# ASDS Statistics, YSU, Fall 2020 Lecture 02

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▶ What is Statistics?

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- ► What is a Statistics (again ¨)?

**Example:** AMS wants to calculate the average salary for all US Mathematicians.

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Can you describe

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If you want to get some trustworthy information, make reliable generalizations and good predictions from your Data, your Data need to be a **good** one.

First, for doing Statistics, Statisticians are modeling the process of Data Collection, they are *Designing the Experiment and the Sampling Methodology*. Correct design is very important for doing a correct analysis.

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**Definition:** We say that our Sample is *Representative* (obtained by a Simple Random Sampling), if it is obtained in the process where all Samples of size k have the same probability of being chosen.

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Which one gives a Representative (Simple Random) Sample?

Simple Random Sampling is not always easy to perform, so people are using different simpler Sampling Strategies (although they are not always giving exactly Representative Samples):

Systematic (Interval) Sampling, we fit the population into a list, enumerate it; choose n; choose at random a starting element from the first n elements in the list; and from that element on, every n-th member of the population is selected;

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- Cluster Sampling, where the total population is divided into subgroups (clusters), then some clusters are randomly chosen. Then we include all elements of chosen clusters into our Sample.

#### Classification of Data wrt its Dimension

#### Data can be

- ► Univariate (1D) here the observations are on a single Variable
- ▶ **Bivariate** (2D) here the observations are on two Variables
- ▶ **Multivariate**  $(n-D, n \ge 2)$  when the observations are on more than a one Variable (usually, more than two)

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**Examples:** Gender is a Categorical Variable, taking values Male, Female; Or Color and Model (of a car) are again Categorical

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- No. of Children, No. of Customers .... are Discrete
- ▶ Height, Weight, Age, ... are Continuous

**Remark:** Of course, we can enumerate Categorical Data, say, instead of *Male, Female* we can just use 0 and 1. It seems that we have already a Numerical, Quantitative (Discrete) Data. But there is a difference:

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Let me give by an example: when talking about the number of children in the family, we can have the following data: 0, 2, 1, 2, 4, 6, and we can calculate, say, the average number of children in families, here 2.5.

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But even if we are enumerating the Gender or the Color, the average Gender or the average Color is not meaningful, we cannot deal with the assigned numbers as above!

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Maybe one of the well-known Ordinal Scale Measurements is the **Likert Scale**: This is our famous

Strongly Disagree | Disagree | Neither | Agree | Strongly Agree

Why we need it?

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This dataset contains a subset of the fuel economy data that the EPA makes available on http://fueleconomy.gov. It contains only models which had a new release every year between 1999 and 2008.

## # A tibble: 3 x 11

## 3 audi

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Lets look at the first 3 rows of our dataset:

a4

```
head(ggplot2::mpg, 3)
```

2

```
manufacturer model displ year
##
                                    cyl trans
                                                  drv
                                                          ctv
##
    <chr>>
                 <chr> <dbl> <int> <int> <chr>
                                                  <chr> <int>
## 1 audi
                        1.8 1999 4 auto(15)
                                                  f
                                                           18
                 a4
                а4
                        1.8 1999 4 manual(m5) f
                                                           21
## 2 audi
```

2008

4 manual(m6) f

20

The variable cty is the *city miles per gallon*, and the variable cyl is the *number of cylinders*. Let's separate that Variables:

```
cty <- ggplot2::mpg$cty
cyl <- ggplot2::mpg$cyl</pre>
```

#### Let's see the results:

cyl

[149] 6 6 6 6 6 8 6 6 6 6 8 4 4 4 4 4 4

[223] 4 4 4 5 5 4 4 4 4 6 6 6

Let's see the results:

cyl

Can you describe this data? What can be said about the No. of Cylinders of these cars?

Let's see the results for cty:

```
cty
     [1] 18 21 20 21 16 18 18 18 16 20 19 15 17 17 15 15 17 16 1
##
    [26] 16 15 15 14 11 11 14 19 22 18 18 17 18 17 16 16 17 17 1
##
    [51] 13 14 14 14 9 11 11 13 13 9 13 11 13 11 12 9 13 13 1
##
    [76] 11 12 14 15 14 13 13 13 14 14 13 13 13 11 13 18 18 17 1
##
   [101] 24 25 23 24 26 25 24 21 18 18 21 21 18 18 19 19 19 20 2
   [126] 14 9 14 13 11 11 12 12 11 11 11 12 14 13 13 13 21 19 2
   [151] 14 15 14 12 18 16 17 18 16 18 18 20 19 20 18 21 19 19 1
   [176] 15 15 16 14 21 21 21 21 18 18 19 21 21 21 22 18 18 18 2
   [201] 15 16 17 15 15 15 16 21 19 21 22 17 33 21 19 22 21 21 2
   [226] 20 20 21 18 19 21 16 18 17
```

cty

Let's see the results for cty:

```
## [1] 18 21 20 21 16 18 18 18 16 20 19 15 17 17 15 15 17 16 1
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## [201] 15 16 17 15 15 15 16 21 19 21 22 17 33 21 19 22 21 21 21
## [226] 20 20 21 18 19 21 16 18 17
```

Again, can you describe this data? What can be said about the City Miles per Gallon values of these cars?

