References with abstracts for QWIM project: Retirement portfolios and strategies in quantitative wealth and investment management

Cristian Homescu

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Abstract

This document includes the list of references (incuding abstracts) for this QWIM project

Contents

1 Motivation for the project

A successful investment process for retirement incorporates many components, including:

- savings and earnings
- asset allocation and location
- goals-based investing
- market returns
- taxation
- longevity
- health care costs
- long-term costs
- inflation
- Social Security
- accumulation and decumulation

2 Relevant references

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Abeliansky, A. and Strulik, H. (2021). "Health and Aging Before and after Retirement." In: *SSRN e-Print*.

We investigate health and aging before and after retirement for specific occupational groups. We use five waves of the Survey of Health, Aging, and Retirement in Europe (SHARE) and construct a frailty index for elderly men and women from 10 European countries. Occupational groups are classified according to low vs. high education, blue vs. white collar color, and high vs. low physical or psychosocial job burden. Controlling for individual fixed effects, we find that, regardless of the used classification, workers from the first (low status) group display more health deficits at any age and accumulate health deficits faster than workers from the second (high status) group. We instrument retirement by statutory retirement ages ("normal" and "early") and find that the health of workers in low status occupations benefits greatly from retirement, whereas retirement effects for workers in high status occupations are small and frequently insignificant. We also find that workers from low status occupations always have higher health deficits, i.e. we find evidence for an occupational health gradient that widens with increasing age, before and after retirement.

Ackerley, A., Nefouse, N., and Nikles, D. (2022). *To spend or not to spend?* Tech. rep. BlackRock.

This was not what we expected to find: on average across all wealth levels, most current retirees still had 80% of their pre-retirement savings after almost two decades of retirement according to research conducted jointly with the Employee Benefit Research Institute (EBRI). One-third even grew their assets over the course of retirement. While that sounds like good news, the reasons why some retirees did not spend their assets may be complicated. Certainly some may have benefited from greater access to defined benefit pensions, more income replacement from Social Security, strong real estate appreciation and an investment market that generally delivered strong returns and high interest rates. On the other hand, some retirees may not have spent their savings due to uncertainty, perhaps even hoarding assets at the expense of fully supporting the retirement lifestyle they might have enjoyed. In order to better understand why retirees either spent down assets or held on to them, BlackRock engaged Greenwald and Associates to conduct 19 in-depth interviews and survey 1,510 retirees.

Interviews for the survey portion of this study were conducted online in February 2018. To qualify for this study participants had to be fully retired for at least five years are between the ages of 60 and 80, and have investable assets between $200,000 and $2,500,000. Survey results have been weighted to reflect the overall make up of Americans retired for five years or longer. In a similarly sized random sample survey of general population respondents, the margin of error (at the 95% confidence level) for the total population in this study would be plus or minus approximately 2.5 percentage points.

We identified six key themes and drivers of retiree spending behavior that emerged from the findings and offer "lessons learned" advice for participants and retirees, employers and advisors.

1) Retirees prefer to keep their assets untouched
2) Retirees more often plan to spend consistently - increasing with age
3) Retirees retain their accumulation mindset
4) Retirees with pension income least likely to spend down. Because they don't need to.
5) Men and women approach finances in retirement differently, and mostly for good reason
6) More recent retirees are less optimistic

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Adcock, C., Areal, N., Armada, M. R., Cortez, M. C., Oliveira, B., and Silva, F. (2017). "Portfolio Performance Measurement: Monotonicity with Respect to the Sharpe Ratio and Multivariate Tests of Correlation." In: *SSRN e-Print*.

This paper reports an investigation into methods of portfolio performance measurement. The work is motivated first by equivocal empirical evidence reported by several authors about the correlation of performance measures with the Sharpe ratio. Secondly it is motivated by recent work which specifies that performance measures will be monotone functions of the Sharpe ratio if portfolio returns follow the same location-scale distribution. The paper demonstrates that the class of location-scale distributions is broader than previously reported. It presents conditions under which monotonicity with respect to the Sharpe ratio will fail. The paper shows that for large sample sizes the correlation between pairs of performance measures that are functions of the Sharpe ratio is unity. The correct null hypothesis for tests of correlation is therefore rho=1. Two multivariate tests of this null hypothesis are presented. The new tests are used to carry out of a comprehensive study of performance measurement for a set over ninety UK investment trusts.

Alderson, M. J. and Betker, B. L. (2017). "Does the Benefit of Deferring Social Security Offset the Opportunity Cost to Do So?" In: *Journal of Financial Planning*.
This study examined the extent to which deferring the claiming date for Social Security benefits preserved an individual's tax-deferred retirement assets, thus enhancing the ability to provide for constant real consumption. The analysis incorporated the relevant reductions and credits for early and late claiming, mortality risk, stochastic investment returns, and the availability of longevity insurance. Results provide guidance for financial planners to help clients make the optimal claiming decision. For single and married individuals with substantial savings, the deferral of benefits reduced the risk of retirement asset exhaustion. The results demonstrate the unique nature of the Social Security asset in the retirement portfolio as well as the value of self-insuring against longevity risk.

Alexandrova, M. and Gatzert, N. (2019). "What Do We Know About Annuitization Decisions?" In: *Risk Management and Insurance Review* 22(1), pp. 57–100.
Against the background of aging societies and increasing life expectancies, the protection of individuals from outliving their savings has become increasingly relevant. Annuities represent insurance against longevity risk and can prevent old-age poverty. The aim of this article is to present the current state of theoretical, empirical and experimental evidence with regard to annuitization decisions. Toward this end, we conduct a systematic literature review that includes 89 articles. Based on this, we study welfare effects of mandatory annuitization, annuitization rates and the optimal fraction of wealth to be annuitized, as well as determinants of retirees' choice to annuitize and their impact on annuity demand. Finally, we present possible solutions for overcoming the low uptake of annuities based on its causes. One main result is that behavioral biases in annuitization decisions particularly require considerably more theoretical research and empirical evidence, and that theoretical models already appear to well explain empirically observed annuitization rates.

Ali, Y., Fang, M., Sota, P. A. A., Taylor, S., and Wang, X. (2019). "Social Security Benefit Valuation, Risk, and Optimal Retirement." In: *Risks* 7(4), p. 124.
We develop valuation and risk techniques for the future benefits of a retiree who participates in the American Social Security program based on their chosen date of retirement, the term structure of interest rates, and forecasted life expectancy. These valuation methods are then used to determine the optimal retirement time of a beneficiary given a specific wage history and health profile in the sense of maximizing the present value of cash flows received during retirement years. We then examine how a number of risk factors including interest rates, disease diagnosis, and mortality risks impact benefit value. Specifically, we utilize principal component analysis in order to assess both interest rate and mortality risk. We then conduct numerical studies to examine how such risks range over distinct income and demographic groups and finally summarize future research directions.

Aliaga-Diaz, R., Ahluwalia, H., Zhu, V., Donaldson, S., Daga, A., and Pakula, D. (2021). *Vanguard's Life-Cycle Investing Model (VLCM): A general portfolio framework for goals-based investing*. Tech. rep. Vanguard.
Investors have multiple goals throughout their lifetime, each requiring them to make complex, interconnected decisions about saving, spending, and asset allocation. We present a framework for making asset allocation decisions based on an investor's goals, preferences, and personal circumstances and factoring in the uncertainty of asset returns. The Vanguard Life-Cycle Investing Model (VLCM) is a proprietary model for glide-path construction that can assist in the creation of custom investment portfolios for retirement as well as nonretirement goals, such as saving for college. The VLCM embodies key principles of life-cycle investing theory, including a utility-based framework encompassing risk aversion and time preference. It also incorporates important behavioral finance considerations such as loss aversion and income shortfall aversion. The use of the VLCM enables cost-benefit analysis of glide-path customization, evaluation of risk-return trade-offs of various asset and sub-asset allocation choices, and multiple portfolio analytics of the probability of success and odds of income sufficiency. Based on VLCM's analytical framework, we find that risk-aversion levels are the dominant factor behind the broad stock-bond split in the glide path, affecting both glide-path slope and ending allocation.

Alleva, B. (2016). "Discount Rate Specification and the Social Security Claiming Decision." In: *SSRN e-Print*.
Choosing the claiming age that maximizes the expected present value of lifetime Social Security retirement benefits requires a survival function to account for an individual's prospective longevity along with the specification of a rate by which to discount the future benefit payments for each claiming age. This article evaluates optimal claiming ages for prospective beneficiaries across a range of 81 real discount rate options (specified in increments of one-tenth of 1 percent) from 0 percent to 8 percent, considering the survival functions for men and women born in 1952. It examines the implications of choosing a given rate as well as the sensitivity of the optimal claiming age to a specific rate choice.

Alleva, B. (2017). "Social Security Retirement Benefit Claiming-Age Combinations Available to Married Couples." In: *SSRN e-Print*.
The rules for claiming Social Security retired-worker benefits are complex in large part because they offer a potential claimant flexibility in choosing a claiming age most to his or her advantage. The complexity of those rules is multiplied for a married couple, as the potential eligibility for spousal benefits, the couple's age difference, and other factors must also be considered. The number of claiming-age combinations available to a married couple varies widely for couples with different circumstances. This note explores the claiming rules, contingent situations, claiming-age combinations, and benefit amounts available to married couples across a range of respective birth years and benefit levels based on respective earnings histories.

Alleva, B. J. (2015). "Minimizing the Risk of Opportunity Loss in the Social Security Claiming Decision." In: *The Journal of Retirement* 3(1), pp. 67–86.
The optimal age to claim Social Security benefits is often defined as that which maximizes expected lifetime benefits. Expectations are not always met however, and this optimal claim age is unlikely to be that which would have maximized lifetime benefits received by the eventual age at death. The optimal claim age then carries the potential for an opportunity loss, or the risk of receiving less than the maximum possible, which varies with age at death. This study evaluates the claiming decision by first computing and presenting the maximizing claim age for each possible death age over the retirement horizon, and then determining the actual opportunity loss for each death age for claiming at the otherwise optimal claim age. The potential for opportunity loss over the retirement horizon is then computed for the optimal claim age. If a participant can target a period within the retirement horizon for which the values total lifetime benefits higher than the remainder, the optimal claim age for that target period will entail less risk, or less potential for opportunity loss, than that for optimizing over the entire retirement horizon. Such a strategy could effectively address gaps in a retirement income portfolio.

Alsabah, H., Capponi, A., Ruiz Lacedelli, O., and Stern, M. (2021). "Robo-Advising: Learning Investors Risk Preferences via Portfolio Choices." In: *Journal of Financial Econometrics* 19(2), pp. 369–292.
We introduce a reinforcement learning framework for retail robo-advising. The robo-advisor does not know the investor risk preference but learns it over time by observing her portfolio choices in different market environments. We develop an exploration-exploitation algorithm that trades off costly solicitations of portfolio choices by the investor with autonomous trading decisions based on stale estimates of investor risk aversion. We show that the approximate value function constructed by the algorithm converges to the value function of an omniscient robo-advisor over a number of periods that is polynomial in the state and action space. By correcting for the investor mistakes, the robo-advisor may outperform a stand-alone investor, regardless of the investor opportunity cost for making portfolio decisions.

An, B.-J. and Sachdeva, K. (2021). "Missing the Target? Retirement Expectations and Target Date Funds." In: *SSRN e-Print*.
This paper measures the cost of biased retirement expectations for investors in target-date funds. Using survey data, we show that respondents systematically underestimate their long-run labor participation on average by 4.8 years, with errors having meaningful cross-sectional relationships. We use these insights to build a life-cycle model of target-date funds to measure the costs of biased expectations. Calibrations suggest that errors in expectations compound over time, costing the median respondent 4% of total wealth, equivalent to 0.2% a year. These estimates suggest that the choice architecture of target-date funds should be modified to improve the financial adequacy of future retirees.

Anantharaman, D. and Henderson, D. (2016). "Understanding Pension Liabilities: A Closer Examination of Discount Rates." In: *SSRN e-Print*.
The discount rate represents a critical choice in accounting for corporate defined-benefit pension plans due to the long-term nature of pension liabilities. U.S. GAAP and IFRS mandate the AA corporate bond rate. Their requirement is subject to much debate, with the risk-free rate and the expected return on pension assets (EROA) often proposed as alternatives. We examine which of these rates best fits pension values as perceived by equity- and debt-market participants. For equity values, the EROA rate dominates, while for credit ratings, the AA rate dominates. For financially healthy firms, however, discounting at the EROA produces the best fit for both equity values and credit ratings. In contrast, for firms nearing financial distress, discounting at the AA rate provides the best fit. We also find that the accumulated benefit obligation measure consistently dominates the projected benefit obligation in explanatory power for credit ratings. Overall, we find that market participants adjust GAAP-recognized pension obligations in both amount and discount rate, in a manner that appears consistent with firm-specific economic realities.

Armour, P. and Hung, A. (2017). "Drawing down Retirement Wealth: Interactions between Social Security Wealth and Private Retirement Savings." In: *SSRN e-Print*.
Individual financial planning for retirement in the US is increasingly important, given the trend away from employer-provided defined benefit (DB) plans, the rising Social Security (SS) Full Retirement Age (FRA), and retiring baby boomers. A key financial decision that Americans make is how to draw on their retirement wealth across various sources, including both privately saved retirement funds and SS benefits. For SS retirement benefits, the main decision is at what age to claim, with claiming before the FRA resulting in lower monthly benefits, and claiming later leading to higher benefits. The terms of this tradeoff have changed in recent years: since 2003, the FRA has risen from 65 and will gradually increase to 67 by 2027, representing a drop in the present value of SS benefits. Meanwhile, defined contribution (DC) plans have gained in popularity, presenting retirees with more control over their private retirement wealth. The changing dynamics of both SS wealth and the private retirement decision space underscore the need for examining how individuals make decisions across their entire portfolio of retirement wealth. We use HRS survey data matched to SS administrative data to study how households integrate SS benefits into their general retirement income plans. We find starkly different non-SS retirement asset decumulation patterns across individuals who claim at different ages, with those claiming before the FRA drawing down pension and IRA wealth faster than those who claim at or after the FRA. However, the earliest claimants, those who claim SS retirement benefits exactly at age 62, are a highly heterogeneous group, consisting of both low-income, high expected mortality individuals as well as individuals with high pension holdings. We further find that this earliest claimant group is more likely to have retired and begun decumulating non-SS retirement assets even before age 62; however, this group's median and average assets are not substantially lower than later claimants. An analysis of claiming behavior by non-SS retirement wealth holdings shows that individuals with more retirement savings were overwhelmingly likely to claim between the ages of 62 and the FRA. On the other hand, those with the least retirement savings are more likely to either claim SS benefits as early as possible, either in the form of disability benefits or retirement benefits, or they would delay claiming SS retirement benefits until after the FRA. Moreover, birth cohorts facing higher FRAs tend to delay claiming SS retirement benefits on average; however, those most affected by this reduction in SS wealth - those with few other retirement assets - are the least reactive. Finally, households that do delay claiming as the FRA rises also tend to delay retirement and drawing down their non-SS retirement assets, indicating complementarity between SS and non-SS decumulation decisions and strong spillovers from changes in both SS and private retirement policy.

Armour, P. and Knapp, D. (2021a). *The Changing Picture of Who Claims Social Security Early*. Tech. rep. AARP Public Policy Institute.
From the early 1990s to 2010s, the fraction of individuals claiming Social Security retirement benefits at the early eligibility age (EEA) of 62 declined from over half of the eligible population to less than a third. Although claiming at 62 can make financial sense for some, Social Security benefit rules result in reduced monthly benefits of up to 30 percent relative to waiting to claim benefits until full retirement age. Such a considerable benefit reduction persists for life- and threatens financial security at later ages. This report highlights differences over time between individuals who claim Social Security retirement benefits at age 62 and those who claim after age 62. It documents how these populations have changed in terms of education, labor force participation, pension plan participation, and wealth-and how demographic, employment, and economic characteristics relate to claiming age. This report is the first in a two-part series; the second report focuses on the later-in-life financial and well-being consequences of early claiming.
The authors identify the demographic, employment, and economic characteristics associated with those who claim at the EEA and relate these characteristics to people who claim after it. For both men and women, individuals who claim Social Security benefits at 62, as compared with those who claim later:

1) Have less education, on average;
2) Are more likely to live in rural areas;
3) Are more likely to already have a worklimiting health condition;
4) Have a lower reported likelihood of living to age 75;
5) Are less likely to have a job before turning age 62;
6) If employed before turning age 62, earn less, on average;
7) If employed before turning age 62, are more likely to have a physically demanding job.

With respect to wealth and pension comparisons of age-62 claimers to later claimers, Armour and Knapp find individuals who claim Social Security benefits at 62:

1) Are substantially more likely to already be receiving pension income;
2) Are less likely to have defined contribution plan assets.

The authors caution that even after accounting for the interrelationship of observable demographic, employment, and economic factors using regression analysis, these characteristics account for only a small fraction of the shift toward later claiming. Other reasons-such as longer life expectancy, a greater ability to work longer, better understanding of the financial benefits of waiting to claim Social Security benefits, and stronger incentives to delay claiming due to changes in Social Security benefit rules-likely influence people to wait to claim their benefits.

Finally, although the novel coronavirus pandemic has not yet been associated with increases in claiming Social Security retirement benefits (Goss and Glenn 2020), as short-term furloughs turn into long-term unemployment, the findings in this report suggest that employment losses may lead to earlier claiming-in particular among those with less education and those living in more rural areas. As the second report in this series makes clear, the financial consequences of early claiming can be long lasting.

Armour, P. and Knapp, D. (2021b). *The Consequences of Claiming Social Security Benefits at Age 62*. Tech. rep. AARP Public Policy Institute.

Deciding when to claim Social Security benefits comes with an important tradeoff: claiming benefits at the early eligibility age (EEA) of 62 allows beneficiaries earlier access to their monthly Social Security payments, but these payments will be lower for life than if the beneficiary had waited. Despite the reduction in monthly benefits, almost one-third of people still claim Social Security retirement benefits at the EEA. Delayed claiming is often promoted by financial advisers and retirement experts for its higher monthly benefits (there is an almost 8 percent boost in monthly benefits for each year someone delays claiming). Nevertheless, little is known about whether claiming Social Security benefits at the EEA leads to substantively different short- and long-term outcomes than does claiming later. This report, the second in a two-part series, focuses on the consequences for individuals who claim at the EEA; it estimates both short- and long-term differences between otherwise similar individuals who claim Social Security retirement benefits at age 62 and those who claim later.

KEY FINDINGS In comparing matched age-62 claimants against later claimants with similar characteristics, Armour and Knapp find:

1) Age-62 claimants stop working earlier and cash out defined contribution plans sooner.
2) Age-62 claimants have lower household income throughout their 60s and 70s (on average, $10,000-$20,000 less in annual household income).
3) Age-62 claimants in their 70s have substantially less liquid wealth than do later claimants at similar ages (on average, 27 percent lower at age 74 and 43 percent lower at age 80).
4) Measures of financial hardship and mortality rates are not statistically or substantively different between the two claiming groups into their 70s.

The findings of this report are limited in a couple of significant ways that may underestimate the effects of claiming early. First, given the time frame, HRS data do not allow the authors to track households past age 80, which likely understates the relative advantage of claiming later since the benefits of delayed claiming grow substantially in claimants' 80s. Second, individuals observed in their late 70s claimed in the 1990s or early 2000s-and thus faced a lower penalty for early claiming than more recent cohorts, who were subject to different Social Security benefit rules. In addition, more recent cohorts are living longer; therefore, someone turning 62 today has a stronger incentive to delay claiming, all else equal.

Armour, P. and Knapp, D. (2022). "The consequences of claiming Social Security benefits at age 62." In: *Journal of Pension Economics and Finance*, pp. 1–27.

Delaying claiming of Social Security old-age benefits past the earliest eligibility age, age 62, raises the monthly benefit for a person's life. Despite arguments from both proponents and opponents of delayed claiming in academia and public discourse, little is known about whether claiming decisions lead to substantively different outcomes. We compare differences in outcomes between age-62 claimants and otherwise similar later claimants that are matched on health, employment, and financial characteristics at age 60. We find that age-62 claimers are substantially less likely to work after 62 and have persistently lower income into their 70s. Differences in assets emerged in the 70s, with early claimants having lower wealth, but we find no differences in mortality or

self-reported financial hardship. The difference in wealth is driven primarily by a growth in wealth among later claimants rather than substantial decumulation by age-62 claimants.

Arnold, T., Earl, J. H., Marshall, C. D., and Schwartz, A. (2017). "Excel Calculators for Determining Retirement Accumulation and Disbursement Information." In: *The Journal of Wealth Management* 20(2), pp. 94–101.
This article provides two different retirement calculators to help investors plan for retirement savings and retirement disbursements. The calculators are created in Excel and require minimal programming, which makes them not only practical but also easy to implement. Furthermore, the calculators can address multiple investor scenarios and can be integrated to produce a reasonably thorough retirement plan.

Arnold, T., Earl, J. H., Marshall, C. D., and Schwartz, A. (2018). "Using "Equivalent Tax Rates" to Determine Tax-Efficient Retirement Investment and Withdrawal." In: *The Journal of Wealth Management* 21(2), pp. 55–69.
The authors propose an equivalent tax rate framework to compare the tax-efficient accumulation and withdrawal of retirement funds across multiple investment vehicles. The method allows for the consideration of a given investment vehicle expected return, time horizon, and tax treatment relative to a tax-deferred retirement account receiving pre-tax contributions and possible employer matching of contributions. The metric is comparable across alternative investment accounts irrespective of the specific marginal tax rate in place during the accumulation or distribution stages.

Arnott, R. D., Harvey, C. R., and Markowitz, H. (2019). "A backtesting protocol in the era of machine learning." In: *The Journal of Financial Data Science* 1(1), pp. 64–74.
Machine learning offers a set of powerful tools that holds considerable promise for investment management. As with most quantitative applications in finance, the danger of misapplying these techniques can lead to disappointment. One crucial limitation involves data availability. Many of machine learning early successes originated in the physical and biological sciences, in which truly vast amounts of data are available. Machine learning applications often require far more data than are available in finance, which is of particular concern in longer-horizon investing. Hence, choosing the right applications before applying the tools is important. In addition, capital markets reflect the actions of people, which may be influenced by others actions and by the findings of past research. In many ways, the challenges that affect machine learning are merely a continuation of the long-standing issues researchers have always faced in quantitative finance. While investors need to be cautious, more cautious than in past applications of quantitative methods new tools offer many potential applications in finance. In this article, the authors develop a research protocol that pertains both to the application of machine learning techniques and to quantitative finance in general.

Arnott, R. D., Sherrerd, K. F., and Wu, L. (2013). "The Glidepath Illusion ... and Potential Solutions." In: *The Journal of Retirement* 1(2), pp. 13–28.
Target-date investment strategies purport to meet the two primary objectives of any retirement savings program: maximizing the real value of investors' nest eggs while minimizing uncertainty around prospective income in retirement. The authors demonstrate that the classic glidepath approach to retirement investing moving from equity-centric to bond-centric investing as we age does not meet these objectives. The authors summarize the flaws in traditional glidepath implementation and explore illustrative changes to the rules-based, mechanistic solution for retirement planning that can improve the expected outcome for investors, using simulations to test alternatives. Their findings show that, even with simple rules-based approaches, there are better ways to achieve our financial objectives for retirement.

Arshanapalli, B. and Nelson, W. (2012). "Asset Allocation Options for Wealth Accumulation." In: *The Journal of Wealth Management* 14(4), pp. 22–27.
What is the best asset allocation scheme for retirement investing and wealth accumulation? The authors examine seven schemes, including the 110 minus age rule of thumb and strategies employed by popular retirement mutual funds. Historically, the 100 percent equity portfolio has offered the highest returns with little extra risk. However, if the authors lower equity premiums or bootstrap historical results, then retirement mutual funds and the 110 minus age rule of thumb perform somewhat better than 100 percent equity portfolios in adverse circumstances. But overall, a 100 percent equity portfolio provides the highest wealth accumulation.

Assabil, S. E. and Mcleish, D. L. (2021). "Assessing the Impact of Longevity Risk for Countries with Limited Data." In: *The Journal of Retirement* 8(3), pp. 62–75.
The impact of longevity risk has not been well studied in most developing countries due to the lack of suitable mortality data. As a result, most pension companies in these countries (especially those on the African continent) do not account for longevity risk in their annual valuation. This can even lead to their collapse if steps are not taken to address it. In this work, we develop a method of assessing longevity risk where there is a

severe lack of mortality data. The method is based on the assumption that there is a nearly linear relationship between annuitant's hazard function and their mortality at higher ages (post-retirement age), which permits approximating with the Gompertz model. We tried the method on mortality data from Ghana and the results are consistent with those in the standard literature. That is, longevity risk is a treat to pension companies in Ghana even though, in the case of Ghana, this treat has partially been mitigated by the high-interest rate in the country. With this method, pension and life companies that are not able to account for longevity risk as a result of lack of data or newly formed pension companies with even two-year mortality data will be able to assess the longevity risk they face without relying on data or models from other countries.

Atkins, A. B. and Caliendo, F. N. (2009). "Strategies for maximizing social security benefits." In: *The Journal of Wealth Management* 12(1), pp. 25–31.

People nearing retirement face a well-known decision: When should Social Security benefits be initiated?

The answer may not seem obvious since there is a key trade-off involved. The retiree can choose low benefits for a longer period of time, or high benefits for a shorter period of time, or something in between. The optimal initiation date that maximizes Social Security wealth is a quantitative question and it depends on the life expectancy of the person and on the real rate of return they expect to earn on their investments, among other things.

The authors attempt to provide some answers to this practical and important question. Their focus throughout is on relatively high-wealth individuals who will receive the maximum Social Security benefits. They show how the government Social Security benefit calculator makes recommendations on the optimal age to initiate benefits be inappropriate for high-wealth individuals.

They conclude with the following rough rules of thumb

Take benefits early (i.e., at age 62): If an individual expects to make a good return on their investments. This is true regardless of their life expectancy.

Take benefits later (i.e., at age 70): If an individual expects to make modest investment returns and they expect to live a long time.

Azman, S. and Pathmanathan, D. (2022). "The GLM framework of the Lee-Carter model: a multi-country study." In: *Journal of Applied Statistics* 49(3), pp. 1752-763–12.

The Lee-Carter model is a well-known model in modeling mortality. We aim to compare three probability models (Poisson, negative binomial and binomial) based on the Generalized Linear Model (GLM) framework of the Lee-Carter model. These models are applied to mortality data for 10 selected countries (Japan, United States, United Kingdom, Australia, Sweden, Spain, Belgium, Canada, Netherlands and Bulgaria) and the fit of these models is assessed using the deviance statistics and standardized residuals against fitted value plot. Among these three models, the negative binomial Lee-Carter model gave the best fit based on the deviance statistics and estimates of the log of deaths.

Bachmann, K., Hens, T., and Stossel, R. (2014). "Designing A Risk Profiler: Which Measures Predict Risk Taking?" In: *SSRN e-Print*.

In this paper we assess the suitability of different risk profiling measures when individuals are involved in a process of discovering their willingness to take risks over different decision modes. The latter involve decisions under ambiguity, decisions after gaining experience and receiving outcome information on previous decisions. We find that risk taking is associated with individuals' risk preferences in all decision modes but not with their investment experience. Although simulated experience improves the risk awareness and supports a higher risk taking, it cannot substitute the assessment of risk preferences and in particular the assessment of individual's loss aversion. In contrast, self-assessed risk tolerance measures are not suitable for predicting risk taking in any decision mode. If risk preferences cannot be assessed, only the gender can be used as a predictor of risk taking.

Bai, Z. and Wallbaum, K. (2020). "Optimizing Pension Outcomes Using Target-Driven Investment Strategies: Evidence from Three Asian Countries with the Highest Old-Age Dependency Ratio." In: *Asia-Pacific Journal of Financial Studies* 49(4), pp. 652–682.

As a response to unforeseeable market turbulence – such as the 2008 financial crisis and the most recent market drawdown triggered by the COVID-19 pandemic – we propose a new pension investment strategy that could better protect a long-term pension plan in volatile market conditions. Over a hypothetical 20-year pension scheme and various target volatility scenarios, we show that our newly proposed strategy, which attaches a target volatility mechanism to a lifecycle strategy, could provide more effective capital protection and risk control for pension investment vehicles. Our results are robust with a consideration of transaction costs.

Bailey, D. H., Borwein, J. M., and Lopez de Prado, M. (2017). "Stock Portfolio Design and Backtest Overfitting." In: *Journal of Investment Management* 15(1), pp. 75–87.
In mathematical finance, backtest overfitting connotes the usage of historical market data to develop an investment strategy, where too many variations of the strategy are tried, relative to the amount of data available. Backtest overfitting is now thought to be a primary reason why investment models and strategies that look good on paper often disappoint in practice. Models and strategies suffering from overfitting typically target the specific idiosyncrasies of a limited dataset, rather than any general behavior, and, as a result, often perform erratically when presented with new data. In this study, we address overfitting in the context of designing a mutual fund or investment portfolio as a weighted collection of stocks. Very often a newly minted equity-based fund of this type has been designed by an exhaustive computer-based search of some sort to obtain an optimal weighting that exhibits excellent performance based, say, on the past 10 or 20 years' historical market data, and the fund often highlights this backtest performance.

Bailey, R., DeShetler, A., Leming, J., Weber, S. M., Youssef, J., and Young, J. A. (2021). *Planning for health care costs in retirement*. Tech. rep. Vanguard.
Vanguard engaged Mercer Health & Benefits to develop a model to forecast health care costs for U.S. retirees. Vanguard believes retirement planning frameworks should be adapted as follows: 1) Planning for annual health care insurance premiums and out-of-pocket expenses at retirement should be distinct from planning for long-term care expenses. 2) Some research estimates health care costs in retirement as a lifetime lump sum. We believe a better planning framework considers these costs as annual expenses personalized to an individual's health status, coverage choices, retirement age, and loss of any employer subsidies. For a typical 65-year-old woman, the Mercer-Vanguard model predicted an annual health care expense of $5,100 for 2020. 3) During their working years, some people should save at higher rates to account for potential future incremental health care spending. This group includes those with generous employer benefits that may not be offered in retirement and those with higher risk of chronic conditions because of family history or current health status 4) Long-term care costs represent a separate and difficult planning challenge because of the wide distribution of potential outcomes. Half of the population will never incur them-but everyone faces a small but meaningful risk of requiring costly care for multiple years.

