ASDS Statistics, YSU, Fall 2020

Lecture 09

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- Name some Statistics for the Spread/Variability of a Dataset

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- What is the idea behind the Quartiles?
- Define the IQR.

Quartiles and IQR

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Note: Recall the idea of Quartiles: the points Q_1, Q_2, Q_3 on the real axis divide our Dataset into (almost) four equal-length portions:

- almost 25% of our Datapoints are to the left to Q_1

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Note: The interval $[Q_1, Q_3]$ contains almost the half of the Datapoints. So the IQR shows the Spread of the middle half of our Dataset, it is a measure of the Spread/Variability.

Quartiles in **R**

In **R**, one can use the commands `quantile(x, 0.25)` and `quantile(x, 0.75)` to find Q_1 and Q_3.

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```r
x <- 1:10
quantile(x,0.25)
```

```
##  25%
## 3.25
```

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quantile(x,0.25)
```

```
##  25%
## 3.25
```

If you will not give a parameter to `quantile`, **R** will calculate 0% (minimum datapoint), 25%, 50%, 75% and 100% (maximum datapoint) quartiles:

```
x <- 1:10
quantile(x)
```

```
##    0%   25%   50%   75%  100%
##  1.00  3.25  5.50  7.75 10.00
```

Quartiles in R

Also, you can use the following commands:

```
x <- 1:10
fivenum(x)
```

```
## [1]  1.0  3.0  5.5  8.0 10.0
```

```
summary(x)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##    1.00    3.25    5.50    5.50    7.75   10.00
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```

To calculate the IQR in **R**, we can use the `IQR` command:

