Note Title 25-Nov-20

· MoM (MME) for Bernoulli(p)

$X \sim \text{Bernoulli}(p)$, $X_1, \dots, X_n \sim \text{Bernoulli}(p)$

Try! 1st order Theo Mom = 1st order Emp Mom

$$\mathbb{E}(X) = \frac{X_1 + X_2 + \dots + X_n}{n} = \overline{X}$$

$$\mathbb{E}(X) = p \Rightarrow \boxed{\hat{p}^{MoM} = \overline{X}}$$

X	0	1	2
$\mathbb{P}$	$\frac{\theta}{10}$	$\frac{\theta}{5}$	$1-\frac{3\theta}{10}$

Try

$$\mathbb{E}(X) = \overline{X}$$

$$\mathbb{E}(X) = 1\cdot\frac{\theta}{5} + 2\cdot\left(1-\frac{3\theta}{10}\right) = 2+\frac{\theta}{5}-\frac{3\theta}{5} = 2-\frac{2\theta}{5}$$

$$2-\frac{2\theta}{5} = \overline{X} \qquad \frac{2\theta}{5} = 2-\overline{X}$$

$$\theta = \frac{5}{2}(2-\overline{X})$$

$$\boxed{\hat{\theta}^{MoM} = \frac{5}{2}(2-\overline{X})}$$

Unbiasedness:

$$\mathbb{E}\left(\hat{\theta}^{MoM}\right) = \frac{5}{2}\left(2-\mathbb{E}(\bar{X})\right) = \frac{5}{2}\left(2-\mathbb{E}(X_1)\right) =$$

$$= \frac{5}{2}\left(2-\left(2-\frac{2\theta}{5}\right)\right) = \theta \quad \Rightarrow \underline{\text{Unbiased}}$$

$$\hat{\theta}^{MoM} = \frac{5}{2}\left(2-\bar{X}\right) \xrightarrow[n\to\infty]{WLLN,\ \mathbb{P}} \frac{5}{2}\left(2-\mathbb{E}(X_1)\right) = \theta$$

$$\Rightarrow \underline{\text{Consistent}}$$

- MoM for $Exp(\lambda)$

$, X \sim Exp(\lambda)$

Try: $\mathbb{E}(X) = \overline{X}$

$$\mathbb{E}(X) = \frac{1}{\lambda} \Rightarrow \frac{1}{\lambda} = \overline{X} \Rightarrow \boxed{\hat{\lambda}^{MoM} = \frac{1}{\overline{X}}}$$

- MoM for $Unif[0,\theta]$, $\theta > 0$

$$X, X_1, \dots, X_n \sim Unif[0,\theta]$$

Try!

$$\mathbb{E}(X) = \overline{X} \quad \mapsto \quad \frac{\theta}{2} = \overline{X} \Rightarrow \boxed{\hat{\theta}^{MoM} = 2\overline{X}}$$

$$\mathbb{E}(X) = \frac{\theta}{2}$$

$$\boxed{\begin{array}{l} \overline{a, b, c} \qquad Unif[0,\theta] \\ \hat{\theta} = 2 \cdot \frac{a+b+c}{3} < \max(a,b,c) \end{array}}$$

· MoM for $Unif[-\theta,\theta]$, $\theta > 0$

$X \sim Unif[-\theta,\theta]$

Try: $\underset{\substack{\parallel \\ 0}}{E(X)} = \overline{X}$

$0 = \overline{X} \Longrightarrow$ no θ here

Try: $E(X^2) = \frac{X_1^2 + X_2^2 + \dots + X_n^2}{n} = \overline{X^2}$

$E(X^2) = \underset{\frac{\theta^2}{3}}{Var(X)} + [\underset{\substack{\parallel \\ 0}}{E(X)}]^2 = \frac{\theta^2}{3}$ $\Longrightarrow$

$\frac{\theta^2}{3} = \overline{X^2}$

$\boxed{\hat{\theta}^{MoM} = \sqrt{3\,\overline{X^2}}}$

• MoM for μ, σ^2 in $N(\mu, \sigma^2)$

$X, X_1, \ldots, X_n \sim N(\mu, \sigma^2)$

Try:

$$\begin{cases} \mathbb{E}(X) = \overline{X} \\ \mathbb{E}(X^2) = \overline{X^2} \end{cases} \qquad \begin{cases} \mu = \overline{X} \\ \sigma^2 + \mu^2 = \overline{X^2} \end{cases} \qquad \boxed{\begin{array}{l} \hat{\mu}^{MoM} = \overline{X} \\ \widehat{\sigma^2}^{MoM} = \overline{X^2} - (\overline{X})^2 \end{array}}$$

$$\mathbb{E}(X^2) = Var(X) + [\mathbb{E}(X)]^2 = \sigma^2 + \mu^2$$

$$\widehat{\sigma^2}^{MoM} = \overline{X^2} - (\overline{X})^2 = \frac{\sum_1^n X_k^2}{n} - \left(\frac{\sum X_k}{n}\right)^2 =$$

$$= \frac{\sum_1^n (X_k - \overline{X})^2}{n}$$