ASDS Statistics, YSU, Fall 2020

Lecture 29

Michael Poghosyan

19 Dec 2020

Contents

Significance and Power of Test

- The Probability of Type I Error is called the **Significance Level** of our Test, and is denoted by α:

Significance and Power of Test

- The Probability of Type I Error is called the **Significance Level** of our Test, and is denoted by α:

$$\alpha = \mathbb{P}(\ \textbf{Type I Error}\ ) = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}).$$

Significance and Power of Test

- The Probability of Type I Error is called the **Significance Level** of our Test, and is denoted by α:

$$\alpha = \mathbb{P}(\textbf{ Type I Error }) = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}).$$

- The Probability of Type II Error is denoted by β:

$$\beta = \mathbb{P}(\textbf{ Type II Error }) = \mathbb{P}(\text{Retain } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is False}).$$

Significance and Power of Test

- The Probability of Type I Error is called the **Significance Level** of our Test, and is denoted by α:

$$\alpha = \mathbb{P}(\textbf{ Type I Error }) = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}).$$

- The Probability of Type II Error is denoted by β:

$$\beta = \mathbb{P}(\textbf{ Type II Error }) = \mathbb{P}(\text{Retain } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is False}).$$

  $(1 - \beta)$ is called the **Power** of the Test.

Significance and Power of Test

- The Probability of Type I Error is called the **Significance Level** of our Test, and is denoted by α:

$$\alpha = \mathbb{P}(\ \textbf{Type I Error}\ ) = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}).$$

- The Probability of Type II Error is denoted by β:

$$\beta = \mathbb{P}(\ \textbf{Type II Error}\ ) = \mathbb{P}(\text{Retain } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is False}).$$

  $(1 - \beta)$ is called the **Power** of the Test.

It is easy to see that

$$\textit{Power} = 1 - \beta = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is False}).$$

Significance and Power

Here are the Probabilities of correct/incorrect decisions for a Hypothesis testing, on a Table:

Significance and Power

Here are the Probabilities of correct/incorrect decisions for a Hypothesis testing, on a Table:

Probabilities of Correct/InCorrect Decisions:

Test Decision \ Reality	$\mathcal{H}_0$ is True	$\mathcal{H}_0$ is False (i.e., $\mathcal{H}_1$ is True)
Reject $\mathcal{H}_0$	$\alpha =$**Significance**	$1-\beta =$**Power**
Do Not Reject $\mathcal{H}_0$	$1-\alpha$	β

Choosing a Test

Test Choice: We want to have a Test with small Error Probabilities, i.e., with small α and β.

Choosing a Test

Test Choice: We want to have a Test with small Error Probabilities, i.e., with small α and β. Unfortunately, making them both small is not always possible. So we first think in controlling α: **our goal will be to choose the desired α and to have a Test of Significance Level α.**

Choosing a Test

Test Choice: We want to have a Test with small Error Probabilities, i.e., with small α and β. Unfortunately, making them both small is not always possible. So we first think in controlling α: **our goal will be to choose the desired α and to have a Test of Significance Level α.**

Common values of α are $0.05, 0.01$ and 0.1 (corresponding to $95\%, 99\%$ and 90% Confidence Level!).

Example

Example: Assume we have a Patient in a Hospital, and we want to see if he is ill or not (with some illness). Then the Null Hypothesis is:

Example

Example: Assume we have a Patient in a Hospital, and we want to see if he is ill or not (with some illness). Then the Null Hypothesis is: **Patient is healthy**. Our Test is *the Doctor*.

Example

Example: Assume we have a Patient in a Hospital, and we want to see if he is ill or not (with some illness). Then the Null Hypothesis is: **Patient is healthy**. Our Test is *the Doctor*.

- What is means that the Significance level of our Test, α, is small ?

Example

Example: Assume we have a Patient in a Hospital, and we want to see if he is ill or not (with some illness). Then the Null Hypothesis is: **Patient is healthy**. Our Test is *the Doctor*.

- What is means that the Significance level of our Test, α, is small ?
- What it means that the Power of our Test, $1 - \beta$, is high ?

