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2.12

2.12.1

The instruction *add \$t0, \$s0, \$s1* performs an addition of the values stored in registers \$s0 and \$s1 and then stores the result in register \$t0. Given: $s0=0x80000000$, and $s1=0xD0000000$.

The result of the addition is: $t0=s0+s1$. For value of t0.

```
# Given values
```

```
s0 = 0x80000000
```

```
s1 = 0xD0000000
```

```
# Compute the addition
```

```
t0 = s0 + s1
```

```
t0_hex = hex(t0) t0_hex
```

```
RESULT : '0x150000000'
```

The value of t0 after executing the add instruction is 0x150000000. However, if we consider the usual 32-bit MIPS architecture, the result would be truncated to 32 bits. In this case, the lower 32 bits of the result would be kept.

```
# Extracting the lower 32 bits
```

```
t0_32bit = t0 & 0xFFFFFFFF
```

```
t0_32bit_hex = hex(t0_32bit)
```

```
t0_32bit_hex
```

```
RESULT: '0x50000000'
```

The value of t0 after truncating to 32 bits is 0x50000000.

2.12.2

In the context of 32-bit arithmetic, "overflow" occurs when the result of an addition does not fit within the 32-bit range. This happens when adding two positive numbers results in a negative value, or adding two negative numbers results in a positive value (considering two's complement representation).

Given: $s0=0x80000000$, and $s1=0xD0000000$ (This is also a negative number). The original result was 0x150000000, which doesn't fit in 32 bits. After truncating to 32 bits, we got $t0=0x50000000$,

which is a positive number. Since the sum of two negative numbers resulted in a positive value, there has been an overflow.

2.12.3

The instruction `sub $t0, $s0, $s1` subtracts the value stored in register `$s1` from the value stored in register `$s0` and then stores the result in register `$t0`. Given: `s0=0x80000000`, and `s1=0xD0000000`

The result of the subtraction is: `t0=s0-s1`. For the value of `t0`:

```
# Compute the subtraction
```

```
t0_sub = s0 - s1
```

```
t0_sub_32bit = t0_sub & 0xFFFFFFFF # Extracting the lower 32 bits
```

```
t0_sub_32bit_hex = hex(t0_sub_32bit)
```

```
t0_sub_32bit_hex
```

```
RESULT: '0xb0000000'
```

The value of `t0` after executing the `sub` instruction and truncating to 32 bits is `0xB0000000`. Given: `s0=0x80000000` (This is a negative number in two's complement), and `s1=0xD0000000` (This is also a negative number). The result `t0=0xB0000000` is also a negative number. For this particular operation, the subtraction of two negative numbers has resulted in a negative value. Therefore, there has been no overflow.

2.12.4

We're performing the subtraction: `s0-s1` Given: `s0=0x80000000`, `s1=0xD0000000`. The result we got was: `t0=0xB0000000` (Also a negative number in two's complement)

For our specific case: Both `s0` and `s1` are negative numbers. Their subtraction resulted in `t0`, which is also a negative number. Thus, the operation did not violate the two overflow conditions mentioned above. Therefore, there has been no overflow, and the result in `t0` is the desired result.

2.12.5

`add $t0, $s0, $s1` This adds the values in `$s0` and `$s1` and stores the result in `$t0`. Given: `s0=0x80000000`, and `s1=0xD0000000` The result of this addition is: `t0=s0+s1`.

`add $t0, $t0, $s0` This adds the value in `$t0` (from the previous operation) with the value in `$s0` and stores the result back in `$t0`. The result of this addition is: `t0=t0+s0`. The values for each step.

```
# First addition
```

```
t0_first_add = s0 + s1
```

```
t0_first_add_32bit = t0_first_add & 0xFFFFFFFF # Extracting the lower 32 bits after the first
```

```
# Second addition
```

```
t0_second_add = t0_first_add_32bit + s0
```

```
t0_second_add_32bit = t0_second_add & 0xFFFFFFFF # Extracting the lower 32 bits after the second addition
```

```
t0_second_add_32bit_hex = hex(t0_second_add_32bit) t0_second_add_32bit_hex
```

```
RESULT: '0xd0000000'
```

After executing the two add instructions, the value of t0 is 0xD0000000.

