Mobile Robot Position Determination Using Data Integration of Odometry and Gyroscope

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MOBILE ROBOT POSITION DETERMINATION USING DATA INTEGRATION OF ODOMETRY AND GYROSCOPE

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ABSTRACT

The objective is to accurately determine mobile robots position and orientation by integrating information received from odometry and an inertial sensor. The position and orientation provided by odometry are subject to different types of errors. To improve the odometry, a fiber optic gyroscope is used to give the orientation information that is more reliable. The information from odometry and gyroscope are integrated using Unscented Kalman Filter (UKF). The position and orientation determined based on the UKF are compared with the results obtained from the commonly used Extended Kalman Filter (EKF).

KEYWORDS: Mobile Robots, Sensor Integration, Position, Unscented Kalman Filter

1. INTRODUCTION

Mobile robot's position determination has been the subject of many studies [1-15,21,23-24,26-31]. Usually a mobile robot's basic positioning system is odometry. The odometry is based on information received from the robot's wheel encoders. Inequality of wheel diameters and wheel slippage are potential causes of odometry errors. In literature [4], these errors are categorized as systematic (e.g. different wheel diameters), and nonsystematic (e.g. wheel slippage) errors. To improve position determination based on odometry, different approaches have been proposed.

One of the most referred techniques is proposed by Bornstein and Feng [6, 7]. The technique intends to correct odometry errors by utilizing a testbench called UMBmark. In the UMBmark, the robot is run on a square path several times. The robot's final position and orientation errors are calculated. Two correction factors are derived based on the calculated errors and the robot geometrical relationships. The derived correction factors are used to estimate the robot position more accurately.

Utilizing another sensor along with odometry is another approach [4,5,9-11]. Bronstein and Feng [5] introduced a method called "Gyrodometry". The approach intends to correct odometry errors when the robot is traversing a bump. The robot position and orientation are calculated based on odometry and gyro. The main positioning system is odometry and the gyro is a standby positioning system. While the robot is traversing over the bump, the odometry fails to provide accurate position and orientation. Therefore, the robot's position and orientation is calculated utilizing the gyro. After traversing over the bump, the odometry is reset by the gyro positioning system. From this point, the odometry will again be utilized as the main positioning system. The gyro data is corrected for drift error utilizing Barshan and Whyte's approach [1].

Another approach is to estimate the robot position by integrating information from multiple sources using different versions of Kalman Filter [1-3,12-15,21,23,24,26,27,29-31]. Barshan and Whyte [1-3] developed an inertial navigation system for guidance of an outdoor mobile robot. A triaxial gyroscope, a triaxial accelerometer, and two electro-level tilt sensors are utilized to calculate the robot's position and orientation. The inertial sensors' drift errors are modeled as exponential functions with respect to time. An Extended Kalman Filter (EKF) is used to estimate the position and orientation of the mobile robot using inertial system outputs. The results indicate that the EKF improved the error on the orientation by factor of five.

Some of the previously discussed approaches either need to be calibrated frequently or to be applied in specific cases. The odometry errors are nonlinear in essence. Previous studies utilized EKF to estimate the odometry's errors. The EKF is a crude approach for approximating nonlinear system through first order linearization (a Taylor series expansion truncated at the first order) of the nonlinear function. This can introduce substantial errors in the estimates of the mean and covariance of the transformed distribution, which can lead to sub-optimal performance and even divergence of the filter. The new version of Kalman filter, the UKF uses Unscented Transformation (UT) which approximates a probability distribution using a small number of carefully chosen test points. These test points are propagated through the true nonlinear system, and allow estimation of the posterior mean and covariance accurate to the third order of any filter [16,19]. The UKF is more accurate and simpler than the EKF applied to nonlinear systems [17,18,32]. It is also shown that the UKF gives better results than the EKF when it is used to improve SLAM (simultaneous localization and mapping) [25].

