CSC D70: Compiler Optimization Dataflow Analysis

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Winter 2021

The content of this lecture is adapted from the lectures of Todd Mowry and Phillip Gibbons

Refreshing from Last Lecture

- Basic Block Formation

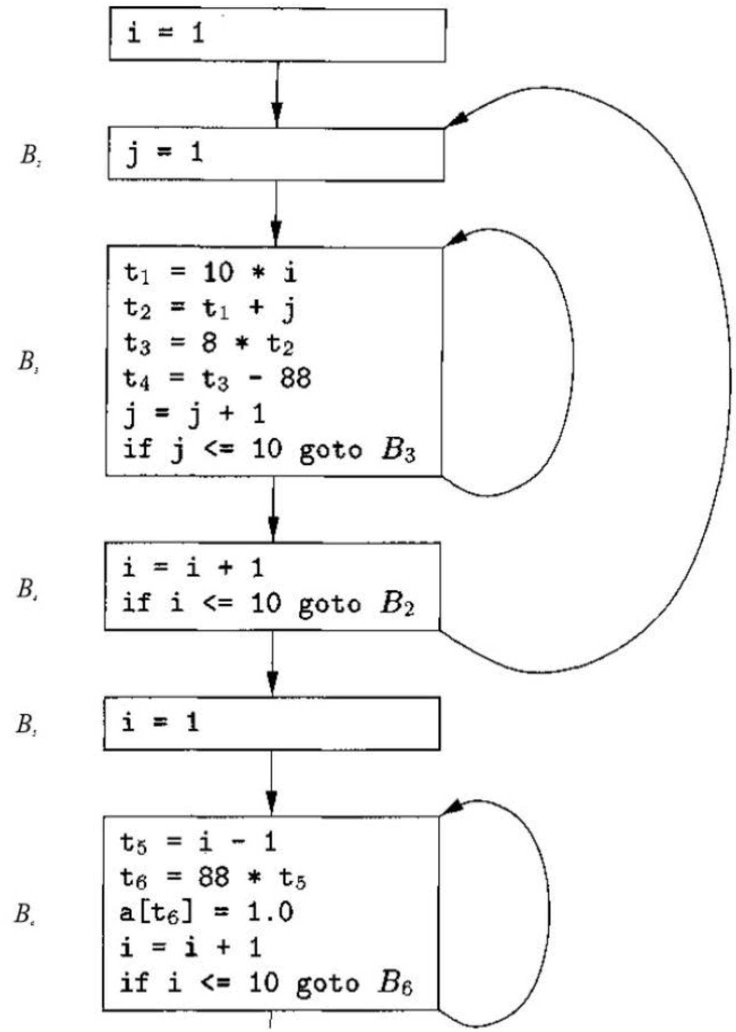
- Value Numbering

Partitioning into Basic Blocks

- Identify the leader of each basic block
  - First instruction
  - Any target of a jump
  - Any instruction immediately following a jump
- Basic block starts at leader & ends at instruction immediately before a leader (or the last instruction)

```
1)  i = 1
2)  j = 1
3)  t1 = 10 * i
4)  t2 = t1 + j
5)  t3 = 8 * t2
6)  t4 = t3 - 88
7)  a[t4] = 0.0
8)  j = j + 1
9)  if j <= 10 goto (3)
10) i = i + 1
11) if i <= 10 goto (2)
12) i = 1
13) t5 = i - 1
14) t6 = 88 * t5
15) a[t6] = 1.0
16) i = i + 1
17) if i <= 10 goto (13)
```

★ = Leader



ALSU pp. 529-531

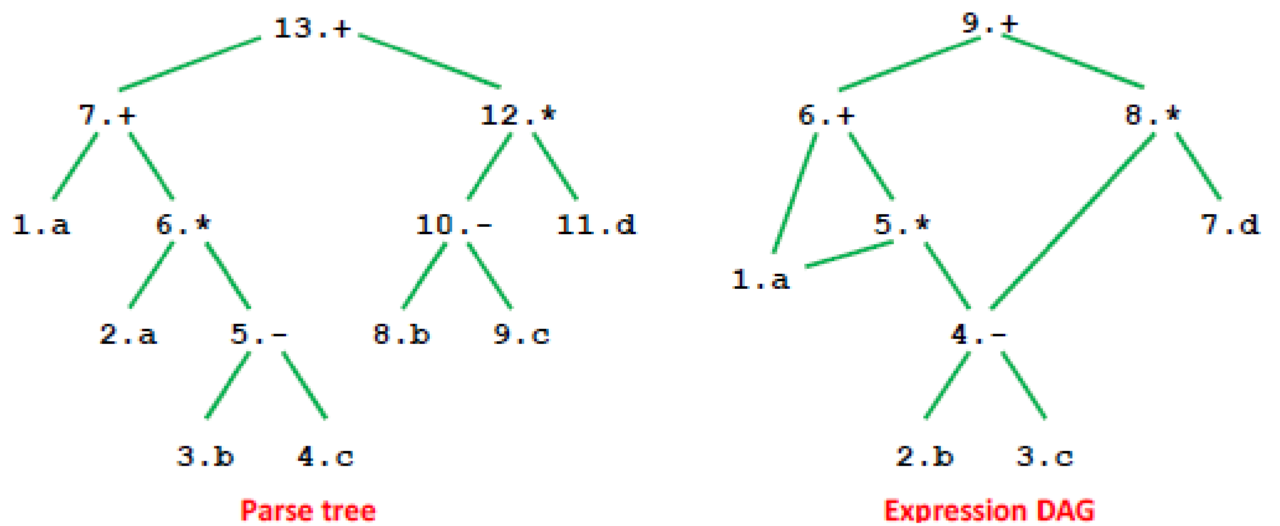
Graph Abstractions

Example 1:

- grammar (for bottom-up parsing):

E -> E + T | E – T | T, T -> T*F | F, F -> (E) | id

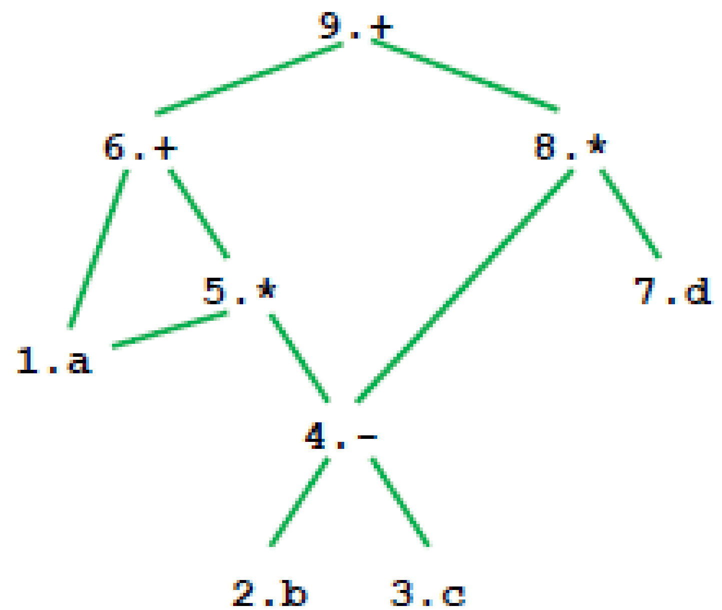
- expression: a+a*(b-c)+(b-c)*d



Graph Abstractions

Example 1: an expression

`a+a*(b-c)+(b-c)*d`



Optimized code:

t1 = b - c

t2 = a * t1

t3 = a + t2

t4 = t1 * d

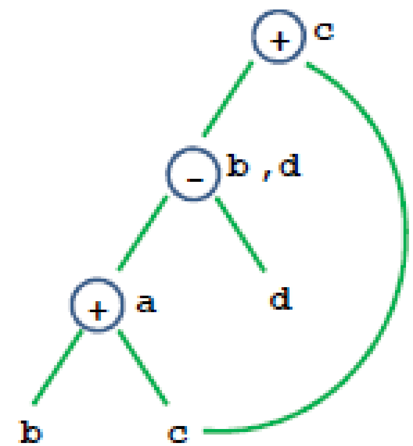
t5 = t3 + t4

How well do DAGs hold up across statements?

- **Example 2**

```
a = b+c;
b = a-d;
c = b+c;
d = a-d;
```

DAG – directed acyclic graph




```
Is this optimized code correct?
a = b+c;
d = a-d;
c = d+c;
```

Critique of DAGs

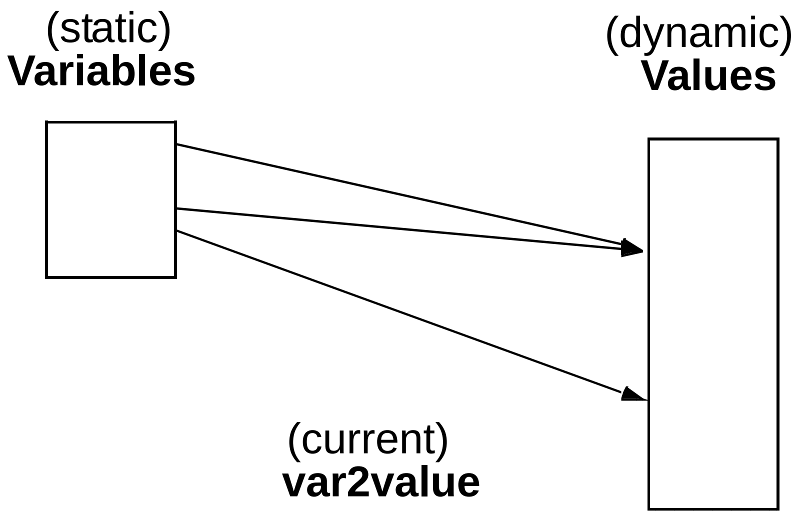
- **Cause of problems**
  - Assignment statements
  - Value of variable depends on TIME

- **How to fix problem?**
  - build graph in order of execution
  - attach variable name to latest value

- **Final graph created is not very interesting**
  - Key: variable->value mapping across time
  - loses appeal of abstraction

Value Numbering (VN)

- More explicit with respect to VALUES, and TIME



- **each value has its own “number”**
  - **common subexpression means same value number**
- var2value: current map of variable to value
  - used to determine the value number of current expression

r1 + r2 => var2value(r1)+var2value(r2)

