ASDS Statistics, YSU, Fall 2020

Lecture 15

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Contents

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Values of X	0	1	...	k	...	n
$\mathbb{P}(X=x)$	$\binom{n}{0}p^0(1-p)^{n-0}$	$\binom{n}{1}p^1(1-p)^{n-1}$	...	$\binom{n}{k}p^k(1-p)^{n-k}$	...	$\binom{n}{n}p^n(1-p)^0$

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- Mean and Variance: $\mathbb{E}(X) = n \cdot p$, $Var(X) = n \cdot p(1 - p)$.

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- Mean and Variance: $\mathbb{E}(X) = n \cdot p$, $Var(X) = n \cdot p(1 - p)$.
- Models: Models the independent repetition of the $Bernoulli(p)$ Experiment.

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- Additional: If $X_1, X_2, ..., X_n \sim Bernoulli(p)$ are independent, then $X_1 + X_2 + ... + X_n \sim Binom(n, p)$.

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- Additional: If $X_1, X_2, ..., X_n \sim Bernoulli(p)$ are independent, then $X_1 + X_2 + ... + X_n \sim Binom(n, p)$.
- Example:

```r
rbinom(10, size = 5, prob = 0.3)
```

```
##  [1] 2 2 1 0 0 1 0 0 0 3
```

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$\mathbb{P}(X = x)$	p	$p(1-p)$	$p(1-p)^2$	...

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- **Note:** **R** is using the second definition of the Geometric Distribution, with the support $\{0, 1, 2, 3, ...\}$, i.e., in **R**, $X \sim Geom(p)$ shows *the number of Failures before the first Success*
- Example:

```
rgeom(10,prob = 0.3)
```

```
##  [1] 7 0 1 3 7 1 0 3 2 6
```

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- Mean and Variance: $\mathbb{E}(X) = \lambda$, $Var(X) = \lambda$.
- Models: Models the number of success in a fixed interval. Say, number of calls, number of customers, number of clicks, number of page visits, . . . λ is the average number of calls, customers, clicks, page visits,. . .

Poisson Distribution

- **R** name: `pois` with the parameter `lambda`

Poisson Distribution

- **R** name: `pois` with the parameter `lambda`
- Example:

```
rpois(10, lambda = 2)
```

```
##  [1] 3 1 2 2 1 2 2 1 1 3
```

Important Continuous Distributions

Uniform Distribution

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- Mean and Variance: $\mathbb{E}(X) = \frac{a+b}{2}$, $Var(X) = \frac{(b-a)^2}{12}$.
- Models: Usually we think about the Uniform Distribution when talking about *picking a random number from an interval*

Uniform Distribution

- **R** name: `unif` with the parameters `min = 0` and `max = 1`

Uniform Distribution

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- Example:

```
runif(10, min = 2, max = 5)
```

```
##  [1] 2.068570 2.785241 4.257182 2.867320 2.457061 4.8752
##  [9] 2.024676 3.168418
```

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- Models: time elapsed until the occurrence of certain event, or the time between events (waiting times), when that time is random. λ is the average "arrival rate", the reciprocal of the average time between the events,
$$\lambda = \frac{1}{\text{average time between events}}.$$

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Exponential Distribution

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- Example:

```r
rexp(10, rate = 2)
```

```
##  [1] 0.05362480 0.67033856 0.04567225 0.47474777 0.1201
##  [7] 0.23659345 0.58521820 0.04410356 0.22873507
```