probability	10
	5      Introduction to Algebra	10
	6      Linear Equations and Inequalities	10
<b>Semester 2</b>	7      Trigonometry 1	10
	8      Linear Graphs	10
	9      Functions	10
	10     Sequences	10
	11     Quadratics	10
	12     Trigonometry 2	10

##### **Texts**

Photocopied chapters dealing with the above topics will be provided. In addition you will be loaned copies of "School Certificate Mathematics Revision" R. Bairstow (streams 1 to 3) and "Form 6 Mathematics Revision" J.R.Barrett ( streams 2 to 4 ). You will have paid a \$20 deposit for Mathematics texts at enrolment and this amount is refundable on return of the books at the end of the course.

##### **Calculator**

You will need a scientific calculator with a fraction button (  $a \frac{b}{c}$  )

## **Course Work:**

**Ten assignments** will be set at regular intervals. Your assignment marks will be counted towards your course work mark.

**Eight short tests** will be set to coincide with assignments and the marks from each of these will count towards your course work mark as well.

**Eight group laboratory sessions**, which will be marked for co-operation and communication as well as for results.

**3 Major Tests** will each be one hour long and held in class on  
**29 April, 16 September and 21 October.**

**A 2 hour Semester Test** will be held at the end of the first semester  
on **23 June**

**Course Work contributes 50% to your final mark.**

The 50% consists of :	Assignments	18%
	Short tests and laboratories	10%
	Major Tests	12%
	Semester Test	10%

## **Examination and Grades**

A 3-hour examination will be held in November. The examination will usually contribute 50% to your final mark, however students have the option of counting the examination 100% if this gives them a higher mark. Grades are awarded from A to D.

## **Streams**

Classes will be streamed to give everyone the best chance of making progress. Early work which you may have forgotten will be covered as far as possible, but the working of problems on your own outside class time will be necessary to cover the ground in the time available.

## **Tutors**

Tutors are available to see students in their office in the Mathematics/Physics Building. Office hours are below, but other mutually convenient times can be arranged.

Streams 1&3 Pam                  Mon 10-1, Fri 12-2

Streams 2&4 Moira                Mon 12-3, Fri 2-3

## Appendix H:

### Two assignments for the Wellesley mathematics course (MATHS 091) in 1996: Assignment 1 and Assignment 9.

#### DEPARTMENT OF MATHEMATICS

#### WELLESLEY PROGRAM

MATHS 091	Assignment 1	Due: 18 March 1996
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Your assignment is due in to the Resource Centre by 4 pm on Monday 18 March. Set out your work neatly and show all working.

1. (a) Calculate

(i)  $2^5 \times 92$  (ii)  $\frac{5 \times 7 - 3}{3 \times 2}$  (2)

(b) Insert the correct symbol,  $<$  or  $>$  or  $=$ , to make a true sentence

(i)  $2 \times 0.8 - 1.45$        $0.305 - 0.05 \times 3$   
(ii)  $(0.2)^3$        $(0.4)^2$  (2)

(c) Calculate

(i)  $15 + 9 \times 3$  (ii)  $(6 - 2)^2 \div 2 \times 3$   
(iii)  $24 + 6 \times 3^2 \div 27 - 22$  (iv)  $(3 \times 4)^2 - 2 \times 4 + 5$  (4)

2. Evaluate as a decimal number

(i)  $5.95 \times 10^4$  (ii)  $6.8 \times 10^{-5}$   
(iii)  $4 \times 10^3 + 6 \times 10^2 + 7 \times 10^0 + 3 \times 10^{-2}$   
(iv)  $10^{-1} + 10^{-3}$  (3)

3. (a) Express

(i) 6392 correct to 1 significant figure  
(ii) 589 875 correct to the nearest thousand  
(iii) 0.005023 in standard form  
(iv) 0.0007523 correct to 4 decimal places (2)

(b) Write  $(111\ 111)^3$  in standard form correct to 2 significant figures

(1)

(c) How many significant figures are there in

(i) 3.409 kg (ii) 0.070 cm (1)

4. (a) (i) The mean distance of the planet Pluto from the sun is 5 920 000 000 km.  
 What is this distance in standard form correct to 3 significant figures?  
 (ii) The mean distance of the planet Mars from the sun is  $2.3 \times 10^8$  km.  
 Which of the two planets, Pluto or Mars, is closer to the sun, and by  
 how many kilometres?  
 (Give your answer as a decimal number correct to 3 significant figures.)

(2)

- (b) A container weighs  $9.26 \times 10^{-4}$  kg. When it is filled with a chemical it weighs  $4.58 \times 10^{-2}$  kg. If 1 kg = 1000 grams, find the weight of the chemical in grams.  
 (Give your answer as a decimal number correct to 3 significant figures.)

(2)

5. Evaluate

$$(i) 1\frac{1}{2} - \frac{8}{11}$$

$$(ii) (2\frac{3}{4} - 1\frac{7}{10}) \div \frac{2}{3}$$

$$(iii) \frac{7}{9} + \frac{3}{4} \times \frac{1}{2}$$

$$(iv) \frac{\frac{2}{1}}{\frac{1}{2} + \frac{1}{4}}$$

(4)

6. (a) Peter has a full tank of petrol. He drives  $\frac{2}{3}$  of the distance from Auckland to Taupo and his tank shows  $\frac{1}{4}$  full. Does he need to stop for more petrol, or will he have enough to get to Taupo? Explain your answer.

(2)

- (b) (i) In an hour's monitoring,  $\frac{4}{5}$  of the 1200 people who enter a certain casino make at least one bet. How many do not bet?  
 (ii) Of those who bet,  $\frac{1}{3}$  win more than \$100. What fraction of the 1200 people who enter the casino in this hour, win more than \$100?

(2)

- (c) A nurse had to measure out a dangerous drug. The correct dose was one-sixth of a unit. He had no measure for one-sixth, but he did have a measure for a half and a quarter. He thought "two plus four is six". I will measure one-half and one-quarter and the sum of these should be one-sixth.  $\frac{1}{2} + \frac{1}{4} = \frac{1}{6}$   
 (i) What is wrong with his fraction calculation? Show the correct solution.  
 (ii) How many times greater than the one-sixth of a unit was the dose given to the patient ?  
 (iii) The nurse also had a measure for one-third of a unit. How could he successfully get a measure of one-sixth? Describe your method exactly.

(3)

**DEPARTMENT OF MATHEMATICS**

**WELLESLEY PROGRAM**

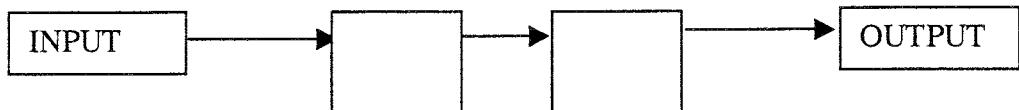
**MATHS 091**

**ASSIGNMENT 9**

**DUE: 9 SEPTEMBER '96**

Your assignment is due in to the Resource Centre by 4 pm on Monday 9 September, 1996.  
Set out your work neatly and show all working.

1. (a) The diagram shows a machine that changes numbers. It multiplies the input by 3 and then adds 2.



- (i) If 5 is input, what is the output?
- (ii) Write the rule for the machine as a function of  $x$ . i.e.  $f(x) =$
- (iii) If 119 is the output, what was the input?
- (iv) Write an expression for the inverse function  $f^{-1}(x)$

(4)

- (b) Describe this function in words  $f(x) = (\sqrt{x} + 5)^2$

Begin "take the input  $x$ ..."

(1)

2. (a)  $f(x) = \frac{2x+1}{x-5}$

- (i) For what value of  $x$  is  $f(x) = 0$ ?
- (ii) For what value of  $x$  is  $f(x)$  undefined?
- (iii) For what value of  $x$  is  $f(x) = \frac{3}{2}$  ?

(3)

(b)  $f(x) = \frac{2}{5}x + 1$

- (i) Sketch a graph of  $y = f(x)$  on graph paper.
- (ii) Find the inverse function  $f^{-1}(x)$
- (iii) Sketch the graph of  $y = f^{-1}(x)$  on the same graph
- (iv) For what value of  $x$  is  $f(x) = f^{-1}(x)$  (use algebra)
- (v) At what point on the graph does  $f(x) = f^{-1}(x)$ ?

(5)

3. Simplify

$$\begin{array}{ll} \text{(i)} & \frac{24x^3y^2}{18x^4y} \\ \text{(iii)} & 3x^3(3x)^3 \end{array} \quad \begin{array}{ll} \text{(ii)} & (xy)^3 \div (2x)^2 \\ \text{(iv)} & 2(2ab^2)^3 \end{array}$$

(4)

4. Tane invests \$1500 at 12% per annum, but the interest is calculated on a monthly basis and added to his investment.

- (i) What percentage interest does he earn per month?
- (ii) Copy and complete the table below to show how his investment grows over a year.

No of months (t)	0	1	2	3	4	5	6	7	8	9	10	11	12
Value of investment (V)	\$1500	\$1515											

- (iii) The table shows a growth function. Find a formula for V in terms of t
- (iv) What would the investment be worth if it is left untouched for 2 years under the same conditions?
- (v) If the interest had been calculated yearly instead of monthly, what would the investment be worth after 2 years?

(6)

5. Simplify

$$\begin{array}{ll} \text{(i)} & \frac{x^2 - 7x}{7x} \\ \text{(ii)} & \frac{x^2 + 5x + 6}{x^2 - 4} \end{array}$$

(3)

6. Simplify

$$\begin{array}{ll} \text{(i)} & (4\sqrt{x})^2 \\ \text{(iii)} & \sqrt{a^3} \times \sqrt{a^5} \\ \text{(ii)} & \sqrt{100p^{100}q^4} \\ \text{(iv)} & \frac{(x\sqrt{3})^2}{9x^2} \end{array}$$

(4)

## Appendix I:

### One test for the Wellesley mathematics course (MATHS 091) in 1996: Test 3.

#### WELLESLEY PROGRAM

MATHS 091

SEMESTER TEST 3

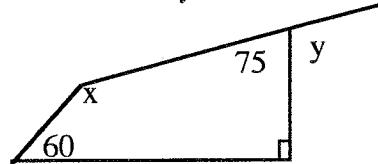
10 SEPTEMBER 96

Time allowed : 1 hour

There are **three** questions. Answer all sections. Set out your work neatly and show all working.

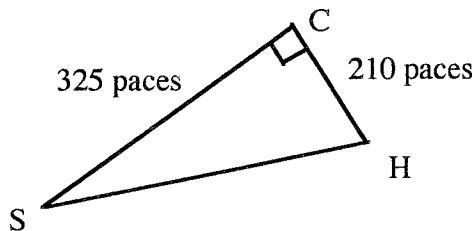
#### Question 1 [17 marks]

- (a) (i) Calculate the sizes of the angles marked  $x$  and  $y$ .



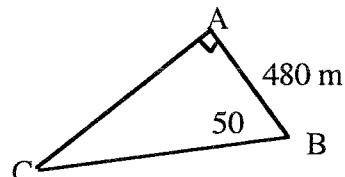
(2)

- (ii) Michael takes a short-cut from home H to school S along the path HS. He knows HC is 210 paces and he once counted CS to be 325 paces. How many paces would it be from H to S and how much shorter is Michael's short-cut?



(3)

- (b) A triangular paddock is to be sown with wheat and the farmer needs to know how much wheat seed to buy. The farmer makes a rough sketch: he measures the length AB and the angle at B.



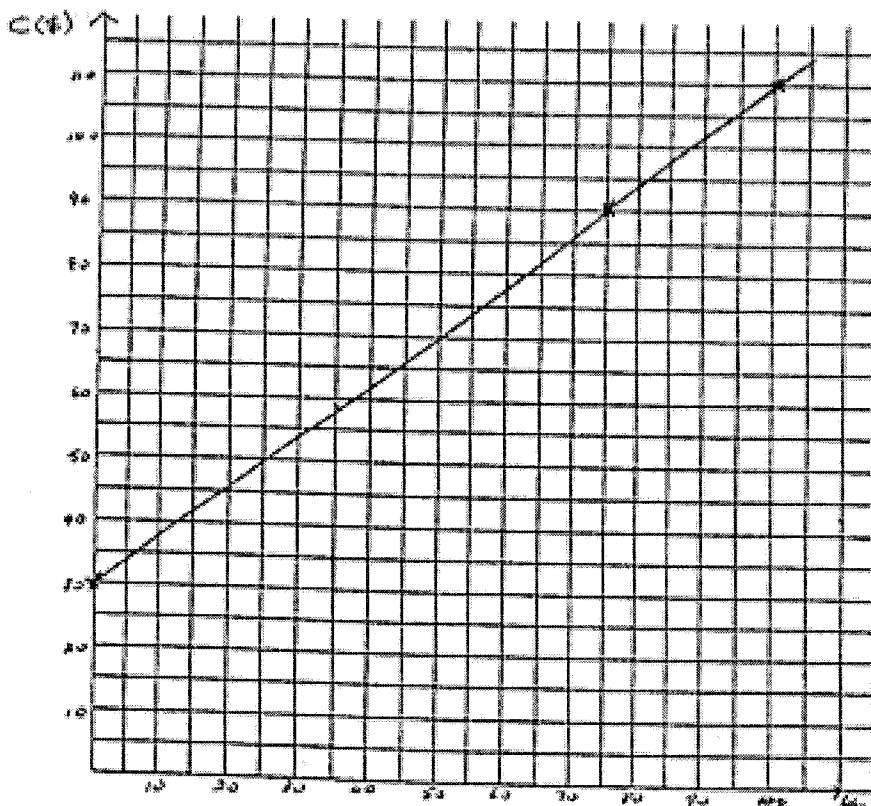
- (i) Find the length of the side AC giving your answer to a sensible degree of accuracy.
- (ii) Find the area of the paddock in  $\text{m}^2$  and also in hectares.

(4)

- (c) From the top of a 90 m cliff, an observer sees a boat at an angle of depression of  $17^\circ$ .
- Draw a small sketch.
  - How far is the boat away from the bottom of the cliff?
- (4)
- (d) A plane flies 7 km after take-off to rise to a height of 3 km. Calculate the angle at which it climbs.
- (3)

**Question 2 [16 marks]**

- (a) The graph represents the cost  $C$  in dollars of a monthly computer bill. The bill is made up of a fixed line charge and a cost per unit ( $u$ ) of data moved.



- Using the graph, estimate the bill for a household which uses 65 units of data.
- If a household bill was \$50, estimate how many units were used.
- What is the fixed line charge?
- What is the charge per unit of data ?
- Write the equation of the straight line graph.
- The graph represents a function which takes units of data as input and produces the cost of a monthly computer bill as output.

Describe in words what the inverse function does.

(6)

- (b) (i) Graph the line A:  $y = 2x - 4$   
 On the same axes, graph the line B:  $3y + 2x = 12$   
 (ii) What are the co-ordinates of the point of intersection?

(5)

- (c) A is the point (4, 9) and B is the point (-2, -3)  
 Find (i) the mid-point of AB  
 (ii) the length of AB  
 (iii) the equation of AB

(5)

### Question 3 [17 marks]

(a)  $f(x) = 3x + 6$  and  $g(x) = 2^x$

Find (i)  $f(-1)$  (ii)  $g(0)$  (iii)  $f^{-1}(x)$

(4)

- (b) An anthropologist can use certain linear functions to estimate the height of a male or female given the length of certain bones.  
 A humerus is the bone from the elbow to the shoulder.  
 Let  $x$  = the length of the humerus in centimetres.  
 Then the height in centimetres of a male with a humerus of length  $x$  is given by

$$M(x) = 2.89x + 70.64$$

and the height in centimetres of a female with a humerus of length  $x$  is given by

$$F(x) = 2.75x + 71.48$$

A humerus 45 cm long was uncovered in a ruins.

