Deep Learning

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Outline

Dilated/Atrous Convolution

Definition 1

Let $F : \mathbb{Z}^2 \to \mathbb{R}$ *be a discrete function. Let* $\Omega_r = [-r, r] \cap \mathbb{Z}^2$ *and let* $k : \Omega_r \to \mathbb{R}$ *be a discrete filter of size* $(2r+1)^2$*. The discrete convolution operator* $*$ *can be defined as*

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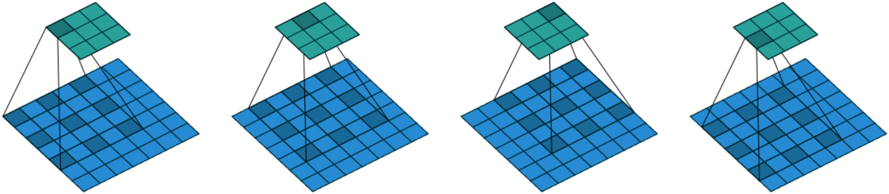
$$(F *_l k)(p) = \sum_{s+lt=p} F(s)\, k(t)$$

Dilated/Atrous Convolution



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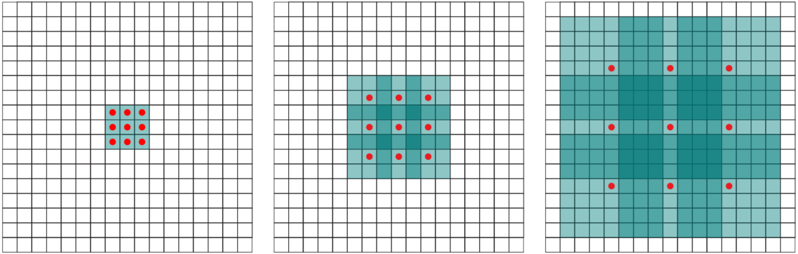


Figure 1: Systematic dilation supports exponential expansion of the receptive field without loss of resolution or coverage. (a) F_1 is produced from F_0 by a 1-dilated convolution; each element in F_1 has a receptive field of 3×3. (b) F_2 is produced from F_1 by a 2-dilated convolution; each element in F_2 has a receptive field of 7×7. (c) F_3 is produced from F_2 by a 4-dilated convolution; each element in F_3 has a receptive field of 15×15. The number of parameters associated with each layer is identical. The receptive field grows exponentially while the number of parameters grows linearly.

1D Dilated Convolution

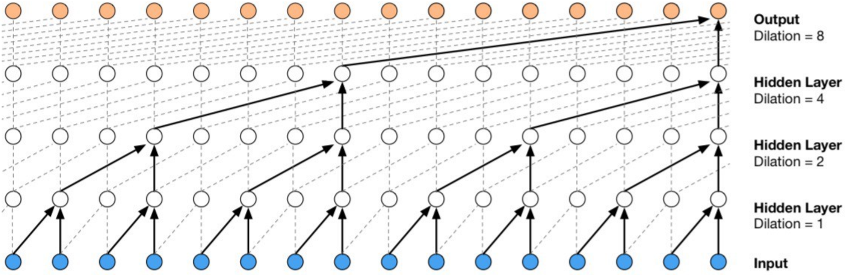
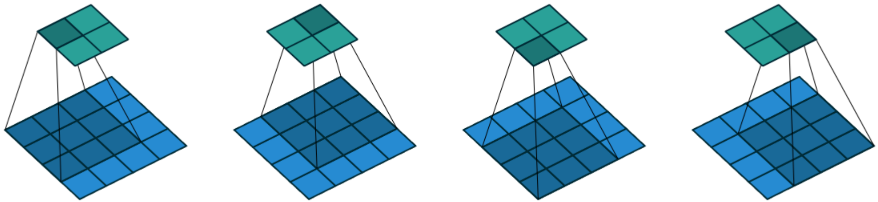


Figure 3: Visualization of a stack of *dilated* causal convolutional layers.

