Think Bayesian

A man is running. Why?

Possible explanations:

1 He is in a hurry

2 He is doing sports

3 He always runs

4 He saw a dragon

A man is running. Why?

Principle 1: Use prior knowledge

1 He is in a hurry

2 He is doing sports

3 He always runs

4 He saw a dragon

A man is running. Why?

Principle 1: Use prior knowledge

1 He is in a hurry

2 He is doing sports

3 He always runs

4 ~~He saw a dragon~~

Low prior probability

A man is running. Why?

Principle 2: Chose answer that explains observations the most

1 He is in a hurry

2 He is doing sports

3 He always runs

4 ~~He saw a dragon~~

A man is running. Why?

Principle 2: Chose answer that explains observations the most

1 He is in a hurry

2 ~~He is doing sports~~

Contradicts the data

3 He always runs

4 ~~He saw a dragon~~

A man is running. Why?

Principle 3: Avoid making extra assumptions

1 He is in a hurry

2 ~~He is doing sports~~

3 He always runs

4 ~~He saw a dragon~~

A man is running. Why?

Principle 3: Avoid making extra assumptions

1

He is in a hurry

2

~~He is doing sports~~

3

~~He always runs~~

Too many assumptions

4

~~He saw a dragon~~

A man is running. Why?

1

He is in a hurry

2

~~He is doing sports~~

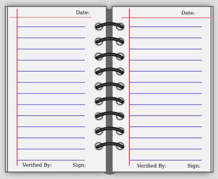
3

~~He always runs~~

4

~~He saw a dragon~~

Main principles



Principle 1:

Use prior knowledge

Principle 2:

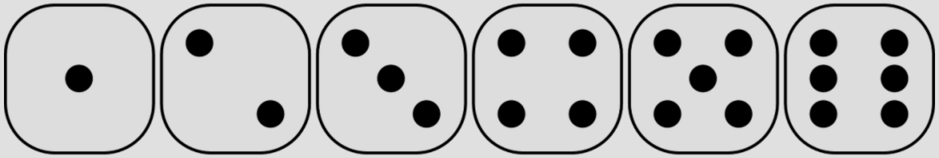
Chose answer that explains observations the most

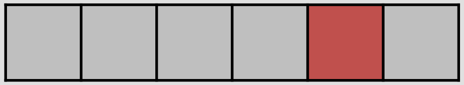
Principle 3:

Avoid making extra assumptions

Review of probability

Probability

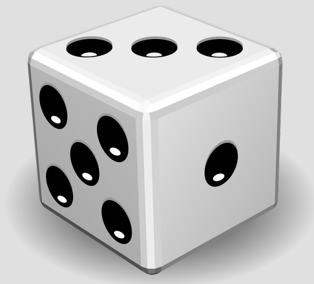




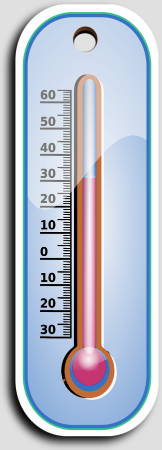
$$P(threw\ 5) = \frac{1}{6}$$

$$P(threw\ odd) = \frac{1}{2}$$

Random variables

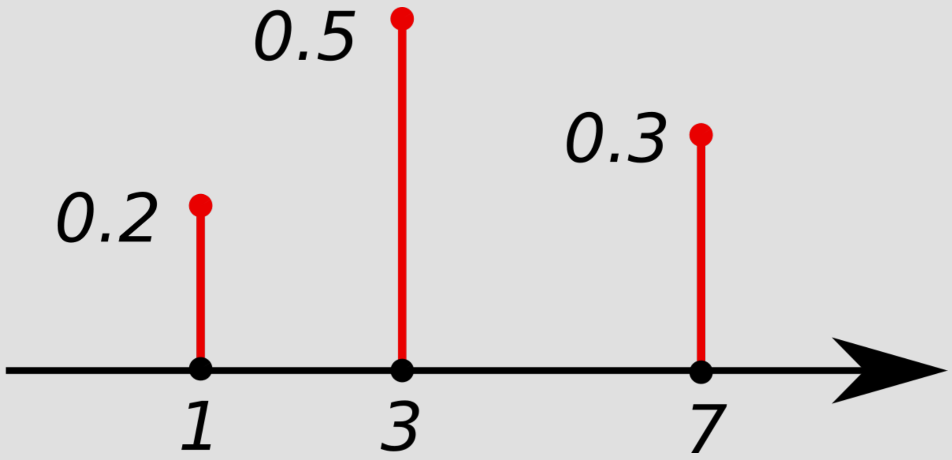


Discrete



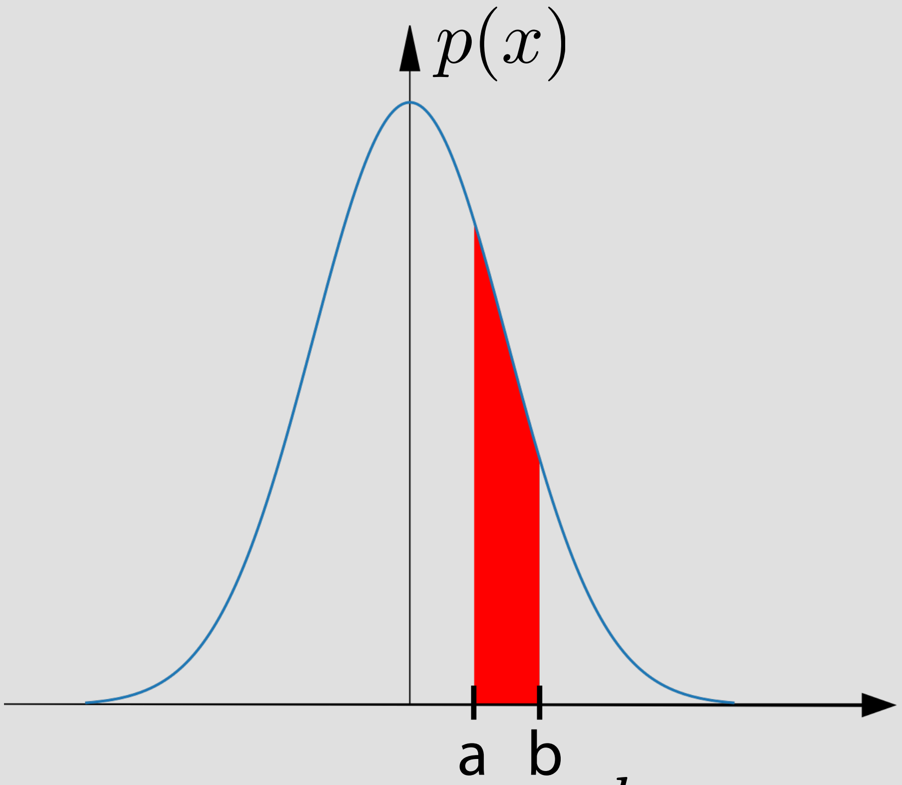
Continuous

Discrete: Probability Mass Function (PMF)



$$P(X) = \begin{cases} 0.2 & X = 1 \\ 0.5 & X = 3 \\ 0.3 & X = 7 \\ 0 & otherwise \end{cases}$$