- (i) If we assume it was from a male, how tall was he?  
 (ii) If we assume it was from a female, how tall was she  
 (iii) What length would the humerus be if the male and female are the same height?

(5)

- (c) Simplify :

$$(i) \frac{16ab^3}{8a^4b^2}$$

$$(ii) \frac{x^2 + x}{x}$$

$$(iii) (3\sqrt{2x})^2$$

(6)

- (d) The rate of inflation is predicted to be 2% per year. If a weekly wage is \$500 at the present time, what would it be in 5 years time if it was adjusted for the predicted rate of inflation ?

(2)



## **Appendix J:**

# The final examination for the compulsory Wellesley mathematics course (MATHS 091) in 1996.

# THE UNIVERSITY OF AUCKLAND

## Mathematics 091

# Wellesley Programme 1996

(Time allowed : THREE hours)

**There are 9 questions each worth 15 marks.**

**Attempt all questions. SHOW YOUR WORKING.**

## Question 1

- (a) State whether each of the following is True or False :

$$(i) \quad \sqrt{5} < 2.24$$

$$(ii) \quad -\frac{7}{3} > -1.5$$

(2)

- (b) Evaluate :

$$(i) \quad \sqrt{7\frac{1}{9}}$$

$$(ii) \quad 16 \times 2 \div \frac{1}{2} + 2(4 + 3 \times 7 - 2^2)$$

(iii)  $(3.0635)^4$  correct to 2 decimal places.

$$(iv) \quad 4 \times 10^3 + 6 \times 10^2 + 7 \times 10^{-1} + 3 \times 10^{-2}$$

(4)

- (c) (i) A TV set was bought for \$550, and sold to a secondhand dealer two years later at a loss of 60%. What price did the dealer pay for the TV ?

- (ii) The dealer re-sold the same TV set for \$345. 00. What percentage profit did he make on the deal ?

(4)

- (d) (i) 4 pumps will empty a school swimming pool in 15 hours. Assuming the pumps work at the same rate, if 1 pump is out of order, how long will it take the other 3 to empty the pool ?

- (ii) A tea blender mixes three kinds of tea costing respectively \$40, \$35, and \$30 per kilogram, in the ratio 1 : 2 : 5. Find the cost of 10kg of the blended tea.

(5)

## Question 2

- (a) (i) Express 0.375 as a fraction in its lowest terms.

(ii) Write 57 659 in standard form correct to 2 significant figures.

(iii) Jay scored 40% in a test marked out of a possible total score of 30.  
What was Jay's score out of 30?

(iv) The energy supplied to a house is  $\frac{2}{5}$  from electricity,  $\frac{1}{3}$  from gas, and the rest from solar power.  
What fraction of the energy supplied is solar power?

(4)

- (b) Given that  $a = 2$ ,  $b = -3$ ,  $c = 1$ ,  $d = 5$ , evaluate :

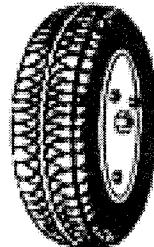
(i)  $ab^2$

$$(ii) \quad \frac{3a - 2b}{3(c - d)}$$

(2)

- (c) A trolley wheel is bolted to its axle by three small nuts of mass  $x$  grams and one large central nut of mass  $y$  grams. A pack contains just enough nuts to bolt on the 4 wheels of a trolley

- (i) Write a formula for the total mass  $M$  of the nuts in a pack.



- (ii) If a pack has a total mass of  $1.1\text{ kg}$ , calculate the value of  $y$  when  $x = 60\text{g}$  (assume the packaging has negligible mass)

(3)

- (d) Simplify :

$$(i) \quad 7(3x - 4) - 5(3 + 4x)$$

$$(ii) \quad \frac{x^2 - xy}{x - y}$$

$$(iii) \quad x\sqrt{4x^{16}}$$

$$(iv) \quad \frac{x^2 - 2x - 3}{x^2 - 4x + 3}$$

(6)

### Question 3

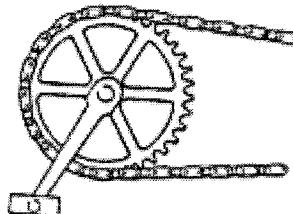
- (a) (i) A piece of paper is  $0.0095\text{cm}$  thick. Express this measurement in  $\text{mm}$ .  
(ii) If 600 sheets of paper are stacked together in a package, what is the height of a package ?  
(iii) How many packages would fit between shelves that are 1 metre apart ?

(3)

- (b) A map has a scale of  $1 : 50\,000$   
(i) How many kilometres are represented by  $1\text{cm}$  on the map ?  
(ii) A forestry plantation has the shape of a square with sides  $3\text{cm}$  long.  
How many hectares does the plantation cover ?

(3)

- (c) The radius of the chain wheel of a bicycle is  $10.4\text{cm}$



- (i) Find the circumference of the sprocketted chain wheel.

The length of the chain in contact with the the wheel is  $38.1\text{cm}$

- (ii) What percentage of the wheel's circumference is in contact with the chain ?  
(iii) What is the angle, to the nearest degree, that this arc forms at the centre of the wheel ?

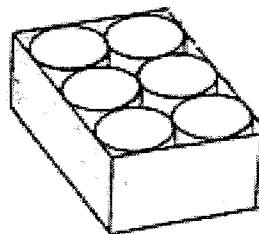
(3)

- (d) A cylindrical jar contains  $462\text{g}$  of marmalade.

- (i) If the mass of  $1\text{cm}^3$  of marmalade is  $1.6\text{g}$ , find the volume of marmalade in the jar.  
(ii) If the same jar is full and its radius is  $3.5\text{cm}$ , find the height of the jar.

The jars are packed in cartons. Each carton contains 12 jars, in two layers of 6, directly on top of each other.

- (iii) Find the dimensions of a carton.



(6)

**Question 4**

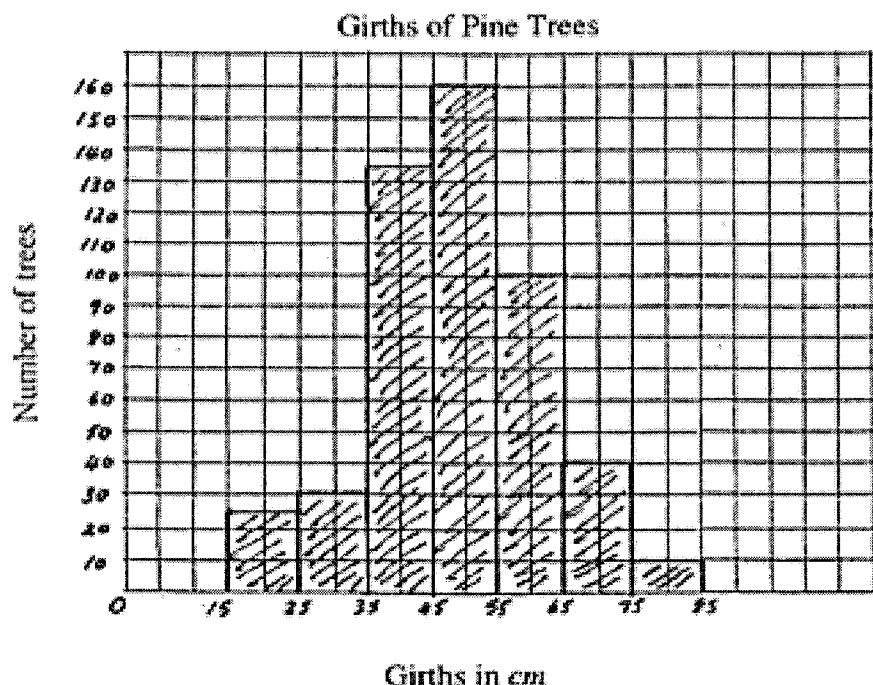
- (a) Statisticians use the words *mean*, *median*, and *mode* to describe "average"

Which of these 3 words is being referred to in the sentences below ?

- (i) The average family has 2.3 children.  
(ii) The average man likes his beer cold.

(2)

- (b) In estimating the value of a plantation of pine trees, the girths of the trees in a sample area were measured and the results displayed in a histogram seen below.



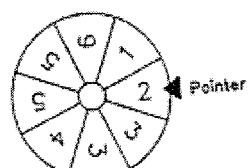
- (i) Is the data continuous or discrete?  
(ii) How many trees were there in the sample area ?  
(iii) Find the modal girth  
(iv) Find an estimate of the mean girth.

(6)

- (c) The disc shown in the diagram is spun until it comes to rest with one of the numbers next to the pointer.

Find the probability that the number obtained is

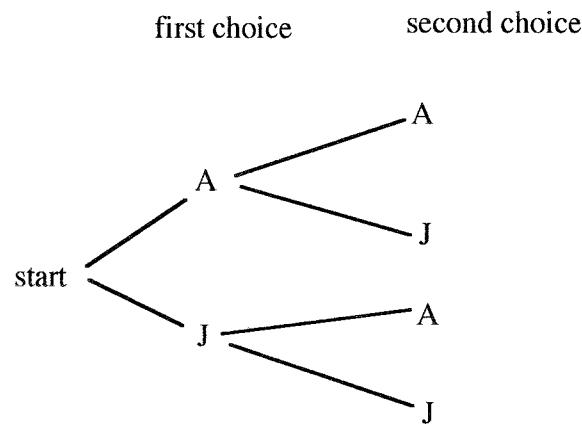
- (i) 3  
(ii) odd  
(iii) a factor of 6



(3)

**Question 4 is continued on page 5**

- (d) A touring mini-bus has 10 passengers, 3 Australian and 7 Japanese.  
On the first day of the tour, the passengers draw lots to choose two people to sit in the front seats beside the driver.
- (i) Copy the probability tree below to show the possible outcomes of the draw. Complete the diagram probabilities .



- (ii) Find the probability that both passengers chosen are Australian.  
(iii) Find the probability that at least one of the passengers chosen is Australian.

(4)

**Question 5**

(a) Expand :

(i)  $3x(x - 9)$

(ii)  $(x + 4)(x - 7)$

(iii)  $(2x + 5)^2$

(3)

(b) Solve for  $x$  :

(i)  $7x + 2 = 2(5x - 8)$

(ii)  $\frac{5x}{3} + 3 = 2$

(iii)  $\frac{3(x - 5)}{4} = \frac{2(x - 4)}{3}$

(4)

(c) At a fast food restaurant, a chicken-burger costs 60 cents more than a hamburger. When 4 hamburgers and 3 chicken-burgers were ordered, the change from a \$10 note was \$1.90. How much does each type of burger cost ?

(3)

(d) Metal strips are placed along each edge of this box. The total length  $L\text{cm}$  of the edging is given by the formula  $L = 4(2x + 3)$ (i) Re-arrange the formula to make  $x$  the subject(ii) If the maximum length of edging available is 60cm, solve the inequality below to find the possible values for  $x$  :

$$4(2x + 3) \leq 60$$

(iii) Graph the possible values for  $x$  on the Real Number line.

(5)

**Question 6**

(a)  $f(x) = \frac{4^x}{x - 4}$

- Find: (i)  $f(3)$   
(ii)  $f(0)$   
(iii) the value of  $x$  for which  $f(x)$  is undefined (3)

(b) The value of a company car is depreciated by 15% per year.

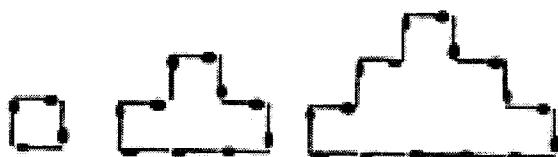
- (i) If the company paid \$37 500 for the car, copy and complete the table below showing how the value drops over 4 years.

Number of years after purchase (t)	0	1	2	3	4
Value in dollars (V)	37 500				

- (ii) If  $V$  is the value of the car after  $t$  years, write a formula for  $V$  in terms of  $t$ .  
(iii) The car is sold after 7 years. What is it valued at then ?  
(iv) A novice mathematician says: "After 7 years depreciation at 15% per year, the car has lost 105% of its value because  $7 \times 15\% = 105\%$  . It would be worth less than nothing. You would have to pay someone \$1875 to take it away."  
As an experienced mathematician, write your reply.

(6)

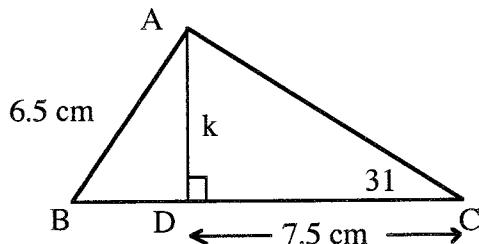
(c) The following patterns are made by arranging matches:



- (i) Draw the next pattern in the sequence.  
(ii) The numbers of matches needed to make each pattern form a sequence.  
Write the first 4 terms of the sequence.  
(iii) Find a formula for  $T(n)$ , the number of matches in the  $n$ th pattern.  
(iv) How many matches would be needed to make the 10th pattern ?  
(v) If 200 matches were available to make the biggest possible pattern in the sequence, would they all be used up, or would there be some left over ? If so, how many ? (6)

**Question 7**

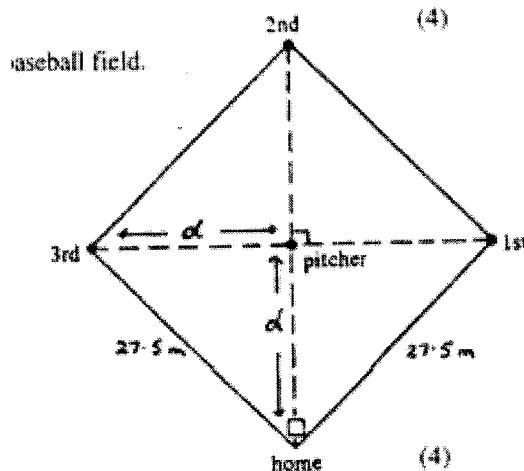
- (a) Use the diagram to find  
 (i)  $k$       (ii)  $\theta$



(4)

- (b) The diagram shows the square shape of a baseball field.

- (i) Calculate  $d$ , the distance from the pitcher's mound to home plate.



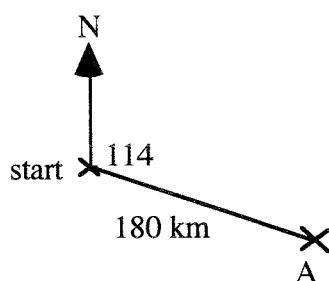
(4)

- (ii) The pitcher gets the ball to reach the home plate in 0.75 seconds. Find the speed of the ball in kilometres per hour.

(4)

- (c) A plane travels on a bearing of  $114^\circ$  for 180 km to the point A, and then changes direction to a bearing of  $85^\circ$ , flying at 600 km per hour for 12 minutes to reach point B.

- (i) Copy and complete the sketch to illustrate the plane's flight path.



- (ii) How far South of its starting point is the plane at point A?  
 (iii) How far North of A is the plane by the time it reaches B?  
 (iv) How far South of its starting point is the plane at point B?

