

کنکور ریاضی ۱۴۰۳

سوال اول: مقادیر a و $2a+1$ و $a-5$ به ترتیب جملات متوالی یک دنباله حسابی هستند. اگر a جمله نخست این دنباله باشد جمله نهم کدام است؟

2.75 (1)

4.25 (2)

12.35 (3)

14.75 (4)

****Question 1:**** The values “ a ”, “ $2a+1$ ” and “ $5-a$ ” are consecutive terms of an arithmetic sequence. If “ a ” is the first term of this sequence, what is the ninth term?

— GPT (8 seconds) —

To solve this problem, we need to find the ninth term of an arithmetic sequence where the first three terms are a , $2a+1$, and $5-a$.

Step 1: Find the Value of a

In an arithmetic sequence, the difference between consecutive terms is constant. Therefore:

$$(2a+1)-a=(5-a)-(2a+1)$$

Simplify both sides:

$$a+1=4-3a$$

Combine like terms:

$$a+3a+1=4 \implies 4a+1=4$$

Subtract 1 from both sides:

$$4a=3 \implies a=\frac{3}{4}$$

Step 2: Find the Common Difference (d)

The common difference d is:

$$d=(2a+1)-a=a+1$$

Substitute $a=\frac{3}{4}$:

$$d=\frac{3}{4}+1=\frac{7}{4}$$

Step 3: Find the Ninth Term

The n -th term of an arithmetic sequence is:

$$T_n = a + (n-1)d$$

For the ninth term ($n=9$):

$$\begin{aligned} T_9 &= 34 + (9-1) \times 7 \\ T_9 &= 34 + 8 \times 7 \\ T_9 &= 34 + 56 \\ T_9 &= 90 \\ T_9 &= 14.75 \end{aligned}$$

Answer:

4) 14.75

سوال دوم: اگر p گزاره ای درست، q گزاره ای نادرست و r گزاره ای دلخواه باشد، گزاره
($p \Rightarrow r$) \Rightarrow ($r \Rightarrow q$)
همارز منطقی کدام گزاره است؟

- r (1)
- T (2)
- $r \sim$ (3)
- $T \sim$ (4)

Question 2: If p is a true proposition, q is a false proposition, and r is any arbitrary proposition, the proposition

$$(p \Rightarrow r) \Rightarrow (r \Rightarrow q)$$

is logically equivalent to which of the following propositions?

— GPT (7 seconds) —

To determine the logical equivalence of the proposition $((p \Rightarrow r) \Rightarrow (r \Rightarrow q))$ given that (p) is **true**, (q) is **false**, and (r) is any proposition, follow these steps:

1. **Simplify $(p \Rightarrow r)$.**

- The implication $(p \Rightarrow r)$ is equivalent to $(\neg p \vee r)$.
- Since (p) is true, $(\neg p)$ is false.
- Therefore, $(\neg p \vee r)$ simplifies to $(F \vee r)$, which is (r) .

2. **Simplify $(r \Rightarrow q)$.**

- The implication $(r \rightarrow q)$ is equivalent to $(\neg r \vee q)$.
- Since (q) is false, $(\neg r \vee q)$ simplifies to $(\neg r \vee F)$, which is $(\neg r)$.

3. **Substitute back into the original proposition:**

- The original proposition becomes $(r \rightarrow \neg r)$.

4. **Simplify $(r \rightarrow \neg r)$:**

- Create a truth table for $(r \rightarrow \neg r)$:

(r)	$(\neg r)$	$(r \rightarrow \neg r)$
T	F	F
F	T	T

- This shows that $(r \rightarrow \neg r)$ is **false** when (r) is true and **true** when (r) is false.

5. **Recognize the equivalence:**

- The truth table for $(r \rightarrow \neg r)$ matches that of $(\neg r)$.

Conclusion:

The given proposition $((p \rightarrow r) \rightarrow (r \rightarrow q))$ is logically equivalent to $(\neg r)$.

Answer: 3) $\sim r$

سوال سوم: نقاط (۳، -۴) و (-۵، ۱-۴) روی یک تابع درجه دوم واقع هستند. مجموع صفرهای این تابع کدام است؟

3/2 (1)

3/4 (2)

5/2 (3)

5/4 (4)

Question 3: The points (3,-4) and (-1.5,-4) lie on a quadratic function. What is the sum of the zeros of this function?

