

# ASDS Statistics, YSU, Fall 2020

## Lecture 28

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12 Dec 2020

# Contents

- ▶ Asymptotic Confidence Intervals
- ▶ Hypothesis Testing

Asymptotic CIs

# Asymptotic CI for the Mean of General Distribution

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**Answer:** The Random Interval (or, rather, the sequence of Intervals)

$$\left( \bar{X}_n - z_{1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}}; \bar{X}_n + z_{1-\alpha/2} \cdot \frac{S_n}{\sqrt{n}} \right)$$

is a  $(1 - \alpha)$ -level Asymptotic CI for  $\mu$ .

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**Note:** We have obtained the following  $(1 - \alpha)$ -level Asymptotic CI for  $\mu$ :

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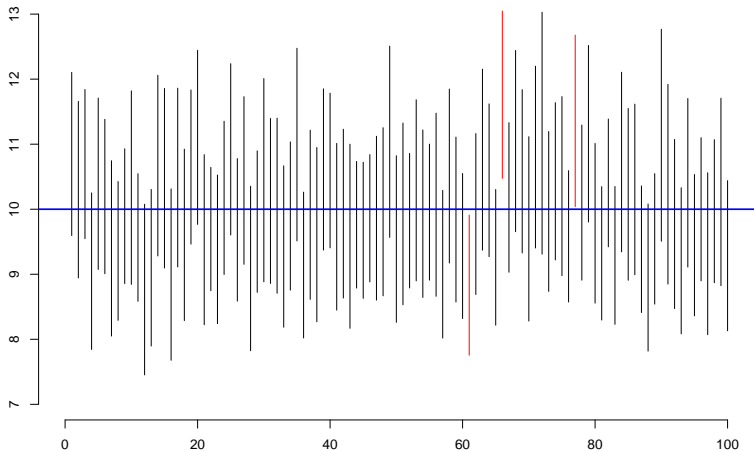
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- ▶ when  $n \geq 30$ , these two almost coincide;
- ▶ although in the theory these intervals work for large  $n$ , but, in practice, the latter one works also for small  $n$

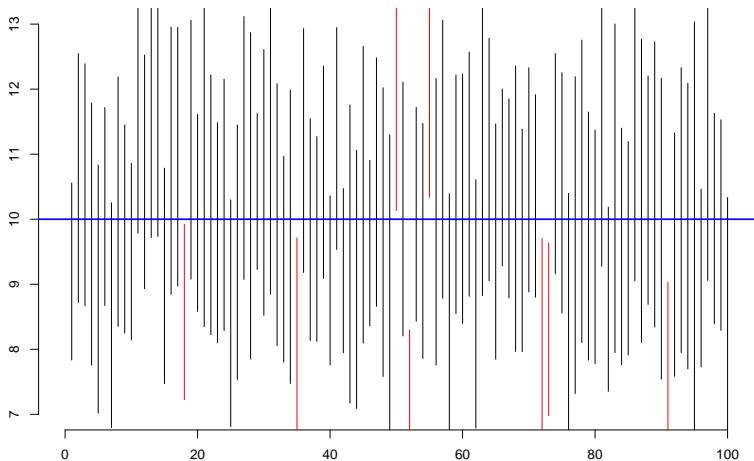
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Asymptotic CI for the Mean with  $z$ ,  $n = 50$

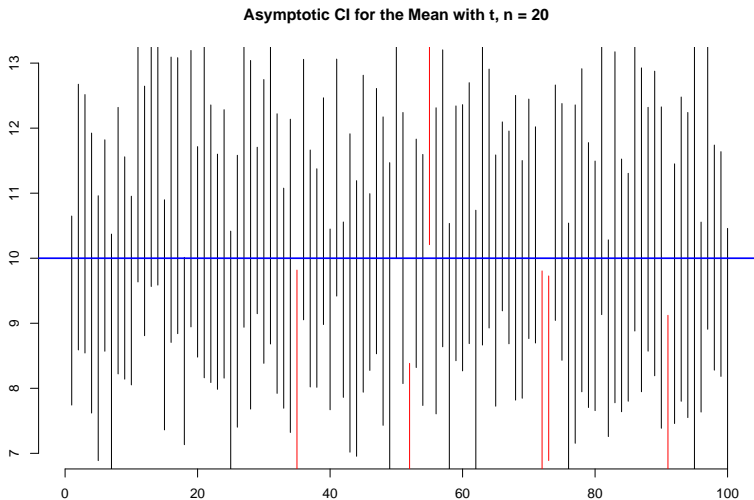


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We model our problem like this: we assume the skull sizes of Italians are coming from some Distribution with some Mean  $\mu$  and Variance  $\sigma^2$ ,  $\sigma^2$  is unknown.

## Example, Cont'd

If we believe that Etruscans are Italians, then we have a Sample from that Distrib:

$$X_1, X_2, \dots, X_{84}.$$

where  $X_k$  is the skull size of  $k$ -th Etruscan person.



