Mathematics Bridging Courses and Success in First Year Calculus

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Students entering university with insufficient mathematics preparation for the courses they intend to study is an increasing problem. We show evidence of how withdrawal rates, failure rates, and final marks in a first year calculus unit are strongly associated with the level of mathematics studied at school, the assumed knowledge published for the degree and enrolment in a bridging course. Bridging course students were, on the whole, able to pass their first semester university calculus-based subject; however, they did not achieve at the level of their mathematically well-prepared peers.

1. Introduction

In 2012, Australia's Chief Scientist in his report to the Prime Minister argued that 'Mathematics, Engineering and Science (MES) are fundamental to shaping the future of Australia, and the future of the world' [1]. Yet, despite the importance of mathematics to society, the number of students studying the higher levels of mathematics in the senior year(s) of secondary school in Australia is falling [2] and similar declines have been reported in many other Western developed countries [3, 4].

In New South Wales (NSW) students can choose to study mathematics in senior secondary school at one of four levels for their Higher School Certificate (HSC) – the qualification students receive at the end of year 12. These levels are an elementary level – with no calculus content and considered not suitable for tertiary study – called General Mathematics, an intermediate level called HSC Mathematics and two advanced levels called HSC Mathematics Extension 1 and HSC Mathematics Extension 2. Of the students eligible for an Australian Tertiary Admission Rank (ATAR), a rank used to select school leavers for admission to university, the percentage of students studying mathematics at an intermediate or advanced level for their HSC has fallen from 61% in 1992 to 35% in 2012 – an alarming trend [5].

In Australia, university entry criteria may also inadvertently exacerbate the problem of low uptake of the higher levels of mathematics in senior secondary school [6]. Students are admitted to university primarily on the basis of their ATAR which allows the comparison of students who have studied different combinations of subjects for their end of Year 12 qualification. At many universities, including the Authors' institution, there are no subject prerequisites for entry into degree programs. Rather students are advised of the level of senior secondary mathematics that is 'assumed knowledge' for each degree, but nevertheless may be offered a place in a degree solely based on their ATAR score. This creates a 'tension' between a student gaining access to a certain degree and being adequately prepared mathematically for that degree [7]. Students, who have elected not to study intermediate or advanced mathematics at senior secondary school, have a difficult choice; either they accept that they have closed their access to quantitative disciplines for the immediate future or they enrol in a degree program for which they are mathematically under-prepared.

The study of mathematics at university features not only in science and engineering degree programs but also in many other tertiary disciplines including pharmacy, economics, agriculture and information technology. Failure to study mathematics at an appropriate level may have serious consequences for a student's success at university. In a study conducted at a Canadian university, Kajander and Lovric [8] concluded that students' performance in a first year calculus course was strongly correlated to the amount of time they spent learning mathematics in the final years of secondary school. In an Australian study, Rylands and Coady [9] analysed data on the performance of first year university students in four mathematics and mathematics-related subjects. They concluded that a student's secondary school mathematics background, and not their ATAR, has a dramatic effect on pass rates: 77% of students with only elementary mathematics failed a basic mathematics subject.

Large universities may address the diversity in the mathematics preparation of commencing students by offering first year mathematics subjects at a number of levels [6]. Many universities, including the Authors' university, attempt to ameliorate the difficulties encountered by mathematically under-prepared students by offering preparatory programs (Bridging Courses) that enable students to obtain prerequisite or assumed knowledge in mathematics *before* commencing their degree program [10].

In Australasia, mathematics bridging courses have been part of the tertiary preparation scene for many years but there has been little research on their effectiveness [9-11]. In 2006, Galligan and Taylor [11] posed two (of four) unanswered questions within bridging mathematics as:

How is success defined in bridging mathematics activities?

Are successful bridging mathematics students successful university students?

For the first question, there are inherent difficulties in defining and measuring success in bridging courses. Godden & Pegg [12] suggest that formal evaluation of bridging mathematics programs may be contrary to the aims of the programs, and undermine their major strengths of flexibility and student-centred approach. They argue that traditional evaluative techniques are 'just not possible' and 'risk losing the essence of the support and assistance so necessary for these students'.

For the second question, internationally, bridging mathematics programs have been shown to be highly effective at resolving skill deficiencies for some students [8, 13]. In a large US study, Bahr [13, p.442] found that 'remediation has the capacity to fully resolve the academic disadvantage of math skill deficiency' for the quarter of students who 'remediated successfully', but the likelihood of successful remediation declined sharply as the 'depth of remedial need' increased. The latter finding echoes Wood's [14] remark that bridging programs do not work for very mathematically weak students.