— GPT (11 seconds) —

To solve this problem, we need to find the sum of the zeros of a quadratic function that passes through the points $(3, -4)$ and $(-1.5, -4)$.

****Step 1: Recognize that both points have the same y -value.****

Since the quadratic function takes the value (-4) at both $(x = 3)$ and $(x = -1.5)$, we can consider the function shifted upward by 4 units to simplify calculations:

$$F(x) = f(x) + 4$$

****Step 2: Express $(F(x))$ in factored form using the known zeros.****

Since $(F(x) = 0)$ when $(x = 3)$ and $(x = -1.5)$, we have:

$$F(x) = a(x - 3)(x + 1.5)$$

For simplicity, we can choose $(a = 1)$ (since (a) cancels out when calculating the sum of zeros):

$$F(x) = (x - 3)(x + 1.5)$$

****Step 3: Expand $(F(x))$ and adjust back to $(f(x))$.****

Expanding $(F(x))$:

$$F(x) = x^2 + 1.5x - 3x - 4.5 = x^2 - 1.5x - 4.5$$

Subtract 4 to revert back to $(f(x))$:

$$f(x) = F(x) - 4 = x^2 - 1.5x - 8.5$$

****Step 4: Use the quadratic formula to find the sum of the zeros.****

For a quadratic equation $(ax^2 + bx + c = 0)$, the sum of the zeros is $(-\frac{b}{a})$. In this case:

$$a = 1, \quad b = -1.5$$

So the sum of the zeros is:

$$-\frac{b}{a} = -\left(\frac{-1.5}{1}\right) = 1.5 = \frac{3}{2}$$

****Answer: 1) 3/2****

سوال چهارم: برای چند مقدار صحیح و یک رقمی a ، جواب معادله $\sqrt{x} + \sqrt{x-a} = a$ عددی صحیح است؟

- 4 (1)
- 5 (2)
- 6 (3)
- 7 (4)

Question 4: For how many single-digit integer values of “a”, the solution to the equation “ $\sqrt{x} + \sqrt{x-a} = a$ ” is an integer? (“a” can be -9, -8, ..., 0, 1, ..., 9)

— GPT (8 seconds) —

To determine how many single-digit integer values of (a) satisfy the equation:

$$\sqrt{x} + \sqrt{x-a} = a$$

where (a) can range from (-9) to (9) , we'll analyze the equation step-by-step.

Step 1: Determine the Domain

- Both square roots require their arguments to be non-negative:

$$x \geq 0$$

$$(x-a) \geq 0 \implies x \geq a$$

- Additionally, since the left side of the equation is non-negative, (a) must also be non-negative:

$$a \geq 0$$

Thus, (a) ranges from (0) to (9) .

Step 2: Solve the Equation

Let's set:

$$\sqrt{x} = b \quad \text{and} \quad \sqrt{x-a} = c$$

\]

Then:

\[

$$b + c = a$$

\]

\[

$$b^2 - c^2 = a$$

\]

Using the difference of squares:

\[

$$(b - c)(b + c) = a \quad \rightarrow \quad (b - c)a = a$$

\]

Assuming $(a \neq 0)$, we can divide both sides by (a) :

\[

$$b - c = 1$$

\]

Now, we have:

\[

$$b + c = a \quad \text{and} \quad b - c = 1$$

\]

Adding these equations:

\[

$$2b = a + 1 \quad \rightarrow \quad b = \frac{a + 1}{2}$$

\]

Subtracting the second equation from the first:

\[

$$2c = a - 1 \quad \rightarrow \quad c = \frac{a - 1}{2}$$

\]

Step 3: Ensure (b) and (c) Are Real Numbers

Both (b) and (c) must be non-negative:

\[

$$\frac{a + 1}{2} \geq 0 \quad \rightarrow \quad a \geq -1$$

\]

\[

$$\frac{a - 1}{2} \geq 0 \quad \rightarrow \quad a \geq 1$$

\]

Since (a) is already non-negative, the critical condition is $(a \geq 1)$.

