ASDS Statistics, YSU, Fall 2020

Lecture 27

Michael Poghosyan

05 Dec 2020

Contents

- Confidence Intervals

CI for the Mean of $\mathcal{N}(\mu, \sigma^2)$, σ is known, Pivotal Method

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Assume σ^2 is known. Given $\alpha \in (0, 1)$, we want to construct a CI of CL $1 - \alpha$ for μ, using a Pivot.

CI for the Mean of $\mathcal{N}(\mu, \sigma^2)$, σ is known, Pivotal Method

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Assume σ^2 is known. Given $\alpha \in (0,1)$, we want to construct a CI of CL $1-\alpha$ for μ, using a Pivot.

Answer: The interval

$$\left(\overline{X} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \;\; \overline{X} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

is a $(1-\alpha)$-level CI for μ, when σ^2 is known (using the Pivoting).

CI for the Mean of $\mathcal{N}(\mu, \sigma^2)$, σ is known, Pivotal Method

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Assume σ^2 is known. Given $\alpha \in (0, 1)$, we want to construct a CI of CL $1 - \alpha$ for μ, using a Pivot.

Answer: The interval

$$\left(\overline{X} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \;\; \overline{X} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

is a $(1 - \alpha)$-level CI for μ, when σ^2 is known (using the Pivoting).

Here z_β is the β-quantile of $\mathcal{N}(0, 1)$.

CI for the Mean of $\mathcal{N}(\mu, \sigma^2)$, σ is known, Pivotal Method

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Assume σ^2 is known. Given $\alpha \in (0,1)$, we want to construct a CI of CL $1-\alpha$ for μ, using a Pivot.

Answer: The interval

$$\left(\overline{X} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \;\; \overline{X} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

is a $(1-\alpha)$-level CI for μ, when σ^2 is known (using the Pivoting).

Here z_β is the β-quantile of $\mathcal{N}(0,1)$.

Note: The Margin of Error in this case is

$$z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}.$$

Example

Example: Assume we want to construct a 95% CI for μ in the $\mathcal{N}(\mu, \sigma^2)$ Model, when σ is given, known.

Example

Example: Assume we want to construct a 95% CI for μ in the $\mathcal{N}(\mu, \sigma^2)$ Model, when σ is given, known. We construct it using a Random Sample

$$X_1, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Example

Example: Assume we want to construct a 95% CI for μ in the $\mathcal{N}(\mu, \sigma^2)$ Model, when σ is given, known. We construct it using a Random Sample

$$X_1, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Since $1 - \alpha = 0.95$, then $\alpha = 0.05$.

Example

Example: Assume we want to construct a 95% CI for μ in the $\mathcal{N}(\mu, \sigma^2)$ Model, when σ is given, known. We construct it using a Random Sample

$$X_1, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Since $1 - \alpha = 0.95$, then $\alpha = 0.05$. By the above formula, we need to calculate the Standard Normal quantile $z_{1-\alpha/2} =$

Example

Example: Assume we want to construct a 95% CI for μ in the $\mathcal{N}(\mu, \sigma^2)$ Model, when σ is given, known. We construct it using a Random Sample

$$X_1, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Since $1 - \alpha = 0.95$, then $\alpha = 0.05$. By the above formula, we need to calculate the Standard Normal quantile $z_{1-\alpha/2} = z_{0.975}$.

Example

Example: Assume we want to construct a 95% CI for μ in the $\mathcal{N}(\mu, \sigma^2)$ Model, when σ is given, known. We construct it using a Random Sample

$$X_1, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Since $1 - \alpha = 0.95$, then $\alpha = 0.05$. By the above formula, we need to calculate the Standard Normal quantile $z_{1-\alpha/2} = z_{0.975}$.