Hypo Testing: Constructing a Test

To test a Hypothesis, one is following the following steps $\ddot{\smile}$:

Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

Hypo Testing: Constructing a Test

To test a Hypothesis, one is following the following steps $\ddot\smile$:

Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

Step 2: We State the Hypotheses: we take Θ_0 and Θ_1 such that $\Theta = \Theta_0 \cup \Theta_1$ and $\Theta_0 \cap \Theta_1 = \varnothing$, and state the Hypothesis:

$$\mathcal{H}_0 : \ \theta \in \Theta_0 \qquad \textit{vs} \qquad \mathcal{H}_1 : \ \theta \in \Theta_1$$

Hypo Testing: Constructing a Test

To test a Hypothesis, one is following the following steps $\ddot{\smile}$:

Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

Step 2: We State the Hypotheses: we take Θ_0 and Θ_1 such that $\Theta = \Theta_0 \cup \Theta_1$ and $\Theta_0 \cap \Theta_1 = \varnothing$, and state the Hypothesis:

$$\mathcal{H}_0: \ \theta \in \Theta_0 \qquad \textit{vs} \qquad \mathcal{H}_1: \ \theta \in \Theta_1$$

Step 3: We choose the desirable Significance Level: we choose α (small enough, say, $\alpha = 0.05$).

[1] If n is free to choose, then we can find it by taking the desirable Power of the Test.

Hypo Testing: Constructing a Test

To test a Hypothesis, one is following the following steps $\ddot\smile$:

Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

Step 2: We State the Hypotheses: we take Θ_0 and Θ_1 such that $\Theta = \Theta_0 \cup \Theta_1$ and $\Theta_0 \cap \Theta_1 = \varnothing$, and state the Hypothesis:

$$\mathcal{H}_0: \ \theta \in \Theta_0 \qquad vs \qquad \mathcal{H}_1: \ \theta \in \Theta_1$$

Step 3: We choose the desirable Significance Level: we choose α (small enough, say, $\alpha = 0.05$).

Step 4: Assume we know how many Observations we will have[1], say, n.

[1] If n is free to choose, then we can find it by taking the desirable Power of the Test.

Hypo Testing: Constructing a Test

To test a Hypothesis, one is following the following steps $\ddot\smile$:

Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

Step 2: We State the Hypotheses: we take Θ_0 and Θ_1 such that $\Theta = \Theta_0 \cup \Theta_1$ and $\Theta_0 \cap \Theta_1 = \varnothing$, and state the Hypothesis:

$$\mathcal{H}_0: \ \theta \in \Theta_0 \qquad vs \qquad \mathcal{H}_1: \ \theta \in \Theta_1$$

Step 3: We choose the desirable Significance Level: we choose α (small enough, say, $\alpha = 0.05$).

Step 4: Assume we know how many Observations we will have[1], say, n. Then we take a Random Sample

$$X_1, X_2, \dots, X_n \sim \mathcal{F}_\theta.$$

[1] If n is free to choose, then we can find it by taking the desirable Power of the Test.

Hypo Testing: Constructing a Test

Step 5: Next, we choose a Test Statistics,

$$T = T(X_1, X_2, ..., X_n)$$

such that

Hypo Testing: Constructing a Test

Step 5: Next, we choose a Test Statistics,

$$T = T(X_1, X_2, ..., X_n)$$

such that

- We can calculate it, if we will have Observations;

Hypo Testing: Constructing a Test

Step 5: Next, we choose a Test Statistics,

$$T = T(X_1, X_2, ..., X_n)$$

such that

- We can calculate it, if we will have Observations;
- the Distribution (or, at least, the Asymptotic Distribution) of T is known **under** $\mathcal{H}_0$.

Hypo Testing: Constructing a Test

Step 5: Next, we choose a Test Statistics,

$$T = T(X_1, X_2, ..., X_n)$$

such that

- We can calculate it, if we will have Observations;
- the Distribution (or, at least, the Asymptotic Distribution) of T is known **under** $\mathcal{H}_0$.