Algorithm

```
Data structure:
     VALUES = Table of
        expression     //[OP, valnum1, valnum2}
        var            //name of variable currently holding expression

For each instruction (dst = src1 OP src2) in execution order

  valnum1 = var2value(src1); valnum2 = var2value(src2);

  IF [OP, valnum1, valnum2] is in VALUES
     v = the index of expression
     Replace instruction with CPY dst = VALUES[v].var
  ELSE
     Add
        expression = [OP, valnum1, valnum2]
        var        = dst
     to VALUES
     v = index of new entry; tv is new temporary for v
     Replace instruction with: tv = VALUES[valnum1].var OP VALUES[valnum2].var
                               CPY dst = tv;

  set_var2value (dst, v)
```

More Details

- **What are the initial values of the variables?**
  – values at beginning of the basic block
- **Possible implementations:**
  – Initialization: create “initial values” for all variables
  – Or dynamically create them as they are used
- **Implementation of VALUES and var2value: hash tables**

Example

```
Assign: a->r1,b->r2,c->r3,d->r4
a = b+c;     ADD t1 = r2,r3
             CPY r1 = t1
b = a-d;     SUB t2 = r1,r4
             CPY r2 = t2
c = b+c;     ADD t3 = r2,r3
             CPY r3 = t3
d = a-d;     SUB t4 = r1,r4
             CPY r4 = t4
```

Conclusions

- **Comparisons of two abstractions**
  - DAGs
  - Value numbering
- **Value numbering**
  - VALUE: distinguish between variables and VALUES
  - TIME
    - Interpretation of instructions in order of execution
    - Keep dynamic state information

VN Example

```
Assign: a->r1,b->r2,c->r3,d->r4
a = b+c;      ADD t1 = r2,r3
              CPY r1 = t1   //(a = t1)
b = a-d;      SUB t2 = r1,r4
              CPY r2 = t2   //(b = t2)
c = b+c;      ADD t3 = r2,r3
              CPY r3 = t3   //(c = t3)
d = a-d;      CPY r~~2~~4 = t2
```

Outline

1. Structure of data flow analysis

2. Example 1: Reaching definition analysis

3. Example 2: Liveness analysis

4. Generalization

What is Data Flow Analysis?

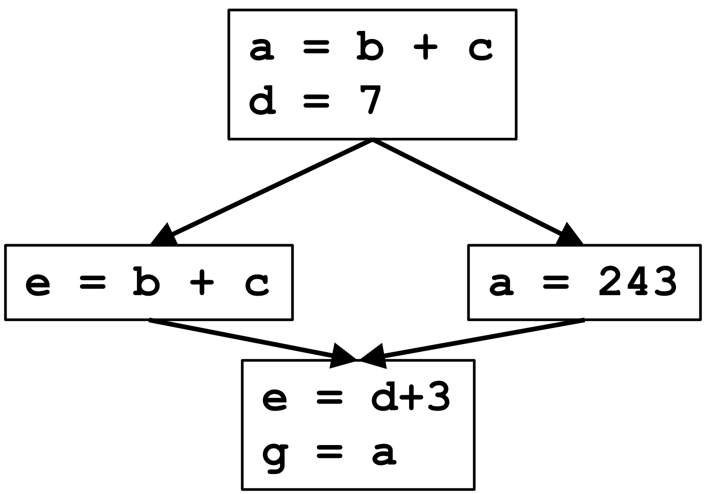
- **Local analysis (e.g., value numbering)**
  - analyze effect of each instruction
  - compose effects of instructions to derive information from beginning of basic block to each instruction

- **Data flow analysis**
  - analyze effect of each basic block
  - compose effects of basic blocks to derive information at basic block boundaries
  - from basic block boundaries, apply local technique to generate information on instructions

What is Data Flow Analysis? (2)

- **Data flow analysis:**
  - Flow-sensitive: sensitive to the control flow in a function
  - intraprocedural analysis
- **Examples of optimizations:**
  - Constant propagation
  - Common subexpression elimination
  - Dead code elimination

What is Data Flow Analysis? (3)



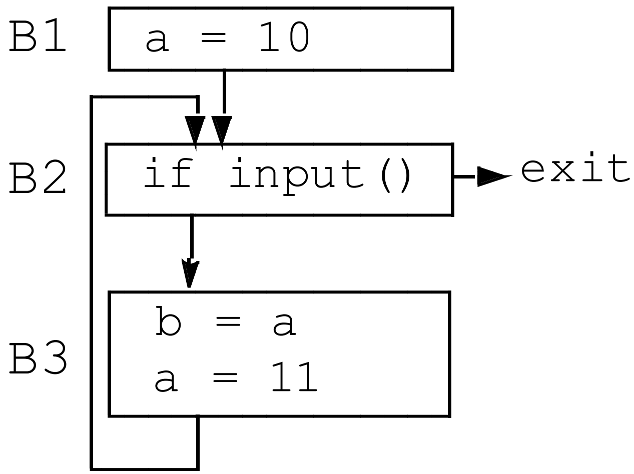
For each variable x determine:

Value of x?

Which “definition” defines x?

Is the definition still meaningful (live)?

Static Program vs. Dynamic Execution



- **Statically**: Finite program
- **Dynamically**: Can have infinitely many possible execution paths
- **Data flow analysis abstraction:**
  - For each point in the program:
    combines information of all the instances of the same program point.
- **Example of a data flow question:**
  - Which definition defines the value used in statement "b = a"?

Effects of a Basic Block

- Effect of a statement: **a = b+c**
  - **Use**s variables (b, c)
  - **Kill**s an old definition (old definition of a)
  - new **definition** (a)
- Compose effects of statements -> Effect of a basic block
  – A **locally exposed use** in a b.b. is a use of a data item which is not preceded in the b.b. by a definition of the data item
  – any definition of a data item in the basic block **kills** all definitions of the same data item reaching the basic block.
  – A **locally available definition** = last definition of data item in b.b.

Effects of a Basic Block

A **locally available definition** = last definition of data item in b.b.

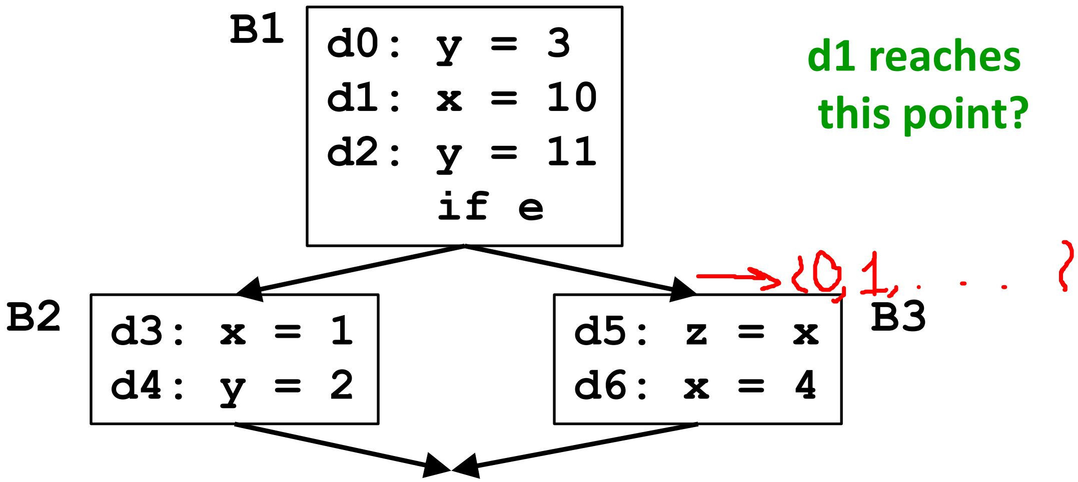
```
t1 = r1+r2
r2 = t1
t2 = r2+r1
r1 = t2
t3 = r1*r1
r2 = t3
if r2>100 goto L1
```

Locally exposed uses? **r1**

Kills any definitions? **Any other definition of t2**

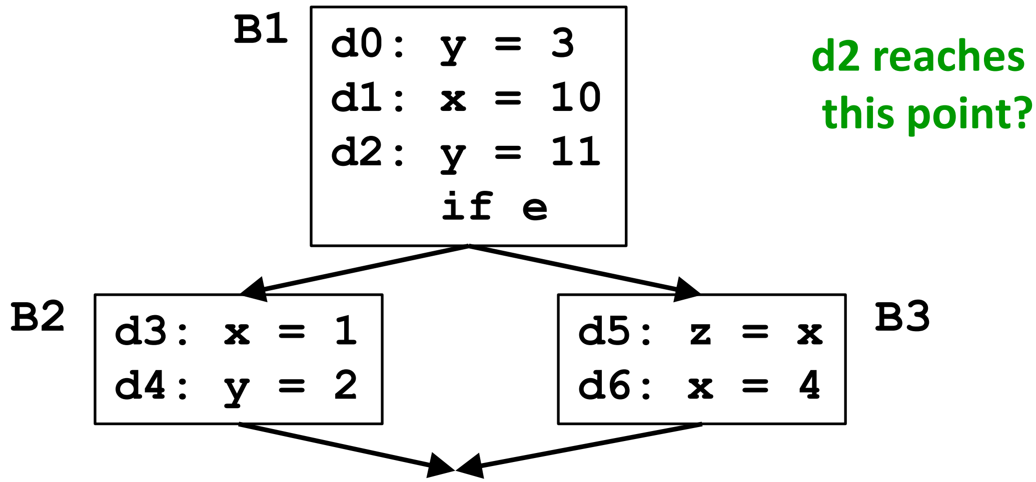
Locally avail. definition? **t2**

Reaching Definitions



- Every assignment is a **definition**
- A **definition** *d* **reaches** a point *p* if **there exists** path from the point immediately following *d* to *p* such that *d* is not killed (overwritten) along that path.
- Problem statement
  - For each point in the program, determine if each definition in the program reaches the point
  - A bit vector per program point, vector-length = #defs

Reaching Definitions (2)