1. For the first addition (add \$t0, \$s0, \$s1): As we determined earlier, there was an overflow.
2. For the second addition (add \$t0, \$t0, \$s0): The result of the first addition was truncated to 0x50000000. Adding s0 (which is 0x80000000) to that value results in 0xD0000000. Given that we are adding a positive number to a negative number and the result is negative, there is no overflow for this operation.

2.12.6

To determine if there's overflow, we'll consider the entire sequence of instructions and their final effect on $\diamond t0$: $t0=s0+s1$, and $t0=t0+s0$.

Given: $s0=0x80000000$ (Negative), and $s1=0xD0000000$ (Negative)

After the first addition: The original result was 0x150000000, which was truncated to 0x50000000 (Positive) due to the 32-bit limitation.

After the second addition: The result is 0xD0000000 (Negative). Here's the sequence of our results:

1. Adding two negative numbers (s0 and s1) gave a positive value (0x50000000), indicating overflow.
2. Adding a negative number (s0) to a positive number (the truncated result from the first addition) gave a negative result (0xD0000000), which is expected and does not indicate overflow.

2.19

2.19.1

sllt2, t0, 44 This is a logical left shift operation. It shifts the value in t0 left by 44 bits and stores the result in t2. However, given that the MIPS architecture is typically 32-bit, a shift by 44 bits effectively means a shift by $44 \bmod 32 = 12$ bits.

Given: $t0=0xAAAAAAAA$ After the shift: $t2=t0 \ll 12$

or t2, t2, t1 This operation computes the bitwise OR of t2 (from the previous operation) and t1 and stores the result back in t2.

Given: $t1=0x12345678$ The result is: $t2=t2 \text{ OR } t1$

The value of t2.

```
# Given register values
```

```
t0 = 0xAAAAAAAA
```

```
t1 = 0x12345678
```

```
# Logical left shift
```

```
t2_sll = t0 << 12 # Since MIPS is 32-bit, it's effectively a shift by 44 mod 32 = 12 bits
```

```
t2_sll_32bit = t2_sll & 0xFFFFFFFF # Extracting the lower 32 bits after the shift
```

```
# Bitwise OR operation
```

```
t2_or = t2_sll_32bit | t1
```

```
t2_or_hex = hex(t2_or)
```

```
t2_or_hex
```

```
RESULT: '0xbabef678'
```

After executing the sequence of instructions, the value of t2 is 0xBABEF678.

2.19.2

sllt2, t0, 4 This is a logical left shift operation. It shifts the value in t0 left by 4 bits and stores the result in t2. Given: t0=0xAAAAAAAA After the shift: t2=t0<<4

andi t2, t2, -1 The "andi" operation computes the bitwise AND of t2 (from the previous operation) and the immediate value, which is -1 in this case. The "andi" operation is used with a 16-bit immediate value, so the -1 would be represented in 16 bits.

Given the two's complement representation, -1 in 16 bits is represented as all 1s. Thus, the operation will effectively be: t2=t2AND0xFFFF

```
# Logical left shift
```

```
t2_sll = t0 << 4
```

```
t2_sll_32bit = t2_sll & 0xFFFFFFFF # Extracting the lower 32 bits after the shift
```

```
# Bitwise AND operation with immediate value -1 (0xFFFF in 16 bits)
```

```
t2_andi = t2_sll_32bit & 0xFFFF
```

```
t2_andi_hex = hex(t2_andi)
```

```
t2_andi_hex
```

```
RESULT: '0xaaa0'
```

After executing the sequence of instructions, the value of t2 is 0xAAA0.

2.19.3

srlt2, t0, 3 This is a logical right shift operation. It shifts the value in t0 right by 3 bits and stores the result in t2. Given: t0=0xAAAAAAAA After the shift: t2=t0>>3

andit2, t2, 0xFFEF The "andi" operation computes the bitwise AND of t2 (from the previous operation) with the immediate value 0xFFEF. The operation will be: t2=t2AND0xFFEF

```
# Logical right shift t2_srl = t0 >> 3 t2_srl_32bit = t2_srl & 0xFFFFFFFF # Extracting the lower
32 bits after the shift # Bitwise AND operation with immediate value 0xFFEF t2_andi =
t2_srl_32bit & 0xFFEF t2_andi_hex = hex(t2_andi) t2_andi_hex
```

RESULT: '0x5545'

After executing the sequence of instructions, the value of t2 is 0x5545.