The method introduced in this paper is to integrate the robot's position and orientation based on an inertial sensor and odometry using the UKF. This method is simple to implement, needless to frequent calibration and applicable to different situations. Mobile robot position and orientation determination based on odometry and gyroscope is presented in Sections 2.1 and 2.2, respectively. In Section 3, the integration of sensory information based on the UKF is explained. The experimental results and conclusions follow in Sections 4 and 5, respectively.

2. POSITION DETERMINATION

To calculate more accurate position and orientation, two positioning systems are utilized. The first and basic positioning system is odometry and the second positioning uses information from a single axis fiber optic gyroscope. The position determination based on odometry and gyroscope is explained next.

2.1. Odometry

The robot's position is defined by $\underline{P}(kT_s) = [\,\underline{p}(kT_s), \psi(kT_s)\,]^T$ at time step kT_s; where k is a positive integer, and T_s is the sampling period. For simplicity T_s is dropped off from notation after this point and the mobile robot is assumed to be traveling on a flat surface. The vector $\underline{p}(k) = [p_x(k), p_y(k)]^T$ specifies the robot's Cartesian position, and $\psi(k)$ defines the robot orientation. In order to characterize the mobile robot's motion, two coordinate frames are defined. The body coordinate frame (x_b, y_b), as is shown in figure 1, is selected so that x is forward and y is lateral with origin located at the robot's kinematics center. The world coordinate frame (x_w, y_w), is stationary and attached to the initial position of the robot. Initially, the body coordinate frame coincides with the world coordinate frame. As shown in figure 1, the position vector $\underline{P}(k)$ is updated during mobile robot motion with respect to the world coordinate frame.

The robot position vector on odometry is estimated incrementally by

$$^{o}\underline{p}(k+1) = {}^{o}\underline{p}(k) + R_{\psi o}(k)\underline{v}^{b}(k) \tag{1}$$

$$\psi_o(k+1) = \psi_o(k) + \dot{\psi}_o(k)T_s \tag{2}$$

where $\underline{v}^b(k) = [v_x^b(k), v_y^b(k)]^T$ is the robot velocity in the body coordinate frame, $R_{\psi_o}(k)$ denotes the rotational transformation matrix from the body coordinate frame to the world coordinate frame, $\psi_o(k)$ and $\dot{\psi}_o(k)$ are the robot orientation and its angular velocity.

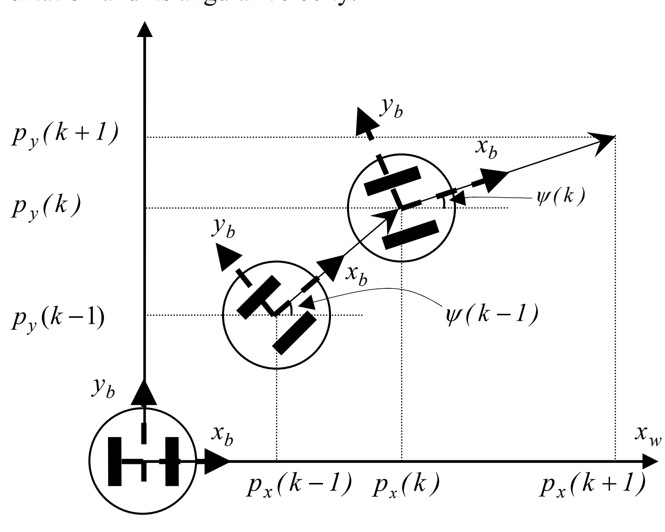

Figure 1. Mobile robot position with respect to the world coordinate frame

The $R_{\psi_o}(k)$ is specified by

$$R_{\psi o}(k)=\begin{bmatrix} cos\psi_o(k) & -sin\psi_o(k) \\ sin\psi_o(k) & cos\psi_o(k) \end{bmatrix} \tag{3}$$

The velocity component $v_y^b(k)$ is assumed to be zero because of the robot's forward motion. The $v_x^b(k)$ and $\psi_o(k)$ are calculated by

$$v_x^b(k)=\frac{1}{2}(v_{er}(k)+v_{el}(k)) \tag{4}$$

$$\dot{\psi}_o(k)=\frac{v_{er}(k)-v_{el}(k)}{b} \tag{5}$$

where b is the robot wheelbase, $v_{er}(k)$ and $v_{el}(k)$ is the robot's right and left wheels translational velocities, respectively. The $v_{er}(k)$, and $v_{el}(k)$ are known from reading the encoder values.

