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Suggestion for a Theoretical Model for Secondary-Tertiary Transition in Mathematics

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One of most notable features of existing body of research in transition seems to be the absence of a theoretical model. The suggestion we present in this paper—to view and understand the high school to university transition in mathematics as a modern-day rite of passage—is an attempt at defining such framework. Although dominantly reflecting North-American reality, we believe that the model could be found useful in other countries as well. Let us emphasize that our model is not new in the sense that it recognizes the transition as such. In this paper, we try to determine whether (and, if so, how) the notion of a rite of passage—which is a well-understood concept in anthropology, as well as in some other disciplines (e.g. culture shock in cultural studies)—can help us understand mathematics transition issues better. Can it help us systematize existing body of research, and enhance our understanding of transition in mathematics; does it point at something new? We believe so, and by elaborating some traditional aspects of rites of passage, we hope to provide a useful lens through which we can examine the process of transition in mathematics, and make suggestions for improved management of some transitional issues.

Introduction

There is no doubt that the secondary-tertiary transition in mathematics is a complex issue, involving a whole spectrum of problems and difficult situations (Barnard 2003; Crawford, Gordon, Nicholas, & Posser, 1994, 1998; Gruenwald, Klymchuk, & Jovanoski, 2003; Guzman, Hodgson, Robert, & Villani, 1998; McInnes, James, & Hartley, 2000; Schoenfeld, 1994; Tall, 1992,1997; Wood, 2001, etc.). Perhaps the most notable feature of existing body of research on transition is the absence of a theoretical model. The suggestion we present—to view and understand the transition in mathematics as a modern-day rite of passage—is an attempt at defining such framework.

Our model is not new in the sense that it recognizes the transition as such. Along with other events related to one's academic life, such as the transition from elementary to high school, or from school to work, or even transition to university professorship, the transition from high school to university has been viewed, for a long time, as a genuine rite of passage, consisting of a wide variety of events, customs and initiation ceremonies (such as orientation activities), and accompanied by numerous carefully conceived strategies (by parents, high school teachers, university instructors, and others) aimed at easing the pains.

There has been an attempt in the past to use rites of passage as a *model* for transition in education (Leemon, 1972). However, since this relates only to admission to a United States fraternity culture it is of limited use as a model for the transition from secondary to tertiary institutions, and in particular within a mathematics education context.

The aim of this paper is to try to determine whether (and, if so, how) the notion of a rite of passage, which is a well-understood concept in anthropology, as well as in some other disciplines (e.g. culture shock in cultural studies), can help us understand mathematics transition issues better. Can it help us systematize the existing body of research, and hence enhance our understanding of transition; does it point at something new? We believe so, and by elaborating some traditional aspects of rites of passage, we hope to provide a useful lens through which we can examine the process of transition, and make suggestions for improved management of some transitional issues.

Anthropological Concept of a Rite of Passage

In his seminal work, French anthropologist Arnold van Gennep (translation, 1960) describes and analyzes certain events that, in one way or another, create a "crisis" in an individual's life. He observed that these "life crises" (e.g., birth, betrothal, marriage, or death) possess a similar general structure, and based on this, developed the theory of what he called rites of passage[1]. When a person undergoes a "life crisis," the usual and customary routines in her/his life are interrupted, changed and distorted. This stage, in which the person experiencing a crisis gets "removed" or isolated from the rest of the community (family, social group, etc.) is called the phase of separation. Although the crisis seems to be an event related to that particular individual, it affects the life of the community that the individual belongs to and will belong to when the rite is completed. This way, the individual's crisis becomes, in many ways, the crisis of the whole community, and the essential purpose of a rite of passage is to re-establish the balance and bring back the customary routines of daily life to the community.

A rite of passage consists of events and activities that assist the person undergoing it to achieve necessary changes, and this process of achieving necessary changes (accompanied by various rituals) constitutes so-called liminal phase of the rite of passage. Changes accomplished, the person is brought back into a community (in all but perhaps a few exceptions different from the one she/he was separated from), and everybody involved resumes their usual life routines. The person undergoing the rite of passage learns about the community that she/he will belong to at the end of the rite. With the support of members belonging to the communities involved, she/he is supposed to find their place in her/his new community. According to Davies (1994, p. 8) "rites of passage often help prepare people for that sense of identity that needs to run alongside the social status according to them," i.e., in our case, as a mathematics student. The 'life equilibrium' has thus been restored, and the communities are ready for the next 'crisis'. This is the final stage (called incorporation) of a rite of passage.