2.20

1. We first shift the desired bits in t0 to the least significant position and then shift them to bits 31 down to 26.

```
sll $t2, $t0, 15 # Shift bits 16-11 to bits 31-26 position
```

2. Then clear bits 31 down to 26 in t1. We can do this using a bitwise AND operation with a mask that has 0s in the positions 31 down to 26 and 1s everywhere else.

```
andi $t3, $t1, 0x03FFFFFF # Clear bits 31-26 in $t1
```

3. Combine the results using a bitwise OR operation.

```
or $t1, $t2, $t3 # Combine the results
```

4. The combined sequence of instructions would be:

```
sll $t2, $t0, 15 andi $t3, $t1, 0x03FFFFFF or $t1, $t2, $t3
```

This sequence extracts bits 16 to 11 from t0 and replaces bits 31 to 26 in t1 with the extracted bits without changing the other 26 bits of t1.

2.22

1. Load the value from the base address of C (i.e., C[0]) into a register.
2. Perform the left shift operation by 4 bits on the loaded value.
3. Store the shifted value back into A.

Here's the MIPS assembly code:

```
lw $t0, 0($s1) # Load C[0] into $t0
```

```
sll $t0, $t0, 4 # Perform the left shift by 4 bits on $t0
```

```
move $t1, $t0 # Store the shifted value in $t1 (A)
```

In this sequence: \$t1 corresponds to A, and \$s1 is the base address of C. \$t0 is a temporary register used for the operation.

2.23

1. *slt \$t2, \$0, \$t0*: This sets register t2 to 1 if 00 (the constant zero value) is less than t0 (which holds the value 0x00101000), and to 0 otherwise. Since 0 is indeed less than 0x00101000, t2 will be set to 1.
2. *bne \$t2, \$0, ELSE*: This instruction branches to the label "ELSE" if t2 is not equal to 00. Since we've established that t2 is 1, the program will branch to "ELSE".
3. *j DONE*: This is a jump instruction to the label "DONE". However, because of the previous branch, this instruction is skipped.
4. *ELSE: addi \$t2, \$t2, 2*: This instruction, under the "ELSE" label, adds 2 to the current value of t2 (which is 1), resulting in t2 being 3.
5. *DONE*: This label marks the end of the sequence.

So, after these instructions, the value of t2 would be 3.

2.26

2.26.1

1. *slt \$t2, \$0, \$t1*: Sets \$t2 to 1 if \$t1 is greater than 0, else it sets \$t2 to 0.
2. *beq \$t2, \$0, DONE*: Exits the loop if \$t2 is 0 (meaning \$t1 is not greater than 0).
3. *subi \$t1, \$t1, 1*: Decreases the value of \$t1 by 1.
4. *addi \$s2, \$s2, 2*: Increases the value of \$s2 by 2.
5. *j LOOP*: Jumps back to the start of the loop.

Given that \$t1 is initialized to 10, the loop will iterate 10 times, decrementing \$t1 until it reaches 0 and incrementing \$s2 by 2 each time. Therefore, the value of \$s2 at the end of the loop can be calculated as follows. Each loop iteration adds 2 to \$s2, and there are 10 iterations in total. Let's calculate the final value of \$s2.

Given that each iteration adds 2 to s2, and knowing there are 10 iterations, we multiply the number of iterations by the value added per iteration: $2 \times 10 = 20$. So, the value of s2 at the end of the loop is 20.

2.26.2

```
int A; // corresponds to $s1, not used in this loop
```

```
int B = 0; // corresponds to $s2, initialized to 0 as per your loop
```

```
int i = 10; // corresponds to $t1, initialized to 10 as per your loop
```

```
int temp; // corresponds to $t2, used for temporary comparisons
```

```
// Start of the loop equivalent to the MIPS assembly loop
```

```
while (i > 0) { // This checks the condition set by "slt $t2, $0, $t1" and "beq $t2, $0, DONE"
```

```
    i--; // Equivalent to "subi $t1, $t1, 1"
```

```
    B += 2; // Equivalent to "addi $s2, $s2, 2"
```

```
} // The loop continues until "i" is not greater than 0, which is checked at the start of each iteration
```

This C loop mimics the logic of the assembly code. The loop will continue until *i* is no longer greater than 0, decrementing *i* and incrementing *B* in each iteration, similar to the assembly instructions.

2.26.3

Based on the structure of the loop, five instructions are executed each iteration.