2.2. Inertial System

A single axis fiber optic gyroscope is used to determine the robot orientation. Gyros measure rotational rate, which can be integrated to yield orientation. The measurements taken from angular rate output of the gyros is represented by

$$\dot{\psi}_G(k)=\dot{\psi}_G^a(k)+e_G(k)+\eta_G(k) \tag{6}$$

where $\dot{\psi}_G^a(k)$ is the robot's actual heading rate, $\dot{\psi}_G(k)$ is the robot's heading rate based on the gyro's reading, $e_G(k)$ is the gyroscope bias drift error, and $\eta_G(k)$ is the associated white noise. The drift error is modeled with an exponential curve and determined experimentally [1]. The robot's actual orientation based on gyro reading is calculated by

$$\psi_G^a(k+1)=\psi_G^a(k)+\dot{\psi}_G^a(k)T_s \tag{7}$$

The position ${}^G\underline{p}(k+1)$ in the world frame based on gyro readings is estimated by

$${}^G\underline{p}(k+1)={}^G\underline{p}(k)+\begin{bmatrix} cos\psi_G^a(k) & -sin\psi_G^a(k) \\ sin\psi_G^a(k) & cos\psi_G^a(k) \end{bmatrix}\underline{v}^b(k) \tag{8}$$

The integration of information received from odometry and the gyroscope using the UKF is discussed in the next section.

3. UNSCENTED KALMAN FILTER

As was mentioned in introduction, previous research in the field of mobile robots used the EKF to determine the robot's position and orientation by integrating information from multiple sensors [1-3,12-15,23,24,26,27,29-31]. The UKF achieves better performance than the EKF in dealing with nonlinear problems [16-19, 32]. Figure 2 shows the block diagram of the approach.

To formulate the UKF [32], the state variables are defined as

$$\underline{E}(k)=\begin{bmatrix} {}^Gp_x(k)-{}^op_x(k) \\ {}^Gp_y(k)-{}^op_y(k) \\ \psi_G^a(k)-\psi_o(k) \end{bmatrix}=\begin{bmatrix} e_x(k) \\ e_y(k) \\ e_\psi(k) \end{bmatrix} \tag{9}$$

$$\underline{E}(k)=\underline{E}_m(k)+\underline{\eta}(k)=\begin{bmatrix} e_{mx}(k) \\ e_{my}(k) \\ e_{m\psi}(k) \end{bmatrix}+\begin{bmatrix} \eta_x(k) \\ \eta_y(k) \\ \eta_\psi(k) \end{bmatrix} \tag{10}$$

where $\underline{E}_m(k)$ represents the modeled part of the predefined data differences and $\underline{\eta}(k)$ is a zero mean white noise. Using equations (1)-(4) and (7)-(8), the following recursive equation is derived

$$\underline{E}(k+1)=\underline{E}(k)+D(k)\begin{bmatrix} v_x^b(k) \\ v_y^b(k) \\ \dot{\psi}_G^a(k)-\dot{\psi}_o(k) \end{bmatrix}=f\left\{\underline{E}(k),\underline{v}^b(k),\psi_G^a(k),\psi_o(k)\right\} \tag{11}$$

