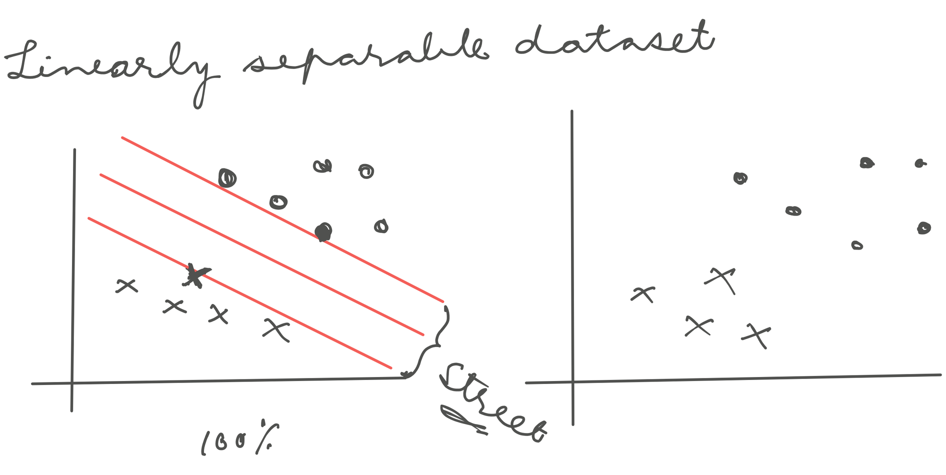
Support Vector Machine

↳ Widest street approach
↳ Pre-deep ML ⟶ most popular

Linearly separable dataset



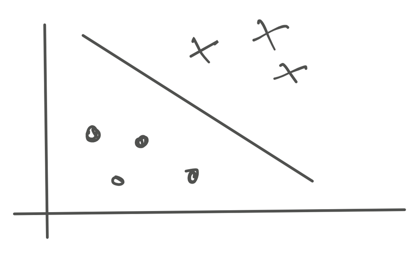
100%

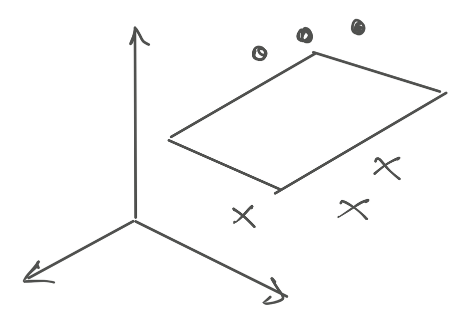
Let me find the decision boundary that maximizes the street.

