2 Unit Bridging Course – Day 1

More on Functions

Collin Zheng

THE UNIVERSITY OF SYDNEY

Previously . . .

In our introduction, we motivated the concept of functions with some intuitive and real-world examples.

This included our 'drug function', which calculated the dosage d of threadworm medication required for a person of weight w, defined precisely by the formula:

$$d = f(w) = \frac{w}{5}.$$

It's important to realise that a function's formula does not need to necessarily arise from real-world phenomena.

Example

For any number x, suppose we stipulate a purely mathematical rule where x is squared and 3 is then added.

This gives rise to a function $f(x)$ defined by the formula:

$$f(x) = x^2 + 3.$$

This is an example of a **quadratic** function, which will be studied in more depth in days 3 and 4.

It's important to realise that a function's formula does not need to necessarily arise from real-world phenomena.

Example

For any number x, suppose we stipulate a purely mathematical rule where x is squared and 3 is then added.

This gives rise to a function $f(x)$ defined by the formula:

$$f(x) = x^2 + 3.$$

This is an example of a **quadratic** function, which will be studied in more depth in days 3 and 4.

Abstract functions

It's important to realise that a function's formula does not need to necessarily arise from real-world phenomena.

Example

For any number x, suppose we stipulate a purely mathematical rule where x is squared and 3 is then added.

This gives rise to a function $f(x)$ defined by the formula:

$$f(x) = x^2 + 3.$$

This is an example of a **quadratic** function, which will be studied in more depth in days 3 and 4.

It's important to realise that a function's formula does not need to necessarily arise from real-world phenomena.

Example

For any number x, suppose we stipulate a purely mathematical rule where x is squared and 3 is then added.

This gives rise to a function $f(x)$ defined by the formula:

$$f(x) = x^2 + 3.$$

This is an example of a **quadratic** function, which will be studied in more depth in days 3 and 4.

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19.$
- $f(-2) = (-2)^2 + 3 = 7.$
- $f(x+h) = (x+h)^2 + 3.$

Practice Questions

For practice, try evaluating the following:

- $f(-5).$
- $f\left(\frac{a-b}{a^2+b^2+c^2}\right).$

THE UNIVERSITY OF SYDNEY

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19.$
- $f(-2) = (-2)^2 + 3 = 7.$
- $f(x+h) = (x+h)^2 + 3.$

Practice Questions

For practice, try evaluating the following:

- $f(-5).$
- $f\left(\frac{a-b}{a^2+b^2+c^2}\right).$

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19.$
- $f(-2) = (-2)^2 + 3 = 7.$
- $f(x + h) = (x + h)^2 + 3.$

Practice Questions

For practice, try evaluating the following:

- $f(-5).$
- $f\left(\frac{a-b}{a^2+b^2+c^2}\right).$

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19$.
- $f(-2) = (-2)^2 + 3 = 7$.
- $f(x+h) = (x+h)^2 + 3$.

Practice Questions

For practice, try evaluating the following:

- $f(-5)$.
- $f\left(\frac{a-b}{a^2+b^2+c^2}\right)$.

THE UNIVERSITY OF SYDNEY

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19.$
- $f(-2) = (-2)^2 + 3 = 7.$
- $f(x+h) = (x+h)^2 + 3.$

Practice Questions

For practice, try evaluating the following:

- $f(-5).$
- $f\left(\frac{a-b}{a^2+b^2+c^2}\right).$

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19$.
- $f(-2) = (-2)^2 + 3 = 7$.
- $f(x+h) = (x+h)^2 + 3$.

Practice Questions

For practice, try evaluating the following:

- $f(-5)$.
- $f(\frac{a-b}{a^2+b^2+c^2})$.

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19$.
- $f(-2) = (-2)^2 + 3 = 7$.
- $f(x + h) = (x + h)^2 + 3$.

Practice Questions

For practice, try evaluating the following:

- $f(-5)$.
- $f(\frac{a-b}{a^2+b^2+c^2})$.

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19$.
- $f(-2) = (-2)^2 + 3 = 7$.
- $f(x + h) = (x + h)^2 + 3$.

Practice Questions

For practice, try evaluating the following:

- $f(-5)$.
- $f(\frac{a-b}{a^2+b^2+c^2})$.

Abstract functions (cont.)