(7)

**Question 8**

- (a) A length  $L$  cm of a spring is increased by adding masses of  $M$  kg to one end so that  $L = 20 + \frac{1}{5} M$

- (i) Copy and complete this table.

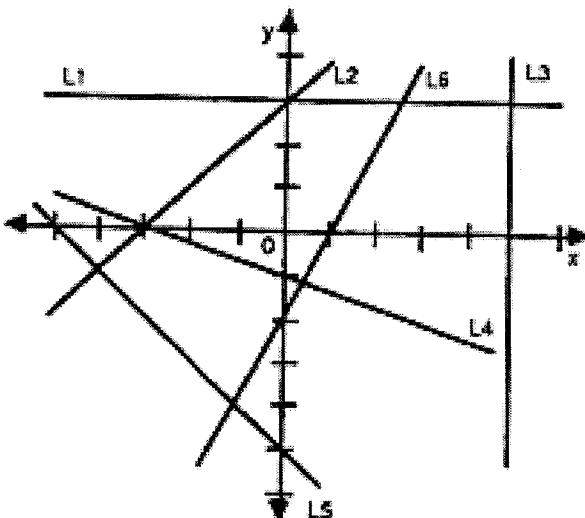
Mass, $M$ (kg)	0	10	20	30	40	50
Length, $L$ (cm)						

- (ii) Draw the graph of  $L$  against  $M$ .  
 (iii) What length is the spring when a mass of 35 kg is hanging on the end ?  
 (iv) What mass is needed for the spring to be 23 cm ?  
 (v) What is the unstretched length of the spring ?  
 (vi)  $f(x) = 20 + \frac{1}{5} x$  is a function which takes a mass  $x$  as input and produces the length of the spring as output. Find  $f^{-1}(x)$  and describe in words what  $f^{-1}(x)$  takes as input, and produces as output.

(7)

- (b) Match the gradients below to four of the lines shown in the diagram, and find the equation of line number 4. (L 4)

- (i) 0                      (ii) -1                      (iii) 2                      (iv) 1



(3)

- (c) A(1,-4), B(-3,-5), C(0,1) are three points on a graph.

- Find (i) the length of the line AB.  
 (ii) the gradient of the line joining A to C  
 (iii) the equation of the line parallel to AC which passes through B.

(5)



## Appendix K:

### Numbers of students in the Mathematics 1 and Wellesley courses and numbers who completed the questionnaires

The following two tables list, for Mathematics 1 and for the Wellesley Mathematics Course respectively, the numbers of students enrolled in the class, numbers who completed the course, numbers who answered the questionnaires in March and October and, of these, the numbers who did not complete the course (DNC) or who gave no student id. Numbers who answered both March and October questionnaires are also listed. This data is given for both 1995 and 1996.

Table 1: Numbers in each group for Mathematics 1.

Year	Students who Answered the March Questionnaire			Students who Answered October Questionnaire			Both Mar & Oct n	Students who completed course (October) n	
	Students enrolled in course (March)	Enrolled	DNC or no id	Total	Enrolled	DNC or no id	Total		
		n	n	n	n	n	n		
1995	84	59	9	50	44	2	42	37	63
1996	76	46	24	22	38	2	36	22	47

Table 2: Numbers in each group for the Wellesley Mathematics Course.

Year	Students who Answered the March Questionnaire			Students who Answered October Questionnaire			Both Mar & Oct n	Students who completed course (October) n	
	Students enrolled in course (March)	Enrolled	DNC or no id	Total	Enrolled	DNC or no id	Total		
		n	n	n	n	n	n		
1995	88	61	15	46	38	4	34	30	58
1996	87	62	13	47	44	8	36	34	51



## **Appendix L:**

### **The demographic profile of Mathematics 1 students who answered questionnaire in March 1996**

Listed in the table below are the numbers of students and percentages in categories of demographic variables for Mathematics 1 students who completed the questionnaire in March 1996. This data was not included in the main data analysis (**Section 5.1.1**) because, although 46 students completed the questionnaire at this time, 22 students withdrew from the course and 2 gave no id, leaving too small a group to compare effectively with the other groups who answered the questionnaire. The data is listed here for completeness.

Variable	Categories	Mar	1996 (n=22)
		n	%
Degree	BSc	6	27
	BA	15	68
	Other	1	5
Gender	Female	13	59
	Male	9	41
Ethnicity	Maori	1	5
	Pakeha	11	50
	PI	1	5
	Asian	8	36
	Others	1	5
Age	≤ 24 years	8	36
Groups	≥ 25 years	14	64
First	English	14	64
Language	Maori	0	0
	PI	0	0
	Chinese	5	23
	Other	3	14
Fluency in English	Fluent	14	64
	Not Fluent	8	36
Maths	≤ Form 5	14	64
Background	≥ Form 6	8	36



## **Appendix M:**

### **The report “Wellesley Program 1989 to 1998: The First Ten Years” (part only)**

#### **Students who completed the Wellesley Program 1989-1998**

<b>Year</b>	<b>Number Completing</b>	<b>Gender</b>		<b>Ethnicity</b>	
1989	33	23	Male	24	Pakeha
			Female	6	Maori
1990	66	37	Male	40	Pakeha
			Female	8	Maori
1991	72	35	Male	47	Pakeha
			Female	7	Maori
1992	66	34	Male	28	Pakeha
			Female	13	Maori
1993	58	31	Male	32	Pakeha
			Female	13	Maori
1994	65	30	Male	37	Pakeha
			Female	14	Maori
1995	59	39	Male	32	Pakeha
			Female	18	Maori
1996	53	25	Male	22	Pakeha
			Female	11	Maori
1997	52	24	Male	22	Pakeha
			Female	10	Maori

Pacific Island Other

9 11

1998	57	31	Male	28	Pakeha
		26	Female	8	Maori
				12	Pacific Island
				9	Other

### Comment

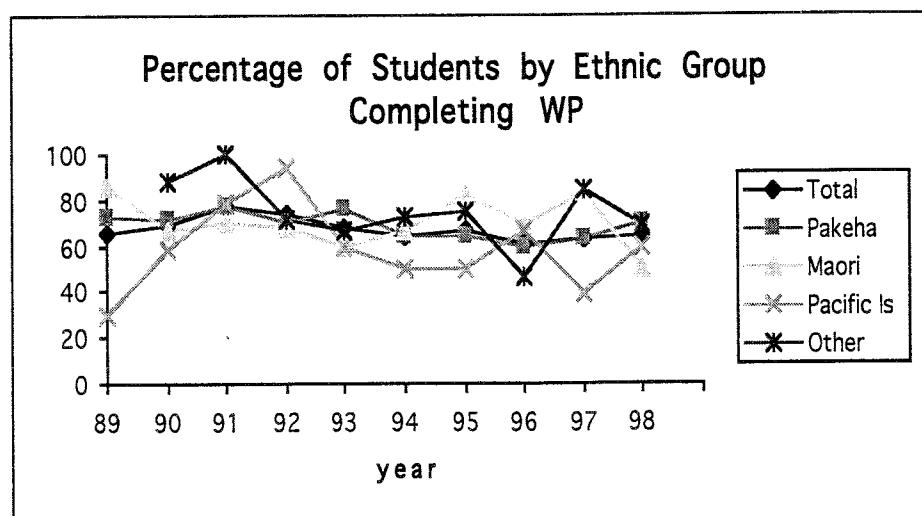
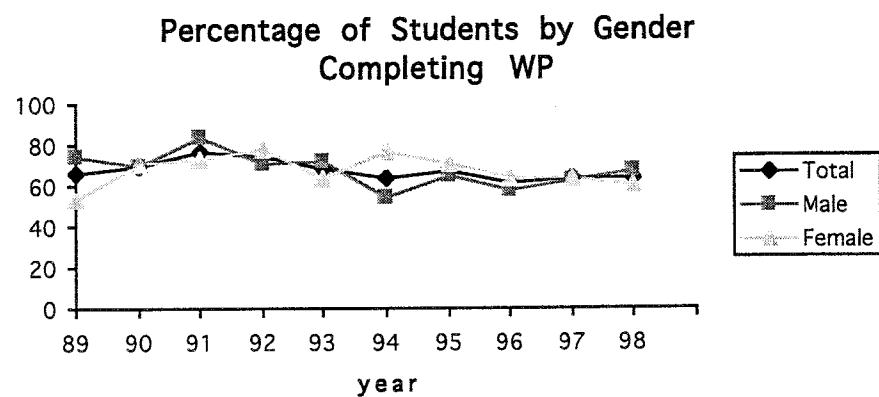
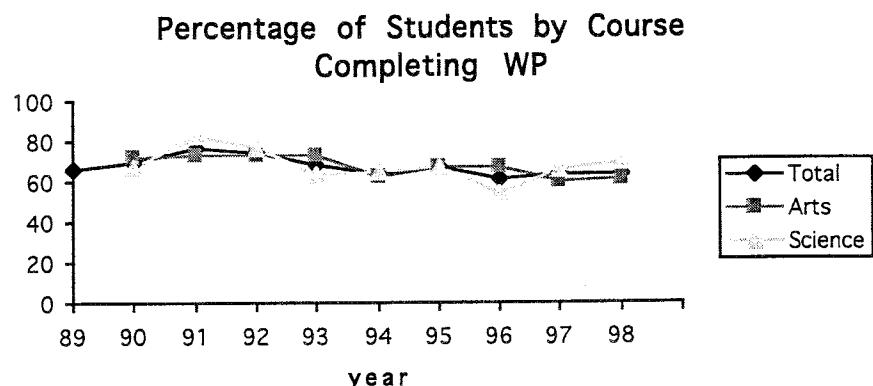
Students in the Wellesley Program are by their nature very vulnerable to the effects of outside pressures.

- As adults it is not easy for them to place themselves back into a position where their work is open to scrutiny and judgement.
- Having been unsuccessful in an educational environment in the past, they are readily overwhelmed by the university system.
- Financial difficulties are often great: suitable part-time work is hard to find, many are ineligible for a student allowance, and many are unwilling to commit themselves to a loan until they discover whether they are suited to tertiary study. Those who are eligible for allowances often have difficulty with the system, and may wait a long period without income.
- There are problems caused by unrealistic expectations for some who are undertaking second-chance education for the benefit of their families.

For all of these reasons, Wellesley Program students drop out and do not complete the course. However, many overcome enormous difficulties, not only financial but also personal, and succeed in their goal of completing a year's preparation for tertiary study.

- ◆ **The Wellesley Program retains 67.5% of students.**

The graphs which follow illustrate the percentage of students who completed the Wellesley Program in each of the categories of interest over the first ten years.



## **Analysis:**

In the first ten years, 861 students have enrolled in the Wellesley Program, and 581 completed it.

### **(i) Gender**

	Male	Female
Enrolment	$\frac{462}{861} = 54\%$	$\frac{399}{861} = 46\%$
Completion	$\frac{309}{581} = 53\%$	$\frac{272}{581} = 47\%$

### **(ii) Ethnic Group**

	Pakeha	Maori	Pacific Island	Other
Enrolment	$\frac{452}{861} = 52.5\%$	$\frac{157}{861} = 18.2\%$	$\frac{166}{861} = 19.3\%$	$\frac{86}{861} = 10\%$
Completion	$\frac{312}{581} = 53.7\%$	$\frac{108}{581} = 18.6\%$	$\frac{99}{581} = 17\%$	$\frac{62}{581} = 10.7\%$

### **(iii) Success Rate**

For students who complete the course, success rates can be calculated on the basis of gender and ethnic group. Success here is the completion of a year's study and the sitting all four examinations, for which the Tertiary Education Foundation Certificate is awarded. Grades earned are an indication of subject strengths but have no bearing on a student's "successful" completion of the course.

The success rate for males:  $\frac{309}{462}$  67%

The success rate for females:  $\frac{272}{399}$  68%

The success rate for Pakeha:  $\frac{312}{452}$  69%

The success rate for Maori is  $\frac{108}{157}$  69%

The success rate for Pacific Islanders:  $\frac{99}{166}$  60%

The success rate for Others:  $\frac{62}{86}$  72%

### **(iv) Target Groups**

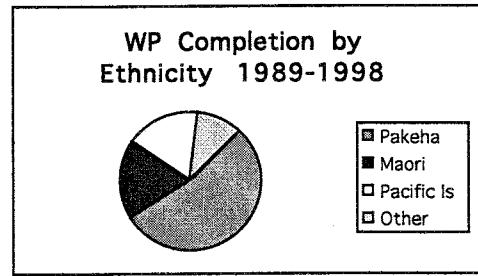
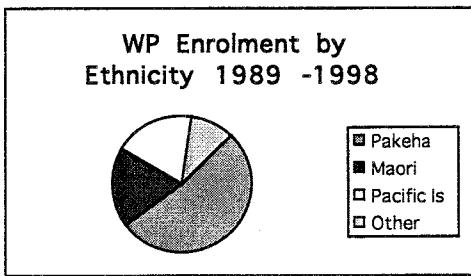
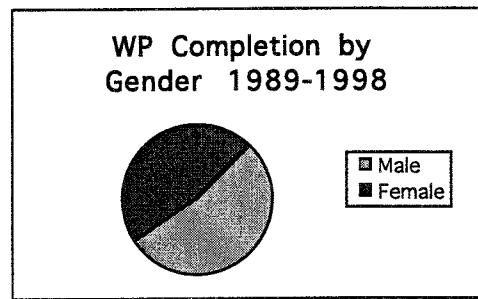
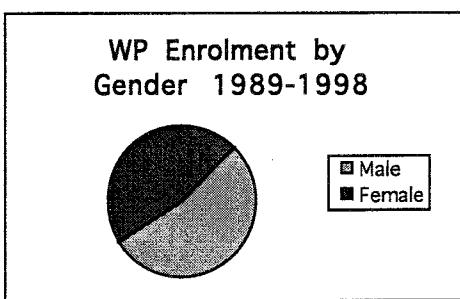
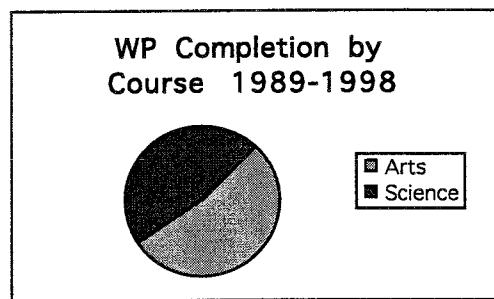
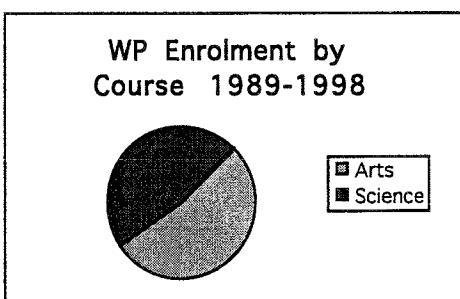
Of the students who complete the Wellesley Program : 21% are Women in Science, 18.6% are Maori, 17% are Pacific Island.

## Conclusions

The data shows that each course, each gender group and each ethnic group, with the possible exception of Pacific Island students, has a reasonably similar success rate.

Students in each group are successfully completing the program in roughly the same proportions as those in which they enrolled.

The Wellesley Program works well for all students regardless of gender or ethnic group. The tutors are conscious of a slightly higher than average drop-out rate for Pacific Island students.





## **Appendix N:**

### **The scales used in the final questionnaires.**

The scales used as a large part of the questionnaire in 1995 and 1996 are listed below.