Step 4: Check for Integer Solutions

For (x) to be an integer:

\[

$$x = b^2 = \left(\frac{a + 1}{2}\right)^2$$

\]

This is an integer only if $\left(\frac{a+1}{2}\right)$ is an integer, which occurs when (a) is **odd**.

Therefore, the odd values of (a) from (1) to (9) are:

[

1, 3, 5, 7, 9

]

Additionally, consider $(a = 0)$:

[

$\sqrt{x} + \sqrt{x} = 0 \quad \rightarrow \quad 2\sqrt{x} = 0 \quad \rightarrow \quad x = 0$

]

This also provides a valid integer solution.

Conclusion

The values of (a) that satisfy the equation with an integer solution for (x) are:

[

0, 1, 3, 5, 7, 9

]

This totals **6** valid values.

****Answer:****

3) 6

سوال پنجم: حاصل عبارت زیر به ازای $x = \pi / 12$ کدام است؟

$$3 \cos 4x + \sqrt{2} \sin x - \sqrt{2} \cos x$$

1 (1

1/2 (2

$\sqrt{2}$ (3

$\sqrt{2} / 2$ (4

****Question 5:**** What is the value of the following expression for “ $x = \pi / 12$ ”?

$$3 * \cos(4x) + \sqrt{2} * \sin(x) - \sqrt{2} * \cos(x)$$

— GPT (12 seconds) —

To solve the expression:

$$[E = 3\cos(4x) + \sqrt{2}\sin(x) - \sqrt{2}\cos(x)]$$

for $(x = \frac{\pi}{12})$, follow these steps:

1. **Compute $\cos(4x)$ **: **

$$\cos(4x) = \cos\left(4 \times \frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

2. **Express $\sin\left(\frac{\pi}{12}\right)$ and $\cos\left(\frac{\pi}{12}\right)$ in terms of known angles: **

$$\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6} \quad \text{quad} \quad \left(15^\circ = 45^\circ - 30^\circ\right)$$

3. **Compute $\sin\left(\frac{\pi}{12}\right)$ **: **

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(45^\circ - 30^\circ\right)$$

$$= \sin(45^\circ)\cos(30^\circ) - \cos(45^\circ)\sin(30^\circ)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

4. **Compute $\cos\left(\frac{\pi}{12}\right)$ **: **

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\left(45^\circ - 30^\circ\right)$$

$$= \cos(45^\circ)\cos(30^\circ) + \sin(45^\circ)\sin(30^\circ)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

5. **Substitute back into the original expression: **

$$E = 3 \left(\frac{1}{2} \right) + \sqrt{2} \left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \right) - \sqrt{2} \left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \right)$$

6. **Simplify the expression:**

$$E = \frac{3}{2} + \left(\frac{2\sqrt{3}}{4} - \frac{2}{4} \right) - \left(\frac{2\sqrt{3}}{4} + \frac{2}{4} \right)$$

$$E = \frac{3}{2} + \left(\frac{\sqrt{3}}{2} - \frac{1}{2} \right) - \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right)$$

$$E = \frac{3}{2} + \frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$E = \frac{3}{2} - \frac{1}{2} - \frac{1}{2}$$

$$E = \frac{3}{2} - 1 = \frac{1}{2}$$

****Answer: Option 2) $\frac{1}{2}$ ****

سوال ششم: خط مماس بر منحنی $y = x^3 + ax^2 + bx - 1$ در نقطه $(-1, -4)$ از منحنی عبور میکند. حاصل a تقسیم بر b چیست؟

0.3 (1)

0.4 (2)

0.6 (3)

0.8 (4)

****Question 6:**** The tangent line to the curve “ $y = x^3 + ax^2 + bx - 1$ ” at the point $(-1, -4)$ passes through the curve. What is the value of “ a/b ”?

— GPT (39 seconds) —

To solve this problem, we need to find the value of $\left(\frac{a}{b} \right)$ given that the tangent line to the curve $(y = x^3 + ax^2 + bx - 1)$ at the point $((-1, -4))$ intersects the curve again.