Step 6: Then, we specify the **Rejection Region (RR)**, usually of the form

$$RR = \{|T| > c\} \qquad or \qquad RR = \{T > c\} \qquad or \qquad RR = \{T < c\}.$$

Hypo Testing: Constructing a Test

Step 5: Next, we choose a Test Statistics,

$$T = T(X_1, X_2, ..., X_n)$$

such that

- We can calculate it, if we will have Observations;
- the Distribution (or, at least, the Asymptotic Distribution) of T is known **under** $\mathcal{H}_0$.

Step 6: Then, we specify the **Rejection Region (RR)**, usually of the form

$$RR = \{|T| > c\} \qquad or \qquad RR = \{T > c\} \qquad or \qquad RR = \{T < c\}.$$

Here c is called **the Critical Value** of the Test. We will choose it (soon).

Hypo Testing: Constructing a Test

Step 5: Next, we choose a Test Statistics,

$$T = T(X_1, X_2, ..., X_n)$$

such that

- We can calculate it, if we will have Observations;
- the Distribution (or, at least, the Asymptotic Distribution) of T is known **under** $\mathcal{H}_0$.

Step 6: Then, we specify the **Rejection Region (RR)**, usually of the form

$$RR = \{|T| > c\} \qquad or \qquad RR = \{T > c\} \qquad or \qquad RR = \{T < c\}.$$

Here c is called **the Critical Value** of the Test. We will choose it (soon). We Reject $\mathcal{H}_0$ if and only if $T \in RR$.

Hypo Testing: Constructing a Test

Now, at this step, we need to choose the Critical Value c.

Hypo Testing: Constructing a Test

Now, at this step, we need to choose the Critical Value c. We choose it to have the desirable Significance Level:

$$\mathbb{P}(\text{ Type I Error }) = \alpha,$$

Hypo Testing: Constructing a Test

Now, at this step, we need to choose the Critical Value c. We choose it to have the desirable Significance Level:

$$\mathbb{P}(\text{ Type I Error }) = \alpha,$$

i.e.,

$$\alpha = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}) = \mathbb{P}(T \in RR \,|\theta \in \Theta_0 \,).$$

Hypo Testing: Constructing a Test

Now, at this step, we need to choose the Critical Value c. We choose it to have the desirable Significance Level:

$$\mathbb{P}(\text{ Type I Error }) = \alpha,$$

i.e.,

$$\alpha = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}) = \mathbb{P}(T \in RR \mid \theta \in \Theta_0).$$

And usually, c is some order quantile of the Distribution of T.

Hypo Testing: Constructing a Test

Now, at this step, we need to choose the Critical Value c. We choose it to have the desirable Significance Level:

$$\mathbb{P}(\text{ Type I Error }) = \alpha,$$

i.e.,

$$\alpha = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}) = \mathbb{P}(T \in RR \mid \theta \in \Theta_0 \text{ }).$$

And usually, c is some order quantile of the Distribution of T.

Step 7: We make an observation, $x_1, x_2, ..., x_n$, calculate the Test Statistics at that Observation, $T = T(x_1, ..., x_n)$.

Hypo Testing: Constructing a Test

Now, at this step, we need to choose the Critical Value c. We choose it to have the desirable Significance Level:

$$\mathbb{P}(\text{ Type I Error }) = \alpha,$$

i.e.,

$$\alpha = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}) = \mathbb{P}(T \in RR \mid \theta \in \Theta_0).$$

And usually, c is some order quantile of the Distribution of T.

Step 7: We make an observation, $x_1, x_2, ..., x_n$, calculate the Test Statistics at that Observation, $T = T(x_1, ..., x_n)$. Now, we will

- Reject $\mathcal{H}_0$, if $T(x_1, ..., x_n) \in RR$;

Hypo Testing: Constructing a Test

Now, at this step, we need to choose the Critical Value c. We choose it to have the desirable Significance Level:

$$\mathbb{P}(\text{ Type I Error }) = \alpha,$$

i.e.,

$$\alpha = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is True}) = \mathbb{P}(T \in RR \mid \theta \in \Theta_0).$$

And usually, c is some order quantile of the Distribution of T.