#### **The Aiken (1974) Mathematics Attitude Scale**

##### **Enjoyment of Mathematics dimension**

- 1 I enjoy going beyond the assigned work and trying to solve the new problems in mathematics.
- 2 Mathematics is enjoyable and stimulating to me.
- 3 I am interested and willing to use mathematics outside my classes and on the job.
- 4 I have always enjoyed studying mathematics.
- 5 I would like to develop my mathematical skills and study this subject more.
- 6 I am interested and willing to acquire further knowledge of mathematics.
- 7 Mathematics is very interesting, and I have usually enjoyed classes in this subject.
- 8 I have never liked mathematics, and it is my most dreaded subject.
- 9 Mathematics makes me feel uneasy and confused.
- 10 Mathematics is dull and boring because it leaves no room for personal opinion.
- 11 Mathematics makes me feel uncomfortable and nervous.

##### **Value of Mathematics dimension**

- 1 Mathematics has contributed greatly to science and other fields of knowledge.
- 2 Mathematics is not important for the advancement of civilization and society.
- 3 Mathematics is a very worthwhile and necessary subject.
- 4 An understanding of mathematics is needed by artists and writers as well as scientists.
- 5 Mathematics helps develop people's minds and teaches them to think.
- 6 Mathematics is needed in designing practically everything.

- 7 Mathematics is needed in order to keep the world running.
- 8 There is nothing creative about mathematics; it's just memorising formulas and things.
- 9 Mathematics is not important in everyday life.
- 10 Mathematics is less important to people than art or literature.

### **Mathematical Self-Concept Scale (Gourgey 1982): a subset**

- 1 It takes me much longer to understand mathematical concepts than the average person.
- 2 If I can understand a maths problem, then it must be an easy one.
- 3 I don't ask questions in maths classes because mine sound so stupid.
- 4 I have never been able to think mathematically.
- 5 I don't have a good enough memory to learn maths.
- 6 Whenever I do a maths problem, I am sure that I have made a mistake.
- 7 I have never felt myself incapable of learning maths.
- 8 I have a good mind for maths.
- 9 I can understand maths better than most people.
- 10 I have no more trouble understanding maths than any other subject.
- 11 When I have difficulties with maths, I know I can handle them if I try.
- 12 When I do maths, I feel confident that I have done it correctly.

### **Beliefs about the Learning of Mathematics (Schoenfeld 1989)**

- 1 Everything important about maths is already known by mathematicians.
- 2 In maths you can be creative and discover things by yourself.
- 3 Maths problems can be done correctly in only one way.
- 4 Real maths problems can be solved by commonsense instead of the maths rules you learn at school.
- 5 To solve maths problems you have to be taught the right procedure, or you cannot do anything
- 6 The best way to do well in maths is to memorise all the formulas.

**Conceptions of Mathematics Scale (Crawford, Gordon, Nicholas & Prosser 1993)**

- 1 For me, mathematics is the study of numbers.
- 2 Mathematics is a lot of rules and equations.
- 3 By using mathematics we can generate new knowledge.
- 4 Mathematics is simply an over complication of addition and subtraction
- 5 Mathematics is about calculations.
- 6 Mathematics is a set of logical systems which have been developed to explain the world and relationships in it.
- 7 What mathematics is about is finding answers through the use of numbers and formulae.
- 8 I think mathematics provides an insight into the complexities of our reality.
- 9 Mathematics is figuring out problems involving numbers.
- 10 Mathematics is a theoretical framework describing reality with the aim of helping us understand the world.
- 11 Mathematics is like a universal language which allows people to communicate and understand the universe.
- 12 The subject of mathematics deals with numbers, figures and formulae.
- 13 Mathematics is about playing around with numbers and working out numerical problems.
- 14 Mathematics uses logical structures to solve and explain real life problems.
- 15 What mathematics is about is formulae and applying them to everyday life and situations.
- 16 Mathematics is a subject where you manipulate numbers to solve problems.
- 17 Mathematics is a logical system which helps explain the things around us.
- 18 Mathematics is the study of the number system and solving numerical problems
- 19 Mathematics is models which have been devised over years to help explain, answer and investigate matters in the world



## **Appendix O:**

### **Semi-structured Interview Schedule for Student Interviews**

Examples of typical questions follow in each section below.

#### **Maths autobiography**

Attitudes of parents:

What was the attitude of your parents to school, to education, to maths?

What was their maths background and confidence doing maths?

What were their expectations of your ability, their encouragement or not?

Attitude of teachers:

Do you remember the attitude of your maths teachers to you and others?

Attitude of partner / spouse (if appropriate):

What is the attitude of your partner / spouse to maths, to your study?

Schooling history – Mathematics history:

When did you leave school?

Have you done any tertiary/vocational training, work (paid or unpaid), since school?

What is the length of time since you have done maths?

Why are you at university? Why are you studying maths now (ie level of motivation)?

What other subjects are you studying / doing well in / like?

What is the extent of your other commitments - work, family?

What was the highest level of maths you have studied? What about your achievement? Did you enjoy learning maths? What do you think about your ability in maths? Focus responses: one bad experience, and one good experience?

What was the type of classroom organization in maths teaching in past eg working individually, working in groups, ... What was a common classroom experience in maths for you? Give one bad experience, and one good experience?

Beliefs about the learning of maths, for example, ask them to talk about the Schoenfeld (1989) statements?

### **Mathematics 1**

What were your expectations of this course?

Do you think you have successfully studied this course? How have you managed the assessment tasks? How much did you use the help available?

Have your ideas about maths changed since you started this course? Are the changes big or small? What do you attribute these shifts too?

What is the most important thing that has been learned about maths in this course?

Look at their answers to open questions in the questionnaire to check if any need any clarification.

What do you feel about your ability to learn maths now? Has this changed since you started this course? How much of a shift?

Do you enjoy studying maths in this course? Did you enjoy studying maths before this experience? How much of a shift?

What do you think about the value of maths - in society & for you personally? Has this changed since you started this course? How much of a shift?

### **Metaphors**

If your experience in this maths course was weather, what would it be like?  
Repeat for food ..., a kitchen utensil ..., a tool ...

If you were to compare your different experiences in maths, this year's compared to your previous experience (in high school), would a different weather come to mind? Repeat for food ..., a kitchen utensil ..., a tool ...

(Other metaphor questions could be chosen from the Mathematics Metaphor Questionnaire (Appendix Z) as required.)

## **Appendix P:**

### **The questionnaire used in the main study in October 1995 for students in Mathematics 1.**

The questionnaire used in the main study in October 1995 for students in Mathematics 1 is included in full below, preceded by the information and consent form. This questionnaire is followed by parts of the October 1995 questionnaire given to students in the Wellesley Program which differ from the above, namely, one demographic variable and the open questions.

*(This first page was on University of Auckland letterhead)*

#### **Information Sheet for Students in Mathematics 1**

##### **Re: Participation in a Research Project**

**Researcher: Barbara Miller-Reilly**                   October 1995

I invite you to participate in a research project which I am conducting to explore the reactions of students to mathematics and the teaching of mathematics. This research project is for my PhD study (supervisors: Dr Brown & Dr Morton). I am interested in finding out the characteristics of students who find the way maths is presented in this course most effective.

If you agree to participate, the type of information of interest to me will be your achievements in and attitudes about mathematics as well as your learning style, gender, first language, ethnicity and age. To protect your anonymity no identifiable information will be given to anyone associated with the grading of your work.

At some stage during this year I will choose a few students who I would invite to participate in an interview, if they are willing to be involved in this additional part of my research project.

If you have any ethical concerns you may contact Dr N. Dawson, Chair, University of Auckland Human Subjects Ethics Committee, telephone 373 7599 extn 6204. I am also available in my office (Room 326, Mathematics/Physics Building) or can be contacted by phone (373 7599 Extn 8790 or 8967).

**THANK YOU**

**I PLAN TO WRITE A REPORT ON THIS PROJECT. IF YOU WOULD LIKE A COPY SENT TO YOU PLEASE WRITE YOUR NAME AND ADDRESS HERE.**

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## **CONSENT TO PARTICIPATION IN RESEARCH/TEACHING CLASSES**

**Title of project:** Students' beliefs about mathematics and reactions to Mathematics1.

**Researcher:** Barbara Miller-Reilly

I have been given and have understood an explanation of this research project. I have had an opportunity to ask questions and have them answered. I understand that I may withdraw myself or any information I have provided from this project (before data collection is completed), without having to give reasons.

I agree to take part in this research.

Signed:

NAME:  
(please print clearly)

Student ID Number:

Date:

APPROVED BY THE UNIVERSITY OF AUCKLAND HUMAN SUBJECTS ETHICS COMMITTEE  
on 9 March 1994 for a period of 2 years, from 9/3/1994 Reference:1994/010

**PLEASE LEAVE THESE PAGES STAPLED TOGETHER**

Code:

(Office use only)

Name:

ID number:

## OPEN QUESTIONS

*Instructions: Answer each of the following questions in a sentence or two. Write your answers in the space below each question.*

1. Explain, with reasons, whether you **liked or disliked** Mathematics 1?

---

---

---

2. What have you **liked best** about this paper?

---

---

---

3. What have you **liked least** about this paper?

---

---

---

4. Could you suggest ways that this paper (26.100) could be improved.

---

---

---

5. What topics did you like **best**? \_\_\_\_\_

6. What topics did you like **least**? \_\_\_\_\_

7. Are there **other topics** you would like to have learned about?

---

---

---

Name:  
ID number:

**8. Explain why you liked or disliked the assigning of marks for participation in tutorial tasks?**

---

---

---

**9. What did you think of the tasks which were given out in tutorials?**

---

---

---

**10. What are your reactions to working with a group on the task in the tutorials?**

---

---

---

**11. What do you think about being able to redo term tests?**

---

---

---

**12. What comment do you have about the style of the term test questions?**

---

---

---

**13. Did you use the printed lecture notes? YES / NO (Circle one)**

*Comment on the usefulness of these notes?* \_\_\_\_\_

Name:

ID number:

**14.** Has this paper affected your **mathematical confidence**? Explain please.

---

---

---

**15.** What do you think about the **maths you have done** in this paper?

---

---

---

**16.** Has this paper affected your **ability to investigate mathematical situations**? If so, in what way?

---

---

---

**17.** What did you think of the **course's approach**?

---

---

---

**18.** Are you studying 26.101 this year? **YES / NO** (Circle one)

Do you intend to continue studying maths next year? \_\_\_\_\_

Name: \_\_\_\_\_  
ID number: \_\_\_\_\_

## STUDENT PROFILE

**Instructions:** Circle the number in front of your answer.

- 1. My age is:**

  1. 17-19
  2. 20-24
  3. 25-29
  4. 30-39
  5. 40+

**2. I am a .....**

  1. Female
  2. Male

**3. Maths Background:**  
*(Highest level at which maths has been studied prior to this paper)*

  1. Form 4 or less
  2. Form 5 (or up to 15 years old)
  3. Form 6 (or up to 16 years old)
  4. Form 7 (or up to 17 years old)
  5. Other \_\_\_\_\_  
(Please specify)

**4. I enjoyed / did not enjoy studying maths at that time.**  
(Please circle one)

**5. I did well / did not do well in maths at that time.**  
(Please circle one)

**6. With what ethnic group do you identify for cultural or lifestyle reasons?**

  1. Maori
  2. Pakeha/European
  3. Pacific Islander
  4. Asian
  5. Indian
  6. Other \_\_\_\_\_  
(Please specify)

**7. First language:**

  1. English
  2. Maori
  3. Pacific Island
  4. Chinese
  5. Other \_\_\_\_\_  
(Please specify)

**8. Fluency in English:**

  1. Not fluent
  2. Fairly fluent
  3. Very fluent

**9. Degree Course:**

  1. BSc
  2. BA
  3. BCom
  4. Other \_\_\_\_\_  
(Please specify)

Name:

ID number:

## MATHS ATTITUDES SURVEY

We are interested in your ideas about and attitudes to maths. Your anonymity will be protected as no identifiable information will be given to anyone associated with the grading of your work.

There are no correct answers for the following statements. Please answer every item quickly by giving your immediate response, **THINKING ABOUT THE MATHS YOU HAVE DONE THIS YEAR IN MATHEMATICS 1.**

		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1	I have never been able to think mathematically. ....	SD	D	U	A	SA
2	To solve maths problems you have to be taught the right procedure, or you cannot do anything. ....	SD	D	U	A	SA
3	Mathematics is not important for the advancement of civilization and society. ....	SD	D	U	A	SA
4	I don't have a good enough memory to learn maths. ....	SD	D	U	A	SA
5	Mathematics makes me feel uncomfortable and nervous. ....	SD	D	U	A	SA
6	I would like to develop my mathematical skills and study this subject more. ....	SD	D	U	A	SA
7	Mathematics is needed in order to keep the world running. ....	SD	D	U	A	SA
8	Mathematics makes me feel uneasy and confused. ....	SD	D	U	A	SA
9	Mathematics is not important in everyday life. ....	SD	D	U	A	SA
10	Everything important about maths is already known by mathematicians. ....	SD	D	U	A	SA
11	Whenever I do a maths problem, I am sure that I have made a mistake. ....	SD	D	U	A	SA

Name:
ID number:

		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
12	The best way to do well in maths is to memorise all the formulas.	SD	D	U	A	SA
13	Mathematics is less important to people than art or literature.	SD	D	U	A	SA
14	Mathematics is needed in designing practically everything.	SD	D	U	A	SA
15	Mathematics is enjoyable and stimulating to me.	SD	D	U	A	SA
16	If I can understand a maths problem, then it must be an easy one.	SD	D	U	A	SA
17	Maths problems can be done correctly in only one way.	SD	D	U	A	SA
18	Mathematics has contributed greatly to science and other fields of knowledge.	SD	D	U	A	SA
19	Real maths problems can be solved by commonsense instead of the maths rules you learn at school.	SD	D	U	A	SA
20	There is nothing creative about mathematics; it's just memorising formulas and things.	SD	D	U	A	SA
21	In maths you can be creative and discover things by yourself.	SD	D	U	A	SA
22	I have a good mind for maths.	SD	D	U	A	SA
23	I don't ask questions in maths classes because mine sound so stupid.	SD	D	U	A	SA
24	Mathematics is a very worthwhile and necessary subject.	SD	D	U	A	SA
25	I have no more trouble understanding maths than any other subject.	SD	D	U	A	SA

Name:

ID number:

		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree	
26	I have always enjoyed studying mathematics.	.....	SD	D	U	A	SA
27	Mathematics helps develop people's minds and teaches them to think.	.....	SD	D	U	A	SA
28	I have never felt myself incapable of learning maths.	.....	SD	D	U	A	SA
29	I have never liked mathematics, and it is my most dreaded subject.	.....	SD	D	U	A	SA
30	It takes me much longer to understand mathematical concepts than the average person.	.....	SD	D	U	A	SA
31	Mathematics is dull and boring because it leaves no room for personal opinion.	.....	SD	D	U	A	SA
32	I enjoy going beyond the assigned work and trying to solve new problems in mathematics.	.....	SD	D	U	A	SA
33	When I do maths, I feel confident that I have done it correctly.	.....	SD	D	U	A	SA
34	When I have difficulties with maths, I know I can handle them if I try.	.....	SD	D	U	A	SA
35	An understanding of mathematics is needed by artists and writers as well as scientists.	.....	SD	D	U	A	SA
36	Mathematics is very interesting, and I have usually enjoyed classes in this subject.	.....	SD	D	U	A	SA
37	I can understand maths better than most people.	.....	SD	D	U	A	SA
38	I am interested and willing to use mathematics outside my classes and on the job.	.....	SD	D	U	A	SA
39	I am interested and willing to acquire further knowledge of mathematics.	.....	SD	D	U	A	SA

**Parts of the October 1995 Questionnaire for the Wellesley Program  
Students Which Differ from the Above: A Demographic Variable and the  
Open Questions**

**STUDENT PROFILE**

*Instructions: Circle the number in front of your answer.*

- 1. My age is:**
  1. 17-19
  2. 20-24
  3. 25-29
  4. 30-39
  5. 40+
  
- 2. I am a .....**
  1. Female
  2. Male
  
- 3. Maths Background:**  
*(Highest level at which maths has been studied prior to this paper)*
  1. Form 4 or less
  2. Form 5 (or up to 15 years old)
  3. Form 6 (or up to 16 years old)
  4. Form 7 (or up to 17 years old)
  5. Other \_\_\_\_\_  
(Please specify)
  
- 4. I enjoyed / did not enjoy studying maths at that time.**  
(Please circle one)
  
- 5. I did well / did not do well in maths at that time.**  
(Please circle one)
  
- 6. With what ethnic group do you identify for cultural or lifestyle reasons?**
  1. Maori
  2. Pakeha/European
  3. Pacific Islander
  4. Asian
  5. Indian
  6. Other \_\_\_\_\_  
(Please specify)
  
- 7. First language:**
  1. English
  2. Maori
  3. Pacific Island
  4. Chinese
  5. Other \_\_\_\_\_  
(Please specify)
  
- 8. Fluency in English:**
  1. Not fluent
  2. Fairly fluent
  3. Very fluent
  
- 9. Wellesley Program Course:**
  1. Arts
  2. Science

Name:

ID number:

## OPEN QUESTIONS

**Instructions:** Answer each of the following questions in a sentence or two. Write your answers in the space below each question.