Rite of passage cannot be successful without a proper social context and without the involvement of (relevant) communities. The primary purpose of ceremonies that accompany a rite of passage is to help the person pass from one well-defined, established and accepted position in life to another, which is equally well-defined, established and accepted. The success of the rite depends, in large measure, on the 'logistical' assistance that the parties interested are able to offer to the individual undergoing the rite of passage.

Secondary-Tertiary Mathematics Transition as a Rite of Passage

Mathematics, its language and reasoning, as well as applications of mathematics are integral parts of our daily life and our (science-based and science-

[1] There are other models developed to explain "life crises," for instance the five-stage model by Maddern (1990), based on the study of Australian Aboriginal rites of passage.

dominated) culture. Since "different groups based on professional, social, cultural characteristics, race, ethnicity, gender, etc. have different views of mathematics and its role in the society" (Gruenwald, Klymchuk, & Jovanoski, 2003, p. 238), transition (as well as many other aspects of teaching mathematics) become a multi-layered phenomena, relevant not only to future university students, but also to their families, broader community, and the society as a whole. The report 'Tackling the Mathematics Problem' prepared by the Institute of Mathematics and its Applications, the London Mathematical Society and the Royal Statistical Society (1995) states that[2]:

> There is unprecedented concern…about the mathematical preparedness of new undergraduates…The serious problems perceived by those in higher education are: (i) a serious lack of essential technical facility—the ability to undertake numerical and algebraic calculation with fluency and accuracy; (ii) a marked decline in analytical powers when faced with simple problems requiring more than one step; (iii) a changed perception of what mathematics is—in particular of the essential place within it of precision and proof.

In the section 'A Brief Outline Of The Problem And The Required Response' of the same report (see http://www.lms.ac.uk/policy/tackling/node4.html#SECTION00040000000000000000), we read:

> An equally serious concern of higher education is its observation of a qualitative change in the mathematical preparation of incoming undergraduates…This is no way restricted to those `new undergraduates´ who ten years ago would not have proceeded to higher education. The problem is more serious; it is not just the case that some students are less well prepared, but many `high-attaining´ students are seriously lacking in fundamental notions of the subject.

Further evidence of significant changes in ability ranges of incoming students can be found in Appleby and Cox (2002), and other references. The wide spectrum and sheer magnitude of these changes (and many others) certainly qualify the secondary-tertiary transition as a rite of passage. In Tall (1991), the author states that,

> Advanced mathematics, *by its very nature,* includes concepts which are subtly at variance with naïve experience. Such ideas require an immense personal reconstruction to build the cognitive apparatus to handle them effectively. It involves a struggle…and a direct confrontation with inevitable conflicts, which require resolution and reconstruction. (p. 252)

The struggle "with inevitable conflicts" is an integral part of the liminal phase of a rite of passage, and the "resolution and reconstruction" characterize the incorporation phase. The pains and struggles of the liminal phase are illustrated in the following excerpt (note the 'pathway' metaphor).

> It is clear that the formal presentation of material to students in university mathematics courses—including mathematics majors, but even more for those who take mathematics as a service subject—involves conceptual obstacles that make the pathway very difficult for them to travel successfully. (Tall, 1991, p. 251)

A 'reborn' individual (i.e., a person who has completed the rite of passage)

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[2] The report can be found at www.lms.ac.uk/policy/tackling/report.html. The quote is from the *Summary* section of the report, http://www.lms.ac.uk/policy/tackling/node2.html #SECTION00020000000000000000

needs to attain skills, knowledge and attitudes that the new 'life' expects of her/him:

> And the changes in technology, that render routine tasks less needful of labour, suggest that the time for turning out students whose major achievement is in reproducing algorithms in appropriate circumstances is fast passing and such an approach needs to move to one which attempts to develop much more productive thinking. (Tall, 1991, p. 251)

Transition from secondary to tertiary education is, in many cases, marked by a physical separation of a student from her/his home, parents (extended family, clan, community, caregivers), siblings, and friends. The student undergoing transition needs to abandon most aspects of her/his life as a member of a family (community), in order to assume new roles that will enable them to function in the university setting. This is clearly, the separation phase in the rite of passage process.