1. The loop starts with $t1=N$.
2. The loop continues until *t1* decrements to 0, which will take *N* iterations.
3. At the end of the *N*th iteration, the "slt" and "beq" instructions are executed, bringing the count to $5N+2$ instructions.
4. After the loop, execution jumps to the "DONE" label without executing more instructions.

So, for *N* iterations, the total number of instructions executed is $5N+2$. If you want an exact number, you would need to provide the specific value of *N*.

2.27

Translating the nested for-loops from C to MIPS assembly:

1. Initialize *i* to 0.
2. Check if *i* is less than *a*; if not, exit the loop.
3. Initialize *j* to 0.
4. Check if *j* is less than *b*; if not, go back to step 1.
5. Calculate the address for $D[4*j]$.
6. Calculate $i + j$ and store the result at the calculated address.
7. Increment *j* and go to step 4.
8. Increment *i* and go to step 2.

```
add $t0, $zero, $zero    # i = 0
```

```
outer_loop:
```

```
    slt $t2, $t0, $s0 # if i >= a, exit the loop
```

```
    beq $t2, $zero, exit_outer
```

```
    add $t1, $zero, $zero # j = 0
```

```
inner_loop:
```

```
    slt $t2, $t1, $s1    # if j >= b, exit the loop
```

```

    beq $t2, $zero, update_i
    sll $t3, $t1, 2      # t3 = 4 * j (calculate the offset for D[4*j])
    add $t3, $s2, $t3    # t3 = address of D[4*j]
    add $t4, $t0, $t1    # t4 = i + j
    sw $t4, 0($t3)      # D[4*j] = i + j
    addi $t1, $t1, 1     # j++
    j inner_loop
update_i:
    addi $t0, $t0, 1    # i++
    j outer_loop
exit_outer:

```

This code assumes that the array D is an array of integers (since we're using an offset of $4*j$, typical of 4-byte integers) and that the base address of D is in $\$s2$. The code uses $\$t0$ and $\$t1$ for i and j , respectively, and $\$s0$ and $\$s1$ for a and b , respectively. It also uses additional temporary registers $\$t2$, $\$t3$, and $\$t4$ for comparisons and calculations.

2.28

The instructions:

- Initialization and comparison instructions: 2 (add for initialization, slt for comparison)
- Branching to exit the loop: 1 (beq)
- Instructions inside the inner loop (including address calculation, addition, memory store, increment, and unconditional jump): 7
- Instructions for incrementing i and unconditional jump to continue the outer loop: 2

This totals 12 unique instructions in the loop's implementation.

Given $a = 10$ and $b = 1$, the inner loop executes once for each iteration of the outer loop, which iterates 10 times. Here's the breakdown:

- The outer loop runs 11 times (10 iterations plus the one failing the condition).
- Each iteration of the outer loop involves 1 comparison and 1 branch instruction, and it initializes j (1 instruction), total 3 instructions per iteration, resulting in $3 \times 11 = 33$ instructions.
- The inner loop runs 10 times (once for each outer iteration) and involves 7 instructions (as listed above), total $7 \times 10 = 70$ instructions.
- The last iteration that checks $j < b$ and increments i contributes 2 instructions per outer loop iteration, resulting in $2 \times 11 = 22$ instructions.

Adding these up gives the total number of executed instructions.

Number of iterations for each loop

outer_loop_iterations = 11 # 10 successful iterations + 1 final failing check

inner_loop_iterations = 10 # Because 'b' is 1, so it runs once for each outer iteration

Number of instructions executed per iteration

instructions_outer = 3 # (slt, beq, add for 'j')

instructions_inner = 7 # All instructions inside the inner loop

instructions_post_outer = 2 # (addi for 'i', j to continue the loop)

Calculating the total number of instructions executed

total_instructions = (outer_loop_iterations * instructions_outer) + \ (inner_loop_iterations * instructions_inner) + \ (outer_loop_iterations * instructions_post_outer)

total_instructions

RESULT: 125

The total number of MIPS instructions executed to complete the nested loops, given that $a = 10$ and $b = 1$, is 125.