Step 7: We make an observation, $x_1, x_2, ..., x_n$, calculate the Test Statistics at that Observation, $T = T(x_1, ..., x_n)$. Now, we will

- Reject $\mathcal{H}_0$, if $T(x_1, ..., x_n) \in RR$;
- Not Reject $\mathcal{H}_0$, if $T(x_1, ..., x_n) \notin RR$.

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known;

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu \neq \mu_0$

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu > \mu_0$

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0: \ \mu = \mu_0$ *vs* $\mathcal{H}_1: \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0: \ \mu = \mu_0$ *vs* $\mathcal{H}_1: \ \mu > \mu_0$
- Case 3: $\mathcal{H}_0: \ \mu = \mu_0$ *vs* $\mathcal{H}_1: \ \mu < \mu_0$

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu > \mu_0$
- Case 3: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu < \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0: \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1: \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0: \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1: \ \mu > \mu_0$
- Case 3: $\mathcal{H}_0: \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1: \ \mu < \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Random Sample: We take $X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2)$;

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0: \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1: \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0: \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1: \ \mu > \mu_0$
- Case 3: $\mathcal{H}_0: \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1: \ \mu < \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Random Sample: We take $X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2)$;

Test Statistics: Idea:

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu > \mu_0$
- Case 3: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu < \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Random Sample: We take $X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2)$;

Test Statistics: Idea:

- what we have to approximate our Parameter (Estimate) μ?

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0 : \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1 : \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0 : \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1 : \ \mu > \mu_0$
- Case 3: $\mathcal{H}_0 : \ \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1 : \ \mu < \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Random Sample: We take $X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2)$;

Test Statistics: Idea:

- what we have to approximate our Parameter (Estimate) μ? - Easy: $\overline{X}$;

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0: \ \mu = \mu_0$ *vs* $\mathcal{H}_1: \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0: \ \mu = \mu_0$ *vs* $\mathcal{H}_1: \ \mu > \mu_0$
- Case 3: $\mathcal{H}_0: \ \mu = \mu_0$ *vs* $\mathcal{H}_1: \ \mu < \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Random Sample: We take $X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2)$;

Test Statistics: Idea:

- what we have to approximate our Parameter (Estimate) μ? - Easy: $\overline{X}$;
- In which case we will not believe that $\mu = \mu_0$? - Piece of Cake:

Test for the Mean of the Normal, σ is known: Z-Test

Model: Our Data comes from $\mathcal{N}(\mu, \sigma^2)$, σ is known; Our (unknown) Parameter is μ

Hypothesis: We are given some μ_0, and we want to Test:

- Case 1: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu \neq \mu_0$
- Case 2: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu > \mu_0$
- Case 3: $\mathcal{H}_0 : \ \mu = \mu_0$ *vs* $\mathcal{H}_1 : \ \mu < \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Random Sample: We take $X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2)$;

Test Statistics: Idea:

- what we have to approximate our Parameter (Estimate) μ? - Easy: $\overline{X}$;
- In which case we will not believe that $\mu = \mu_0$? - Piece of Cake: if $\overline{X}$ will be far from μ_0

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z =$$

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check that it satisfies the requirements:

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check that it satisfies the requirements:

- If we will have the Observation, we can calculate it, for sure;

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check that it satisfies the requirements:

- If we will have the Observation, we can calculate it, for sure;
- If the Null Hypo is True, then $\mu = \mu_0$. Recall what is the connection between our Statistics, Z and μ:

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check that it satisfies the requirements:

- If we will have the Observation, we can calculate it, for sure;
- If the Null Hypo is True, then $\mu = \mu_0$. Recall what is the connection between our Statistics, Z and μ: the Distribution of X_k in Z is $\mathcal{N}(\mu, \sigma^2)$.