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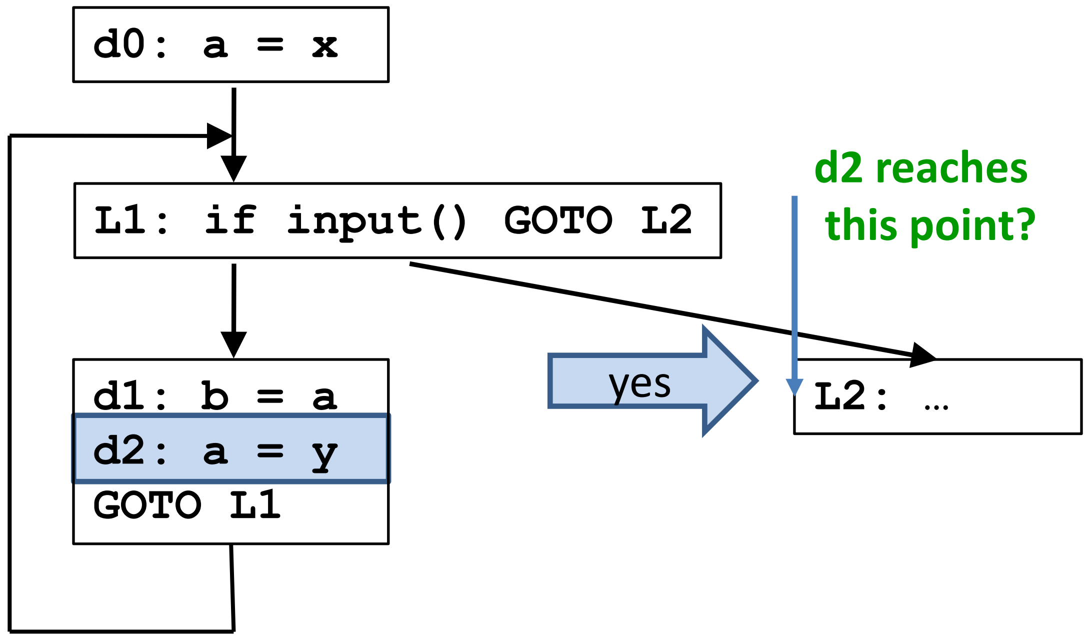
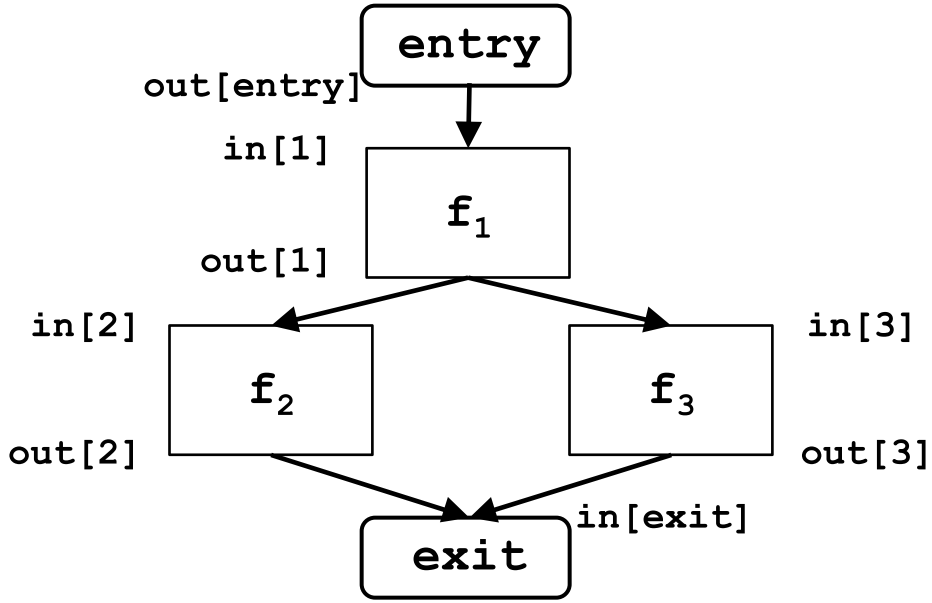
Reaching Definitions (3)

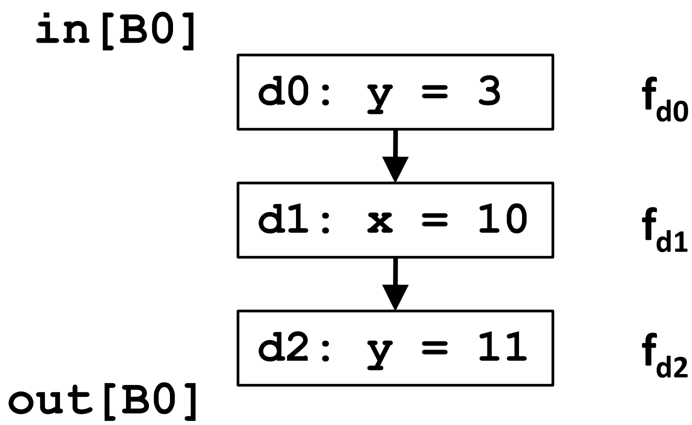


Data Flow Analysis Schema



- Build a flow graph (nodes = basic blocks, edges = control flow)
- Set up a set of equations between in[b] and out[b] for all basic blocks b
  - Effect of code in basic block:
    - Transfer function f_b relates in[b] and out[b], for same b
  - Effect of flow of control:
    - relates out[b_1], in[b_2] if b_1 and b_2 are adjacent
- Find a solution to the equations

Effects of a Statement



- f_s : A transfer function of a statement
  - abstracts the execution with respect to the problem of interest
- For a statement s (d: x = y + z)
  out[s] = f_s(in[s]) = Gen[s] U (in[s]-Kill[s])
  - **Gen[s]**: definitions generated: Gen[s] = {d}
  - **Propagated** definitions: in[s] - Kill[s],
    where **Kill[s]**=set of all other defs to x in the rest of program

Effects of a Basic Block

```
in[B0]
d0: y = 3      f_d0
d1: x = 10     f_d1
d2: y = 11     f_d2
out[B0]
```

$f_B = f_{d2} \cdot f_{d1} \cdot f_{d1}$

- Transfer function of a statement s:
  - $out[s] = f_s(in[s]) = Gen[s] \cup (in[s]-Kill[s])$
- Transfer function of a basic block B:
  - Composition of transfer functions of statements in B
- $out[B] = f_B(in[B]) = f_{d2}f_{d1}f_{d0}(in[B])$

$= Gen[d_2] \cup (Gen[d_1] \cup (Gen[d_0] \cup (in[B]-Kill[d_0]))-Kill[d_1]))\ -Kill[d_2]$

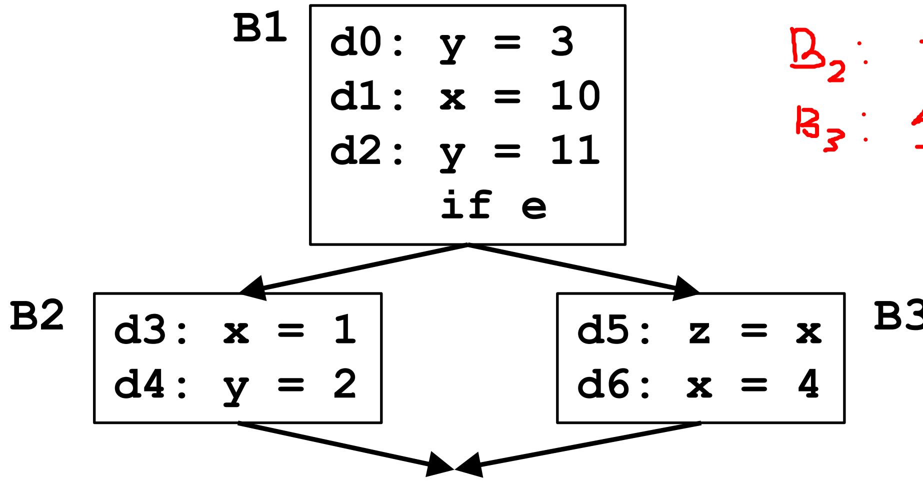
$= Gen[d_2] \cup (Gen[d_1] \cup (Gen[d_0] - Kill[d_1]) - Kill[d_2]) \cup$
$in[B] - (Kill[d_0] \cup Kill[d_1] \cup Kill[d_2])$

$= Gen[B] \cup (in[B] - Kill[B])$

- Gen[B]: locally exposed definitions (available at end of bb)
- Kill[B]: set of definitions killed by B

Example

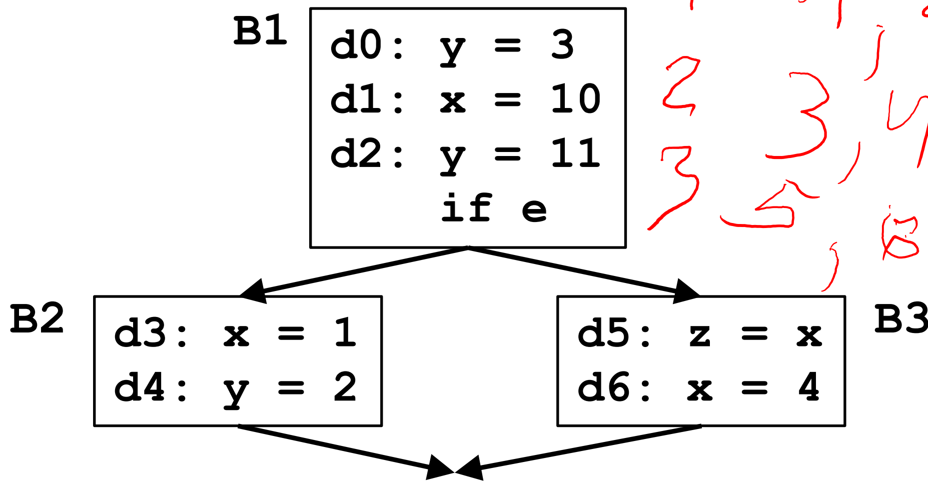
	Gen	Kill
B_1:	1, 2	0, 3, 4, 6, 2
B_2:	3 4	0 1 2 6
B_3:	5 6	1 3