In this paper, we do not attempt to define or measure success within the bridging courses themselves. Rather, we first examine the results of students enrolled in a first semester university mathematics subject known to have a sizeable number of students who are demonstrably mathematically under-prepared. We then ask and answer the questions:

How do the results of mathematics bridging course students compare to those of similarly under-prepared students who do not take the mathematics bridging course?

How do the results of mathematics bridging course students compare to those of students who studied the appropriate level of mathematics in senior secondary school?

2. Methodology

2.1 Context of the study

The University of Sydney is a major research-intensive university in Australia with over 50,000 students. Mathematics is taught in first year at four levels ranging from an introductory calculus subject to advanced mathematics subjects. Students are advised of the level of mathematics appropriate for them, but some degrees have requirements that lead to students enrolling in subjects for which they are demonstrably mathematically under-prepared [15].

For this reason our study concentrates on the performance of students in the first semester subject called Differential Calculus. This subject has HSC Mathematics Extension 1 (advanced mathematics) as assumed knowledge and relies on that content knowledge. Its first semester companion subject is Linear Algebra and students usually go on to complete second semester subjects called Integral Calculus and Modelling and Statistics. The four subjects together make up one-quarter of a full-time load for a year.

Our university offers two different mathematics bridging courses; 2 Unit and Extension 1 in February each year. The bridging courses include 24 hours of class teaching held over 12 days. In 2013, they had a total enrolment of 365 students; 42% (n=155) in the Extension 1 course. The courses are fee-paying and open to all, including students enrolling at other universities. The Extension 1 bridging course is for students who have previously studied HSC Mathematics (intermediate) and wish to enrol in subjects that have an assumed knowledge of HSC Mathematics Extension 1(advanced) including Differential Calculus. The 2 Unit bridging course is for students who have previously studied General Mathematics (elementary) or no mathematics in senior secondary school and are commencing degrees or enrolling in subjects with an assumed knowledge of HSC Mathematics (intermediate). As many of the 2 Unit bridging course students subsequently enrol in university mathematics or statistics subjects in other faculties or are eligible to enrol in our introductory calculus subject, we will not consider these students further.

2.2 Data collection

Data were collected from the University's database on all students who were enrolled in Differential Calculus for the years 2012 and 2013. Further information was obtained from internal School of Mathematics and Statistics databases and the Faculty of Science bridging course database.

Students were only included in the study, if:

- they had applied to the Universities Admission Centre (UAC) for the first time in the two years previous to their enrolment at the University, and
- the subjects they studied for the HSC were recorded together with their marks.

Any additional criteria will be given at the beginning of a section.

2.3 Limitations

Our bridging course students were self-selecting and were prepared to commit the time and money to learning mathematics prior to entering university. Many other variables including motivation levels and the amount of mathematics support sought and provided are unknown and cannot be controlled. Our bridging course students may not be representative of bridging course students at other universities.

3. HSC Background in Mathematics vs. Success

In this section, we will report on students who studied some level of mathematics for their HSC and completed Differential Calculus. Figure 1 shows the distribution of final marks in Differential calculus grouped by the highest level of mathematics studied for the HSC. The results for students who studied General Mathematics (elementary) or no mathematics are not shown due to the very small number of students (<10 students). We found very strong evidence ($p < 0.0001$) that students who had completed higher levels of mathematics at school attained appreciably higher final marks in first semester calculus. In 2012, the means for students who had completed HSC Mathematics, Extension 1 and Extension 2 were 49.9, 59.5 and 69.5 respectively. The corresponding means for 2013 were 50.4, 58.4 and 67.1 respectively. The estimated differences in means of between 8 and 10 marks between these groups were all statistically significant. In the subsequent sections, we divide the students who had studied HSC Mathematics for their HSC into two groups; those students who enrolled in the Extension 1 mathematics bridging course and those who did not enrol in the bridging course. We then explore to what extent enrolling in a bridging course can ameliorate this difference in marks.