Bairoliya, N. and McKiernan, K. (2021). "Revisiting Retirement and Social Security Claiming Decisions." In: *SSRN e-Print*.
Why do individuals retire and claim their Social Security benefits at the age they do? Understanding the key drivers of these decisions has been an important topic of research as it can help guide policy discussions on the impact of potential reforms to the Social Security program. We revisit this crucial question by exploring new sources of heterogeneity in these decisions as well as novel mechanisms governing these trade-offs. Using data from the Health and Retirement Study and the Understanding America Survey, we first document (1) important heterogeneities in social security claiming behavior of men by their education and marital status, (2) strong correlations between health, labor supply and benefit claiming decisions and (3) significant misinformation related to Social Security program knowledge and survival chances at older ages. We then build a life-cycle model of consumption, savings, labor supply, and Social Security application decisions as well as heterogeneity in education, marital status and SS program knowledge. The model includes uncertainty in health, subjective survival, wages, and job separation as well as rich details of the U.S. Social Security program to understand why a majority of individuals claim Social Security benefits prior to their normal retirement age, despite large penalties associated with these early benefit claims. We show that the estimated model can closely match the claiming behavior as seen in the data and also produce differences in SS claims along the dimensions of heterogeneity considered. Counterfactual experiments indicate that precautionary motives, misinformation, and preferences governing future discounting as well as altruism, together, go a long way in explaining overall claiming behavior. Together, these forces can explain a third of the overall early benefit claims and two-thirds of age 62 claims-with varying intensities across education and marital groups.

Banerjee, S. (2019). *Asset Decumulation or Asset Preservation? What Guides Retirement Spending?* Tech. rep. Employee Benefit Research Institute EBRI.
The Employee Benefit Research Institute (EBRI) undertook a study examining the extent to which the non-housing assets of certain retirees changed during their first 20 years of retirement (or until death, if earlier).
The study relied on income and asset data from the Health and Retirement Study (HRS), the most comprehensive survey of older Americans in the country, and on spending data from the Consumption and Activities Mail Survey

(CAMS), a supplement to HRS.
Highlights:

1) The study shows that retirees generally exhibit very slow decumulation of assets.
2) More specifically, within the first 18 years of retirement, individuals with less than $200,000 in non-housing assets immediately before retirement had spent down (at the median) about one-quarter of their assets; those with between $200,000 and $500,000 immediately before retirement had spent down 27.2 percent. Retirees with at least $500,000 immediately before retirement had spent down only 11.8 percent within the first 20 years of retirement at the median.
3) While some retirees do spend down most of their assets in the first eighteen years following retirement, about one-third of all sampled retirees had increased their assets over that period.
4) Pensioners were much less likely to have spent down their assets than non-pensioners. During the first 18 years of retirement, the median non-housing assets of pensioners (who started retirement with much higher levels of assets) had gone down only 4 percent, compared to 34 percent for non-pensioners.
5) The median ratio of household spending to household income for retirees of all ages hovered around one, inching slowly upward with age. This suggests that majority of retirees had limited their spending to their regular flow of income and had avoided drawing down assets, which explains why pensioners, who had higher levels of regular income, were able to avoid asset drawdowns better than others

All numbers are measured in 2015 dollars.

Basu, A. K., Byrne, A., and Drew, M. E. (2011). "Dynamic Lifecycle Strategies for Target Date Retirement Funds." In: *The Journal of Portfolio Management* 37(2), pp. 83–96.
Lifecycle funds offered to retirement plan participants gradually reduce exposure to stocks as the funds approach the target date of the participants' retirement.The authors show that such deterministic switching rules produce inferior wealth outcomes for the investor compared to strategies that dynamically alter the allocation between growth and conservative assets based on cumulative portfolio performance relative to a set target.The dynamic allocation strategies proposed in this article exhibit almost stochastic dominance over strategies that unidirectionally switch assets without consideration of portfolio performance.

Basu, A. K. and Wiafe, O. K. (2017). "Impact of persistent bad returns and volatility on retirement outcomes." In: *Finance Research Letters* 21, pp. 201–205.
We examine wealth outcomes and risk of ruin faced by retirees due to persistent bad returns and high volatility in equity markets occurring at different stages of their retirement. Our results show poor equity returns persisting over long periods can put retirement security to serious risk but volatile market conditions actually have the opposite impact. The timing of such persistent bad returns and volatility (early or late stages of retirement) is critical and has differing effects on retirement outcomes. The results are robust to varying portfolio allocations to equities although the precise impacts are different.

Bateman, H., Stevens, R., and Lai, A. (2015). "Risk Information and Retirement Investment Choice Mistakes Under Prospect Theory." In: *Journal of Behavioral Finance* 16(4), pp. 279–296.
We assess alternative presentations of investment risk using a discrete choice experiment which asked subjects to rank three investment portfolios for retirement savings across nine risk presentation formats and four underlying risk levels. Using Prospect Theory utility specifications, we estimate individual-specific parameters for risk preferences in gains and losses, loss aversion, and error propensity variability. Our results support presentations that describe investment risk using probability tails. Risk preferences and error propensity were found to vary significantly across sociodemographic groups and levels of financial literacy. Our findings should assist regulatory efforts to disclose risk information to the mass market.

Bell, D. N. F., Comerford, D. A., and Douglas, E. (2020). "How do subjective life expectancies compare with mortality tables? Similarities and differences in three national samples." In: *The Journal of the Economics of Ageing* 16 (100241).
Estimates of personal longevity play a vital role in decisions relating to asset accumulation and decumulation. Subjective life expectancy (SLE) is a measure of individuals' expectation of remaining years of life. Either explicitly or implicitly, it is a key determinant of consumption and savings behaviour, and may be guided by a person's own health and health behaviours. The Gateway to Global Aging, a platform for the Health and Retirement Study's (HRS) family of population surveys, provides harmonised longitudinal datasets for many countries, each based on individual survey responses from respondents aged 50 and above. In this paper, we analyse SLE three of these datasets: the English Longitudinal Study of Ageing (ELSA), The Irish Longitudinal

Study of Ageing (TILDA) and Healthy Ageing in Scotland (HAGIS). First, we focus on measurement of SLE, followed by the SLE differential - the discrepancy between SLE and mortality risk indicated by population life tables. One novel finding from our analysis is that the SLE differential is positive for Ireland and is negative for Scotland and England. This difference does not appear to be explained by differences of survey design or population characteristics.

Bengen, W. P. (2021). "The Planner's Toolkit for Managing Retirement Withdrawal Plans." In: *Journal of Financial Planning* 34(4), pp. 74–80.

Since 1993, I've been researching sustainable withdrawals from retirement portfolios and searching for something: a complete, soup-to-nuts approach to managing the withdrawal process-from construction of the original withdrawal plan, through monitoring the plan and identifying deviations, to applying corrections, as needed. I present in this article my version of the financial planner's complete toolkit for managing withdrawal plans. As with any area of financial planning, you can't launch the initial plan and expect it to be appropriate for the client forever. A withdrawal plan needs to be actively managed to determine if the plan no longer suits a client's needs, and to make appropriate changes to bring the plan back into focus.

In a way, this article is more of an overview rather than a complete delineation of the process. My approach relies heavily on graphical elements: dozens of charts are required as part of the toolkit. Furthermore, owing to the large number of variables involved, withdrawal planning is imbued with an inherent complexity. Accordingly, there simply isn't room in an article of this length to provide you with the level of detail demanded for a full exposition of my methods. So, consider this a test drive and please wait for the production models to be released in the near future.

I'll proceed in three steps:

1) Establishing the withdrawal plan (determining, among other variables, the withdrawal scheme and initial withdrawal rate).
2) Monitoring the plan for deviations.
3) Making corrections to the plan, as required (or as desired by the client).

These three steps may be repeated several times during the life of the client's plan.

Benz, C. (2021). *When Is the 'Right' Time to File for Social Security Benefits?* Tech. rep. Morningstar.

Author Mike Piper discusses when it makes sense to claim Social Security early and when delaying is advisable. Many consumers have heard the advice to delay claiming Social Security until later in life–past age 62 and perhaps up until age 70–with the goal of claiming a higher lifetime benefit. But is delaying Social Security always advisable? Not necessarily. In a recent episode of The Long View podcast, author Mike Piper shared instances when claiming earlier is advisable, as well as when claiming benefits later makes sense. He also discussed the free tool he created to help with Social Security claiming decisions, Open Social Security. Piper writes the popular Oblivious Investor blog, and he is also the author of several books about Social Security, taxes, and retirement, including Can I Retire?, Social Security Made Simple, Investing Made Simple, and Taxes Made Simple.

Beracha, E., Downs, D. H., and MacKinnon, G. (2017). "The 4% rule: Does real estate make a difference?" In: *Journal of Property Research* 34(3), pp. 181–210.

This paper examines the wealth maximisation and preservation effects of including commercial real estate in retirement-phase portfolio management. Prior research addresses the role of real estate during the wealth-accumulation phase of the investor lifecycle; however, little is known about the contribution of real estate during the invest-and-spend, or decumulation, phase. To address this issue, we estimate short-fall risk based on the widely known 4% Rule. We use pricing data for multiple asset classes and simulation techniques, combined with a robust correlation structure, to examine: short-fall risk sensitivity to alternative spending rules; the impact of public vs. private real estate allocations; wealth preservation as an investment objective; and the effect of real estate on upside, or wealth maximisation, potential. We find short-fall risk in a decumulation portfolio decreases with substantial allocations to real estate. This result holds for a portfolio including either public or private real estate. Additionally, and under most conditions, the best performing decumulation-phase portfolios include a real estate allocation with both public and private real estate exposure. These results have significant implications for investors, whether they be retirees, plan administrators or endowments, as well as financial economists studying the lifecycle of investment decisions.

Bernhardt, T. and Donnelly, C. (2018). *Pension Decumulation Strategies: A State-of-the-Art Report*. Tech. rep. Risk Insight Lab, Heriot-Watt University.

This report is focused on income withdrawal and investment strategies that are proposed for decumulation in retirement. Popular strategies which are used in practice and specifications of how to decide the income and investment strategy are reviewed.

One of the most popular rules, the SWR or "4% rule", is very simple and easy to implement. The retiree invests in a constant-mix strategy and withdraws an inflation-indexed income for 30 years. There is intended to be a 90% chance of the strategy being successful.

However the constant-mix investment strategy is criticized by Scott et al. (2009) as being too expensive. In many possible future scenarios, there is an excess of assets at the end of 30 years. That means that the retiree has followed a wasteful strategy. They have paid for surpluses in excess of their stated goal. Moreover, there is a chance that the retiree could exhaust their funds before 30 years are up. Scott et al. (2009) show that options or a different dynamic strategy could be employed to give either better outcomes for the same cost or the same outcomes for a lower cost.

As shown repeatedly in the literature, optimal investment strategies are dynamic strategies. They are usually not constant-mix strategies, although their exact dynamics depend on the optimization problem. In contrast, many drawdown products in the market are constant-mix strategies. Optimal withdrawal rates, when allowed to be a decision variable, also vary over time. How much they vary will depend on the problem specification. However, if the problem set-up allows for risk aversion, then the withdrawal rate will vary less as the investor becomes more risk averse.

Annuities or modern tontines should be part of any decumulation strategy, particular at older ages. They are a cost-efficient strategy which either eliminate or significantly reduce the chance of running out of money. In general, the annuity or modern tontine should be incorporated into the retiree's assets when their mortality is the main driver of the decumulation strategy. This is the case when buying these products has a higher value than trading solely in the financial market.

There are many approaches to specifying the decumulation problem. Utility theory-based methods, such as Expected Utility Theory and Cumulative Prospect Theory, allow for investor risk aversion. This latter property can give intuitively appealing results. However, the problem specification is not easy to communicate.

Minimizing the difference between the actual consumption and a desired consumption via the minimization of a quadratic function has also been studied. Closed-form solutions are difficult to obtain.

Probabilistic methods, such as minimizing the probability of ruin, appear more objective. However, as they don't allow for investor risk aversion, the results can potentially be unattractive. The problems have to be carefully specified in order to obtain an agreeable solution.

Habit formation tries to match actual consumption behaviour closely. The investor seeks to keep their consumption above a weighted average of their past consumption. It may be difficult to justify such a formulation to the investor. Similarly, a problem specification (confusingly called drawdown) where the probability of the investor's wealth falling below some fraction of their running maximum wealth is minimized, may also be hard to explain as a decumulation goal.

Bessler, W., Opfer, H., and Wolff, D. (2017). "Multi-asset portfolio optimization and out-of-sample performance: an evaluation of Black Litterman, mean-variance, and naive diversification approaches." In: *The European Journal of Finance* 23(1), pp. 1–30.

The Black Litterman model aims to enhance asset allocation decisions by overcoming the problems of mean-variance portfolio optimization. We propose a sample-based version of the Black Litterman model and implement it on a multi-asset portfolio consisting of global stocks, bonds, and commodity indices, covering the period from January 1993 to December 2011. We test its out-of-sample performance relative to other asset allocation models and find that Black Litterman optimized portfolios significantly outperform na ive-diversified portfolios (1/N rule and strategic weights), and consistently perform better than mean-variance, Bayes Stein, and minimum-variance strategies in terms of out-of-sample Sharpe ratios, even after controlling for different levels of risk aversion, investment constraints, and transaction costs. The BL model generates portfolios with lower risk, less extreme asset allocations, and higher diversification across asset classes. Sensitivity analyses indicate that these advantages are due to more stable mixed return estimates that incorporate the reliability of return predictions, smaller estimation errors, and lower turnover.

Bessler, W. and Wolff, D. (2017). "Portfolio Optimization with Industry Return Prediction Models." In: *SSRN e-Print*.

We postulate that utilizing return prediction models with fundamental, macroeconomic, and technical indicators instead of using historical averages should result in superior asset allocation decisions. We investigate the

predictive power of individual variables for forecasting industry returns in-sample and out-of-sample and then analyze multivariate predictive regression models including OLS, a regularization technique, principal components, a target-relevant latent factor approach, and forecast combinations. The gains from using industry return predictions are evaluated in an out-of-sample Black-Litterman portfolio optimization framework. We provide empirical evidence that portfolio optimization utilizing industry return prediction models significantly outperform portfolios using historical averages and those being passively managed.

Bhansali, V. (2015). "Tail-Risk Management for Retirement Investments." In: *The Journal of Retirement* 2(3), pp. 78–86.

One of the insights of behavioral finance is the tendency for small investors to overreact to market swings. Even a well-structured portfolio may be vulnerable to panic selling in a downturn.

The article explains how a policy of tail-risk hedging could deter such behavior by putting a floor on drops in the portfolio. Instead of maintaining an overly conservative stance and an unnecessarily low share of equities, investors could hedge their portfolios by relying on options, which have become a much more feasible tool for the small investor in recent years.

The article discusses the relationship between the time to retirement and the need to hedge. Investors nearing retirement have to adopt a more defensive position or hedge explicitly, but younger investors can be more aggressive and buy options with lower strike prices.

Bianchi, M. and Briere, M. (2021a). "Robo-Advising for Small Investors." In: *SSRN e-Print*.

We study the effects of robo-advising on investors' attention, trading, and performance on a large set of Employees Saving Plans covering a representative sample of French employees. We find that relative to self-managing, accessing the robo services is associated to an increase in the time investors spend to follow their portfolios and to an increase in trading activities. After having taken up the robo, investors are willing to increase their investment, bear more risk, and to rebalance their portfolio in a way to keep their allocation closer to the target. They also experience higher risk-adjusted returns. These effects tend to be stronger for investors with smaller portfolios, who are less likely to have access to traditional advice. Our results shed light on the dynamics of investors' trust towards the robo service and suggest that automated advice can promote financial inclusion.

Bianchi, M. and Briere, M. (2021b). "Robo-Advising: Less AI and More XAI?" In: *SSRN e-Print*.

We start by considering some of the key reasons behind the academic and industry interest in robo-advisors. We discuss how robo-advice could potentially address some fundamental problems in investors' decision making as well as in traditional financial advice. We then move on to some of the ongoing issues regarding the future of robo-advice. Firstly, the role Artificial Intelligence (AI) plays, and should play, in robo-advice. Secondly, how far should the personalisation of robo-advice recommendations go. Third, how trust in automated financial advice can be generated and maintained. Fourth, whether robots are perceived as complements or substitutes to human decision-making. Our conclusion outlines some thoughts on what the next generation of robo-advisors might look like. We highlight the importance of recent insights in Explainable AI and how new forms of AI applied to financial services would benefit from importing insights from economics and psychology to design effective human/robot interaction.

Biggs, A. G., Chen, A., and Munnell, A. (2021). "The Consequences of Current Benefit Adjustments for Early and Delayed Claiming." In: *SSRN e-Print*.

Workers have the option of claiming Social Security retirement benefits at any age between 62 and 70, with later claiming resulting in higher monthly benefits. These higher monthly benefits reflect an actuarial adjustment designed to keep lifetime benefits equal, for an individual with average life expectancy, regardless of when benefits are claimed. The actuarial adjustments, however, are decades old. Since then, interest rates have declined; life expectancy has increased; and longevity improvements have been much greater for high earners than low earners. This paper explores how changes in longevity and interest rates have affected the fairness of the actuarial adjustment over time and how the disparity in life expectancy affects the equity across the income distribution. It also looks at the impact of these developments on the costs of the program and the progressivity of benefits. The paper found that:

1) The increases in life expectancy and the decline in interest rates argue for smaller reductions for early claiming and a smaller delayed retirement credit for later claiming.

2) Specifically, the benefit at 62 should equal 77.5 percent, as opposed to 70.0 percent, of the full age-67 benefit, and the benefit at 70 should equal 119.9 percent, instead of 124.0 percent, of the full benefit.

3) The outdated actuarial adjustments are a modest moneymaker for the program - about $1.9 billion in 2018, with most of the gains coming from those claiming at 62, who are typically lower earners.

4) Surprisingly, the correlations between earnings and life expectancy and between earnings and claiming behavior have only modest implications for both the cost and progressivity of Social Security benefits.

5) Finally, the cost and distributional effects of earnings-related life expectancy and claiming cannot be addressed through the actuarial adjustments for early and late claiming. They reflect the fact that high earners get their large benefits for a long time and low earners get their more modest benefits for a shorter time.

The policy implications of the findings are:

- Increases in life expectancy and the decline in interest rates suggest smaller reductions for early claiming and a smaller delayed retirement credit for later claiming.
- Accounting for differential mortality would involve changing benefits, and is not a problem that can be solved by tinkering with the actuarial adjustments.

This study considered Consequences of Current Benefit Adjustments for Early and Delayed Claiming.

Biggs, A. G. (2017). "The life cycle model, replacement rates, and retirement income adequacy." In: *The Journal of Retirement* 4(3), pp. 96–110.

The key insight of the life cycle model in economics is that a household consumption at any given time is determined not so much by its current income as by the total income available to the household over its lifetime. A replacement rate can be a useful tool in approximating the life cycle model predictions for how households prepare for retirement. The Social Security Administration Office of the Chief Actuary (OACT) publishes two different calculations of retirement income replacement rates, each of which finds that Social Security benefits replace about 40% of a typical retiree pre-retirement earnings. Some interpret these figures as indicating that Social Security benefits are insufficiently generous and that U.S. households total retirement saving is inadequate. But OACT two methods for calculating replacement rates both violate the life cycle model in a meaningful way. OACT career-average earnings replacement rates, in which lifetime earnings are first indexed upward by the rate of economywide wage growth, exaggerates by roughly one-fifth the real value of earnings available to a household for consumption over its lifetime. This overstatement lowers a household measured ability to replace their pre-retirement earnings. OACT final-earnings replacement rates effectively compare Social Security retirement benefits to pre-retirement earnings only in the years in which the individual worked, ignoring the life cycle model prediction that household consumption is a function of long-term average earnings, including years in which a household member was not employed. A replacement-rate calculation more consistent with the life cycle model would compare retirement income to an average of real earnings calculated over a significant number of years. Such an approach would find substantially higher replacement rates for the typical retiree. It is important both for Social Security policy and the analysis of overall retirement savings adequacy that replacement-rate calculations build on the insights of the life cycle model that guides most economic analysis of retirement saving.

Bjerring, T., Ross, O., and Weissensteiner, A. (2017). "Feature selection for portfolio optimization." In: *Annals of Operations Research* 256, pp. 21–40.

Most portfolio selection rules based on the sample mean and covariance matrix perform poorly out-of-sample. Moreover, there is a growing body of evidence that such optimization rules are not able to beat simple rules of thumb, such as 1/N. Parameter uncertainty has been identified as one major reason for these findings. A strand of literature addresses this problem by improving the parameter estimation and/or by relying on more robust portfolio selection methods. Independent of the chosen portfolio selection rule, we propose using feature selection first in order to reduce the asset menu. While most of the diversification benefits are preserved, the parameter estimation problem is alleviated. We conduct out-of-sample back-tests to show that in most cases different well-established portfolio selection rules applied on the reduced asset universe are able to improve alpha relative to different prominent factor models.

Blake, D. and Cairns, A. J. G. (2020). "Longevity risk and capital markets: the 2018-19 update." In: *Annals of Actuarial Science* 14(2), pp. 219–261.

This Special Issue of the Annals of Actuarial Science contains 12 contributions to the academic literature all dealing with longevity risk and capital markets. Draft versions of the papers were presented at Longevity 14: The Fourteenth International Longevity Risk and Capital Markets Solutions Conference that was held in Amsterdam on 20-21 September 2018. It was hosted by the Pensions Institute at City, University of London and the Netspar Network for Studies on Pensions, Ageing and Retirement.

Blanchett, D. (2014). "Addressing Key Retirement Risks." In: *The Journal of Retirement* 2(2), pp. 67–80.
This article introduces a new model to jointly determine the optimal allocation to equities, real assets, and annuities for a retiree with a given set of preferences. This model improves on a number of existing retirement income models by directly incorporating a cost associated with deviating from the investor's target risk preference, jointly testing the effects of four preference variables aversion to volatility, strength of a bequest motive, preference for sustainable expenditure, and sensitivity to the risk of future inflation using a dynamic withdrawal strategy and taking a probabilistic approach to mortality.

Blanchett, D. (2018). "Health shocks and subsequent retiree spending." In: *The Journal of Retirement* 6(1), pp. 55–69.
Healthcare expenses are receiving increasing attention from retirees (and the media) today and many financial planners are starting to think about how to potentially incorporate these expenses into a financial plan. This article explores some of the key considerations associated with modeling healthcare-related expenses for retirees with a focus on the impact of these expenses on future retiree spending (and consumption) using data from the Health and Retirement Study. We find that households that experience unexpected out-of-pocket healthcare expenses tend to decrease spending by more than those who do not. For example, if healthcare expenses in a given year exceed 10% of total spending (excluding health insurance premiums, which we define as a health shock), then future total spending, nondurable spending, and consumption drop by approximately 5% more than households that do not experience the shock. Financial advisors (and households) interested in including healthcare expenses as part of a financial plan should be aware of this effect since it can potentially reduce the actual financial implications associated with a retiree health shock (or other retiree healthcare expenditures).

Blanchett, D. (2020a). "The Benefit of Diversified Guaranteed Income for Retirees: Combining Immediate Fixed and Variable Annuities." In: *Retirement Management Journal* 2(3).
This paper explores the potential benefits of developing a retirement income that considers both immediate fixed annuities (IFA) and immediate variable annuities (IVA) using a stochastic utility model combined with a scenario framework. Optimal annuity allocations vary considerably across household type, but certainty equivalent retirement income increases by 20 percent, on average, when incorporating annuities. Total annuity allocations increase when both IFAs and IVAs are considered, and retirees realize only approximately two-thirds of the benefits of annuitization when just one annuity type is considered. IVA allocations were typically higher than IFA allocations because most households already have a base level of fixed guaranteed income (through Social Security); therefore, IVAs can be a unique diversifier from a retirement-income perspective. Overall, this analysis strongly suggests retirees (and financial advisors) should consider annuities as part of a retirement-income strategy, and that they should consider different types of annuities to create the best possible plan.

Blanchett, D. (2020b). "The value of allocating to annuities." In: *The Journal of Retirement* 8(1), pp. 40–52.
This article explores the costs and benefits associated with different approaches a financial advisor could take toward using (or not using) annuities across his or her entire book of clients (i.e., for an entire cohort of retirees). Fees are used to proxy the implied total expenses (or costs) for the two approaches, and not surprisingly, the relative fees between the portfolio and the annuity has a significant impact on the respective benefit of each. For example, a financial advisor would be better off not recommending annuities at all (on average), if the investment-only approach is considered to be low cost and the annuity approach is assumed to be high cost. If the fees for both approaches are assumed to be moderate (i.e., the implied total expenses of the strategies are equal), which is the most realistic scenario for financial advisors providing high-quality advice to clients, the average annuity allocation across scenarios is approximately 30% and helping retirees allocate to annuities optimally generates a total of 73 basis points of alpha-equivalent benefit compared with a financial advisor that does not consider annuities as part of the retirement plan. The marginal alpha-equivalent benefit associated with optimal annuitization strategies, which is estimated by focusing on the total additional benefit generated from allocating to annuities based on the percentage of assets actually used to purchase the annuity, is closer to 150 basis points-a level of increased risk-adjusted performance that would be almost impossible for a financial advisor to overcome using traditional investments (i.e., ignoring annuities).

Blanchett, D. (2021a). "Do Advisors Improve 401(k) Plans?" In: *The Journal of Retirement* 8(4), pp. 26–42.
Research on the impact of 401(k) plan advisors has focused predominately on the investment menu or participant-level allocation decisions. This research takes a more holistic perspective and reviews the differences between advised plans across a variety of domains with the use of data from 5500 Employee Benefits Security Administration filings from 2016. Advised plans with assets from $1 million to $50 million were clearly performing "better" than those without, and the advisor impact generally declined as plan assets increased. Although this

research suggests advisors are adding value, correlation is not causation, and additional research on the actual impact of 401(k) advisors is warranted.

Blanchett, D. (2021b). "Minding the Gap in Subjective Mortality Estimates." In: *The Journal of Retirement* 9(2), pp. 8–20.
This article explores the accuracy of subjective life expectancy estimates using data primarily from the from the Health and Retirement Study (HRS). Although individuals appear to some sense about their likelihood of survival (i.e., their subjective mortality), there are notable gaps in these estimates, consistent with past research. Evidence suggests that although subjective estimates may be relatively accurate, on average, and that households appear to do a relatively good job considering various objective factors (for example, health status), there are often significant errors in individual forecasts, and households do not appear to correctly consider all the relevant objective factors (such as income and smoking). Therefore, financial planners need to educate themselves on how to better model and personalize mortality assumptions into financial plans versus relying on purely subjective estimates to ensure that planning assumptions are as accurate as possible.

Blanchett, D. and Cormier, W. (2021). "Right-Sizing Retirement: Exploring the Retirement Consumption Gap in Early Retirement." In: *Journal of Financial Planning* 34(2), pp. 68–81.
Research suggests some households underspend in retirement, resulting in a "retirement consumption gap." This paper explores this effect, specifically during the first 10 years of retirement, and incorporates both household assets and pre-retirement spending using data from the Health and Retirement Study. Only 18 percent of households have enough wealth to cover pre-retirement consumption when they retire, which suggests most households will not be able to maintain their pre-retirement lifestyle in retirement-a finding consistent with other general estimates of the retirement readiness of Americans. Real financial assets decline for 65 percent of households during the first 10 years of retirement, with a median real decline of 35 percent. Real retiree spending declines for 75 percent of households during the first 10 years of retirement, with a median annual decline of approximately 2 percent per year. This suggests financial planners should consider changes in retirement spending that are less than inflation as part of a retirement plan. The percentage of households that can fund their retirement consumption increases dramatically during the first 10 years of retirement, from 18 percent to 48 percent, primarily due to reductions in spending. This suggests households "right-size" their spending early in retirement to better align with available resources. It is not clear, though, to what extent this behavior persists further into retirement (due to data limitations). Many well-funded households could increase consumption, but appear not to do so (i.e., exhibit a retirement consumption gap). Potential reasons include the desire to leave a bequest, uncertain medical expenses (especially late in retirement), uncertain life expectancy, etc. While this group is a minority of retiree households, understanding what drives this behavior is especially important to financial planners since this group tends to have the most accumulated wealth and are, therefore, more likely to seek the services of, or use, a financial planner.

Blanchett, D., Finke, M., and Pfau, W. (2018). "Low Returns and Optimal Retirement Savings." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.
Lifetime financial outcomes relate closely to the sequence of investment returns earned over the life cycle. Higher return assumptions allow individuals to save at a lower rate, withdraw at a higher rate, retire with a lower wealth accumulation, and enjoy a higher standard of living. While analysis of this topic is often based on historical investment performance, present bond yields are historically low and equity prices are quite high, suggesting that individuals will likely experience lower returns in the future. This implies the need for higher savings rates, lower withdrawal rates, a larger nest egg at retirement, and a lower lifetime standard of living. We show that lower-income workers will need to save about 50 percent more if low rates of return persist in the future, and higher-income workers will need to save nearly twice as much in a low return environment compared to the optimal savings using historical returns.

Blanchett, D. and Finke, M. S. (2019). "Should Annuities be Purchased from Tax-Sheltered Assets?" In: *SSRN e-Print*.
Retirees who purchase an annuity may assume that retirement savings accounts are ideal for funding retirement income. Annuities, however, are a tax-favored investment. We investigate the relative benefits of purchasing an annuity from tax-deferred and taxable accounts for various payout levels, tax rates, asset tax efficiency, and assumed portfolio rates of return. We find considerable evidence that investors are better off using non-qualified accounts to purchase annuities, although the benefits vary significantly by investor characteristics and the tax efficiency of investments held in non-qualified savings. In some cases, selecting the right account increases the after-tax income by over 10% which is equivalent to approximately 50 basis points of added portfolio return. A

15-year deferred annuity purchased from non-qualified vs. qualified bonds earning 4% provides over 30% more after-tax income for a 65-year old with a 40% marginal tax rate.

Blanchett, D. and Finke, M. S. (2021). "Guaranteed Income: A License to Spend." In: *SSRN e-Print*.
Prior research finds that retirees don't spend nearly as much as they could from their investments. Economic theory provides both rational and behavioral explanations for under-spending among retirees with high non-annuitized wealth. Longevity risk will result in lower spending among rational, risk-averse retirees. Retirees may also exhibit behavioral preferences that make them far more comfortable spending from income than assets. We explore how the composition of retirement assets is related to retirement spending and find that retirees who hold a higher percentage of their wealth in guaranteed income spend more than retirees whose wealth consists primarily of non-annuitized assets. Marginal estimates suggest that investment assets generate about half of the amount of additional spending as an equal amount of wealth held in guaranteed income. In other words, retirees will spend twice as much each year in retirement if they shift investment assets into guaranteed income wealth. The size of the effect suggests that the explanation for under-spending non-annuitized savings is likely both a behavioral and a rational response to longevity risk.