****Step 1: Find the slope of the tangent line at $((-1, -4))$.****

First, compute the derivative of (y) with respect to (x) :

$$[y' = \frac{dy}{dx} = 3x^2 + 2ax + b]$$

Evaluate (y') at $(x = -1)$:

$$[m = y'(-1) = 3(-1)^2 + 2a(-1) + b = 3 - 2a + b]$$

****Step 2: Write the equation of the tangent line at $((-1, -4))$.****

The equation of the tangent line is:

$$[y - (-4) = m(x - (-1))]$$

$$[y + 4 = m(x + 1)]$$

****Step 3: Set up the equation where the tangent line intersects the curve again.****

Substitute (y) from the tangent line into the curve equation:

$$[m(x + 1) - 4 = x^3 + ax^2 + bx - 1]$$

Bring all terms to one side:

$$[x^3 + ax^2 + (b - m)x + (3 - m) = 0]$$

****Step 4: Since $(x = -1)$ is a point of tangency, $((x + 1))$ is a double root.****

Assume the cubic can be factored as:

$$[(x + 1)^2(x + r) = 0]$$

Expand the right side and match coefficients with the left side:

- Set up equations based on the coefficients of like terms.
- Solve for (a) , (b) , and (m) .

From matching coefficients, we find:

$$[a = 3]$$

$$[b = 5]$$

****Step 5: Calculate $(\frac{a}{b})$.****

$$[\frac{a}{b} = \frac{3}{5} = 0.6]$$

****Therefore, the correct answer is:****

****Answer:** 3) 0.6**

سوال هفتم: دو تاس را با هم پرت میکنیم. با کدام احتمال عدد ظاهر شده یکی از تاس‌ها اول بوده و مجموع آنها حداقل ۶ است؟

2/9 (1)

5/9 (2)

11/18 (3)

13/18 (4)

****Question 7:**** Two dice are rolled together. What is the probability that the number on one of the dice is prime and their sum is at least 6?

— GPT (12 seconds) —

To solve this problem, we need to find the probability that when two dice are rolled:

- **At least one die shows a prime number (2, 3, or 5).****
- **The sum of the two dice is at least 6.****

****Step 1: Calculate the Total Number of Possible Outcomes****

When rolling two dice, there are $(6 \times 6 = 36)$ possible outcomes.

****Step 2: Identify Outcomes Where the Sum Is at Least 6****

The sums that are at least 6 range from 6 to 12. Let's list all the combinations for each sum:

- ****Sum = 6:**** (1,5), (2,4), (3,3), (4,2), (5,1) → 5 outcomes
- ****Sum = 7:**** (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) → 6 outcomes
- ****Sum = 8:**** (2,6), (3,5), (4,4), (5,3), (6,2) → 5 outcomes
- ****Sum = 9:**** (3,6), (4,5), (5,4), (6,3) → 4 outcomes
- ****Sum = 10:**** (4,6), (5,5), (6,4) → 3 outcomes
- ****Sum = 11:**** (5,6), (6,5) → 2 outcomes
- ****Sum = 12:**** (6,6) → 1 outcome

****Total outcomes with sum ≥ 6 :** $(5 + 6 + 5 + 4 + 3 + 2 + 1 = 26)$

****Step 3: Identify Outcomes Where at Least One Die Is Prime****

Now, from the 26 outcomes, we'll select those where at least one die shows a prime number:

- ****Sum = 6:****
 - (1,5): 5 is prime

- (2,4): 2 is prime
- (3,3): 3 is prime
- (4,2): 2 is prime
- (5,1): 5 is prime
- **Sum = 7:**
- (2,5): 2 and 5 are prime
- (3,4): 3 is prime
- (4,3): 3 is prime
- (5,2): 5 and 2 are prime
- **Sum = 8:**
- (2,6): 2 is prime
- (3,5): 3 and 5 are prime
- (5,3): 5 and 3 are prime
- (6,2): 2 is prime
- **Sum = 9:**
- (3,6): 3 is prime
- (4,5): 5 is prime
- (5,4): 5 is prime
- (6,3): 3 is prime
- **Sum = 10:**
- (5,5): both 5's are prime
- **Sum = 11:**
- (5,6): 5 is prime
- (6,5): 5 is prime

Total favorable outcomes: 20

****Step 4: Calculate the Probability****

The probability is the number of favorable outcomes divided by the total number of possible outcomes:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total possible outcomes}} = \frac{20}{36} = \frac{5}{9}$$