Convolution as a Matrix Operation



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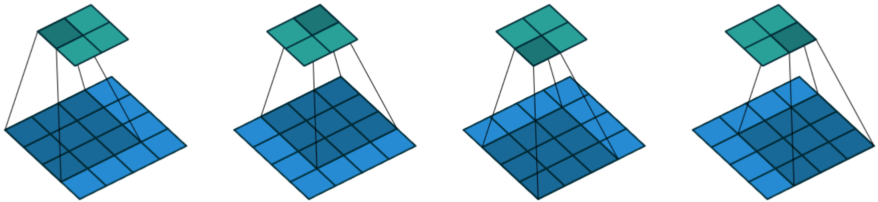
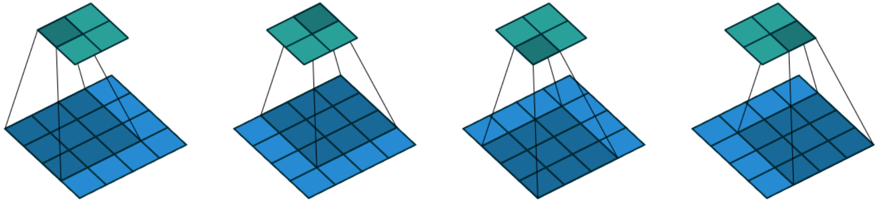


$$\begin{pmatrix} w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} \end{pmatrix}$$

This linear operation takes the input matrix flattened as a 16-dimensional vector and produces a 4-dimensional vector that is later reshaped as the 2×2 output matrix.

Transposed Convolution (stride=0)



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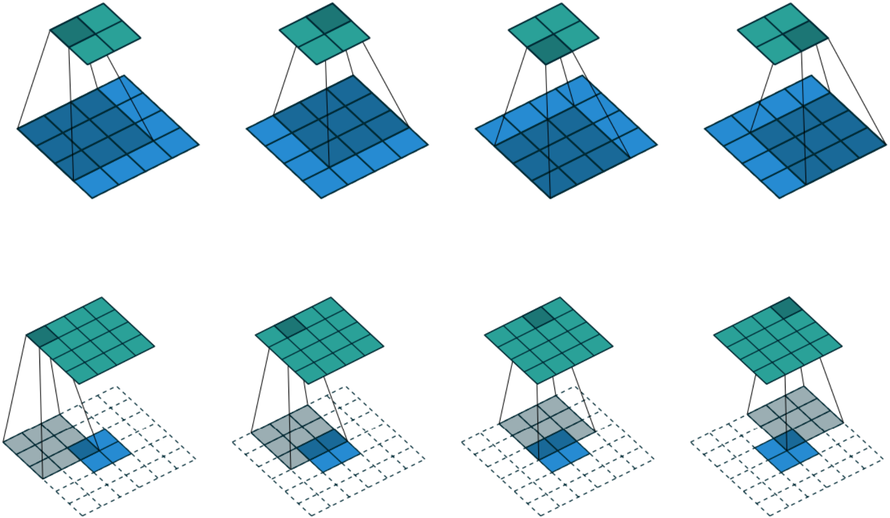
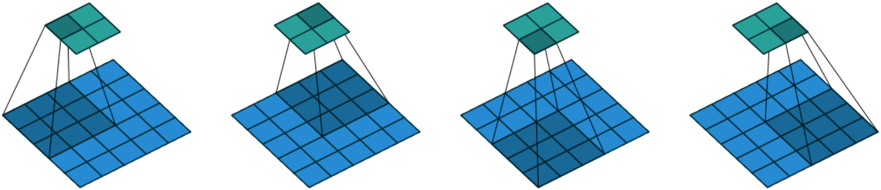


Transposed Convolution (stride=1)