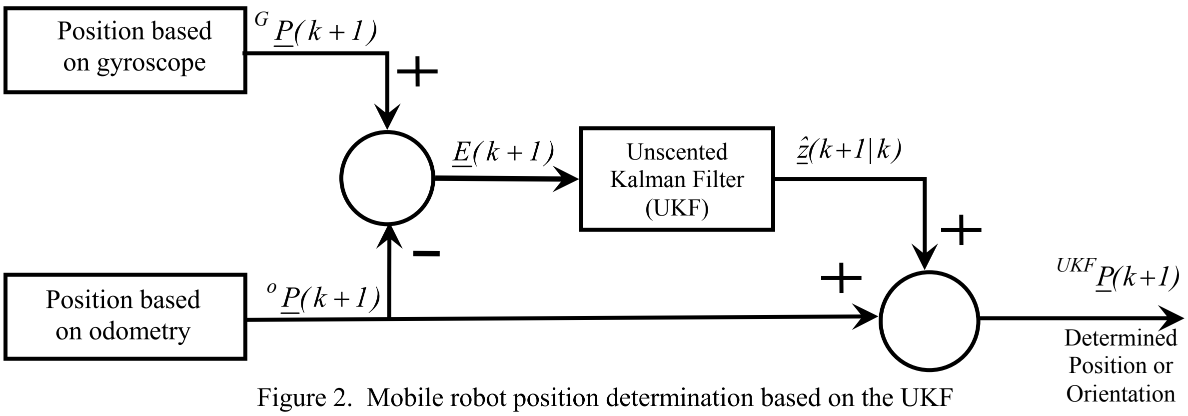

Figure 2. Mobile robot position determination based on the UKF

Where $f(k)$ is the input vector function and $D(k)$ is defined by

$$D(k)=\begin{bmatrix} \cos\psi_G^a(k)-\cos\psi_o(k) & -\sin\psi_G^a(k)+\sin\psi_o(k) & 0 \\ \sin\psi_G^a(k)-\sin\psi_o(k) & \cos\psi_G^a(k)-\cos\psi_o(k) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

The output vector function is specified by

$$\underline{Z}(k)=\underline{E}(k)=h\left\{\underline{E}(k),\underline{v}^b(k),\psi_G^a(k),\psi_o(k)\right\} \tag{13}$$

The vectors $\hat{\underline{X}}^a(k|k)$, its mean $\hat{\underline{X}}(k|k)$ and covariance $P^a(k|k)$ are defined as

$$\hat{\underline{X}}^a(k|k)=\begin{bmatrix} e_{mx}(k) \\ e_{my}(k) \\ e_{m\psi}(k) \end{bmatrix} \tag{14}$$

$$\hat{\underline{X}}(k|k)=E\left[\hat{\underline{X}}^a(k|k)\right] \tag{15}$$

$$P^a(k|k)=cov(\hat{\underline{X}}^a(k|k))+cov(\underline{\eta}(k)) \tag{16}$$

The translated sigma points are calculated as

$$\underline{\chi}_0(k|k)=\hat{\underline{X}}(k|k) \tag{17}$$

$$\underline{\chi}_i(k|k)=\sigma_i^a(k|k)+\hat{\underline{X}}(k|k) \tag{18}$$

where $\sigma_i^a(k|k)$ is the *i*-th column of matrix $\pm\sqrt{(I_3+\lambda)P^a(k|k)}$, I_3 is an 3×3 identity matrix and λ is a 3×3 diagonal matrix which indicates filter's scaling factors. The diagonal elements of λ are defined as

$$\lambda_{jj}=\alpha_j^2(1+\kappa_j)-1,\ j=1,2,3 \tag{19}$$

where α_j, $j=1,2,3$ determines the spread of sigma points around the corresponding state variable of $\hat{\underline{X}}^a(k|k)$ and κ_j is the secondary scaling parameter which usually set to zero. The predicted mean for each sigma point is calculated as follow

$$\underline{\chi}_i(k+1|k)=f\left\{\underline{\chi}_i(k|k),\underline{v}^b(k),\psi_G^a(k),\psi_o(k),k\right\} \tag{20}$$

The priori state estimation in each step is calculated as follow

$$\underline{\hat{x}}(k+1|k)=\sum_{i=0}^{2}\underline{W}_i^{(m)}\underline{\chi}_i(k+1|k) \tag{21}$$

where $\underline{W}_i^{(m)}$, $i=0,1,2$ are weighting factors defined as 3×3 diagonal matrices for the UKF. The diagonal elements of these matrices are

$$W_{0(jj)}^{(m)}=\lambda_{jj}/(1+\lambda_{jj}),\ j=1,2,3 \tag{22}$$

$$W_{i(jj)}^{(m)}=W_{i(jj)}^{(c)}=1/\{2(1+\lambda_{jj})\},\ i=1,2,\ j=1,2,3 \tag{23}$$