Some example evaluations for $f(x) = x^2 + 3$:

- $f(4) = 4^2 + 3 = 19$.
- $f(-2) = (-2)^2 + 3 = 7$.
- $f(x+h) = (x+h)^2 + 3$.

Practice Questions

For practice, try evaluating the following:

- $f(-5)$.
- $f(\frac{a-b}{a^2+b^2+c^2})$.

Solutions

As with the example evaluations above, the procedure is to simply replace x wherever it occurs in the formula $f(x) = x^2 + 3$ with the input. Do not feel intimidated if the input is complicated – the procedure remains the same! Finally you should simplify your answer if possible.

- $f(-5) = (-5)^2 + 3 = 25 + 3 = 28.$
- $f(\frac{a-b}{a^2+b^2+c^2}) = (\frac{a-b}{a^2+b^2+c^2})^2 + 3.$

Abstract functions (cont.)

Solutions

As with the example evaluations above, the procedure is to simply replace x wherever it occurs in the formula $f(x) = x^2 + 3$ with the input. Do not feel intimidated if the input is complicated – the procedure remains the same! Finally you should simplify your answer if possible.

- $f(-5) = (-5)^2 + 3 = 25 + 3 = 28.$
- $f\left(\frac{a-b}{a^2+b^2+c^2}\right) = \left(\frac{a-b}{a^2+b^2+c^2}\right)^2 + 3.$

THE UNIVERSITY OF SYDNEY

Abstract functions (cont.)

Solutions

As with the example evaluations above, the procedure is to simply replace x wherever it occurs in the formula $f(x) = x^2 + 3$ with the input. Do not feel intimidated if the input is complicated – the procedure remains the same! Finally you should simplify your answer if possible.

- $f(-5) = (-5)^2 + 3 = 25 + 3 = 28.$
- $f(\frac{a-b}{a^2+b^2+c^2}) = (\frac{a-b}{a^2+b^2+c^2})^2 + 3.$

Abstract functions (cont.)

Solutions

As with the example evaluations above, the procedure is to simply replace x wherever it occurs in the formula $f(x) = x^2 + 3$ with the input. Do not feel intimidated if the input is complicated – the procedure remains the same! Finally you should simplify your answer if possible.

- $f(-5) = (-5)^2 + 3 = 25 + 3 = 28.$
- $f(\frac{a-b}{a^2+b^2+c^2}) = (\frac{a-b}{a^2+b^2+c^2})^2 + 3.$

Naming *variables*

It's important to note that the x in $f(x) = x^2 + 3$ is only a 'dummy variable' that symbolises the input for the function, whether it be a simple number or something very complicated like the $\frac{a-b}{a^2+b^2+c^2}$ term above.

Thus, we certainly could have written the function as $f(a) = a^2 + 3$ or $f(\alpha) = \alpha^2 + 3$. Although x is often preferred by convention, which letter or symbol one uses is ultimately unimportant. That is, the x in $f(x) = x^2 + 3$ is *interchangeable*.

So ultimately, $f(x) = x^2 + 3$, $f(a) = a^2 + 3$ and $f(\alpha) = \alpha^2 + 3$ are all the *same* functions!

Naming *variables*

It's important to note that the x in $f(x) = x^2 + 3$ is only a 'dummy variable' that symbolises the input for the function, whether it be a simple number or something very complicated like the $\frac{a-b}{a^2+b^2+c^2}$ term above.

Thus, we certainly could have written the function as $f(a) = a^2 + 3$ or $f(\alpha) = \alpha^2 + 3$. Although x is often preferred by convention, which letter or symbol one uses is ultimately unimportant. That is, the x in $f(x) = x^2 + 3$ is *interchangeable*.

So ultimately, $f(x) = x^2 + 3$, $f(a) = a^2 + 3$ and $f(\alpha) = \alpha^2 + 3$ are all the *same* functions!

Naming *variables*

It's important to note that the x in $f(x) = x^2 + 3$ is only a 'dummy variable' that symbolises the input for the function, whether it be a simple number or something very complicated like the $\frac{a-b}{a^2+b^2+c^2}$ term above.

Thus, we certainly could have written the function as $f(a) = a^2 + 3$ or $f(\alpha) = \alpha^2 + 3$. Although x is often preferred by convention, which letter or symbol one uses is ultimately unimportant. That is, the x in $f(x) = x^2 + 3$ is *interchangeable*.