Aside from cultural and social changes - the changes that affect the transition to tertiary mathematics are numerous.[3] To name a few:

- Changes in: teaching and learning styles, type of mathematics taught, levels of conceptual understanding, and amount of advanced mathematical thinking required.
- In tertiary mathematics courses, students are exposed to introduction and/or routine use of abstract concepts, ideas and abstract reasoning; they witness an increased emphasis on multiple representations of mathematical objects, precision of mathematical language required and the central role of proofs.
- Students in transition are exposed to numerous didactical differences in approaches to teaching, as well as to a large variety in individual instructors' teaching styles, and changing features of knowledge and knowing.
- Students in transition undergo personal changes requiring an adjustment of learning strategies, time management skills and a shift to more independent living and studying.
- Social and cultural changes with the advent of large class sizes, constant change of groups (unlike in high school), the required level of maturity, exposure to culture different from one's own, the climate in the classroom and the degree of competitiveness.

Following our idea—to view this wide range of changes through the lens of a rite of passage framework—we arrived at several conclusions that we mention and discuss in the forthcoming section. While many of these conclusions are not new, they do confirm that the rite of passage model might be the right one to use. New (or, not so well known) insights that our model generates (such as the time gap that we will discuss) suggest several directions for future research, since some of the issues identified are far from being fully understood.

Implications of the Model

The first, and certainly one of most important messages we need to take from

[3] Borrowed from Guzman, Hodgson, Robert, & Villani, 1998. In this paper, we merely list them without trying to organize or rank according to importance for transition.

the model, is that there is no such thing as a smooth transition, and that this is not even necessarily desirable. Shock is inevitable, we must acknowledge it and deal with it. We must tell our students, in no uncertain terms, that (for most, if not for all) the first semester/first year in university (and longer!) will be a stressful, demanding, life-changing experience, requiring many changes and adjustments, and it will be painful in many ways. But, we should also convince our students that all this, in the end, will be worth it.

The shock of passage from informal to formal language and reasoning is a very common cognitive problem that students experience when learning mathematics (quite often, it surfaces in first-year university courses). Exploring the ways in which students work with definitions in mathematics, authors Alcock and Simpson state that:

> …certain reasoning strategies are inadequate when applied to university mathematics, although they might be efficient and sufficient in non-technical contexts and in the kind of reasoning with specific objects required by [high] school mathematics. (Alcock & Simpson, 2002, p. 33)

University instructors sometimes intensify this problem by mixing colloquialisms and formal mathematics language in their lectures. Textbooks seem to be showing similar attitudes; good illustrations can be found in Kajander and Lovric (in press), where authors examine presentation of mathematics in high school and university calculus textbooks used in Ontario, Canada. For instance: many textbooks introduce and discuss the formal definition of the tangent (limit of secant lines), but the narratives that follow include phrases such as 'tangent touches the curve', 'tangent has one point in common with the graph', which, being vague or only partially correct, could (and do) lead to creation and/or strengthening of students' misconceptions. Or, instead of trying to examine carefully, and deal with, the limits that lead to indeterminate forms (such as $\infty - \infty$), many textbooks use metaphors such as 'conflicting influences', 'competing forces', 'contest', etc. (Kajander & Lovric, in press). The motivation—to try to be "reader-friendly," to "simplify" mathematical reasoning, or to provide "intuitive" explanations to perhaps present the material closer to high-school approach[4]—could turn out to be quite counterproductive.

The rite-of-passage model suggests that we abandon these and similar approaches. Instead, it might be more beneficial to expose entry-level university students to precise mathematical language and rigour of mathematical reasoning (of course, together with helping them develop sound, theory-supported intuition about mathematics concepts and objects they study), and to insist on proper use of mathematical symbols and notation. An example of this is a new course, Math 1C03, at McMaster University, which has been designed with this idea in mind, and no doubt there are courses at other universities where a similar approach has been taken. Informing university instructors in an important part of this shift in teaching, as some seem to be unaware of students' problems in working with formal objects (in this case definitions):

> The student must learn to override these strategies [strategies mentioned in the previous quotation, [op.a.] with the new approach of working from the dictated definitions, but since the role of mathematical definitions usually remains below

[4] As in the case of tangents (indeterminate forms are not taught in high schools in Ontario, Canada).