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By plugging the values for our case, we'll obtain the following Asymptotic CI of level  $(1 - \alpha)$  for  $p$ :

$$\left( \bar{X}_n - z_{1-\alpha/2} \cdot \sqrt{\frac{\bar{X}_n(1 - \bar{X}_n)}{n}}; \bar{X}_n + z_{1-\alpha/2} \cdot \sqrt{\frac{\bar{X}_n(1 - \bar{X}_n)}{n}} \right).$$

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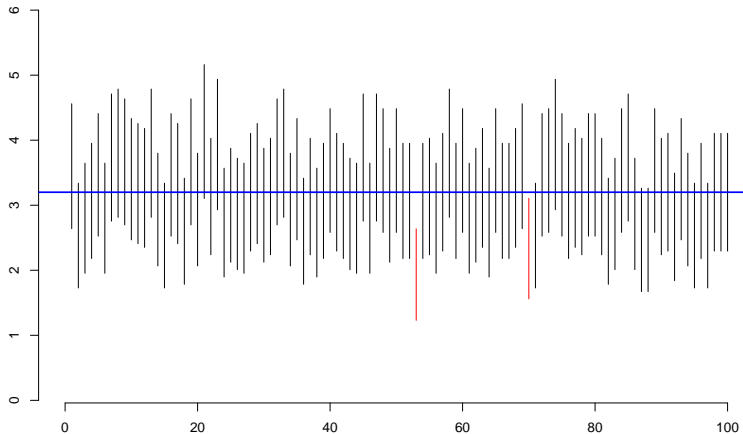
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We know that  $\hat{\lambda}_n^{MLE} = \bar{X}_n$ , and also,  $\mathcal{I}(\lambda) = \frac{1}{\lambda}$ . Then, using the formula above, we'll obtain the following Asymptotic CI of level  $(1 - \alpha)$  for  $\lambda$ :

$$\left( \bar{X}_n - z_{1-\alpha/2} \cdot \sqrt{\frac{\bar{X}_n}{n}}; \quad \bar{X}_n + z_{1-\alpha/2} \cdot \sqrt{\frac{\bar{X}_n}{n}} \right).$$

## Example, in R

Asymptotic CI for the Pois lambda, n = 50



## Example, in R, Code

```
lambda <- 3.2
conf.level <- 0.95; a = 1 - conf.level
sample.size <- 15; no.of.intervals <- 100
z <- qnorm(1-a/2)

plot.new()
plot.window(xlim=c(0,no.of.intervals),ylim=c(lambda-3,lambda+3))
axis(1); axis(2)
title("Asymptotic CI for the Pois lambda, n = 50")
for(i in 1:no.of.intervals){
  x <- rpois(sample.size, lambda = lambda)
  ME <- z*sqrt(mean(x)/sample.size) #Margin of Error
  lo <- mean(x) - ME; up <- mean(x) + ME
  if(lo > lambda || up < lambda){
    segments(c(i), c(lo), c(i), c(up), col = "red")
  }
  else{
    segments(c(i), c(lo), c(i), c(up))
  }
}
abline(h = lambda, lwd = 2, col = "blue")
```

# Hypothesis Testing

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As always, we assume we have a Dataset coming as a realization of a Random Sample from some unknown Parametric Distribution  $\mathcal{F}_\theta$ :

$$X_1, X_2, \dots, X_n \sim \mathcal{F}_\theta.$$

In this case we want to Test a Hypothesis about  $\theta$ : say, see whether  $\theta = \theta_0$ , a given number, or not.

## Example

**Example:** We have a coin, and we want to see if it is fair or not.



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and using this Sample, we want to Test the Hypothesis, we want to see if we can **Reject  $\mathcal{H}_0$  in favor of  $\mathcal{H}_1$  or not**, i.e., we want to see if **we have enough evidence in our Data to Reject  $\mathcal{H}_0$** .

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**Example:** In the above example about the coin fairness, if  $p$  is the Probability of a Head, then our Hypotheses are:

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So, when we will have our Data, we will see if we can **Reject**  $\mathcal{H}_0$ .

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So the conclusion of the Hypothesis Testing need to be either:

**Reject  $\mathcal{H}_0$**    or   **Fail to Reject  $\mathcal{H}_0$ .**

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## Moral, and Choosing Null Hypotheses

**Moral:** In Hypothesis testing, if we have enough evidence from Data against  $\mathcal{H}_0$ , we Reject it, otherwise, we say that we do not have enough evidence to Reject  $\mathcal{H}_0$ , so we Fail to Reject it, and keep believing in it.

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One is using this general idea to choose the Null and Alternative Hypotheses: **we will keep believing in Null, if the Data will not show strong evidence against.**



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This is an example of **A/B Testing**.

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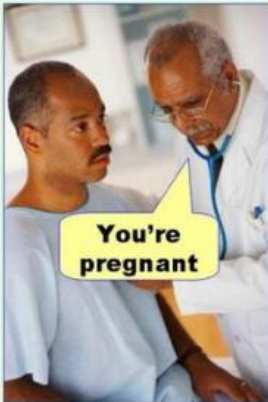
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Do Not Reject $\mathcal{H}_0$	Correct Decision (True Positive)	<b>Type II Error (False Negative)</b>

Can you guess the Null Hypo?

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**Type I error**  
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