- a **transfer function** f_b of a basic block b:
  OUT[b] = f_b(IN[b])
  incoming reaching definitions -> outgoing reaching definitions
- A basic block b
  - **generates** definitions: Gen[b],
    – set of locally available definitions in b
  - **kills** definitions: in[b] - Kill[b],
    where Kill[b]=set of defs (in rest of program) killed by defs in b
- **out[b] = Gen[b] U (in(b)-Kill[b])**

Example

Gen Kill



1   1, 2   B1   0, 2, 3, 4, 6

2   3, 4   B2   0, 1, 2, 6

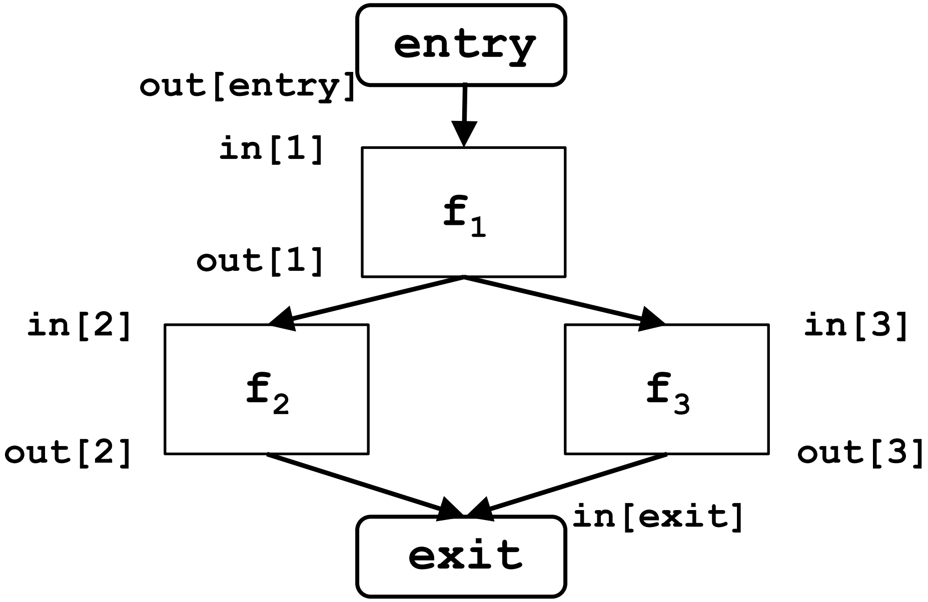
3   5, 6   B3   1, 3

- a **transfer function** f_b of a basic block b:

  $$OUT[b] = f_b(IN[b])$$

  incoming reaching definitions -> outgoing reaching definitions
- A basic block b
  - **generates** definitions: Gen[b],
    - set of locally available definitions in b
  - **kills** definitions: in[b] - Kill[b],
    where Kill[b]=set of defs (in rest of program) killed by defs in b
- **out[b] = Gen[b] U (in(b)-Kill[b])**

Effects of the Edges (acyclic)

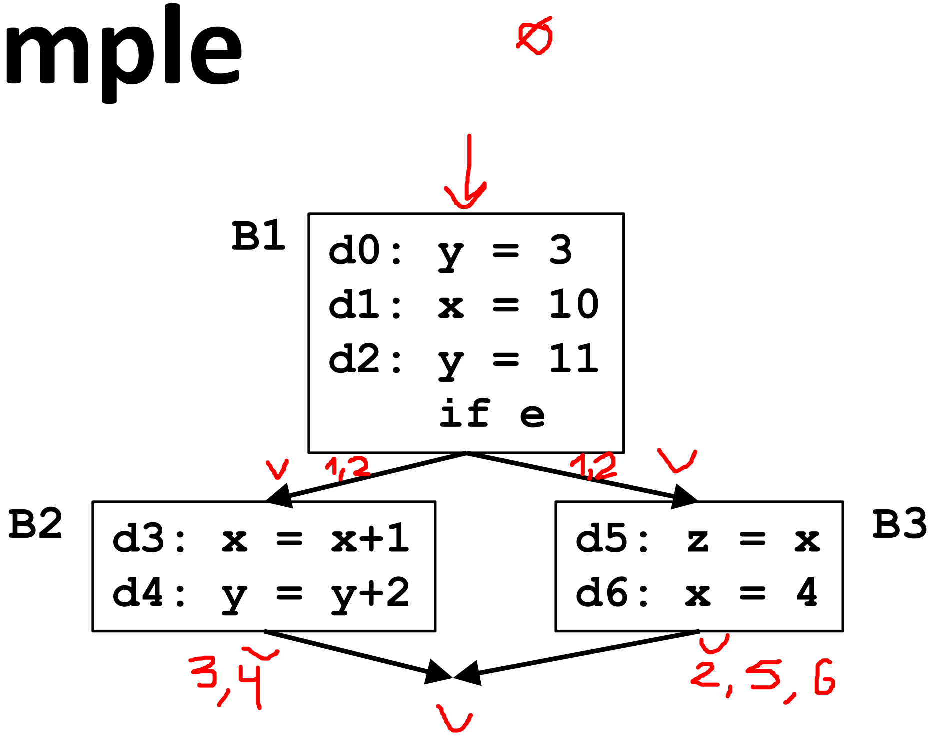


- out[b] = f_b(in[b])
- Join node: a node with multiple predecessors
- **meet** operator:

  in[b] = out[p_1] U out[p_2] U ... U out[p_n], where

  p_1, ..., p_n are all predecessors of b

Example



f	Gen	Kill
1	{1,2}	{0,2,3,4,6}
2	{3,4}	{0,1,2,6}
3	{5,6}	{1,3}

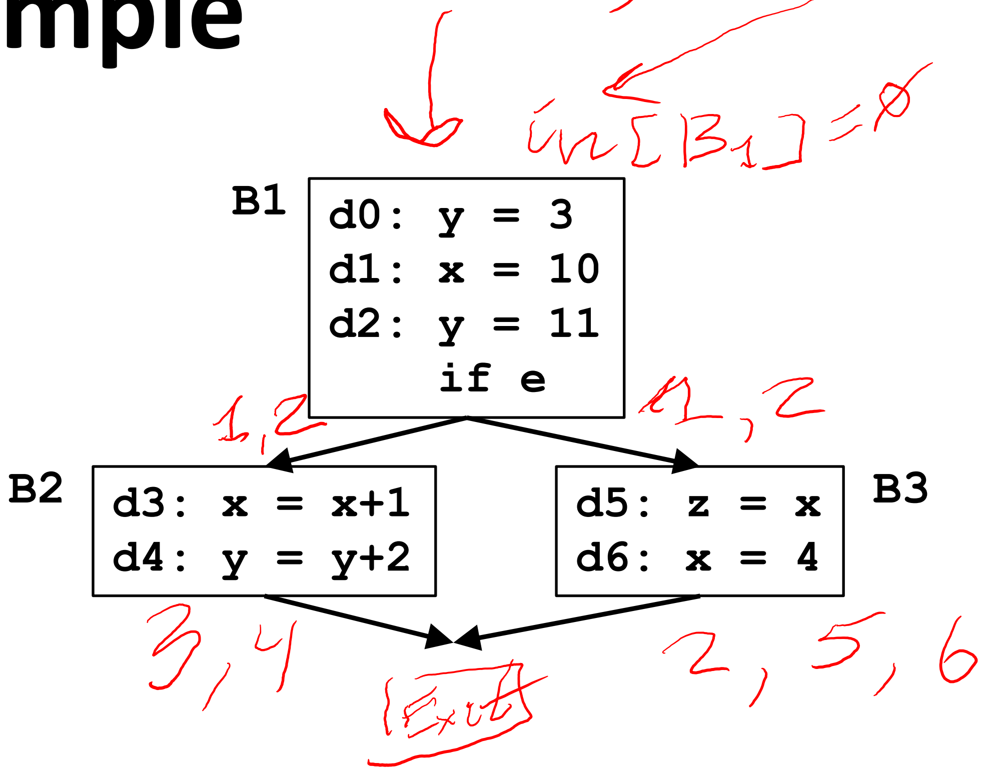
- out[b] = f_b(in[b])
- Join node: a node with multiple predecessors
- **meet** operator:

  in[b] = out[p_1] U out[p_2] U ... U out[p_n], where

  p_1, ..., p_n are all predecessors of b

Example

Entry = out[E] = ∅



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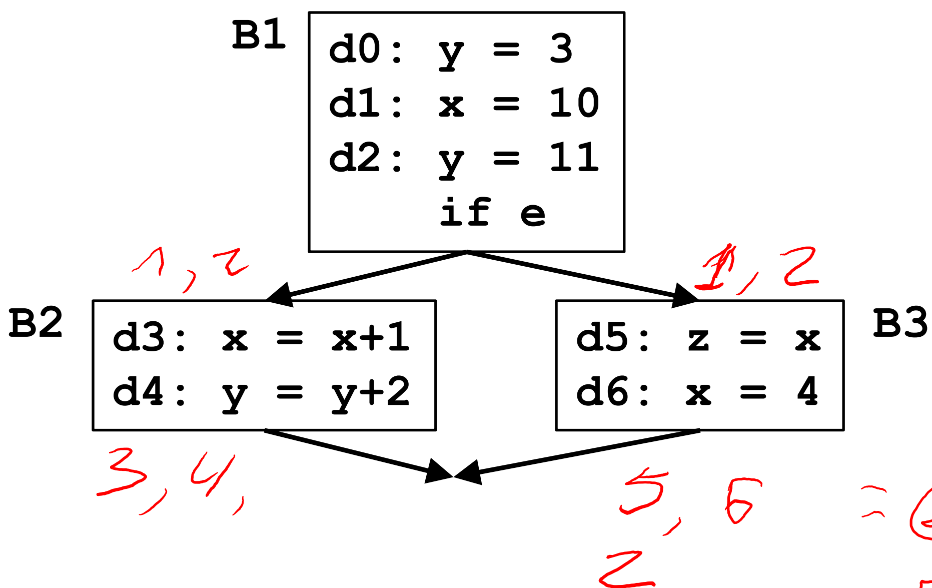
Example