Continuous: Probability Density Function (PDF)



$$P(x \in [a,b]) = \int_a^b p(x)dx$$

Independence

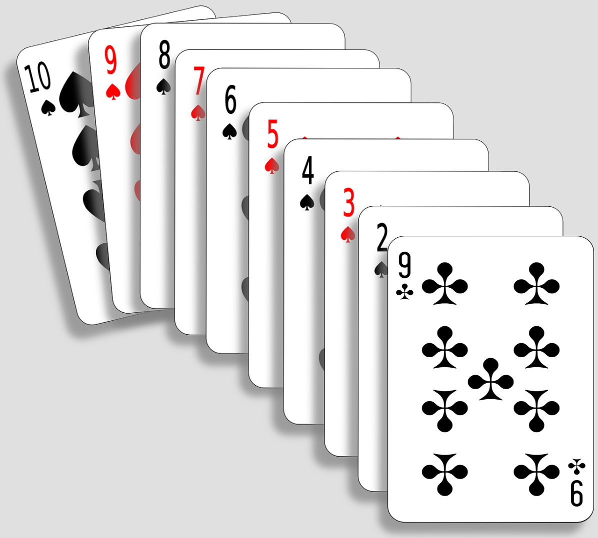
X and Y are independent if:

Marginals

$$\underbrace{P(X,Y)}_{\text{Joint}} = P(X)P(Y)$$

Joint

Draws from the deck



52 cards

$$P(X_1 = 9\clubsuit, X_2 = 9\clubsuit) = 0$$

$$P(X_1 = 9\clubsuit)P(X_2 = 9\clubsuit) = \frac{1}{52^2}$$

Two coins

$$P(X_1 = \mathrm{h}, X_2 = \mathrm{t}) = P(X_1 = \mathrm{h})P(X_2 = \mathrm{t})$$

Conditional probability

Probability of **X** given that **Y** happened:

Joint

$$P(X|Y) = \frac{P(X,Y)}{P(Y)}$$

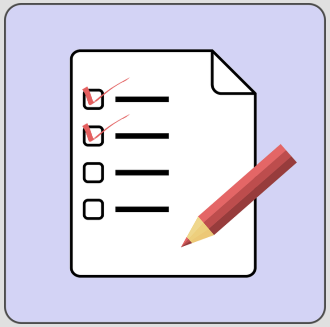
Conditional

Marginal

Conditional probability

$$P(M) = 0.4$$
$$P(M \,\&\, F) = 0.25$$

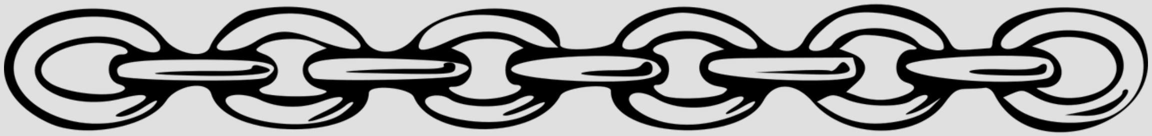
Midterm



Final

$$P(F|M) = \frac{P(M \,\&\, F)}{P(M)} = \frac{0.25}{0.4} = 0.625$$

Chain rule



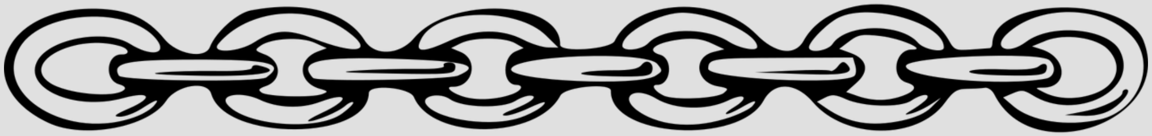
$$P(X, Y) = P(X|Y)P(Y)$$

Chain rule

$$P(X,Y) = P(X|Y)P(Y)$$

$$P(X,Y,Z) = P(X|Y,Z)P(Y|Z)P(Z)$$

Chain rule



$$P(X, Y) = P(X|Y)P(Y)$$

$$P(X, Y, Z) = P(X|Y, Z)P(Y|Z)P(Z)$$

$$P(X_1, \ldots, X_N) = \prod_{i=1}^{N} P(X_i|X_1, \ldots, X_{i-1})$$

Sum rule

Marginalization

$$p(X) = \int_{-\infty}^{\infty} p(X, Y) dY$$

Bayes theorem

θ — parameters

X — observations

Posterior Likelihood Prior

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Evidence