> the level of consciousness of working mathematicians, this is rarely communicated and might be far from transparent. (Alcock & Simpson, 2002, p. 33)

Working with theorems—understanding what a theorem says, understanding the implication (or equivalence) established by the statement, applying the theorem correctly, dealing with a proof—is equally challenging for students (Tall, 1991, 1992). In terms of learning strategies, perhaps the hardest obstacle is the passage from surface to deep learning. The rite-of-passage model tells us that we should probably not be looking for a 'smooth' way of addressing these problems. Instructors need to spend a considerable amount of time discussing proper, creative ways of learning and understanding mathematics, including examples of situations where surface learning approach fails. Bringing students' attention to misconceptions in their learning process will further help in this respect (Kajander & Lovric, in press).

Furthermore, the rite-of-passage framework suggests that it does not make much sense to try to simulate high school situations within a university context. Recall that a rite of passage is a change from one well-defined situation (high school, or other secondary institution) to another, equally well defined (university or college). No effort is made to 'bring the two situations closer'. Thus, certain practices and attitudes, such as expectations of similarity of coursework and examination questions (Appleby & Cox, 2002, pp. 14-15) may have to be abandoned. Large classes and the use of lecturing as a teaching style should not be viewed as negative aspects of university life that make the secondary-tertiary transition more difficult. There is no evidence to suggest that large classes present an obstacle to learning, or that one cannot learn by listening and taking notes. We believe that it is a mistake to label lecturing as traditional ("sage on the stage," 'talk and chalk') and thus imply that it is outdated and inappropriate. Lecturing should be viewed as an important aspect of the totality of learning and teaching experiences, that spans from inquiry and problem-solving to experiential education, to invited lectures by professionals. One important message we get from rites of passage across cultures is that there is no one, unique way of dealing with "life crises." The abundance of ideas, methods, resources, and creative solutions is amazing.

Rites of passage involve situations that are clearly defined and transparent to everyone involved, so, while still in high school, students should be told (directly and in detail) about their future life as university students. Commonly used phrases such as 'in university you learn from textbooks' or 'your grades will fall by 30 percent in university' have no real meaning or information value for future university students. Evidence, based on a survey of incoming students conducted at McMaster University and anecdotal evidence, suggests that high school students know very little about numerous aspects of university life, such as changes in academic requirements or in teaching style. Furthermore, a rite of passage does not get adjusted for individual needs; expectations are the same of everyone[5]. Its success depends (among other factors) on preparation. Of course, it is not possible to define, in a unique way, what a 'successful' rite of passage in mathematics is, or what it consists of. A 'success' in a transition includes a wide variety of parameters (depending on the individual's experience, expectations, programme of study, institution, etc.), each of which could assume one of many possible values. What

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[5] Although not directly related to transition—we believe that one of major reasons why students from Finland score consistently high in international testing lies in the fact that all students in Finland are taught according to the *exact same* curriculum.

constitutes adequate preparation in mathematics transition? Certain shocks cannot be prevented; however, really good background preparation in an academic sense could significantly help one's transition process.

Secondly, a rite of passage takes time; it cannot, and should not, be accelerated. Organizing an orientation session (or several sessions) or a lecture to incoming students on note-taking, seeking help, time management, or academic dishonesty, may not be really helpful. Students need time to digest thousands of bits of information and to reflect upon them, in order to understand their meaning, ramifications, and to internalize all of it. This is not to suggest that we should not have such sessions, but that we should not expect them to be the end of the matter.