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check that it satisfies the requirements:

- If we will have the Observation, we can calculate it, for sure;
- If the Null Hypo is True, then $\mu = \mu_0$. Recall what is the connection between our Statistics, Z and μ: the Distribution of X_k in Z is $\mathcal{N}(\mu, \sigma^2)$. So, under $\mathcal{H}_0$, $X_k \sim \mathcal{N}(\mu_0, \sigma^2)$, so

$$\overline{X} \sim \mathcal{N}\left(\mu_0, \frac{\sigma^2}{n}\right)$$

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check that it satisfies the requirements:

- If we will have the Observation, we can calculate it, for sure;
- If the Null Hypo is True, then $\mu = \mu_0$. Recall what is the connection between our Statistics, Z and μ: the Distribution of X_k in Z is $\mathcal{N}(\mu, \sigma^2)$. So, under $\mathcal{H}_0$, $X_k \sim \mathcal{N}(\mu_0, \sigma^2)$, so

$$\overline{X} \sim \mathcal{N}\left(\mu_0, \frac{\sigma^2}{n}\right) \qquad \text{and} \qquad Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim$$

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check that it satisfies the requirements:

- If we will have the Observation, we can calculate it, for sure;
- If the Null Hypo is True, then $\mu = \mu_0$. Recall what is the connection between our Statistics, Z and μ: the Distribution of X_k in Z is $\mathcal{N}(\mu, \sigma^2)$. So, under $\mathcal{H}_0$, $X_k \sim \mathcal{N}(\mu_0, \sigma^2)$, so

$$\overline{X} \sim \mathcal{N}\left(\mu_0, \frac{\sigma^2}{n}\right) \qquad \text{and} \qquad Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim \mathcal{N}(0,1).$$

Test for the Mean of the Normal, σ is known: Z-Test

Test Statistics: So, we will base our Test-Statistics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check that it satisfies the requirements:

- If we will have the Observation, we can calculate it, for sure;
- If the Null Hypo is True, then $\mu = \mu_0$. Recall what is the connection between our Statistics, Z and μ: the Distribution of X_k in Z is $\mathcal{N}(\mu, \sigma^2)$. So, under $\mathcal{H}_0$, $X_k \sim \mathcal{N}(\mu_0, \sigma^2)$, so

$$\overline{X} \sim \mathcal{N}\left(\mu_0, \frac{\sigma^2}{n}\right) \qquad \text{and} \qquad Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim \mathcal{N}(0, 1).$$

  Great!

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \mu = \mu_0$ vs $\mathcal{H}_1: \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad$ vs $\quad \mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$.

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad$ vs $\quad \mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$. The Critical Value c is yet to be determined.

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$. The Critical Value c is yet to be determined.

Case 2: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu > \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$. The Critical Value c is yet to be determined.

Case 2: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu > \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if Z will be far to the **Right** to 0, i.e., we choose $RR = \{Z > c\}$.

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$. The Critical Value c is yet to be determined.

Case 2: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu > \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if Z will be far to the **Right** to 0, i.e., we choose $RR = \{Z > c\}$. Again, the Critical Value c is yet to be determined.

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$. The Critical Value c is yet to be determined.

Case 2: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu > \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if Z will be far to the **Right** to 0, i.e., we choose $RR = \{Z > c\}$. Again, the Critical Value c is yet to be determined.

Case 3: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu < \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$. The Critical Value c is yet to be determined.

Case 2: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu > \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if Z will be far to the **Right** to 0, i.e., we choose $RR = \{Z > c\}$. Again, the Critical Value c is yet to be determined.

Case 3: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad \text{vs} \quad \mathcal{H}_1: \ \mu < \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if Z will be far to the **Left** to 0, i.e., we choose $RR = \{Z < c\}$.

Test for the Mean of the Normal, σ is known: Z-Test

Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to 0

We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad$ vs $\quad \mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$. The Critical Value c is yet to be determined.

Case 2: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad$ vs $\quad \mathcal{H}_1: \ \mu > \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if Z will be far to the **Right** to 0, i.e., we choose $RR = \{Z > c\}$. Again, the Critical Value c is yet to be determined.

Case 3: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad$ vs $\quad \mathcal{H}_1: \ \mu < \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if Z will be far to the **Left** to 0, i.e., we choose $RR = \{Z < c\}$. Again, the Critical Value c is yet to be determined.

Choosing the Critical Value

Now, let us choose c.