1. Explain, with reasons, whether you **liked or disliked** Wellesley Program Maths?

---

---

---

2. What have you **liked best** about this paper?

---

---

---

3. What have you **liked least** about this paper?

---

---

---

4. Could you suggest ways that Wellesley Program Maths **could be improved**.

---

---

---

5. Has the maths you have learned in this paper **been of any use in your other papers**? Explain.

---

---

Name:

ID number:

6. Has this paper affected your **mathematical confidence**? Explain please.

---

---

---

7. What do you think about the **maths** you have done in this paper?

---

---

---

8. Has this paper affected your **ability to think mathematically**? If so, in what way?

---

---

---

9. What did you think of the paper's approach?

---

---

---

10. Do you intend to continue studying maths next year? \_\_\_\_\_

## **Appendix Q:**

### **Final marks for Mathematics 1 students in March 1996**

Numbers of students and medians of final marks, in categories of the demographic variables, for Mathematics 1 students in March 1996 who answered the questionnaire and who also completed the course.

Variable	Categories	March 1996	
		n	n=22 Med
Degree	BSc	6	84
	BA	15	77
	Other	1	-
Gender	Female	13	78
	Male	9	75
Ethnicity	Maori	1	-
	Pakeha	11	79
	PI	1	-
	Asian	8	82
	Others	1	-
Age	≤ 24 years	8	80
Groups	≥ 25 years	14	77
Fluency in English	Fluent	14	78
	Not Fluent	8	82
Maths	≤ Form 5	14	78
Background	≥ Form 6	8	78
Years Since Maths Studied	< 10 years		NA
	≥ 10 years		



## Appendix R:

### Achievement of students by two-way interactions of demographic variables in Mathematics 1

Achievement of students in categories of the two-way interactions of the demographic variables gender, age, degree and fluency in English for Mathematics 1 students in October 1995 and 1996.

Categories of two-way interactions			October 1995 n=44		October 1996 n=38	
			n	Median	n	Median
Age by Fluency in English	$\leq 24$ years	Fluent	14	71	9	80
		Not Fluent	11	53	9	69
	$\geq 25$ years	Fluent	14	65	13	78
		Not Fluent	5	69	5	77
Gender by Fluency in English	Female	Fluent	13	72	14	81
		Not Fluent	14	58	7	74
	Male	Fluent	13	63	8	70
		Not Fluent	2	-	7	75
Degree by Fluency in English	BSc	Fluent	13	72	12	82
		Not Fluent	1	-	3	-
	BA	Fluent	10	64	10	78
		Not Fluent	14	61	10	71
Age by Degree	$\leq 24$ years	BSc	9	74	7	82
		BA	14	60	10	67
	$\geq 25$ years	BSc	6	60	6	89
		BA	10	70	10	77
Gender by Degree	Female	BSc	9	69	7	86
		BA	14	66	12	77
	Male	BSc	5	74	7	86
		BA	10	59	8	71
Age by Gender	$\leq 24$ years	Female	17	64	11	82
		Male	8	66	7	64
	$\geq 25$ years	Female	10	69	10	77
		Male	9	59	7	84



## Appendix S:

### Students' scores on scales measuring attitudes to mathematics in Mathematics 1

The analyses presented in the following four tables are for the demographic groups where the achievement differences were greatest, by fluency in English (Table 1), by age (Table 2) and by gender (Table 3). Each table lists the numbers of students in each group in 1995 and the medians of their scores, in March and October, for Enjoyment of Mathematics and Value of Mathematics, the two dimensions of Aiken's (1974) Mathematics Attitudinal Scale. The differences between these two median values, if negative, were expressed as a percentage of the median score in March, otherwise as a percentage of ( $40 - \text{March score}$ ).

Table 1. Analysis by Fluency in English

<i>Enjoyment of Mathematics Scale Scores</i>					
Variable		n	March 1995 Median	October 1995 Median	Difference as Percentage of March Score
			(out of 40)		
Younger Age Group by Fluency					
≤ 24 yrs	Fluent	11	21.8	22.7	5%
	Not fluent	9	21.4	19.1	-11%
Females by Fluency					
Female	Fluent	12	25.0	24.6	-2%
	Not fluent	12	24.1	24.0	0%
<i>Value of Mathematics Scale Scores</i>					
Younger Age Group by Fluency					
≤ 24 yrs	Fluent	11	30.0	32.0	20%
	Not fluent	9	26.0	26.0	0%
Females by Fluency					
Female	Fluent	12	31.0	32.0	11%
	Not fluent	12	27.0	26.8	-1%

Table 2. Analysis by Age

<i>Enjoyment of Mathematics Scale Scores</i>					
Variable	n	March 1995 Median	October 1995 Median	Difference as Percentage of March Score	
Non-Fluent Group by Age					
Non-Fluent	≤ 24 yrs	9	21.4	19.1	-11%
	≥ 25 yrs	5	25.5	27.3	12%
<i>Value of Mathematics Scale Scores</i>					
Non-Fluent Group by Age					
Non-Fluent	≤ 24 yrs	9	26.0	26.0	0%
	≥ 25 yrs	5	30.2	30.0	-1%

Table 3. Analysis by Gender

<i>Enjoyment of Mathematics Scale Scores</i>					
Variable	n	March 1995 Median	October 1995 Median	Difference as Percentage of March Score	
Fluent Group by Gender					
Fluent	Female	12	25.0	24.6	-2%
	Male	11	26.4	27.3	7%
<i>Value of Mathematics Scale Scores</i>					
Fluent Group by Gender					
Fluent	Female	12	31.0	32.0	11%
	Male	11	29.0	32.0	27%

## Appendix T:

### Students' beliefs about themselves as learners of mathematics: Mathematics 1

Students' beliefs about themselves as learners of mathematics were measured using a Mathematical Self-Concept Scale. An analysis of scores on a Mathematical Self-Concept Scale, is presented in the following table, for the demographic groups where the achievement differences were greatest, by fluency in English, by age and by gender. This table lists the numbers of students in each group in 1995 and the medians of their Mathematical Self-Concept Scale scores in March and October. The differences between these two median values were expressed as a percentage of the median score in March.

Variable	n	March 1995 Median	October 1995 Median	Difference as Percentage of March Score (out of 40)
<b>Analysis by Fluency in English</b>				
<b>Younger Age Group by Fluency</b>				
≤ 24 yrs	Fluent	11	22.5	22.5
	Not fluent	9	20.0	18.3
<b>Females by Fluency</b>				
Female	Fluent	12	20.0	22.1
	Not fluent	12	19.7	18.8
<b>Analysis by Age</b>				
<b>Non-Fluent Group by Age</b>				
Non-Fluent	≤ 24 yrs	9	20.0	18.3
	≥ 25 yrs	5	21.7	21.7
<b>Analysis by Gender</b>				
<b>Fluent Group by Gender</b>				
Fluent	Female	12	20.0	22.1
	Male	11	23.3	22.5



## **Appendix U:**

### **Students' beliefs about the learning of mathematics in Mathematics 1**

Students' beliefs about the learning of mathematics were gathered in the questionnaire from responses to six statements (Schoenfeld, 1989):

Maths problems can be done correctly in only one way.

The best way to do well in maths is to memorise all the formulas.

In maths you can be creative and discover things by yourself.

Everything important about maths is already known by mathematicians.

To solve maths problems you have to be taught the right procedure, or you cannot do anything.

Real maths problems can be solved by commonsense instead of the maths rules you learn at school.

The responses to the statement *Real maths problems can be solved by commonsense instead of the maths rules you learn at school* are difficult to interpret hence this statement will not be included in the analysis.

Statements related to students' beliefs about the learning of mathematics are listed in turn: each arrow links an individual student's response category (*Strongly Disagree, Disagree, Undecided, Agree or Strongly Agree* in March to their response category in October, 1995. Adjacent figures allow comparison of two demographic groups.

Analysis by Fluency (for females) in Figures 1-5.

Figure 1. Maths problems can be done correctly in only one way.

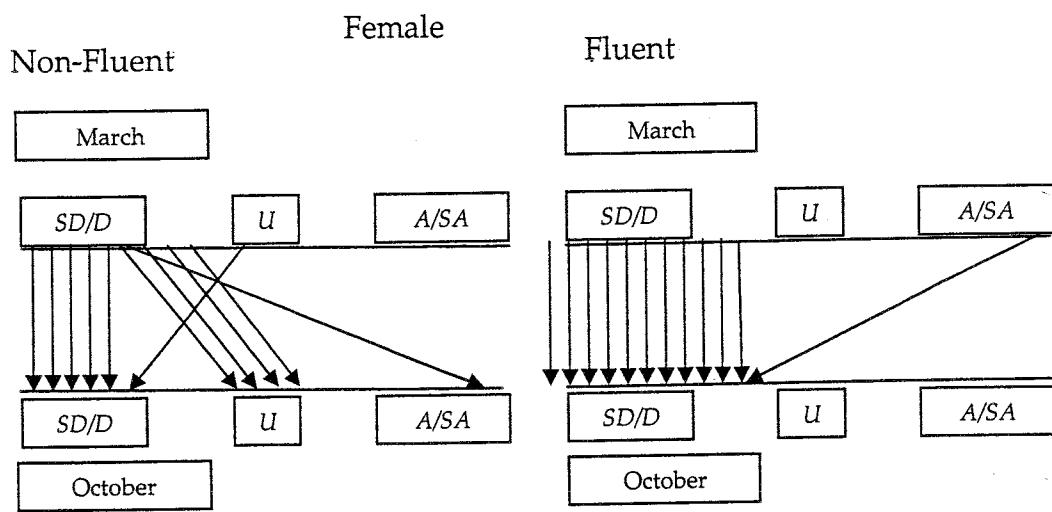


Figure 2. The best way to do well in maths is to memorise all the formulas.

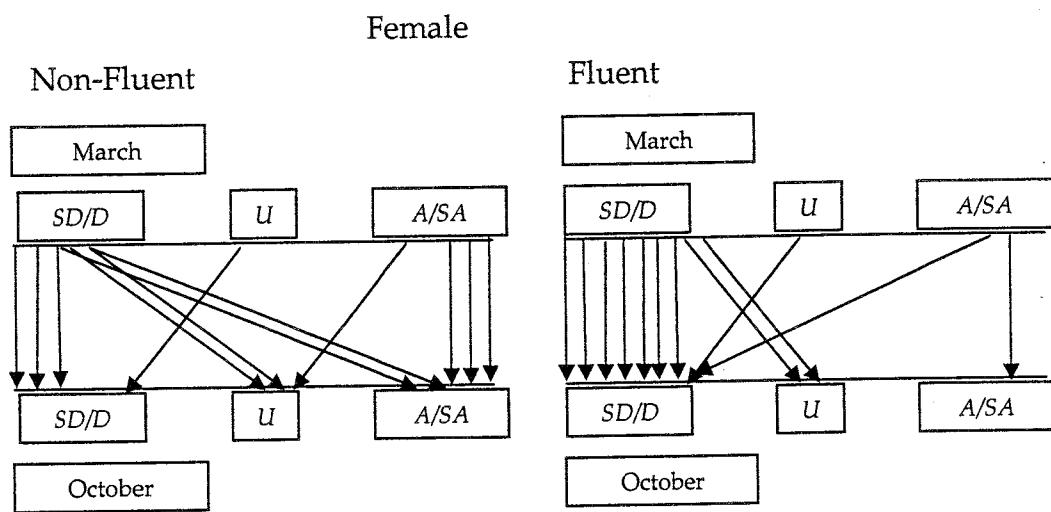


Figure 3. In maths you can be creative and discover things by yourself.

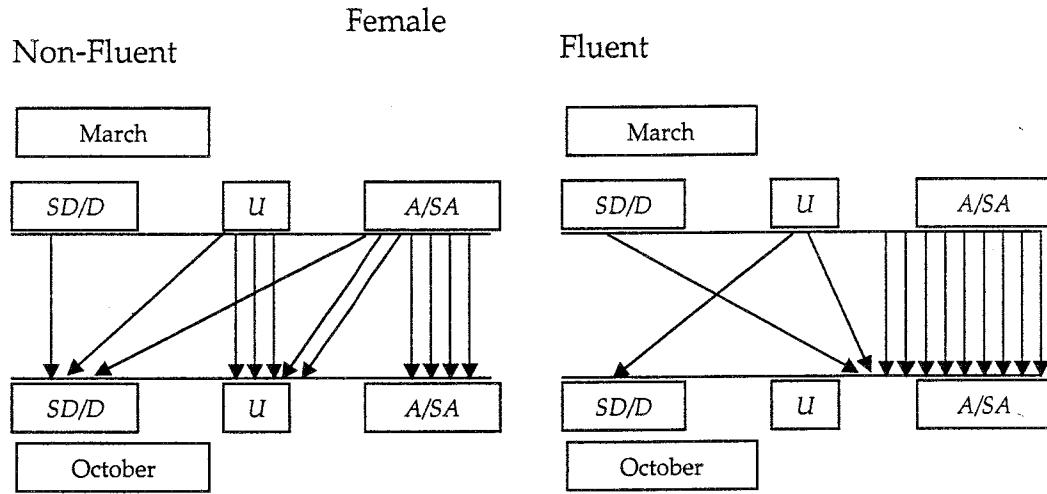


Figure 4. Everything important about maths is already known by mathematicians.