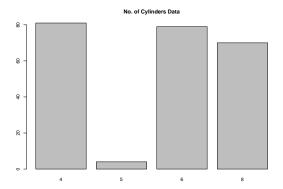
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For example, let us draw the BarPlot for the frequencies of the cyl variable:

```
barplot(table(cyl), main = "No. of Cylinders Data")
```



Now, let us give some numerical summaries for cty: calculate the average Miles per Gallon for a City, and its max and min.

```
cat("mean = ", mean(cty))
## mean = 16.85897
cat("Max = ", max(cty))
## Max = 35
cat("Min = ", min(cty))
## Min = 9
```

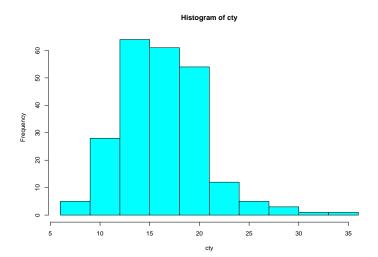
summary(cty)

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## Max = 35
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## Min = 9
And we can use the summary command to get some numerical info:
```

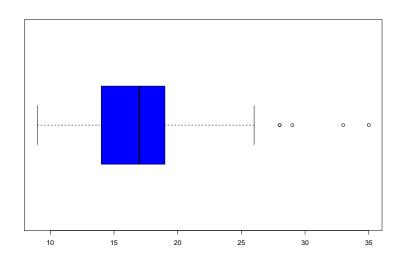
## Min. 1st Qu. Median Mean 3rd Qu. Max. ## 9.00 14.00 17.00 16.86 19.00 35.00

To get some visual information about the Variable cty, its distribution, we can draw the Histogram:



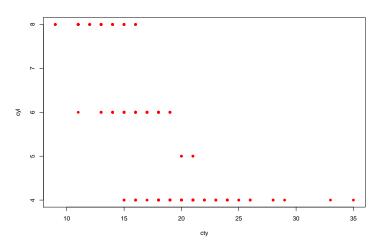
Now, we can draw the BoxPlot of the cty data:

```
boxplot(cty, horizontal = T, col = "blue")
```



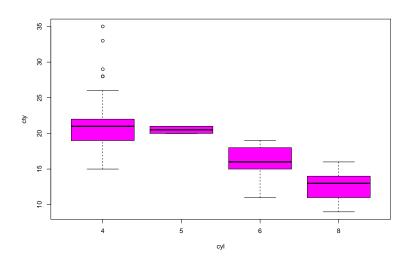
Now, instead of just getting information about cyl and cty separately, let us give visually the relationship between them:

```
plot(cty, cyl, pch=16, col = "red")
```



... or draw a BoxPlot of cty for each type of the cylinder:

```
boxplot(cty~cyl, col="magenta")
```



Moral: our brain cannot get an insight from the list of numbers, but Descriptive Statistics can help  $\ddot{-}$ 

How to do it?

Descriptive Statistics

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And we start by describing some of the *Graphical Summaries*.

Here, for the beginning, we will assume that we have a univariate (mostly numerical) data (dataset),  $x_1, x_2, ..., x_n$ . In this case we will say that we are given a (univariate, 1D) dataset x.

# Frequency Tables

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**Definition:** The **relative frequency** (or percentage) of a value t in observations  $x_1, x_2, ..., x_n$  is the ratio of frequency of t divided by the total number of observations, n:

Relative Frequency of 
$$t = \frac{\text{Frequency of } t}{\text{Total Number of Observations}} = \frac{\text{Frequency of } t}{n}.$$

# Frequency Tables, Example

**Example:** Given the following Dataset:

$$1, 2, 4, 7, 2, 3, 2, 1, 2, 1, 4, 1, -1\\$$

obtain the Frequency and Relative Frequency Tables.

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**Example:** Let's construct the Frequency Table of the above Dataset using **R**:

```
x \leftarrow c(1, 2, 4, 7, 2, 3, 2, 1, 2, 1, 4, 1, -1)
table(x)
```

```
## x
## -1 1 2 3 4 7
## 1 4 4 1 2 1
```

# Cumulative Frequency and Relative Frequency Tables

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We will meet, in fact, the Cumulative Relative Frequency Table soon, under the name Empirical CDF, ECDF.