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha =$$

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha = \mathbb{P}(\text{Type I Error}) =$$

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha = \mathbb{P}(\text{Type I Error}) = \mathbb{P}(\text{Reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is True}) =$$

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha = \mathbb{P}(\text{Type I Error}) = \mathbb{P}(\text{Reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is True}) =$$

$$= \mathbb{P}(|Z| > c \mid \mu = \mu_0) =$$

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha = \mathbb{P}(\text{Type I Error}) = \mathbb{P}(\text{Reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is True}) =$$

$$= \mathbb{P}(|Z| > c \mid \mu = \mu_0) = \mathbb{P}(|Z| > c \mid Z \sim \mathcal{N}(0, 1)).$$

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha = \mathbb{P}(\text{Type I Error}) = \mathbb{P}(\text{Reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is True}) =$$

$$= \mathbb{P}(|Z| > c \mid \mu = \mu_0) = \mathbb{P}(|Z| > c \mid Z \sim \mathcal{N}(0,1)).$$

This means, geometrically, that

$$c =$$

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha = \mathbb{P}(\text{Type I Error}) = \mathbb{P}(\text{Reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is True}) =$$

$$= \mathbb{P}(|Z| > c \mid \mu = \mu_0) = \mathbb{P}(|Z| > c \mid Z \sim \mathcal{N}(0,1)).$$

This means, geometrically, that

$$c = z_{1-\alpha/2},$$

the $1 - \frac{\alpha}{2}$-level quantile of the Standard Normal Distribution.

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha = \mathbb{P}(\text{Type I Error}) = \mathbb{P}(\text{Reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is True}) =$$

$$= \mathbb{P}(|Z| > c \mid \mu = \mu_0) = \mathbb{P}(|Z| > c \mid Z \sim \mathcal{N}(0, 1)).$$

This means, geometrically, that

$$c = z_{1-\alpha/2},$$

the $1 - \frac{\alpha}{2}$-level quantile of the Standard Normal Distribution.

So, finally, we have the Test for the Case 1: given μ_0, σ, Observations and Significance Level α, calculate $Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$.

Choosing the Critical Value

Now, let us choose c. We consider only **Case 1**:

$$RR = \{|Z| > c\}.$$

We choose c from the requirement to have a Test with Significance Level α:

$$\alpha = \mathbb{P}(\text{Type I Error}) = \mathbb{P}(\text{Reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is True}) =$$

$$= \mathbb{P}(|Z| > c \mid \mu = \mu_0) = \mathbb{P}(|Z| > c \mid Z \sim \mathcal{N}(0,1)).$$

This means, geometrically, that

$$c = z_{1-\alpha/2},$$

the $1 - \frac{\alpha}{2}$-level quantile of the Standard Normal Distribution.

So, finally, we have the Test for the Case 1: given μ_0, σ, Observations and Significance Level α, calculate $Z = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}$.

- If $|Z| > z_{1-\alpha/2}$, **Reject** $\mathcal{H}_0$;
- If $|Z| \leq z_{1-\alpha/2}$, **Do Not Reject** $\mathcal{H}_0$.

Z-Test, Complete Version

Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is known**, the Parameter (our unknown) is μ;

Z-Test, Complete Version

Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is known**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0: \ \mu = \mu_0$

Z-Test, Complete Version

Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is known**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0 : \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Z-Test, Complete Version

Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is known**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0 : \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Test Statistics: $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$

Z-Test, Complete Version

Model: $X_1, X_2, ..., X_n \stackrel{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is known**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0: \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Test Statistics: $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$

Distrib of the Test-Statistics Under $\mathcal{H}_0$: $Z \sim \mathcal{N}(0, 1)$;

Z-Test, Complete Version

Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is known**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0 : \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Test Statistics: $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$

Distrib of the Test-Statistics Under $\mathcal{H}_0$: $Z \sim \mathcal{N}(0, 1)$;