2.38

Given the 32-bit hexadecimal value 0x11223344 at address 0x10000000, the byte layout in memory would be:

- 0x10000000: 0x11 (MSB)
- 0x10000001: 0x22
- 0x10000002: 0x33
- 0x10000003: 0x44 (LSB)

The given MIPS instructions perform the following operations:

lbu \$t0, 0(\$t1): This instruction loads the byte at the address contained in register \$t1 (0x10000000) into register \$t0. Because it's a "load byte unsigned" (lbu) instruction, it will retrieve the byte at that address without sign extension. Given the big-endian memory organization, the byte loaded into \$t0 is 0x11.

sw \$t0, 0(\$t2): This instruction stores the word in register \$t0 at the address contained in register \$t2 (0x10000010). Since \$t0 was populated by the "lbu" instruction and holds the value 0x11, this value is stored into the word at the new address. However, because it's a "store word" (sw) instruction, it deals with words (4 bytes), not single bytes. The 0x11 value in \$t0 will be stored in the least significant byte of the word at the address in \$t2, while the remaining more significant

bytes will be zeroes, resulting in the word 0x00000011 being stored at 0x10000010. So, the value stored at the address pointed to by register \$t2 (0x10000010) is 0x00000011.

2.39

To create the 32-bit constant 0010 0000 0000 0001 0100 1001 0010 0100₂ and store it in register t1, we would first convert this binary number to hexadecimal for easier handling in MIPS assembly, and then use the appropriate instructions to load this value into the register.

Convert the binary number to hexadecimal:

0010 0000 0000 0001 0100 1001 0010 0100₂ equals 0x20014924 in hexadecimal.

In MIPS, there is no single instruction that directly loads a 32-bit immediate value into a register. Instead, we use a combination of instructions to achieve this. One common approach is to use the lui (load upper immediate) and ori (bitwise or immediate) instructions.

```
lui $t1, 0x2001 # Load the upper 16 bits of the constant into $t1
```

```
ori $t1, $t1, 0x4924 # Bitwise-OR to insert the lower 16 bits into $t1
```

The lui instruction shifts the 16-bit immediate value left by 16 bits and stores the result in register t1, effectively loading the upper half of the 32-bit constant. The ori instruction then inserts the lower 16 bits. The combination of these two instructions effectively constructs the full 32-bit constant in register t1.

2.46

2.46.1

To determine if this is a good design choice, we need to calculate the total execution time before and after the proposed changes and then compare these times.

1. Before the changes:
 - a. Calculate the total number of cycles for each type of instruction and then sum them to get the total number of cycles for the program.
 - b. Use the clock cycle time to find the total execution time.
2. After the changes:
 - a. Reduce the number of arithmetic instructions by 25%.
 - b. Adjust the CPI for the arithmetic instructions due to the increased complexity (as the clock cycle time increases by 10%, the CPI for arithmetic instructions might increase correspondingly).
 - c. Calculate the new total number of cycles and the new execution time using the increased clock cycle time.
3. Comparison:
 - a. Compare the execution times before and after the changes to evaluate the effectiveness of the design choice.

```
# Constants for the initial scenario
```

```
initial_arithmetic_instructions = 500e6
```

```
initial_load_store_instructions = 300e6
```

```
initial_branch_instructions = 100e6
```

```
CPI_arithmetic = 1
```

```
CPI_load_store = 10
```

```
CPI_branch = 3
```

```
# Total cycles calculation before the changes
```

```
total_cycles_before = (initial_arithmetic_instructions * CPI_arithmetic +  
initial_load_store_instructions * CPI_load_store + initial_branch_instructions * CPI_branch) #  
Assumptions for the new scenario
```

```
reduction_in_arithmetic_instructions = 0.25 # 25% reduction
```

```
increase_in_clock_cycle_time = 0.10 # 10% increase
```

```
# New number of arithmetic instructions
```

```
new_arithmetic_instructions = initial_arithmetic_instructions * (1 -  
reduction_in_arithmetic_instructions)
```

```
# Total cycles calculation after the changes (assuming CPI for arithmetic operations doesn't  
change)
```

```
total_cycles_after = (new_arithmetic_instructions * CPI_arithmetic +  
initial_load_store_instructions * CPI_load_store + initial_branch_instructions * CPI_branch)
```

```
# The execution time is directly proportional to the number of cycles and the clock cycle time.
```

```
# We assume an initial arbitrary clock cycle time to make this comparison, as we're comparing the  
proportionality.
```

```
arbitrary_clock_cycle_time = 1 # This can be any non-zero number since we're comparing the  
ratio.
```

```
execution_time_before = total_cycles_before * arbitrary_clock_cycle_time
```

```
execution_time_after = total_cycles_after * (arbitrary_clock_cycle_time * (1 +  
increase_in_clock_cycle_time))
```

```
# We are interested in the change in execution time, not the exact execution time.
```

```
change_in_execution_time = execution_time_after - execution_time_before
```

```
total_cycles_before, total_cycles_after, change_in_execution_time
```

```
RESULT :(3800000000.0, 3675000000.0, 242500000.00000048)
```