E out[E] = ∅
in[B_1] = ∅



f	Gen	Kill
1	{1,2}	{0,2,3,4,6}
2	{3,4}	{0,1,2,6}
3	{5,6}	{1,3}

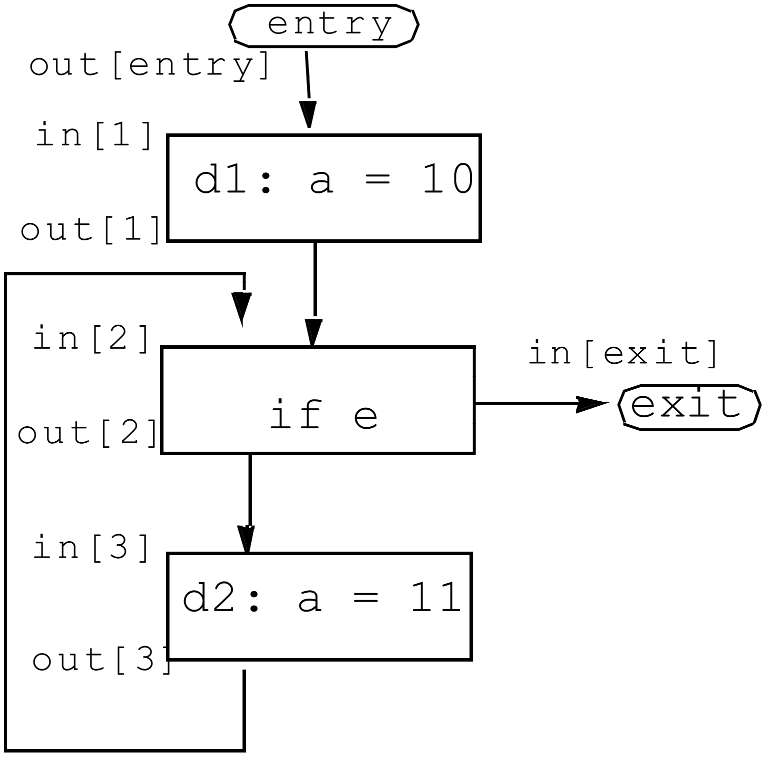
out[B_1] = Gen[B1] ∪ [in[B1] - Kill[B_1]]

- out[b] = f_b(in[b])
- Join node: a node with multiple predecessors
- **meet** operator:

  in[b] = out[p_1] U out[p_2] U ... U out[p_n], where

  p_1, ..., p_n are all predecessors of b

Cyclic Graphs



- Equations still hold
  - out[b] = f_b(in[b])
  - in[b] = out[p_1] U out[p_2] U ... U out[p_n], p_1, ..., p_n pred.
- Find: fixed point solution

Reaching Definitions: Iterative Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
   out[Entry] = ∅

// Initialization for iterative algorithm
   For each basic block B other than Entry
      out[B] = ∅

// iterate
   While (Changes to any out[] occur) {
      For each basic block B other than Entry {
         in[B] = ∪ (out[p]), for all predecessors p of B
         out[B] = f_B(in[B])    // out[B]=gen[B]∪(in[B]-kill[B])
      }
```

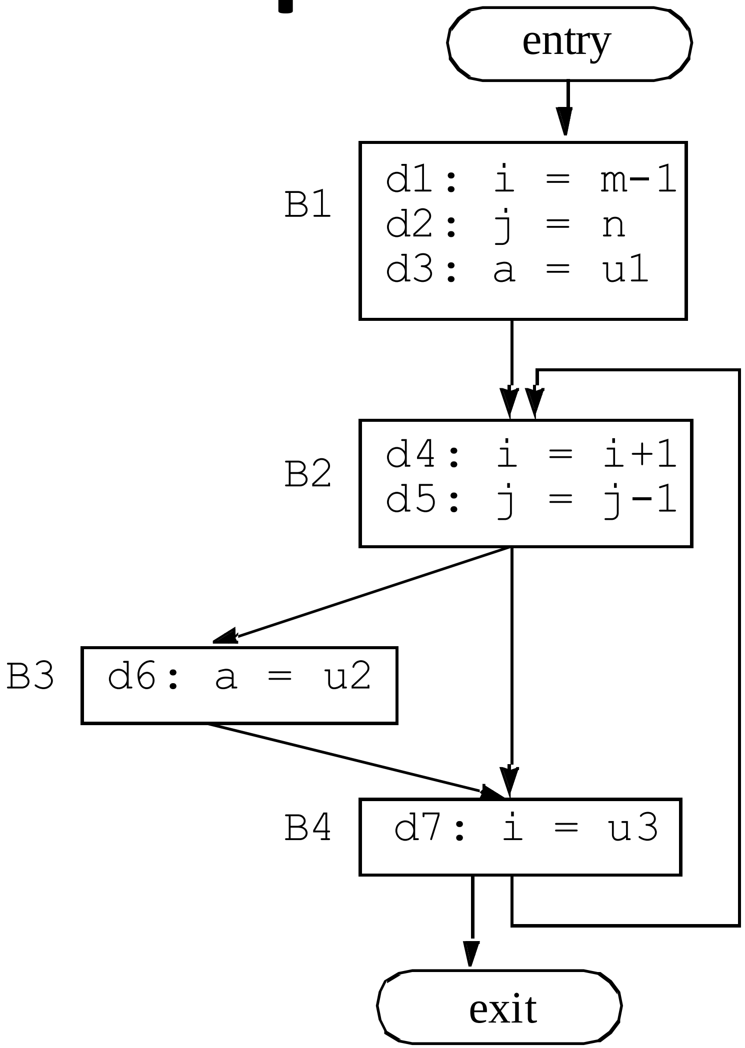
Reaching Definitions: Worklist Algorithm

```
input: control flow graph CFG = (N, E, Entry, Exit)

// Initialize
    out[Entry] = ∅          // can set out[Entry] to special def
                            // if reaching then undefined use
    For all nodes i
        out[i] = ∅          // can optimize by out[i]=gen[i]
    ChangedNodes = N

// iterate
    While ChangedNodes ≠ ∅ {
        Remove i from ChangedNodes
        in[i] = U (out[p]), for all predecessors p of i
        oldout = out[i]
        out[i] = f_i(in[i])     // out[i]=gen[i]U(in[i]-kill[i])
        if (oldout ≠ out[i]) {
            for all successors s of i
                add s to ChangedNodes
        }
    }
```

Example

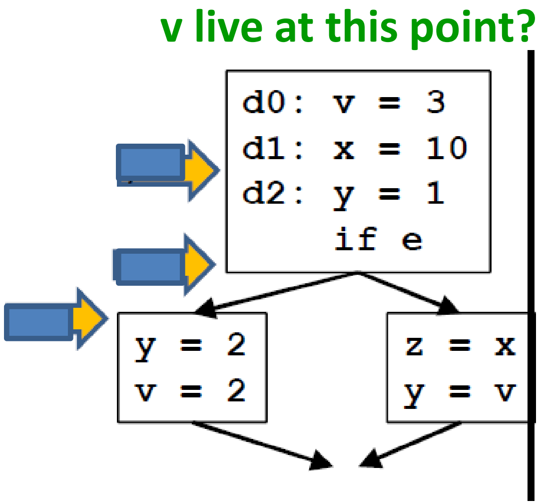


	First Pass	Second Pass
IN[B1]	000 00 0 0	000 00 0 0
OUT[B1]	111 00 0 0	111 00 0 0
IN[B2]	111 00 0 0	111 01 1 1
OUT[B2]	001 11 0 0	001 11 1 0
IN[B3]	001 11 0 0	001 11 1 0
OUT[B3]	000 11 1 0	000 11 1 0
IN[B4]	001 11 1 0	001 11 1 0
OUT[B4]	001 01 1 1	001 01 1 1
IN[exit]	001 01 1 1	001 01 1 1

Live Variable Analysis

- **Definition**
  - A variable **v** is live at point *p* if
    - the value of **v** is used along some path in the flow graph starting at *p*.
  - Otherwise, the variable is **dead**.
- **Motivation**
    - e.g. register allocation

```
    for i = 0 to n
      … i …
  …
  for i = 0 to n
      … i …
```

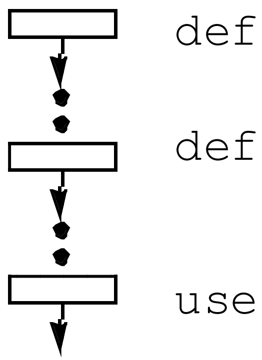


- **Problem statement**
  - For each basic block
    - determine if each variable is live in each basic block
  - Size of bit vector: one bit for each variable

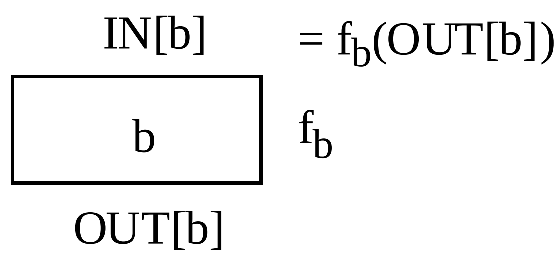
Transfer Function

- **Insight: Trace uses backwards to the definitions**

an execution path



control flow



example

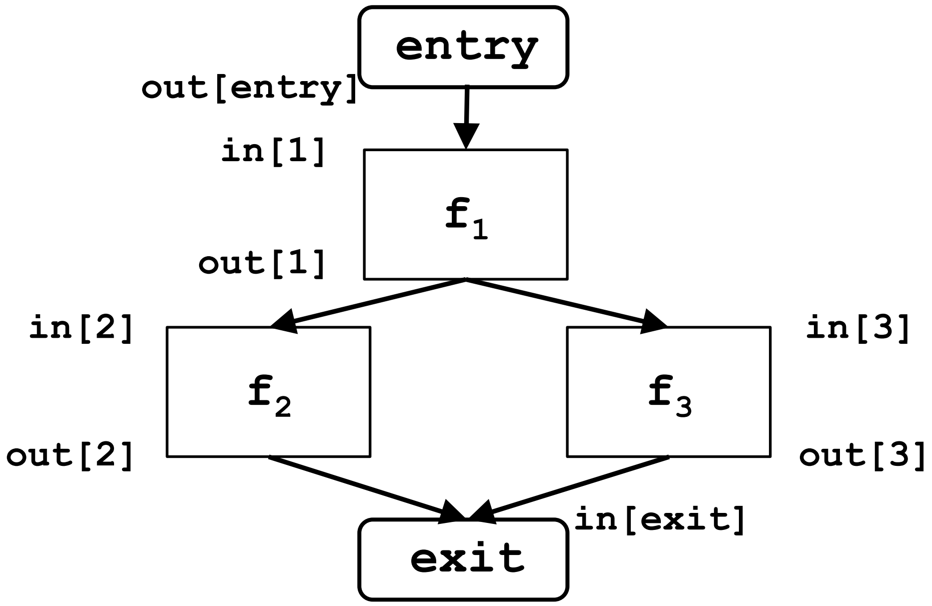
```
d3: a = 1
d4: b = 1