Blanchett, D., Finke, M. S., and Guillemette, M. A. (2016a). "Who Exhibits Time Varying Risk Aversion?" In: *SSRN e-Print*.
There is growing empirical evidence that risk preferences change based on financial market conditions. This paper explores individual predictors of time varying risk aversion among participants in U.S. defined contribution plans using a unique dataset with daily responses to a risk tolerance questionnaire. We find that older investors (ages 51-65) are more susceptible to time varying risk aversion. Among older investors, variable risk preference was greatest for participants with the smallest account balances and the lowest incomes, but was unrelated to equity allocation within their retirement portfolio. Much of the variation in the aggregate risk tolerance score can be attributed to variation in future long-term equity performance expectations among older investors. We discuss how target-date funds have the potential to reduce losses from poor market timing that may result from time varying risk aversion.

Blanchett, D., Finke, M. S., and Pfau, W. D. (2016b). "Required Retirement Savings Rates Today." In: *SSRN e-Print*.
Recent asset pricing studies suggest that demand for stocks since 1980 has driven expected returns below their historical average. The current yield of risk-free assets in the U.S. is also well below historical bond yields. This decrease in bond yields, coupled with increases in longevity, has doubled the cost of funding a real dollar of income in retirement since 1980 for a 65-year-old retiree. Many common financial planning practices are surprisingly sensitive to asset returns, and advisors need to understand the challenges clients will face if high asset prices persist. Results from a life cycle planning model show that savings rates would need to rise sharply for households hoping to maintain the same standard of living in retirement if real asset returns are low. Low expected returns also have a surprisingly strong impact on the amount of savings needs to fund legacy goals and a negative impact on client spending throughout their life cycle. Advisors may need to modify expected returns in planning software to provide clients with more realistic projections on meeting long-term spending goals.

Blanchett, D. and Kaplan, P. D. (2018). "Beyond the Glide Path:The Drivers of Target-Date Fund Returns." In: *The Journal of Retirement* 5(4), pp. 25–39.
The authors explore the relative importance of the three primary drivers of target-date fund (TDF) performance: equity (or market) exposures (which, across a series vintages, combine to form a glide path), style exposures (intrastock and intrabond allocations), and other investment selection decisions (e.g., manager selection and the active/passive decision, as well as any other residual risk exposures). They find that overall equity exposure drives only about 25% of the variation in returns across TDFs versus approximately 30% for style and 45% for selection, on average. Consequently, the analysis of the riskiness of a given TDF must be based on more than the overall weight given to equites.

Blanchett, D. M. (2017). "The Impact of Guaranteed Income and Dynamic Withdrawals on Safe Initial Withdrawal Rates." In: *Journal of Financial Planning* 30(4), pp. 42–52.
Optimal safe initial withdrawal rates can differ significantly across relatively similar retiree household scenarios and to a much larger extent than has been commonly noted in retirement income research. Important variables to consider are the amount of guaranteed income, the extent to which the household can adjust consumption, the risk of the investment portfolio, the return assumptions used for projections, and the degree of income stability desired by the retiree. Guaranteed income had the largest impact on the estimated safe initial withdrawal rates among the variables considered. Those withdrawal rates varied by more than 4 percentage points for different

levels of guaranteed income. Although recent research using forward-looking returns suggested that 3 percent is a more appropriate safe initial withdrawal rate for retirees today, this analysis suggested that 4 percent is likely still a much better starting place for many retirees, although the optimal target varies considerably. For financial planners using probability of success-based metrics, targeting a lower success level (e.g., a 75 percent success rate versus a 95 percent success rate) is likely more appropriate, although the optimal target is highly dependent on retiree characteristics.

Blanchett, D. M., Finke, M., and Pfau, W. D. (2013). "Low Bond Yields and Safe Portfolio Withdrawal Rates." In: *The Journal of Wealth Management* 16(2), pp. 55–62.
The majority of research on sustainable withdrawal strategies has used either historical rolling time periods or a stochastic (Monte Carlo) simulation process based on long-term averages, where the expected return of an asset class is the same for each year of the simulation. While these approaches may be reasonable to describe long-term averages, we believe they are less useful when there is a significant and sustained deviation, such as the current low bond yield market. This article introduces a model that takes into account current bond yields and allows them to drift toward a higher value during retirement, using an autoregressive model based primarily on historical relationships between asset classes. This approach can better replicate the actual bond returns a current or near retiree can expect during retirement both now and in the future. Using this model, we find a significant reduction in safe initial withdrawal rates, with a 4 percent initial real withdrawal rate having approximately a 50 percent probability of success over a 30-year period. It finds that a retiree who wants a 90 percent probability of achieving a retirement income goal with a 30-year time horizon and a 40 percent equity portfolio would only have an initial withdrawal rate of 2.8 percent. Such a low withdrawal rate would require 42.9 percent more savings if the retiree wanted to pull the same dollar value out of the portfolio annually as he or she would get with a 4 percent withdrawal rate from a smaller portfolio.

Bogan, V. L., Geczy, C. C., and Grable, J. E. (2020). "Financial planning: A research agenda for the next decade." In: *Financial Planning Review* 3(2).
We provide an informed discussion about challenges, opportunities and the future of research and practice in the field of financial planning over the next 10 years. As editors of Financial Planning Review, using a mix-methods approach and a survey of subject-matter expert views, we outline what we believe are some of the future key themes of financial planning. We also present an overview of the challenges and opportunities facing researchers who are working to build, inform, and expand the financial planning body of literature. Further, we discuss the financial planning-related topics that would benefit most from increased research and study.

Bollen, N. P. B. and Posavac, S. (2018). "Gender, risk tolerance, and false consensus in asset allocation recommendations." In: *Journal of Banking & Finance* 87, pp. 304–317.
We study the impact of gender on asset allocation recommendations. Graduate business students and professional wealth managers are randomly assigned a male or female client. Participants recommend an allocation and choose an allocation for themselves. Male students choose a riskier allocation than female students, consistent with existing evidence of a gender difference in risk tolerance, and recommend a riskier allocation. In contrast, male and female wealth managers choose and recommend the same allocation, indicating that male and female finance professionals feature similar risk preferences. In both samples, a subject's allocation choice is the strongest predictor of the recommendation provided.

Boon, L.-N., Briere, M., and Werker, B. J. M. (2020). "Systematic longevity risk: to bear or to insure?" In: *Journal of Pension Economics and Finance* 19(3), pp. 409–441.
We compare two contracts for managing systematic longevity risk in retirement: a collective arrangement that distributes the risk among participants, and a market-provided annuity contract. We evaluate the contracts' appeal with respect to the retiree's welfare, and the viability of the market solution through the financial reward to the annuity provider's equityholders. We find that individuals prefer to bear the risk under a collective arrangement than to insure it with a life insurers' annuity contract subject to insolvency risk (albeit small). Under realistic capital provision hypotheses, the annuity provider is incapable of adequately compensating its equityholders for bearing systematic longevity risk.

Boreiko, D. and Massarotti, F. (2020). "How Risk Profiles of Investors Affect Robo-Advised Portfolios." In: *Frontiers in Artificial Intelligence* 3.
Automated financial advising (robo-advising) has become an established practice in wealth management, yet very few studies have looked at the cross-section of the robo-advisors and the factors explaining the persistent variability in their portfolio allocation recommendations. Using a sample of 53 advising platforms from the US and Germany, we show that the underlying algorithms manage to identify different risk profiles, although

substantial variability is evident even within the same investor types' groups. The robo-advisor expertise in a particular asset class seems to play a significant role, as does the geographical location, while the breadth of the offered investment choice (number of portfolios) across the robo-advisors under study does not seem to have an effect.

Branning, J. and Grubbs, M. (2019). "Crafting Retirement Income that is Stable, Secure, and Sustainable." In: *SSRN e-Print*.

In retirement planning, advisors should consider a logical, definitional based framework as convictional where empirical, evidence-based information is not available or is unclear. This paper presents a 3-S model framework for retirement income. The 3-S model seeks retirement income that is simultaneously stable, secure, and sustainable. Categorizing income sources for retirement as Agreement-based (Time Bound 3-S), Mortality-based (Risk Pooled 3-S), or Historically based portfolio income (Probabilistic 3-S). This paper asserts that any of these three categories is definitionally capable of offering stable, secure, and sustainable retirement income.

Brauer, K. (2021). "Nudged into Better Portfolios and Lower Risk: Robo-Advice and Savings Decisions." In: *SSRN e-Print*.

Mutual fund and ETF savings plans (SPs) are becoming an increasingly important part of retail investor portfolios and many investors now adopt robo-advisors to obtain guidance on SP choices. Using data from a large online bank that introduced a robo-advising tool, I explore how robo-advice changes investors' SP choices and document three main results. First, default options improve robo-advice users' fund choices towards lower-cost and more diversified funds. Second, many investors - also those who previously held all-equity portfolios - adhere to the default asset allocation that is associated with a 50% equity exposure, although they could construct riskier SP portfolios through the robo-advisor. Third, I document considerable heterogeneity in longer-term adherence to robo-advisor recommendations. First-time SP users are more inert and stick to the robo-advisor's proposed asset allocation while experienced SP users quickly readjust their equity exposure away from the robo-advisor's recommendation. My results emphasize the power of defaults in all-digital robo-advisory services and highlight how they can improve fund choices while at the same time push investors into unsuitable asset allocations.

Bravo, J. M. (2019). "Funding for longer lives. Retirement wallet and risk-sharing annuities." In: *Ekonomiaz: Revista vasca de economia*.

Longevity increases and population ageing create challenges for all societal institutions, particularly those providing retirement income, health care, and long-term care services. At the individual level, an obvious question is how to ensure all retirees have an adequate, secure, stable and predictable lifelong income stream that will allow them to maintain a target standard of living for however long the individual lives. In this paper we introduce and discuss the concept of retirement wallet representing the multiple income and service sources individuals and their families will have to fund for longer lives. We then address the main decumulation risks and options, including the adoption of a given longevity insurance strategy, of a programmed withdrawal strategy and of an investment strategy. The main payout options available for allocating assets accumulated in pension plans are discussed, particularly the role of traditional and innovative investment and longevity risk-sharing structures. We provide illustrative results for the price of innovative participating longevity-linked life annuities (PLLAs) that link benefits to the dynamics of both a longevity index and an interest rate adjustment factor using Spanish mortality and financial market data.

Bravo, J. M. V. (2020). "Addressing the Pension Decumulation Phase of Employee Retirement Planning." In: *Who Wants to Retire and Who Can Afford to Retire?* IntechOpen.

Longevity increases and population ageing create challenges for all societal institutions, particularly those providing retirement income, healthcare, and long-term care services. At the individual level, an obvious question is how to ensure all retirees have an adequate, secure, stable, and predictable lifelong income stream that will allow them to maintain a target standard of living for, however, long the individual lives. In this chapter, we review and discuss the main pension decumulation options by explicitly modelling consumers' behaviour and objectives though an objective function based on utility theory accounting for consumption and bequest motives and different risk preferences. Using a Monte-Carlo simulation approach calibrated to US financial market and mortality data, our results suggest that purchasing a capped participating longevity-linked life annuity at retirement including embedded longevity and financial options that allow the annuity provider to periodically revise annuity payments if observed survivorship and portfolio outcomes deviate from expected (or guaranteed) values at contract initiation deliver superior welfare results when compared with classical annuitization and non-annuitization decumulation strategies.

Breach, T., D'Amico, S., and Orphanides, A. (2020). "The term structure and inflation uncertainty." In: *Journal of Financial Economics* 138(2), pp. 388–414.
To assess the importance of inflation risk for nominal Treasury yields, a novel quadratic term structure model with time-varying inflation risk is estimated using survey-based inflation uncertainty. The resulting yield decomposition captures very diverse macroeconomic dynamics of inflation and real risk premiums (large and positive during the 1980s but small and negative post-2008) and generates sensible high-frequency estimates of expected inflation and real short rates over a long sample. The explicit link between the model-implied factors and macro fundamentals reveals that short- but not long-run fluctuations are unspanned by yields, consistent with an interest rate policy unresponsive to transient inflation shocks.

Bronshtein, G., Scott, J., Shoven, J. B., and Slavov, S. N. (2020). "Leaving big money on the table: Arbitrage opportunities in delaying social security." In: *The Quarterly Review of Economics and Finance* 78, pp. 261–272.
Even though delaying Social Security is equivalent to purchasing a very favorably priced annuity, almost everyone takes Social Security at or before their full retirement age. Many who take Social Security early simultaneously report additional annuity income. This combination can create an arbitrage opportunity where an individual could explicitly or implicitly sell their relatively high priced annuity, use the proceeds to delay Social Security and secure higher income for life. Several hundred thousand, perhaps millions, of households fail to take advantage of this arbitrage opportunity, with a resulting loss that ranges up to $250,000.

Browning, C., Guo, T., Cheng, Y., and Finke, M. S. (2020). "Spending in Retirement." In: *SSRN e-Print*.
Retirement savings adequacy estimates are often based on the assumption that individuals spend the same amount every year in retirement, and that the withdrawal rate to fund spending is based on spending down a percentage of retirement savings. We simulate safe consumption rates and compare the amount that wealthy retirees are spending to the amount they could safely spend given different asset return assumptions and investment portfolio allocations. Retirees in the top quintile of financial wealth are spending nowhere near an amount that would place them in danger of running out of money. In fact, the average financial assets of wealthy retirees increased during this time period and most retirees spent less than their income. Setting aside 40% of financial assets to cover uncertain longevity, medical costs, and bequests still results in a consumption gap as high as 47.3.% among higher-wealth retirees.

Bruni, R., Cesarone, F., Scozzari, A., and Tardella, F. (2016). "Real-world datasets for portfolio selection and solutions of some stochastic dominance portfolio models." In: *Data in Brief* 8, pp. 858–862.
A large number of portfolio selection models have appeared in the literature since the pioneering work of Markowitz. However, even when computational and empirical results are described, they are often hard to replicate and compare due to the unavailability of the datasets used in the experiments. We provide here several datasets for portfolio selection generated using real-world price values from several major stock markets. The datasets contain weekly return values, adjusted for dividends and for stock splits, which are cleaned from errors as much as possible. The datasets are available in different formats, and can be used as benchmarks for testing the performances of portfolio selection models and for comparing the efficiency of the algorithms used to solve them. We also provide, for these datasets, the portfolios obtained by several selection strategies based on Stochastic Dominance models (see "On Exact and Approximate Stochastic Dominance Strategies for Portfolio Selection" (Bruni et al. [2])). We believe that testing portfolio models on publicly available datasets greatly simplifies the comparison of the different portfolio selection strategies.

Bruni, R., Cesarone, F., Scozzari, A., and Tardella, F. (2017). "On exact and approximate stochastic dominance strategies for portfolio selection." In: *European Journal of Operational Research* 259(1), pp. 322–329.
New type of approximate stochastic dominance designed for portfolio selection. Equivalent to minimizing the expected shortfall of the portfolio below the benchmark. An easily solvable LP model for the practical implementation of our approach. Extensive empirical comparison of stochastic dominance models for portfolio selection. One recent and promising strategy for Enhanced Indexation is the selection of portfolios that stochastically dominate the benchmark. We propose here a new type of approximate stochastic dominance rule which implies other existing approximate stochastic dominance rules. We then use it to find the portfolio that approximately stochastically dominates a given benchmark with the best possible approximation. Our model is initially formulated as a Linear Program with exponentially many constraints, and then reformulated in a more compact manner so that it can be very efficiently solved in practice. This reformulation also reveals an interesting financial interpretation. We compare our approach with several exact and approximate stochastic dominance models for portfolio selection. An extensive empirical analysis on real and publicly available datasets shows very good out-of-sample performances of our model.

Bryzgalova, S., Huang, J., and Julliard, C. (2021). "Bayesian solutions for the factor zoo: we just ran two quadrillion models." In: *SSRN e-Print*.
We propose a novel, and simple, Bayesian estimation and model selection procedure for cross-sectional asset pricing. Our approach, that allows for both tradable and non-tradable factors, and is applicable to high dimensional cases, has several desirable properties. First, weak and spurious factors lead to diffuse, and centered at zero, posteriors for their market price of risk, making such factors easily detectable. Second, posterior inference is robust to the presence of such factors. Third, we show that flat priors for risk premia lead to improper marginal likelihoods, rendering model selection invalid. Therefore, we provide a novel prior, that is diffuse for strong factors but shrinks away useless ones, under which posterior probabilities are well behaved, and can be used for factor and (non necessarily nested) model selection, as well as model averaging, in large scale problems. We apply our method to a very large set of factors proposed in the literature, and analyse 2.25 quadrillion possible models, gaining novel insights on the empirical drivers of asset returns.

Buetow, G. W. and Hanke, B. (2020). "How Long is Long Enough?" In: *The Journal of Retirement* 88(22), pp. 39–48.
Defined contribution (DC) plan (DCP) fiduciaries are often faced with conflicting perspectives when executing their responsibilities and terminating underperforming active managers are involved. Consultants, investment managers, and capital market intermediaries generally argue that more time is needed to prove that an investment strategy is suboptimal. These parties often have inherent conflicts of interest to argue for active management over passive management. Most conflicts are economic in nature, but some are steeped in intellectual hubris. In this article the authors enter the fray from the plan participants' (PPs) side. Ultimately, the very essence of a DCP is to offer a menu of investment options that enable PPs to optimize wealth aggregation in a diversified manner using a multiasset class solution. We show that PPs are better served when fiduciaries monitor active investment managers and replace them with passive alternatives in a timely manner if they underperform. Too often plan fiduciaries churn a DCP by replacing underperforming active funds with other active funds.

Buetow, G. W., Hanke, B., and Zagonov, M. (2020). "Active management in defined contribution plans." In: *The Journal of Retirement* 7(4), pp. 61–79.
We analyze the problem that fiduciaries face when monitoring and selecting from a universe of active mutual funds within a defined contribution (DC) plan. In a DC plan, a fiduciary must recognize that there are two levels of decision makers, namely the fiduciary, who decides which funds will comprise the DC plan and the individual plan participants, who must decide which funds to invest in and the timing of their investment. Moreover, plan participants, and to some degree the fiduciary, need to be able to make investment decisions without being investment professionals. We find that due to the general lack of consistency in performance of mutual funds, fiduciaries and plan participants would be better served by selecting passive rather than active funds across the US equity mutual fund space. Moreover, the most consistently outperforming funds tend to have meaningfully higher tracking errors relative to their stated benchmarks, which makes effective asset allocation in a DC plan more difficult.

Butt, A., Khemka, G., and Warren, G. J. (2021). "Principles and Rules for Translating Retirement Objectives into Strategies." In: *SSRN e-Print*.
This article sets out principles and decision rules for setting appropriate drawdown and investment strategies during retirement given an individual's objectives and risk tolerance. In particular, we highlight how the suitable drawdown strategy can relate to the objective, and how annuities might be combined with other investments to help manage risk. Our aim is to offer research-based guidance to product providers, financial advisers and individuals in formulating retirement strategies.

Cairns, A. J. G., Blake, D., Dowd, K., and Coughlan, G. (2020). "A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration." In: *SSRN e-Print* (s).
We use a case study of a pension plan wishing to hedge the longevity risk in its pension liabilities at a future date. The plan has the choice of using either a customised hedge or an index hedge, with the degree of hedge effectiveness being closely related to the correlation between the value of the hedge and the value of the pension liability. The key contribution of this paper is to show how correlation and, therefore, hedge effectiveness can be broken down into contributions from a number of distinct types of risk factors. Our decomposition of the correlation indicates that population basis risk has a significant influence on the correlation. But recalibration risk as well as the length of the recalibration window are also important, as is cohort effect uncertainty. Having accounted for recalibration risk, additional parameter uncertainty has only a marginal impact on hedge effectiveness. Finally, the inclusion of Poisson risk only starts to become significant when the smaller population falls

below about 10,000 members over age 50. Our case study shows that, at least for medium and large pension plans, longevity risk can be substantially hedged using index hedges as an alternative to customised longevity hedges. As a consequence, when the hedger's population involves more than about 10,000 members over age 50, index longevity hedges (in conjunction with the other components of an ALM strategy) can provide an effective and lower cost alternative to both a full buy-out of pension liabilities or even to a strategy using customised longevity hedges.

Campbell, J. Y. and Sigalov, R. (2022). "Portfolio choice with sustainable spending: A model of reaching for yield." In: *Journal of Financial Economics* 143(1), pp. 188–206.
We show that reaching for yield-a tendency to take more risk when the real interest rate declines while the risk premium remains constant-results from imposing a sustainable spending constraint on an otherwise standard infinitely lived investor with power utility. When the interest rate is initially low, reaching for yield intensifies. The sustainable spending constraint also affects the response of risk-taking to a change in the risk premium, which can even change sign. In a variant of the model where the sustainable spending constraint is formulated in nominal terms, low inflation also encourages risk-taking.

Canarella, G., Gil-Alana, L. A., Gupta, R., and Miller, S. M. (2020). "Modeling US historical time-series prices and inflation using alternative long-memory approaches." In: *Empirical economics* 58, pp. 1491–1511.
We consider two important features of the historical US price data (1774-2015), namely the data persistence and cyclical structure. We first consider the persistence of the series and focus on standard long-memory models that incorporate a peak at the zero frequency. We examine different models with respect to the deterministic terms, including nonlinear deterministic trends of the Chebyshev form. Then, we investigate a more general model that includes both persistence and cyclicality of the series and, thus, includes two fractional integration parameters, one at the zero (long-run) frequency and the other at the nonzero (cyclical) frequency. We model the cyclical structure as a Gegenbauer process. This specification outperforms the standard long-memory specifications. We find that the order of integration at the zero frequency is about 0.5, and the one at the cyclical frequency is about 0.2 with cycles repeating approximately every 6 years, producing mean-reverting long-memory effects at both the zero and cyclical frequencies. Fitting the values to this model, however, we discover the presence of a break that, according to the methods employed, takes place at around 1940-1941. The results indicate the prevalence of the long-run or zero component with a much higher degree of persistence during the second post-1940-1941 subsample, suggesting important implications for monetary policy.

Capolongo, A. and Pacella, C. (2021). "Forecasting inflation in the euro area: countries matter!" In: *Empirical Economics* 61, pp. 2477–2499.
We construct a Bayesian vector autoregressive model with three layers of information: the key drivers of inflation, cross-country dynamic interactions, and country-specific variables. The model provides good forecasting accuracy with respect to the popular benchmarks used in the literature. We perform a step-by-step analysis to shed light on which layer of information is more crucial for accurately forecasting medium-run euro area inflation. Our empirical analysis reveals the importance of including the key drivers of inflation and taking into account the multi-country dimension of the euro area. The results show that the complete model performs better overall in forecasting inflation excluding energy and unprocessed food over the medium term. We use the model to establish stylized facts on the euro area and cross-country heterogeneity over the business cycle.

Capponi, A., Olafsson, S., and Zariphopoulou, T. (2020). "Personalized Robo-Advising: Enhancing Investment through Client Interaction." In: *arXiv e-Print*.
Automated investment managers, or robo-advisors, have emerged as an alternative to traditional financial advisors. The viability of robo-advisors crucially depends on their ability to offer personalized financial advice. We introduce a novel framework, in which a robo-advisor interacts with a client to solve an adaptive mean-variance portfolio optimization problem. The risk-return tradeoff adapts to the client's risk profile, which depends on idiosyncratic characteristics, market returns, and economic conditions. We show that the optimal investment strategy includes both myopic and intertemporal hedging terms which are impacted by the dynamics of the client's risk profile. We characterize the optimal portfolio personalization via a tradeoff faced by the robo-advisor between receiving client information in a timely manner and mitigating behavioral biases in the risk profile communicated by the client. We argue that the optimal portfolio's Sharpe ratio and return distribution improve if the robo-advisor counters the client's tendency to reduce market exposure during economic contractions when the market risk-return tradeoff is more favorable.

Capponi, A. and Zhang, Z. (2020). "Risk Preferences and Efficiency of Household Portfolios." In: *arXiv e-Print*.

We propose a novel approach to infer investors' risk preferences from their portfolio choices, and then use the implied risk preferences to measure the efficiency of investment portfolios. We analyze a dataset spanning a period of six years, consisting of end of month stock trading records, along with investors' demographic information and self-assessed financial knowledge. Unlike estimates of risk aversion based on the share of risky assets, our statistical analysis suggests that the implied risk aversion coefficient of an investor increases with her wealth and financial literacy. Portfolio diversification, Sharpe ratio, and expected portfolio returns correlate positively with the efficiency of the portfolio, whereas a higher standard deviation reduces the efficiency of the portfolio. We find that affluent and financially educated investors as well as those holding retirement related accounts hold more efficient portfolios.

Cassidy, D. P., Peskin, M. W., Siegel, L. B., and Sexauer, S. (2013). "Be Kind to Your Retirement Plan - Give It a Benchmark." In: *The Journal of Retirement* 1(1), pp. 81–90.

The performance of any retirement plan needs to be gauged against an appropriately chosen benchmark. We explain the importance of making transparent the risks taken in retirement portfolios, especially QDIA portfolios, and indicate how benchmarks help with this task. Benchmarks should be chosen or constructed for both the accumulation and decumulation phases of a plan, and should have a risk level appropriate to the age and situation of investors. For example, the S&P 500 is a poor benchmark to use when investors are five years from retirement. Given the importance of income to retirees, a benchmark for decumulation should be based on matching assets to liabilities or planned spending, and it should minimize longevity, investment, counterparty, and liquidity risks. We give an example of a decumulation benchmark, which is based on a strategy of holding TIPS and deferred life annuities. This is a call to action: Only by adopting the best practice of choosing and using an appropriate benchmark for retirement portfolios-accumulation and decumulation-can fiduciaries execute the responsibility they have agreed to take on.

Cesarone, F., Moretti, J., and Tardella, F. (2018). "Why Small Portfolios Are Preferable and How to Choose Them." In: *SSRN e-Print*.

One of the fundamental principles in portfolio selection models is minimization of risk through diversification of the investment. However, this principle does not necessarily translate into a request for investing in all the assets of the investment universe. Indeed, following a line of research started by Evans and Archer almost 50 years ago, we provide here further evidence that small portfolios are sufficient to achieve almost optimal in-sample risk reduction with respect to variance and to some other popular risk measures, and very good out-of-sample performances. While leading to similar results, our approach is significantly different from the classical one pioneered by Evans and Archer. Indeed, we describe models for choosing the portfolio of a prescribed size with the smallest possible risk, as opposed to the random portfolio choice investigated in most of the previous works. We find that the smallest risk portfolios generally require no more than 15 assets. Furthermore, it is almost always possible to find portfolios that are just 1% more risky than the smallest risk portfolios and contain no more than 10 assets. The preference for small optimal portfolios is also justified by recent theoretical results on the estimation errors for the parameters required by portfolio selection models. Our empirical analysis is based on some new and on some publicly available benchmark data sets often used in the literature.

Cesarone, F., Mottura, C., Ricci, J. M., and Tardella, F. (2019). "On the stability of portfolio selection models." In: *SSRN e-Print*.

One of the main issues in portfolio selection models consists in assessing the effect of the estimation errors of the parameters required by the models on the quality of the selected portfolios. Several studies have been devoted to this topic for the minimum variance and for several other minimum risk models. However, no sensitivity analysis seems to have been reported for the recent popular Risk Parity diversification approach, nor for other portfolio selection models requiring maximum gain-risk ratios.Based on artificial and real-world data, we provide here empirical evidence showing that the Risk Parity model is always the most stable one in all the cases analyzed. Furthermore, the minimum risk models are typically more stable than the maximum gain-risk models, with the minimum variance model often being the preferable one.

Chalmers, J. and Reuter, J. (2020). "Is conflicted investment advice better than no advice?" In: *Journal of Financial Economics* 138(2), pp. 366–387.

The benefit of investment advice depends on the quality of advice and the investor's counterfactual portfolio. We use changes in the Oregon University System Optional Retirement Plan to highlight the impact of plan design on the counterfactual portfolios of advice seekers. When brokers are available and target date funds (TDFs) are not, brokers help participants with high predicted demand for advice bear market risk, but they recommend higher-commission options. When brokers are removed and TDFs are added, new high-predicted-

demand participants primarily invest in TDFs, which offer similar market risk but higher Sharpe ratios than the broker-advised portfolios within our sample.

Chang, H.-Y., Sheu, D.-F., and Chen, S.-Y. (2010). "A Framework for Assessing Individual Retirement Planning Investment Policy Performance." In: *The Journal of Wealth Management* 13(3), pp. 38–49.
Because there are more and more older people in Taiwan, how retirees can maintain their past consumption level is an important issue. This article assesses the performance of individual investment planning policies for retirement. Because factors such as investment performance, risk, and taxation must be considered in making investment policy choices, the authors apply the analytic hierarchy process (Satty [1980]) to design a framework for evaluating individual investment policy performance. The research findings show that the most important criterion is investment performance. These results provide some suggestions for retirement planning.

Changwony, F. K., Campbell, K., and Tabner, I. T. (2021). "Savings goals and wealth allocation in household financial portfolios." In: *Journal of Banking & Finance* 124 (106028).
We investigate how savings goals relate to wealth allocation and how this relationship is moderated by financial advice and numerical ability. Using panel data from a large household survey we find that as the number and the time horizon of savings goals increases, portfolios shift from safe assets to both fairly safe assets and risky assets. We also find that households with access to multiple sources of financial advice and independent financial advice hold more fairly safe and risky assets and that independent financial advice enhances the influence of savings goals on wealth allocation to fairly safe and risky assets. Overall we find that the possession of savings goals is associated with long term saving activity, and this is particularly evident for those with low levels of numerical ability. By enabling the formation of savings goals, the financial planning process can facilitate long-term investment in risky assets.

Chaudhuri, S. E. and Lo, A. W. (2019). "Dynamic Alpha: A Spectral Decomposition of Investment Performance Across Time Horizons." In: *Management Science* 65(9), pp. 4440–4450.
The value added by an active investor is traditionally measured using alpha, tracking error, and the information ratio. However, these measures do not characterize the dynamic component of investor activity, nor do they consider the time horizons over which weights are changed. In this paper, we propose a technique to measure the value of active investment that captures both the static and dynamic contributions of an investment process. This dynamic alpha is based on the decomposition of a portfolio's expected return into its frequency components using spectral analysis. The result is a static component that measures the portion of a portfolio's expected return resulting from passive investments and security selection and a dynamic component that captures the manager's timing ability across a range of time horizons. Our framework can be universally applied to any portfolio and is a useful method for comparing the forecast power of different investment processes. Several analytical and empirical examples are provided to illustrate the practical relevance of this decomposition.