```
x <- 1:10
IQR(x)
```

```
## [1] 4.5
```

Note

Note: Please note that **R** is not using our definition of the Quartiles, so sometimes we will get different results when calculating by a hand or by **R**.

BoxPlot

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- The Quartiles $Q_1, Q_2 = Median, Q_3$
- the Lower and Upper Fences
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$$\left[Q_1 - \frac{3}{2}IQR,\ Q_3 + \frac{3}{2}IQR\right];$$

the lines joining that fences to corresponding quartiles are the *Whiskers*;

- the set of all Outliers

$$O = \left\{x_i : x_i \notin \left[Q_1 - \frac{3}{2}IQR, Q_3 + \frac{3}{2}IQR\right]\right\}$$

BoxPlot, Example

Then we draw the points W_1, Q_1, Q_2, Q_3, W_2 on the real line and add all outliers, and make a box over $[Q_1, Q_3]$.

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Example: Draw the Boxplot of

$$x: \;0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12$$

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Example: Draw the Boxplot of

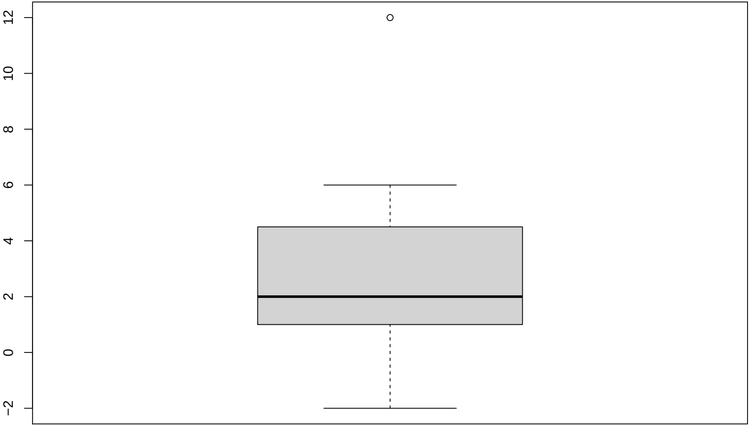
$$x: \; 0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12$$

Solution: OTB

BoxPlot, Example

Now, using **R**:

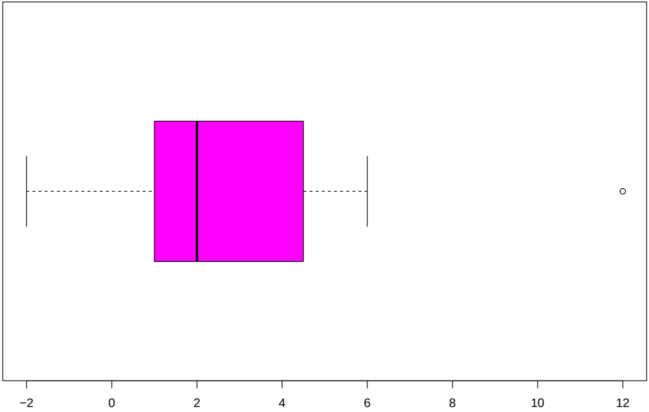
```r
x <- c(0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12)
boxplot(x)
```



BoxPlot, Example

Another view:

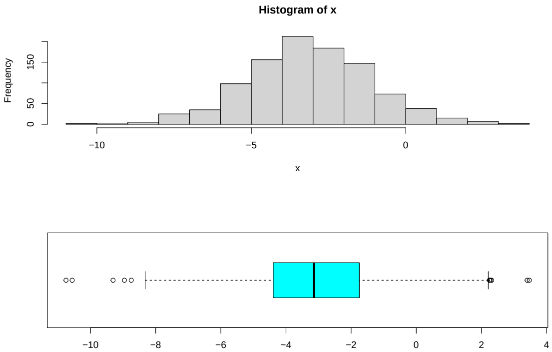
```
x <- c(0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12)
boxplot(x, horizontal = T, col = "magenta")
```



BoxPlot, Example

Here are some Datasets' Histograms along with the BoxPlots:

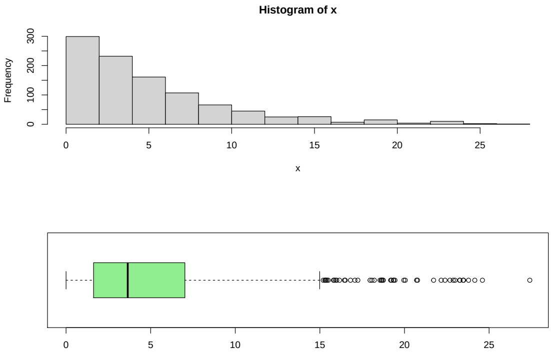
```
x <- rnorm(1000, mean = -3, sd = 2)
par(mfrow=c(2,1)); hist(x)
boxplot(x, horizontal = T, col = "cyan")
```



BoxPlot, Example

Here are some Datasets' Histograms along with the BoxPlots:

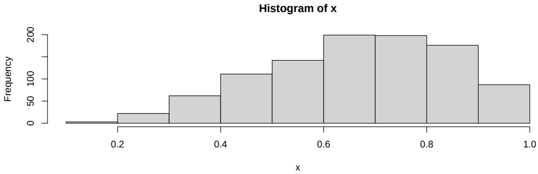
```
x <- rexp(1000, rate = 0.2)
par(mfrow=c(2,1)); hist(x)
boxplot(x, horizontal = T, col = "lightgreen")
```

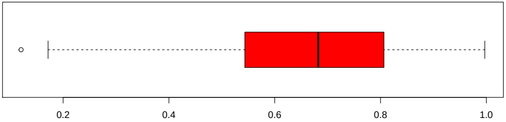


BoxPlot, Example

Here are some Datasets' Histograms along with the BoxPlots:

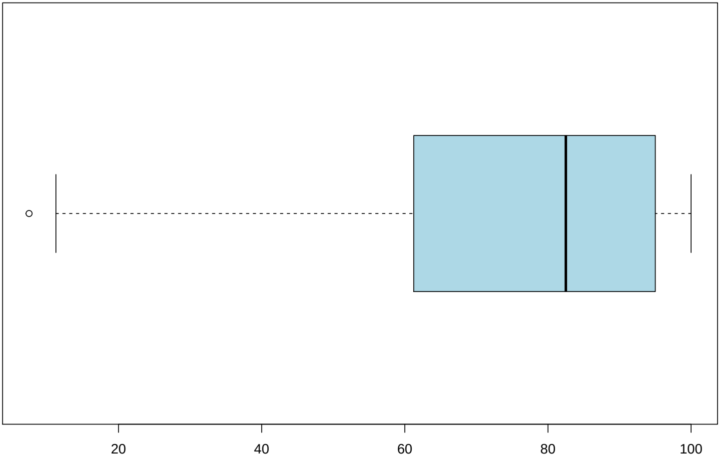
```
x <- rbeta(1000, shape1 = 4, shape2 = 2)
par(mfrow=c(2,1)); hist(x)
boxplot(x, horizontal = T, col = "red")
```





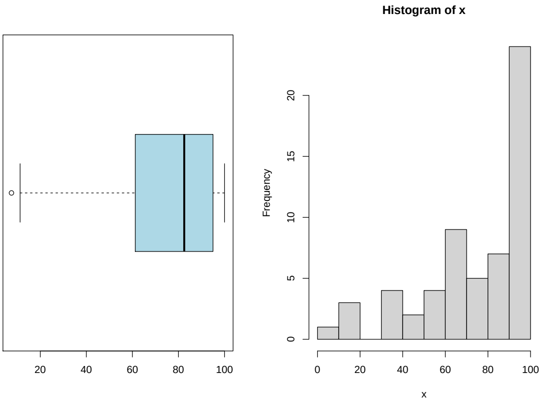
BoxPlot, Example

Here is the BoxPlot of the AUA Stat Quiz grades: can you describe the result?



BoxPlot, Example

And here is the BoxPlot of the same Quiz grades along with the Histogram:




```
##     Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     7.50   61.25   82.50   74.63   95.00  100.00
```

BoxPlot, Common Error

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Here is a common error when Plotting the BoxPlot:

- One uses $W_1 = Q_1 - 1.5 \cdot IQR$ and $W_2 = Q_3 + 1.5 \cdot IQR$. This is **not correct**! W_1 and W_2 need to be from our Dataset!

Take as W_1 and W_2 the smallest and largest **Datapoints**, respectively, in

$$\left[Q_1 - \frac{3}{2} IQR,\ Q_3 + \frac{3}{2} IQR \right].$$

Additions/Variations:

Some Variations:

- Variable Width BoxPlot

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See, for Example, this page.

Boxplot, Why we use it

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Boxplot, Why we use it

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Boxplot, Why we use it

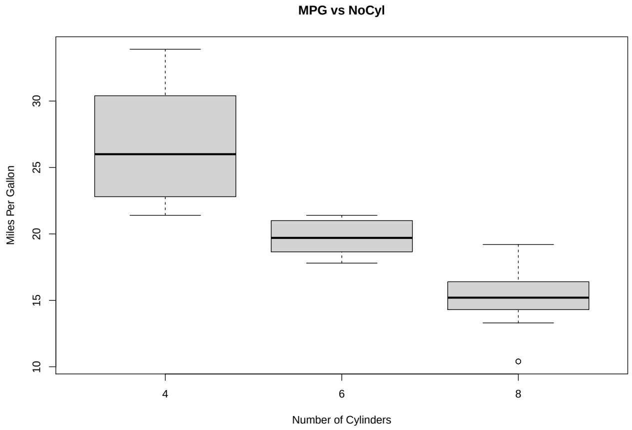
We use BoxPlots to:

- Visualize the distribution of the Dataset
- To compare two or more Datasets

Example

Here we use the `mtcars` Dataset:

```
boxplot( mpg~cyl, data=mtcars, main="MPG vs NoCyl",
   xlab="Number of Cylinders", ylab="Miles Per Gallon")
```

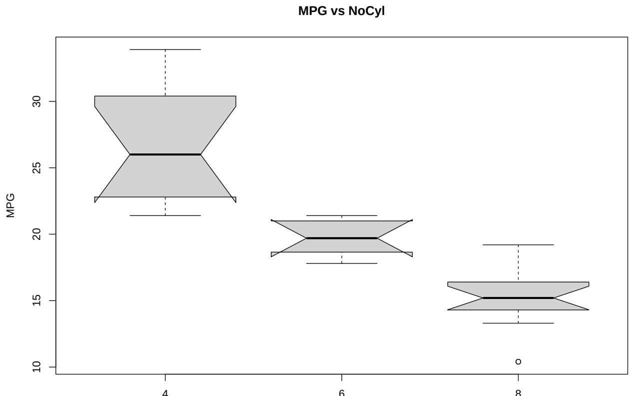


Example

Again,

```
boxplot( mpg~cyl, data=mtcars, notch = T,
         main="MPG vs NoCyl", xlab="Number of Cylinders", y
```

```
## Warning in bxp(list(stats = structure(c(21.4, 22.8, 26,
## notches went outside hinges ('box'): maybe set notch=FAL
```



Note

Recall that an **Outlier** in the BoxPlot sense is a Datapoint x_k with

$$x_k \notin \left[Q_1 - \frac{3}{2} IQR,\ Q_3 + \frac{3}{2} IQR \right].$$

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Another way to define an **Outlier:** Datapoint x_k is an Outlier, if

$$|x_k - \bar{x}| \geq 3 \cdot sd(x).$$

Note: Where the coefficient $\dfrac{3}{2}$ in front of the IQR comes from?

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This comes from the Normal Distribution: if our r.v. X is Normally Distributed, then (with theoretical Quartiles)

$$\mathbb{P}(X \in [Q_1 - 1.5 \cdot IQR, Q_3 + 1.5 \cdot IQR]) \approx 0.993,$$

so the chances that an Observation will be outside of this interval are very small.

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so the chances that an Observation will be outside of this interval are very small. So if we see that kind of Observation, we think that this number is an Outlier.

BoxPlot, Notes

Note: Sometimes, BoxPlot's Whiskers span to the Max and Min Datapoints, so in this case BoxPlot doesn't show Outliers.