Before the changes, the program requires 3,800,000,000 cycles to complete. After implementing the more powerful arithmetic instructions and increasing the clock cycle time by 10%, the program requires 3,675,000,000 cycles.

However, due to the increase in clock cycle time, the actual execution time increases by 242,500,000 units (in the same time unit used for the clock cycle time).

So, while the total number of cycles has decreased, the execution time has increased due to the longer clock cycle. This is crucial because the user experiences execution time, not the number of cycles.

Therefore, this might not be a good design choice if the goal is to decrease execution time. The 25% reduction in arithmetic instructions does not compensate for the 10% increase in clock cycle time. In performance-sensitive applications, this trade-off could lead to a slower perceived performance, even though the processor is performing fewer overall computations.

2.46.2

Here, the enhancement is doubling the performance of arithmetic instructions, meaning they would take half the cycles per instruction (CPI) as before. First, we'll calculate the initial execution time based on the CPI values and the number of instructions. Then, we'll calculate the new execution time after the improvement and determine the speedup from these values.

1. Initial Execution Time (ET_initial): This is calculated as the product of the total cycles (calculated from the CPI and the number of instructions) and the clock cycle time (CCT). We assume CCT remains constant, where Total Cycles is the sum of the products of CPI and the instruction count for each type of instruction.
2. New Execution Time (ET_new): After the enhancement, arithmetic instructions are executed faster (half the CPI or one-tenth the CPI, depending on the scenario). We recalculate the total cycles with the new CPI for arithmetic instructions and then determine the new execution time using the same CCT.
3. Speedup: This is the ratio of the initial execution time to the new execution time.

Constants for the initial scenario

CPI_arithmetic_initial = 1 # Initial CPI for arithmetic instructions

For the scenario of doubling the performance (half the CPI)

CPI_arithmetic_double = CPI_arithmetic_initial / 2 # CPI is halved

For the scenario of a tenfold increase in performance (one-tenth the CPI)

CPI_arithmetic_tenfold = CPI_arithmetic_initial / 10

CPI is one-tenth # Total cycles calculation before the changes (initial scenario)

total_cycles_initial = (initial_arithmetic_instructions * CPI_arithmetic_initial +
initial_load_store_instructions * CPI_load_store + initial_branch_instructions * CPI_branch)

```
# Total cycles calculation after doubling arithmetic performance
```

```
total_cycles_double = (initial_arithmetic_instructions * CPI_arithmetic_double +  
initial_load_store_instructions * CPI_load_store + initial_branch_instructions * CPI_branch)
```

```
# Total cycles calculation after tenfold increase in arithmetic performance
```

```
total_cycles_tenfold = (initial_arithmetic_instructions * CPI_arithmetic_tenfold +  
initial_load_store_instructions * CPI_load_store + initial_branch_instructions * CPI_branch)
```

```
# Speedup calculation: It's the ratio of the number of cycles before and after the improvement.
```

```
# This works under the assumption that clock cycle time remains constant.
```

```
speedup_double = total_cycles_initial / total_cycles_double
```

```
speedup_tenfold = total_cycles_initial / total_cycles_tenfold
```

```
speedup_double, speedup_tenfold
```

```
RESULT :(1.0704225352112675, 1.1343283582089552)
```

Doubling the performance of arithmetic instructions (i.e., halving the CPI for these instructions) results in an overall speedup of approximately 1.07. This means the machine would be about 7% faster with the enhanced arithmetic instructions.

Improving the performance of arithmetic instructions tenfold (i.e., making the CPI one-tenth of its original value) results in an overall speedup of approximately 1.13. This means the machine would be about 13% faster with these highly enhanced arithmetic instructions.

2.47

2.47.1

The average CPI (Cycles Per Instruction) is calculated based on the individual CPIs of different types of instructions and their respective proportions in the total instruction mix.