Chen, A., Fuino, M., Sehner, T., and Wagner, J. (2022a). "Valuation of long-term care options embedded in life annuities." In: *Annals of Actuarial Science*, pp. 1–27.
In most industrialised countries, one of the major societal challenges is the demographic change coming along with the ageing of the population. The increasing life expectancy observed over the last decades underlines the importance to find ways to appropriately cover the financial needs of the elderly. A particular issue arises in the area of health, where sufficient care must be provided to a growing number of dependent elderly in need of long-term care (LTC) services. In many markets, the offering of life insurance products incorporating care options and LTC insurance products is generally scarce. In our research, we therefore examine a life annuity product with an embedded care option potentially providing additional financial support to dependent persons. To evaluate the care option, we determine the minimum price that the annuity provider requires and the policyholder's willingness to pay for the care option. For the latter, we employ individual utility functions taking account of the policyholder's condition. We base our numerical study on recently developed transition probability data from Switzerland. Our findings give new and realistic insights into the nature and the utility of life annuity products proposing an embedded care option for tackling the financing of LTC needs.

Chen, A., Hieber, P., and Rach, M. (2021a). "Optimal retirement products under subjective mortality beliefs." In: *Insurance: Mathematics and Economics* 101(A), pp. 55–69.
Many empirical studies confirm that policyholder's subjective mortality beliefs deviate from the information given by publicly available mortality tables. In this study, we look at the effect of subjective mortality beliefs on the perceived attractiveness of retirement products, focusing on two extreme products, conventional annuities (where the insurance company takes the longevity risk) and tontines (where a pool of policyholders shares the longevity risk). If risk loadings and charges are neglected, a standard expected utility framework, without

subjective mortality beliefs, leads to the conclusion that annuities are always preferred to tontines (Yaari (1965), Milevsky and Salisbury (2015)). In the same setting, we show that this result is easily reversed if an individual perceives her peer's life expectancies to be lower than the ones used by the insurance company. We prove that, assuming such subjective beliefs, there exists a critical tontine pool size from which the tontine is always preferred over the annuity. This suggests that tontines might be perceived as much more attractive than suggested by standard expected utility theory without subjective mortality beliefs.

Chen, A., Li, H., and Schultze, M. B. (2022b). "Tail index-linked annuity: A longevity risk sharing retirement plan." In: *Scandinavian Actuarial Journal*, pp. 1–26.
This paper proposes an innovative retirement product focusing on longevity risk sharing, a contract we refer to as tail index-linked annuity (TILA). Specifically, the proposed TILA pays out variable annual payments, which will be equal to a regular nominal amount when a reference survival index is lower than a predetermined threshold (i.e. normal evolution of longevity risk), and a reduced, index-dependent payment when the threshold is passed (i.e. highly unfavorable evolution of longevity risk). The proposed TILA aims at not only improving the benefits of the policyholders, which has been the focus in recent literature on innovative retirement products, but also reducing the longevity risk exposure of the insurer, particularly for advanced retirement ages. Using real-world mortality data and a stochastic multi-population mortality model, we find that the proposed TILA leads to higher expected lifetime utility than regular annuities for policyholders with different degrees of risk aversions. Meanwhile, numerical analysis shows that the proposed TILA could greatly mitigate the solvency risk of the insurer, leading to a substantially lower loss probability and expected (tail-) loss than regular annuities in the presence of a longevity shock, and therefore could reduce the insurer's required solvency capital under the latest solvency regulations.

Chen, A., Rach, M., and Sehner, T. (2019). "On the Optimal Combination of Annuities and Tontines." In: *SSRN e-Print*.
Tontines, retirement products constructed in such a way that the longevity risk is shared in a pool of policyholders, have recently gained vast attention from researchers and practitioners. Typically, these products are cheaper than annuities, but do not provide stable payments to policyholders. This raises the question whether, from the policyholders' viewpoint, the advantages of annuities and tontines can be combined to form a retirement plan which is cheaper than an annuity and carries less risk than a tontine. In this article, we analyze and compare three approaches of combining annuities and tontines in an expected utility framework: The introduced in Chen et al. (2019), a product very similar to the tonuity which we call and a portfolio consisting of an annuity and a tontine. We show that the payoffs of a tonuity or an antine can be replicated by a portfolio consisting of an annuity and a tontine. Consequently, policyholders achieve higher expected utility levels when choosing the portfolio over the novel retirement products tonuity and antine.

Chen, A. and Munnell, A. H. (2021). *How Much Taxes Will Retirees Owe on Their Retirement Income?* Tech. rep. Center for Retirement Research at Boston College.
To evaluate their retirement resources, households approaching retirement will examine their Social Security statements, defined benefit pensions, defined contribution balances, and other financial assets. However, many households may forget that not all of these resources belong to them; they will need to pay some portion to federal and state government in taxes. It is unclear, however, just how large the tax burden is for the typical retired household and for households with different income levels. This project aims to shed light on the tax burdens that retirees face by estimating lifetime taxes for a group of recently retired households. The project uses data from the Health and Retirement Study (HRS) linked to administrative earnings to determine Social Security benefits and administrative records on state of residence to estimate state tax liabilities. Income is then projected over the expected retirement of each household. Federal and state taxes, are estimated with TAXSIM, for each household on its reported and projected income.
The paper found that:

- These estimates show that households in the aggregate will have to pay about 6 percent of their income in federal and state income taxes.
- But this liability rests primarily with the top quintile of the income distribution.
- For the lowest four quintiles, taxes are negligible - ranging from 0 percent to 1.9 percent.
- In contrast, the average liability is 3 percent for the top quintile, 16.4 percent for the top 5 percent, and 22.7 percent for the top 1 percent.

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Chen, A., Haberman, S., and Thomas, S. (2020). "The implication of the hyperbolic discount model for the annuitisation decisions." In: *Journal of Pension Economics and Finance* 19(3), pp. 372–391.
The low demand for immediate annuities at retirement has been a long-standing puzzle. We show that a hyperbolic discount model can explain this behaviour and results in the attractiveness of long-term deferred annuities. With a set of benchmark assumptions, we find that retirees would be willing to pay a much higher price than the actuarial fair price for annuities with longer deferred periods. Moreover, if governments were to introduce a pre-commitment device which requires pensioners to make annuitisation decisions around 10 years before retirement, the take up rate of annuities could become higher.

Chen, A., Haberman, S., and Thomas, S. H. (2021b). "Combining Flexible Asset Allocation, Sustainable Withdrawals, and Deferred Annuities to provide an Adaptive Lifelong Investing Solution." In: *SSRN e-Print*.
In this paper, we integrate investment decisions in the post-retirement decumulation period with that of the deferred annuity purchase to provide a lifetime decumulation solution. Based on Monte Carlo simulation and historical experience, we use the Perfect Withdrawal Rate (PWR) as a tool to make recommendations on withdrawal rates and asset allocations for different levels of risk preferences. We have a few potentially important findings. First, we illustrate how cheap it is to use a deferred annuity (especially with a deferred period of more than 15 years) as a solution to deal with longevity risk and maintain control of retirement wealth with the investor. Second, we find that if an individual wants to maximise median PWR, he/she should allocate almost 100% in stocks regardless of the length of chosen decumulation period. If an individual wants to maximise minimum PWR, he/she should allocate around 40% - 60% in stocks; therefore, a substantial stocks component should be maintained even if the individual is very risk averse. This then links to our final conclusion on a re-defined Glidepath: if an individual can accept a lower than 50% risk of failure, he/she should move from stocks to bonds as he/she becomes older; however a certain percentage in stocks should be maintained through the decumulation phase.

Chen, A., Haberman, S., and Thomas, S. (2017). "Optimal Decumulation Strategies During Retirement with Deferred Annuities." In: *SSRN e-Print*.
Since greater flexibility in accessing pension savings has been given to defined contribution pensioners, retirees are in need of advice on how to spend down their savings to make retirement income last throughout their lifetime. Deferred annuities have been discussed extensively in recent years as a retirement solution and have been recommended in the OECD Roadmap for the Good Design of Defined Contribution Pension Plans (OECD2016). Assuming a world where deferred annuities are available, we propose two utility maximising decumulation strategies comprising a deferred annuity purchased at retirement and optimal consumption and savings before the commencement of the annuity. A retiree who is concerned about longevity risk and wants to retain a certain level of liquidity is advised to spend 21.6% on a 15-year deferred annuity or 9.13% on a 20-year deferred annuity. A retiree who simply wants to use annuities to maximise overall satisfaction from retirement consumption is advised to spend 61.83% on a 6-year deferred annuity. We compare our strategies with other available decumulation strategies in the market, hence verifying the merits of the design. Moreover, the stability of our results are examined after allowing for consumption smoothness, social income benefits, a target replacement ratio and a bequest motive.

Chen, W. and Langrene, N. (2020). "Deep neural network for optimal retirement consumption in defined contribution pension system." In: *arXiv e-Print*.
In this paper, we develop a deep neural network approach to solve a lifetime expected mortality-weighted utility-based model for optimal consumption in the decumulation phase of a defined contribution pension system. We formulate this problem as a multi-period finite-horizon stochastic control problem and train a deep neural network policy representing consumption decisions. The optimal consumption policy is determined by personal information about the retiree such as age, wealth, risk aversion and bequest motive, as well as a series of economic and financial variables including inflation rates and asset returns jointly simulated from a proposed seven-factor economic scenario generator calibrated from market data. We use the Australian pension system as an example, with consideration of the government-funded means-tested Age Pension and other practical aspects such as fund management fees. The key findings from our numerical tests are as follows. First, our deep neural network optimal consumption policy, which adapts to changes in market conditions, outperforms deterministic drawdown rules proposed in the literature. Moreover, the out-of-sample outperformance ratios increase as the number of training iterations increases, eventually reaching outperformance on all testing scenarios after less than 10 minutes of training. Second, a sensitivity analysis is performed to reveal how risk aversion and bequest motives change the consumption over a retiree's lifetime under this utility framework. Third, we provide the

optimal consumption rate with different starting wealth balances. We observe that optimal consumption rates are not proportional to initial wealth due to the Age Pension payment. Forth, with the same initial wealth balance and utility parameter settings, the optimal consumption level is different between males and females due to gender differences in mortality.

Chen, W., Minney, A., Toscas, P., Koo, B., Zhu, Z., and Pantelous, A. A. (2021c). "Personalised drawdown strategies and partial annuitisation to mitigate longevity risk." In: *Finance Research Letters* 39 (101644).
Despite the importance of drawdown strategies under a defined contribution system with increased longevity risk, little guidance to retired and retiring members has been forthcoming from superannuation funds. This paper provides a do-it-yourself drawdown design for members of superannuation funds along with comparison studies on a range of retirement income strategies under an array of realistic scenarios. A stochastic economic scenario generator is used to simulate the uncertain outcomes of different drawdown strategies during retirement. The impact of annuitisation for mitigating longevity risk under government pension rules and the selection of personalised drawdown and annuitisation strategies for retirement are examined.

Cheng, Y., Gibson, P., and Guo, T. (2017). "Life Quality and Health Costs in Late Retirement." In: *SSRN e-Print*.
Individuals are living longer due to the advancement of medical technology and nutrition quality. Are the elderly enjoying retirement in those extended years with good quality of life, or, are they simply alive? Using data from the Health and Retirement Study (HRS) and the Consumption and Activities Mail Survey (CAMS), this study contributes to the literature by presenting empirical evidence on how individuals spend time in retirement. The results show that retirees on average do not spend their time significantly different throughout retirement. Most life tasks such as reading the paper or magazines, listening to music, playing sports or exercising, visiting others, and house cleaning are similar among retirees in different age groups. We also present evidence that retirees on average experience a spike in medical expenses late in retirement. We compare systematic withdrawal strategies with and without health costs risk quantifying the impact on portfolio sustainability.

Chien, C.-L. (2019). *Enhancing Retirement Success Rates in the United States*. Springer International Publishing. 113 pp.
Dr. Chien's book aims to help middle class retirees get a better handle on their retirement finances by ensuring that all of their key household assets are considered in the planning process. It is essential reading for retirees and their advisors who are seeking how to best position assets for a successful retirement.

Chiu, Y.-F., Hsieh, M.-H., and Tsai, C. (2019). "Valuation and analysis on complex equity indexed annuities." In: *Pacific-Basin Finance Journal* 57 (101175).
Equity-indexed annuities (EIAs) are popular products that eliminate the downside risk while still providing upside potential. Among the three major categories of EIAs, ratchet EIAs are the most popular. Ratchet EIAs with quanto features emerge due to differences in asset returns across countries. The literature covers the pricing of the EIAs that are not quantos, and this paper fills the hole. In deriving pricing formulas, we add an exchange rate model as well as a foreign risk-free rate model to the pricing framework of Black and Scholes. Our formulas cover quanto ratchet EIAs for both compound and simple versions that may have a return cap and employ two types of geometric return averaging. The numerical analyses illustrate how contract features and market parameters affect contract values. The results also highlight the significance of quantos in contract pricing.

Clare, A., Glover, S., Seaton, J., Smith, P. N., and Thomas, S. (2020). "Measuring sequence of returns risk." In: *The Journal of Retirement* 8(1), pp. 65–79.
The authors discuss the nature and importance of the concept of sequence risk, the risk that a bad return occurs at a particularly unfortunate time, such as around the point of maximum accumulation or the start of decumulation. This concept is especially relevant in the context of retirement savings, where the implications for withdrawal rates of a bad return can be particularly severe. They show how the popular glidepath or target-date savings products are very exposed to such risk. Three different measures of sequence risk are proposed, each of which is intended to inform investors of the probability that a chosen investment strategy may not deliver desired withdrawal rates, and hence these measures are intended to aid investment choices; conventional performance measures such as Sharpe or Sortino ratios are only indirectly related to this ability to achieve a given withdrawal experience. Finally, the authors note that, using US data, very simple portfolios comprising equities and bonds can achieve very low probabilities of failure to achieve popular desired withdrawal rates, such as 5% a year, as long as the equity component is smoothed by switching in and out of cash using a simple trend-following rule.

Clare, A., Seaton, J., Smith, P. N., and Thomas, S. (2017). "Reducing Sequence Risk Using Trend Following and the CAPE Ratio." In: *Financial Analysts Journal* 73(4), pp. 91–103.

The risk of experiencing bad investment outcomes at the wrong time, or sequence risk, is a poorly understood but crucial aspect of the risk investors face -particularly those in the decumulation phase of their savings journey, typically over the period of retirement financed by a defined contribution pension scheme. Using US equity return data for 1872-2014, we show how this risk can be significantly reduced by applying trend-following investment strategies. We also show that knowing a valuation ratio, such as the cyclically adjusted price-to-earnings (CAPE) ratio, at the beginning of a decumulation period is useful for enhancing sustainable investment income.

Clare, A., Seaton, J., Smith, P. N., and Thomas, S. (2021a). "Perfect Withdrawal in a Noisy World: Investing Lessons with and without Annuities while in Drawdown between 2000 and 2019." In: *The Journal of Retirement* 9(1), pp. 9–39.
This article shows how the relatively new concept of Perfect Withdrawal Rate can be used in assessing the appropriate sustainable withdrawal amounts from a pot of wealth. This concept can be applied equally to private retirement funds, endowments, and charities - and indeed in any context requiring regular withdrawals from an initial source of funds. The subject of estimating sustainable withdrawal rates usually falls back on describing the likely minimum safe withdrawal possibilities for various portfolio constructions over different decumulation periods. This analysis employs either a long period of historical data or a recombination of data in the form of Monte Carlo simulations. To illustrate the power of the Perfect Withdrawal concept, the article considers the case of someone who initiated retirement on January 1, 2000, at age 65 and, with the benefit of actual investment returns, assesses investment and withdrawal rate options and lessons to be learned from this experience. The article also introduces the concept and a methodology for purchasing a delayed annuity so that at age 85 (on December 31, 2019), the hypothetical retiree is fully transitioned from investment income to annuity income for the rest of their life, no matter how long that may be.

Clare, A. D., Seaton, J., Smith, P. N., and Thomas, S. H. (2019). "Absolute Momentum, Sustainable Withdrawal Rates and Glidepath Investing in US Retirement Portfolios from 1925." In: *SSRN e-Print*.
A significant part of the development in pension provision in many countries is the emergence of Date Funds or TDFs. In this paper we examine the proposition of de-risking through life and the guidance offered by TDFs in the decumulation phase following retirement. We investigate the withdrawal experience associated with Glidepath Investing in the US since 1925 for conventional bond-equity portfolios. We find one very powerful conclusion: that smoothing the returns on individual assets by simple absolute momentum or trend following techniques is a potent tool to enhance withdrawal rates, often by as much as 50% per annum! And, perhaps of even greater social relevance is that it removes the -tail of unfortunate withdrawal rate experiences, i.e. the bad luck of a poor sequence of returns early in decumulation. We show that diversifying assets over time by switching between an asset and cash in a systematic way is potentially more important for the retirement income experience than diversifying one portfolio across asset classes. We also show that Glidepath investing is only sensible within a few years of the target date. This finding provides succour to enthusiasts for target date investing in the face of the growing hostility in the literature.

Clare, A. D., Seaton, J., Smith, P. N., and Thomas, S. H. (2021b). "Can sustainable withdrawal rates be enhanced by trend following?" In: *International Journal of Finance & Economics* 26(1), pp. 27–41.
We examine the consequences of alternative popular investment strategies for the decumulation of fundsinvested for retirement through a defined contribution pension scheme. We examine in detail the viability of specific 'safe' withdrawal rates including the '4%-rule' of Bengen. We find two powerful conclusions. First that smoothing the returns on individual assets by simple trend following techniques is a potent tool to enhance withdrawal rates.Second, we show that while diversification across asset classes does lead to higher withdrawal rates than simple equity/bond portfolios, "smoothing" returns in itself is far more powerful a tool for raising withdrawal rates. In fact,smoothing the popular equity/bond portfolios (such as the 60/40 portfolio) is in itself an excellent and simple solution to constructing a retirement portfolio. Alternatively, trend following enables portfolios to contain more risky assets, and the greater upside they offer, for the same level of overall risk compared to standard portfolios. To anticipate our empirical findings, we find two powerful conclusions: Smoothing the returns on individual assets by simple trend following techniques (or similar) is a potent tool to enhance withdrawal rates. Although diversification across asset classes does lead to higher withdrawal rates than simple equity/bond portfolios, "smoothing" returns is a far more powerful tool for raising withdrawal rates; in fact, smoothing the popular equity/bond portfolios (such as the 60/40 portfolio) is an excellent and simple solution to constructing a retirement portfolio.

Clark, R., Maurer, R., and Mitchell, O. S. (2018a). "How Persistent Low Returns Will Shape Saving and Retirement." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.

Financial market developments over the past decade have undermined what was once thought to be conventional wisdom about saving, investment, and retirement spending. Foremost among these is the depressingly persistent and extended period of low capital market returns, driving concerns about how to rethink saving and investments in what can be called the "new normal." This chapter introduces the themes of the book: how we arrived at our current state of affairs and what changes need to be made to achieve adequate retirement incomes for future retirees, and exploring new designs for pension plan sponsors. With increasing life expectancy adding to the problem of low market returns, the chapter urges policymakers to start reforming now to ensure retirement financial security.

Clark, R. L., Hammond, R. G., Morrill, M. S., and Vanderweide, D. (2018b). "Annuity options in public pension plans: the curious case of social security leveling." In: *The Journal of Retirement* 6(1), pp. 33–44.
Social Security Leveling is an annuity option that allows defined benefit pension participants to receive a level income throughout retirement by taking a larger pension benefit before they reach Social Security eligibility. This option can enable retirees to access a greater proportion of their employer pension wealth at younger ages and have a smoother income stream in retirement, at the cost of a lower pension payment during later retirement years. To evaluate the desirability of this option, a retiree must consider pricing, anticipated life expectancy, and income needs throughout retirement. We show that plan design features of this annuity option favor those who expect a shorter-than-average period of retirement and who have higher personal discount rates. This study uses detailed administrative records on all North Carolina state and local government retirees from 2009 through 2014. Among individuals claiming retirement benefits before age 62, over 20% chose the Social Security Leveling annuity option. Using multivariate regression analysis, this study finds evidence that North Carolina public sector retirees choose the Social Security Leveling option in a manner that is consistent with predictions.

Collins, P. J. (2016). "Annuities and retirement income planning." In: *CFA Foundation Research Foundation Briefs*.
Annuitization is one asset management strategy for retirees seeking to secure lifetime income. The US annuity marketplace offers a variety of annuity contracts, including single premium annuities, advanced life deferred annuities, variable annuities with lifetime income guarantee riders, and ruin contingent deferred annuities. Advisers seeking to provide guidance to clients in or near retirement can benefit by understanding (1) the arguments both for and against annuitization and (2) how a client's interests might be best represented in the marketplace. Important annuity contract provisions are highlighted and briefly discussed so the adviser can become more familiar with retirement-planning options.

Collins, P. J., Lam, H. D., and Stampfli, J. (2015). "How Risky is Your Retirement Income Risk Model?" In: *Financial Services Review* 24(3).
Adequately sustaining lifetime income is a critical portfolio objective for retired investors. This article provides a brief review of various retirement income modeling approaches including historical back testing, Monte Carlo simulations, and other risk modeling techniques. Implausible assumptions underlying risk models may mislead investors concerning the risk and return expectations of their investment strategies. We compare risk models, evaluate their credibility, and demonstrate how overly-simplified models may distort the risks retired investors face. Failure rate differences are stark: 4 percent at the low end versus 49 percent at the high end. The article ends with general comments regarding model risk and practitioner investment advice.

Collins, P. J. and Stampfli, J. (2019). "Sequence of Returns Risk Reconsidered." In: *Retirement Management Journal* 8(1).
In the process of evaluating various retirement income strategies, it appears that, for investors not endowed with substantial wealth relative to consumption demands, sequence of returns risk is operative throughout retirement. We explore, in an asset-liability modeling context, the reasons why sequence risk exists throughout the planning horizon and why it can be particularly acute at the end of an investor's life span. Given the nature of this risk, prudent asset management benefits from developing appropriate risk metrics, and from implementing credible monitoring, evaluation, and communication procedures. Two case studies focus on sequence of returns risk. They present risk metrics designed to answer the following questions: (1) is the investor's retirement income strategy feasible; (2) if yes, is it sustainable; and (3) does it allow sufficient flexibility to provide security in the face of financial shocks? The risk metrics employ information derived from both investment simulation models and actuarial calculations.

Consigli, G. and Di Tria, M. (2018). "Asset-liability management and goal-based investing for retail business." In: *International Journal of Financial Engineering and Risk Management* 2(4), p. 308.
The industry of online personal financial services is expected over the next years to absorb an increasing share of households and individuals' savings and investment decisions with a parallel expansion of tailored decision tools

and underlying methodological developments. In this article we present a dynamic stochastic optimisation model formulated to tackle a long-term optimal wealth management problem-based explicitly on the introduction of consumption and investment goals with a terminal inflation-adjusted retirement target. By embedding a goal-based investing philosophy in a dynamic framework we provide a reference modelling approach for increasingly popular households asset-liability management services. In a discrete setting we show that a dynamic stochastic programming formulation will lead to a highly realistic representation and solution of an otherwise hardly manageable optimisation problem and it is consistent with computer-aided decision support tools' operational requirements.

Consigli, G., Iaquinta, G., Moriggia, V., di Tria, M., and Musitelli, D. (2012). "Retirement planning in individual asset liability management." In: *IMA Journal of Management Mathematics* 23(4), pp. 365–396.
Increasing financial pressure on State-controlled pension systems has caused, over the last two decades or so, an unprecedented effort by private pension funds (PFs) and insurance companies to issue new types of retirement vehicles. This article investigates the effects of such widespread phenomenon from the perspective of individual asset liability management. A multistage stochastic programming problem has been formulated with investment opportunities including PFs, unit-linked contracts and variable life annuities. The introduction of a specific risk measure with respect to a desirable retirement income stream and a planning horizon spanning the entire individuals' working life helps to analyse the implications of observed market dynamics on retirement strategies. We present comparative results focusing on the retirement planning problem for three representative individuals carrying different time horizons but common retirement goals. The results show the benefits over traditional pension accumulation plans of dynamic strategies based on mixed portfolios of retirement products.

Consiglio, A., Cocco, F., and Zenios, S. (2006). "Scenario optimization asset and liability modelling for individual investors." In: *Annals of Operations Research* 152(1), pp. 167–191.
We develop a scenario optimization model for asset and liability management of individual investors. The individual has a given level of initial wealth and a target goal to be reached within some time horizon. The individual must determine an asset allocation strategy so that the portfolio growth rate will be sufficient to reach the target. A scenario optimization model is formulated which maximizes the upside potential of the portfolio, with limits on the downside risk. Both upside and downside are measured vis-a-vis the goal. The stochastic behavior of asset returns is captured through bootstrap simulation, and the simulation is embedded in the model to determine the optimal portfolio. Post-optimality analysis using out-of-sample scenarios measures the probability of success of a given portfolio. It also allows us to estimate the required increase in the initial endowment so that the probability of success is improved.

Coppola, M., Russolillo, M., and Simone, R. (2020). "On the management of retirement age indexed to life expectancy: a scenario analysis of the Italian longevity experience." In: *The Journal of Risk Finance* 21(3), pp. 217–231.
Purpose This paper aims to measure the financial impact on social security system of a recently proposed indexation mechanism for retirement age by considering the Italian longevity experience. The analysis is motivated by the progressive increase in life expectancy at advanced age, which is rapidly bringing to the fore noticeable socio-economic consequences in most industrialized countries. Among those, the impact on National Social Security systems is particularly relevant if people live longer than expected; this will lead to greater financial exposure for pension providers. Design/methodology/approach Referring to the Italian population for illustrative purposes, the authors contemplate different scenarios for mortality projection methods and for the implementation of pension age shift while accounting for gender and cohort gaps and model risk. Synthetic indicators to measure the impact of the indexation mechanism on social security system are introduced on the basis of pension cash flows. Findings An indexation policy that manages gender gap while adjusting retirement age for varying life expectancy is proposed. As a result, sustainability of public retirement expenditure is improved. Originality/value The paper is a concise scenario analysis of the reduction of costs and risks that pension providers would have if the system resorted to link retirement age to life expectancy. The ideas fostered by the paper follow a recent proposal of the Authors on a flexible retirement scheme that deals with model risk for mortality projection and accounts for gender gap in mortality rates.

Correia, C. L., Sayre, R., and Allen, J. C. (2017). "Avoiding the Medicaid Trap: A Step-by-Step Guide to Estate Planning." In: *The Journal of Retirement* 5(2), pp. 96–103.
Until an elderly relative suddenly needs long-term care and cannot afford to pay for it, most Americans do not appreciate how difficult the Medicaid legal system is to navigate. With long-term services ranging from a median monthly average of USD1,473 for limited services to USD7,698 for comprehensive services, providing

long-term care for a loved one forces the majority of Americans to apply for Medicaid benefits. Many individuals find themselves trapped to transfer any of their estate prior to applying for Medicaid because of the five-year lookback penalty. This article discusses seven estate planning steps that will help individuals protect their net worth while also securing eligibility for Medicaid benefits.

Cosares, S. (2013). "Generating a Family of Optimal Glide Paths for Investment Planning with Target Dates." In: *The Journal of Wealth Management* 16(3), pp. 43–53.
The article presents an optimization model for an approach to financial planning whereby investment decisions over time follow a (decreasing) glide path. With this strategy, an investment advisor would recommend riskier assets for the immediate future in order to take advantage of their larger expected returns. The author presumes that, over time, the proportion of more conservative investments in the portfolio will increase. Thus, it is likely to achieve expected investment-gain targets while reducing the risk of severe losses in wealth at the time when the money is actually needed (the target dates). There are a number of benefits to front-loading investment risk in this way, even though it comes with an increase in overall risk, as measured by the variance in the random variable representing total accumulated wealth. To accommodate varying objectives among investors regarding this trade-off, the model includes a parameter for glide-path intensity that allows the investor to select the level of deviation from the risk-minimizing solution, which would tend to recommend investments having roughly the same risk return level for every year in the planning horizon (i.e., a straight-line strategy). The model then focuses on the traditional multi-goal and multitime-horizon problem and uses simulations to demonstrate the benefits associated with a flexible implementation of the glide-path strategy that further minimizes the risk of missing a goal.

Costa, P., Pakula, D., and Clarke, A. S. (2021). *Fuel for the F.I.R.E.: Updating the 4% rule for early retirees*. Tech. rep. Vanguard.
Followers of the F.I.R.E. movement-Financial Independence Retire Early-have long relied on the "4% rule" to determine what they can withdraw from their portfolios over perhaps 40 or 50 years in retirement. The 4% rule, however, was originally designed for investors with a 30-year retirement horizon. And the simplifying assumptions it embeds about future returns, diversification, and fees can limit a retirement plan's viability over both shorter and longer horizons. Using Vanguard's principles for investing success, this research paper illustrates how F.I.R.E. investors can improve their chances of financing an early retirement.

Crook, M. and Baredes, M. (2015). "Total Wealth Allocation: Liquidity, Longevity, and Legacy." In: *The Journal of Wealth Management* 18(3), pp. 18–26.
In general, households use old technology for asset allocation decisions. Target-risk portfolio models are pervasive across the industry, and risk-profile questionnaires continue to form the foundation for strategic asset allocation decision-making. Better technology would utilize liability relative optimization and a holistic balance sheet and incorporate behavioral finance considerations. However, these concepts have not been integrated into most practitioners' businesses, even though the core work on each was completed 20-30 years ago. The authors propose a scalable, total wealth model for integrating liability relative optimization into households' portfolios. The basis for the model is an asset segmentation approach that refl ects the nuances of a households' balance sheet and future objectives and enables liability relative optimization and total wealth considerations to be applied appropriately.

Crook, M. and Sutedja, R. (2017). "Will Long-Term Care Ruin Retirement Plans?" In: *The Journal of Retirement* 4(3), pp. 42–50.
Long-term care expenses represent a known unknown in retirement planning. A large majority of households will need some sort of long-term care support as they age, and costs for certain types of long-term care, like nursing homes, can run over USD100,000 per year. However, most families will not incur expenses of this order. Who is at risk? We use a simulation-based framework to analyze how often long-term care expenses are likely to cause ruin in an otherwise prudently constructed financial plan. Our framework also enables us to estimate long-term care usage by type of service and calculate a distribution of total care costs for a particular household. We find that roughly 85% of older couples will utilize long-term care. Long-term care expenses impact financial plan sustainability at a declining rate as wealth increases from USD1 million to USD10 million. At a portfolio value of USD1 million, adding long-term care expenses to the simulation results in ruin (defined as depleting the portfolio entirely prior to the death of both members of the couple) about 30% of the time. With certain caveats discussed in the article, we estimate that failure rates increase by 9 percentage points for USD5 million households, and increase by about 5 percentage points for USD10 million households. We also find that female same-sex households are particularly at risk due to greater longevity and higher incidence of long-term care usage.