R gives:

```r
qnorm(0.975)
```

```
## [1] 1.959964
```

so our 95% CI will be

$$\left(\overline{X} - 1.96 \cdot \frac{\sigma}{\sqrt{n}}, \;\; \overline{X} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}\right).$$

Example

Example with R: We generate random numbers from $\mathcal{N}(2.31, 4)$ (so here we assume we know the true parameter value of μ).

```
sigma <- 2
n <- 20
smpl <- rnorm(n, mean = 2.31, sd = sigma)
smpl
```

```
##  [1]  1.17818311  3.44746051  0.04385609 -0.04980528  2.
##  [7]  3.41368211  6.49536059  2.89622846 -0.37201655  6.
## [13]  3.78522113  1.02518955  2.46148062  2.74349331  1.
## [19]  0.84583401 -0.25556437
```

Example

Example with R: We generate random numbers from $\mathcal{N}(2.31, 4)$ (so here we assume we know the true parameter value of μ).

```
sigma <- 2
n <- 20
smpl <- rnorm(n, mean = 2.31, sd = sigma)
smpl
```

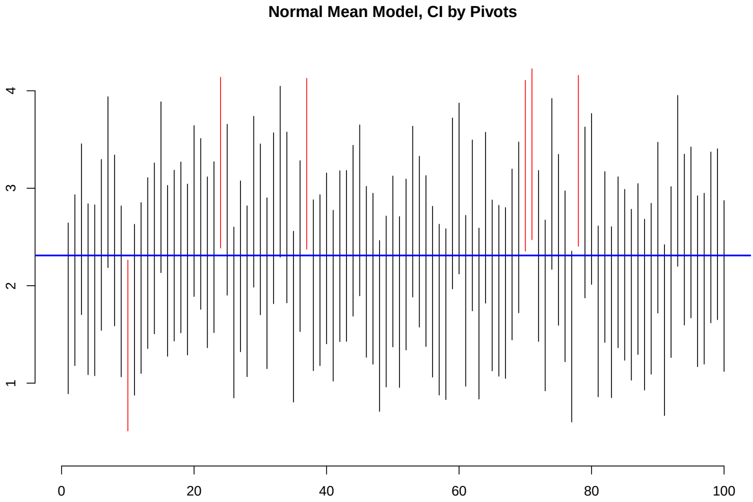
```
##  [1]  1.17818311  3.44746051  0.04385609 -0.04980528  2.
##  [7]  3.41368211  6.49536059  2.89622846 -0.37201655  6.
## [13]  3.78522113  1.02518955  2.46148062  2.74349331  1.
## [19]  0.84583401 -0.25556437
```

Now we construct the 95% CI by the above formula:

```
me <- 1.96* sigma/sqrt(n) #the Margin of Error
c(mean(smpl) - me, mean(smpl) + me)
```

```
## [1] 1.235312 2.988389
```

Example, Simulation



Example, Simulation, Code

```
mu <- 2.31; sigma <- 2
conf.level <- 0.95; a = 1 - conf.level
sample.size <- 20; no.of.intervals <- 100
z <- qnorm(1-a/2) ## our quantile
ME <- z*sigma/sqrt(sample.size) #Margin of Error

plot.new()
plot.window(xlim = c(0,no.of.intervals), ylim = c(mu-2,mu+2))
axis(1); axis(2)
title("Normal Mean Model, CI by Pivots")
for(i in 1:no.of.intervals){
  x <- rnorm(sample.size, mean = mu, sd = sigma)
  lo <- mean(x) - ME; up <- mean(x) + ME
  if(lo > mu || up < mu){
    segments(c(i), c(lo), c(i), c(up), col = "red")
  }
  else{
    segments(c(i), c(lo), c(i), c(up))
  }
}
abline(h = mu, lwd = 2, col = "blue")
```

CI for the Mean of $\mathcal{N}(\mu, \sigma^2)$, σ is **unknown**, PivMe

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Assume σ^2 is **unknown**, which is more realistic.

CI for the Mean of $\mathcal{N}(\mu, \sigma^2)$, σ is **unknown**, PivMe

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Assume σ^2 is **unknown**, which is more realistic. Given $\alpha \in (0, 1)$, we want to construct a CI of CL $1 - \alpha$ for μ, using a Pivot.

CI for the Mean of $\mathcal{N}(\mu, \sigma^2)$, σ is **unknown**, PivMe

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Assume σ^2 is **unknown**, which is more realistic. Given $\alpha \in (0, 1)$, we want to construct a CI of CL $1 - \alpha$ for μ, using a Pivot.

Answer: The following interval:

$$\left(\overline{X} - t_{n-1,1-\alpha/2} \cdot \frac{S}{\sqrt{n}}, \quad \overline{X} + t_{n-1,1-\alpha/2} \cdot \frac{S}{\sqrt{n}}\right)$$

is a $(1 - \alpha)$-level CI for Normal μ, when σ^2 is unknown.

