ASDS Statistics, YSU, Fall 2020

Lecture 05

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- What we need to have to plot a Histogram?

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- What we need to have to plot a Histogram?
- Give the Definition of the Frequency Histogram.

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- What we need to have to plot a Histogram?
- Give the Definition of the Frequency Histogram.
- Give the Definition of the Density Histogram.

Histogram Example

Example: Plot the Frequency, Relative Frequency and Density Histograms for

$$0,\ 4,\ 2,\ 2,\ 0,\ 0.5,\ 1,\ 3$$

Example

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We use here the *discoveries* Standard Dataset from **R**, which gives us the numbers of "great" inventions and scientific discoveries in each year from 1860 to 1959:

Example

To draw the Density Histogram in **R**, we will use the *freq=FALSE* parameter in the *hist* command.

We use here the *discoveries* Standard Dataset from **R**, which gives us the numbers of “great” inventions and scientific discoveries in each year from 1860 to 1959:

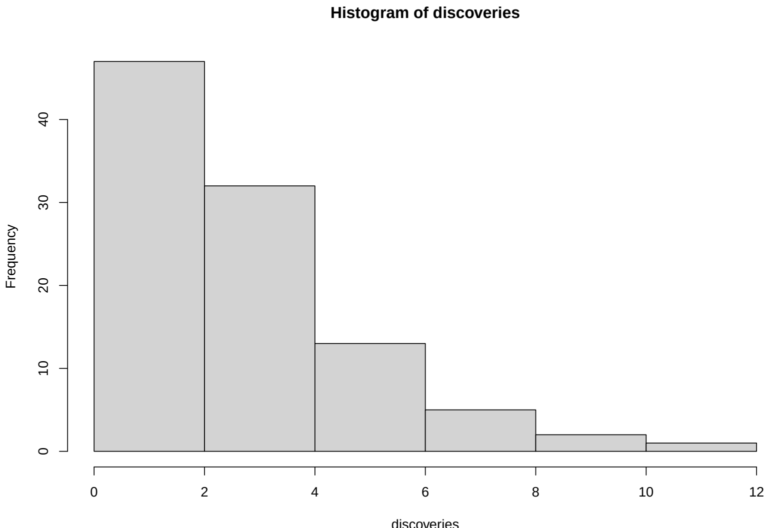
```
discoveries
```

```
## Time Series:
## Start = 1860
## End = 1959
## Frequency = 1
##   [1]  5  3  0  2  0  3  2  3  6  1  2  1  2  1  3  3
##  [26] 12  3 10  9  2  3  7  7  2  3  3  6  2  4  3  5
##  [51]  3  6  5  8  3  6  6  0  5  2  2  2  6  3  4  4
##  [76]  2  2  1  3  4  2  2  1  1  1  2  1  4  4  3  2
```

Example

First, the Frequency Histogram:

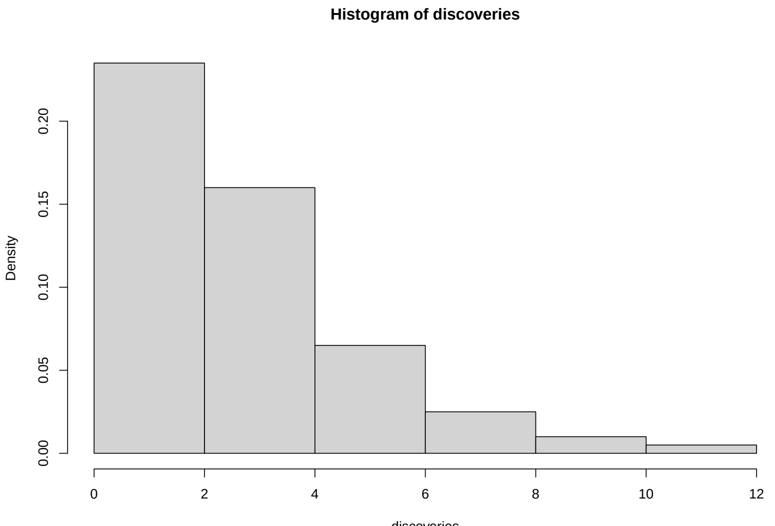
```
hist(discoveries)
```



Example

Now, the Density Histogram:

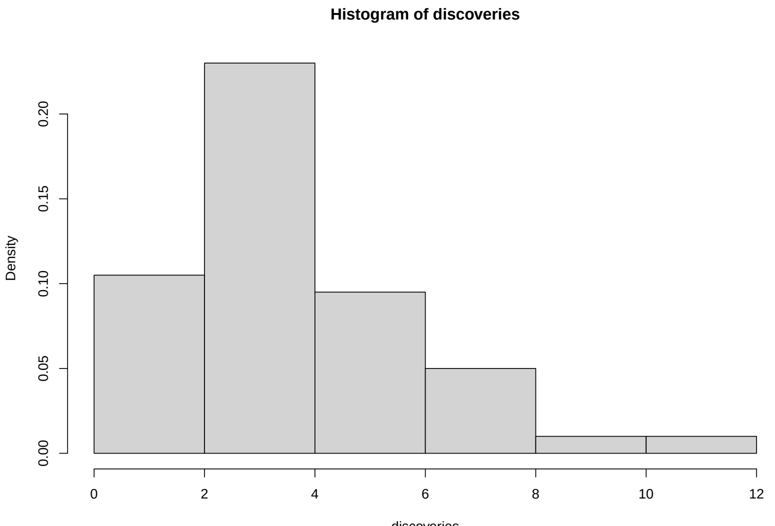
```r
hist(discoveries, freq = FALSE)
```



Example

Finally, the Density Histogram with the Bins left-endpoints included:

```
hist(discoveries, freq = FALSE, right = FALSE)
```



Example

Now let us change the default bins for a Histogram.

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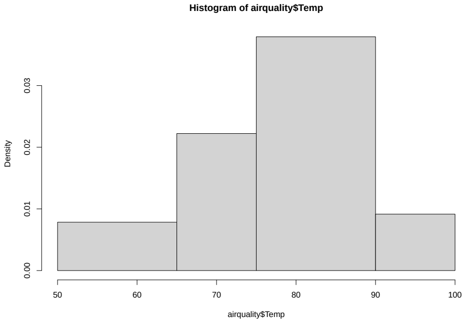
Now let us change the default bins for a Histogram. We can use the following - first define the vector of our class interval (Bins) endpoints: (note that you need to cover all Datapoints!)

```
bins.endpoitns <- c(50, 65, 75, 90, 100)
hist(airquality$Temp, breaks = bins.endpoitns)
```



Notes

- By default, if we give custom bins with non-equal lengths, **R** is plotting the Density Histogram!

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- By default, if we give custom bins with non-equal lengths, **R** is plotting the Density Histogram!
- You can give the *breaks* parameter either the vector of Bins' endpoints or the number of (equal-length) intervals

Estimation of the PDF through the Density Histogram

As it was stated above, the Density Histogram is an approximation (estimate) of the PDF of the Data unknown Distribution. To check this, let us take a synthetic Dataset from the Distribution we know:

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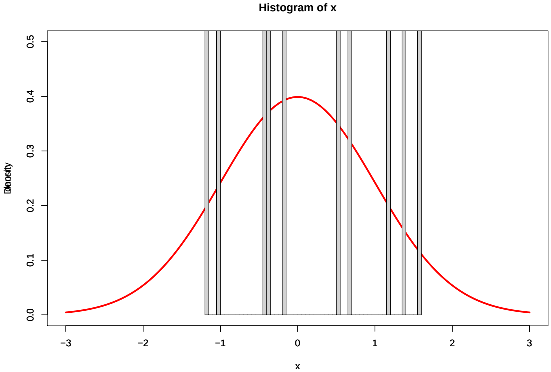
```
plot(dnorm, lwd = 3, col= "red", xlim=c(-3,3), ylim=c(0,0.5))
x <- rnorm(10)
par(new = TRUE)
hist(x, breaks = 40, freq = FALSE, xlim=c(-3,3), ylim=c(0,0.5))
```