In summary, financial plans that do not incorporate long-term care expenses can significantly overestimate the long-term sustainability of the plan.

Crook, M. W. (2019). "Liabilities Matter: Improving Target Date Glidepath Construction through Liability Adaptive Asset Allocation." In: *The Journal of Retirement* 7(1), pp. 44–57.
Target date glidepath funds have become extraordinarily popular in defined contribution plans during the past decade. Most glidepath strategies are based, fundamentally, on the idea that investors have bond-like human capital that slowly depletes during their career, and therefore investors should complement their human capital depletion with a declining equity glidepath as they progress toward retirement. Despite the theoretical attractiveness, the human capital approach unfortunately falls flat from an empirical and practical standpoint. In this article the author proposes a new liability-adaptive approach for target date glidepath construction. By moving away from a human capital approach and instead focusing on future retirement liabilities, the author shows that plan sponsors can improve outcomes for participants. The author also believes this framework enables sponsors to better communicate the underlying assumptions in their glidepath offering to participants who are planning for retirement. Finally, the author shows how financial advisors can use the same framework with individual investors and families to improve outcomes on a customized basis.

Cvitanic, J., Kou, S., Wan, X., and Williams, K. L. (2020). "Pi portfolio management: reaching goals while avoiding drawdowns." In: *SSRN e-Print*.
We propose an approach to portfolio selection that explicitly takes into account investors simultaneous investment objectives, such as achieving target return levels and avoiding specific drawdowns. Our approach is consistent with both standard and non-standard risk preferences, such as those of prospect theory. Instead of asking the investor to choose between lotteries, transforming this into an estimate of the risk preferences, and then selecting the portfolio accordingly, we propose to directly offer investors a choice between lotteries" with varying probabilities of experiencing target levels of profits and losses. Our approach enables investors to flexibly assess the effectiveness of portfolio choices under various conditions. We discuss implementation considerations and compare our approach to traditional mean-variance portfolio selection.

D'Acunto, F. and Rossi, A. G. (2021). "New Frontiers of Robo-Advising: Consumption, Saving, Debt Management, and Taxes." In: *SSRN e-Print*.
Traditional forms of robo-advice were targeted to help individuals make portfolio allocation decisions. Based on the balance-sheet view of households, the scope for robo-advising has been expanding to many other personal-finance choices, such as households' saving and consumption decisions, debt management, mortgage uptake, tax management, and lending. This chapter reviews existing research on these new functions of robo-advising with a special emphasis on the questions that are still open for researchers across several disciplines. We also discuss the attempts to optimize jointly all personal-finance decisions, which we label "holistic robo-advisors". We conclude by assessing fruitful avenues for research and practice in finance, computer science, marketing, decision science, information systems, law, and sociology.

Dadashi, H. (2020a). "Optimal investment strategy post retirement without ruin possibility: A numerical algorithm." In: *Journal of Computational and Applied Mathematics* 363, pp. 325–336.
We give a numerical algorithm based on the policy iteration method to find approximations of solution of an HJB equation arising from the portfolio optimization problem post retirement for a member of a defined contribution plan. In our numerical algorithm, we apply a fully implicit finite difference scheme to the HJB equation on a bounded domain. In formulating the optimal control problem, the loss function is written using a target function during the decumulation phase and at the terminal time. Following Di Giacinto et al. (2014), a minimum guarantee for the final annuity is considered. Finally, we give the simulation results for the final annuity, optimal investment strategy and optimal wealth process, for different levels of risk aversion and different loss functions.

Dadashi, H. (2020b). "Optimal investment–consumption problem: Post-retirement with minimum guarantee." In: *Insurance: Mathematics and Economics* 94, pp. 160–181.
We study the optimal investment-consumption problem for a member of defined contribution plan during the decumulation phase. For a fixed annuitization time, to achieve higher final annuity, we consider a variable consumption rate. Moreover, to have a minimum guarantee for the final annuity, a safety level for the wealth process is considered. To solve the stochastic optimal control problem via dynamic programming, we obtain a Hamilton-Jacobi-Bellman (HJB) equation on a bounded domain. The existence and uniqueness of classical solutions are proved through the dual transformation. We apply the finite difference method to find numerical approximations of the solution of the HJB equation. Finally, the simulation results for the optimal investment-consumption

strategies, optimal wealth process and the final annuity for different admissible ranges of consumption are given. Furthermore, by taking into account the market present value of the cash flows before and after the annuitization, we compare the outcomes of different scenarios.

Daher, M., Dahling, D., Pritchard, R., and Tseng, B. (2020). *From Accumulation to Decumulation – Why It Matters and What Plan Sponsors Should Know*. Tech. rep. Fidelity Investments.
Through conversations with Fidelity clients – and consistent with industry surveys – we have noted increasing interest from DC plan sponsors in allowing retirees to stay in plan.
Retirees and pre-retirees need help balancing risks – including investment, longevity, liquidity, and utilization risk – to translate accumulated savings into sustainable income.
Fidelity's recordkeeping data shows that older DC participants may not fully appreciate the importance of appropriate asset allocation and diversification as they age.
Sponsors wanting to allow participants to stay in plan should consider offering post-retirement tools and education along with strategies and products that can help retirees to allocate their assets appropriately, generate income, and safely draw down savings during retirement.

Dai, W. and Medhat, M. (2021). "US Inflation and Global Asset Returns." In: *SSRN e-Print*.
We study the relation between US inflation and the performance of global asset classes (including bonds, stocks, industry portfolios, factor premiums, commodities, and REITs), both over a long sample period (1927-2020) and over the most recent 30 years (1991-2020). We find that most assets had positive average real returns in both low- and high-inflation years. While average real returns were lower in years with higher inflation for most assets, many of the differences are not statistically reliable, especially among non-bond assets and in more recent times. We also find mostly weak correlations over time between nominal returns and inflation, including contemporaneous, lagged, expected, and unexpected inflation. The notable exceptions are energy stocks and commodities, where there are reliably positive correlations with both expected and unexpected inflation, but our results also suggest both assets are too volatile to be an effective inflation hedge. Our results confirm the potential of most asset classes to outpace inflation over the long term and suggest that, for investors prioritizing the preservation of purchasing power, inflation-indexed securities may be a more appropriate inflation hedge than commonly suggested alternatives.

Dang, D.-M., Forsyth, P. A., and Vetzal, K. R. (2017). "The 4 percent strategy revisited: a pre-commitment mean-variance optimal approach to wealth management." In: *Quantitative Finance* 17(3), pp. 335–351.
In contrast to single-period mean-variance (MV) portfolio allocation, multi-period MV optimal portfolio allocation can be modified slightly to be effectively a down-side risk measure. With this in mind, we consider multi-period MV optimal portfolio allocation in the presence of periodic withdrawals. The investment portfolio can be allocated between a risk-free investment and a risky asset, the price of which is assumed to follow a jump diffusion process. We consider two wealth management applications: optimal de-accumulation rates for a defined contribution pension plan and sustainable withdrawal rates for an endowment. Several numerical illustrations are provided, with some interesting implications. In the pension de-accumulation context, Bengen (1994)?s [J. Financial Planning, 1994, 7, 171?180], historical analysis indicated that a retiree could safely withdraw 4 percent of her initial retirement savings annually (in real terms), provided that her portfolio maintained an even balance between diversified equities and U.S. Treasury bonds. Our analysis does support 4 percent as a sustainable withdrawal rate in the pension de-accumulation context (and a somewhat lower rate for an endowment), but only if the investor follows an MV optimal portfolio allocation, not a fixed proportion strategy. Compared with a constant proportion strategy, the MV optimal policy achieves the same expected wealth at the end of the investment horizon, while significantly reducing the standard deviation of wealth and the probability of shortfall. We also explore the effects of suppressing jumps so as to have a pure diffusion process, but assuming a correspondingly larger volatility for the latter process. Surprisingly, it turns out that the MV optimal strategy is more effective when there are large downward jumps compared to having a high volatility diffusion process. Finally, tests based on historical data demonstrate that the MV optimal policy is quite robust to uncertainty about parameter estimates. In contrast to single-period mean-variance (MV) portfolio allocation, multi-period MV optimal portfolio allocation can be modified slightly to be effectively a down-side risk measure. With this in mind, we consider multi-period MV optimal portfolio allocation in the presence of periodic withdrawals. The investment portfolio can be allocated between a risk-free investment and a risky asset, the price of which is assumed to follow a jump diffusion process. We consider two wealth management applications: optimal de-accumulation rates for a defined contribution pension plan and sustainable withdrawal rates for an endowment. Several numerical illustrations are provided, with some interesting implications. In the pension de-accumulation

context, Bengen (1994)?s [J. Financial Planning, 1994, 7, 171?180], historical analysis indicated that a retiree could safely withdraw 4 percent of her initial retirement savings annually (in real terms), provided that her portfolio maintained an even balance between diversified equities and U.S. Treasury bonds. Our analysis does support 4 percent as a sustainable withdrawal rate in the pension de-accumulation context (and a somewhat lower rate for an endowment), but only if the investor follows an MV optimal portfolio allocation, not a fixed proportion strategy. Compared with a constant proportion strategy, the MV optimal policy achieves the same expected wealth at the end of the investment horizon, while significantly reducing the standard deviation of wealth and the probability of shortfall. We also explore the effects of suppressing jumps so as to have a pure diffusion process, but assuming a correspondingly larger volatility for the latter process. Surprisingly, it turns out that the MV optimal strategy is more effective when there are large downward jumps compared to having a high volatility diffusion process. Finally, tests based on historical data demonstrate that the MV optimal policy is quite robust to uncertainty about parameter estimates.

Das, S. and Varma, S. (2020). "Dynamic Goals-Based Wealth Management using Reinforcement Learning." In: *Journal of Investment Management* 18 (2), pp. 1–20.
We present a reinforcement learning (RL) algorithm to solve for a dynamically optimal goal-based portfolio. The solution converges to that obtained from dynamic programming. Our approach is model-free and generates a solution that is based on forward simulation, whereas dynamic programming depends on backward recursion. This paper presents a brief overview of the various types of RL. Our example application illustrates how RL may be applied to problems with path-dependency and very large state spaces, which are often encountered in finance.

Das, S. R., Ostrov, D., Radhakrishnan, A., and Srivastav, D. (2020a). "Dynamic portfolio allocation in goals-based wealth management." In: *Computational Management Science* 17, pp. 613–640.
We report a dynamic programming algorithm which, given a set of efficient (or even inefficient) portfolios, constructs an optimal portfolio trading strategy that maximizes the probability of attaining an investor's specified target wealth at the end of a designated time horizon. Our algorithm also accommodates periodic infusions or withdrawals of cash with no degradation to the dynamic portfolio's performance or runtime. We explore the sensitivity of the terminal wealth distribution to restricting the segment of the efficient frontier available to the investor. Since our algorithm's optimal strategy can be on the efficient frontier and is driven by an investor's wealth and goals, it soundly beats the performance of target date funds in attaining investors' goals. These optimal goals-based wealth management strategies are useful for independent financial advisors to implement behavioral-based FinTech offerings and for robo-advisors.

Das, S. R., Ostrov, D., Radhakrishnan, A., and Srivastav, D. (2022a). "Dynamic optimization for multi-goals wealth management." In: *Journal of Banking & Finance* 140 (106192).
We develop a dynamic programming methodology that seeks to maximize investor outcomes over multiple, potentially competing goals (such as upgrading a home, paying college tuition, or maintaining an income stream in retirement), even when financial resources are limited. Unlike Monte Carlo approaches currently in wide use in the wealth management industry, our approach uses investor preferences to dynamically make the optimal determination for fulfilling or not fulfilling each goal and for selecting the investor's investment portfolio. This can be computed quickly, even for numerous investor goals spread over different or concurrent time periods, where each goal may be all-or-nothing or may allow for partial fulfillment. The probabilities of attaining each (full or partial) goal under the optimal scenario are also computed, so the investor can ensure the algorithm accurately reflects their preference for the relative importance of each of their goals. This approach vastly outperforms static portfolio strategies and target-date funds, widely used in the wealth management industry.

Das, S. R., Ostrov, D. N., Casanova, A., Radhakrishnan, A., and Srivastav, D. (2020b). "Combining Investment and Tax Strategies for Optimizing Lifetime Solvency under Uncertain Returns and Mortality." In: *SSRN e-Print*.
We consider an investor who is looking to maximize their probability of remaining solvent throughout their lifetime by using an algorithm that aims to optimize their investment allocation strategy and optimize their tax strategy for withdrawal allocations between tax deferred accounts (TDAs), Roth accounts, and taxable stock and bond accounts. Our optimization works with stochastic investment returns and stochastic mortality. We find that optimizing the investment strategy (via dynamic programming) has a much larger impact on the investor remaining solvent than optimizing the tax strategy (via Monte Carlo and numerical optimization). This result is key to effectively optimizing both strategies simultaneously. We show that our optimized investment strategy soundly beats a standard target date fund strategy, while our novel optimized tax strategy displays the optimal desired properties suggested by non-stochastic tax optimization research.

Das, S. R., Ostrov, D. N., Casanova, A., Radhakrishnan, A., and Srivastav, D. (2022b). "Optimal Goals-Based Investment Strategies For Switching Between Bull and Bear Markets." In: *The Journal of Wealth Management* 24(4), pp. 8–36.
We solve a dynamic, long-horizon, goals-based wealth management problem, given different investment regimes. In a world with a good regime (bull market) and a bad regime (bear market), an investor who is cognizant that regime switching occurs has the potential to do better than an investor who assumes only one regime. However, models with more than one regime incur the additional risk of regime uncertainty. Investors must be able to predict which regime is governing the market with reasonable levels of confidence, or they can be worse off than investors who assume just one regime. Using data from recent history, we develop a framework that determines how accurate regime prediction needs to be to achieve gains from a regime-cognizant goals-based investing approach.

Das, S. R. and Ross, G. (2021). "The Role of Options in Goals-Based Wealth Management." In: *SSRN e-Print*.
We develop a facile methodology using dynamic programming for goals-based wealth management over long horizons where rebalancing uses the standard securities and also derivative securities. A kernel density estimation approach is developed to accommodate any number of derivative assets, solving a high dimensional problem with fast computation. The approach easily accommodates skewed and fat-tailed distributions. Portfolio performance is much better with the use of options, especially for investors with aggressive goals.

De La Pena, J. I., Fernandez-Ramos, M. C., and Garayeta, A. (2021). "Cost-Free LTC Model Incorporated into Private Pension Schemes." In: *International Journal of Environmental Research and Public Health* 18(5) (2268).
Long-term care coverage is not integrated into an individual's retirement strategy. It is an additional public health service that is not considered into private pension funds. Nevertheless, this coverage is not sufficient due to the problems of financial sustainability of the public pension systems. However, there are large sums in pension plans dedicated to paying retirement pensions that can be transformed into support for long-term care coverage. This paper develops a mechanism of pension transformation through the different mortality of the beneficiary when becoming a dependent beneficiary. This mechanism allows the beneficiary to convert their pension to LTC support at their own choice, without increasing the cost of the private pension scheme. The proposed model provides consistency in the pension that a retiree receives and adapts it to a retiree's life expectancy: the retiree receives a higher pension when he/she needs it most.

de Mendonça, H. F., Garcia, P. M., and Vicente, J. V. M. (2021). "Rationality and anchoring of inflation expectations: An assessment from survey-based and market-based measures." In: *Journal of Forecasting* 40(6), pp. 1027–1053.
The aim of this paper is twofold. Firstly, we test the rationality of survey-based and market-based inflation expectations. Secondly, we investigate whether they indicate a different performance of the central bank in anchoring inflation expectations. Briefly, this paper verifies if inflation expectations' proxies display the same features regarding rationality and anchoring. Using data from the Brazilian market, we present robust evidence that both survey-based and market-based inflation expectations have useful content to explain realized inflation. Moreover, we find that these proxies of inflation expectations provide different assessments of the central bank's ability to anchor inflation expectations. The findings point out that central banks must monitor both survey-based and market-based inflation expectations to improve their monetary policy conduct.

de Villiers, J. U. and Roux, E.-M. (2019). "Reframing the Retirement Saving Challenge: Getting to a Sustainable Lifestyle Level." In: *Journal of Financial Counseling and Planning* 30(2), pp. 277–288.
An increasing number of individuals will be unable to retire comfortably amidst an international retirement savings crisis. Research suggests that behavioral factors contribute to inadequate retirement savings. We present a procedure that reframes the retirement savings decision, aimed at alleviating some of the negative effects of the behavioral factors. This procedure shifts the focus from the required wealth at retirement (the future) to the lifestyle an individual can afford to maintain now (the present). A sustainable lifestyle level (SLL) approach is expressed mathematically and illustrated with practical examples. The SLL approach offers a practical tool for retirement planning professionals to present recommendations that are simple and easy to understand for individuals faced with complex retirement planning decisions.

Deelstra, G., Devolder, P., and Melis, R. (2021). "Optimal annuitisation in a deterministic financial environment." In: *Decisions in Economics and Finance* 44(1), pp. 161–175.
The global reforms to public pension schemes over the last thirty years have progressively reduced individuals' post-retirement social security income. In order to compensate for this, individuals join pension funds and individual plans to increase their wealth at retirement. These types of fully funded plans generally give individuals

the opportunity to withdraw the capital accumulated into their scheme or to convert it into an annuity. In this paper, we analyse individuals' post-retirement choices to allocate the wealth at retirement between consumption, risk-free investments and a life annuity. We develop a discrete time optimisation model, in a deterministic framework, with a constant relative risk aversion (CRRA) utility function. We study the effect of a bequest motive and the annuity rate used by the insurer on the optimal choice. Several numerical applications are presented to illustrate the optimal annuitisation decision results and the optimal consumption paths.

Dehm, C. (2020). "Stochastic mortality : modelling and optimal investment." PhD thesis. University of Ulm.
Nothing is more certain than death, nothing more uncertain than its hour. Even today the timepoint of death is unknown, but death will occur with certainty. In this work, the uncertainty of death time is included in our considerations of mortality modelling, optimal investment as insured under uncertain death time and optimal investment as insurance company under mortality risk. This work provides three contributions to scientific literature: In the first part, we suggest causality-based modifications of the Gompertz law and the Lee Carter model with focus on main death causes. In an application on German health data, we identify three main death causes, namely 'diseases of circulatory system', 'neoplasms' and 'diseases of the respiratory system', completed by modelling 'accumulated remaining diseases'. Using VAR(p)-time series models, we forecast life expectancy and life annuity for the total population as well as for the genders separately. The causal Lee Carter model for men matches the holistic Lee Carter model best, with nearly same life expectancy and life annuity. Moreover, we find that the predominant marginal impact on life expectancy are 'neoplasms' with an average loss of about 5 years. In the second contribution, we consider an optimal investment problem inspired by a life insurance contract. These contracts usually have an option-like form with the additional difficulty that the timepoint of benefit payment (e.g. death of the insured) is uncertain. We generalize the known Merton problem in two directions: we consider non-concave optimization with uncertain investment maturity. In fact, we consider general non-concave utility functions and prove an existence and optimality result. To illustrate our findings, we do a numerical study, which suggests that the occurrence of a possible premature stop leads to a reduced performance of the optimal solution in terms of certainty equivalent. Due to the long term of an insurance contract, a typical life insurance company is subject to interest and investment risks as well as mortality risk. In our third contribution, we consider these risks as jump-diffusion processes. Through (interest) zero-coupon bond and longevity bond, interest rate risk and mortality risk can be (partially) traded on the capital market. In this Levy-market, we derive an extended HJB equation system to be able to find closed-form solutions of the optimal trading strategy and the expected terminal value. We prove necessity and sufficiency of the HJB equation approach. Moreover, we show a general L2-asymptotic of truncated Levy processes.

DeJong, J. C. and Robinson, J. H. (2022). "A Case Study in Sequence Risk: A 20-Year Retrospective on the Impact of the 2000-2002 and 2007-2009 Bear Markets on Retirement Nest Egg Sustainability." In: *The Journal of Wealth Management* 24(4), pp. 37–68.
The first decade of the 2000s saw two prolonged bear markets, both of which produced declines of over 50% in value in the S&P 500 Index from peak to trough. In total, the ten-year period from March 1999 through February 2009 saw the stock market lose more than 30 percent of its value. This period represented the worst decade for the U.S. stock market since the Great Depression. For consumers who had the misfortune of retiring in 1999, this "lost decade" embodied the definition of sequence of returns risk. Researchers at the time could only speculate as to the impact such sharply negative returns might have on the long-term sustainability of retiree spending portfolios.

del Carmen Valls Martínez, M., Santos-Jaén, J. M., Amin, F.-u., and Martín-Cervantes, P. A. (2021). "Pensions, Ageing and Social Security Research: Literature Review and Global Trends." In: *Mathematics* 9(24), p. 3258.
Pension systems are one of the fundamental pillars of the welfare state. The ageing of the population caused by longer life expectancy and low birth rates has led to a crisis in the public pension system in developed countries. Changes for the system's sustainability are necessary, and the scientific literature on the subject is abundant, especially in recent years. This article aims to carry out a bibliometric analysis of the research carried out to date, highlighting, in turn, future lines of research. The study was carried out on a total of 1287 articles published from 1936 to 2021 and found in the Scopus database. The SciMAT, VOSviewer, and Datawrapper tools were used to analyse the most important articles, authors, countries, and institutions by volume of production and citations, as well as the relationships between them. Likewise, the most important keywords and their evolution over time were highlighted, obtaining the main focus of the research. In addition to the general analysis, a specific study was carried out in the area of Mathematics. The results show that the leading countries are the United Kingdom, the USA, and the Netherlands. On the other hand, the lead subject area in which these articles have

been published is Economics, Econometrics, and Finance. The research trends are sustainability, pension reform related to ageing, and pension insurance.

DeLibero, R. and Pfau, W. D. (2017). "Life Insurance as a Retirement Income Tool." In: *SSRN e-Print*.
Given its tax-preferential treatment, careful study is warranted to determine whether life insurance can play an important role in an overall retirement portfolio. This study develops hypothetical scenarios for different types of individuals with varying ages and distribution periods, while using a historical outlook to determine the proper structure of a variable universal life insurance policy. We compare a variable universal life policy to different investment vehicles (both in qualified and non-qualified accounts) on an after-tax basis in order to better understand the potential tradeoff for tax-deferral and insurance fees within the life insurance.

Delong, L. and Chen, A. (2016). "Asset allocation, sustainable withdrawal, longevity risk and non-exponential discounting." In: *Insurance: Mathematics and Economics* 71, pp. 342–352.
The present paper studies an optimal withdrawal and investment problem for a retiree who is interested in sustaining her retirement consumption above a pre-specified minimum consumption level. Apparently, the withdrawal and investment policy depends substantially on the retiree's health condition and her time preferences (subjective discount factor). We assume that the health of the retiree can worsen or improve in an unpredictable way over her lifetime and model the retiree's mortality intensity by a stochastic process. In order to make the decision about the consumption and investment policy more realistic, we assume that the retiree applies a non-exponential discount factor (an exponential discount factor with a small amount of hyperbolic discounting) to value her future income. In other words, we consider an optimization problem by combining four important aspects: asset allocation, sustainable withdrawal, longevity risk and non-exponential discounting. Due to the non-exponential discount factor, we have to solve a time-inconsistent optimization problem. We derive a non-local HJB equation which characterizes the equilibrium optimal investment and consumption strategy. We establish the first-order expansions of the equilibrium value function and the equilibrium strategies by applying expansion techniques. The expansion is performed on the parameter controlling the degree of discounting in the hyperbolic discounting that is added to the exponential discount factors. The first-order equilibrium investment and consumption strategies can be calculated in a feasible way by solving PDEs.

Dempster, M. A. H., Kloppers, D., Medova, E., Osmolovsky, I., and Ustinov, P. (2016). "Lifecycle Goal Achievement or Portfolio Volatility Reduction?" In: *The Journal of Portfolio Management* 42(2), pp. 99–117.
This article is concerned with the use of currently available technology to offer individuals, financial advisors, and pension fund financial planners detailed prospective financial plans tailored to an individual's financial goals and obligations. By taking account of all an individual's prospective cash flows, including servicing current liabilities, and simultaneously optimizing prospective spending, saving, asset allocation, tax, and insurance, etc. using dynamic stochastic optimization, the authors compare the results of their goal-based fully dynamic strategy with the financial advisory industry's representative current best practices. These include piecemeal fixed-allocation portfolios for specific goals, target-date retirement funds, and fixed real-income post-retirement financial products, all using Markowitz mean-variance optimization based on the very general goal of minimizing portfolio volatility for a specific portfolio expected return over a finite horizon. Making use of the same data and marketcalibrated Monte Carlo stochastic simulation for all the alternative portfolio strategies, the authors find that flexibility is of key importance for both individual portfolio and spending decisions. The authors measure superiority by the certainty-equivalent increase in expected utility of individual lifetime consumption (gamma) and the extra initial capital required by an individual to put the dominated strategy on the same expected-utility footing as the optimal dynamic strategy (initial capital gap). They find that the adaptive dynamic goal-based portfolio strategy's performance is far superior to all the industry's Markowitz-based approaches. These empirical results should put paid to the commonly held view that the extra complexity of holistic dynamic stochastic models is not worth the marginal extra value obtained from their use.

Dempster, M. A. H. and Medova, E. A. (2011). "Asset Liability Management for Individual Households." In: *British Actuarial Journal* 16(2).
Personal finance is a challenging topic which can benefit from a scientific approach to individual financial planning. This paper presents an individual asset liability management (iALM) model for life cycle planning which uses the methodology of dynamic stochastic optimisation and incorporates ideas from both classical and behavioural finance. Its implementation is in the form of a decision support tool for use by financial advisers or wealth managers. The investment universe is given by a set of indices for major asset classes and their returns are simulated forward over the lifetime of a household. On the liability side the foreseen cash flows of incomes and outgoings are simulated and punctuated by life events such as illness and death. The household's utility

function is constructed for each time period over a range of monetary values in terms of household financial goals and preferences. Taxes and pension savings are treated using the tax shielded saving accounts specific to a national jurisdiction in terms of constraints in the optimisation sub-models. The paper goes on to present an analysis of iALM model recommendations for a representative UK household, together with an evaluation of the sensitivity of the financial plan generated to changes in market environments such as the 2007-9 crisis. The promise of this new technology is to bring modern decision support tools to individual investors in order to facilitate custom designed consumption, savings and investment policies.

Diamond, S., Boyd, S., Greenberg, D., Kochenderfer, M., and Ang, A. (2021). "Optimal Claiming of Social Security Benefits." In: *SSRN e-Print*.
Using a lifecycle framework with Epstein-Zin (1989) utility and a mixed-integer optimization approach, we compute the optimal age to claim Social Security benefits. Taking advantage of homogeneity, a sufficient statistic is the ratio of wealth to the primary insurance amount (PIA). If the investor's wealth to PIA ratio exceeds a certain threshold, individuals should defer Social Security for at least a year. The optimal threshold depends on mortality assumptions and an individual's utility preferences, but is less sensitive to capital market assumptions. The threshold wealth to PIA ratio increases from 5.5 for men and 5.2 for women at age 62 to 11.1 for men and 10.4 for women at age 69. Below the threshold wealth to PIA ratio, individuals claim Social Security to raise consumption. Above this level, investors can afford to fund consumption out of wealth for at least one year, and then claim a higher benefit.

Dickson, J. M., Bruno, M. A., and Wong, B. C. (2018). *A "BETR" approach to Roth conversions*. Tech. rep. Vanguard.
Investors typically decide whether to convert to a Roth IRA from a traditional IRA by comparing their current and expected future marginal tax rates. The rule of thumb has been that higher future tax rates make a conversion more desirable, while lower ones make it less so. (Given that future tax rates are uncertain for many reasons, many investors may want to diversify this tax risk through partial conversions.) We introduce a break-even tax rate (BETR) that yields a more accurate view of what future tax rate would make an investor indifferent to a conversion.

DiJoseph, M. A. (2020). *Spending guidelines to help ease retirees' market worries*. Tech. rep. Vanguard.
Staying the course does not have to mean standing still. In fact, Vanguard research explains how you can help clients build adaptable, resilient, and sustainable retirement spending plans. Our research paper From assets to income: A goals-based approach to retirement spending shows you how to help clients implement a personalized spending strategy. The resulting plan is a customized, responsive formula that can help reduce clients' anxiety and stress about their ability to meet retirement income goals, regardless of the market environment.

DiLellio, J. and Ostrov, D. N. (2019). "Constructing Tax Efficient Withdrawal Strategies for Retirees with Traditional 401(k)/IRAs, Roth 401(k)/IRAs, and Taxable Accounts." In: *SSRN e-Print*.
We construct an algorithm for United States retirees that computes individualized tax-efficient annual withdrawals from IRAs/401(k)s, Roth IRAs/Roth 401(k)s, and taxable accounts. Our algorithm applies a new approach that generates an individualized strategy that results in consistent improvements over non-individualized withdrawal strategies currently advocated by financial institutions and academics. Among other results, we quantifiably demonstrate why retirees should avoid, not seek, dividend producing stocks in their taxable accounts. Our model, which can work to optimize either portfolio longevity or the bequest to an heir, accommodates many salient tax code features, including dividends, different taxable lots, conversions, and required minimum distributions.

Dillschneider, Y., Maurer, R., and Schober, P. (2020). "Dynamic Portfolio Choice with Annuities When the Interest Rate Is Stochastic." In: *SSRN e-Print*.
This paper studies the optimal life cycle consumption and portfolio choice problem taking into account annuity risk due to stochastic interest rates. When the purchase of annuities is restricted to the retirement date, the annuitant is exposed to the risk of meeting low interest rates at the purchase date. This annuity risk can be diversified by spreading annuity purchases over the whole pre-retirement period. The numerical results of our life cycle model show that such temporal diversification enhances welfare for retirees up to 9% of certainty equivalent consumption and that welfare gains increase with higher interest rate risk.

Ding, G. and Marazzina, D. (2021). "Sensitivity of Optimal Retirement Problem to Liquidity Constraints." In: *arXiv e-Print*.
In this work we analytically solve an optimal retirement problem, in which the agent optimally allocates the risky investment, consumption and leisure rate to maximise a gain function characterised by a power utility

function of consumption and leisure, through the duality method. We impose different liquidity constraints over different time spans and conduct a sensitivity analysis to discover the effect of this kind of constraint.