CI for the Mean of $\mathcal{N}(\mu, \sigma^2)$, σ is **unknown**, PivMe

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2).$$

Assume σ^2 is **unknown**, which is more realistic. Given $\alpha \in (0, 1)$, we want to construct a CI of CL $1 - \alpha$ for μ, using a Pivot.

Answer: The following interval:

$$\left(\overline{X} - t_{n-1,1-\alpha/2} \cdot \frac{S}{\sqrt{n}}, \quad \overline{X} + t_{n-1,1-\alpha/2} \cdot \frac{S}{\sqrt{n}}\right)$$

is a $(1 - \alpha)$-level CI for Normal μ, when σ^2 is unknown.

Here $t_{n-1,\beta}$ is the β-quantile of the Student's T-Distribution with $n - 1$ degrees of freedom, which we denote by $t(n - 1)$.

CI for μ, Normal Model, Notes

Note: To compare:

- If σ is known, $(1-\alpha)$-level CI for μ is
$$\overline{X} \pm z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$
- If σ is unknown, $(1-\alpha)$-level CI for μ is
$$\overline{X} \pm t_{n-1,1-\alpha/2} \cdot \frac{S}{\sqrt{n}}$$

CI for μ, Normal Model, Notes

Note: To compare:

- If σ is known, $(1-\alpha)$-level CI for μ is

$$\overline{X} \pm z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

- If σ is unknown, $(1-\alpha)$-level CI for μ is

$$\overline{X} \pm t_{n-1,1-\alpha/2} \cdot \frac{S}{\sqrt{n}}$$

Note: If we will compare the quantiles of the same level of $\mathcal{N}(0,1)$ with $t(n-1)$, we will see that CIs for the case when σ is unknown are wider than for the case when σ is known. This is intuitive, of course - to compensate the uncertainty in σ, we need to take a wider interval:

```
c(qnorm(0.975), qt(0.975, df = 3), qt(0.975, df = 20))
```

```
## [1] 1.959964 3.182446 2.085963
```

Example

Example: Assume we want to estimate the average time μ our Stat students are spending for a homework. We collect information from 10 students for the last hw, and get the following responses (in hrs):

```
[1] 2.17 1.42 2.13 0.56 1.21 2.22 1.35 2.37 1.47 1.70
```

Example

Example: Assume we want to estimate the average time μ our Stat students are spending for a homework. We collect information from 10 students for the last hw, and get the following responses (in hrs):

```
[1] 2.17 1.42 2.13 0.56 1.21 2.22 1.35 2.37 1.47 1.70
```

Now we assume that the time spent for solving a hw is Normally Distributed with the Mean μ and some variance σ^2.

Example

Example: Assume we want to estimate the average time μ our Stat students are spending for a homework. We collect information from 10 students for the last hw, and get the following responses (in hrs):

```
[1] 2.17 1.42 2.13 0.56 1.21 2.22 1.35 2.37 1.47 1.70
```

Now we assume that the time spent for solving a hw is Normally Distributed with the Mean μ and some variance σ^2.

So $X \sim \mathcal{N}(\mu, \sigma^2)$ shows

Example

Example: Assume we want to estimate the average time μ our Stat students are spending for a homework. We collect information from 10 students for the last hw, and get the following responses (in hrs):

```
[1] 2.17 1.42 2.13 0.56 1.21 2.22 1.35 2.37 1.47 1.70
```

Now we assume that the time spent for solving a hw is Normally Distributed with the Mean μ and some variance σ^2.

So $X \sim \mathcal{N}(\mu, \sigma^2)$ shows the time spent solving a hw for a (randomly chosen) student.

Example

Example: Assume we want to estimate the average time μ our Stat students are spending for a homework. We collect information from 10 students for the last hw, and get the following responses (in hrs):

```
[1] 2.17 1.42 2.13 0.56 1.21 2.22 1.35 2.37 1.47 1.70
```

Now we assume that the time spent for solving a hw is Normally Distributed with the Mean μ and some variance σ^2.

So $X \sim \mathcal{N}(\mu, \sigma^2)$ shows the time spent solving a hw for a (randomly chosen) student. Our unknown, μ is

Example

Example: Assume we want to estimate the average time μ our Stat students are spending for a homework. We collect information from 10 students for the last hw, and get the following responses (in hrs):

```
[1] 2.17 1.42 2.13 0.56 1.21 2.22 1.35 2.37 1.47 1.70
```

Now we assume that the time spent for solving a hw is Normally Distributed with the Mean μ and some variance σ^2.