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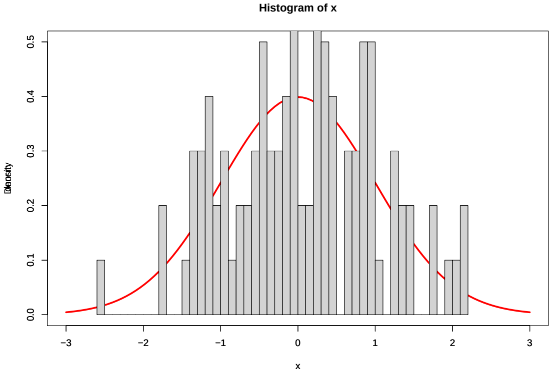
```
plot(dnorm, lwd = 3, col= "red", xlim=c(-3,3), ylim=c(0,0.5))
x <- rnorm(100)
par(new = TRUE)
hist(x, breaks = 40, freq = FALSE, xlim=c(-3,3), ylim=c(0,0.5))
```



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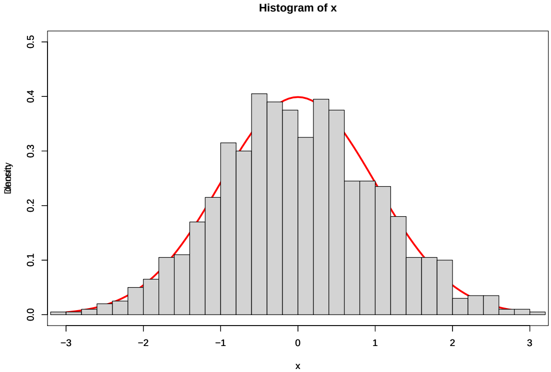
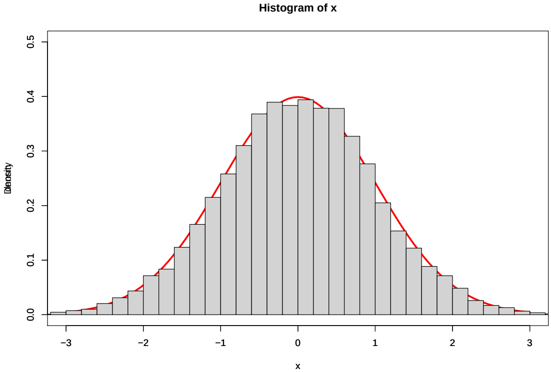
```
plot(dnorm, lwd = 3, col= "red", xlim=c(-3,3), ylim=c(0,0.5))
x <- rnorm(1000)
par(new = TRUE)
hist(x, breaks = 40, freq = FALSE, xlim=c(-3,3), ylim=c(0,0.5))
```



Estimation of the PDF through the Density Histogram

As it was stated above, the Density Histogram is an approximation (estimate) of the PDF of the Data unknown Distribution. To check this, let us take a synthetic Dataset from the Distribution we know:

```
plot(dnorm, lwd = 3, col= "red", xlim=c(-3,3), ylim=c(0,0.5))
x <- rnorm(10000)
par(new = TRUE)
hist(x, breaks = 40, freq = FALSE, xlim=c(-3,3), ylim=c(0,0.5))
```



Choosing Bin sizes correctly

It is important to choose the Bin sizes (lengths of the Bin, class, intervals) wisely. Otherwise you will skip some info or you will not get any valuable info.

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Let us use another **R** standard dataset to show the effect of the choice of the bin size: *precip*. This Dataset shows the average amount of precipitation (rainfall) in inches for each of 70 United States (and Puerto Rico) cities.