$\mathcal{H}_1$ is	RR is
$\mu \neq \mu_0$	$\lvert Z \rvert > z_{1-\frac{\alpha}{2}}$
$\mu > \mu_0$	$Z > z_{1-\alpha}$
$\mu < \mu_0$	$Z < z_\alpha$

t-Test

Model: $X_1, X_2, ..., X_n \stackrel{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is unknown**, the Parameter (our unknown) is μ;

t-Test

Model: $X_1, X_2, ..., X_n \stackrel{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is unknown**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0: \ \mu = \mu_0$

t-Test

Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is unknown**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0 : \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

t-Test

Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is unknown**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0 : \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Test Statistics: $t = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$, where

$$S^2 = \frac{\sum_{k=1}^n (X_k - \overline{X})^2}{n-1}.$$

t-Test

Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is unknown**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0: \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Test Statistics: $t = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$, where

$$S^2 = \frac{\sum_{k=1}^{n} (X_k - \overline{X})^2}{n-1}.$$

Distrib of the Test-Statistics Under $\mathcal{H}_0$**:** $t \sim t(n-1)$;

t-Test

Model: $X_1, X_2, ..., X_n \stackrel{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is unknown**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0 : \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Test Statistics: $t = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$, where

$$S^2 = \frac{\sum_{k=1}^{n}(X_k - \overline{X})^2}{n-1}.$$

Distrib of the Test-Statistics Under $\mathcal{H}_0$: $t \sim t(n-1)$;

$\mathcal{H}_1$ is	RR is
$\mu \neq \mu_0$	$\lvert t \rvert > t_{n-1,1-\frac{\alpha}{2}}$
$\mu > \mu_0$	$t > t_{n-1,1-\alpha}$
$\mu < \mu_0$	$t < t_{n-1,\alpha}$

t-test Example

Example: I have generated in **R** a Sample of Size 20 from $\mathcal{N}(3.12, 2^2)$ and made some rounding:

```
set.seed(20112019)
n <-20; sigma <- 2
obs <-  rnorm(n, mean = 3.12, sd = sigma)
obs <- round(obs, digits = 2); obs
```

```
##  [1]  1.80  5.60  1.10  3.20  4.91  5.15  1.76  2.47  0.
## [13]  3.98  4.79  1.98  4.50  3.52  4.13 -0.08  3.87
```

t-test Example

Example: I have generated in **R** a Sample of Size 20 from $\mathcal{N}(3.12, 2^2)$ and made some rounding:

```
set.seed(20112019)
n <-20; sigma <- 2
obs <-  rnorm(n, mean = 3.12, sd = sigma)
obs <- round(obs, digits = 2); obs
```

```
##  [1]  1.80  5.60  1.10  3.20  4.91  5.15  1.76  2.47  0.
## [13]  3.98  4.79  1.98  4.50  3.52  4.13 -0.08  3.87
```

Now, let us forget about the fact that the actual value of μ is 3.12 and that $\sigma = 2$, and do some Testing, just assuming that our Observation is coming from a Normal Distribution.

t-test Example

Example: I have generated in **R** a Sample of Size 20 from $\mathcal{N}(3.12, 2^2)$ and made some rounding:

```
set.seed(20112019)
n <-20; sigma <- 2
obs <-  rnorm(n, mean = 3.12, sd = sigma)
obs <- round(obs, digits = 2); obs
```

```
##  [1]  1.80  5.60  1.10  3.20  4.91  5.15  1.76  2.47  0.
## [13]  3.98  4.79  1.98  4.50  3.52  4.13 -0.08  3.87
```

Now, let us forget about the fact that the actual value of μ is 3.12 and that $\sigma = 2$, and do some Testing, just assuming that our Observation is coming from a Normal Distribution. Say, let us Test, at the 5% Significance Level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 4 \qquad vs \qquad \mathcal{H}_1: \ \mu \neq 4.$$

Example, Cont'd

First, we calculate *t*-statistic:

```r
mu0 <- 4;
x.bar <- mean(obs); s <- sd(obs);
t <- (x.bar - mu0)/(s/sqrt(n)); t
```

```
## [1] -1.795358
```

Example, Cont'd

First, we calculate t-statistic:

```
mu0 <- 4;
x.bar <- mean(obs); s <- sd(obs);
t <- (x.bar - mu0)/(s/sqrt(n)); t
```

```
## [1] -1.795358
```

Now, we calculate the critical value, the quantile $t_{n-1,1-\alpha/2}$:

```
a <- 0.05
c <- qt(1-a/2, df = n-1); c
```

```
## [1] 2.093024
```