So ultimately, $f(x) = x^2 + 3$, $f(a) = a^2 + 3$ and $f(\alpha) = \alpha^2 + 3$ are all the *same* functions!

Naming *functions*

Suppose in a maths question you were asked to plot the graphs of three different functions together on one xy-plane:

- $y = 2x$
- $y = 1$
- $y = x^2 - 1$

It would be unwise to name all three functions $f(x)$, since any verbal or written reference to "the function $f(x)$" will only cause confusion, given that it's the name given to all three functions!

Naming *functions*

Suppose in a maths question you were asked to plot the graphs of three different functions together on one xy-plane:

- $y = 2x$
- $y = 1$
- $y = x^2 - 1$

It would be unwise to name all three functions $f(x)$, since any verbal or written reference to "the function $f(x)$" will only cause confusion, given that it's the name given to all three functions!

Naming *functions*

Suppose in a maths question you were asked to plot the graphs of three different functions together on one xy-plane:

- $y = 2x$
- $y = 1$
- $y = x^2 - 1$

It would be unwise to name all three functions $f(x)$, since any verbal or written reference to "the function $f(x)$" will only cause confusion, given that it's the name given to all three functions!

Naming *functions*

The convention used to overcome this problem is to rename the f in $f(x)$. For instance, we can name:

- $y = 2x$ as $\boldsymbol{f}(x) = 2x$
- $y = 1$ as $\boldsymbol{g}(x) = 1$
- $y = x^2 - 1$ as $\boldsymbol{h}(x) = x^2 - 1$

Under this tradition, the f in $y = f(x)$ no longer just means that y is a function of x, but also as a **name** for the function itself.

This convention is ubiquitous throughout maths, providing us the capability to assign distinct names to different functions whenever they are to be considered under the same setting.

Naming *functions*

The convention used to overcome this problem is to rename the f in $f(x)$. For instance, we can name:

- $y = 2x$ as $\boldsymbol{f}(x) = 2x$
- $y = 1$ as $\boldsymbol{g}(x) = 1$
- $y = x^2 - 1$ as $\boldsymbol{h}(x) = x^2 - 1$

Under this tradition, the f in $y = f(x)$ no longer just means that y is a function of x, but also as a **name** for the function itself.

This convention is ubiquitous throughout maths, providing us the capability to assign distinct names to different functions whenever they are to be considered under the same setting.

Naming *functions*

The convention used to overcome this problem is to rename the f in $f(x)$. For instance, we can name:

- $y = 2x$ as $\boldsymbol{f}(x) = 2x$
- $y = 1$ as $\boldsymbol{g}(x) = 1$
- $y = x^2 - 1$ as $\boldsymbol{h}(x) = x^2 - 1$

Under this tradition, the f in $y = f(x)$ no longer just means that y is a function of x, but also as a **name** for the function itself.

This convention is ubiquitous throughout maths, providing us the capability to assign distinct names to different functions whenever they are to be considered under the same setting.

Naming *functions*

The convention used to overcome this problem is to rename the f in $f(x)$. For instance, we can name:

- $y = 2x$ as $\boldsymbol{f}(x) = 2x$
- $y = 1$ as $\boldsymbol{g}(x) = 1$
- $y = x^2 - 1$ as $\boldsymbol{h}(x) = x^2 - 1$

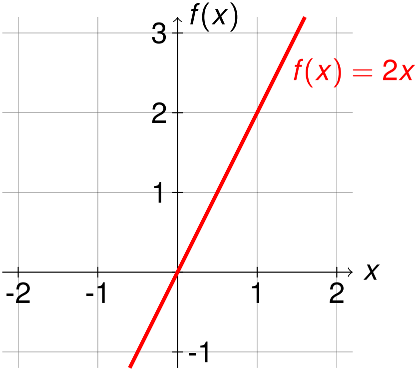
Under this tradition, the f in $y = f(x)$ no longer just means that y is a function of x, but also as a **name** for the function itself.

This convention is ubiquitous throughout maths, providing us the capability to assign distinct names to different functions whenever they are to be considered under the same setting.