The predicted observation is determined by

$$\underline{\hat{z}}(k+1|k)=\sum_{i=0}^{2}W_i^{(m)}\underline{\hat{Z}}_i(k+1|k) \tag{24}$$

where $\underline{\hat{Z}}_i(k+1|k)$ are instantiations of projected sigma points for each variable and calculated by

$$\underline{\hat{Z}}_i(k+1|k)=h\{\underline{\chi}_i(k+1|k),\dot{\psi}_G^a(k),\dot{\psi}_o(k),k\}=\underline{\chi}_i(k+1|k) \tag{25}$$

The results calculated in equation (24) are used to estimate the robot position as

$$^{UKF}\underline{P}(k+1)={}^{o}\underline{P}(k+1)+\underline{\hat{z}}(k+1|k) \tag{26}$$

where $^{UKF}\underline{P}(k)$ is the corrected position using UKF along the corresponding axis.

4. EXPERIMENTAL RESULTS

In order to verify the applicability of the approach, two sets of experiments are conducted. The experimental set-up includes a commercially available Pioneer 2-Dxe mobile robot [28] and a KVH E-core gyroscope [22], which has the input rate of ±30 °/sec and the resolution of 0.014 °/sec. The software program is written in VC++ to read the odometry information, C++ program to read the gyro output, and Matlab to determine position of the mobile robot based on the UKF and the EKF.

The UKF approach is implemented utilizing real-time information from odometry and gyroscope as presented in Section 3. The optimum values used for coefficients α_x,α_y and α_ψ in equation (19) are 1.5, 2.05 and 2.0, respectively and the κ_j coefficients are chosen as 0.001. These optimum values are chosen such that they provide the best estimates for robot position and orientation for all experiments [32]. The position and orientation of the robot is estimated by UKF using equation (26). The EKF approach implemented for comparison is the same as the work in [1].

The experimental emphasis is on the robot's final position and orientation due to ease of taking accurate measurements. Position and orientation errors for each run are defined as the differences between the actual position and orientation and the calculated position and orientation for each positioning system. The common approach used for result presentation of each experiment, include a table and two figures. The table shows the MSE (Mean Square Error) for each positioning system, and figures show the robot position error and the orientation error for each run. The results for the experiments conducted are presented next.

4.1. Experiment-1

In this experiment, the effect of unequal wheel diameters on robot position and orientation is investigated to exaggerate the odometry systematic errors for a short run. For this purpose, the robot's left wheel has a slightly lower air pressure than the right wheel. The robot's travel path in this experiment is to move forward three meters, stop and then immediately turn 180^o, move forward again three meters, and then turn 180^o. As shown in table-1, the UKF provide more accurate position and orientation which is better than the EKF. Figures 3, and 4 show that the odometry provides erroneous information about robot position and orientation as expected.

ERRORS	UKF	EKF	GYRO	ODOMETRY
x-axis (mm)	15.6	19.6	12.8	222
y-axis (mm)	33.5	56.8	34.6	778
Position MSE(mm)	36.9	60.1	36.9	809
Heading (^{o})	0.65	1	1.6	29

Table 1. Average of final position and orientation errors (ten runs)