First-year university can be viewed as the incorporation phase of a rite of passage. Relevant community members (course instructors, senior students, teaching assistants, and also peer students, university administrators) need to be involved in this process. It is suggested that a successful transition programme is a continuous effort, where activities and efforts at incorporation are built into all aspects of students' life, from lectures and tutorials to social events and free time. The main reason why we decided to consider such a long incorporation phase in our transition model (ending roughly at the end of first year university) lies in the fact that any significant change requires time to be fully accepted and built into one's cognitive, social, psychological (and other) frameworks. Although not often, we do find the recognition of this fact in literature:

> Also note context and variety of forms $1\text{-}x(2\text{-}x)$ may be misread as $(1\text{-}x)(2\text{-}x)$, and $(2\text{-}x)(1\text{-}x)$ is significantly more difficult than $(x\text{-}2)(x\text{+}1)$ for weaker students, though to us they probably seem equivalent. So in this case students probably need lots of exercises (with answers), rather than teaching … (Appleby & Cox, 2002, p. 13)

Having to do *lots of exercises* takes time. One needs to recognize the fact that

> …large amount of routine basic skills of mathematics is learnt only by repetitive practice; why should mathematics be different from any worthwhile activity? The motivation for such drudgery is that it will be useful in the end. (Appleby & Cox, 2002, p. 16)

Furthermore, we need to keep in mind the time students will need to recall relevant background material. Losses due to absence of mathematics activity can be quite significant (Howson, 1989). According to a study about the summer losses (Cooper, Nye, Charlton, Lindsay, & Greathouse, 1996), as reported on the webpage of the Center for Summer Learning, Johns Hopkins University[6]: "Most students lose about two months of grade level equivalency in mathematical computation skills over the summer months."

Researchers speculate that there is no significant difference between lower and middle-income students when it comes to summer learning losses in mathematics.

It takes time to recall background knowledge and bring one's learning skills to a level necessary to follow a university mathematics course. At McMaster University, introductory 6-7 lectures of the science calculus course[7] (about 20% of the course) are devoted to a review of basic facts about functions, with an emphasis on transcendental functions. First stages of instructor's analysis of students' success in the course suggest that those students who were initially weak in

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[6] http://www.summerlearning.org/ ('About the Center – Know the Facts' link).

[7] Math 1A3; taught by a co-author (Miroslav Lovric) for the last 12 years.

understanding and working with trigonometric, logarithm and exponential functions, on average, experience difficulty in every situation (graphing, critical points, conceptual understanding of differentiation and definite integration, exponential growth and decay, etc.) where these functions appear in the (university calculus) course. In other words, the review that is done initially in the course does not help (all) students. We are certain that the major reason that this is so is the time spent; a brief 2-3 hour review of trigonometry cannot replace studying trigonometry, at an appropriate level and pace, in a high school course.

An issue that requires more attention relates to university mathematics success of students who followed an accelerated scheme in high school (e.g., skipping a grade, or taking a mathematics course at some alternative institution—'going to school after school'). There is data that shows that these students, although by many standards advanced in high school, do not do well in university (Easter, 1990). By following the 'shortest path' (to where?) these students have no time to actually *do* mathematics, to think creatively about it, nor to internalize important ideas and concepts ('accelerated' in some cases means just going over the material faster, and does not involve necessary adjustments, say, in pedagogical approaches). Moreover, evidence shows that a very small number of "accelerated" students opt for a major or minor in mathematics—the subject that they were good at in high school. Quite often, they end up in computer science, economics, etc., with no desire to pursue mathematics seriously (Cohen, 1978). Acceleration is rarely used in Japan, yet their secondary school pupils do very well in comparison with other students (Howson, 1989). Likewise, the purpose of "accelerated" sessions organized at universities ("cram" sessions, review sessions, exam preparation sessions, etc.) may need to be examined. Although these sessions might work for some students, they might not be very helpful (or not helpful at all) for students who did not learn the material in the first place. Furthermore, do they work in the long term, or is it a temporary fix at best?

Time spent learning mathematics in high school (in general; but even more so in the final years of high school) is crucial for students' success in university. Let us briefly mention a study on transition conducted at McMaster University (Kajander & Lovric, 2005). A major new factor contributing to the need for a good understanding of the mathematics transition in Ontario (Canada) lies in the massive reform of secondary education: not only did the secondary curriculum change from a five-year to a four-year programme but significant changes took place in the curriculum itself. These changes in the secondary curriculum resulted in universities experiencing the so-called 'double cohort' of two years of high school students (i.e., those that completed high school in 5 years and those that completed it in 4 years) entering university at the same time. The 5-year cohort students not only spent one extra year in high school, but they also took more mathematics courses during their final year in high school (average of 2.39 courses per student for a 5-year cohort student group, vs. average of 1.81 courses for a 4-year cohort student group). Comparison of the performance of the two cohorts in the same university calculus course was hardly a surprise: the average university calculus mark of a student from a 5-year cohort was two grades[8] higher than the average mark of a student from a 4-year cohort. The 5-year cohort students, on average, are (not surprisingly!) one year older than the average 4-year cohort student. This one-year difference is also believed to play a significant role in

