ASDS Statistics, YSU, Fall 2020

Lecture 20

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Parametric Statistics: General Problem

One of the general Problems of Statistics is the following: we have a Sample, a Dataset $x: \ x_1, ..., x_n$, and our aim is to get an insight from these numbers, to get an information about the Population, about the *process* generating that Dataset.

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We can, of course, calculate the Sample Mean and the Sample Variance of our Dataset. Or, we can plot the Histogram or KDE. But will this give an info about the Population or the process generating the Dataset? Well, no, in general.

Parametric Statistics: Modeling

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str(cars)
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## 'data.frame':    50 obs. of  2 variables:
##  $ speed: num  4 4 7 7 8 9 10 10 10 11 ...
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then we think about this as a Sample, and we realize that if we would make another Sampling, we'd get other numbers. Even with the same cars!

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Example: If we consider the weights (in Kg) of 10 persons:

$$69.5, 77.1, ..., 109,$$

then we make the following model: let X_1 be the weight of the first person (say, the first person we will meet when performing the experiment), X_2 be the weight of the second person,. . . , X_{10} be the weight of the 10-th person.

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Our Dataset of weights is just one of the possible realizations of $X_1, ..., X_{10}$.

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rnorm(6, mean = 155, sd = sqrt(30))
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So, again, having a Dataset $x_1, ..., x_n$, statisticians work with a r.v.s $X_1, X_2, ..., X_n$ to work not only with a particular Sample, but with **all possible samples** from the Distribution (Process) behind the phenomenon.

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Now, if we have a Random Sample $X_1, ..., X_n$, then, because they are IID, we will have that all X_k-s are coming from the same Distribution:

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We will consider one of the main Problems of the Parametric Statistics: **Using the observations from our Random Sample, estimate the value of the Parameter θ.**

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$$X_1, X_2, ..., X_n \sim Pois(\lambda).$$

After collecting that data, we will get the Dataset $x_1, x_2, ..., x_n$ of the daily number of car accidents for day $1, 2, ..., n$.

- Here our Parameter is $\lambda = \theta$; it is 1D;
- The set of Parameters is $(0, +\infty) = \Theta \subset \mathbb{R}$;
- The Parametric Family of Distributions is $Pois(\lambda)$.

And our problem here will be to estimate our unknown λ, using the realizations $x_1, x_2, ..., x_n$.

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Point Estimates

Motivating Example $\ddot\smile$

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Example: I have generated the following Data from a Normal Distribution:

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Moral: Statistics is like a Detective Story: you need to find the Unknown (murderer?) using some (small?) amount of Observations, Data you have $\ddot\smile$

Statistics, Estimator and Estimate

Let us recall what is our Problem: assume we have a Dataset $x_1, ..., x_n$. We assume that this is a realization of a Random Sample $X_1, ..., X_n$, coming from one of the Distributions from some Parametric Family:

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This is our third meaning of the term *Statistics*.

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The value of the Estimator at our observations, $g(x_1, x_2, ..., x_n)$, is called an **Estimate** for θ, and it is again (unfortunately) denoted by $\hat{\theta} = \hat{\theta}_n$.

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And the following is not an estimator:

- $\hat{\lambda} = \frac{\lambda}{X_1 + X_n}$, since it depends on λ - the unkown parameter value.

Estimators and Estimates

Note: We require our Estimator to be independent of the Parameter θ, since we want to be able to calculate the value of the Estimator on the observation, i.e., we want to be able to calculate the Estimate, we want to make our Estimator *observable*. Since θ is unknown, then we cannot use it in the Estimator - we will not be able to calculate the Estimate.

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- **Estimate** is a number, it is the result of plugging the observation into the Estimator.

Example

Example: Assume we want to estimate the probability of a birth (birthrate) of a girl child in Armenia. To that end, say, we ask a maternity hospital to provide the birth data for a day. And say, our data is:

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First we need to build a Model. To that end, we encode our data by

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where $b = 0$ and $g = 1$: this is to be able to use one of our standard Distributions. Next, from a Dataset we pass, for a generalization, to a Random Sample

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7,$$

where X_k is the gender of the k-th child *before the observation was made* ($X_k = 1$ if the child will be a girl, and 0 otherwise).

Example, cont'd

Then we will have

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7 \sim \mathit{Bernoulli}(p),$$

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To estimate p, let us take the following **Estimator**:

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This is a r.v. . The **Estimate** for p, using our Observation, will be

$$\hat{p} = \frac{0 + 1 + 1 + 0 + 0 + 1 + 0}{7} = \frac{3}{7}.$$

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Or, maybe

$$\hat{p} = \frac{X_{(1)} + X_{(n)}}{2} \qquad or \qquad \hat{p} = Median(X_1, ..., X_n)?$$

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good enough to estimate the unknown μ?

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And what about estimating σ^2? Can you suggest Estimators? Say, which one to choose:

$$\widehat{\sigma^2} = \left(\frac{\sum_{k=1}^{n} |X_k - \overline{X}_n|}{n}\right)^2 \qquad \textit{or} \qquad \widehat{\sigma^2} = \frac{\sum_{k=1}^{n}(X_k - \overline{X}_n)^2}{n} \qquad \textit{or}$$

$$\widehat{\sigma^2} = \frac{\sum_{k=1}^{n}(X_k - \overline{X}_n)^2}{n-1} \qquad \textit{or} \qquad \widehat{\sigma^2} = \text{other Estimator?}$$

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Question: Which Estimator to use?

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And we will use $Var_\theta(X)$ for the Variance of X.

Properties of Estimators

Risk, Mean Squared Error of the Estimator

Assume we have a Random Sample

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and we use the Estimator $\hat{\theta}$ to estimate θ.

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Definition: The **Mean Squared Error** or the **Quadratic Risk** of the estimator $\hat{\theta}_n$ of θ is

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