

# Think Bayesian



# A man is running. Why?

Possible explanations:

1



He is in a hurry

2



He is doing sports

3



He always runs

4



He saw a dragon



# A man is running. Why?

Principle 1: Use prior knowledge

1



He is in a hurry

2



He is doing sports

3



He always runs

4



He saw a dragon



# A man is running. Why?

Principle 1: Use prior knowledge

1



He is in a hurry

2



He is doing sports

3



He always runs

4



He saw a dragon

**Low prior probability**



# A man is running. Why?

Principle 2: Choose answer that explains observations the most

1



He is in a hurry

2



He is doing sports

3



He always runs

4



~~He saw a dragon~~



# A man is running. Why?

Principle 2: Choose answer that explains observations the most

1



He is in a hurry

3



He always runs

2



He is doing sports  
**Contradicts the data**

4



He saw a dragon



# A man is running. Why?

Principle 3: Avoid making extra assumptions

1



He is in a hurry

2



He is doing sports

3



He always runs

4



He saw a dragon



# A man is running. Why?

Principle 3: Avoid making extra assumptions

1



He is in a hurry

2



He is doing sports

3



He always runs

**Too many assumptions**

4



He saw a dragon



# A man is running. Why?

1



He is in a hurry

2



He is doing sports

3



He always runs

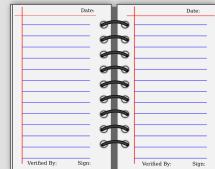
4



He saw a dragon



# Main principles



**Principle 1:**  
Use prior knowledge

**Principle 2:**  
Choose answer that explains observations the most

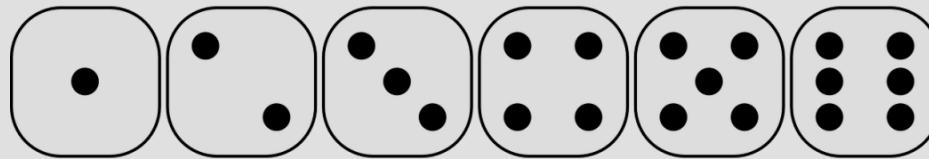
**Principle 3:**  
Avoid making extra assumptions