[8] 8, versus 6 (final grades are given on scale 0-12; 12 is the highest, and 1 the lowest passing grade).

transition, for a number of reasons (difficulties in adjustment, maturity, requirements of social life, etc.). We note that a rite of passage occurs when the time is right, i.e., when the participants reach the maturity (mental, physical) deemed necessary to deal with the transition. Finally, to conclude this section—we have already asserted the value and importance of existence of community for individuals in transition—it is evident that community building is a long-term project, and certainly cannot be accomplished at an accelerated pace.

Thirdly, a rite of passage assumes a certain (well-defined) level of maturity of the individual involved, and demands that they take responsibility. Thus, we should not be shy in demanding that our students be responsible, in particular, and most important, in the context of their learning. Rather that expecting students to accept some responsibility for negotiating the transition, quite often university instructors are blamed, or it is suggested that 'traditional' methods (such as lecturing) be abandoned and replaced by ones that involve students more (inquiry, problem-solving, etc.). However, the latter methods need more skilled instructor input than the former, which is not generally available. Whatever methods of instruction are chosen, ultimately it is the student who has to negotiate the transition.

In many situations, instructors may go too far in trying to 'help' their students learn. For instance, instructors provide class notes and make sure that all material is covered in lectures (thus not stimulating their students to maintain good notes and/or to read relevant material from a textbook); sometimes, they organize students into groups, to facilitate group work (why not provide a bulletin board forum and ask students to do it themselves?); or, they provide extra practice or review problems (thus removing the need for students to consult additional resources—libraries are stacked with books students rarely (if ever) use to find extra problems). Too much help seems to disempower. Uncomfortable as the transition may be, if a student successfully manages it then she/he will certainly improve their skills and confidence over time.

Fourthly, some rites of passage (such as initiation rites) involve simulation of dying and rebirth. In this symbolic way, certain attitudes, behaviour, beliefs, knowledge, that belong to 'old' life are abandoned, making way for new attitudes, behaviour, beliefs and knowledge that are appropriate for the new stage in individual's life. Without this form of dying and rebirth a rite of passage cannot be accomplished.

In the case of mathematics, students may acquire a number of attitudes, practices and beliefs in high school that prove to be damaging to their university academic life (a typical, and very common example is the surface learning approach). Sometimes, in an attempt to simplify mathematics, high school students are taught wrong 'facts' (e.g., a "tangent is a line that touches the graph only once," or "a graph cannot cross its horizontal asymptote"). Successful transition programme, thus, must incorporate a process of 'unlearning', i.e., abandoning, or at least weakening cognitive models inhabited by misconceptions[9] (Kajander & Lovric, in press).

One other issue is the 'fear of new'. Students feel somewhat comfortable when they are taught something that they learned (and possibly remember a bit of) in high school. The first time that new material is introduced (in case of the first-year

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[9] Robustness of some of these models suggests that it is easier to learn something when there is no prior knowledge.

calculus course at McMaster University it is the inverse trigonometric functions) there is a considerable level of discomfort among the students. It is normal to feel discomfort during a rite of passage but much easier to deal with if this is expected.

Suggestions for Further Research

Here we briefly outline several possible directions of research in transition that the rite-of-passage model suggests. First and foremost, very little is known about relation between critical periods in the life history of a student and corresponding cognitive processes. For instance, several authors have studied misconceptions in understanding mathematics and the dynamics of cognitive models that contain them (Biza, Souyoul, & Zachariades, 2005 ; Tsamir, 2007; Vosniadou & Verschaffel, 2004). However, very few have focused on the period of transition from high school to university (some work has been done in Kajander & Lovric, in press, and in Tall, 1991).

