Note Title 02-Dec-20

CI by Cheby method

- Bernoulli(p)

$\alpha \in (0,1)$ CI for p of CL$(1-\alpha)$

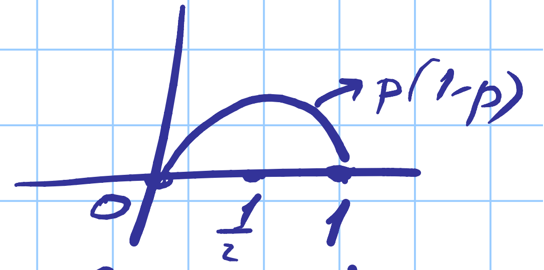
$$\mathbb{P}(|X-\mathbb{E}(X)| \geq \varepsilon) \leq \frac{Var(X)}{\varepsilon^2} \quad \text{(Chebyshov Ineq)}$$

$$\mathbb{P}(|X-\mathbb{E}(X)| < \varepsilon) \geq 1 - \frac{Var(X)}{\varepsilon^2}$$

$$X_1, X_2, \ldots, X_n \sim \text{Bernoulli}(p)$$

$$\mathbb{P}(|\bar{X}-\mathbb{E}(\bar{X})| < \varepsilon) \geq 1 - \frac{Var(\bar{X})}{\varepsilon^2}$$

$$E(\bar{X}) = E(X_1) = p$$



$$Var(\bar{X}) = \frac{Var(X_1)}{n} = \frac{p(1-p)}{n}$$

$$p(1-p) \le \frac{1}{4}$$

$$\mathbb{P}\left(|\bar{X} - p| < \varepsilon \right) \ge 1 - \frac{p(1-p)}{n\varepsilon^2} \ge 1 - \underbrace{\frac{1}{4n\varepsilon^2}}_{=\alpha}$$

$$\alpha = \frac{1}{4n\varepsilon^2} \qquad \varepsilon = \frac{1}{2\sqrt{n\alpha}}$$

$$\mathbb{P}\left(|\bar{X} - p| < \frac{1}{2\sqrt{n\alpha}} \right) \ge 1 - \alpha$$

$$\mathbb{P}\left(\bar{X} - \frac{1}{2\sqrt{n\alpha}} < p < \bar{X} + \frac{1}{2\sqrt{n\alpha}} \right) \ge 1 - \alpha$$

$\overline{X} \pm \frac{1}{2\sqrt{n\alpha}}$ is a $(1-\alpha)$ CI for p.

- 120 → Sample Size → 55 Smokers

$p =$ Prop of Smokers in AUA − ?

$$\hat{p} = \overline{X} = \frac{55}{120}$$

95% CL

$\alpha = 1 - 0.95 = 0.05$

$$\overline{X} \pm \frac{1}{2\sqrt{n \cdot \alpha}}$$

$$\frac{55}{120} \pm \frac{1}{2\sqrt{120 \cdot 0.05}}$$

$ME \leq 0.1$

$$\frac{1}{2\sqrt{n\alpha}} \leq 0.1$$

$\alpha = 0.05$

$$2\sqrt{n\alpha} \geq 10$$

$n\alpha \geq 25$

$$n \geq \frac{25}{0.05} = \underline{\underline{500}}$$