ASDS Statistics, YSU, Fall 2020

Lecture 14

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or, in a table form,

Values of X	x_1	x_2	...
$\mathbb{P}(X = x)$	p_1	p_2	...

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- the Expected Value (Mean):

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$$\mathbb{E}(g(X)) = \int_{-\infty}^{+\infty} g(x) \cdot f(x) dx \; (cont.) \quad | \quad \mathbb{E}(g(X)) = \sum_k g(x_k) \cdot \mathbb{P}(X = x_k) \; ($$

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- The Variance

$$Var(X) = \mathbb{E}\Big((X - \mathbb{E}(X))^2\Big) = \mathbb{E}(X^2) - \Big[\mathbb{E}(X)\Big]^2.$$

Important Discrete Distributions

Bernoulli Distribution

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- Mean and Variance: $\mathbb{E}(X) = p$, $\mathit{Var}(X) = p(1-p)$.
- Models: Models binary output, "success-failure" type Experiments, a lot of examples.

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- Example:

```r
rbinom(10, size = 1, prob = 0.3)
```

```
##  [1] 0 1 0 0 0 0 1 0 0 1
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- PMF:

Values of X	0	1	...	k	...	n
$\mathbb{P}(X=x)$	$\binom{n}{0}p^0(1-p)^{n-0}$	$\binom{n}{1}p^1(1-p)^{n-1}$	...	$\binom{n}{k}p^k(1-p)^{n-k}$	...	$\binom{n}{n}p^n(1-p)^0$

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- Mean and Variance: $\mathbb{E}(X) = n \cdot p$, $Var(X) = n \cdot p(1 - p)$.

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- Mean and Variance: $\mathbb{E}(X) = n \cdot p$, $Var(X) = n \cdot p(1-p)$.
- Models: Models the independent repetition of the $Bernoulli(p)$ Experiment.

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- Additional: If $X_1, X_2, ..., X_n \sim Bernoulli(p)$ are independent, then $X_1 + X_2 + ... + X_n \sim Binom(n, p)$.
- Example:

```r
rbinom(10, size = 5, prob = 0.3)
```

```
##  [1] 2 1 1 1 1 1 3 3 2 0
```