d5: c = a
d6: a = 4
```

- **A basic block b can**
  - generate live variables: **Use[b]**
    - set of locally exposed uses in b
  - propagate incoming live variables: **OUT**[b] - **Def[b]**,
    - where Def[b]= set of variables defined in b.b.
- **transfer function** for block b:

  in[b] = Use[b] U (out(b)-Def[b])

Flow Graph

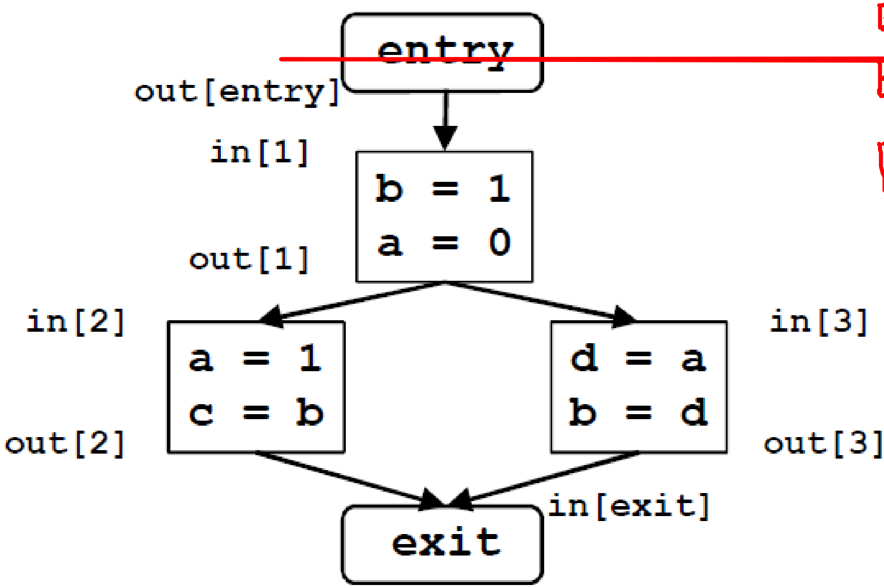


- in[b] = f_b(out[b])
- Join node: a node with multiple successors
- **meet** operator:

  out[b] = in[s_1] U in[s_2] U ... U in[s_n], where

  s_1, ..., s_n are all successors of b

Flow Graph (2)



Use Def

B_1: ∅
B_2: b
B_3: a

f	Use	Def
1	{}	{a,b}
2	{b}	{a,c}
3	{a}	{b,d}

- in[b] = f_b(out[b])
- Join node: a node with multiple successors
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Liveness: Iterative Algorithm

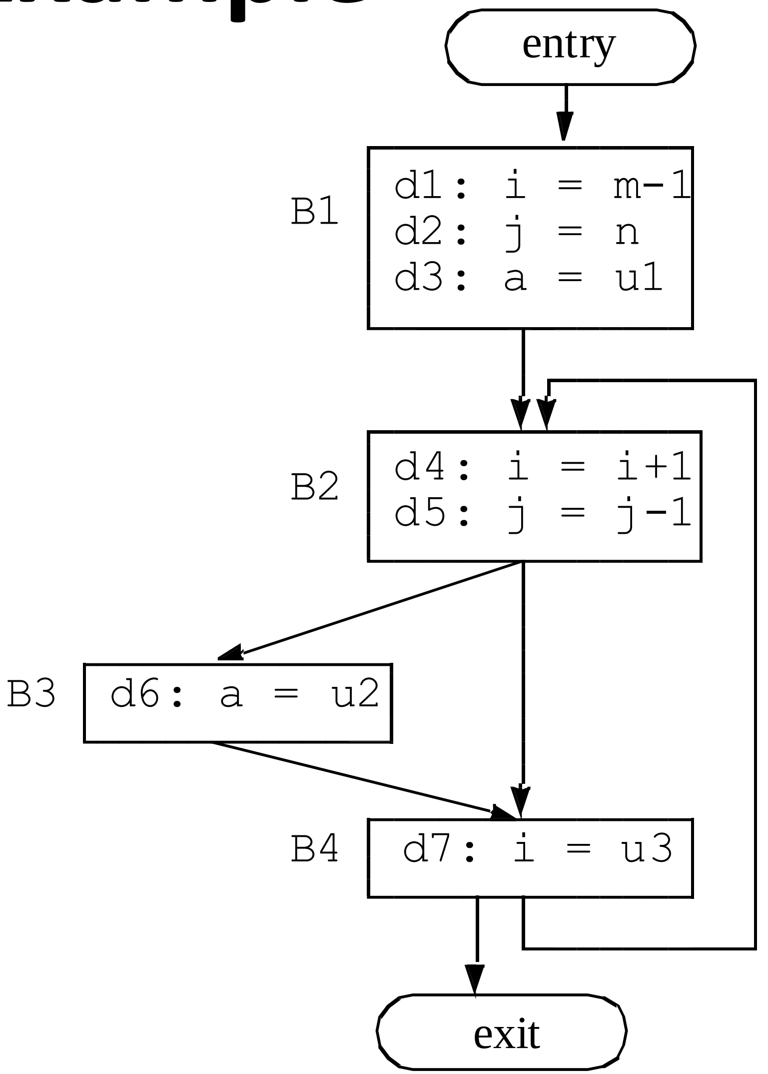
```
input: control flow graph CFG = (N, E, Entry, Exit)

// Boundary condition
   in[Exit] = ∅

// Initialization for iterative algorithm
   For each basic block B other than Exit
      in[B] = ∅