max S ⟵ math

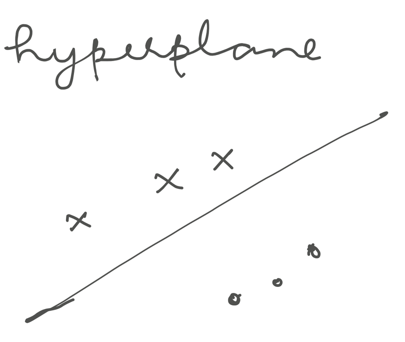
Plane.
↳ hyperplane

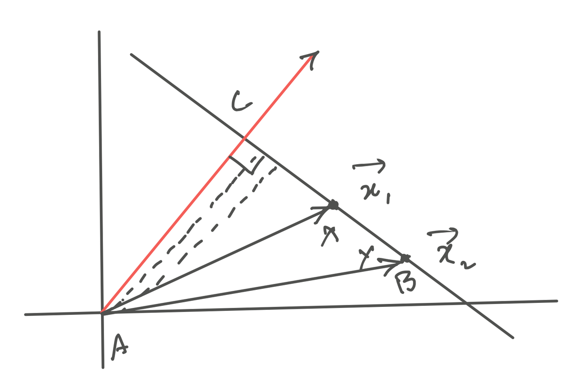
1D o o o o ☆ × × × point

2D  line

3D  plane

>3D hyperplane





AC ⟵ AX cos θ
⟵ AY cos φ

Dot product ⟶ $\vec{A}\cdot\vec{B} = \|A\|\,\|B\|\cos\theta$

$$\frac{\vec{A}\cdot\vec{B}}{\|A\|} = \vec{B}\cdot\frac{\vec{A}}{\|A\|}$$

Unit vector ⟶ Direction
⟶ Mag = 1

Vector $\vec{w}$ which normal to our plane

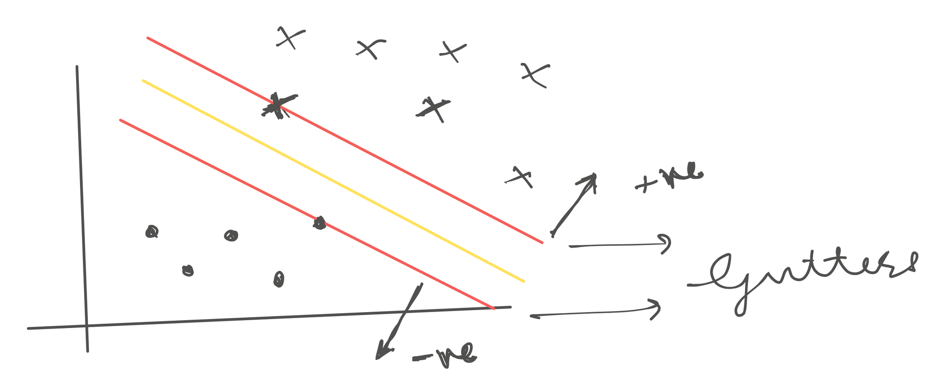
$$\frac{\vec{W}\vec{x}}{\|W\|} = c$$

$$\Rightarrow \vec{W}\cdot\vec{x} = \|W\|\,c$$

$$\Rightarrow \vec{W}\cdot\vec{x} - \|w\|\,c = 0$$

$$\Rightarrow \boxed{\vec{W}\cdot\vec{x} - b = 0}$$

$\forall x$ lie on the plane



$W \longrightarrow \infty$ choices

$W,\ 2W,\ 3W$

$\boxed{Wx_+ + b \geq 1}$ & $\boxed{Wx_- + b \leq -1}$

$\downarrow$

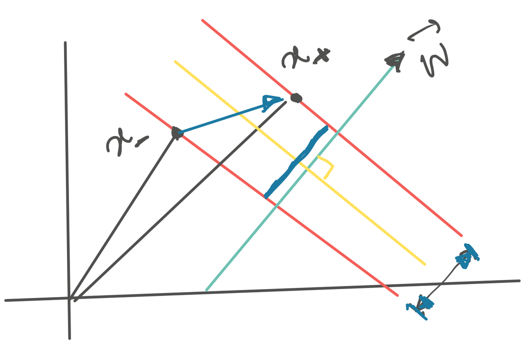
$-(Wx_- + b) \geq 1$

$y = 1 \}\ x_+$
$y = -1 \}\ x_-$

① $\boxed{y_i(Wx_i + b) \geq 1}$ $\forall i$ not in the street

② $\boxed{y_i(Wx_i + b) = 1}$ $\forall i$ on the gutter

Goal : Max. width of street
↳ Max. gap between gutters



$\vec{x}_+ - \vec{x}_- = \vec{y}$

$\vec{x}_- + \vec{y} = x_+$

Project $(\vec{x}_+ - \vec{x}_-)$ on $\vec{w}$.

③ $\boxed{(\vec{x}_+ - \vec{x}_-)\cdot\frac{\vec{W}}{\|W\|} = \text{Street width}}$

② ⟶ $y_i(\vec{w}\vec{x}_i + b) = 1$ ⟵ on gutters

+ve class

$+1(\vec{W}\vec{x}_+ + b) = 1$

$\boxed{\vec{W}\vec{x}_+ = 1 - b}$

Wx_+

−ve class

$-1(\vec{W}\vec{x}_- + b) = 1$

$-\vec{W}\vec{x}_- - b = 1$

$\boxed{\vec{W}\vec{x}_- = -1 - b}$

Wx_-

Substituting in ③

$$(\vec{x}_+ - \vec{x}_-)\frac{\vec{W}}{\|W\|}$$

$$= \frac{\vec{W}\vec{x}_+ - \vec{W}\vec{x}_-}{\|W\|}$$

$$= \frac{(1-b) - (-1-b)}{\|W\|}$$

$$= \frac{1 - b + 1 + b}{\|W\|}$$

$$\boxed{= \frac{2}{\|W\|}\ \Big\}\ \text{Street width}}$$

Max

SVM ⟶ Max $\frac{2}{\|W\|}$ st. $\boxed{\begin{array}{l} y_i(\vec{w}\vec{x}_i + b) = 1 \leftarrow \text{on gutters} \\ y_i(\vec{w}\vec{x}_i + b) \geq 1 \leftarrow \text{outside the street} \end{array}}$

$\simeq$ min $\|W\|$ st.

$\simeq$ min $\frac{1}{2}\|W\|^2$ st.

$\simeq$ $\boxed{\min \frac{1}{2}\vec{W}\cdot\vec{W}}$ st.

Constrained Optimization problem
→ Quadratic obj
→ Linear constraints } QP } QP solver

Lagrange multipliers