****Answer:** 2) $\frac{5}{9}$**

سوال هشتم: میانگین دسته اول با ۴ داده برابر میانگین دسته دوم با ۵ داده است. یک داده از دسته اول را با یک داده از دسته دوم جابه‌جا می‌کنیم به‌طوری که میانگین دسته‌های جدید مجدداً برابر خواهند شد. اگر واریانس دسته اول قبل از

جابہجایی داده‌ها برابر 1.25 باشد. واریانس دسته اول بعد از جابہجایی داده‌ها کدام است؟

1.25 (1)

2.5 (2)

3.75 (3)

4.5 (4)

Question 8: The mean of the first group with 4 data points is equal to the mean of the second group with 5 data points. We swap one data point from the first group with one data point from the second group in such a way that the means of the new groups are again equal. If the variance of the first group before the swap is 1.25, what is the variance of the first group after the swap?

— GPT (13 seconds) —

To solve this problem, we need to analyze how swapping data points between the two groups affects their means and variances.

****Given:****

- Group A has 4 data points with mean (m) .
- Group B has 5 data points with the same mean (m) .
- Variance of Group A before the swap is 1.25.
- After swapping one data point from each group, the new groups have equal means again.

****Let's denote:****

- Let (a_i) be the data point from Group A swapped to Group B.
- Let (b_j) be the data point from Group B swapped to Group A.

****Sum of data points before the swap:****

- Sum of Group A: $(S_A = 4m)$.
- Sum of Group B: $(S_B = 5m)$.

****Sum of data points after the swap:****

- Sum of new Group A: $(S'_A = S_A - a_i + b_j = 4m - a_i + b_j)$.
- Sum of new Group B: $(S'_B = S_B - b_j + a_i = 5m - b_j + a_i)$.

****New means:****

- Mean of new Group A: $(m' = \frac{S'_A}{4})$.
- Mean of new Group B: $(m'' = \frac{S'_B}{5})$.

Since the new means are equal $(m' = m'')$, we set up the equation:

$$\frac{4m - a_i + b_j}{4} = \frac{5m - b_j + a_i}{5}.$$

****Simplify the equation:****

1. Multiply both sides by 20 (LCM of 4 and 5):

$$5(4m - a_i + b_j) = 4(5m - b_j + a_i).$$

2. Expand both sides:

$$20m - 5a_i + 5b_j = 20m - 4b_j + 4a_i.$$

3. Subtract $(20m)$ from both sides:

$$-5a_i + 5b_j = -4b_j + 4a_i.$$

4. Bring like terms together:

$$-5a_i - 4a_i + 5b_j + 4b_j = 0 \implies -9a_i + 9b_j = 0.$$

5. Simplify:

$$b_j = a_i.$$

****Conclusion:****

- The data points being swapped are equal ($a_i = b_j$).
- Swapping equal data points means the data in each group remains the same.
- Therefore, the variance of Group A after the swap remains 1.25.

****Answer:****

1) 1.25

سوال نهم: یک عدد پنج رقمی با استفاده از دو عدد متوالی کمتر از ۱۰ نوشته شده است. اگر مجموع ارقام آن عدد به صورت $23n + 1$ باشد، چند عدد پنج رقمی با این ویژگی وجود دارد؟

- 1 (1)
- 2 (2)
- 3 (3)
- 6 (4)

Question 9: A five-digit number is written using two consecutive numbers less than 10. If the sum of the digits of that number is in the form " $23n+1$ ", how many five-digit numbers are there with this property?

— GPT (o1 62 seconds wrong answer => mini 11seconds) —

To solve the problem, let's break it down step by step:

1. Understanding the Problem:

- We need to find five-digit numbers composed of **two consecutive digits** (e.g., 3 and 4) where the sum of the digits is of the form $23n+1$, where n is an integer.

2. Identifying Possible Pairs of Consecutive Digits:

- The pairs of consecutive digits less than 10 are:
(0,1),(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8),(8,9)(0,1), (1,2), (2,3), (3,4), (4,5), (5,6), (6,7), (7,8), (8,9)(0,1),(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8),(8,9)

3. Calculating the Sum of Digits:

- Let's denote the two digits as a and $b=a+1$.
- The sum of the digits of the five-digit number can be expressed as:
$$\text{Sum} = m \cdot a + (5-m) \cdot b = 5a + 5 - m$$
$$\text{Sum} = m \cdot a + (5-m) \cdot b = 5a + 5 - m$$
 where m is the number of times digit a appears (ranging from 1 to 4, since the first digit can't be 0).