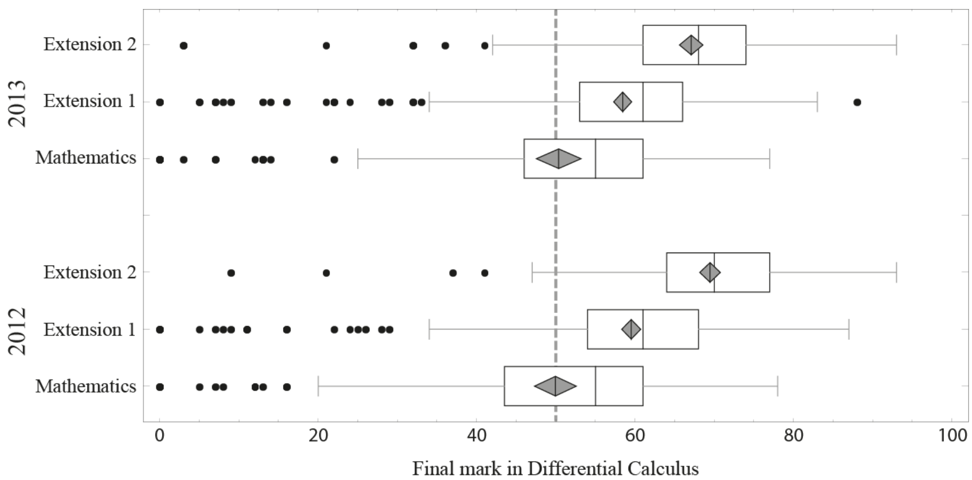

Figure 1. Boxplots of the distribution of final marks in Differential Calculus by the highest level of mathematics studied for the HSC. The shaded diamonds indicate the mean and its 95% confidence limits. The pass-fail boundary at 50% is indicated by a vertical dashed line.

4. Mathematics Bridging Course Students

In this section, we will report on all students who studied HSC Mathematics (intermediate) as the highest level of mathematics for their HSC and enrolled in (but not necessarily completed) Differential Calculus. All percentages will be rounded to the nearest whole number. Table 1 shows summary demographics for the students in 2012 and 2013 that satisfied these criteria. The demographics are very consistent from one year to the next. Each year's sample has about 200 students; almost all are 20 years old or younger, about three-quarters are male, over 80% are enrolled in science, engineering or computer science degrees and about 40% chose to attend a bridging course. Throughout this paper we will refer to the students who enrolled in the Extension 1 Mathematics Bridging Course as the BC group, and those who did not as the non-BC group. Note that there are no 2 Unit bridging course students in the non-BC group.

Table 1. Background information on all students who had studied HSC Mathematics (intermediate) as the highest level of mathematics for their HSC and enrolled in (but not necessarily completed) Differential Calculus.

	2012	2013
Total number of students satisfying criteria for this study	200	196
Male students	146	141
Students 20 years old or younger	198	196
Students enrolled in degrees or combined degrees with an engineering, science or computer science component	176	164
Students who enrolled in the bridging course	81	78

We compared some pre-existing attributes of students in the BC group with those in the non-BC group and found no evidence ($p > 0.05$) of a difference in gender, ATAR or mark achieved in HSC Mathematics (intermediate) at school. This suggests that these factors are not associated with whether students chose to enrol in a bridging course. However, the published assumed knowledge for their degrees was an important factor. The students were enrolled in over 20 degree programs, which we classified into two groups according to whether or not the programs had HSC Mathematics Extension 1 (advanced) published as the assumed knowledge in UAC Guide 2013. The degrees with Extension 1 as assumed knowledge included all engineering degrees and combined degrees such as the Bachelor of Engineering/Bachelor of Commerce, the Bachelor of Resource Economics and the Bachelor of Project Management. The degrees that did not have Mathematics Extension 1 as assumed knowledge included the Bachelor of Science, Bachelor of Computer Science and Technology and the Bachelor of Liberal Arts and Science. Most, but not all, the degrees in this latter group have HSC Mathematics (intermediate) as assumed knowledge. The number of students in each category is given in Table 2. The data show very strong evidence of an association between bridging course enrolment and the assumed knowledge for the student's degree (Fisher's Exact Test, $p \leq 0.0005$). For the degrees with assumed knowledge of HSC Mathematics Extension 1 (advanced), slightly over half the students with only an HSC Mathematics (intermediate) background chose to do the bridging course.

Table 2. Number of BC and non-BC students grouped according to the level of assumed knowledge published for their degree. The *p*-values are given for Fisher's Exact test.

	2012		2013	
	Non-BC	BC	Non-BC	BC
HSC Extension 1 not assumed	74	30	64	22
HSC Extension1 assumed	45	51	54	56
	$p \leq 0.0005$		$p \leq 0.0005$	

5. Impact of Bridging Course

We analysed three indicators that might be related to the impact of enrolling in bridging courses: withdrawal rates, failure rates and final marks. We show below that participation in a bridging course was correlated with favourable changes in each of these indicators. We also then compared the final mark of bridging course students to those who had studied HSC Mathematics Extension 1 at school. We will refer to the latter students as the HSC Extension 1 group.