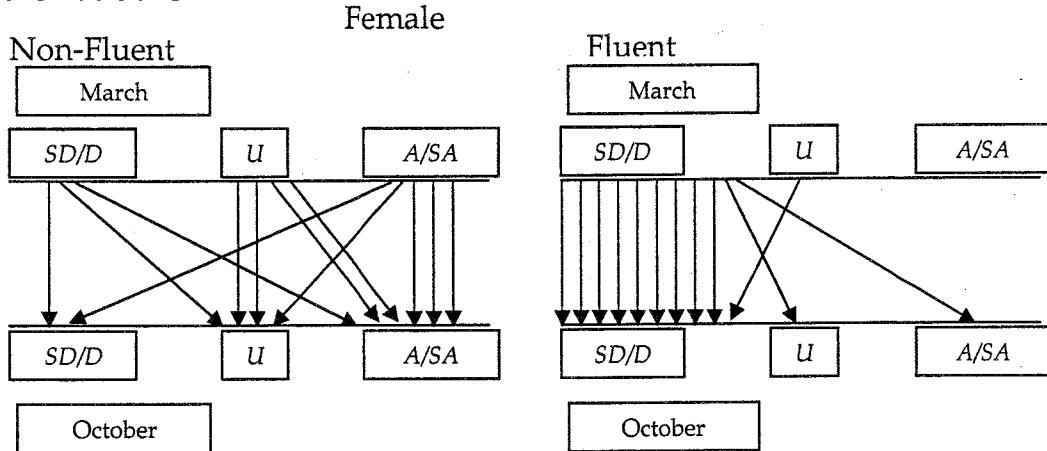
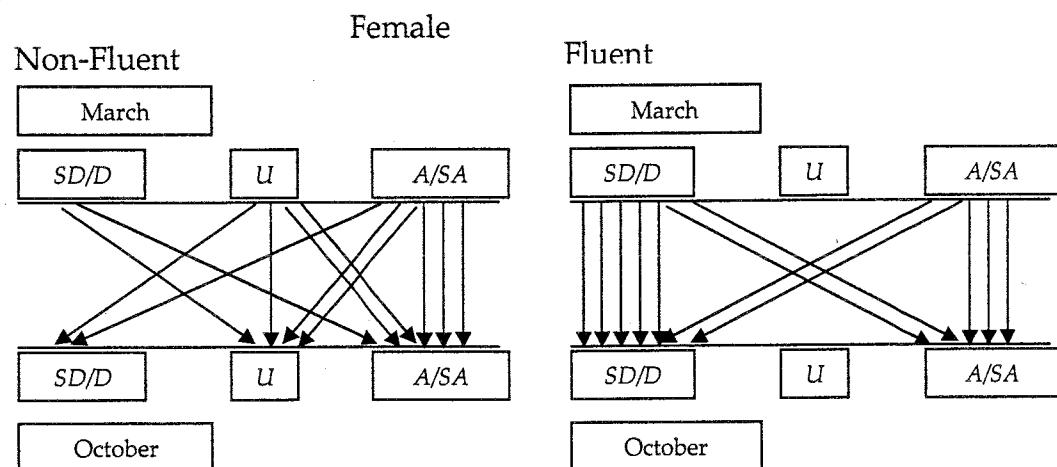


Figure 5. To solve maths problems you have to be taught the right procedure, or you cannot do anything.



Non-fluent and fluent female groups entered the course, in March 1995, with largely similar beliefs about the learning of mathematics. (Only in one of the five statements analysed does the distribution of responses vary between the two groups. Eleven of the twelve fluent females disagree or strongly disagree with the statement *Everything important about maths is already known by mathematicians* in March, compared to three of the twelve non-fluent female students.) By October, for all the five statements, the distributions of responses differed between the two groups. Fluent females, by October, indicate they have more mathematical confidence than the non-fluent female group. For example, in October, eleven of the twelve fluent students, compared to four of the twelve non-fluent students, agree or strongly agree with the statement *In maths you can be creative and discover things by yourself*. Also, all twelve fluent students, compared to only seven of the twelve non-fluent students, disagree or strongly disagree with the statement *The best way to do well in maths is to memorise all the formulas*. These two examples illustrate the differences between the fluent and non-fluent female groups that are occurring for all five statements. In other words, the lower achieving non-fluent students are exhibiting a more instrumental approach in their response pattern in October than they did in March, and also in comparison to the higher achieving fluent group who indicate they are more confident relational learners of mathematics. In October, it is also common to see four or five of the non-fluent students mark the undecided response, compared to between zero and two of the fluent students. This pattern may indicate non-fluent students are uncertain, possibly confused by the approach or by the statement in the questionnaire.

Analysis by Age (for non-fluent students) in Figures 6-10.

Figure 6. Maths problems can be done correctly in only one way.

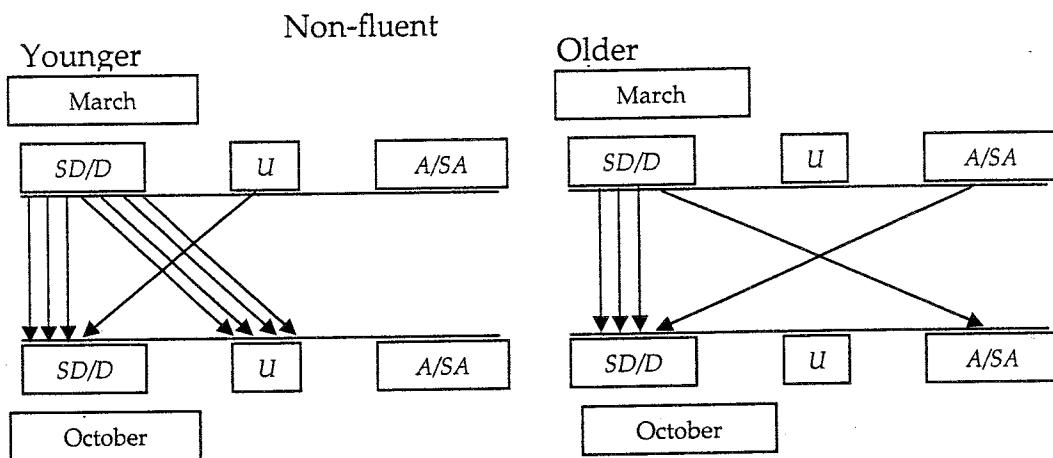


Figure 7. The best way to do well in maths is to memorise all the formulas.

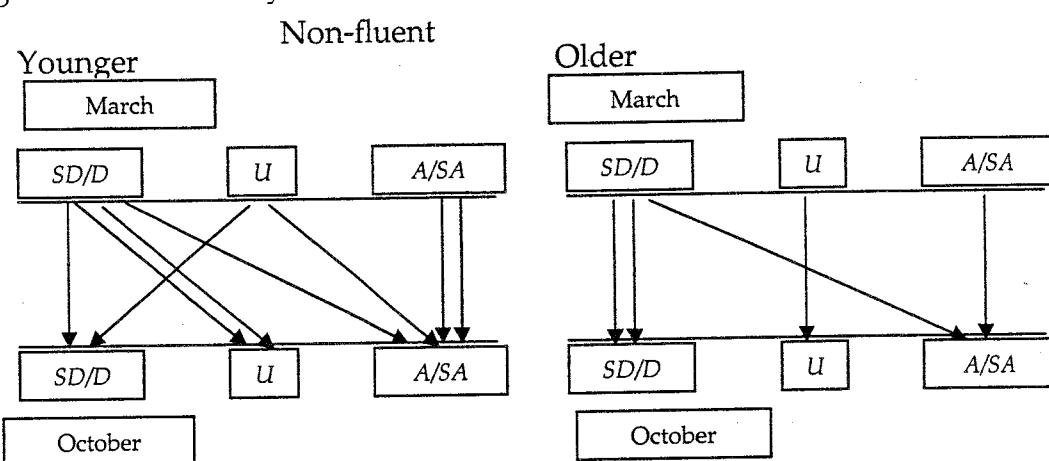


Figure 8. In maths you can be creative and discover things by yourself.

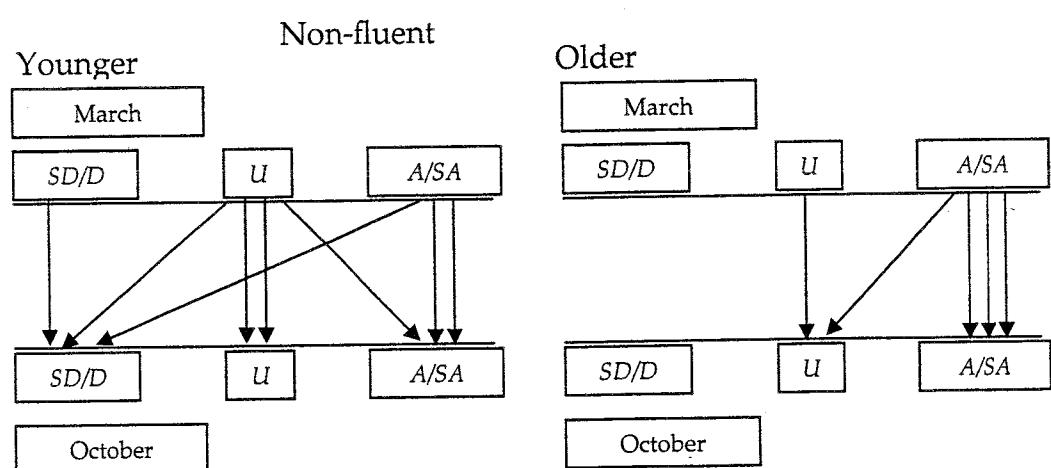


Figure 9. Everything important about maths is already known by mathematicians.

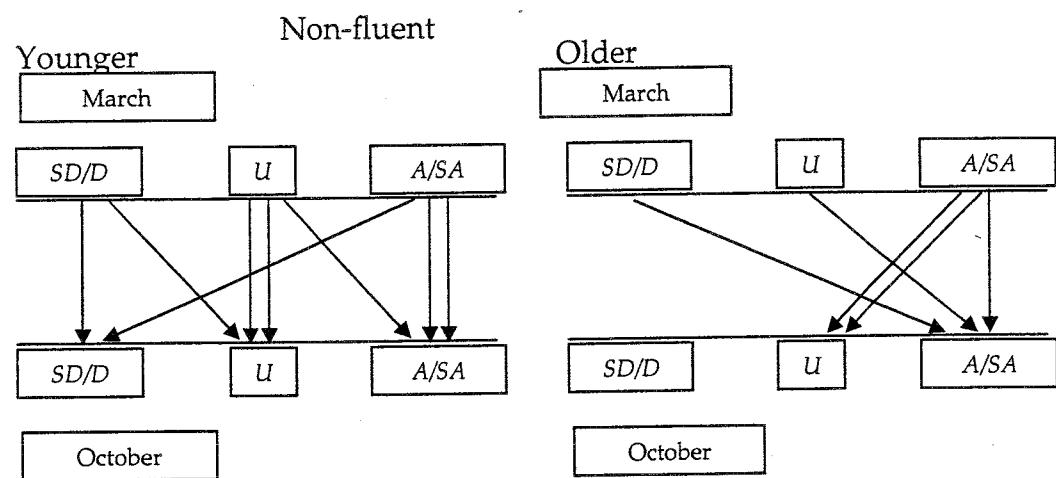
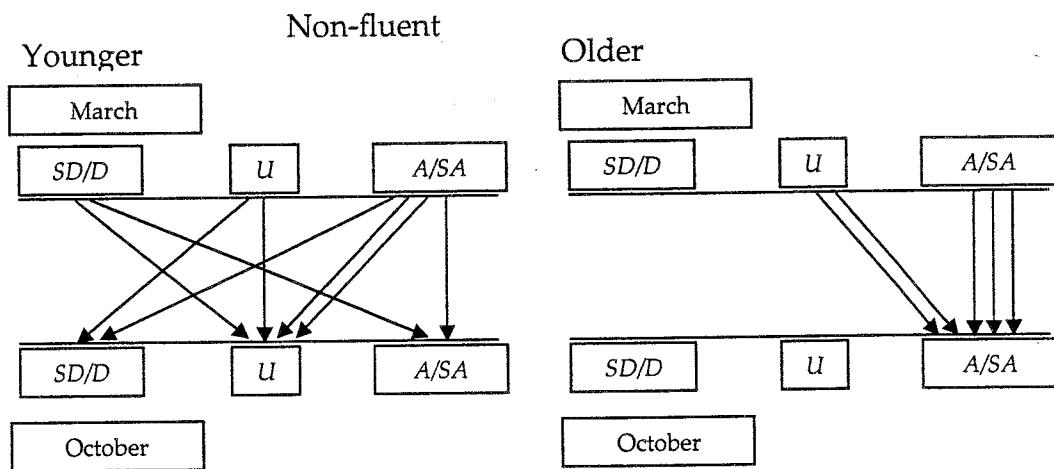


Figure 10. To solve maths problems you have to be taught the right procedure, or you cannot do anything.



I consider the age effect within the non-fluent group, since these demographic groups is where achievement difference by age is greatest. The sizes of these groups are smaller than those analysed in the fluency and gender discussion, so results are more tentative. For the younger non-fluent age group ( $n=8$ ), for four of the five statements there is an even distribution of responses, indicating a mixture of students, some who are exhibiting an instrumental approach and some a more relational approach to learning mathematics, as well as some who give an undecided response. Only in one statement is the response distribution skewed, in the more relational learning direction, as four of the eight students disagree or strongly disagree with the statement *Maths problems can be done correctly in only one way* although four students move from disagreement in March to being undecided about this statement by October. For the older non-fluent age group ( $n=5$ ), for four of the five statements the response distribution skewed, two indicating a relational approach to learning mathematics (*Maths problems can be done correctly in only one way* and *In maths you can be creative and discover things by yourself*) and two indicating an instrumental approach to learning mathematics (*Everything important about maths is already known by mathematicians* and *To solve maths problems you have to be taught the right procedure, or you cannot do anything*). The response pattern to the statement although skewed (towards disagreement) in March, by October is symmetric, indicating a more instrumental approach by end of the year. In summary, small numbers in each age group (for non-fluent students) mean the results of this analysis are unclear. There seems to be a mixture of instrumental and relational learners of mathematics in both age groups.

Analysis by Gender (for fluent students) in Figures 11-15.

Figure 11. Maths problems can be done correctly in only one way.

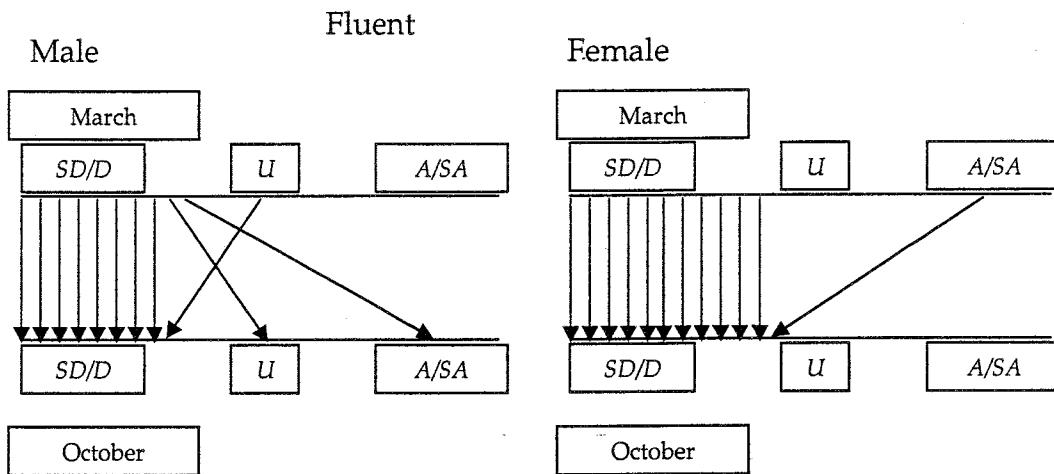


Figure 12. The best way to do well in maths is to memorise all the formulas.