So $X \sim \mathcal{N}(\mu, \sigma^2)$ shows the time spent solving a hw for a (randomly chosen) student. Our unknown, μ is the average time to solve a hw.

Example

Example: Assume we want to estimate the average time μ our Stat students are spending for a homework. We collect information from 10 students for the last hw, and get the following responses (in hrs):

```
[1] 2.17 1.42 2.13 0.56 1.21 2.22 1.35 2.37 1.47 1.70
```

Now we assume that the time spent for solving a hw is Normally Distributed with the Mean μ and some variance σ^2.

So $X \sim \mathcal{N}(\mu, \sigma^2)$ shows the time spent solving a hw for a (randomly chosen) student. Our unknown, μ is the average time to solve a hw. And we have an observation from a Random Sample

$$X_1, X_2, ..., X_{10} \sim \mathcal{N}(\mu, \sigma^2).$$

Example

Example: Assume we want to estimate the average time μ our Stat students are spending for a homework. We collect information from 10 students for the last hw, and get the following responses (in hrs):

```
[1] 2.17 1.42 2.13 0.56 1.21 2.22 1.35 2.37 1.47 1.70
```

Now we assume that the time spent for solving a hw is Normally Distributed with the Mean μ and some variance σ^2.

So $X \sim \mathcal{N}(\mu, \sigma^2)$ shows the time spent solving a hw for a (randomly chosen) student. Our unknown, μ is the average time to solve a hw. And we have an observation from a Random Sample

$$X_1, X_2, ..., X_{10} \sim \mathcal{N}(\mu, \sigma^2).$$

We construct a 95% CI for μ, the average time to solve the hw, by the above formula:

Example

```r
n <- 10
cl <- 0.95; a = 1-cl
t <- qt(1-a/2, df = n-1)
smpl<-c(2.17,1.42,2.13,0.56,1.21,2.22,1.35,2.37,1.47,1.70)
s <- sd(smpl)
me <-  t*s/sqrt(n) #the Margin of Error
c(mean(smpl) - me, mean(smpl) + me)
```

```
## [1] 1.253748 2.066252
```

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **known**

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2),$$

where μ is **known**.

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **known**

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2),$$

where μ is **known**. We take an α, and want to construct a CI of CL $1-\alpha$ for σ^2, using a Pivotal Quantity.

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **known**

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2),$$

where μ is **known**. We take an α, and want to construct a CI of CL $1-\alpha$ for σ^2, using a Pivotal Quantity.

Answer: The $(1-\alpha)$-level CI for σ^2, when μ is known, is

$$\left(\frac{\sum_{k=1}^{n}(X_k-\mu)^2}{\chi^2_{n,1-\frac{\alpha}{2}}}, \quad \frac{\sum_{k=1}^{n}(X_k-\mu)^2}{\chi^2_{n,\frac{\alpha}{2}}} \right).$$

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **known**

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2),$$

where μ is **known**. We take an α, and want to construct a CI of CL $1-\alpha$ for σ^2, using a Pivotal Quantity.

Answer: The $(1-\alpha)$-level CI for σ^2, when μ is known, is

$$\left(\frac{\sum_{k=1}^n (X_k - \mu)^2}{\chi^2_{n,1-\frac{\alpha}{2}}}, \quad \frac{\sum_{k=1}^n (X_k - \mu)^2}{\chi^2_{n,\frac{\alpha}{2}}} \right).$$

Here $\chi^2_{n,\beta}$ is the β-quantile of the $\chi^2(n)$ Distribution.

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **unknown**

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2),$$

where μ is **unknown**, but we are not interested in μ.

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **unknown**

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2),$$

where μ is **unknown**, but we are not interested in μ. We take an α, and want to construct a CI of CL $1 - \alpha$ for σ^2, using a Pivotal Quantity.

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **unknown**

Problem: Assume we have a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{N}(\mu, \sigma^2),$$

where μ is **unknown**, but we are not interested in μ. We take an α, and want to construct a CI of CL $1-\alpha$ for σ^2, using a Pivotal Quantity.