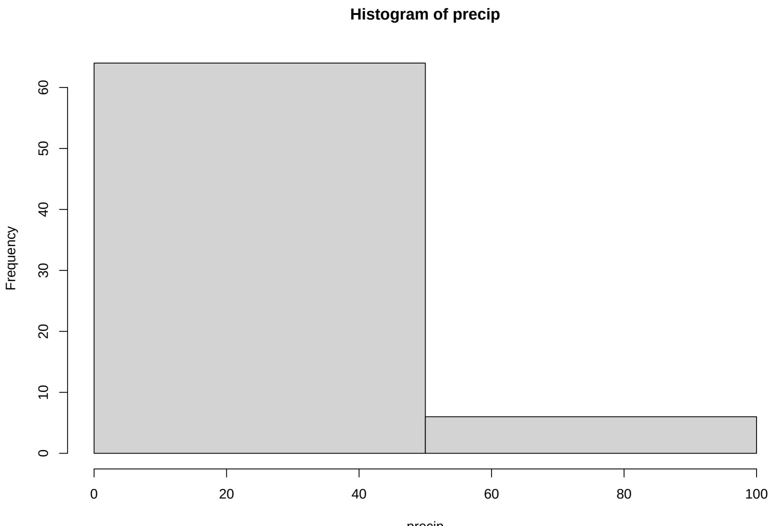
```r
head(precip)
```

```
##       Mobile       Juneau      Phoenix Little Rock Los Ange
##         67.0         54.7          7.0        48.5        1
```

Version 1, Small bins

Here, we just use 2 bins:

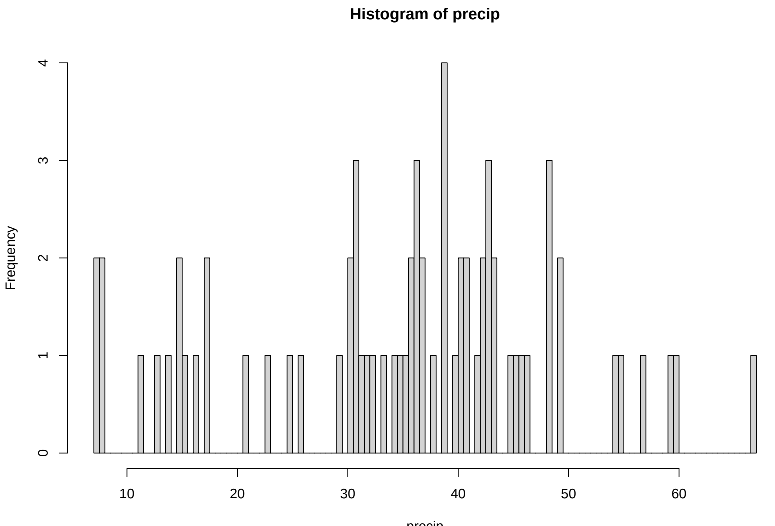
```
hist(precip, breaks = 2)
```



Version 2, large bins

Here, we use 200 bins:

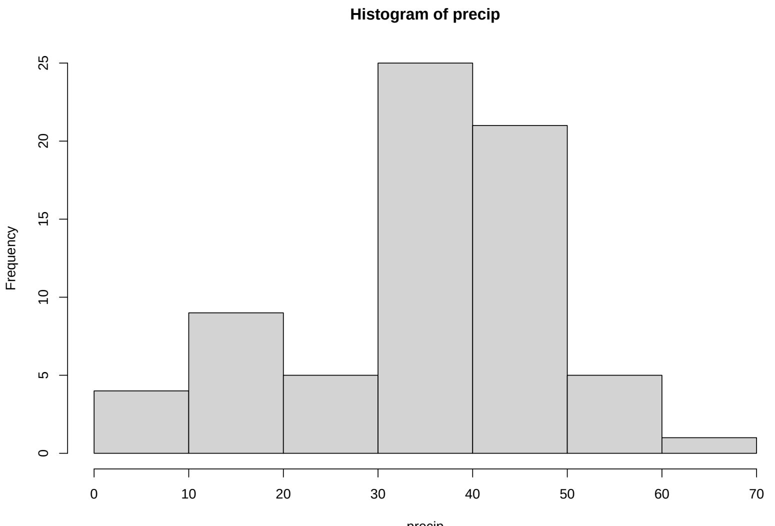
```r
hist(precip, breaks = 200)
```



Version 2, large bins

Now, the default:

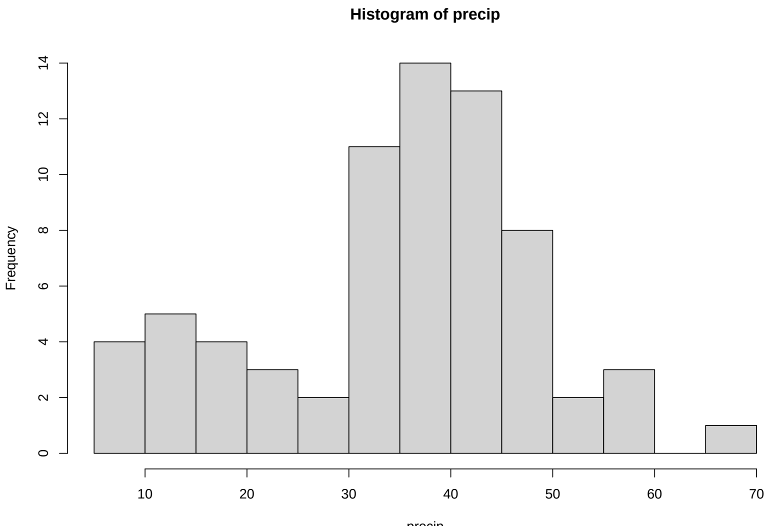
```r
hist(precip)
```



Version 3

Now, let us change to 20 bin intervals:

```r
hist(precip, breaks = 20)
```



Choosing the Bin Length

In fact, choosing the correct Bin width is not an easy job. See, for example, the Histogram Wiki page.

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- *Barplot*'s rectangles widths are arbitrary, do not mean anything, rectangles are not adjacent; *Histogram*'s rectangles are adjacent, and the choice of the Bin widths is changing the graph
- *Barplot* is for a categorical or Discrete Data, *Histogram* is for both Discrete and Continuous
- We can exactly reconstruct the Dataset from the *Barplot*, but not the *Histogram*

Addition to the Histogram

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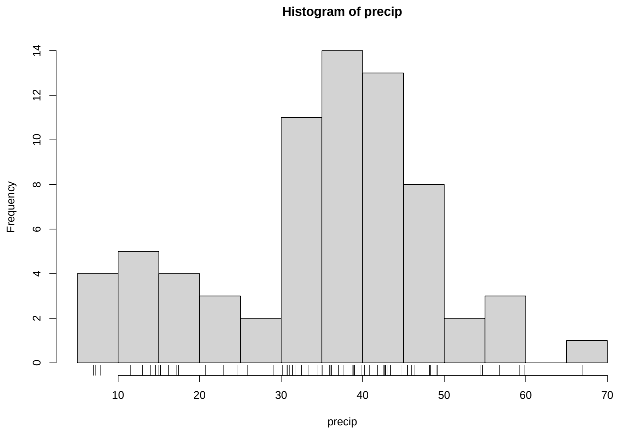
```
hist(precip, breaks = 20)
rug(precip)
```



What we can see from the Histogram

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- has values far apart from others, has outliers (anomalies)
- is unimodal, bimodal or multimodal

KDE

Another estimate for the unknown Distribution PDF is the **Kernel Density Estimator**, KDE.

KDE

Another estimate for the unknown Distribution PDF is the **Kernel Density Estimator**, KDE. It is, in some sense, the smoothed version of the Histogram: Histogram is a piecewise-constant function, with jumps, so it is not a smooth function.

KDE

To define the KDE, we first choose a smooth Kernel function $K(t)$, here, a function with

$$K(t) \geq 0, t \in \mathbb{R}, \qquad \text{and} \qquad \int_{-\infty}^{+\infty} K(t)dt = 1.$$

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Next, one defines the Kernel Density Estimator with Kernel K as

$$KDE_K(x) = KDE(x) = \frac{1}{nh} \cdot \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right).$$