Given: Arithmetic instructions: 70% of total, 2 cycles each, Load/Store instructions: 10% of total, 6 cycles each, and Branch instructions: 20% of total, 3 cycles each.

```
# Percentages of each type of instruction
```

```
percentage_arithmetic = 0.70
```

```
percentage_load_store = 0.10
```

```
percentage_branch = 0.20
```

```
# Cycles per instruction for each type of instruction
```

```
cycles_arithmetic = 2
```

```
cycles_load_store = 6
```

```
cycles_branch = 3
```

```
# Calculating the average CPI
```

```
average_CPI = (percentage_arithmetic * cycles_arithmetic + percentage_load_store *  
cycles_load_store + percentage_branch * cycles_branch)
```

```
average_CPI
```

```
RESULT: 2.6
```

The average CPI (Cycles Per Instruction) given the instruction mix and individual cycles for each type of instruction is 2.6.

2.47.2

To achieve a 25% improvement in performance, we need to understand that performance is inversely related to execution time, which is directly proportional to both the CPI and the number of instructions. If we want to improve performance by 25%, we're effectively saying we want to reduce the execution time to 75% of its current value.

Given that we are only improving the arithmetic instructions and not the load/store or branch instructions, we need to focus on the impact that the arithmetic instructions have on the overall CPI and, therefore, the execution time. The formula for the new average CPI, given a 25% improvement in performance, can be derived from the relationship:

New Execution Time = 0.75 × Old Execution Time

Since execution time is directly related to the product of CPI and the instruction count (and the instruction count remains constant), we can say:

New Average CPI = 0.75 × Old Average CPI

We have already established that the average CPI with the old performance is 2.6. Therefore, the new average CPI would be 0.75 × 2.6.

Now, we want to find out the new CPI for arithmetic instructions that would give us this new average CPI. The average CPI is composed of the contributions from each type of instruction. The contribution from the arithmetic instructions is what we are allowed to change. The maximum CPI that the improved arithmetic instructions can have to achieve this overall performance improvement.

```
# The target is a 25% improvement in performance, which means the new execution time should  
be 75% of the old execution time.
```

```
target_execution_time_factor = 0.75
```

```
target_average_CPI = target_execution_time_factor * average_CPI
```

```
fixed_CPI_contribution = (percentage_load_store * cycles_load_store + percentage_branch *  
cycles_branch)
```

```
new_CPI_arithmetic = (target_average_CPI - fixed_CPI_contribution) / percentage_arithmetic
```

```
new_CPI_arithmetic
```

RESULT

```
1.0714285714285714
```

To achieve a 25% improvement in overall performance, the new cycles per arithmetic instruction must be reduced, on average, to approximately 1.07. This means the arithmetic instructions need to execute nearly twice as fast as they did previously, assuming the performance of load/store and branch instructions remains unchanged.

2.47.3

A 50% improvement in performance means we want to make the program run in half the time it currently takes. This is equivalent to saying that we want the new execution time to be 50% of the current execution time, or conversely, that we want a performance increase of 100% from the current state.

Given the direct relationship between execution time and CPI (since the number of instructions does not change), achieving a 50% reduction in execution time implies we need a corresponding reduction in the average CPI.

The new average CPI target with a 50% improvement in performance, and then find out what the new CPI for arithmetic instructions needs to be to achieve this new average CPI, assuming the CPIs for load/store and branch instructions remain unchanged.

```
# The target is a 50% improvement in performance, which means the new execution time should be 50% of the old execution time.
```

```
target_execution_time_factor_50 = 0.50
```

```
# Calculate the new target average CPI to meet the desired performance improvement.
```

```
target_average_CPI_50 = target_execution_time_factor_50 * average_CPI
```

```
new_CPI_arithmetic_50 = (target_average_CPI_50 - fixed_CPI_contribution) / percentage_arithmetic
```

```
new_CPI_arithmetic_50
```

RESULT :0.14285714285714268

To achieve a 50% improvement in overall performance, the new cycles per arithmetic instruction must be reduced, on average, to approximately 0.143. This means the arithmetic instructions need to execute significantly faster than they did previously, assuming the performance of load/store and branch instructions remains unchanged.

Furthermore:

2.13

2.13.1

Given s_0 holds the value 1281012810: If s_0 and s_1 are both positive and their sum is negative, then overflow has occurred. If s_0 is positive and s_1 is negative (or vice versa) and their sum is positive, then overflow has occurred. But since s_0 is positive and relatively small, it's impossible for its addition with a negative number to overflow in a 32-bit system.