Looking at existing mechanics of the transition process we notice that, in many cases (Guzman, Hodgson, Robert, & Villani, 1998; Wood, 2001):

- very little information about university, and almost no 'tangible' experience of tertiary academic life and requirements are given in high school;
- the time between the end of high school and the start of university can range from several months to many years; during this time interval, very little thought or action about university, beyond purely logistical issues, occurs;
- universities attempt to smooth the transition by offering bridging courses, and variety of orientation activities.

Perhaps the most striking feature of this process (that contradicts basic principles of the rite-of-passage model) is its discontinuity—both temporal and in terms of community. The fact that almost every paper on transition calls for establishment of a closer communication between high school and university mathematics instructors means that this important part of transition community-building is far from resolved. Our model (and common sense, as well as experiences of many instructors) suggests that this is a crucial issue in transition. Although it is addressed in literature and implemented to an extent (see, for instance, Demana (1990) for a local initiative, and Howe (1998), or Wood (2001) for global (country-wide) efforts), more work is needed, as effective and productive high school-university communication is still not common practice.

An equally important direction of research (and action!) concerns the temporal gap between high school and university. Students' mathematics activities (or lack of) in the actual gap (time between last day in high school and first day at university) are rarely the focus of research or action, with programmes such as 'Semester Zero' at University of Technology in Sydney, Australia (Wood, 2001, p. 96) still rare.

Universities offer a variety of bridging or remedial courses. In some cases, such courses have a stigma attached to them (Appleby & Cox, 2002; Clark, 1995) which, of course, lowers students' self-confidence and motivation (perhaps such courses should be rethought or removed; no rite of passage ever leaves a participant in a state of shame or embarrassment, or lets her/him down—on the contrary, it empowers the participants in multiple ways). Referring students to use computers to do remedial work is rarely beneficial (Appleby & Cox, 2002). Even with our best intentions we still lack conclusive research on ways in which modern technology possibly enhances one's learning; i.e., the environment, in the sense of rite of passage, is unclear and not well understood, which no rite of passage allows to happen. Bridging courses do not (in many cases) help really weak students (Wood, 2001), for many reasons, which indicates a failure in design of such courses; a bridging course (like a rite of passage) needs to be for all, not just for some or for a majority.

The rite-of-passage model applied to mathematics suggests that one could analyse problems and issues in transition by studying their dynamics within the three constituent stages of transition:

- separation (from high school); this stage takes place while students are still in high school, and includes anticipation of forthcoming university life;
- liminal phase (from high school to university) includes the end of high school, the time between high school and university, and the start of first year at a university;
- incorporation (into university) includes, roughly, first year at a university.

Consider, for instance, previously mentioned case of transcendental functions. Separation: how do we teach these functions to students who will be coming to university? Are certain topics of particular importance? Liminal phase: How do we encourage/motivate students to do work on their own, to prepare themselves for university study? Are there good materials (online, or printed?) that could help them recall and brush up on graphs of trigonometric functions, laws of logarithms, etc.? How can we prepare students in the separation phase so that they can study/review transcendental functions on their own in the liminal phase? What knowledge and skills related to these functions are robust (i.e., harder to forget)? Incorporation: what are effective ways of building up on students' prerequisite knowledge, to introduce possible new ways of thinking, as well as new facts and ideas about transcendental functions?

Our society is highly individualized. It is important to discover how we can build a community that will make sense to an individual, and that will help the individual navigate the passage with the maximum benefit to them and to the academic community they join.

A Critique of the Model

Possible critiques of this model could include the remark that a rite of passage relates to an individual's physical growth and life (birth, initiation and social puberty, marriage, etc.). Furthermore, a rite of passage takes place 'when the time is right', and that is determined by a number of factors, most of which are not extrinsic. On the other hand, one could claim that secondary to tertiary transition is primarily related to one's academic life, and is therefore related primarily to the mind and not to (adequate) physical maturity. However, in most Western societies the vast majority of students have this transition pegged to age. Moreover, the transition has to begin when a student finishes secondary education, which is not internal to the individual, but uniform to all and imposed by a number of external factors.

This paper and the examples we have used, reflect mostly North-American reality. We do not claim that the model is universal, but believe that some issues and ideas presented here could be applied in contexts much beyond North America.

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