Ding, Y., Li, Y., and Song, R. (2022). "Statistical Learning for Individualized Asset Allocation." In: *arXiv e-Print*.
We establish a high-dimensional statistical learning framework for individualized asset allocation. Our proposed methodology addresses continuous-action decision-making with a large number of characteristics. We develop a discretization approach to model the effect from continuous actions and allow the discretization level to be large and diverge with the number of observations. The value function of continuous-action is estimated using penalized regression with generalized penalties that are imposed on linear transformations of the model coefficients. We show that our estimators using generalized folded concave penalties enjoy desirable theoretical properties and allow for statistical inference of the optimal value associated with optimal decision-making. Empirically, the proposed framework is exercised with the Health and Retirement Study data in finding individualized optimal asset allocation. The results show that our individualized optimal strategy improves individual financial well-being and surpasses benchmark strategies.

Diris, B., Palm, F., and Schotman, P. (2015). "Long-Term Strategic Asset Allocation: An Out-of-Sample Evaluation." In: *Management Science* 61(9), pp. 2185–2202.
We evaluate the out-of-sample performance of a long-term investor who follows an optimized dynamic trading strategy. Although the dynamic strategy is able to benefit from predictability out-of-sample, a short-term investor using a single-period market timing strategy would have realized an almost identical performance. The value of intertemporal hedge demands in strategic asset allocation appears negligible. The result is caused by the estimation error in predicting the predictors. A myopic investor only needs to predict one-period-ahead expected returns, but hedge demands also require accurate predictions of the predictor variables. To reduce the problem of errors in optimized portfolio weights, we consider Bayesian procedures. Myopic and dynamic portfolios are similarly affected by such modifications, and differences in performance become even smaller.

Dixon, M. F. and Halperin, I. (2021). "Goal-based wealth management with generative reinforcement learning." In: *Risk (Cutting edge)*.
A combination of machine learning techniques provides multi-period portfolio optimisation. Matthew Dixon and Igor Halperin develop a reinforcement learning (RL) approach to goal-based wealth management problems such as optimisation of retirement plans or target date funds. They present G-Learner: a reinforcement learning algorithm that does not assume a data generation process and is suitable for noisy data. Their approach is based on G-learning, a probabilistic extension of the Q-learning method of reinforcement learning. In addition to G-Learners, which solves the direct RL problem, they develop GIRL, a G-learning inverse RL algorithm to infer the investor reward function from the observed trading actions.

Dong, B., Xu, W., Sevic, A., and Sevic, Z. (2020). "Efficient willow tree method for variable annuities valuation and risk management." In: *International Review of Financial Analysis* 68, p. 101429.
Variable annuities (VAs) with various guarantees are popular retirement products in the past decades. However, due to the sophistication of the embedded guarantees, most existing methods only focus on the one of embedded guarantees underlying one specified stochastic model. The method to evaluate VAs with all guarantees and manage its risk is very limited, except for the Monte Carlo method. In this paper, we propose an efficient willow tree method to evaluate VAs embedded with all popular guarantees on the market underlying various stochastic models. Moreover, our tree structure is also applicable to compute dollar delta, value at risk (VaR) and conditional tail expectation (CTE) in hedging and risk-based capital calculation. Numerical experiments demonstrate the accuracy and efficiency of our method in pricing and managing the risk of VAs.

Dong, Z.-L., Zhu, M.-X., and Xu, F.-M. (2022). "Robo-advisor using closed-form solutions for investors' risk preferences." In: *Applied Economics Letters*.
In this article, we design a robo-advisor which has a bi-level framework. The framework enables it to handle a large amount of assets using fast algorithms in the lower level. The proposed robo-advisor can utilize the closed-form solutions for investors' risk preferences based on corresponding portfolio choices. A dynamic weight is applied to update investors' risk preferences. Numerical results based on real data in Chinese stock market show that our proposed robo-advisor can accurately estimate the risk preferences of investors and outperform the benchmark formed by market indexes.

Donnelly, C. and Bernhardt, T. (2018). *Pension decumulation strategies: a state-of-the-art report*. Tech. rep. Heriot Watt University.
The authors are part of the ARC research project "Minimising Longevity and Investment Risk while Optimising Future Pension Plans". The goal is to develop new pension product designs that keep the customers' needs at

the forefront. As a first step, this report was written to familiarize the project team with the existing knowledge on decumulation strategies. There is a UK-focus in the report. The report is aimed at the reader who is familiar with the actuarial world, but are not necessarily familiar with stochastic control techniques. For this reason, technical descriptions of the mathematical methods used to solve many of the problems studied in the cited literature are not included. Questions relating to property, taxes, regulations and solvency have been ignored in order to keep the focus on what is the state-of-the-art on the fundamental question of "how should I decumulate in retirement"? The literature on decumulation strategies is vast and only a fraction of it is included. Omission of relevant material is unintentional.

Donnelly, C., Khemka, G., and Lim, W. (2022). "Investing for retirement: Terminal wealth constraints or a desired wealth target?" In: *European Financial Management*.
We investigate how well different investment strategies can give pre-retirees more certainty about their income in retirement, whilst allowing them to benefit from taking investment risk. Under an expected utility-maximizing framework, we find that a loss aversion utility function gives a high degree of certainty about its desired wealth target and is robust to different market models. Imposing terminal wealth constraints does not improve the certainty of achieving the desired target enough to counter-balance the increased chance of obtaining a lower income. The power utility function is not robust to different market models and becomes too risk-averse with wealth constraints.

Drew, M. E., Walk, A. N., West, J., and Cameron, J. (2014). "Improving retirement adequacy through asset class prioritization." In: *Journal of Financial Services Marketing* 19(4), pp. 291–303.
Highly risk-averse retirees are generally advised to adopt a fixed spending strategy such as the 4% withdrawal rule. To prevent the premature depletion of a retirement portfolio, the rule attempts to proxy as the "safe withdrawal rate". But a constant withdrawal rate means that retirees accumulate unspent surpluses when markets outperform and face spending shortfalls when markets underperform. While a safe withdrawal rate can prevent spending shortfalls, the opportunity cost of unspent surpluses associated with this strategy can be extreme. We apply a range of basic investment decision rules to a retirement portfolio applying various withdrawal rates and examine the probability of shortfalls over a retirement horizon. Using a block bootstrap simulation technique, we examine decision rules relating to stock and bond investments. Our results show that retirement portfolios with a bias towards stocks coupled with a decision rule that sources withdrawals from bonds and cash before stocks significantly outperforms alternative withdrawal strategies, despite the inherent increase in volatility. This finding is in direct contrast to the safe withdrawal rate conventions used in contemporary financial advice models.

Drew, M. E., Walk, A. N., and West, J. (2015). "Conditional Allocations to Real Estate: An Antidote to Sequencing Risk in Defined Contribution Retirement Plans." In: *The Journal of Portfolio Management* 41(6), pp. 82–95.
In this article, the authors investigate the potential for real estate as an asset class to be exploited to protect against sequencing risk (or path dependency) in defined contribution retirement funds. Their results suggest that allocating both listed and unlisted real estate assets to retirement portfolios, even if very minor, can enhance the risk-return profile and probability of successfully achieving retirement outcomes. Using a bootstrap simulation approach, the authors test for a range of asset allocations that include real estate. In addition, they examine the sensitivity of real estate performance to changes in monetary policy to optimize portfolio outcomes for fund managers who actively seek exposure to real estate assets. Their findings indicate that the performance of real estate is highly dependent on monetary policy settings that, when used in a dynamic asset allocation process, have the potential to enhance portfolio returns while limiting the impact of downside risk.

Drew, M. E., Walk, A. N., and West, J. M. (2016). "Withdrawal Capacity in the Face of Expected and Unexpected Health and Aged-Care Expenses During Retirement." In: *The Journal of Retirement* 3(3), pp. 77–94.
We examine the consequences of taking account of costs associated with age-related health treatment and agedcare services during the retirement phase. Simulating asset return data using historical bootstrap simulation, we derive an optimal withdrawal income during retirement using dynamic optimization techniques. The greatest risk to income sustainability occurs when unexpected health costs combine with above-average longevity for conservative investors. High costs of health treatment without a commensurate adjustment in asset allocation toward assets with a less conservative risk-return profile risk premature wealth depletion. The risk of ruin can be mitigated through a dynamic life-cycle strategy during the retirement phase.

Drew, M. E. and West, J. M. (2021). "Retirement Income Sufficiency through Personalised Glidepaths." In: *Financial Analysts Journal* 77(2), pp. 5–20.

Portfolio "glidepaths" accommodate growth in a worker's early working life and transition to lower risk settings as the worker nears retirement. The success of this design hinges on its objective of amassing wealth at the date of retirement, but the design offers little in the way of a solution for the provision of income during retirement. The relevant risk for workers is retirement income uncertainty. Allocating investments through time to satisfy an income goal is not equivalent to the maximisation of wealth at retirement. We demonstrate that glidepaths can be personalised for individuals to maximise expected retirement income sufficiency under a range of assumptions, including longevity risk.

Dreyer, A. and Pogorelova, L. (2018). "The Effect of Income-Dependent Mortality on Withdrawal Strategies." In: *The Journal of Retirement* 6(1), pp. 70–81.
Recent literature documented significant dependency between individual lifetime earnings and life expectancy. In light of this phenomenon, we examine the effect of using salary-fitted mortality tables on optimized withdrawal strategies. We find that although earnings rank meaningfully affects expected retirement length, the impact of modeling salary-fitted mortality is more nuanced and depends on the investor risk-aversion parameter. In the base case, a risk-neutral investor is willing to adjust her optimal withdrawal strategy roughly in proportion to the change in expected retirement length and can therefore experience a substantially larger misallocation of withdrawals. In contrast, for a risk-averse investor, a 10% income-related difference in expected retirement length does not necessitate a sizable adjustment of an optimized withdrawal strategy; the unconditional mortality assumption causes annual withdrawal misallocation of only about 1% to 3% of pre-retirement consumption. Given this difference in results, we stress the need for a comprehensive and cohesive framework when analyzing the full implications of a retirement horizon assumption.

Du, Y. (2021). "Essays on Portfolio Choice and Health over the Life Cycle." PhD thesis. Temple University.
This dissertation examines the effect of health and its associated variables on households' consumption and portfolio choices over life cycle. The first two essays constitute my job market paper, which explains why the risky portfolio share rises in wealth from two health mechanisms: endogenous health investment and medical expenditure risk. The third chapter explores the effect of health and health risk on households' optimal consumption and portfolio decisions over life cycle.
Chapter 1 titled "PORTFOLIO CHOICE AND HEALTH ACROSS WEALTH: EMPIRICAL EVIDENCE" illustrates the empirical relationship between the portfolio puzzle and the heterogeneity of health variables across wealth. Classic financial theory suggests that under the assumption of no borrowing constraints and no mean-reverting stock returns, households should hold a constant risky portfolio in spite of their wealth, ages and life horizons (Samuelson (1969) and Merton (1969, 1971)). Yet data from the Survey of Consumers Finances (SCF) show that the risky portfolio share of financial assets increases in wealth. In the literature, this is called the "portfolio puzzle". Meanwhile, various sources of data indicate that, compared with the non-wealthy households, the wealthy people have better health, longer life horizon, higher out of pocket medical spending with lower uncertainty, and more health care time. All these facts suggest a novel correlation between the portfolio puzzle and the heterogeneity of health variables across wealth and provide a robust empirical foundation to explain the portfolio puzzle from a health perspective.
In Chapter 2 titled "A LIFE CYCLE MODEL OF PORTFOLIO CHOICE AND HEALTH", a life cycle model with endogenous health investment and medical expenditure risk is proposed to capture the key empirical features in the first chapter. This calibrated model remarkably matches the U.S. data. I find that endogenous health investment is essential to explain the portfolio puzzle: if health is exogenous without investment, the model can only could deliver 7.2% of the risky share gap across wealth. Medical expenditure risk is less important and has a larger effect on the non-wealthy group. If I abstract from medical expenditure risk, the risky shares increase for both groups: 24% for the low wealth group and 5% for the wealthy group. This life cycle model provides new insights into how health affects households' financial behavior.
Chapter 3 titled "OPTIMAL CONSUMPTION AND PORTFOLIO CHOICE WITH HEALTH RISK" investigates the effect of health and health risk on households' optimal consumption and portfolio allocations over the life cycle. The simulation results show that consumption, savings in bonds, and savings in stocks all increase with health. The risky portfolio share, which is the ratio of savings in stocks to the total financial assets, demonstrates the same tendency for both health states over the life cycle: at the very young age, the risky portfolio share is relatively high. Starting from the middle age, this share falls significantly and keeps steady until the end of life. For most of the lifetime, the risky portfolio share is positively related with health. These results emphasize the importance of health and its associated risk in consumption and portfolio decisions.

Duffy, S., Finke, M. S., and Blanchett, D. (2021). "The Value of Delayed Social Security Claiming for Higher-Earning Women." In: *SSRN e-Print*.
This study estimates the expected benefit from delayed Social Security claiming for higher-earning, healthier women who can expect to receive more future income payments than other Americans. The expected net present value of Social Security payments from delayed claiming for healthy women is $179,999, or more than twice the value of delayed claiming for a male of average health. The benefit from delayed claiming ranges from a low of $16,548 per year to $29,395 per year for healthy women. Even average health women gain at least $132,202 in Social Security wealth by waiting until age 70 to claim Social Security income benefits. Significant gains to women who delay claiming are robust to a 2% increase in real discount rates and to a 21% reduction in income that could occur if the Social Security trust fund depletes in 2026. The added benefits from reduced longevity and inflation risk suggest that the failure to delay claiming results in a significant loss in both retirement wealth and expected welfare.

Dunham, L. M. and Washer, K. M. (2020). "Using Life Tables for Retirement Planning." In: *The Journal of Wealth Management* 23(3), pp. 28–36.
This study examines two methods for determining retirement accumulations. The first method assumes that the client will live to a very old age. This method may be appropriate for clients who are highly risk averse and/or want to self-insure the risk of outliving their savings. The second method employs life expectancy tables to weight retirement cash flows by survival probabilities. The target accumulation under this method is much lower and more easily attainable for clients than the first method. The primary drawback of the second method is that, from a statistical point of view, about 55% of clients will outlive their savings. Because of this, risk-sharing, perhaps through a deferred life annuity, would be advisable unless clients have other viable options available (e.g., family support) in the event that their savings are used up. We believe there is value in both methods and that some clients will prefer a higher, more ambitious target, whereas others will prefer a lower, more attainable target.

Dziwisch, A., Krahnhof, P., and Zureck, A. (2021). "Empirical determination of sustainable withdrawal rates considering historical yields and inflation rates in Germany." In: *Zeitschrift fur die gesamte Versicherungswissenschaft* 110, pp. 117–132.
On account of the current low interest rate phase, which is most likely to continue in the coming years, the average yields to be achieved in the bond, time deposit and savings product sectors are declining, so that risk-averse investors in particular have few opportunities to generate return-oriented retirement provisions.
This scientific article analyzes the level of a possible safe withdrawal rate for diversified pension portfolios, considering historical returns and inflation rates. Consequently, this article provides immediate practical added value for a possible retirement provision.
The evaluation is based on the consideration of historical returns of the stock and bond market in Germany. To determine a safe withdrawal rate, the development of portfolios with different compositions and inflation-adjusted withdrawal rates are simulated over periods of 15 to 35 years. In this simulation, the risky part of the portfolio is represented by German equities, the low risk part by German government bonds.
To sum up, the empirical results show a maximum safe withdrawal rate of 4%. The underlying portfolio is composed of 50% equities and 50% government bonds. Particularly due to the outlined demographic change in Germany as well as the ongoing low-interest phase, the empirical study can provide significant theoretical and practical insights.

Elkins, K. (2019). *Stanford analyzed 292 retirement strategies to determine the best one – here's how it works*. Tech. rep. CNBC.
In 2017, the Stanford Center on Longevity analyzed 292 different retirement income strategies and determined the best way for most people to withdraw their savings. It's called the "spend safely in retirement strategy" (SSiRS) and involves two basic components: delaying Social Security benefits and creating an "automatic retirement paycheck." In a new 2019 report conducted by the Stanford Center on Longevity and the Society of Actuaries, "Viability of the Spend Safely in Retirement Strategy," the research team took a deeper dive into the SSiRS and explored different ways to implement it. The strategy is designed to be used by middle-income workers and retirees, which the report defines as those having USD1 million or less in retirement savings. Here's a closer look at how the two key components of the SSiRS work.

Elnekave, R. (2007). "The mathematics of savings and retirement planning." In: *The Journal of Wealth Management* 10(3), pp. 87–92.

Saving and retirement planning currently rely on techniques that have proven to be failure prone at best, and completely inadequate under the worst circumstances. Further, existing methods do not provide an understanding of the drivers behind the success or failure of savings plans. This lack of insight, combined with a deficient framework, contributes to the adverse outcomes experienced by many pension plans and individuals. This article introduces a new perspective on modeling savings that borrows extensively from the field of hydrology. To accomplish this, the author applies a modified version of mathematical techniques used by hydrologists in dam building to savings and retirement planning. The goal is to present an intuitive decision framework, based on a novel concept. In the process, the author hopes to open a new path in the dialogue to improve the state of the art in savings and retirement planning.

Envestnet Research (2020). *Capital Sigma: The Advisor Advantage*. Tech. rep. Envestnet.
We recently updated our quantitative research study, "Capital Sigma: The Advisor Advantage," which explores how select sources of advisor-created value can produce meaningful levels of excess return. For advisors to deliver the most alpha to client portfolios, we recommend a focus on five sources: financial planning, asset selection and allocation, investment selection, systematic rebalancing and tax management. The sum total of these five sources creates what we call Capital Sigma. In fact, the combination of successfully implementing these five sources has produced around 3% of value add annually –results that are consistent with PMC research first published in 2014.

Erdemlioglu, D. and Joliet, R. (2019). "Long-term asset allocation, risk tolerance and market sentiment." In: *Journal of International Financial Markets, Institutions and Money* 62, pp. 1–19.
This paper studies optimal equity portfolios with long-term horizon under heterogeneous risk aversion levels. We focus on European stocks and empirically show that contemporaneous excess returns of semi-active strategies are negatively associated with market conditions and sentiment. Consistent with our long-horizon perspective, we find that the effects of sentiment measures on semi-active portfolio returns are sizeable and economically relevant, particularly in bull (post-crisis) periods, even after controlling for the five Fama-French factors, momentum, macro indicators and political uncertainty shocks either globally or country-wise. By contrast, the effects of sentiment measures on the passive (benchmark) portfolio appear to be negligible. The results further indicate that realized portfolio returns generated from our long-term strategies are considerably resilient to the episodes of flight-to-safety (risk-off) regimes.

Eschenbach, T. G. and Lewis, N. A. (2019). "Risk, standard deviation, and expected value: when should an individual start social security?" In: *The Engineering Economist* 64(1), pp. 24–39.
In choosing when to start collecting Social Security, the differences in expected net present values (NPVs) are small-but the corresponding standard deviations are not. Starting earlier is less risky. The case analyzed is single individuals in the U.S. system, but the methodology can be applied to couples and to the systems of other nations. Considering risk and return together places Social Security in the same risk/return framework as other capital investments. Behavioral, situational, and qualitative factors that often dominate decisions on when to start are linked with quantitative approaches to longevity risk and mortality risk.

Estrada, J. (2014). "The Glidepath Illusion: An International Perspective." In: *The Journal of Portfolio Management* 40(2), pp. 52–64.
Target-date funds have become a core product for investors who are saving for retirement. These funds periodically reduce their allocations to stocks and increase their allocations to bonds and cash, becoming more conservative as retirement approaches. This lifecycle strategy implies that investors are aggressive with little capital and conservative with much more capital, which may be suboptimal in terms of wealth accumulation. This article evaluates three alternative types of strategies, including contrarian strategies that follow a glidepath opposite that of target-date funds; that is, they become more aggressive as retirement approaches. The results from a comprehensive sample that spans more than 19 countries, two regions, and 110 years suggest that, relative to lifecycle strategies, the alternative strategies considered here provide investors with higher upside potential, more limited downside potential, and higher uncertainty although that uncertainty is largely limited to how much higher their terminal wealth is expected to grow with these strategies.

Estrada, J. (2016). "The Retirement Glidepath: An International Perspective." In: *The Journal of Investing* 25(2), pp. 28–54.
All individuals need to decide how much to save during their working years, how much to spend during retirement, and the asset allocation of their portfolio in both periods. A portfolio's exposure to stocks and bonds affects key variables, such as the probability of portfolio failure, degree of downside protection, and expected bequest. How this exposure should evolve during retirement is the ultimate issue explored in this article. After considering

declining-equity, rising-equity, and static glidepaths, the comprehensive international evidence from 19 countries and the world market over 110 years ultimately suggests that both an all-equity portfolio and a 60/40 stock/bond allocation are simple and very effective strategies for retirees to implement.

Estrada, J. (2017a). "Maximum Withdrawal Rates: A Novel and Useful Tool." In: *Journal of Applied Corporate Finance* 29(4), pp. 134–137.
The failure rate is arguably the tool most widely used to evaluate retirement strategies. This tool has several shortcomings, one of which is that it does not account for the very different bequests that different strategies may leave behind. The maximum withdrawal rate is a novel tool that, besides dealing with this problem, is useful for 1) providing a comprehensive evaluation of retirement strategies, and 2) evaluating the likelihood of sustaining a target sequence of inflation adjusted withdrawals.

Estrada, J. (2017b). "Refining the Failure Rate." In: *The Journal of Retirement* 4(3), pp. 63–76.
The importance of determining how often a retirement strategy failed in the past, and how likely it is to fail in the future, can hardly be overstated. For this reason, the failure rate has become an essential tool when evaluating these strategies. However, this variable is silent about how long into the retirement period a strategy failed; two strategies that sustained withdrawals for 10 and 25 years of a 30-year retirement period have both failed, but a retiree would be far from indifferent between them. The two variables proposed here, which seek to refine and complement, not to replace, the failure rate aim to correct this shortcoming.

Estrada, J. (2017c). "Replacing the Failure Rate: A Downside Risk Perspective." In: *SSRN e-Print*.
The evaluation of retirement strategies is typically based on the failure rate, although some alternatives have been proposed recently, such as risk-adjusted success (RAS), which aims to capture not just the failure but also the success of the strategies evaluated. The main shortcoming of RAS is that it measures risk with the standard deviation, which penalizes strategies that leave large bequests. The variable proposed here, downside risk-adjusted success (D-RAS), addresses this shortcoming by taking a downside risk perspective, measuring risk with the semideviation, and therefore with volatility below a chosen benchmark. This modification leads to the selection of more plausible strategies than those selected by both the failure rate and RAS.

Estrada, J. (2018a). "From Failure to Success: Replacing the Failure Rate." In: *The Journal of Wealth Management* 20(4), pp. 9–21.
The failure rate is arguably the variable most widely used in the evaluation of retirement strategies. Its main shortcoming, evaluating how often a strategy fails but not by how much it does, is overcome by shortfall years, which considers precisely this information. The joint use of the failure rate and shortfall years is an improvement over using just the failure rate but implies the use of two variables, rather than just one, which do not always point in the same direction. This article introduces a new variable, years sustained, that focuses on success rather than on failure. The ratio between its mean and standard deviation, risk-adjusted success, is the single variable proposed here to be used in a comprehensive evaluation of retirement strategies.

Estrada, J. (2018b). "Maximum withdrawal rates: an empirical and global perspective." In: *The Journal of Retirement* 5(3), pp. 57–71.
Standard analysis of retirement strategies involves evaluating their failure rate. One of the shortcomings of this approach is that a strategy may have a low failure rate and at the same time leave a large unintended bequest. Maximum withdrawal rates, by definition, exhaust a portfolio by the end of the retirement period, thus leaving no bequest; they can be used both to assess the likelihood of sustaining any chosen level of inflation-adjusted withdrawals, and more generally to evaluate retirement strategies. This article provides a comprehensive historical perspective on maximum withdrawal rates in 21 countries over 115 years with 11 asset allocations ranging from 100% stocks to 100% bonds.

Estrada, J. (2019). "Managing to target: dynamic adjustments for accumulation strategies." In: *SSRN e-Print*.
Planning for retirement, particularly during the accumulation period, largely consists of setting a target value for the retirement portfolio and implementing a policy aimed at hitting that target. Financial plans are inevitably based on expected returns, which are likely to be different from those an individual experiences during the accumulation period. Thus, when the portfolio deviates from the path outlined in the plan, the individual can choose between a static policy of sticking to his plan and simply hope to hit the target, or dynamic policies designed to keep the portfolio close to its path. This article evaluates three types of such dynamic policies, broadly referred to as to target (M2T), that adjust the periodic contributions or the portfolio asset allocation. The results reported show that some of the dynamic policies outlined outperform a static policy, and adjusting contributions is far superior to adjusting the asset allocation.

Estrada, J. (2020a). "Retirement Planning: From Z to A." In: *The Journal of Retirement* 88(22), pp. 8–22.

Retirement planning is an issue that must be tackled early and solved backward. It must be tackled early because little can be done if an individual is not on the right path and has only a few years left to work. It must be solved backward because it makes little sense to aim for a portfolio that may not be able to sustain the desired lifestyle in retirement. This article introduces an approach that integrates the working period and the retirement period; leads individuals to consider all relevant variables at the beginning of their journey; and enables them to start saving early to build a target portfolio designed specifically to sustain a desired retirement. The analytical framework introduced yields closed-form solutions for the target retirement portfolio and the contributions that need to be made during the working years to hit that target. The framework proposed is illustrated with an empirical base case, sensitivity analysis, and Monte Carlo simulations.

Estrada, J. (2020b). "Sequence Risk: Is It Really a Big Deal?" In: *SSRN e-Print*.

Financial planners are keenly aware of, and routinely warn clients about, sequence risk; that is, the possibility of facing a sequence of low returns early in retirement that may force retirees to scale down the plans they had made. This really is a scary scenario, but one that the evidence here shows that retirees are not very likely to encounter. A new and refined definition of sequence risk is advanced in this article, linking this type of risk to the sustainability of a withdrawal strategy. Furthermore, three ways of assessing sequence risk are proposed, among them one that enables a retiree to monitor the sustainability of his withdrawal strategy periodically and to introduce adjustments when necessary. The ultimate message of this article is that retirees should be informed, but not obsess, about sequence risk.

Estrada, J. (2020c). "Target-Date Funds, Glidepaths, and Risk Aversion." In: *SSRN e-Print*.

Target-date funds feature asset allocations that become increasingly conservative as investors approach retirement. An important shortcoming of this strategy is that it is suboptimal in terms of capital accumulation, which begs the question of why these funds are so popular. A possible answer is that investors become more risk averse as they age, gradually favoring more downside protection as they approach retirement. The main issue explored in this article is how much more risk averse would investors need to become during their working years to select asset allocations similar to those in target-date funds; the evidence here shows that investors would have to roughly double their risk aversion during the last 25 years of their working period. An intuitive interpretation of this result, based on how much an individual would pay to avoid a gamble, is also discussed.

Estrada, J. (2020d). "Target-Date Funds, Glidepaths, and Risk Aversion." In: *The Journal of Wealth Management* 23(3), pp. 50–60.

Target-date funds feature asset allocations that become increasingly conservative as investors approach retirement. An important shortcoming of this strategy is that it is suboptimal in terms of capital accumulation, which begs the question of why these funds are so popular. A possible answer is that investors become more risk averse as they age, gradually favoring more downside protection as they approach retirement. The main issue explored in this article is how much more risk averse would investors need to become during their working years to select asset allocations similar to those in target-date funds; the evidence here shows that investors would have to roughly double their risk aversion during the last 25 years of their working period. An intuitive interpretation of this result, based on how much an individual would pay to avoid a gamble, is also discussed.

Estrada, J. (2021a). "Sequence Risk: Is It Really a Big Deal?" In: *The Journal of Investing* 30(6), pp. 47–69.

Financial planners are keenly aware of, and routinely warn clients about, sequence risk; that is, the possibility of facing a sequence of low returns early in retirement that may force retirees to scale down significantly the plans they had made. This really is a scary scenario, but one that the evidence here shows that retirees are not very likely to encounter. A new and refined definition of sequence risk is advanced in this article, linking this type of risk to the sustainability of a withdrawal strategy. Furthermore, three ways of assessing sequence risk are proposed, among them one that enables a retiree to monitor the sustainability of his withdrawal strategy periodically and to introduce adjustments when necessary. The ultimate message of this article is that retirees should be informed, but not obsess, about sequence risk.

Estrada, J. (2021b). "The Gain-Pain Index: Asset Allocation for Individual (And Other?) Investors." In: *SSRN e-Print*.

Individual investors typically determine their asset allocation using investor questionnaires, which can be viewed as black boxes that generate a result without highlighting the benefits and costs of the portfolios considered. This article introduces an asset allocation tool, the gain-pain index (GPI), that overcomes this shortcoming. The tool proposed incorporates two critical variables found in investor questionnaires, the portfolio holding period and the investor's risk tolerance, and broadens the definition of risk beyond volatility by also considering the

probability of suffering a loss and the magnitude of the loss. The model is used to determine optimal asset allocations for 21 countries and the world market, for different holding periods and levels of risk aversion.

Estrada, J. (2021c). "The Sustainability of (Global) Withdrawal Strategies." In: *Journal of Financial Planning* 34(11), pp. 82–98.

The most important financial issue retirees have to deal with is whether their strategy will be able to sustain all the withdrawals they expect to make and a bequest they aim to leave. It is critical to periodically monitor the evolution of a financial plan in order to detect early signs of trouble, which may lead a retiree to introduce dynamic adjustments to a strategy.

To that purpose, this article features two tools, a sustainability test and the sustainable withdrawal, and shows how to apply them. This article also discusses the empirical evidence on both tools based on a comprehensive sample of 22 markets over a 120-year period.

Estrada, J. and Kritzman, M. (2018). "Evaluating Retirement Strategies: A Utility-Based Approach." In: *SSRN e-Print*.

Retirees need to make two critical financial decisions, namely, the withdrawal rate and the asset allocation of their portfolios. We propose a methodology that retirees, and particularly advisors, could use to make these decisions in an optimal way. We introduce a new variable, the coverage ratio, and a theoretical approach, based on utility. Our approach can be used to make optimal decisions during both the accumulation and the retirement period, but we illustrate it by focusing on the latter, and particularly on the choice of an optimal asset allocation. We find that the strategies selected by our utility-based approach are in general somewhat more aggressive than those selected by the failure rate and other existing approaches.

Estrada, J. and Kritzman, M. (2019). "Toward Determining the Optimal Investment Strategy for Retirement." In: *The Journal of Retirement* 7(1), pp. 35–42.