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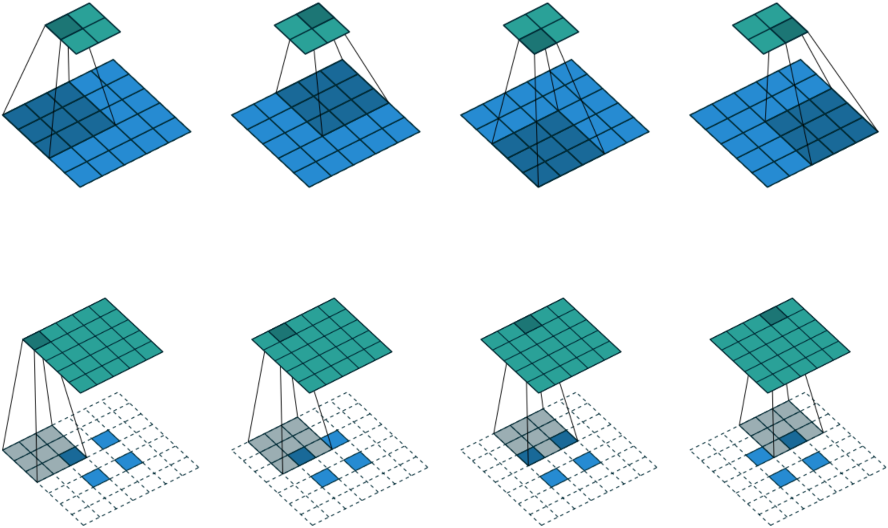
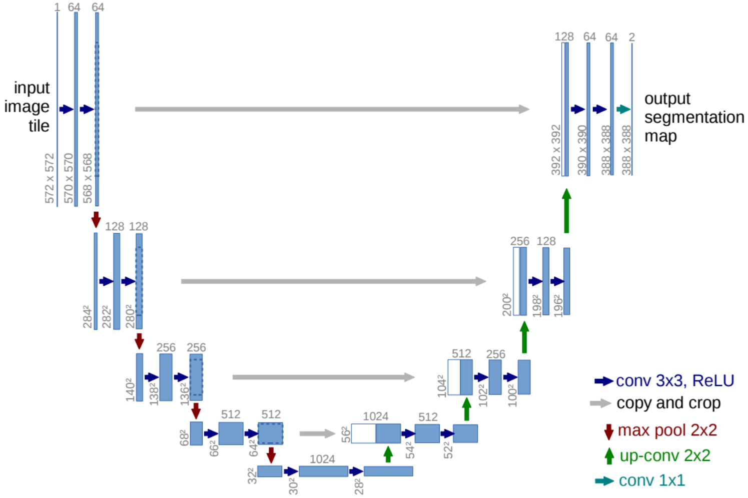


UNet



input image tile
output segmentation map
1
64
64
572 x 572
570 x 570
568 x 568
128
128
284²
282²
280²
256
256
140²
138²
136²
512
512
68²
66²
64²
1024
32²
30²
28²
1024
56²
512
54²
52²
512
256
104²
102²
100²
256
128
200²
198²
196²
128
64
64
2
392 x 392
390 x 390
388 x 388
388 x 388
conv 3x3, ReLU
copy and crop
max pool 2x2
up-conv 2x2
conv 1x1

Outline

KL Divergence

The KL divergence (also called relative entropy) is a measure of how one probability distribution is different from a second, reference probability distribution:

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but there is no symmetry, i.e. $KL\left(P||Q\right) \neq KL\left(Q||P\right)$.

Jensen-Shannon Divergence

JS Divergence is the following

$$JS\left(P||Q\right) = \frac{1}{2}KL\left(P||M\right) + \frac{1}{2}KL\left(M||Q\right),$$

where $M = \dfrac{P+Q}{2}$.

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Recall that probability density function (if it exists) of multivariate normal distribution with mean μ and with (non-singular, symmetric, positive definite) covariance matrix Σ is the following function:

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$$KL(\mathcal{N}_1, \mathcal{N}_2) = \frac{1}{2}\left(\mathrm{tr}\left(\Sigma_2^{-1}\Sigma_1\right) + (\mu_2-\mu_1)^{\mathsf{T}}\Sigma_2^{-1}(\mu_2-\mu_1) - k + \ln\frac{|\Sigma_2|}{|\Sigma_1|}\right).$$

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In one dimensional case we will have

$$KL(\mathcal{N}_1, \mathcal{N}_2) = \log\frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1-\mu_2)^2}{2\sigma_2^2} - \frac{1}{2}.$$

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Examples?