5.1 Withdrawal rates

Students who were awarded a final mark in Differential Calculus will be considered to have completed, those who were not will be classified as withdrawals. Table 3 shows strong evidence (Fisher's Exact test: $p<0.0001$) of an association between the BC group and low withdrawal rates. In 2012, only 5% of the BC group compared to 34% of the non-BC group withdrew before completing Differential Calculus. The corresponding withdrawal rates for 2013 were 9% and 37% respectively.

Table 3. Student withdrawals and completions for BC and non-BC students. Withdrawals include students who discontinued not fail and discontinued fail.

	2012		2013	
	BC	Non-BC	BC	Non-BC
Withdrawals	4	40	7	44
Completions	77	79	71	74
	$p \leq 0.0001$		$p \leq 0.0001$	

Tables 4 and 5 subdivide these groups further depending on whether or not HSC Mathematics Extension 1 was the assumed knowledge for their degrees. While substantial fewer students withdrew from the subject Differential Calculus if Mathematics Extension 1 was the assumed for their degree, nevertheless for both groups there was an association between the BC group and low withdrawal rates. The evidence suggests that enrolling in a bridging course has a positive impact on a students' retention in university mathematics subjects irrespective of the assumed knowledge for their degrees.

Table 4. As in Table 3 but only for students in degrees with HSC Mathematics Extension 1 as assumed knowledge

	2012		2013	
	BC	Non-BC	BC	Non-BC
Withdrawals	2	12	3	11
Completions	49	33	53	43
	p=0.023		p=0.003	

Table 5. As in Table 3 but only for students in degrees where HSC Mathematics Extension 1 was not assumed knowledge.

	2012		2013	
	BC	Non-BC	BC	Non-BC
Withdrawals	2	28	4	33
Completions	28	46	18	31
	p=0.002		p=0.007	

5.2 Failure rates

Table 6 summarises the pass and failure rates for those students who completed Differential Calculus. For the 2012 data, the failure rate for students who had enrolled in the bridging course was less than half that for those who had not enrolled; 17% and 38% respectively. The 2013 data also shows a smaller failure rate for bridging course students; 23% compared to 36%, but the evidence for an association is somewhat weaker. Table 7 compares the pass and failure rates of the students in the BC group to those students in the HSC Extension 1 group. Although the failure rates for the HSC Extension 1 groups are about 5% lower, there is no statistical evidence of an association between these groups and passing or failing Differential Calculus.

Table 6. Number of students failing and passing for the BC and non-BC groups. The category Fail includes Absent fail.

	2012		2013	
	BC	Non-BC	BC	Non-BC
Fail	13	30	16	27
Pass or above	64	49	55	47
	p=0.004		p=0.072	

Table 7 compares the pass and failure rates of the students in the BC group to those students in the HSC Extension 1 group. Although the failure rates for the HSC Extension 1 groups are about 5% lower, there is no statistical evidence of an association between these groups and passing or failing Differential Calculus.

Table 7. Number of students failing and passing for the BC group and students who studied HSC Mathematics Extension 1 at high school.

	2012		2013	
	BC	Extension 1	BC	Extension 1
Fail	13	49	16	79
Pass or above	64	415	55	414
	p=0.12		p=0.18	

5.3 Final marks

The results for students in the BC group and non-BC group, and the BC group and HSC Extension 1 group were compared. We found evidence that the students' background and enrolment in a bridging course are each associated with appreciably higher final marks. Figure 2 shows the distribution of final marks for the non-BC, the BC and HSC Extension 1 groups. In 2012 the mean marks for these three groups were 46.7, 53.2 and 59.6 respectively. The estimated mean difference of 6.5 marks between the non-BC and BC groups was statistically significant (t=2.50, df=154, p=0.015, with a 95% confidence interval from 1.3 to 11.8 marks). The estimated mean difference of 6.3 marks between the BC and Extension 1 groups was also statistically significant (t=3.87, df=539, p=0.0001, with a 95% confidence interval 3.1 to 9.5 marks).

Similarly, in 2013 the mean marks for these three groups were 47.3, 53.5 and 58.4 respectively. The estimated mean difference of 6.2 marks between the non-BC and BC groups was statistically significant (t=2.17, df=143, p=0.031 with 95% confidence interval 0.6 to 11.8 marks). The estimated mean difference of 4.9 marks between the BC and Extension 1 groups was also statistically significant (t=3.00, df=562, p=0.003, with a 95% confidence interval 1.7 to 8.2 marks).

In summary, the mean marks of the BC group were approximately halfway between those of the non-BC group and the Extension 1 group.