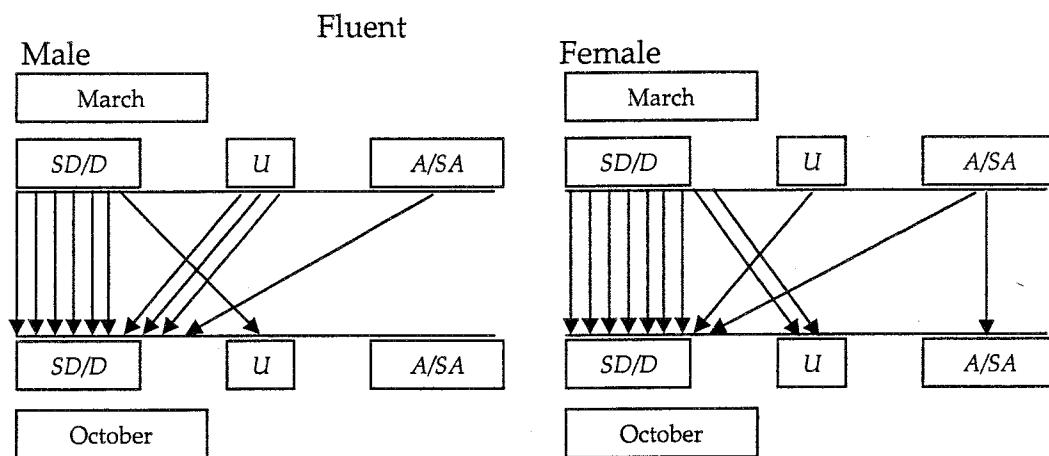


Figure 13. In maths you can be creative and discover things by yourself.

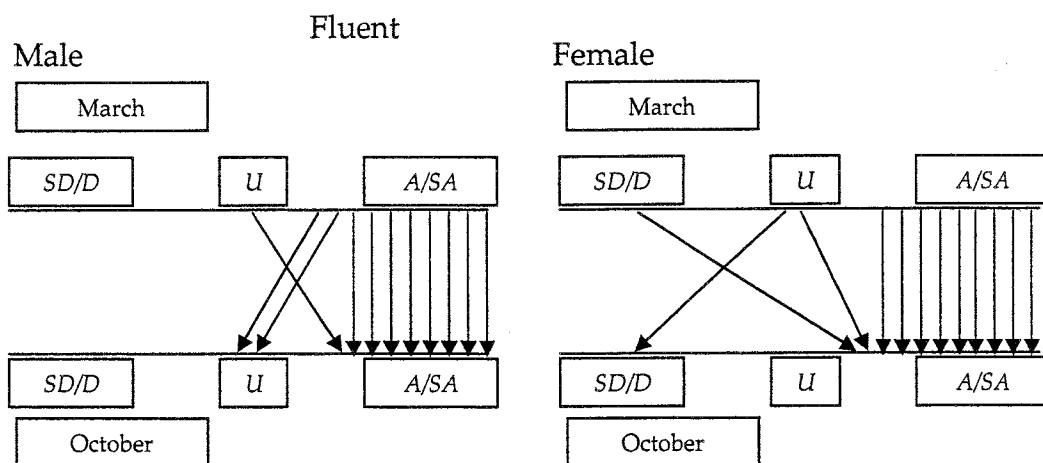


Figure 14. Everything important about maths is already known by mathematicians.

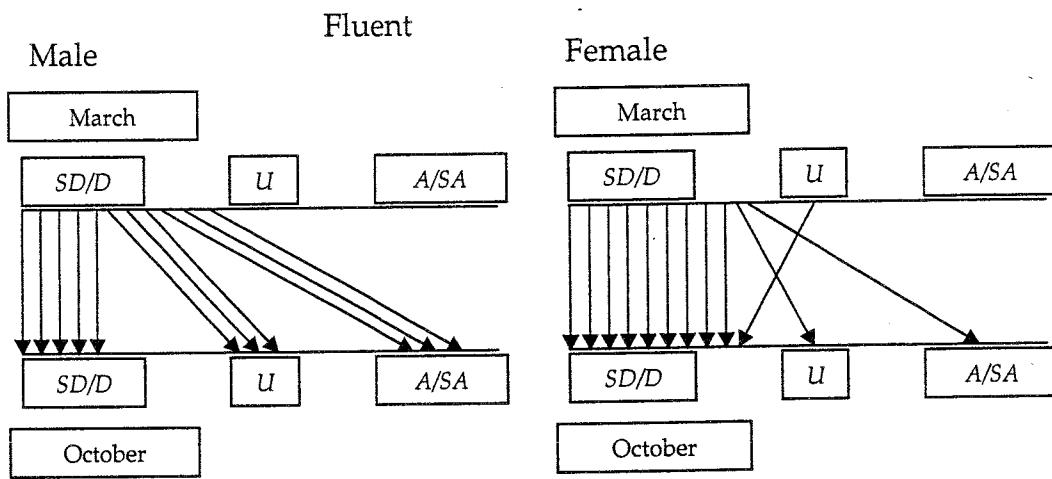
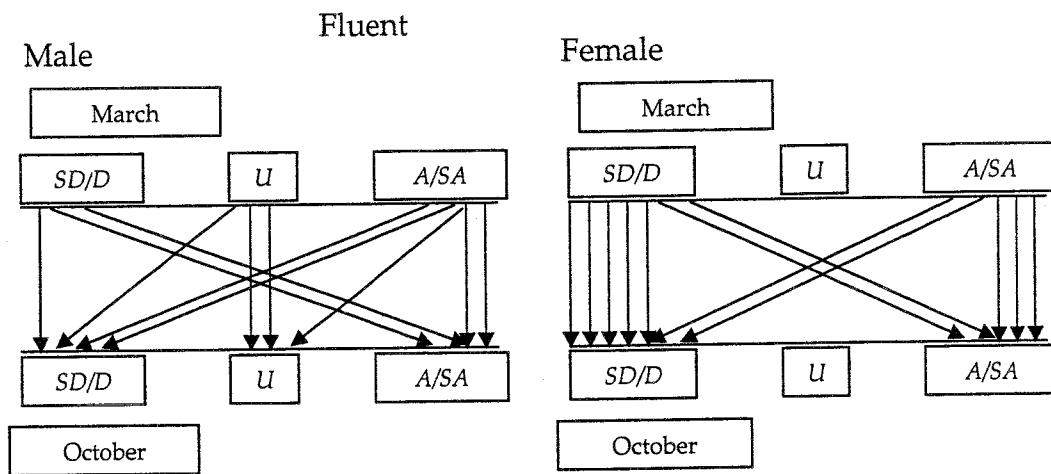


Figure 15. To solve maths problems you have to be taught the right procedure, or you cannot do anything.



For both female and male fluent students, in three of the five statements the response distribution is skewed, indicating a relational approach to learning mathematics, that is, for the statements *Maths problems can be done correctly in only one way*, *In maths you can be creative and discover things by yourself* and *The best way to do well in maths is to memorise all the formulas*. For both female and male fluent students, the response distribution is symmetric to the statement *To solve maths problems you have to be taught the right procedure, or you cannot do anything*, indicating a mixture of instrumental and relational learners of mathematics in the fluent group, as well as some who give an undecided response, more fluent males are undecided. For the fluent male group, the response pattern to the statement *Everything important about maths is already known by mathematicians* although skewed (towards disagreement) in March, by October is symmetric, indicating more students using an instrumental approach by end of the year. The response pattern remains skewed (towards disagreement) for the fluent female group, indicating more using a relational approach to learning mathematics. To summarise, the higher achieving fluent female students indicate a relational learning approach in their response patterns to four of the five statements whereas the lower achieving fluent males show a relational approach to learning in three of the five statements. More males than females respond in the undecided category.

## Appendix V:

### **Medians of final marks, for the demographic variables also available on the class roll, for students in the Wellesley mathematics course who answered a questionnaire in 1995 or 1996.**

Medians of final marks are listed, for students who completed the Wellesley mathematics course in 1995 and 1996, for categories of the demographic variables available on the class roll. These achievement statistics are calculated for the groups who completed the initial questionnaire (March) and/or the final questionnaire (October) for each year.

Variable	Categories	March 1995 n=46		October 1995 n=34		March 1996 n=47 <sup>#</sup>		October 1996 n=36	
		n	Median	n	Median	n	Median	n	Median
Options	Science	26	81	21	82	18	73	16	84
	Arts	20	36	13	53	29	51	20	51
Gender	Female	15	71	12	63	26	58	23	58
	Male	31	58	22	68	21	60	13	69
Ethnicity	Maori	11	53	7	53	8	51	7	50
	Pakeha	27	69	21	82	21	69	14	82
	PI	6	39	4	51	11	57	10	59
	Others	2	-	2	-	6	67	5	65
Streams	1	7	25	6	27	14	46	13	46
	2	13	53	5	53	12	59	8	63
	3	12	67	11	68	11	60	6	71
	4	14	87	12	90	10	84	9	84

<sup>#</sup> In March 1996, one student gave no ethnicity information.



## Appendix W:

### Students' beliefs about themselves as learners of mathematics and two of their attitudes to mathematics: Wellesley mathematics course

Students' beliefs about themselves as learners of mathematics were measured using a subset of Gourgey's (1982) Mathematical Self-Concept Scale. An analysis of scores on this Scale, is presented in the following table, for students in streams 1 and 4, the groups where achievement differences were greatest. These means are listed for both March and October in 1995 and 1996, and the percentage gain (or loss) in these scores over each year is listed in the final column.

Streams/ Year	n	Maths Self-Concept Mean Score March (out of 40)	Maths Self-Concept Mean Score October (out of 40)	Difference as Percentage
1	1995	5	24.3	-21.2%
	1996	15	20.6	6.3% <sup>†</sup>
4	1995	10	28.7	5.3%
	1996	8	28.8	-1.4%*

<sup>†</sup>In 1996, in stream 1, five students passed the course. The means of their scores are 24 (March), 28 (October), so the percentage gain for this group is 16.7%. For the eight who failed, the corresponding statistics are 18.4, 18.3 and -0.5%.

\*In 1996, in stream 4, two students found the pace too fast and were the only students in stream 4 with a lower score on this scale in October. If these two scores are excluded the percentage difference is +6.9%.

The means of the Mathematics Self-Concept scores in stream 1 (the lowest achieving stream) are lower than those in stream 4 (the highest achieving stream) in 1995 and 1996, both in March and October. Percentage changes in stream 1 scores from March to October are negative in 1995 and positive in 1996, however, if the group who passed in 1996 are excluded (see footnote to Table) percentage changes are negative both years. Stream 4 students have started the course with higher scores, so it might be expected that there would be little change, however they have increased in 1995, with little change in 1996. The mathematical self-concept of students seems to depend on the mathematics background knowledge of students and these beliefs are influenced somewhat negatively by the Wellesley mathematics course for those who failed the course in stream 1, but positively if they passed the course. Students are influenced largely positively by the course in stream 4.

A more detailed examination was made of the percentage differences in Mathematical Self-concept Scale scores in 1995. Examination of the lowest

quartile of these differences, students whose mathematical self-concept scores were lowered the most by the end of the course, showed over 50% were from stream 1. Examination of the highest quartile, the group of students whose scores gained most, indicate that over 60% were from stream 4, the top stream.

An analysis of measures of two dimensions of students' attitudes to mathematics follows, for stream 1 and 4, the demographic groups where achievement differences were greatest. Enjoyment of Mathematics and Value of Mathematics are these two dimensions, in Aiken's (1974) Mathematics Attitudinal Scale.

The following table contains the means of the scores for Aiken's (1974) Enjoyment of Mathematics Scale (out of 40), in March and in October, 1995 and 1996 (*in italics*), and the percentage gain (or loss) over the year for students in each stream.

Streams/ Year	n	Enjoyment of Maths Mean Score in March	Enjoyment of Maths Mean Score in October	Difference as Percentage
1	1995	5	24.6	-4.3%
	1996	15	24.1	+3.1% <sup>t</sup>
4	1995	10	30.9	+5.2%
	1996	8	30.1	+1.0 %

<sup>t</sup>In 1996, in stream 1, five students passed the course. The means of their scores are 25.5 (March), 24.9 (October), so the percentage difference for this group is -2.3%. For the eight who failed, the corresponding statistics are 22.7, 24.0 and +5.9%.

The means of the Enjoyment of Mathematics scores in stream 1 (the lowest achieving stream) in 1996 are lower than those in stream 4 (the highest achieving stream) in both March and October in 1995 and 1996. The percentage changes in scores from March to October are small and vary in stream 1, they are small and positive in stream 4 both years.

Enjoyment of mathematics seems to depend on the mathematics background knowledge of students and these attitudes are influenced, to some degree, positively by the Wellesley mathematics course in stream 4.

A more detailed examination was made of the percentage differences in Enjoyment of Mathematics scores in 1996. Examination of the lowest quartile of these differences, students whose scores were lowered the most by the end of the course, showed almost 50% were from stream 1. Examination of the highest quartile, the group of students whose scores gained most, indicates that 50% were from stream 1, the bottom stream, an unexpected result, although it fits

with the figures in the footnote to the table above. The percentage differences in stream 1, 1996, for the groups of students who passed and those who didn't pass were the opposite of what we might expect. However, the means of this scale are higher for those who passed, than those who didn't.

The following table contains the numbers of students and means of the scores for Aiken's (1974) Value of Mathematics Scale (out of 40), in March and in October, 1995 and 1996 (*in italics*) as well as the percentage gain (or loss) over the year for students in streams 1 and 4.

Streams/Year	n	Value of Maths Mean Score in March	Value of Maths Mean Score in October	Difference as Percentage
1	1995	5	27.0	+4.2%
	1996	13	28.9	+2.3%
4	1995	10	32.0	+8.4%
	1996	8	30.8	-1.2%

Scores on the Value of Mathematics Scale for streams 1 and 4 are all reasonably high, approximately 30 out of 40, with a small consistent difference between stream 1 and stream 4, where stream 4 scores are somewhat higher. Attitudes about the (extrinsic) value of mathematics do not seem to depend much on mathematics background knowledge and these attitudes are usually influenced, to some degree, positively by the Wellesley mathematics course in both streams under consideration.



## **Appendix X:**

### **Students' beliefs about the learning of mathematics: Wellesley mathematics course**

Students' beliefs about the learning of mathematics were gathered in the questionnaire from responses to six statements (Schoenfeld, 1989):

Maths problems can be done correctly in only one way.

The best way to do well in maths is to memorise all the formulas.

In maths you can be creative and discover things by yourself.

Everything important about maths is already known by mathematicians.

To solve maths problems you have to be taught the right procedure, or you cannot do anything.

Real maths problems can be solved by commonsense instead of the maths rules you learn at school.

The responses to the statement *Real maths problems can be solved by commonsense instead of the maths rules you learn at school* are difficult to interpret hence this statement will not be included in the analysis.