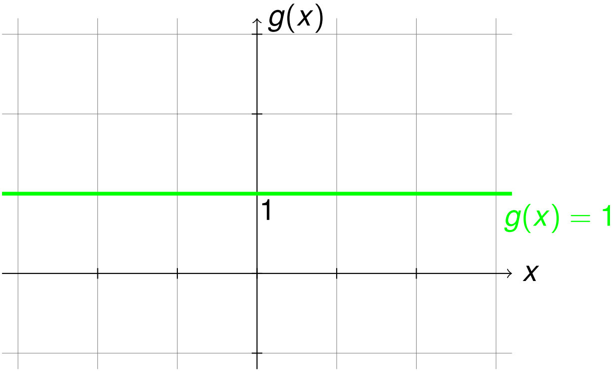
Let's now try to sketch the functions f, g and h together on the same plane. Firstly, here's the graph of $f(x) = 2x$:

Naming *functions*

Let's now try to sketch the functions f, g and h together on the same plane. Firstly, here's the graph of $f(x) = 2x$:



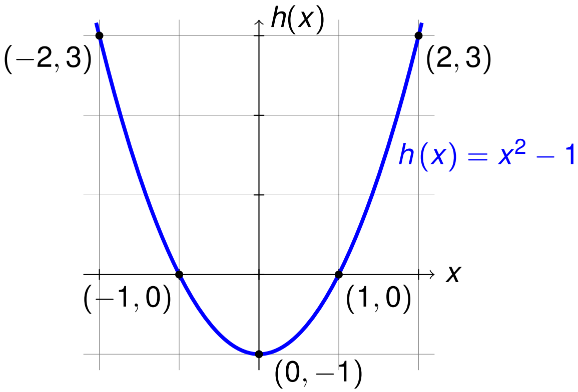
Here's the graph of $g(x) = 1$:



$f(x)$ and $g(x)$ are examples of **linear functions**, which will be studied in more depth in Day 2.

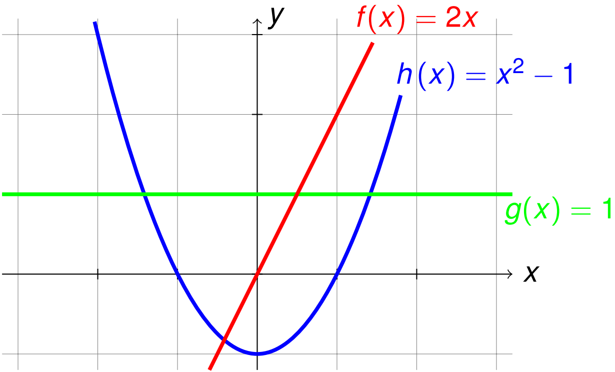
Naming *functions*

Finally, here's the graph of $h(x) = x^2 - 1$, an example of a **quadratic function.**



We'll look at quadratic functions in more detail in Day 3.

Finally, plotting all three functions together on the same graph:



Summary

- Functions may arise naturally from real-world phenomena or they may be defined by abstract mathematical formulas.
- The x in $f(x)$ is an *interchangeable* symbol used to represent the independent variable, while the f in $f(x)$ is often used to denote the *name* of the function.
- A wide range of different classes of functions will be studied throughout this bridging course:

  – Linear (Day 2)
  – Quadratic (Days 3-4)
  – General polynomials (Days 5-6)
  – Exponential (Day 8)
  – Circular (Day 10)
  – Logarithmic (Day 11)

Summary

- Functions may arise naturally from real-world phenomena or they may be defined by abstract mathematical formulas.
- The x in $f(x)$ is an *interchangeable* symbol used to represent the independent variable, while the f in $f(x)$ is often used to denote the *name* of the function.
- A wide range of different classes of functions will be studied throughout this bridging course:
  - Linear (Day 2)
  - Quadratic (Days 3-4)
  - General polynomials (Days 5-6)
  - Exponential (Day 8)
  - Circular (Day 10)
  - Logarithmic (Day 11)

Summary

- Functions may arise naturally from real-world phenomena or they may be defined by abstract mathematical formulas.
- The x in $f(x)$ is an *interchangeable* symbol used to represent the independent variable, while the f in $f(x)$ is often used to denote the *name* of the function.
- A wide range of different classes of functions will be studied throughout this bridging course:
  - Linear (Day 2)
  - Quadratic (Days 3-4)
  - General polynomials (Days 5-6)
  - Exponential (Day 8)
  - Circular (Day 10)
  - Logarithmic (Day 11)