Answer: The following is an $(1-\alpha)$-level CI for σ^2, when μ is unknown:

$$\left(\frac{\sum_{k=1}^{n}(X_k - \overline{X})^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{\sum_{k=1}^{n}(X_k - \overline{X})^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **unknown**

Let me give the CI for σ^2 again:

$$\left(\frac{\sum_{k=1}^{n}(X_k - \overline{X})^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{\sum_{k=1}^{n}(X_k - \overline{X})^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **unknown**

Let me give the CI for σ^2 again:

$$\left(\frac{\sum_{k=1}^{n} (X_k - \overline{X})^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{\sum_{k=1}^{n} (X_k - \overline{X})^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

Usually, you will see this in the following form:

$$\left(\frac{(n-1) \cdot S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{(n-1) \cdot S^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right),$$

CI for σ^2, $\mathcal{N}(\mu, \sigma^2)$ Model, μ is **unknown**

Let me give the CI for σ^2 again:

$$\left(\frac{\sum_{k=1}^{n}(X_k - \overline{X})^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{\sum_{k=1}^{n}(X_k - \overline{X})^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

Usually, you will see this in the following form:

$$\left(\frac{(n-1)\cdot S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{(n-1)\cdot S^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right),$$

where S is the Sample Standard Deviation given by

$$S^2 = \frac{\sum_{k=1}^{n}(X_k - \overline{X})^2}{n-1}.$$

Example

Example: Assume we have scale of very high precision, and we want to measure the precision of our scale. Say, we weight a 50AMD coin, using that scale, 10 times, and we obtain the following measurements (in gramms):

```
 [1] 3.449243 3.450802 3.453054 3.448778 3.452541 3.448835 3.448
 [9] 3.450314 3.449047
```

Our aim is to Estimate the Precision of the Scale.

Example

Example: Assume we have scale of very high precision, and we want to measure the precision of our scale. Say, we weight a 50AMD coin, using that scale, 10 times, and we obtain the following measurements (in gramms):

```
 [1] 3.449243 3.450802 3.453054 3.448778 3.452541 3.448835 3.448
 [9] 3.450314 3.449047
```

Our aim is to Estimate the Precision of the Scale.

We model this as: say, we weight, by using our scale, something with **exact** weight μ, but we do not know μ.

Example

Example: Assume we have scale of very high precision, and we want to measure the precision of our scale. Say, we weight a 50AMD coin, using that scale, 10 times, and we obtain the following measurements (in gramms):

```
 [1] 3.449243 3.450802 3.453054 3.448778 3.452541 3.448835 3.448
 [9] 3.450314 3.449047
```

Our aim is to Estimate the Precision of the Scale.

We model this as: say, we weight, by using our scale, something with **exact** weight μ, but we do not know μ. Our scale will show a number around, very close to μ, let it be, let it be W.

Example

Example: Assume we have scale of very high precision, and we want to measure the precision of our scale. Say, we weight a 50AMD coin, using that scale, 10 times, and we obtain the following measurements (in gramms):

```
 [1] 3.449243 3.450802 3.453054 3.448778 3.452541 3.448835 3.448
 [9] 3.450314 3.449047
```

Our aim is to Estimate the Precision of the Scale.

We model this as: say, we weight, by using our scale, something with **exact** weight μ, but we do not know μ. Our scale will show a number around, very close to μ, let it be, let it be W. So we will assume

$$W \sim$$

Example

Example: Assume we have scale of very high precision, and we want to measure the precision of our scale. Say, we weight a 50AMD coin, using that scale, 10 times, and we obtain the following measurements (in gramms):

```
 [1] 3.449243 3.450802 3.453054 3.448778 3.452541 3.448835 3.448
 [9] 3.450314 3.449047
```

Our aim is to Estimate the Precision of the Scale.

We model this as: say, we weight, by using our scale, something with **exact** weight μ, but we do not know μ. Our scale will show a number around, very close to μ, let it be, let it be W. So we will assume

$$W \sim \mathcal{N}(\mu, \sigma^2),$$

where σ^2 (or, better, σ) is measuring our scale Precision.

Example

Example: Assume we have scale of very high precision, and we want to measure the precision of our scale. Say, we weight a 50AMD coin, using that scale, 10 times, and we obtain the following measurements (in gramms):

```
 [1] 3.449243 3.450802 3.453054 3.448778 3.452541 3.448835 3.448
 [9] 3.450314 3.449047
```

Our aim is to Estimate the Precision of the Scale.