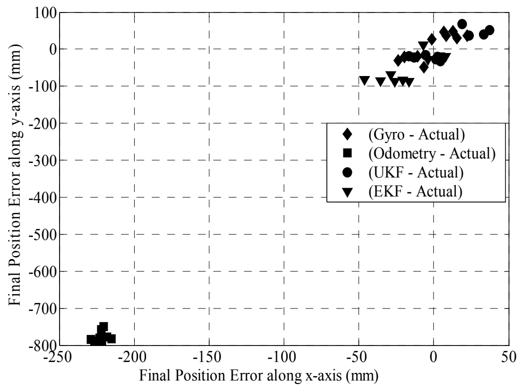

Figure 3. Robot's final position for ten runs

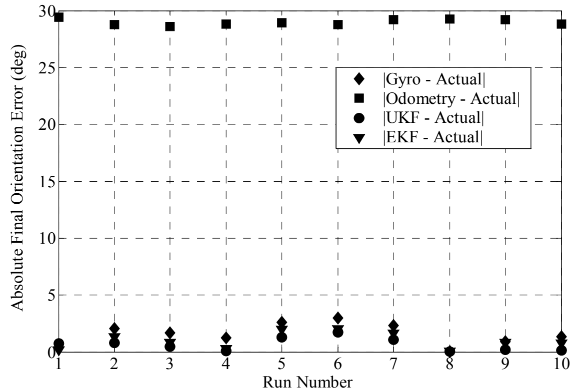

Figure 4. Robot's final orientation error

4.2. Experiment-2

In this experiment, the effect of the mobile robot wheels sliding is examined. The robot travels a distance of three meters forward. The robot has equalized wheel air pressure but in the middle of the path, a sliding surface is placed on the floor. Upon traversing over the sliding surface, which is a sheet of paper, the surface is rotated manually along the floor's normal vector to simulate the wheel slippage. The simulated slippage, changes the robot orientation that will not be detected by the odometry and will contribute to the odometry's nonsystematic error. This experiment repeated for ten times.

As shown in table-2, the UKF provides the best estimation compare to the other positioning systems for robot position along x- and y-axes. The robot orientation based on gyro, the EKF and UKF are almost the same result and are much better than the odometry. Figures 5 and 6 show that the UKF is a better tool to estimates the robot position and orientation for all individual runs.

ERRORS	UKF	EKF	GYRO	ODOMETRY
x-axis (mm)	8.4	84.8	77.9	35.5
y-axis (mm)	9.4	39.1	33.6	639.1
Position MSE (mm)	12.6	89.7	84.9	640.1
Heading(^{o})	0.58	0.59	0.58	17.3

Table 2. Average of final position and orientation errors (ten runs)

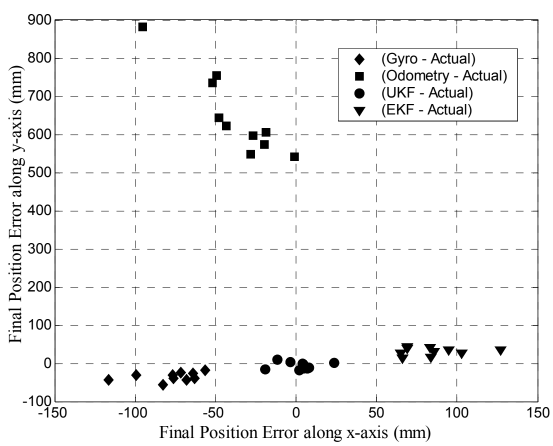

Figure 5. Robot's final position for ten runs

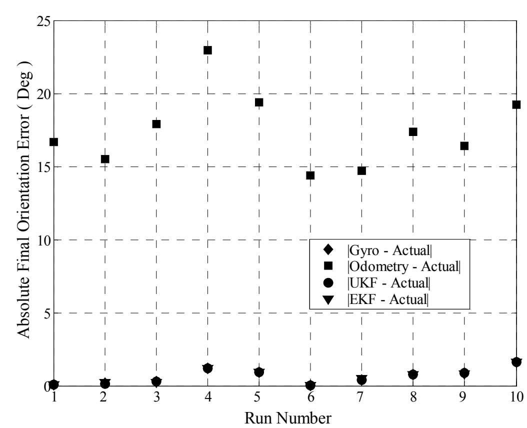

Figure 6. Robot's final orientation error for ten runs

5. CONCLUSIONS

The robot position and orientation are estimated by integrating information from gyro and odometry. In this research, the UKF is used to integrate data from gyro and odometry. In order to validate the applicability of the approach, two sets of experiments are designed and executed. The experiments intend to examine the approach in estimating the odometry's systematic, nonsystematic errors. The results show that the UKF estimates the robot's position and orientation more accurately than the previously used approach of EKF.

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