Example, Cont'd

First, we calculate t-statistic:

```
mu0 <- 4;
x.bar <- mean(obs); s <- sd(obs);
t <- (x.bar - mu0)/(s/sqrt(n)); t
```

```
## [1] -1.795358
```

Now, we calculate the critical value, the quantile $t_{n-1,1-\alpha/2}$:

```
a <- 0.05
c <- qt(1-a/2, df = n-1); c
```

```
## [1] 2.093024
```

Finally, we check if t is in RR, i.e., if $|t| > t_{n-1,1-\alpha/2}$:

```
abs(t) > c
```

```
## [1] FALSE
```

Example, Cont'd

First, we calculate t-statistic:

```
mu0 <- 4;
x.bar <- mean(obs); s <- sd(obs);
t <- (x.bar - mu0)/(s/sqrt(n)); t
```

```
## [1] -1.795358
```

Now, we calculate the critical value, the quantile $t_{n-1,1-\alpha/2}$:

```
a <- 0.05
c <- qt(1-a/2, df = n-1); c
```

```
## [1] 2.093024
```

Finally, we check if t is in RR, i.e., if $|t| > t_{n-1,1-\alpha/2}$:

```
abs(t) > c
```

```
## [1] FALSE
```

So the decision is:

Example, Cont'd

First, we calculate t-statistic:

```
mu0 <- 4;
x.bar <- mean(obs); s <- sd(obs);
t <- (x.bar - mu0)/(s/sqrt(n)); t
```

```
## [1] -1.795358
```

Now, we calculate the critical value, the quantile $t_{n-1,1-\alpha/2}$:

```
a <- 0.05
c <- qt(1-a/2, df = n-1); c
```

```
## [1] 2.093024
```

Finally, we check if t is in RR, i.e., if $|t| > t_{n-1,1-\alpha/2}$:

```
abs(t) > c
```

```
## [1] FALSE
```

So the decision is: **Fail to Reject** $\mathcal{H}_0$ at 5% level.

Example, Cont'd

Now, the same, but with an **R** built-in function `t.test`:

```r
t.test(obs, mu = mu0, conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  obs
## t = -1.7954, df = 19, p-value = 0.08852
## alternative hypothesis: true mean is not equal to 4
## 95 percent confidence interval:
##  2.524009 4.112991
## sample estimates:
## mean of x
##    3.3185
```

Example, Cont'd

Now, for the same Data, let us Test, at the 10% Significance Level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 3 \qquad \textit{vs} \qquad \mathcal{H}_1: \ \mu > 3.$$

Example, Cont'd

Now, for the same Data, let us Test, at the 10% Significance Level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 3 \qquad vs \qquad \mathcal{H}_1: \ \mu > 3.$$

```r
t.test(obs, mu=3,alternative="greater", conf.level=0.9)
```

```
##
##  One Sample t-test
##
## data:  obs
## t = 0.83906, df = 19, p-value = 0.2059
## alternative hypothesis: true mean is greater than 3
## 90 percent confidence interval:
##  2.814508      Inf
## sample estimates:
## mean of x
##    3.3185
```

Note

Note: In **R** `t.test` command, the default values for parameters are:

- `mu = 0`
- `alternative = "two.sided"`
- `conf.level = 0.95`

Note

Note: In **R** `t.test` command, the default values for parameters are:

- `mu = 0`
- `alternative = "two.sided"`
- `conf.level = 0.95`

Important Note: There are different ways to test a Hypothesis: one is by Test Statistics, another one is by CIs, and the next, easiest one is by *p*-Values.

Note

Note: In **R** `t.test` command, the default values for parameters are:

- `mu = 0`
- `alternative = "two.sided"`
- `conf.level = 0.95`

Important Note: There are different ways to test a Hypothesis: one is by Test Statistics, another one is by CIs, and the next, easiest one is by *p*-Values. All Statistical Software are calculating *p*-Values when doing testing. And the Decision based on the *p*-Value is:

- If *p*-Value$< \alpha$, then we Reject $\mathcal{H}_0$
- If *p*-Value$\geq \alpha$, then we Fail to Reject $\mathcal{H}_0$