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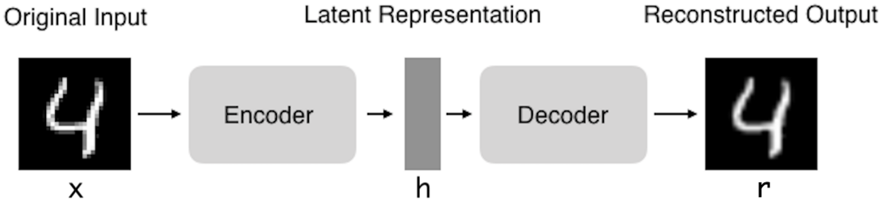
Autoencoders



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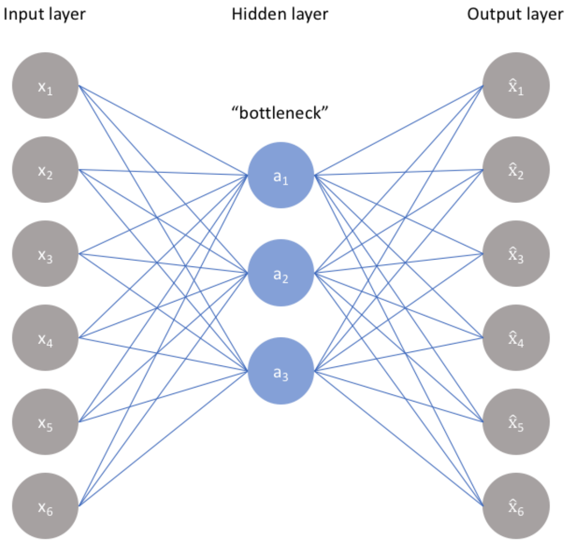
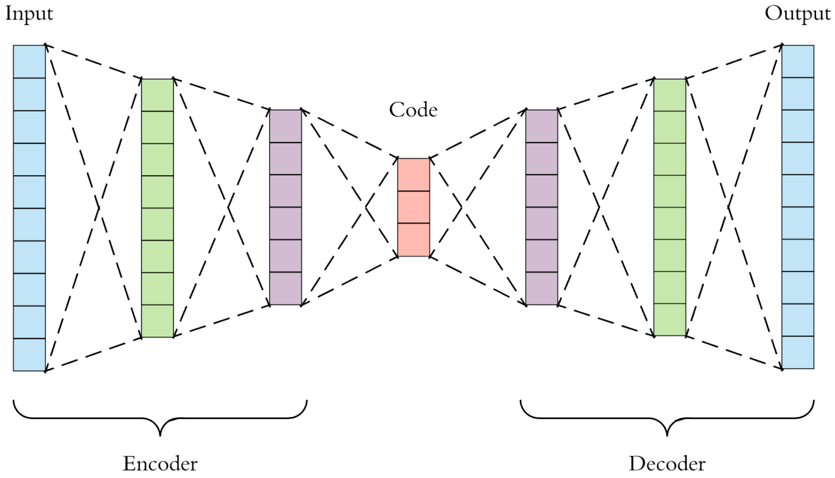
- Variational Autoencoders

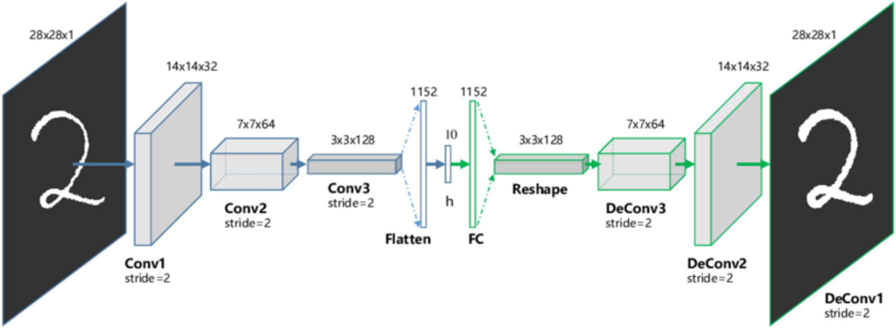
Vanilla Autoencoders



Multilayer/Deep Autoencoders



Convolutional Autoencoders



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- The end result is to reduce the learned representation's sensitivity towards the training input.