Deep Learning

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- This is accomplished by adding a regularizer, or penalty term, to whatever cost or objective function the algorithm is trying to minimize.
- The end result is to reduce the learned representation's sensitivity towards the training input.

Contractive Autoencoders

Let f is our encoder, g is the decoder and D is our training dataset. In the previous cases we minimize this kind of loss function:

$$\sum_{x \in D} L\left(x, g\left(f\left(x\right)\right)\right).$$

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In the case of contractive autoencoders we will minimize this one

$$\sum_{x \in D} \left(L\left(x, g\left(f\left(x\right)\right)\right) + \lambda \left\|J_f\left(x\right)\right\|_F^2\right),$$

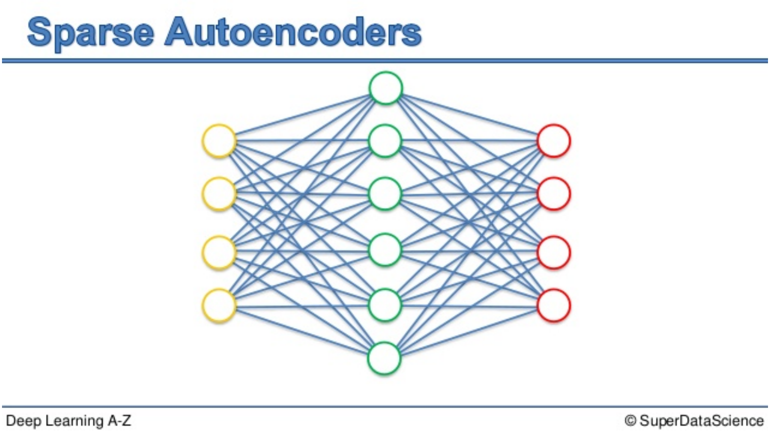
where the added summand is the square of Frobenius norm of the following Jacobian matrix:

$$\left[J_f\left(x\right)\right]_{i,j} = \frac{\partial f_j\left(x\right)}{\partial x_i}$$

i.e.

$$\left\|J_f\left(x\right)\right\|_F^2 = \sum_{i,j} \left(\frac{\partial f_j\left(x\right)}{\partial x_i}\right)^2.$$

Sparse Autoencoders



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- Sparsity penalty is introduced on the hidden layer. This is to prevent output layer copy input data. This prevents overfitting.

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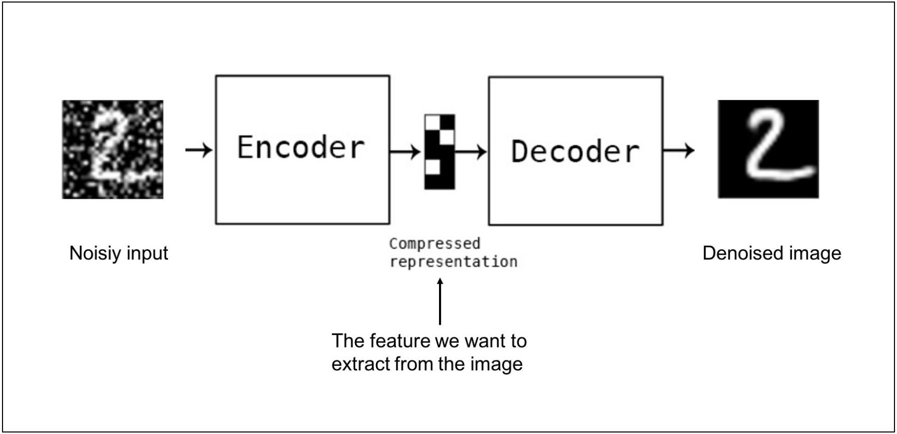
where

$$KL(\rho || \rho_j) = -\rho \log \frac{\rho_j}{\rho} - (1 - \rho) \log \frac{1 - \rho_j}{1 - \rho}.$$

Denoising Autoencoders



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- This helps to avoid the autoencoders to copy the input to the output without learning features about the data.
- The model learns a vector field for mapping the input data towards a lower dimensional manifold which describes the natural data to cancel out the added noise.

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- Can we generate content with autoencoders?
- Variational autoencoder can be defined as being an autoencoder whose training is regularised to avoid overfitting and ensure that the latent space has good properties that enable generative process.
- Instead of encoding an input as a single point, we encode it as a distribution over the latent space.

Variational Autoencoders

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- Let minimize the function

$$L(w) = KL(q_w(z|x) || p(z|x)).$$

Note that

$$KL\left(q_w\left(z|x\right)||p\left(z|x\right)\right) = \mathbb{E}_{q_w(z|x)}\left[\log \frac{q_w\left(z|x\right)}{p\left(z|x\right)}\right]$$

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$$\underset{w,w'}{\operatorname{argmin}}\left(KL\left(q_w\left(z|x\right)||p\left(z\right)\right) + \mathbb{E}_{q_w(z|x)}\left[\|x - f_{w'}\left(z\right)\|^2\right]\right)$$

Problem: how to evaluate the term $KL\left(q_w\left(z|x\right)||p\left(z\right)\right)$?

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Solution: we will assume that $q_w = \mathcal{N}\left(\mu, \Sigma\right)$ and $p = \mathcal{N}\left(0, I\right)$.

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Assumptions

- In practice, the encoded distributions are chosen to be normal so that the encoder can be trained to return the mean and the covariance matrix that describe these Gaussians.
- The distributions returned by the encoder are enforced to be close to a standard normal distribution.

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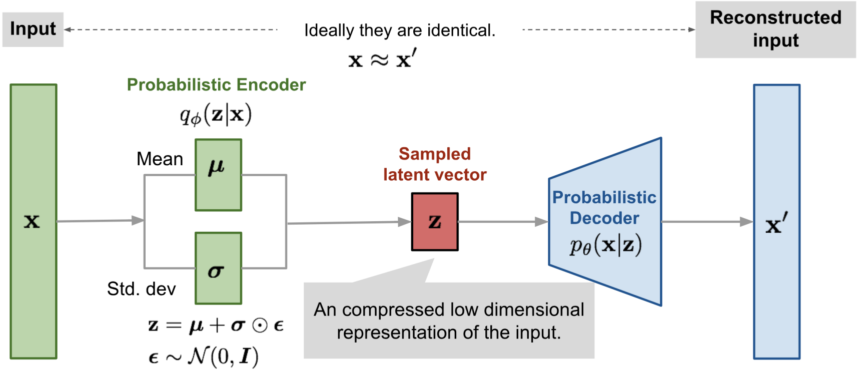

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