Optimization

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Example

Find all stationary points of f and check if these points are local maximum, minimum or saddle points for that function if

a. $f(x_1, x_2) = 4x_1^4 + x_2^4 + 4x_1 x_2$;

b. $f(x_1, x_2) = \frac{1}{3}x_1^3 - 4x_1 + \frac{1}{3}x_2^3 - 16x_2$;

c. $f(x_1, x_2, x_3) = 3x_1^3 - 9x_1 + x_2^3 + x_3^3 - 6x_3^2 - 10.$

Theorem

When f is convex, any local minimizer x^* is a global minimizer of f. If in addition f is continuously differentiable, then any stationary point x^* is a global minimizer of f.

As f is convex

$$f(x) \geq f(x^*) + \nabla f(x^*)^T (x - x^*) = f(x^*), \quad \forall x,$$

therefore x^* is a global minimizer.

Example

Find the global minimizer of f on Ω if

$$f(x_1, x_2, x_3) = x_1^4 + x_2^4 + x_1^2 x_2^2 + x_3^2, \quad \Omega = \mathbb{R}^3.$$

The Rate Convergence of Numerical Sequence

Definition

A sequence x_n exhibits **linear** convergence to a limit x if there is a constant C in the interval $(0\ ,\ 1)$ and an integer N such that

$$|x_{n+1} - x| \leq C|x_n - x|, \quad \forall n \geq N.$$

Example

$$x_n = \frac{1}{2^n}.$$

Definition

A sequence x_n exhibits **superlinear** convergence to a limit x if there is a sequence β_n, which converges to 0, and an integer N such that

$$|x_{n+1} - x| \leq \beta_n |x_n - x|, \quad \forall n \geq N.$$

Example

$$x_n = \frac{n}{2^{n^2}} + 1.$$