// iterate
   While (Changes to any in[] occur) {
      For each basic block B other than Exit {
         out[B] = ∪ (in[s]), for all successors s of B
         in[B] = f_B(out[B])    // in[B]=Use[B]∪(out[B]-Def[B])
      }
```

Example



	First Pass	Second Pass
OUT[entry]	{m,n,u1,u2,u3}	{m,n,u1,u2,u3}
IN[B1]	{m,n,u1,u2,u3}	{m,n,u1,u2,u3}
OUT[B1]	{i,j,u2,u3}	{i,j,u2,u3}
IN[B2]	{i,j,u2,u3}	{i,j,u2,u3}
OUT[B2]	{u2,u3}	{j,u2,u3}
IN[B3]	{u2,u3}	{j,u2,u3}
OUT[B3]	{u3}	{j,u2,u3}
IN[B4]	{u3}	{j,u2,u3}
OUT[B4]	{}	{i,j,u2,u3}

Framework

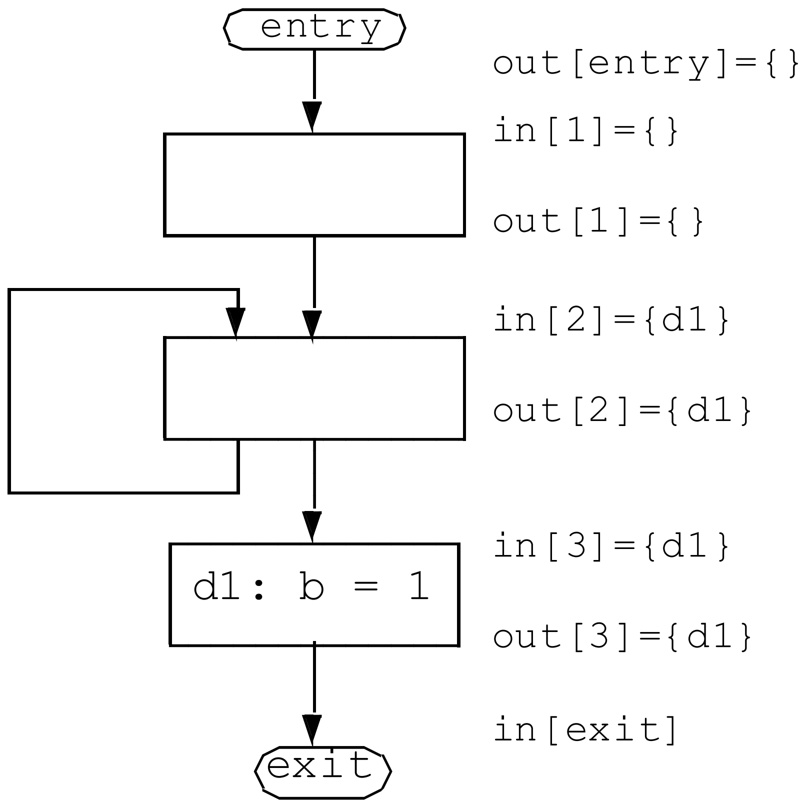
	Reaching Definitions	**Live Variables**
Domain	Sets of definitions	Sets of variables
Direction	forward: $out[b] = f_b(in[b])$ $in[b] = \wedge\ out[pred(b)]$	backward: $in[b] = f_b(out[b])$ $out[b] = \wedge\ in[succ(b)]$
Transfer function	$f_b(x) = Gen_b \cup (x - Kill_b)$	$f_b(x) = Use_b \cup (x - Def_b)$
Meet Operation ($\wedge$)	$\cup$	$\cup$
Boundary Condition	$out[entry] = \emptyset$	$in[exit] = \emptyset$
Initial interior points	$out[b] = \emptyset$	$in[b] = \emptyset$

Other examples (e.g., Available expressions), defined in ALSU 9.2.6

Thought Problem 1. “Must-Reach” Definitions

- **A definition D (a = b+c) <u>must</u> reach point P iff**
  - D appears at least once along on all paths leading to P
  - a is not redefined along any path after last appearance of D and before P
- **How do we formulate the data flow algorithm for this problem?**

Thought Problem 2: A legal solution to (May) Reaching Def?



- Will the worklist algorithm generate this answer?

Questions

- **Correctness**
  - equations are satisfied, if the program terminates.
- **Precision: how good is the answer?**
  - is the answer ONLY a union of all possible executions?
- **Convergence: will the analysis terminate?**
  - or, will there always be some nodes that change?
- **Speed: how fast is the convergence?**
  - how many times will we visit each node?

Foundations of Data Flow Analysis

1. **Meet operator**
2. **Transfer functions**
3. **Correctness, Precision, Convergence**
4. **Efficiency**

- Reference: ALSU pp. 613-631
- Background: Hecht and Ullman, Kildall, Allen and Cocke[76]
- Marlowe & Ryder, Properties of data flow frameworks: a unified model. Rutgers tech report, Apr. 1988

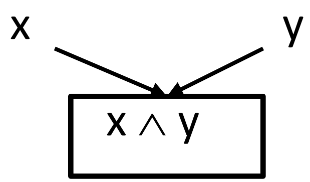
A Unified Framework

- **Data flow problems are defined by**
  - Domain of values: V
  - Meet operator ($V \wedge V \rightarrow V$), initial value
  - A set of transfer functions ($V \rightarrow V$)

- **Usefulness of unified framework**
  - To answer questions such as correctness, precision, convergence, speed of convergence for a family of problems
    - If meet operators and transfer functions have properties X, then we know Y about the above.
  - Reuse code

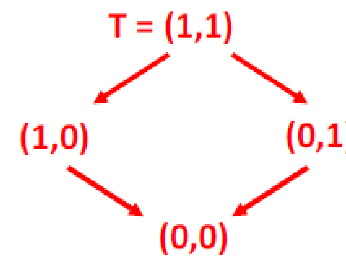
Meet Operator

- **Properties of the meet operator**
  - commutative: $x \wedge y = y \wedge x$



  - idempotent: $x \wedge x = x$
  - associative: $x \wedge (y \wedge z) = (x \wedge y) \wedge z$
  - there is a Top element T such that $x \wedge T = x$

Partial Order

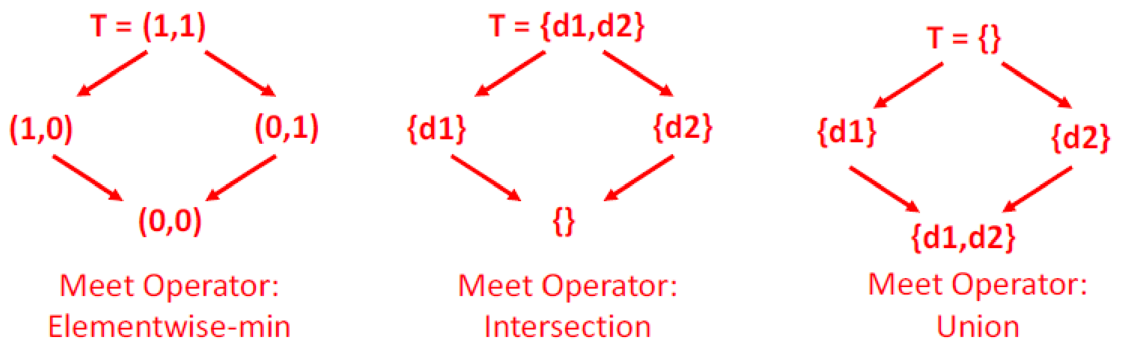


Meet Operator:
Elementwise-min

- **Meet operator defines a partial ordering on values**
  - $x \leq y$ if and only if $x \wedge y = x$ (y -> x in diagram)
    - Transitivity: if $x \leq y$ and $y \leq z$ then $x \leq z$
    - Antisymmetry: if $x \leq y$ and $y \leq x$ then $x = y$
    - Reflexivity: $x \leq x$

Partial Order

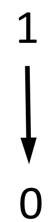
- Example: let **V** = {x | such that x $\subseteq$ { $\mathbf{d_1}$, $\mathbf{d_2}$}}, $\wedge$ = $\cap$



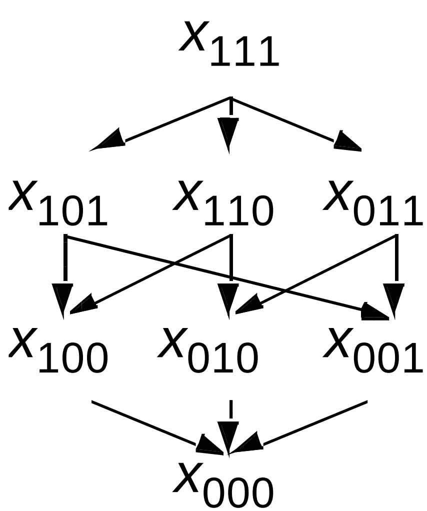
- Top and Bottom elements
  - Top T such that: $x \wedge T = x$
  - Bottom $\perp$ such that: $x \wedge \perp = \perp$
- Values and meet operator in a data flow problem define a semi-lattice:
  – there exists a T, but not necessarily a $\perp$.
- x, y are ordered: $x \leq y$ then $x \wedge y = x$ (y -> x in diagram)
- what if x and y are not ordered?
  - $x \wedge y \leq x$, $x \wedge y \leq y$, and if $w \leq x$, $w \leq y$, then $w \leq x \wedge y$

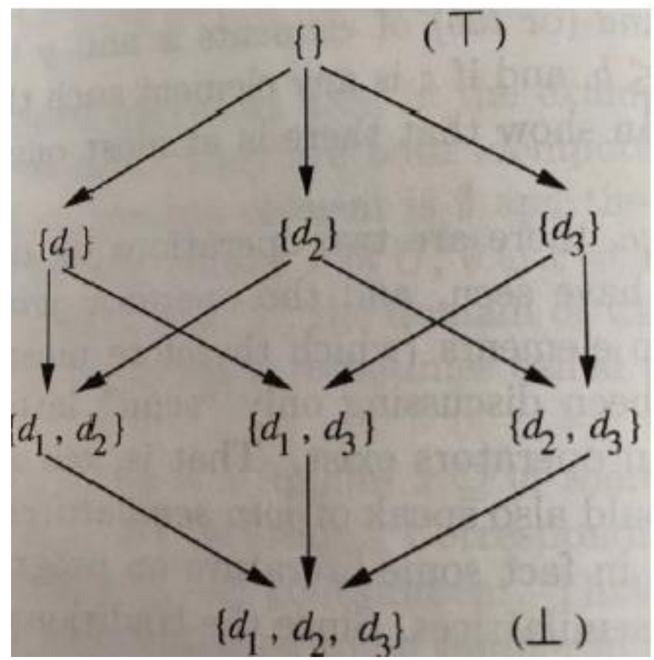
One vs. All Variables/Definitions

- **Lattice for each variable: e.g. intersection**



- **Lattice for three variables:**





Descending Chain

- **Definition**
  - The **height** of a lattice is the largest number of **> relations** that will fit in a descending chain.

    $x_0 > x_1 > x_2 > \ldots$

- **Height of values in reaching definitions?**

  Height n – number of definitions

- **Important property: finite descending chain**
- **Can an infinite lattice have a finite descending chain?**

  yes

- **Example: Constant Propagation/Folding**
  - To determine if a variable is a constant
- **Data values**
  - undef, ... -1, 0, 1, 2, ..., not-a-constant

Transfer Functions

- **Basic Properties** f**: V $\rightarrow$ V**
  - Has an identity function
    - There exists an f such that f (x) = x, for all x.
  - Closed under composition
    - if $f_1, f_2 \in F$, then $f_1 \cdot f_2 \in F$

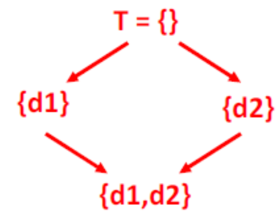
Monotonicity

- A framework (F, V, $\wedge$) is monotone if and only if
  - $x \leq y$ implies $f(x) \leq f(y)$
  - i.e. a "smaller or equal" input to the same function will always give a "smaller or equal" output
- Equivalently, a framework (F, V, $\wedge$) is monotone if and only if
  - $f(x \wedge y) \leq f(x) \wedge f(y)$
  - i.e. merge input, then apply *f* is **small than or equal to** apply the transfer function individually and then merge the result

Example

- **Reaching definitions: f(x) = Gen ∪ (x - Kill), ∧ = ∪**
  - Definition 1:
    - $x_1 \leq x_2$, Gen ∪ (x_1 - Kill) ≤ Gen ∪ (x_2 - Kill)
  - Definition 2:
    - (Gen ∪ (x_1 - Kill)) ∪ (Gen ∪ (x_2 - Kill))

      = (Gen ∪ (($x_1 \cup x_2$) - Kill))



Meet Operator: Union

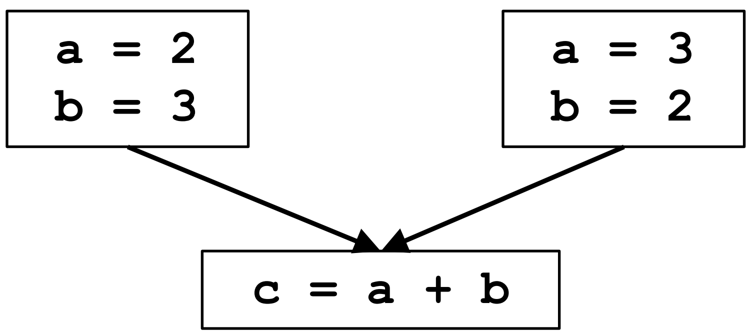
$[x_1 \leq x_2 \text{ iff } x_2 \rightarrow x_1]$

- **Note: Monotone framework does not mean that f(x) ≤ x**
    - e.g., reaching definition for two definitions in program
    - suppose: f_x: $Gen_x = \{d_1, d_2\}$; $Kill_x = \{\}$

- **If input(second iteration) ≤ input(first iteration)**
    - result(second iteration) ≤ result(first iteration)

Distributivity

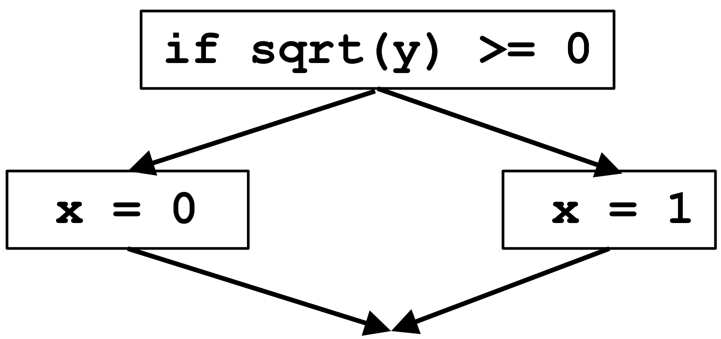
- A framework ($F, V, \wedge$) is **distributive** if and only if
  - $f(x \wedge y) = f(x) \wedge f(y)$
  - i.e. merge input, then apply f is **equal to** apply the transfer function individually then merge result
- Example: Constant Propagation is NOT distributive



Data Flow Analysis

- **Definition**
  - Let $f_1, \ldots, f_m : \in F$, where f_i is the transfer function for node i
    - $f_p = f_{n_k} \cdot \ldots \cdot f_{n_1}$, where p is a path through nodes $n_1, \ldots, n_k$
    - f_p = identify function, if p is an empty path
- **Ideal data flow answer:**
  - For each node n:

    $\wedge f_{p_i}(\mathrm{T})$, for all possibly executed paths p_i reaching n.



- **But determining all possibly executed paths is undecidable**

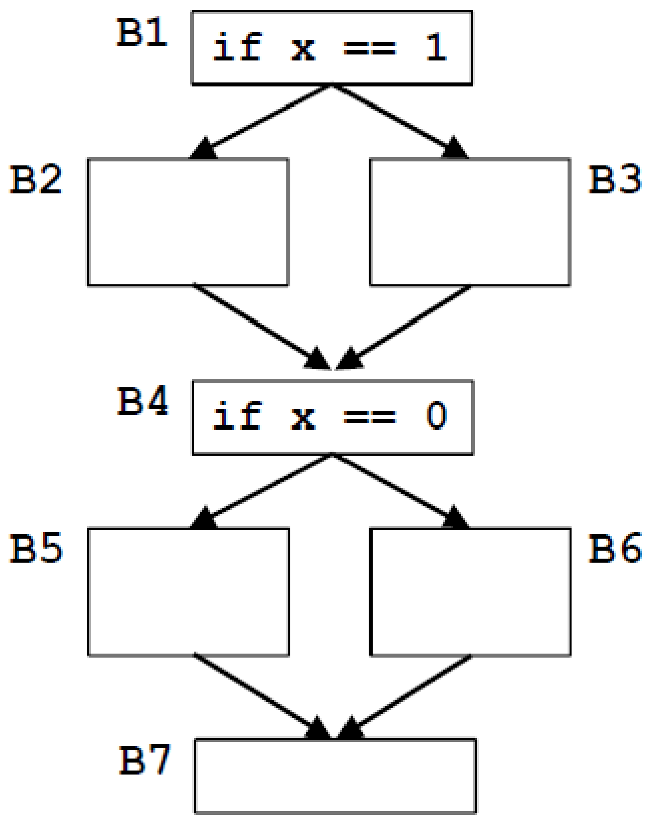
Meet-Over-Paths (MOP)

- **Error in the conservative direction**
- **Meet-Over-Paths (MOP):**
  - For each node *n:*

    $\text{MOP}(n) = \wedge f_{p_i}(\text{T})$, for all paths p_i reaching n

  - a path exists as long there is an edge in the code
  - consider more paths than necessary
  - MOP = Perfect-Solution $\wedge$ Solution-to-Unexecuted-Paths
  - MOP ≤ Perfect-Solution
  - Potentially more constrained, solution is small
    - hence *conservative*
  - It is not **safe** to be > Perfect-Solution!
- **Desirable solution: as close to MOP as possible**

MOP Example



Assume: B2 & B3 do not update x

Ideal: Considers only 2 paths
B1-B2-B4-B6-B7 (i.e., x=1)
B1-B3-B4-B5-B7 (i.e., x=0)

MOP: Also considers unexecuted paths
B1-B2-B4-B5-B7
B1-B3-B4-B6-B7

Solving Data Flow Equations

- **Example: Reaching definitions**
  - out[entry] = {}
  - Values = {subsets of definitions}
  - Meet operator: $\cup$
    - in[b] = $\cup$ out[*p*], for all predecessors *p* of b
  - Transfer functions: out[b] = $gen_b \cup (in[b] - kill_b)$
- **Any solution satisfying equations = Fixed Point Solution (FP)**
- **Iterative algorithm**
  - initializes out[b] to {}
  - if converges, then it computes Maximum Fixed Point (MFP):
    - MFP is the largest of all solutions to equations
- **Properties:**
  - FP ≤ MFP ≤ MOP ≤ Perfect-solution
  - FP, MFP are safe
  - in(b) ≤ MOP(b)

Partial Correctness of Algorithm

- **If data flow framework is monotone, then if the algorithm converges, IN[b] ≤ MOP[b]**
- **Proof: Induction on path lengths**
  - Define IN[entry] = OUT[entry] and transfer function of entry = Identity function
  - Base case: path of length 0
    - Proper initialization of IN[entry]
  - If true for path of length k, $p_k = (n_1, ..., n_k)$, then true for path of length k+1: $p_{k+1} = (n_1, ..., n_{k+1})$
    - Assume: $IN[n_k] \le f_{n_{k-1}}(f_{n_{k-2}}(\ldots f_{n_1}(IN[entry])))$
    - $IN[n_{k+1}] = OUT[n_k] \wedge \ldots$

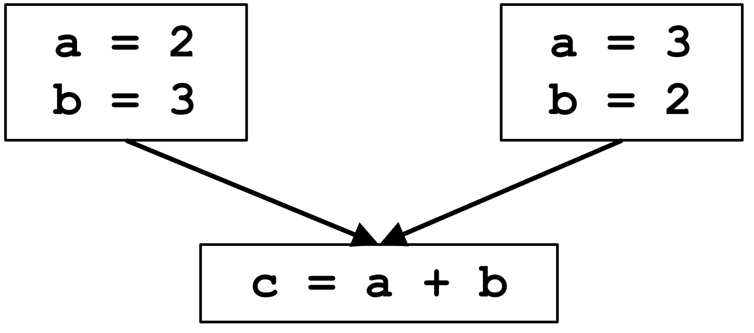
      $\le OUT[n_k]$

      $\le f_{n_k}(IN[n_k])$

      $\le f_{n_{k-1}}(f_{n_{k-2}}(\ldots f_{n_1}(IN[entry])))$

Precision

- **If data flow framework is distributive,then if the algorithm converges, IN[b] = MOP[b]**



- Monotone but not distributive: behaves as if there are additional paths

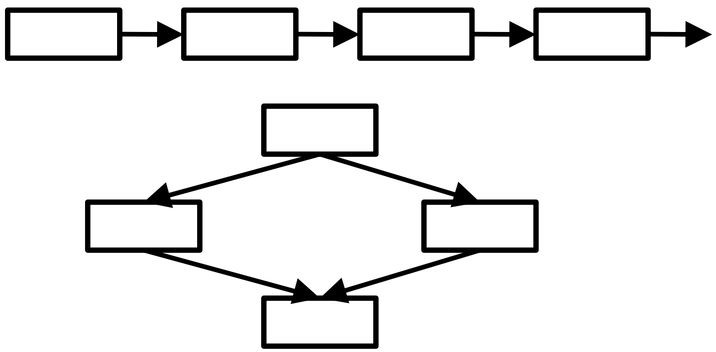
Additional Property to Guarantee Convergence

- **Data flow framework (monotone) converges if there is a finite descending chain**

- For each variable IN[b], OUT[b], consider the sequence of values set to each variable across iterations:
  - if sequence for in[b] is monotonically decreasing
    - sequence for out[b] is monotonically decreasing
      - (out[b] initialized to T)
  - if sequence for out[b] is monotonically decreasing
    - sequence of in[b] is monotonically decreasing

Speed of Convergence

- Speed of convergence depends on order of node visits



- Reverse “direction” for backward flow problems

Reverse Postorder

- **Step 1: depth-first post order**