We model this as: say, we weight, by using our scale, something with **exact** weight μ, but we do not know μ. Our scale will show a number around, very close to μ, let it be, let it be W. So we will assume

$$W \sim \mathcal{N}(\mu, \sigma^2),$$

where σ^2 (or, better, σ) is measuring our scale Precision.

So now, using the above observations (weighting results), we will construct a 90% CI for σ^2.

Example, Cont'd

Recall the $(1-\alpha)$-level CI for σ^2:

$$\left(\frac{(n-1)\cdot S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}},\ \ \frac{(n-1)\cdot S^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

Example, Cont'd

Recall the $(1-\alpha)$-level CI for σ^2:

$$\left(\frac{(n-1)\cdot S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{(n-1)\cdot S^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

Now, we use **R** to do the rest:

Example, Cont'd

Recall the $(1-\alpha)$-level CI for σ^2:

$$\left(\frac{(n-1)\cdot S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}},\quad \frac{(n-1)\cdot S^2}{\chi^2_{n-1,\frac{\alpha}{2}}}\right).$$

Now, we use **R** to do the rest: we want to construct 90% CI, so our $\alpha =$

Example, Cont'd

Recall the $(1-\alpha)$-level CI for σ^2:

$$\left(\frac{(n-1)\cdot S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{(n-1)\cdot S^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

Now, we use **R** to do the rest: we want to construct 90% CI, so our $\alpha = 0.1$.

Example, Cont'd

Recall the $(1-\alpha)$-level CI for σ^2:

$$\left(\frac{(n-1) \cdot S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{(n-1) \cdot S^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

Now, we use **R** to do the rest: we want to construct 90% CI, so our $\alpha = 0.1$. We have 10 observations, so $n = 10$.

Example, Cont'd

Recall the $(1-\alpha)$-level CI for σ^2:

$$\left(\frac{(n-1)\cdot S^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \quad \frac{(n-1)\cdot S^2}{\chi^2_{n-1,\frac{\alpha}{2}}} \right).$$

Now, we use **R** to do the rest: we want to construct 90% CI, so our $\alpha = 0.1$. We have 10 observations, so $n = 10$. We calculate S^2:

```
w <- c(3.449243, 3.450802, 3.453054, 3.448778, 3.452541, 3.
s2 <- var(w)
s2
```

```
## [1] 4.605341e-06
```

Example, Cont'd

The Degrees of Freedom in our case, since we do not know μ, is

Example, Cont'd

The Degrees of Freedom in our case, since we do not know μ, is $n-1=9$, and we calculate the cooresponding quantiles for the $\chi^2(9)$:

```
alpha <- 0.1
lq<-qchisq(alpha/2, df=9); uq<-qchisq(1-alpha/2, df=9)
c(lq,uq)
```

```
## [1]  3.325113 16.918978
```

Example, Cont'd

The Degrees of Freedom in our case, since we do not know μ, is $n - 1 = 9$, and we calculate the cooresponding quantiles for the $\chi^2(9)$:

```
alpha <- 0.1
lq<-qchisq(alpha/2, df=9); uq<-qchisq(1-alpha/2, df=9)
c(lq,uq)
```

```
## [1]  3.325113 16.918978
```

Finally, we calculate our CI endpoints:

```
n <- 10
c((n-1)*s2/uq, (n-1)*s2/lq)
```

```
## [1] 2.449797e-06 1.246516e-05
```

Example, Cont'd

The Degrees of Freedom in our case, since we do not know μ, is $n-1=9$, and we calculate the cooresponding quantiles for the $\chi^2(9)$:

```
alpha <- 0.1
lq<-qchisq(alpha/2, df=9); uq<-qchisq(1-alpha/2, df=9)
c(lq,uq)
```

```
## [1]  3.325113 16.918978
```

Finally, we calculate our CI endpoints:

```
n <- 10
c((n-1)*s2/uq, (n-1)*s2/lq)
```

```
## [1] 2.449797e-06 1.246516e-05
```

Note: The actual value of `sd` I was using was: $sd=0.002$, so the true value of my σ^2 was

$$\sigma^2 = 4 \cdot 10^{-6}.$$