Optimization

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Example

Determine whether the matrix

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 3 & 10 & 2 \\ 0 & 2 & 5 \end{pmatrix};$$

is positive definite (semidefinite), negative definite (semidefinite) or indefinite by using only definition.

Example

Determine whether the matrix A is positive definite (semidefinite), negative definite (semidefinite) or indefinite if

$$A = \begin{pmatrix} -1 & 3 & -2 \\ 3 & -9 & -4 \\ -2 & -4 & -5 \end{pmatrix}.$$

Example

Check whether f is convex (strictly convex), concave (strictly concave) on Ω if

$$f(x_1, x_2, x_3) = e^{x_1 x_2} + x_3^2, \quad \Omega = \mathbb{R}^3.$$

Example

Find all stationary points of f and check if these points are local maximum, minimum or saddle points for that function:

a. $f(x_1, x_2, x_3) = x_1^2 - 4x_1x_2 + 5x_2^2 + x_3^4 - 4x_3^3$;

b. $f(x_1, x_2) = e^{x_1x_2}$.

Example

Find the minimizers of f in Ω if

a.

$$f(x_1, x_2) = e^{(x_1-1)^4} + x_2^4, \quad \Omega = \mathbb{R}^2.$$

b.

$$f(x_1, x_2) = x_1^6 + x_2^6 + 2x_1^2 + 4x_2^2 - x_1 x_2, \quad \Omega = \mathbb{R}^2;$$

Example

Let $f(x) = e^{(2-x)^2} + 4x$. Our aim is to find the global minimizer x^* of f over $[0, 8]$.

a. Show that $f(x)$ is unimodal in $[0, 8]$.

b. Calculate x_2 approximation of the minimum point using the Golden Section (Ratio) Search Method with $\gamma = 1/4$.

c. Calculate x_2 approximation of the minimum point using the Bisection Method.

Example

Find the limit and the rate of convergence to that limit for the following sequences:

a. $x_n = \frac{1}{n2^n}$;

b. $x_n = \frac{5 \cdot 6^{3^n} + 1}{6^{3^n}}$.

Example

Solve the problem

$$\begin{aligned} \textit{minimize} \quad & f(x) \\ \textit{subject to} \quad & x \in \Omega, \end{aligned}$$

i.e., find the global minimum points of $f(x)$ on Ω, if

a. $f(x) = x^2$, $\Omega = (1, 2)$;

b. $f(x) = -x^2 + x + 10$, $\Omega = [-1, 1]$;

c. $f(x) = \frac{x+1}{x^2+3}$, $\Omega = [0, +\infty)$.