```
main() {
    count = 1;
    Visit(root);
}
Visit(n) {
    for each successor s that has not been visited
        Visit(s);
    PostOrder(n) = count;
    count = count+1;
}
```

- **Step 2: reverse order**

```
For each node i
    rPostOrder = NumNodes - PostOrder(i)
```

Depth-First Iterative Algorithm (forward)

```
input: control flow graph CFG = (N, E, Entry, Exit)

/* Initialize */
    out[entry] = init_value
    For all nodes i
       out[i] = ⊤
    Change = True

/* iterate */
    While Change {
       Change = False
       For each node i in rPostOrder {
          in[i] = ∧(out[p]), for all predecessors p of i
          oldout = out[i]
          out[i] = f_i(in[i])
          if oldout ≠ out[i]
             Change = True
       }
    }
```

Speed of Convergence

- **If cycles do not add information**
  - information can flow in one pass down a series of nodes of increasing order number:
    - e.g., 1 -> 4 -> 5 -> 7 -> 2 -> 4 ...
  - passes determined by number of back edges in the path
    - essentially the nesting depth of the graph
  - Number of iterations = number of back edges in any acyclic path + 2
    - (2 are necessary even if there are no cycles)

- **What is the depth?**
  – corresponds to depth of intervals for “reducible” graphs
  – in real programs: average of 2.75

A Check List for Data Flow Problems

- **Semi-lattice**
  - set of values
  - meet operator
  - top, bottom
  - finite descending chain?
- **Transfer functions**
  - function of each basic block
  - monotone
  - distributive?
- **Algorithm**
  - initialization step (entry/exit, other nodes)
  - visit order: rPostOrder
  - depth of the graph

Conclusions

- Dataflow analysis examples
  - Reaching definitions
  - Live variables

- Dataflow formation definition
  - Meet operator
  - Transfer functions
  - Correctness, Precision, Convergence
  - Efficiency

CSC D70: Compiler Optimization Dataflow Analysis

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University of Toronto

Winter 2021

The content of this lecture is adapted from the lectures of Todd Mowry and Phillip Gibbons