For the first case, the maximum positive value for a 32-bit integer in two's complement representation is $2^{31}-1$ (since one bit is reserved for the sign). To determine the range for s_1 that causes an overflow, we need to find values for which the sum s_0+s_1 exceeds this maximum.

The overflow for the addition of two positive numbers will occur if the sum exceeds the maximum positive 32-bit two's complement value. Given: $s_0=128$ max positive= $2^{31}-1$ max positive= $2^{31}-1$

```
# Given value for s0
```

```
s0_value = 128
```

```
# Maximum positive value for a 32-bit integer in two's complement
```

```
max_positive_32bit = 2**31 - 1
```

```
# Calculate the smallest value of s1 that causes overflow
```

```
min_s1_overflow = max_positive_32bit - s0_value + 1
```

```
# The range for s1 that would cause an overflow is [min_s1_overflow, 2**31)
```

```
range_s1_overflow = (min_s1_overflow, 2**31)
```

```
range_s1_overflow
```

```
RESULT: (2147483520, 2147483648)
```

The range of values for s_1 that would result in overflow when added to s_0 (which has a value of 1281012810) is: [2147483520,2147483648)

This means any value for s_1 within this range, when added to s_0 , will cause an overflow in a 32-bit system.

2.13.2

Given $s_0=128$ (which is positive):

1. For the first case, there will be no overflow since s_0 is small, and subtracting any negative number from it will not cause the result to overflow in a 32-bit system.
2. For the second case, the condition for overflow is: $s_0-s_1 < \text{minimum 32-bit integer}$ Where the minimum 32-bit integer in two's complement representation is -2^{31} . From this, we can determine the range of values for s_1 that would cause an overflow.

```
# Minimum 32-bit value in two's complement
```



```
min_32bit = -2**31
```

```
# To determine the maximum value of s1 that causes overflow:
```

```
# s0 - s1 < min_32bit
```

```
# => s1 > s0 - min_32bit
```

```
max_s1_overflow = s0_value - min_32bit
```

```
# The range for s1 that would cause an overflow is [max_s1_overflow, 2**31)
```

```
# as values equal to or above 2**31 are not valid 32-bit two's complement integers.
```

```
range_s1_overflow_subtraction = (max_s1_overflow, 2**31)
```

```
range_s1_overflow_subtraction
```

```
RESULT: (2147483776, 2147483648)
```

For the subtraction instruction sub t0, s0,s1, the range of values for s1 that would result in overflow when subtracted from s0 (which has a value of 1281012810) is: [2147483776,2147483648)

This means any value for s1 within this range, when subtracted from s0, will cause an overflow in a 32-bit system. However, note that the range is somewhat counter-intuitive since the start of the range is larger than the end. This is because the value 231231 (2147483648) is not a valid 32-bit two's complement integer, so any value equal to or above this would be invalid. Thus, the only valid value in this range that would cause an overflow is s1=2147483776.

2.13.3

Given s0=128 (positive):

1. For the first case: The condition for overflow is $s1 - s0 > \text{max positive 32-bit integer}$ where the max positive 32-bit integer in two's complement representation is $2^{31} - 1$.
2. For the second case: There will be no overflow since s0 is positive and small, subtracting it from any positive number will not cause the result to overflow in a 32-bit system.

```
# Given value for s0
```

```
s0_value = 128
```

```
# Maximum positive value for a 32-bit integer in two's complement
```

```
max_positive_32bit = 2**31 - 1
```

```
# Calculate the minimum value of s1 that causes overflow for the reversed subtraction
```

```
min_s1_overflow_subtraction_reverse = s0_value + max_positive_32bit + 1
```

```
# The range for s1 that would cause an overflow is (-2**31, min_s1_overflow_subtraction_reverse]
```

```
range_s1_overflow_subtraction_reverse = (-2**31, min_s1_overflow_subtraction_reverse)
```

range_s1_overflow_subtraction_reverse

RESULT: (-2147483648, 2147483776)

For the instruction sub0, s1, s0, the range of values for s1 that would result in overflow when s0 (which has a value of 1281012810) is subtracted from s1 is: (-2147483648, 2147483776]

This means any value for s1 within this range, when s0 is subtracted from it, will cause an overflow in a 32-bit system.