Investors who are about to retire are first and foremost concerned with supporting their spending needs throughout retirement. But they also derive satisfaction from growing their wealth beyond what is needed to support consumption in order to leave a bequest to their heirs or chosen charities. The predominant metric for evaluating retirement investment strategies is the failure rate. However, it doesn't distinguish between strategies that fail early in retirement from those that fail near the end of retirement, and it doesn't account for potential bequests. To overcome these shortcomings the authors propose a new metric, the coverage ratio, which is more comprehensive and informative than the failure rate. In addition, they propose a utility function to evaluate the coverage ratio, which penalizes shortfalls more than it rewards surpluses. Finally, the authors use their proposed framework to determine the optimal allocation to stocks and bonds using historical and simulated returns.

Fabozzi, F. J. and Lopez de Prado, M. (2018). "Being Honest in Backtest Reporting: A Template for Disclosing Multiple Tests." In: *The Journal of Portfolio Management* 45(1), pp. 141–147.

Selection bias under multiple testing is a serious problem. From a practitioner perspective, failure to disclose the impact of multiple tests of a proposed investment strategy to clients and senior management can lead to the adoption of a false discovery. Clients will lose money, senior management will misallocate resources, and the firm may be exposed to reputational, legal, and regulatory risks. From the perspective of academic journals that publish evidence supporting an investment strategy, the failure to address selection bias under multiple testing threatens to invalidate large portions of the literature in empirical finance. In this article, the authors propose a template that practitioners should use to fairly disclose multiple tests involved in an alleged discovery when pitching strategies to clients and senior management. The same template could be used by contributors to academic journals so that referees, and ultimately readers, can assess the strategy. By disclosing this information, those who are charged with making the final decision about a discovery can evaluate the probability that the purported discovery is false.

Fan, J., Weng, H., and Zhou, Y. (2021). "Optimal estimation of functionals of high-dimensional mean and covariance matrix." In: *arXiv e-Print*.

Motivated by portfolio allocation and linear discriminant analysis, we consider estimating a functional $mu^T\Sigma^{-1}\mu$ involving both the mean vector μ and covariance matrix Σ. We study the minimax estimation of the functional in the high-dimensional setting where $\Sigma^{-1}\mu$ is sparse. Akin to past works on functional estimation, we show that the optimal rate for estimating the functional undergoes a phase transition between regular parametric rate and some form of high-dimensional estimation rate. We further show that the optimal rate is attained by a carefully designed plug-in estimator based on de-biasing, while a family of naive plug-in estimators are proved to fall short. We further generalize the estimation problem and techniques that allow robust inputs of mean and covariance matrix estimators. Extensive numerical experiments lend further supports to our theoretical results.

Fan, Y.-A., Murray, S., and Pittman, S. (2013). "Optimizing retirement income: an adaptive approach based on assets and liabilities." In: *The Journal of Retirement* 1(1), pp. 124–135.
Many retirees financial resources are insufficient to sustain their desired expenditures, which makes clear the need for more efficient retirement asset allocation strategies. We propose an approach to retirement investing that directly incorporates spending targets within a multiperiod framework that improves performance with respect to spending sustainability and bequest motives. We propose an model, which is dynamic and allows for recourse decisions and permits changes in asset allocation in response to market outcomes as the retiree ages. Practitioners can use the adaptive model to increase the sustainability of their clients spending and the funding of bequests, thereby enhancing their clients welfare in their later years.

Faria, M. C. (2021). "An Examination of Target Date Fund Glidepath Construction." In: *SSRN e-Print*.
The paper examines Target Date Fund glidepath construction as well as asset allocation strategies. We consider characteristics, features, and assumptions in solving the multi-period portfolio selection problem. This paper discusses the theoretical underpinnings of deterministic, adaptive, and stochastic models as well as some interesting alternative strategies. Furthermore, we also propose research topics for future consideration that the author believes may improve upon the existent model frameworks.

Fellowes, M., Fichtner, J. J., Plews, L., and Whitman, K. (2019). *The Retirement Solution Hiding in Plain Sight: How Much Retirees Would Gain by Improving Social Security Decisions*. Tech. rep. Capital One Investing.
Social Security now accounts for about one-third of all income annually received by U.S. retirees, amounting to $1 trillion in annual benefits. While impactful, research consistently finds that the financial effect of Social Security could be even greater if more people waited to enroll, since monthly benefits can increase in value if retirees delay claiming. But, we don't know how much is annually lost from households making the sub-optimal decision about when to claim Social Security, how many are making mistakes, or who is making those wrong decisions. To explore these questions, we utilize new technology invented by United Income and data sponsored by the Social Security Administration, finding:

- Retirees will collectively lose $3.4 trillion in potential income that they could spend during their retirement because they claimed Social Security at a financially sub-optimal time, or an average of $111,000 per household. The average Social Security recipient would receive 9 percent more income in retirement if they made the financially optimal decision about when to claim this retirement benefit.
- Current retirees will collectively lose an estimated $2.1 trillion in wealth because they made the sub-optimal decision about when to claim Social Security, or an average of $68,000 per household. Most retirees will lose wealth in their 60s and early 70s if they choose to optimize Social Security, but will be wealthier in their late 70s through the rest of their lives.
- Only 4 percent of retirees make the financially optimal decision about when to claim Social Security. About 57 percent of retirees would build more wealth through their life if they waited to claim until they were 70 years old (when only 4 percent of retirees currently claim), while only 6.5 percent of retirees would have more wealth if they claimed prior to turning 64 (when over 70 percent of retirees currently claim benefits).
- About 21 percent of those at risk of not affording retirement (or having enough income to cover their expected cost of living) would see an improvement in their chances if they claimed Social Security at the optimal time. Among those retirees at risk that start with a greater than 10 percent chance of affording retirement, 95 percent see their chances of affording retirement improve by an average of 28 percent.
- Elderly poverty could be cut by nearly 50 percent if all retirees claimed Social Security at the financially optimal time. In particular, about 13 percent of people over the age of 70 are expected to live in poverty at some point, which is estimated to fall to 7 percent if retirees had claimed Social Security at the optimal time -a rate that could potentially fall even further if they earned additional income while they waited to claim Social Security.

This report finds that nearly no retirees are making the financially optimal decision about Social Security, and that the costs of those mistakes are high for retiring households, particularly those at risk of not being able to afford retirement. In addition, since making the optimal decision means sacrificing wealth in the near-term, we think it is unlikely more people will make the right decision without a policy intervention. There are numerous difficulties associated with solving this problem, though, which will require a thorough and diverse process for addressing. Among the topics for consideration should be the eligibility age range rules, which were last materially modified in 1983. Since 92 percent of retirees are expected to be better off waiting to claim until at least their 65th birthday, claiming before should ideally be an exception for those who demonstrably need to

claim benefits before the full retirement age. Means-testing rules may be one way to address this, though an easier place to start would be to change how the Social Security Administration frames claiming age options to the public. Instead of portraying age 62 as the "early eligibility age," for instance, claiming at age 62 could instead be labeled as the "minimum benefit age" while age 70 could be labeled as the "maximum benefit age." The Social Security Administration could also be provided with resources to improve utilization of the policy it administers, perhaps in partnership with third-party fiduciaries. With the potential to put $2.1 trillion wealth and $3.4 trillion in income in the pockets of retirees, policymakers should be focused on improving this program.

Feng, R. and Jing, X. (2017). "Analytical valuation and hedging of variable annuity guaranteed lifetime withdrawal benefits." In: *Insurance: Mathematics and Economics* 72, pp. 36–48.
Variable annuity is a retirement planning product that allows policyholders to invest their premiums in equity funds. In addition to the participation in equity investments, the majority of variable annuity products in today's market offer various types of investment guarantees, protecting policyholders from the downside risk of their investments. One of the most popular investment guarantees is known as the guaranteed lifetime withdrawal benefit (GLWB). In current market practice, the development of hedging portfolios for such a product relies heavily on Monte Carlo simulations, as there were no known closed-form formulas available in the existing actuarial literature. In this paper, we show that such analytical solutions can in fact be determined for the risk-neutral valuation and delta-hedging of the plain-vanilla GLWB. As we demonstrate by numerical examples, this approach drastically reduces run time as compared to Monte Carlo simulations. The paper also presents a novel technique of fitting exponential sums to a mortality density function, which is numerically more efficient and accurate than the existing methods in the literature.

Fichtner, J. J. and Seligman, J. S. (2018). "Retirement Saving and Decumulation in a Persistent Low-Return Environment." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.
The current retirement environment presents challenges, not only over the period for which interest rates remain low, but also once interest rates appreciably increase. This chapter addresses two related questions: first, how have households responded to the current low interest rate environment, and second, are there alternative responses or investments which households might do well to consider? We employ the Health and Retirement Study to first investigate impacts of the low interest rate on savings, wealth, and asset allocation. We also report on a subset of households who were relatively successful at building and preserving wealth over this period. Following this, we consider alternative portfolio and wealth management strategies targeting increases in equities and delayed claiming of Social Security in terms of their potential to add value in persistent low return environments.

Fidelity Research (2019). *Why work with a financial advisor*. Tech. rep. Fidelity.
Industry studies estimate that professional financial advice can add between 1.5% and 4% to portfolio returns over the long term, depending on the time period and how returns are calculated. A one-on-one relationship with an advisor is not just about money management. A financial advisor can help provide ongoing financial planning so you can have peace of mind while pursuing your life goals. The financial planning process includes defining your goals, understanding your current situation, and identifying the key steps to move forward. Beyond long-term goals like retirement, and shorter-term ones like buying a house, education, or travel, holistic financial planning can also include legacy planning, family support, health care, insurance, and charitable giving.

Finefrock, C. J., Gradisher, S. M., and Nitz, C. M. (2015). "Long-Term Care Insurance: Comparisons for Determining the Best Options for Clients." In: *Journal of Financial Planning* 38(2), pp. 36–43.
An estimated 8.36 million people need some type of long-term care (LTC) service annually. The American Association for Long-Term Care Insurance (AALTCI) recommends that individuals purchase LTC policies in their mid-50s. The cost for LTC is increasing and currently averages $43,472 annually for adult day service center, to $87,600 for a private nursing home room. Studies show that most Americans are not financially prepared for LTC needs. This study evaluates various LTC solutions, including traditional LTC, variable annuity combination products, fixed annuity combination products, variable life insurance combinations, universal life combinations, and "hybrid" LTC. Results suggest the areas of differentiation include: cost per unit of coverage, case design flexibility, certainty of coverage, liquidity, investment upside, death benefit to heirs, and degree of medical underwriting required.

Finke, M. (2019). *Bill Sharpe Seeks a Better Retirement Income Solution*. Tech. rep. ThinkAdvisor.
Sharpe still comes up with simple truths that can rock even a veteran's understanding of retirement planning. To Sharpe, the role of a financial advisor is to understand the range of options all clients face in order to build an income that efficiently balances risk and return. The advisor needs to understand these tradeoffs, and he or she needs to be able to explain them to a client. He's even written a book that's available on his Stanford

website to help advisors "focus more on communicating possible outcomes and helping the clients understand the options." Sharpe's critique of the 4% rule is that the methodology is supposed to safely provide a straight line of after-inflation income using a portfolio of random returns over an unknown lifetime. But when you don't know what asset returns will look like and how long you'll live, a retiree has two lifestyle choices: (1) live well and risk running out of money, or (2) be conservative and risk giving too much to your heirs.

Finke, M. and Pfau, W. (2015). "Reduce Retirement Costs with Deferred Income Annuities Purchased before Retirement." In: *Journal of Financial Planning* 28(7), pp. 40–49.
Most deferred income annuities (DIAs) have a relatively short deferral period, making them particularly appealing to clients nearing retirement who value the ability to plan on a fixed, nominal income stream after retirement. Unlike long-term deferral period annuities (longevity insurance) that are primarily meant to protect against longevity risk, a short-term deferral period annuity can provide a steady income to pre-fund retirement spending over the entire retirement life cycle. The purchase of a DIA before retirement is particularly valuable for clients who would have maintained a stock allocation lower than 70 percent. Retirees would have been better off without a DIA in simulations where portfolio returns are very high or when retirees die early. Findings suggest that short deferral period annuities can reduce the cost of funding retirement, provide longevity protection, and offer behavioral benefits to clients concerned about near-retirement market performance.

Finke, M. S. and Blanchett, D. (2016). "An Overview of Retirement Income Strategies." In: *Journal of Investment Consulting* 17(1), pp. 22–30.
This paper provides an introduction to strategies that can be used to turn a retirement portfolio into income. Defined contribution savings present a retirement planning challenge when asset returns, longevity, and spending needs are uncertain. We present the fundamental trade-offs of managing an investment portfolio and decumulating investments in retirement, and we discuss the potential benefit of financial products that address portfolio and spending risks. Simulations illustrate the trade-offs among legacy, liquidity, and shortfall risk. We dis-cuss a comprehensive strategy that combines longevity protection, portfolio allocation, and dynamic spending based on realized portfolio returns to provide a guideline for professionals building retirement income plans for clients. We then provide an overview of the benefits of adding products such as annuities to a traditional investment portfolio to reduce longevity risk and increase expected spending. The primary purpose of this paper is to provide a fundamental overview of current research and theory on retirement income planning.

Fisch, J. E. and Turner, J. A. (2018). "Making a Complex Investment Problem Less Difficult:Robo Target-Date Funds." In: *The Journal of Retirement* 5(4), pp. 40–45.
Investing is a complicated affair, particularly for people with low financial literacy. Target-date funds are designed to make investing easy for pension participants. To simplify the employee decision, many defined contribution plans offer employees a target-date fund based on only one piece of data employee expected retirement date. The employee may be placed in a target-date fund as the default plan if the employee does not make an active choice. Target-date funds one-size-fits-all approach generally does not provide the appropriate level of risk for all employees who plan to retire in a given year. The authors address that issue in this article. Their proposal has three parts. First, they propose that target-date funds should allow greater personalization of investments by offering participants a conservative, moderate, and risky fund for each target-date. While pension participants currently have the option of choosing a more or less risky target-date fund by choosing a later or earlier target-date than their actual retirement date, many pension participants lack the sophistication to take advantage of that option. Second, they suggest that pension plans incorporate robo advisers to help participants identify the appropriate level of risk and appropriate target-date fund based on their personal circumstances. Third, they propose on-the-spot financial education, to be provided when a participant is selecting a target-date fund, to help participants understand the implications of risk level and target-date fund choice for both pension growth and the range of possible outcomes.

Fonseca, R., Michaud, P.-C., Galama, T., and Kapteyn, A. (2021). "Accounting for the Rise of Health Spending and Longevity." In: *Journal of the European Economic Association* 19(1), pp. 536–579.
We estimate a stochastic life-cycle model of endogenous health spending, asset accumulation, and retirement to investigate the causes behind the increase in health spending and longevity in the United States over the period 1965-2005. Accounting for changes over time in taxes, transfers, Social Security, income, health insurance, smoking and obesity, and technological progress, we estimate that technological progress is responsible for half of the increase in life expectancy over the period. Substantial growth in health spending over the period is largely the result of growth in economic resources and the generosity of health insurance, with a modest role for medical technological progress. The growth in spending does not come from changes in a single source, but

sources jointly interacted to increase spending: complementarity effects explain up to 26.3% of the increase in health spending. Overall, for those born in 1940, the combined changes in resources and health insurance that occurred over the period are valued at 35.7% of lifetime consumption.

Forsyth, P. (2021). "Two stage decumulation strategies for DC plan investors." In: *International Journal of Theoretical and Applied Finance* 24(01) (2150007).
Optimal stochastic control methods are used to examine decumulation strategies for a defined contribution (DC) plan retiree. An initial investment horizon of 15 years is considered, since the retiree will attain this age with high probability. The objective function reward measure is the expected sum of the withdrawals. The objective function tail risk measure is the expected linear shortfall with respect to a desired lower bound for wealth at 15 years. The lower bound wealth level is the amount which is required to fund a lifelong annuity 15 years after retirement, which generates the required minimum cash flows. This ameliorates longevity risk. The controls are the withdrawal amount each year, and the asset allocation strategy. Maximum and minimum withdrawal amounts are specified. Specifying a short initial decumulation horizon, results in the optimal strategy achieving: (i) median withdrawals at the maximum rate within 2-3 years of retirement (ii) terminal wealth larger than the desired lower bound at 15 years, with greater than 90% probability and (iii) median terminal wealth at 15 years considerably larger than the desired lower bound. The controls are computed using a parametric model of historical stock and bond returns, and then tested in bootstrap resampled simulations using historical data. At the 15 year investment horizon, the retiree has the option of (i) continuing to self-manage the decumulation policy or (ii) purchasing an annuity.

Forsyth, P., Vetzal, K. R., and Westmacott, G. (2017a). "Target Wealth: The Evolution of Target Date Funds." In: *SSRN e-Print*.
Target Date Funds have become very popular with investors saving for retirement. The main feature of these funds is that investors are automatically switched from high risk to low risk assets as retirement approaches. However, our analysis brings into question the rationale behind these funds. Based on a model with parameters fitted to historical returns, and also on model independent bootstrap resampling, we find that constant proportion strategies give virtually the same results for terminal wealth at the retirement date as target date strategies. This suggests that the vast majority of Target Date Funds are serving investors poorly. However, if we allow the asset allocation strategy to adapt to the current level of the total portfolio value, significantly lower risk of terminal wealth can be achieved, at no cost to its expected value.

Forsyth, P., Vetzal, K. R., and Westmacott, G. (2018). "Management of Withdrawal Risk Through Optimal Life Cycle Asset Allocation." In: *SSRN e-Print*.
Retirees who do not have defined benefit pension plans typically must fund spending from accumulated savings. This leads to the risk of depleting these savings, i.e. withdrawal risk. We analyze this risk through full life cycle optimal dynamic asset allocation, including the accumulation and decumulation phases. We pose the asset allocation strategy as a problem in optimal stochastic control. Various possible objective functions are tested and compared using metrics such as the probability of portfolio depletion, the median of the remaining portfolio value, and conditional value at risk (CVAR). The control problem is solved using a Hamilton-Jacobi-Bellman formulation, based on a parametric model of the underlying stochastic processes and a variety of objective functions. Monte Carlo simulations which use the optimal controls are presented to evaluate the performance metrics. These simulations are based on both the parametric model and bootstrap resampling of 91 years of historical data. Based primarily on the resampling tests, we conclude that target-based approaches which seek to establish a safety buffer of wealth at the end of the decumulation period appear to be superior to strategies which directly attempt to minimize risk measures such as the probability of portfolio depletion.

Forsyth, P. A. (2022a). "A Stochastic Control Approach to Defined Contribution Plan Decumulation: "The Nastiest, Hardest Problem in Finance"." In: *North American Actuarial Journal* 26(2), pp. 227–251.
We pose the decumulation strategy for a defined contribution (DC) pension plan as a problem in optimal stochastic control. The controls are the withdrawal amounts and the asset allocation strategy. We impose maximum and minimum constraints on the withdrawal amounts, and impose no-shorting no-leverage constraints on the asset allocation strategy. Our objective function measures reward as the expected total withdrawals over the decumulation horizon, and risk is measured by expected shortfall (ES) at the end of the decumulation period. We solve the stochastic control problem numerically, based on a parametric model of market stochastic processes. We find that, compared to a fixed constant withdrawal strategy, with minimum withdrawal set to the constant withdrawal amount the optimal strategy has a significantly higher expected average withdrawal, at the cost

of a very small increase in ES risk. Tests on bootstrapped resampled historical market data indicate that this strategy is robust to parametric model misspecification.

Forsyth, P. A. (2022b). "Short term decumulation strategies for underspending retirees." In: *Insurance: Mathematics and Economics* 102, pp. 56–74.
There is growing empirical evidence that many retirees are decumulating their assets very slowly, if at all. This fact is in stark contrast to the usual lifecycle models of spending. It appears that these underspending retirees adjust their withdrawals to avoid reducing their assets. In order to appeal to this class of retirees, we use optimal stochastic control techniques which maximize a multi-objective risk-reward problem. The reward is the total of withdrawals (over a five year period), while risk is based on a left tail measure. Our controls for this problem are the withdrawal amount per quarter, and the stock-bond asset allocation. We allow flexible withdrawals (even zero). This added flexibility results in a high probability of (i) retaining 90% of real wealth at the end of five years, and (ii) significant total spending over the five years. We suggest that these types of strategies will be appealing this underspending group of retirees.

Forsyth, P. A., Li, Y., and Vetzal, K. R. (2017b). "Are target date funds dinosaurs? Failure to adapt can lead to extinction." In: *arXiv e-Print*.
Investors in Target Date Funds are automatically switched from high risk to low risk assets as their retirements approach. Such funds have become very popular, but our analysis brings into question the rationale for them. Based on both a model with parameters fitted to historical returns and on bootstrap resampling, we find that adaptive investment strategies significantly outperform typical Target Date Fund strategies. This suggests that the vast majority of Target Date Funds are serving investors poorly.

Forsyth, P. A. and Vetzal, K. R. (2019). "Optimal asset allocation for retirement saving: deterministic vs. time consistent adaptive strategies." In: *Applied Mathematical Finance* 26(1), pp. 1–37.
We consider optimal asset allocation for an investor saving for retirement. The portfolio contains a bond index and a stock index. We use multi-period criteria and explore two types of strategies: deterministic strategies are based only on the time remaining until the anticipated retirement date, while adaptive strategies also consider the investor accumulated wealth. The vast majority of financial products designed for retirement saving use deterministic strategies (e.g., target date funds). In the deterministic case, we determine an optimal open loop control using mean-variance criteria. In the adaptive case, we use time consistent mean-variance and quadratic shortfall objectives. Tests based on both a synthetic market where the stock index is modelled by a jump-diffusion process and also on bootstrap resampling of long-term historical data show that the optimal adaptive strategies significantly outperform the optimal deterministic strategy. This suggests that investors are not being well served by the strategies currently dominating the marketplace.

Forsyth, P. A., Vetzal, K. R., and Westmacott, G. (2019). "Management of Portfolio Depletion Risk through Optimal Life Cycle Asset Allocation." In: *North American Actuarial Journal* 23(3), pp. 447–468.
Members of defined contribution (DC) pension plans must take on additional responsibilities for their investments, compared to participants in defined benefit (DB) pension plans. The transition from DB to DC plans means that more employees are faced with these responsibilities. We explore the extent to which DC plan members can follow financial strategies that have a high chance of resulting in a retirement scenario that is fairly close to that provided by DB plans. Retirees in DC plans typically must fund spending from accumulated savings. This leads to the risk of depleting these savings, that is, portfolio depletion risk. We analyze the management of this risk through life cycle optimal dynamic asset allocation, including the accumulation and decumulation phases. We pose the asset allocation strategy as an optimal stochastic control problem. Several objective functions are tested and compared. We focus on the risk of portfolio depletion at the terminal date, using such measures as conditional value at risk (CVAR) and probability of ruin. A secondary consideration is the median terminal portfolio value. The control problem is solved using a Hamilton-Jacobi-Bellman formulation, based on a parametric model of the financial market. Monte Carlo simulations that use the optimal controls are presented to evaluate the performance metrics. These simulations are based on both the parametric model and bootstrap resampling of 91 years of historical data. The resampling tests suggest that target-based approaches that seek to establish a safety margin of wealth at the end of the decumulation period appear to be superior to strategies that directly attempt to minimize risk measures such as the probability of portfolio depletion or CVAR. The target-based approaches result in a reasonably close approximation to the retirement spending available in a DB plan. There is a small risk of depleting the retiree funds, but there is also a good chance of accumulating a buffer that can be used to manage unplanned longevity risk or left as a bequest.

Forsyth, P. A., Vetzal, K. R., and Westmacott, G. (2020). "Optimal Asset Allocation for DC Pension Decumulation with a Variable Spending Rule." In: *ASTIN Bulletin* 50(2), pp. 419–447.
We determine the optimal asset allocation to bonds and stocks using an annually recalculated virtual annuity (ARVA) spending rule for DC pension plan decumulation. Our objective function minimizes downside withdrawal variability for a given fixed value of total expected withdrawals. The optimal asset allocation is found using optimal stochastic control methods. We formulate the strategy as a solution to a Hamilton-Jacobi-Bellman (HJB) Partial Integro Differential Equation (PIDE). We impose realistic constraints on the controls (no-shorting, no-leverage, discrete rebalancing) and solve the HJB PIDEs numerically. Compared to a fixed-weight strategy which has the same expected total withdrawals, the optimal strategy has a much smaller average allocation to stocks and tends to de-risk rapidly over time. This conclusion holds in the case of a parametric model based on historical data and also in a bootstrapped market based on the historical data.

Forsyth, P. A., Vetzal, K. R., and Westmacott, G. (2021). "Optimal control of the decumulation of a retirement portfolio with variable spending and dynamic asset allocation." In: *ASTIN Bulletin* 51(3), pp. 905–938.
We extend the Annually Recalculated Virtual Annuity (ARVA) spending rule for retirement savings decumulation (Waring and Siegel (2015) Financial Analysts Journal, 71(1), 91-107) to include a cap and a floor on withdrawals. With a minimum withdrawal constraint, the ARVA strategy runs the risk of depleting the investment portfolio. We determine the dynamic asset allocation strategy which maximizes a weighted combination of expected total withdrawals (EW) and expected shortfall (ES), defined as the average of the worst 5% of the outcomes of real terminal wealth. We compare the performance of our dynamic strategy to simpler alternatives which maintain constant asset allocation weights over time accompanied by either our same modified ARVA spending rule or withdrawals that are constant over time in real terms. Tests are carried out using both a parametric model of historical asset returns as well as bootstrap resampling of historical data. Consistent with previous literature that has used different measures of reward and risk than EW and ES, we find that allowing some variability in withdrawals leads to large improvements in efficiency. However, unlike the prior literature, we also demonstrate that further significant enhancements are possible through incorporating a dynamic asset allocation strategy rather than simply keeping asset allocation weights constant throughout retirement.

Fox, S. (2020). "Linking metrics to objectives: retirement saving, spending, and active management." In: *The Journal of Retirement* 7(3), pp. 6–29.
Although financial advisors and institutions are increasingly urged to adopt objectives or goals-based investment strategies, there is a need to more clearly define what is meant by investment objectives and their effects on outcomes. In a controlled modeling experiment, I formalize two investor personas (Accumulator and Decumulator) and two investment strategies (Target Cash Flows and Target Wealth) to provide a deeper understanding of the effects on investor "success" or "failure." In addition, applying characteristics that are potentially available in actively managed funds, such as differences among investments in terms of beta, alpha, and sequence of returns, enables the demonstration of several surprising effects, including changes in the distribution of investment returns and, of importance, the trajectory of investor outcomes in an objectives-based framework.

Frank, L. R. and Brayman, S. (2016). "Combining Stochastic Simulations and Actuarial Withdrawals into One Model." In: *Journal of Financial Planning* 29(11), pp. 44–53.
This paper explored a method that combined actuarial approaches for calculating withdrawals in retirement with Monte Carlo simulations. The model recalculated withdrawals for each scenario within each simulation, with a new simulation beginning for each year based on individual capital remaining and adjusted time horizons using mortality tables. The model approach made a direct connection between the various elements making up spending decisions. There were important differences between a simulation that used decision rules to shape income and altered initial withdrawal assumptions, and a model that recast each subsequent year in isolation. The combination of stochastic simulations and period-adjusting actuarial withdrawal techniques proposed here has been termed Continually Adjusting Stochastic Actuarial Model (CASAM).

Frank, L. R., Mitchell, J. B., and Blanchett, D. M. (2012). "An Age-Based, Three-Dimensional Distribution Model Incorporating Sequence and Longevity Risks." In: *Journal of Financial Planning* 25(3).
The authors develop an age-based, three-dimensional distribution model that illustrates a retiree's yearly transition through retirement, including superannuated years (very old age), to demonstrate the impact of longevity risk
The model builds on "Probability-of-Failure-Based Decision Rules to Manage Sequence Risk in Retirement" by Frank, Mitchell, and Blanchett (2011) to simultaneously:

1) Establish an age-based distribution model

2) Incorporate current-age life expectancy
3) Address survivorship into superannuated ages
4) Address market sequence risk
5) Develop a method to rationally incorporate retiree goals of either consumption or inheritance, and switch between the two as desired

Past research has primarily focused on fixed, non-age-specific distribution periods. The model developed in this paper demonstrates:

- How longevity probability can be developed into dynamic, yearly adjustable distribution periods
- These dynamic distribution periods can then be combined with stochastic (Monte Carlo) real returns
- The distribution period (DP) can be dynamically managed, as the retiree ages, by varying the percentiles of longevity
- The withdrawal rate is very sensitive to DP, and the DP is very sensitive to life expectancy; thus more attention should be paid to longevity risk than has been in past research
- The probability of failure (POF) remains a useful metric to evaluate exposure of a retiree's portfolio to market sequence risk, and a POF-based decision rule applies to both the asset allocation effect and the age effect on withdrawal rates

A working paper, which includes appendices with data that demonstrate how these figures are developed, and additional data, is available at SSRN: www.ssrn.com/abstract=1849983.

Franklin, M. B. (2021). *Maximizing Social Security Retirement Benefits*. Tech. rep. InvestmentNews.

This is the guide for consumers and financial advisers about how this critical source of guaranteed income fits into an overall retirement plan.
Topics include:

- The ABC's of claiming Social Security
- How Social Security benefits are taxed
- What survivors need to know to maximize their benefits
- What claiming strategies are disappearing for most new retirees
- How recent changes to Social Security claiming rules affect married couples and divorced spouses

Plus, much more including new rules & strategies to help maximize benefits.

Fraser, S. P. and Payne, B. C. (2018). "Bond Tents: Reshaping the Equity Glide Slope at the End of Wealth Accumulation." In: *The Journal of Wealth Management* 21(2), pp. 27–38.

Asset allocation is widely accepted as a primary driver of portfolio returns over time. Conventional investment practice recommends that investors reduce their risk and thus increase their allocation to fixed-income investments as they approach retirement. This research extends the literature by examining the shape of the fixed-income glide slope prior to retirement. The results suggest that portfolios with high fixed-income allocations, or bond tents, can serve investors well, and nonlinear glide slopes can produce favorable portfolio characteristics for investors.

French, E. and Jones, J. B. (2017). "Health, Health Insurance, and Retirement: A Survey." In: *Annual Review of Economics* 9(1), pp. 383–409.