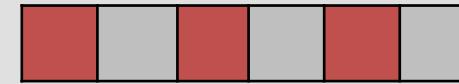
# **Review of probability**



# Probability



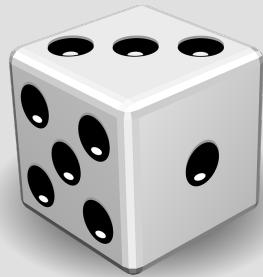
$$P(\text{threw 5}) = \frac{1}{6}$$



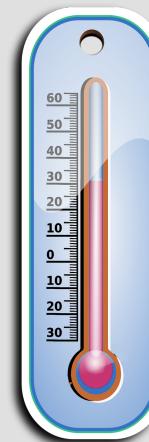
$$P(\text{threw odd}) = \frac{1}{2}$$



# Random variables



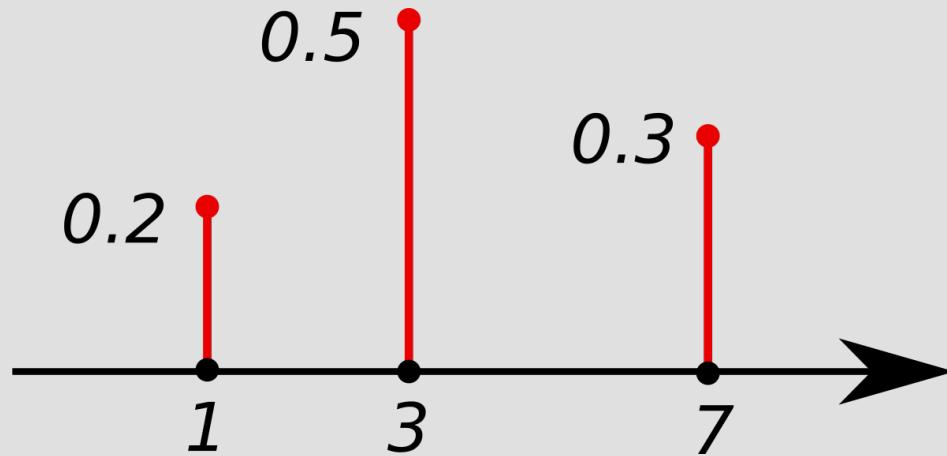
Discrete



Continuous



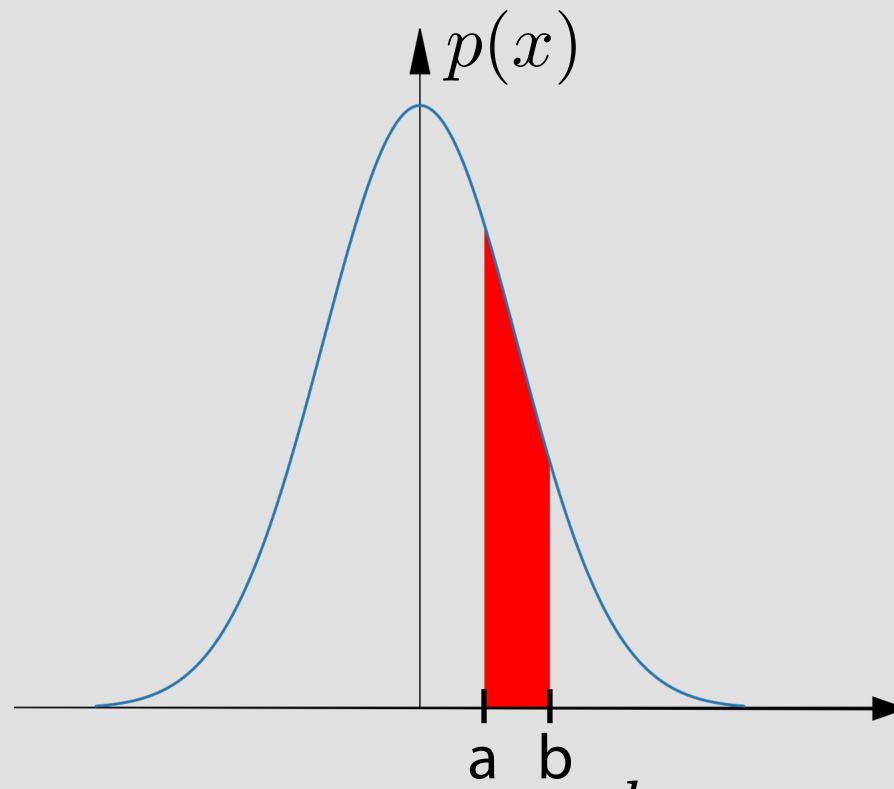
# Discrete: Probability Mass Function (PMF)



$$P(X) = \begin{cases} 0.2 & X = 1 \\ 0.5 & X = 3 \\ 0.3 & X = 7 \\ 0 & otherwise \end{cases}$$



# Continuous: Probability Density Function (PDF)



$$P(x \in [a, b]) = \int_a^b p(x) dx$$



# Independence

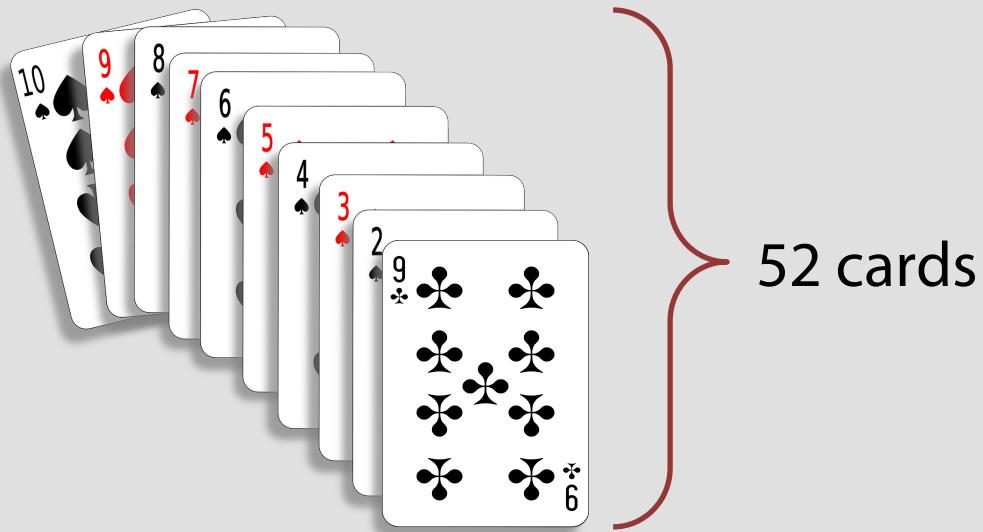
X and Y are independent if:

$$P(X, Y) = P(X)P(Y)$$

↑  
Joint      Marginals  
↓ ↓



# Draws from the deck



$$P(X_1 = 9\clubsuit, X_2 = 9\clubsuit) = 0$$

$$P(X_1 = 9\clubsuit)P(X_2 = 9\clubsuit) = \frac{1}{52^2}$$



# Two coins



$$P(X_1 = \text{h}, X_2 = \text{t}) = P(X_1 = \text{h})P(X_2 = \text{t})$$



# Conditional probability

Probability of **X** given that **Y** happened:

$$P(X|Y) = \frac{P(X, Y)}{P(Y)}$$

Joint

Conditional

Marginal

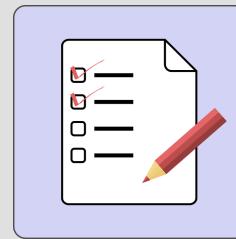
The diagram illustrates the formula for conditional probability. It features a central fraction  $P(X, Y) / P(Y)$ . A red bracket on the left side of the numerator  $P(X, Y)$  is labeled "Conditional". A red bracket on the right side of the denominator  $P(Y)$  is labeled "Marginal". Above the fraction, a red bracket spanning both the numerator and the denominator is labeled "Joint".



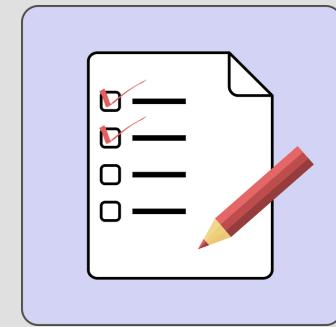
# Conditional probability

$$P(M) = 0.4$$

$$P(M \& F) = 0.25$$



Midterm



Final

$$P(F|M) = \frac{P(M \& F)}{P(M)} = \frac{0.25}{0.4} = 0.625$$



# Chain rule



$$P(X, Y) = P(X|Y)P(Y)$$



# Chain rule



$$P(X, Y) = P(X|Y)P(Y)$$

$$P(X, Y, Z) = P(X|Y, Z)P(Y|Z)P(Z)$$



# Chain rule



$$P(X, Y) = P(X|Y)P(Y)$$

$$P(X, Y, Z) = P(X|Y, Z)P(Y|Z)P(Z)$$

$$P(X_1, \dots, X_N) = \prod_{i=1}^N P(X_i | X_1, \dots, X_{i-1})$$



# Sum rule

Marginalization

$$p(X) = \int_{-\infty}^{\infty} p(X, Y) dY$$



# Bayes theorem

$\theta$  — parameters

$X$  — observations

$$P(\theta|X) = \frac{P(X, \theta)}{P(X)} = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Evidence

The diagram illustrates the components of Bayes' theorem. At the top, four labels are positioned: 'Posterior' on the left, 'Likelihood' in the center, 'Prior' on the right, and 'Evidence' at the bottom. Red arrows point from each label to its corresponding term in the equation below. The 'Posterior' arrow points to the first term in the numerator. The 'Likelihood' arrow points to the first term in the fraction. The 'Prior' arrow points to the second term in the numerator. The 'Evidence' arrow points to the denominator.

