ASDS Statistics, YSU, Fall 2020

Lecture 06

Michael Poghosyan

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Contents

Last Lecture Recap

- Give the Definition of the KDE.

KDE

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$$\int_{-\infty}^{+\infty} KDE(x)dx = \frac{1}{nh} \cdot \sum_{i=1}^{n} \int_{-\infty}^{+\infty} K\left(\frac{x - x_i}{h}\right) dx =$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} \int_{-\infty}^{+\infty} K\left(\frac{x - x_i}{h}\right) d\frac{x - x_i}{h} \overset{u=\frac{x-x_i}{h}}{=}$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} \int_{-\infty}^{+\infty} K(u)du = \frac{1}{n} \cdot \sum_{i=1}^{n} 1 = 1.$$

KDE

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Theorem: Assume we are constructing the $KDE = KDE(\cdot, h_n)$ for the IID r.v $X_1, X_2, ..., X_n$, coming from an unknown PDF f, and with the bandwidth h_n.

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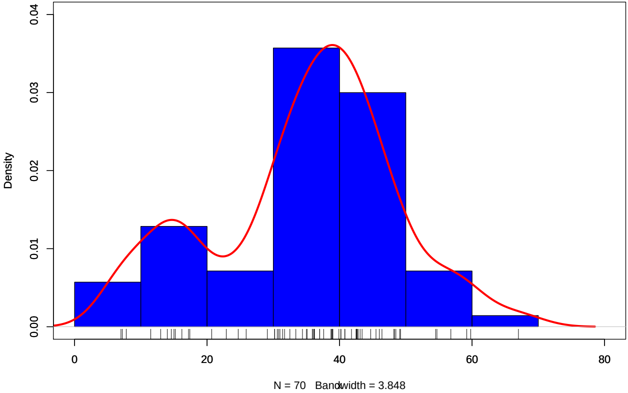
Theorem: Assume we are constructing the $KDE = KDE(\cdot, h_n)$ for the IID r.v $X_1, X_2, ..., X_n$, coming from an unknown PDF f, and with the bandwidth h_n. If the PDF f is continuous at the point x, and if $h_n \to 0$ and $n \cdot h_n \to \infty$, then

$$KDE(x, h_n) \to f(x) \qquad in \quad \mathbb{P}.$$

KDE Example

```
x <- precip; d <- density(x)
hist(x, freq = FALSE, xlim = c(0, 80), ylim = c(0,0.04),
     col = "blue", main = "")
rug(x); par(new = TRUE)
plot(d, lwd = 3, col = "red", xlim = c(0,80), ylim = c(0,0.04),
     main = "")
```



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Here is the idea behind the theory of choosing the bin size for the Histogram, and bandwidth for KDEs.

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$$Risk(\hat{f}, f) = MISE(\hat{f}, f) = \mathbb{E} \int_{\mathbb{R}} (\hat{f}(x) - f(x))^2 dx.$$

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Now, the idea is to choose h in such a way that the Risk of $\hat{f}$ will be the minimal.

Visualizing 2D Data

In case we have a 2D numerical Dataset

$$(x_1, y_1), (x_2, y_2), ..., (x_n, y_n),$$

we usually do the ScatterPlot - the plot of all points (x_i, y_i), $i = 1, ...n$.

Example

Example: Graph the ScatterPlot for the following data:

Person ID	Age	Weight
1	20	69
2	22	57
3	40	65
4	20	70

Example

Say, consider again the *cars* Dataset:

```r
head(cars, 3)
```

```
##   speed dist
## 1     4    2
## 2     4   10
## 3     7    4
```

```r
str(cars)
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## 'data.frame':    50 obs. of  2 variables:
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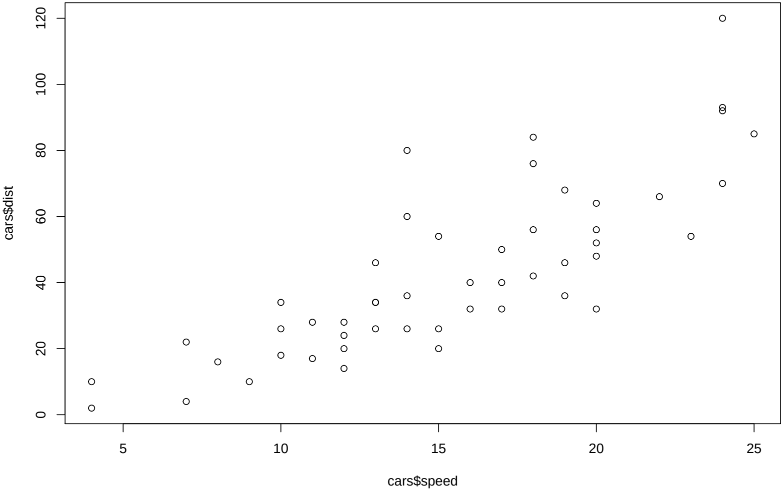
It has 2 Variables: *Speed* and *Distance*, and 50 Observations. Let us do the ScatterPlot of Observations:

ScatterPlot

```
plot(cars$speed, cars$dist)
```



Notes

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- In this graph you can see that there is some relationship between the *Speed* and *Distance*, there is a *trend*: if the speed gets larger, the (stopping) distance is tending to increase.
- Here we have visualized 2D Dataset of the (N, N) type, i.e., both Variables were N=Numerical. Think about how we can visualize a 2D Dataset of the type, say (C=Categorical)

$$(N, C),\ (C, N),\ (C, C),\ (N, N, C),\ \ldots$$

Additions: Multidimensional Graphs

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And some ideas for multidimensional Visualizations:

- One can draw the 2D (N, N) in 3D, by constructing 2D Histograms and KDEs

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- One can add the 4th Dimension by using the Size of Points
- And add the 5-th one by using the Shape of Points, . . .

Examples

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Or, the following one: Gender Gap in Earnings per University

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- etc ...