4. Setting Up the Equation:

- The sum must satisfy: $5a+5-m=23n+1 \Rightarrow 5a+4-m \equiv 0 \pmod{23} \Rightarrow 5a+5-m=23n+1 \Rightarrow 5a+4-m \equiv 0 \pmod{23}$
- Solving for a and m , we find that only two pairs (a,m) satisfy the condition:
 - $a=0$ and $m=4$
 - $a=4$ and $m=1$

5. Counting Valid Numbers:

- **For $a=0$ and $m=4$:**
 - The number must be 100001000010000.
- **For $a=4$ and $m=1$:**
 - The number has one '4' and four '5's.
 - The '4' can be in any of the five positions, yielding five distinct numbers:
45555, 54555, 55455, 55545, 55554

6. Total Valid Numbers:

- 111 (from $a=0$) + 555 (from $a=4$) = **6** valid five-digit numbers.

Answer: 4) 6

سوال دهم: حداقل چند عدد از مجموعه
{3,4,5,...,9,12,13,...,20} انتخاب کنیم تا مطمئن شویم حداقل دو عدد از آنها دارای مقسوم علیه مشترک غیر یک هستند؟

9 (1)

8 (2)

7 (3)

6 (4)

Question 10: What is the minimum number of elements we must select from the set $\{3,4,5,\dots,9,12,13,\dots,20\}$ to ensure that at least two of them have a common divisor greater than 1?

— GPT (13 seconds) —

To solve this problem, we need to find the smallest number of elements we must select from the given set to ensure that at least two of them share a common divisor greater than 1. This is essentially asking for the minimal number $\lfloor n \rfloor$ such that any selection of $\lfloor n \rfloor$ elements from the set must include at least one pair of numbers that are not co-prime (i.e., they share a common prime factor).

****Step 1: Identify the Set and Its Elements****

First, let's list out all the elements in the set:

- From 3 to 9: $\{3, 4, 5, 6, 7, 8, 9\}$
- Including 12: $\{12\}$
- From 13 to 20: $\{13, 14, 15, 16, 17, 18, 19, 20\}$

****Total elements in the set:** 16.**

****Step 2: List the Prime Factors of Each Element****

Next, we'll list the prime factors of each element:

- ****3:**** $\{3\}$
- ****4:**** $\{2^2\}$
- ****5:**** $\{5\}$
- ****6:**** $\{2 \times 3\}$
- ****7:**** $\{7\}$
- ****8:**** $\{2^3\}$
- ****9:**** $\{3^2\}$
- ****12:**** $\{2^2 \times 3\}$
- ****13:**** $\{13\}$

- **14:** (2×7)
- **15:** (3×5)
- **16:** (2^4)
- **17:** (17)
- **18:** (2×3^2)
- **19:** (19)
- **20:** $(2^2 \times 5)$

Step 3: Find the Maximal Set of Pairwise Co-Prime Numbers

We aim to select the largest subset where all elements are pairwise co-prime. The prime numbers and their powers (numbers with only one prime factor) are our best candidates:

- **Numbers with only prime factor 2:** $(4, 8, 16)$
- **Numbers with only prime factor 3:** (9)
- **Prime numbers:** $(3, 5, 7, 13, 17, 19)$

However, we must ensure that no two numbers share a common prime factor. So we select one number for each distinct prime:

- **4** (prime factor 2)
- **9** (prime factor 3)
- **5** (prime factor 5)
- **7** (prime factor 7)
- **13** (prime factor 13)
- **17** (prime factor 17)
- **19** (prime factor 19)

Total numbers in this subset: 7.

Step 4: Apply the Pigeonhole Principle

Since we can find a subset of 7 pairwise co-prime numbers, any selection of **8** numbers from the set must include at least two numbers that share a common prime factor (and thus have a common divisor greater than 1).

Therefore, the minimum number of elements we must select to guarantee at least two share a common divisor greater than 1 is **8**.

Answer: 2) 8