ASDS Statistics, YSU, Fall 2020

Lecture 04

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23 Sep 2020

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- Histogram

Last Lecture Recap

- For which type of variables BarPlot will work?

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- Give the Definition of the Empirical CDF.

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- Give the Definition of the Empirical CDF.
- State the Glivenko-Cantelli theorem.

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To define the Histogram, first we divide the range of our Dataset into *class intervals* or *bins*:

- we take first the range: either $I = [\min_i\{x_i\}, \max_i\{x_i\}]$ or I is an interval containing $[\min_i\{x_i\}, \max_i\{x_i\}]$;

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- we calculate the number n_j of datapoints x_i lying in I_j:

$$n_j = \text{the number of data points in } I_j \qquad j = 1, 2, ..., k.$$

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Histograms

Definition: The **frequency histogram** of our continuous (or a grouped) data $x_1, ..., x_n$ is the piecewise constant function

$$h_{freq}(x) = n_j, \qquad \forall x \in I_j, \quad j = 1, 2, ..., k.$$

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Frequency histogram shows the number of observations in our dataset in each bin, in each class interval. One also defines $h_{freq}(x) = 0$ for all $x \notin I$.

Example

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Here is the header:

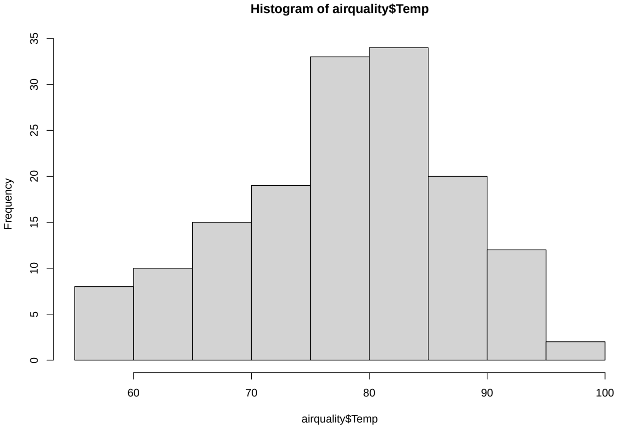
```r
head(airquality)
```

```
##   Ozone Solar.R Wind Temp Month Day
## 1    41     190  7.4   67     5   1
## 2    36     118  8.0   72     5   2
## 3    12     149 12.6   74     5   3
## 4    18     313 11.5   62     5   4
## 5    NA      NA 14.3   56     5   5
## 6    28      NA 14.9   66     5   6
```

Example

Let's Plot the histogram of the *Temp* (Temperature) Variable:

```r
hist(airquality$Temp)
```



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- **R**, by default, is choosing some appropriate bins;
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- **R** is adding the default *OX* axis name and the Figure Title.

Histograms

Next is the Relative Frequency Histogram definition:

Definition The **relative frequency histogram** of our continuous data $x_1, ..., x_n$ is the piecewise constant function

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The Default **R** package has no Relative Frequency Histogram Plotting command (or I do not know $\ddot{\smile}$). But you can use, say, the *lattice* library's *histogram* command:

```
library(lattice)
histogram(airquality$Temp)
```

The Density or Normalized Relative Frequency Histogram

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$$h_{dens}(x) = \frac{n_j}{n} \cdot \frac{1}{length(I_j)}, \qquad \forall x \in I_j.$$

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$$h_{dens}(x) = \frac{n_j}{n} \cdot \frac{1}{length(I_j)}, \qquad \forall x \in I_j.$$

Here $length(I_j)$ is the length of the interval I_j. Also we define $h_{dens}(x) = 0$, if $x \notin I$.

Note

In the case (which is the mostly used one) when all intervals I_j have the same length:

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$$h_{dens}(x) = \frac{h_{relfreq}(x)}{h} = \frac{n_j}{n \cdot h}, \qquad \forall x \in I_j.$$

Idea of the Density Histogram

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The idea of dividing to the lenght of the corresponding interval, in the definition of the Density Histogram, is that in this case, the Total Area of all rectangles of our Histogram is 1.

Recall that all PDF functions integrate to 1. And the Density Histogram is approximating (estimating) the unknown PDF behind our Data!