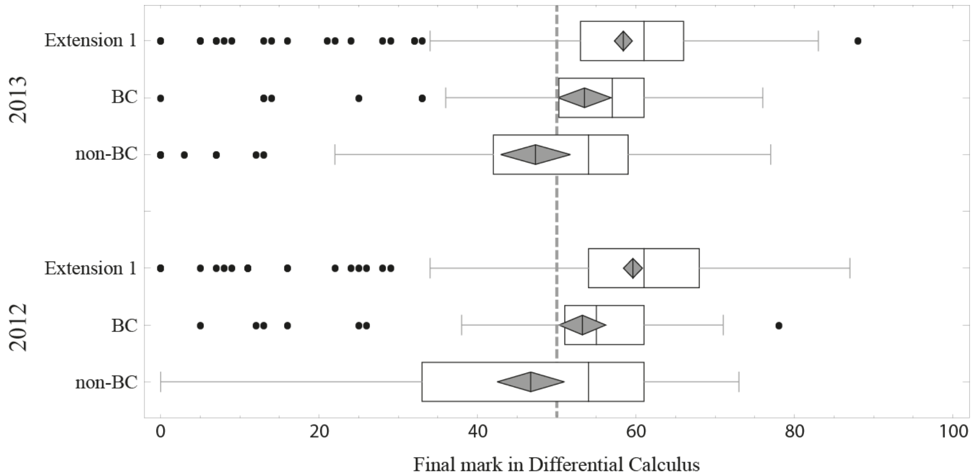

Figure 2. Boxplots of the distribution of marks for BC, non-BC and HSC Extension 1 students. The data is represented as in Figure 1.

6. Discussion

In this study, we do not examine students' success during the mathematics bridging course but look beyond the bridging course to investigate the outcomes for a group of bridging course students in their university mathematics subjects relative to their cohort.

We found that, in general, students who commenced university mathematically under-prepared for their university mathematics subjects were significantly more likely to withdraw from or fail those subjects. This will come as no surprise to university mathematics teachers and is consistent with the research of Rylands and Coady [9]. It is of concern, however, that students may make decisions not to study the higher levels of mathematics in senior secondary school based on the short term goal of gaining a university place, not realising that by doing so they may severely compromise their future success at university.

Consistent with previous research [8], we found that the students, who had previously studied intermediate mathematics at senior secondary school and were mathematically under-prepared for their studies but had invested their time and money prior to university by enrolling in a mathematics bridging course, were significantly less likely to withdraw from and more likely to succeed at their university mathematics subjects compared to students who did not enrol in a bridging course. While improving student success is pleasing, reducing the rate of withdrawal for some students may be more problematic. For many students, not withdrawing from their mathematics subjects may be the appropriate action even at the risk of failing, but for others it may be more prudent to withdraw and enrol in a subject for which they are better suited mathematically. This highlights the need for good and timely advice for students as they select their university mathematics subjects.

While the mathematics bridging course students in this study were generally successful in their university mathematics subjects, our bridging course students did not achieve at the level of those who studied advanced mathematics at senior secondary school. Furthermore, our study does not consider whether attending a mathematics bridging course improves the outcomes in university mathematics subjects for students who studied elementary mathematics (or no mathematics) at senior secondary school. For these students, the research suggests that the gap between elementary mathematics and intermediate mathematics may be a bridging too far for attendance at short bridging courses prior to university to realistically address [13, 14]. Therefore, secondary school students should be discouraged from thinking that attending a mathematics bridging course is a panacea for failing to study an appropriate level of mathematics in secondary school – it is not an adequate alternative.

We do not claim a causal link between mathematics bridging course attendance and a student's success in his or her university mathematics subjects and hence a measure of its efficacy. Previous research suggests that students who enrol in mathematics bridging courses value the bridging courses not only as a means of ameliorating prior difficulties with mathematics and improving their mathematics learning but as a means to facilitate their transition to university [16]. The success of our bridging course students in their university mathematics studies may point to the vital contribution that mathematics bridging courses play as a vehicle to nurture the student's 'will to learn' which Barnett [17] suggests is of crucial importance to students' academic persistence 'ahead of both knowledge and skill' in the face of uncertainty and difficulty [17, 18].

7. Conclusion

Our study highlights the importance of universities accurately informing prospective students of the appropriate level of mathematics to study at senior secondary school for their degree programs and the possible consequences for students of not doing so. Although 75% of underprepared students who enrolled in a bridging course were able to pass their first semester university calculus-based subject, they still did not achieve at the level of their well-prepared colleagues. Whether or not this disadvantage persists in non-calculus based university mathematics subjects or to later semesters is a subject for our future research.

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