Statements related to students' beliefs about the learning of mathematics are listed in turn: each arrow links an individual student's response category (*Strongly Disagree, Disagree, Undecided, Agree or Strongly Agree* in March to their response category in October, 1995. Adjacent figures allow comparison of two demographic groups, namely, stream 1 and stream 4<sup>55</sup>.

The diagram for stream 1 is on the left and the corresponding diagram for stream 4 is on the right in each figure.

Analysis by Stream: Figures 1-5.

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<sup>55</sup> Because we are comparing each student's response in March with that given in October, this comparison can only be done for students who completed both the March and October questionnaires, in either 1995 or 1996 (combining both years, n= 18 in stream 1 and n=18 in stream 4).

Figure 1. Maths problems can be done correctly in only one way.

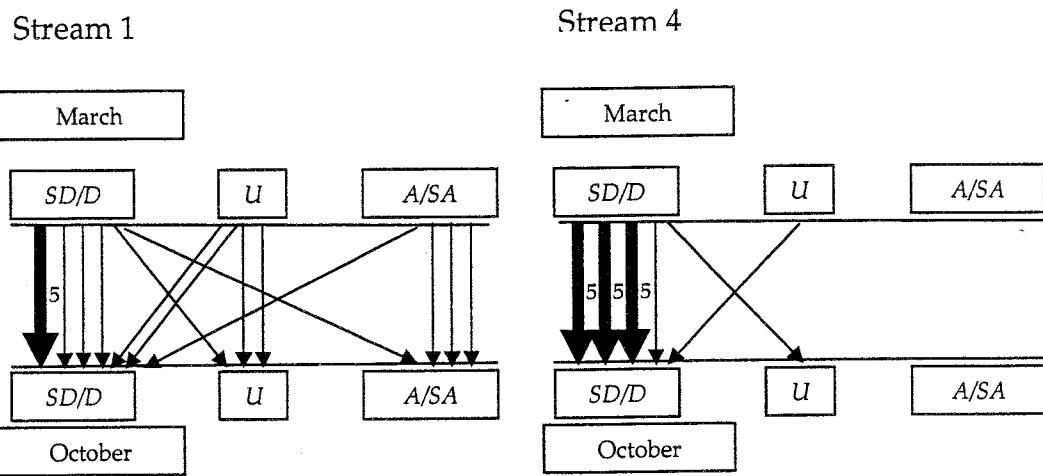


Figure 2. The best way to do well in maths is to memorise all the formulas.

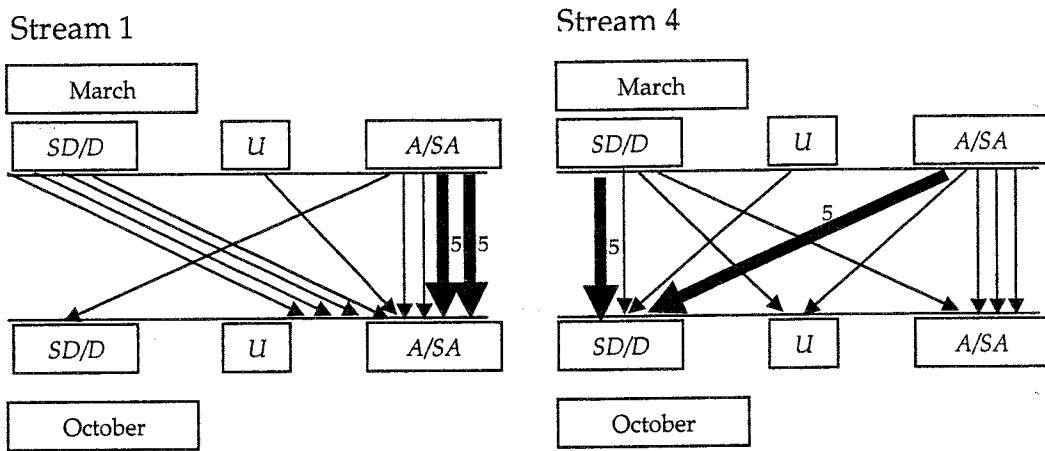


Figure 3. In maths you can be creative and discover things by yourself.

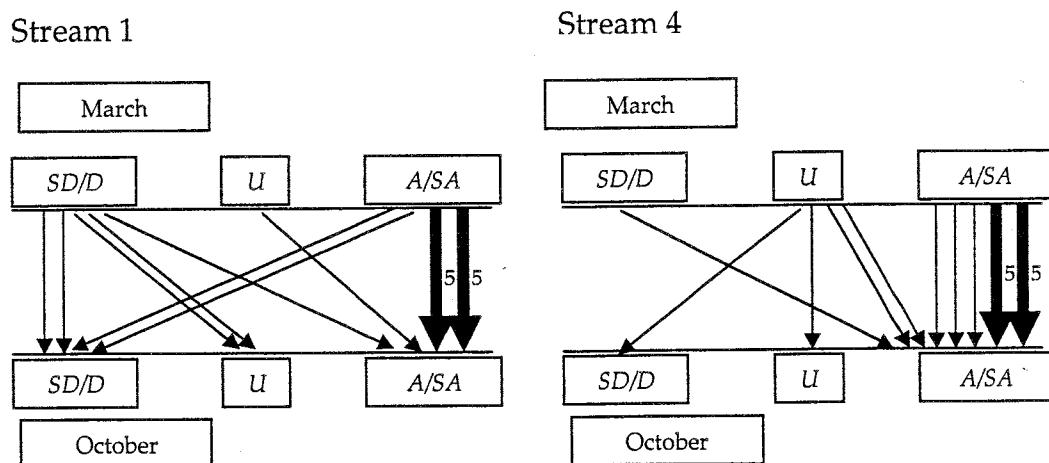


Figure 4. Everything important about maths is already known by mathematicians.

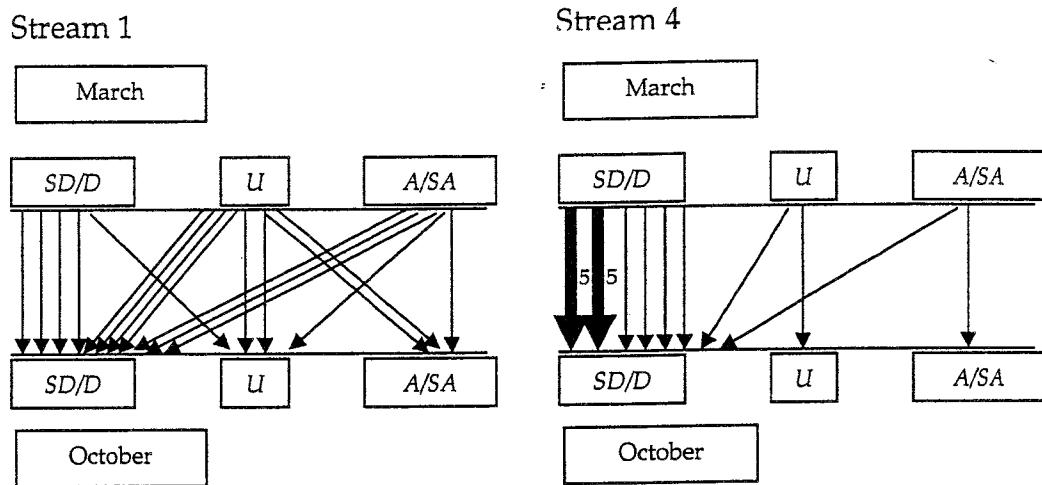
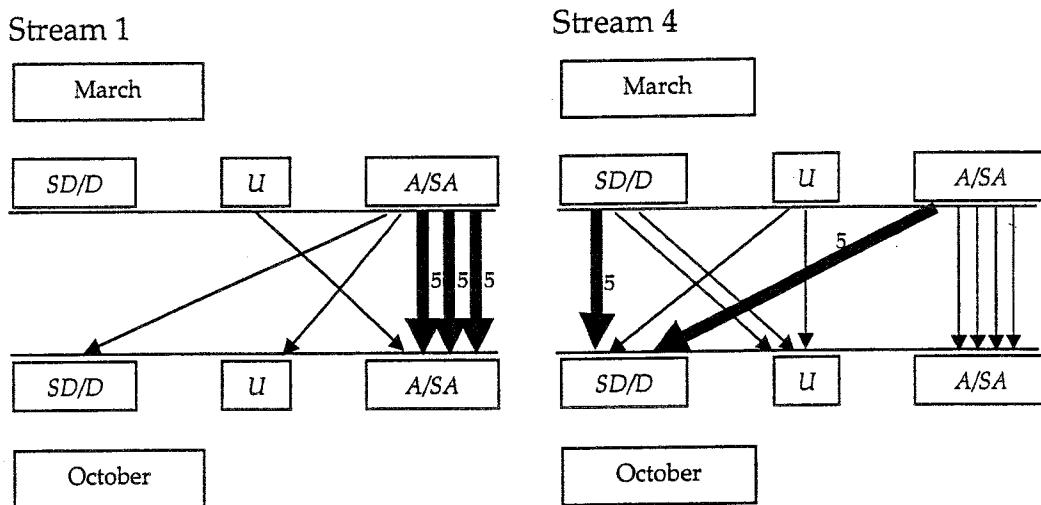


Figure 5. To solve maths problems you have to be taught the right procedure, or you cannot do anything.



In March, the response distribution for stream 4 students is symmetric to two statements, *To solve maths problems you have to be taught the right procedure, or you cannot do anything* and *The best way to do well in maths is to memorise all the formulas*, indicating a mixture of less and more mathematically confident responses from this group, but by October almost two-thirds of the students in stream 4 give responses indicating a focus on conceptual ways of learning mathematics ('disagreeing' or 'strongly disagreeing' with these statements). In contrast, the response distribution for stream 1 students is skewed in March and October in the opposite direction, indicating a focus on an instrumental approach to learning mathematics. About 90% of the students in stream 1

'agree' or 'strongly agree' with the first statement in March and in October. The corresponding percentages for the second statement increase from about 70% in March to over 90% in October, indicating that more lower achieving stream 1 students are focusing on an instrumental approach to learning mathematics in October than in March.

The response distributions are similar in each stream (although different for each) for the three other statements *Maths problems can be done correctly in only one way*, *In maths you can be creative and discover things by yourself* and *Everything important about maths is already known by mathematicians*. A high proportion of the higher achieving stream 4 students indicate a conceptual focus to learning mathematics by their responses to all three statements, sometimes this proportion increases from March to October. The percentages in October are 94%, 89% and 89% respectively, indicating that stream 4 students are predominantly conceptual learners of mathematics. In contrast, a smaller proportion of stream 1 students indicate they are conceptual learners of mathematics. By October, 61%, 67% and 61% of the responses, respectively, indicate this focus. For two statements this percentage is the same in March and October however, for the third statement (*Everything important about maths is already known by mathematicians*) there has been an increase from 27% in March, to 61% who 'disagree' or 'strongly disagree' with this statement by October, indicating a higher proportion feel that mathematics is more accessible to them.

## **Appendix Y:**

### **The Mathematical Autobiography Questionnaire**

A questionnaire developed at the University of Central Queensland was adapted to gather data about the previous mathematical experiences of the individual undertaking the supervised study in mathematics.

#### **Why a Mathematical Autobiography?**

A mathematical autobiography is a unique process where we are able to reflect on our Experiences of Mathematics. The process begins with us remembering our personal experience. When we are able to focus on our specific experiences, i.e. mathematics, we can often learn more precisely about

- a. patterns in our experience
- b. our beliefs and values
- c. our perceptions about what occurred to develop our beliefs/ values
- d. our feelings
- e. the relationship between our learning experiences and our actual world.

On being able to reflect on our own experience, it is suggested that we may be encouraged to re-evaluate, change, improve, embrace ourselves and mathematics. This may have a profound effect on our view of ourselves as users of mathematics or our future career pathway, where we use mathematics or how we relate to mathematical experiences in our current lives.

## Mathematics

Remember back to your earliest experiences of mathematics:

How old were you?

What environment were you in?

I remember ...

I was using/playing with/ experimenting...

Remember back to your earliest experiences where you were exploring mathematical concepts:

How old were you?

I remember...

The context was...

I was using/playing with /experimenting...

Remember when you started primary school:

Primary School:

Age:

Recall some experiences you had with mathematics in your early years of schooling:

On a scale of 1 – 10, rate how these early experiences had an influence on your attitude and perceptions of mathematics.

Not significant 1—————10 Very significant

Experience 1:

Experience 2:

Recall any significant experiences that changed or influenced your attitude/  
perceptions of mathematics during your schooling.

How old were you?

Context:

I remember...

Recall any significant experiences that changed or influenced your attitude/  
perceptions about yourself understanding mathematics / using mathematics.

How old were you?

Context:...

I remember...

Recall any times during your experiences as a worker, paid or non-paid, where you have specifically used mathematical concepts, skills or understandings.

Experience 1:

Age:

Type of work:

I recall...

Experience 2:

Age:

Type of work:

I recall...

How you have used mathematical concepts, skills and understandings in the past month.

Age:

Type of work:

I recall...

Write your mathematical experience diary here:

60 years

50 years

40 years

30 years

20 years

15 years

10 years

5 years

3 years

Age

Your feelings and the study of mathematics

How are you feeling about commencing/re-commencing the study of mathematics?

Write down some of your concerns about commencing/re-commencing the study of mathematics?

Write down some of your expectations about commencing/re-commencing the study of mathematics?

Expectations about your ability:

Expectations about mathematics:

Expectations about learning mathematics:

What were some of your first experiences in mathematics?

What were some of the things you learned about mathematics?

How do you recall feeling about mathematics at that time?

Do you recall any specific negative experiences when you were studying mathematics?

What did you do with any negative feelings that you developed as a result of studying mathematics?

Do you recall any positive experiences that you had in the study of mathematics?

What specifically made your experiences positive for you?

Thank you.



## **Appendix Z:**

### **The Mathematics Metaphor Questionnaire**

The questionnaire entitled "Personal Comparisons For Mathematics", developed by Gibson (1994), was adapted and used to gather data about students' feelings about mathematics, particularly the individual undertaking supervised study in mathematics. It was also adapted for the interviews to gauge students' reactions to a teaching approach.

Write your responses to the following questions.

1      Pretend that you have to describe mathematics to someone. List all the words or phrases you can think of to use.

2      Imagine yourself doing or using maths. What does doing or using maths feel like? List all the words or phrases you would use to describe what doing maths feels like.

3.      Think about the things that maths is like. List all the things or objects that you think that maths is like.

4. Here are some categories that you might want to think about to extend your list of things that you think maths is most like.

i If maths were weather, what kind of weather would it be? Explain why.

ii If maths were food, what kind of food would it be? Explain why.

iii If maths were food, how would you eat it? Explain why.

iv If maths were a hobby, what kind of hobby would it be? Explain why.

v If maths were a way to travel, what means of travel would it be? Explain why.

vi If maths were a colour, what colour would it be? Explain why.

vii If maths were a way to communicate, what way would it be? Explain why.

viii If maths were an animal, what kind of animal would it be? Explain why.

viii If maths were a building, what kind of building would it be? Explain why.

ix If maths were a plant, what kind of plant would it be? Explain why.

x If maths were music, what kind of music would it be? Explain why.

xi If maths were a kitchen utensil, what kind of a kitchen utensil would it be? Explain why.

xii If maths were a garden tool, what kind of a garden tool would it be? Explain why.

5. Read through your list of words that describe maths, saying, "Maths is .....". Choose an object from your list that best fits maths. Write a paragraph describing the ways that maths and this object are alike for you.

*For me, maths is most like a(n)*