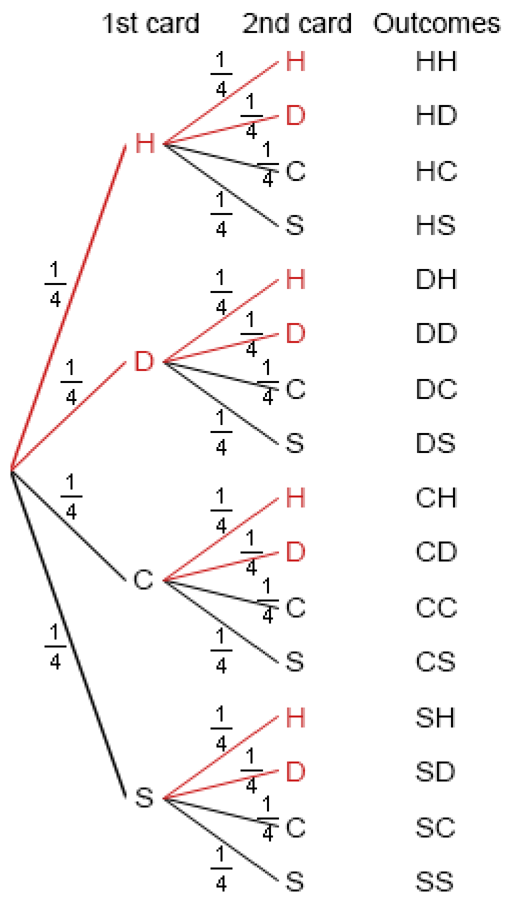
PROBABILITY TREES WITH REPLACEMENT

SOLUTIONS

TASK 1 Card selection

1 The probability of selecting a card from each suit is $\frac{1}{4}$ since each suit makes up $\frac{1}{4}$ the cards in the pack.

Angelo returns the card to the pack each time so the probabilities do not change from step to step. These are **independent events**.



2 **a** $P(HH) = \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{16}$

b P(two red cards)

$= P(HH) + P(HD) + P(DH) + P(DD)$

$= (\frac{1}{4} \times \frac{1}{4}) + (\frac{1}{4} \times \frac{1}{4}) + (\frac{1}{4} \times \frac{1}{4}) + (\frac{1}{4} \times \frac{1}{4})$

$= \frac{4}{16}$ or $\frac{1}{4}$

c P(1red, 1 black, any order) = P(HC) + P(HS) + P(DC) + P(DS) + P(CH) + P(CD) + P(SH) + P(SD)

$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$

$= \frac{1}{2}$

3 $P(SSS) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

4 P(10 spades in row) $= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \ldots \times \frac{1}{4} = (\frac{1}{4})^{10}$ (ie $\frac{1}{1048576}$)

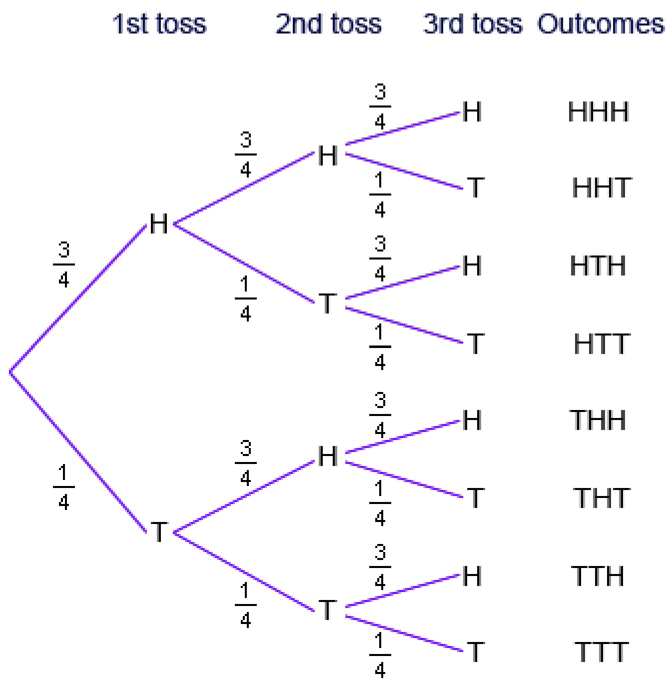
5 In the answers for questions 3 and 4, the probability is the product of repeated $\frac{1}{4}$. The number of factors of $\frac{1}{4}$ equals the number of steps in the tree. So the probability of selecting 100 spades in a row would be $(\frac{1}{4})^{100}$.

TASK 2 Coin flips

1 **a** P(H) = $\frac{3}{4}$

b P(T) = $\frac{1}{4}$

2 See diagram.



3 **a** $P(HHH) = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

$= \frac{27}{64}$

b $P(TTT) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{64}$

c $P(HHT) = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$

$= \frac{9}{64}$

d P(2 heads and 1 tail in any order) = P(HHT) + P(HTH) + P(THH)

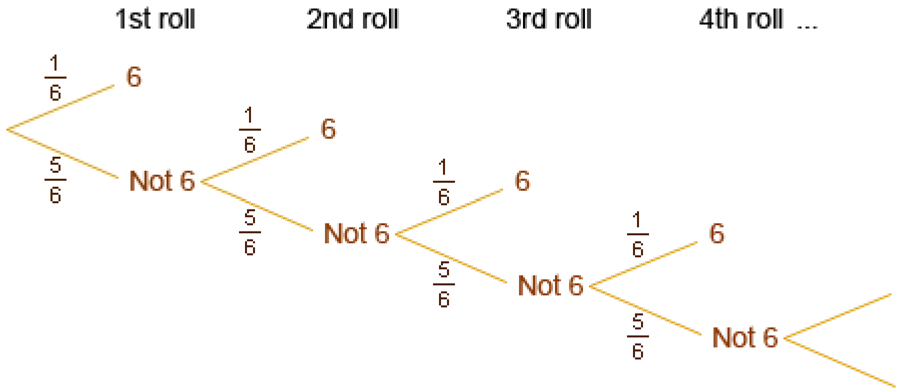
$= (\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}) + (\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4})$

$= \frac{9}{64} + \frac{9}{64} + \frac{9}{64}$

$= \frac{27}{64}$

CHALLENGE Roll a six

Note: A probability tree showing this information is not symmetrical. Once Milu rolls a 6, she doesn't have another roll. So the tree branches out each step from the lower branch only (not 6).



1 $P(6) = \frac{1}{6}$

2 $P(\text{not } 6, 6) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

3 $P(\text{not } 6, \text{not } 6, 6) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

4 The pattern shows repeated factors of $\frac{5}{6}$ followed by one factor of $\frac{1}{6}$.

a (P not getting 6 until 10th roll) $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \ldots\ldots \times \frac{5}{6} \times \frac{1}{6}$ [There are **9** factors of $\frac{5}{6}$ here.]

$= (\frac{5}{6})^9 \times \frac{1}{6}$

b (P not getting 6 until 24th roll) $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \ldots\ldots \times \frac{5}{6} \times \frac{1}{6}$ [There are **24** factors of $\frac{5}{6}$ here.]

$= (\frac{5}{6})^{24} \times \frac{1}{6}$