The degree to which retirement decisions are driven by health is a key concern for both academics and policy makers. In this review, we survey the economic literature on the health-retirement link in developed countries. We describe the mechanisms through which health affects labor supply and discuss how these mechanisms interact with public pensions and public health insurance. The historical evidence suggests that health is not the primary source of variation in retirement across countries and over time. Furthermore, the decline of health with age can only explain a small share of the decline in employment near retirement age. Health considerations nonetheless play an important role, especially in explaining cross-sectional variation in employment and other outcomes within countries. We review the mechanisms through which health affects retirement and discuss recent empirical analyses.

Friedenthal, M. (2020). "The Third Generation of Financial Planning." In: *Advisor Perspectives*.
By introducing additional variables to the simulation process within a financial plan, a more realistic and reliable representation of probable outcomes will be achieved. The third generation of financial planning incorporates dynamic correlations between stocks, bonds, and inflation as well as dynamic volatilities of each data series. It incorporates dynamic risk directive expectations that evolve as clients migrate towards and through their cash-flow series. Layering into the simulation process the customizable, last-survivor probabilities per annum facilitates an encompassing probability of outliving one's money, currently the most robust metric in planning technology.

Fuino, M. and Wagner, J. (2018). "Long-term care models and dependence probability tables by acuity level: New empirical evidence from Switzerland." In: *Insurance: Mathematics and Economics* 81, pp. 51–70.
Abstract Due to the demographic changes and population aging occurring in many countries, the financing of long-term care (LTC) poses a systemic threat. The scarcity of knowledge about the probability of an elderly person needing help with activities of daily living has hindered the development of insurance solutions that complement existing social systems. In this paper, we consider two models: a frailty level model that studies the evolution of a dependent person through mild, moderate and severe dependency states to death and a type of care model that distinguishes between care received at home and care received in an institution. We develop and interpret the expressions for the state- and time-dependent transition probabilities in a semi-Markov framework. Then, we empirically assess these probabilities using a novel longitudinal dataset covering all LTC needs in Switzerland over a 20-year period. As a key result, we are the first to derive dependence probability tables by acuity level, gender and age for the Swiss population. We find that the transition probabilities differ significantly by gender, age and time spent in the frailty level and type of care states.

Fullmer, R. K. (2009). "A Framework for Portfolio Decumulation." In: *Journal of Investment Consulting* 10(1), pp. 63–71.
Before the bear market of 2008-09, the conventional wisdom was that conservative asset allocations might not provide sufficient returns for the long horizons that many retirees face. Therefore, new retirees needed significant allocations to equity. The advice often came with a call to invest more aggressively in the early years of retirement, then taper the equity exposure later in retirement. Arguments for this approach typically present results of Monte Carlo simulation showing that higher equity allocations, particularly early in retirement, can increase the probability of sustaining higher withdrawal rates from the portfolio. But these arguments fall victim to at least two fallacies.
The first fallacy is tragically ironic. Even though the advice is based on probability theory, it overlooks a primary tenet emphasized by Blaise Pascal, the founder of probability theory. Pascal realized that while probability matters, so do consequences. In fact, Pascal emphatically warned against playing the odds without considering the consequences. While it may be true that higher equity exposures can increase the likelihood of sustaining higher withdrawal rates, they also increase the magnitude of failure in unsuccessful cases. Students of investment theory may recognize this oversight as a form of "the fallacy of time-diversification of risk." By making the portfolio more aggressive, the risk of achieving poor returns puts the entire financial plan in peril. The retiree may go broke early, face a lower standard of living, or abandon retirement altogether. So everyone who advises retirees must seriously ponder the following question: Even though higher equity allocations may raise the probability of success for a retirement income plan, does this necessarily mean that higher equity allocations are less risky?
The second fallacy involves an assumption underlying most simulation methodologies. The assumption is that the investment strategy is predetermined at the start of the investment horizon. Examples of predetermined strategies include allocations that remain static over the entire horizon or that use glide paths similar to that of target date funds, in which the equity exposure tapers over time. The only way such an assumption makes sense, however, is if the investor's risk exposure is static over time. But for retirees who depend on nest eggs for living expenses, this assumption is never correct. Holding more-aggressive portfolios early in retirement and less-aggressive allocations later in retirement leads to the retiree taking the most investment risk precisely when it is most risky to do so - when the risk of outliving the portfolio (longevity risk) is greatest.

Fullmer, R. K. (2012). "Modern Portfolio Decumulation A New Strategy for Managing Retirement Income." In: *Someday Rich: Planning for Sustainable Tomorrows Today*. Wiley, pp. 249–271.
This chapter presents Richard K. Fullmer's paper, titled "Modern Portfolio Decumulation." It is a logical extension of his mis-measurement of risk work. It describes a new framework for efficient portfolio construction in the decumulation phase of the investment lifecycle, in which an investor who has accumulated assets over

time wishes to use those assets to fund ongoing living expenses. It combines elements of both investment theory and actuarial science, introducing an effective way to manage longevity risk in the portfolio. The chapter discusses that the conventional standard for accumulation of wealth is not optimal for decumulation. It outlines an alternative approach for investing in the decumulation phase, bringing focus upon dynamic asset allocation models.

Fullmer, R. K. and Tzitzouris, J. A. (2014). "Evaluation of Target-Date Glide Paths withinDefined Contribution Plans." In: *The Journal of Retirement* 1(4), pp. 75–94.
In this article, the authors present a methodology to support sponsors of defined-contribution (DC) plans when evaluating the allocation to equity over a target-date fund's glide path. When evaluating glide paths, sponsors must pay heed to two primary goals: The first is to generate lifetime income consistently over the course of retirement; the second is to limit the risk of capital loss near and during retirement, which is particularly important for participants who withdraw balances over short horizons. The challenge of glide path selection is striking a compromise between these competing goals. This compromise cannot be achieved via objective analysis alone and must also be informed by the subjective horizon and risk preferences of the sponsor acting as agent for the plan participants. The authors find that higher-equity glide paths offer greater efficacy for lifetime income replacement, while lower-equity glide paths offer greater efficacy for stable account balances with lower risk of capital losses. These conclusions hold over a wide range of plan characteristics and assumptions, although the relative magnitude of the trade-off depends on characteristics unique to each plan.

Fulton, C. and Hubrich, K. (2021). "Forecasting US Inflation in Real Time." In: *Econometrics* 9(4), p. 36.
We analyze real-time forecasts of US inflation over 1999Q3-2019Q4 and subsamples, investigating whether and how forecast accuracy and robustness can be improved with additional information such as expert judgment, additional macroeconomic variables, and forecast combination. The forecasts include those from the Federal Reserve Board's Tealbook, the Survey of Professional Forecasters, dynamic models, and combinations thereof. While simple models remain hard to beat, additional information does improve forecasts, especially after 2009. Notably, forecast combination improves forecast accuracy over simpler models and robustifies against bad forecasts; aggregating forecasts of inflation's components can improve performance compared to forecasting the aggregate directly; and judgmental forecasts, which may incorporate larger and more timely datasets in conjunction with model-based forecasts, improve forecasts at short horizons.

Gabudean, R., Gomes, F., Michaelides, A., and Zhang, Y. (2021). "On Optimal Allocations of Target-Date Funds." In: *The Journal of Retirement* 9(2), pp. 58–79.
We study optimal life-cycle portfolio allocation and its application to target-date fund (TDF) design. We show that optimal TDF allocation must be explicitly linked to a savings rate; for example, a higher savings rate generating higher financial wealth accumulation should necessarily come with a more conservative TDF. Further, we quantify the extent to which accumulated wealth, the investing environment, and participant characteristics affect TDF allocations and compare the resulting optimal allocations against the observed universe of US TDF products. Plan sponsors and asset managers can use these findings to improve TDF selection and design.

Gabudean, R. C. and Weiss, R. A. (2019). "How to Evaluate Target-Date Funds: A Practical Guide." In: *The Journal of Retirement* 6(4), pp. 68–81.
The authors describe and showcase a framework to analyze life-cycle, or target-date, investment strategies with intuitive, practical metrics. Anchored in the complex theory of life-cycle investing, we propose a comprehensive collection, or , of measures to capture various aspects of the problem: wealth-focused versions of return and risk concepts; the link between retirement contributions and portfolio returns; the strategy ability to support income in retirement; and behavior around retirement, a period when risk to invested wealth is greatest. These metrics are synthesized into one holistic view based on investor-specific preferences, highlighting the connection between investor characteristics and the life-cycle investment problem.

Gan, G. (2013). "Application of data clustering and machine learning in variable annuity valuation." In: *Insurance: Mathematics and Economics* 53(3), pp. 795–801.
We study the pricing of a large portfolio of VA policies. A clustering method is used to select representative policies. A machine learning method is used to estimate the guarantee value. The proposed method performs well in terms of accuracy and speed. The valuation of variable annuity guarantees has been studied extensively in the past four decades. However, almost all the studies focus on the valuation of guarantees embedded in a single variable annuity contract. How to efficiently price the guarantees for a large portfolio of variable annuity contracts has not received enough attention. This paper fills the gap by introducing a novel method based on

data clustering and machine learning to price the guarantees for a large portfolio of variable annuity contracts. Our test results show that this method performs very well in terms of accuracy and speed.

Gao, X. and Sun, L. (2021). "Modeling retirees' investment behaviors in the presence of health expenditure risk and financial crisis risk." In: *Economic Modelling* 94, pp. 442–454.
This paper examines the impact of both health risks and financial market turmoil on individual investment decisions during retirement. Using a panel data set compiled from the U.S. Health and Retirement Survey, we find that health shocks increase retirees' disposable income uncertainty and compromise their risk-taking capacity. This is especially the case for those with moderate levels of wealth. However, we find that health risk alone cannot explain the empirically observed lower level of risky asset investment behaviors. It is the presence of extreme events in the financial markets, in combination with health risks, that significantly reduce retirees' incentives to invest in risky assets. Compared with younger investors, retirees' shorter investment horizon and higher health risks render them more vulnerable to the detrimental consequences of wealth evaporation caused by financial crises. Our results indicate that effective portfolio management for retirees must take into account both their health risks and the occurrence of financial market tail risks.

Gard, R. and Gremm, M. (2018). "Two Measures of Financial Risk Tolerance from Questionnaire Data." In: *SSRN e-Print*.
Set-based principal component analysis extracts two independent measures of financial risk tolerance from questionnaire responses. These measures are less noisy than most scoring methods. They are also more robust because they provide a redundant assessment of financial risk tolerance. This analysis reveals that typical risk questionnaires measure financial risk tolerance and income-related risk tolerance. These two factors are unrelated, which suggests that income-related questions should be dropped from financial risk questionnaires. We also examine how financial risk tolerance depends on financial literacy and gender.

Ge, W. (2019a). "Optimal Glide Path Selection for Low-Volatility Assets." In: *The Journal of Index Investing* 10(3), pp. 70–84.
Target-date funds (TDFs) have become a popular choice for retirement plans, and the concept of a glide path is essential for TDFs. One of the key assumption behind glide paths is that the main component of a retirement plan should be growth assets invested in the overall equity market example, the S&P 500 Index. Researchers have recently challenged this assumption and argued for using smart beta factors for retirement purposes. Among them, the factor of low volatility may be uniquely suitable for retirement investing. This article studies the use of low-volatility assets for the purpose of retirement planning and the choice of ideal glide paths. This study is agnostic about the means by which the low-volatility risk/return profile is achieved and analyzes four series with diminished volatility constructed with different methodologies. The article concludes that low-volatility assets may indeed help retirement investors achieve their objectives, though such investors must choose ideal glide paths carefully to suit the selected low-volatility series. When equipped with proper low-volatility assets and carefully chosen glide paths, retirement plan managers may both improve the odds that their plans succeed and increase the expected final wealth levels.

Ge, W. (2019b). "Utilizing Low-Volatility Assets To Mitigate Sequence Risk In Retirement Investing." In: *The Journal of Investing* 28(5), pp. 85–100.
Sequence risk is a subtle risk factor retirement investment managers face that can hinder their plans for success. This article illustrates the damaging effects that sequence risk can have on retirement plans and how asset volatility can exacerbate this risk. The study analyzes an asset-centric approach that utilizes low-volatility assets to mitigate sequence risk, as their diminished volatility implies more certainty for retirement plans. It further examines the return-risk profiles of four candidate low-volatility series, compares their performance either as stand-alone series or in the context of 60/40 portfolios with the traditional equity represented by the S&P 500 Index, and finally employs two sets of Monte Carlo simulations to assess the effectiveness of such assets for the task of sequence risk mitigation. The study demonstrates that 60/40 portfolios with low-volatility components can significantly reduce the uncertainty of retirement investment outcomes in terms of both narrower final wealth ranges and reduced failure rates.

Geisler, G., Harden, B., and Hulse, D. (2021). "A Comparison of the Tax Efficiency of Decumulation Strategies." In: *Journal of Financial Planning* 34(3), pp. 72–89.
This article examines several decumulation strategies for a client approaching retirement with a mix of tax-favored retirement accounts and taxable accounts, each holding appreciated stocks and taxable bonds.
The analysis applies these strategies to three "nest egg" scenarios to determine their resulting portfolio lives, while examining tax consequences throughout those lives.

In these scenarios, the more tax-efficient decumulation strategies last from 1 to over 11 percent longer than both the Conventional Wisdom and Schwab's recently marketed decumulation strategies.
The results provide insights that financial planning professionals can use to tailor tax-efficient decumulation recommendations that better fit a client's particular situation, increasing how long their wealth lasts during retirement.

Geisler, G. and Hulse, D. (2016). "The Taxation of Social Security Benefits and Planning Implications." In: *Journal of Financial Planning* 29(5), pp. 52–63.
Up to 85 percent of Social Security benefits could be taxable, with the percentage increasing as income increases. Additional income can cause additional Social Security benefits to be taxable, a so-called "tax torpedo." The tax effect of this tax torpedo depends on the tax bracket in which the benefits are taxed. This effect can be as high as 21.25 percent of the additional income, but it is less than 10 percent in many circumstances. This tax effect is an important factor to consider when deciding whether to start Social Security benefits at age 62 or age 70.

Gemmo, I., Rogalla, R., and Weinert, J.-H. (2020). "Optimal portfolio choice with tontines under systematic longevity risk." In: *Annals of Actuarial Science* 14(2), pp. 302–315.
We derive optimal portfolio choice patterns in retirement (ages 66-105) for a constant relative risk aversion utility maximising investor facing risky capital market returns, stochastic mortality risk, and income-reducing health shocks. Beyond the usual stocks and bonds, the individual can invest his assets in tontines. Tontines are cost-efficient financial contracts providing age-increasing, but volatile cash flows, generated through the pooling of mortality without guarantees, which can help to match increasing financing needs at old ages. We find that a tontine invested in the risk-free asset dominates stock investments for older investors without a bequest motive. However, with a bequest motive, it is optimal to replace the tontine investment over time with traditional financial assets. Our results indicate that early in retirement, a tontine is only an attractive investment option, if the tontine funds are invested in a risky asset. In this case, they crowd out stocks and risk-free bonds in the optimal portfolios of younger investors. Over time, the average optimal portfolio weight of tontines decreases. Introducing systematic mortality risks noticeably reduces the peak allocation to tontines.

Ghilarducci, T., Radpour, S., and Webb, A. (2019). "New evidence on the effect of economic shocks on retirement plan withdrawals." In: *The Journal of Retirement* 6(4), pp. 7–19.
Using data from the Survey for Income and Program Participation (SIPP), this study investigates the relationship between withdrawals from 401(k) and IRA accounts and household level economic shocks such as job loss, job change, divorce, and the onset of poor health. Workers in low-wage households are more likely to withdraw from their accounts than those in middle and high income households, in part because they are more likely to withdraw when they experience a shock and also experience more shocks. Shocks are associated with about 20% of all retirement account withdrawals and exacerbate pre-existing inequalities in financial preparation for retirement.

Glickman, M. M. and Hermes, S. (2015). "Why Retirees Claim Social Security at 62 and How It Affects Their Retirement Income: Evidence from the Health and Retirement Study." In: *The Journal of Retirement* 2(3), pp. 25–39.
Despite higher monthly benefits for those who delay, many people still claim Social Security retirement benefits at age 62, the earliest age of eligibility. This study uses data from the Health and Retirement Study to examine the demographic and occupational characteristics associated with early claiming, as well as the retirement income of early claimers compared with those who delay. The authors find that several work-related factors may cause people to claim Social Security benefits early and suggest they may face challenges in continuing to work at older ages. For example, those who worked in physically demanding blue-collar jobs were 55% more likely to claim benefits prior to their full retirement age, after controlling for other factors, compared with those in all other occupations. Other factors, such as having lower expectations of living to age 75 significantly increase the likelihood of claiming early. The median income for those who claim at full retirement age or later was 45% higher after claiming benefits than for those who claimed early and 33% higher at age 72. Even when comparing early and delayed claimers with similar total income after claiming, average household income for delayed claimers was higher at age 72 than for early claimers.

Goldberg, L. R., Cai, T., and Hand, P. (2021). "Tax-Rate Arbitrage: Realization of Long-Term Gains to Enable Short-Term Loss Harvesting." In: *SSRN e-Print*.
We look at an enhanced loss-harvesting strategy, tax-rate arbitrage, which exploits the differential between short- and long-term tax rates. Our study relies on ATBAT, an After-Tax Back-Testing Analysis Tool that lets us examine tax-managed strategies over numerous historical periods. For the ideal tax-rate arbitrage investor,

one who is subject to federal only tax rates, who has a long horizon and a planned liquidation date, and who launches the strategy from all cash, tax-rate arbitrage generated an average of 0.78% in excess after-tax active return at a 10-year horizon relative to a standard loss-harvesting strategy. Other investors with different profiles may benefit from tax-rate arbitrage but typically to a lesser extent.

Goldman Sachs Asset Management Research (2021). *Retirement Realities: Time For Change: Retirement Survey & Insights Report 2021*. Tech. rep. Goldman Sachs Asset Management.

Goldman Sachs Asset Management released the findings of its Retirement Survey & Insights Report, which seeks to learn directly from plan participants about their experience preparing for, transitioning to, and managing their finances in retirement.

The survey revealed that financial hardships have affected retirement savings for 89% of workers, compared to 17% of retirees. More than half of the retirees polled retired earlier than expected, and health was the top concern among retirees. The youngest respondents are expecting to leave the workforce much earlier, however, with 25% of Generation Z respondents planning to retire before the age of 55.

Overall, survey responses highlight the importance of employer retirement programs for both retired and currently working individuals, with 56% of retired respondents (and 38% of working respondents) surveyed stating that they derive their source of retirement education from employer retirement programs.

Responses were analyzed from two distinct populations: those currently in retirement and those currently working, with the latter cohort spanning a range of age groups, including Generation X, Millennials and Generation Z individuals, in geographies across the US.

Notable highlights from the report include:

1) More than half of retired respondents retired earlier than expected and health was the top reason listed for retired individuals. Health reasons topped the list of reasons for retirement, with 24% of respondents forced into retirement due to health concerns, and with only 3% of respondents having worked longer than expected. Notably, only 9% of participants indicated that having sufficient retirement savings is a key trigger for retirement.

2) Interrupted savings may be causing shortfalls in retirement plans. Eighty-nine percent of working individuals say financial hardships impacted their retirement savings, compared to 17% of retired individuals. Eighty-three percent of workers say paying down existing loans impacted their retirement savings, compared to 15% of retirees.

3) Younger workers may need to plan with more realistic assumptions. Twenty-five percent of Generation Z respondents plan to retire before age 55, while roughly 30% of those under age 40 believe they need 60% or less of their pre-retirement income in retirement, compared to Generation X respondents, who feel they need 80% of their pre-retirement income in retirement.

4) Employer-sponsored retirement programs are the most used source of retirement education. Survey responses highlight the importance of employer retirement programs for both retired and currently working individuals, with 56% of retired respondents surveyed stating that they derive their source of retirement education from employer retirement programs. Additionally, offering new programs or continuing to expand on existing ones is an important value driver for any organization.

The survey was conducted between July and August 2021 in the United States, with responses from 1,237 participants (613 working Generation X, Millennials and Generation Z individuals and 624 retired individuals (aged 60-75)).

Goldstein, B. (2018). "The glide path less traveled: A critical examination of assumptions, outcomes, and glide path specification." In: *SSRN e-Print*.

This paper defines key assumptions, derived from empirical data, to deliver highly accurate and representative simulations of participant outcomes. The paper constructs an alternative glide path design that we believe more efficiently balances major retirement savings risks - longevity/shortfall, return sequence, and inflation - than the industry average glide path. The paper demonstrates that an alternative glide path - one that begins with higher equity allocations than the industry average, follows a steeper slope of asset shifts, and arrives at a risk-parity solution at the point of retirement– has less variability in outcomes than the industry average.

Goodman, B. and Richardson, D. P. (2019). "Achieving Retirement Income Security: A Comparison of Guaranteed Lifetime Withdrawal Benefit, Systematic Withdrawal and Partial Variable Annuity Strategies." In: *SSRN e-Print*.

Many retirement income products attempt to satisfy multiple and sometimes conflicting objectives because retirees desire products that provide retirement security, inflation protection, liquidity, asset growth and the potential for an estate. In this paper, we used historical data over the past 90 years to conduct simulations and analyze how a Guaranteed Lifetime Withdrawal Benefit (GLWB), systematic withdrawal, and partial Variable Immediate Annuity (VIA) strategies performed in meeting these multiple objectives. With the exception of retirement income starting dates at the beginning of the Great Depression, all three strategies performed well in providing income throughout retired life. The partial VIA and GLWB strategies provided better "peace of mind" retirement income products, while the systematic withdrawal strategy offered the greatest flexibility in managing retirement assets. Overall, we conclude that a partial VIA income strategy comprised of a VIA and supplemental liquid asset account would have provided the best mixture of income generation, risk management, and estate potential for the majority of cohorts.

Grable, J. E., Lyons, A. C., and Heo, W. (2019). "A test of traditional and psychometric relative risk tolerance measures on household financial risk taking." In: *Finance Research Letters* 30, pp. 8–13.
This article provides a comparative test of two ways in which individual relative risk tolerance is commonly measured in research and practice. Results suggest that the popular method of measuring gambles across lifetime income provides less insight into household investment behavior than the psychometric approach that is often used to derive a risk profile measure.

Grealish, A. and Kolm, P. N. (2021). "Robo-Advisors Today and Tomorrow: Investment Advice Is Just an App Away." In: *The Journal of Wealth Management* 24(3), pp. 144–155.
Over the past decade robo-advisors have gone from offerings from a handful of start-ups to an established and fast-growing segment of the wealth management industry. Robo-advisors use technology to translate core retail investing principles-establishing an investment plan, seeking broad diversification, weighting cost and value, and accounting for taxes-into automated platforms with easy-to-use interfaces. Intuitive user experience backed by well-established investment processes allow individual investors to create and execute investment strategies with little or no interaction with a financial professional, resulting in large economies of scale for robo-advisors and low costs for clients. In this article, we discuss how robo-advisors assess client risk tolerance, build, and recommend portfolios, manage risk, and optimize taxes. We discuss the implementation of goal-based investing, socially responsible (ESG) investing, and smart beta strategies on robo-advisory platforms. Additionally, we examine robo-advisor performance during the market downturn in March 2020, the first significant market drawdown since their introduction, and find portfolio performance in agreement with broadly diversified stock and bond holdings. Consistent with expectations, robo-advisors also reported increased tax-loss harvesting activity during this downturn.

Greiner, S. P. and Stoyanov, S. V. (2019). "Portfolio scoring by expected risk premium." In: *The Journal of Portfolio Management* 45(4), pp. 83–90.
In this article, the authors discuss a general method for ranking portfolios that places few limitations on the portfolio constituents other than using publicly traded assets. The ranking scores reflect the expected reward investors would require for accepting the risks of the portfolio in the context of an asset pricing framework. The scores are computed through a factor model that acknowledges the factor return correlations. The authors illustrate the approach with a large universe of exchange-traded funds assuming a linear model with Fama-French-Carhart factors wherein factor premiums (i.e., expected returns) are proportional to factor volatilities. The empirical analysis implies that the most significant factors from the Fama-French-Carhart factor set driving the premiums are the market and the momentum factors.

Grennon, T. (2015). "A Dynamic Asset Allocation Approach for Selecting a 401K QDIA." In: *SSRN e-Print*.
The selection of the asset allocation and target date mutual funds that are used as qualified default investment alternative (QDIA) options for 401k participants are driven today by strategic asset allocation, risk and target retirement start dates. This paper details a dynamic asset allocation approach to be used for selecting a QDIA which uses forward looking return assumptions and the Sharpe Ratio for selection.

Grennon, T. (2016). "A Dynamic Approach to Retirement Income." In: *SSRN e-Print*.
We are all well aware of the pitfalls of a tradition systematic withdrawal approach to retirement income especially in light of what we know about sequence return risk. What if instead of simply assuming we earn 10% on stocks, 5% on bonds, and 2.5% on cash equivalents and withdrawing 4% we utilize forward looking return assumptions based on fundamentals (stocks) and yields (bonds, cash)? The following paragraphs explore the idea and test the concept of a safe relative withdrawal rate.

Gudkov, N., Ignatieva, K., and Ziveyi, J. (2019). "Pricing of guaranteed minimum withdrawal benefits in variable

annuities under stochastic volatility, stochastic interest rates and stochastic mortality via the componentwise splitting method." In: *Quantitative Finance* 19(3), pp. 501–518.
This paper values guaranteed minimum withdrawal benefit (GMWB) riders embedded in variable annuities assuming that the underlying fund dynamics evolve under the influence of stochastic interest rates, stochastic volatility, stochastic mortality and equity risk. The valuation problem is formulated as a partial differential equation (PDE) which is solved numerically by employing the operator splitting method. Sensitivity analysis of the fair guarantee fee is performed with respect to various model parameters. We find that (i) the fair insurance fee charged by the product provider is an increasing function of the withdrawal rate; (ii) the GMWB price is higher when stochastic interest rates and volatility are incorporated in the model, compared to the case of static interest rates and volatility; (iii) the GMWB price behaves non-monotonically with changing volatility of variance parameter; (iv) the fair fee increases with increasing volatility of interest rates parameter, and increasing correlation between the underlying fund and the interest rates; (v) the fair fee increases when the speed of mean-reversion of stochastic volatility or the average long-term volatility increases; (vi) the GMWB fee decreases when the speed of mean-reversion of stochastic interest rates or the average long-term interest rates increase. We investigate both static and dynamic (optimal) policyholder's withdrawal behaviours; we present the optimal withdrawal schedule as a function of the withdrawal account and the investment account for varying volatility and interest rates. When incorporating stochastic mortality, we find that its impact on the fair guarantee fee is rather small. Our results demonstrate the importance of correct quantification of risks embedded in GMWBs and provide guidance to product providers on optimal hedging of various risks associated with the contract.

Guidolin, M., Hansen, E., and Lozano-Banda, M. (2018). "Portfolio performance of linear SDF models: an out-of-sample assessment." In: *Quantitative Finance* 18(8), pp. 1425–1436.
We evaluate linear stochastic discount factor models using an ex-post portfolio metric: the realized out-of-sample Sharpe ratio of mean-variance portfolios backed by alternative linear factor models. Using a sample of monthly US portfolio returns spanning the period 1968-2016, we find evidence that multifactor linear models have better empirical properties than the CAPM, not only when the cross-section of expected returns is evaluated in-sample, but also when they are used to inform one-month ahead portfolio selection. When we compare portfolios associated to multifactor models with mean-variance decisions implied by the single-factor CAPM, we document statistically significant differences in Sharpe ratios of up to 10 percent. Linear multifactor models that provide the best in-sample fit also yield the highest realized Sharpe ratios.

Guo, D. (2019). "A Statistical Response to Challenges in Vast Portfolio Selection." PhD thesis. University of Waterloo.
The thesis is written in response to emerging issues brought about by an increasing number of assets allocated in a portfolio and seeks answers to puzzling empirical findings in the portfolio management area. Over the years, researchers and practitioners working in the portfolio optimization area have been concerned with estimation errors in the first two moments of asset returns. The thesis comprises several related chapters on our statistical inquiry into this subject. Chapter 1 of the thesis contains an introduction to what will be reported in the remaining chapters. A few well-known covariance matrix estimation methods in the literature involve adjustment of sample eigenvalues. Chapter 2 of the thesis examines the effects of sample eigenvalue adjustment on the out-of-sample performance of a portfolio constructed from the sample covariance matrix.

Guo, D., Boyle, P. P., Weng, C., and Wirjanto, T. S. (2019). "When Does The 1/N Rule Work?" In: *SSRN e-Print*.
We propose a "1/N favorability index" to measure how favorable a market is to holding a 1/N portfolio. This index reflects the extent of difficulty for an optimized portfolio to outperform the 1/N portfolio in a specific market. A single-factor model predicts that bull markets are accompanied by a high 1/N favorability index and vice versa. We validate the model implication that the 1/N portfolio is more difficult to beat in bull markets using stock return datasets from a number of countries as well as the classic datasets used by DeMiguel et al. (2009). Our results imply that the reported good performance of the 1/N portfolio in the US equity market can be partially attributed to the long-run bullish trend in the market which gives rise to the high favorability of the market to the 1/N portfolio.

Gupta, K. (2020). "Optimal investment and consumption strategy for a retiree under stochastic force of mortality." MA thesis. Simon Fraser University.
With an increase in the self-driven retirement plans during past few decades, more and more retirees are managing their retirement portfolio on their own. Therefore, they need to know the optimal amount of consumption they can afford each year, and the optimal proportion of wealth they should invest in the financial market. In this

project, we study the optimization strategy proposed by Delong and Chen (2016). Their model determines the optimal consumption and investment strategy for a retiree facing (1) a minimum lifetime consumption, (2) a stochastic force of mortality following a geometric Brownian motion process, (3) an annuity income, and (4) non-exponential discounting of future income. We use a modified version of the Cox, Ingersoll, and Ross (1985) model to capture the stochastic mortality intensity of the retiree and, subsequently, determine a new optimal consumption and investment strategy using their framework. We use an expansion method to solve the classic Hamilton-Jacobi-Bellman equation by perturbing the non-exponential discounting parameter using partial differential equations.

Habib, F., Huaxiong, H., Mauskopf, A., Nikolic, B., and Salisbury, T. S. (2018). "Optimal allocation to deferred income annuities." In: *SSRN e-Print*.
In this paper we employ a lifecycle model that uses utility of consumption and bequest to determine an optimal Deferred Income Annuity (DIA) purchase policy. We lay out a mathematical framework to formalize the optimization process. The method and implementation of the optimization is explained, and the results are then analyzed. We extend our model to control for asset allocation and show how the purchase policy changes when one is allowed to vary asset allocation. Our results indicate that (i.) refundable DIAs are less appealing than non-refundable DIAs because of the loss of mortality credits; (ii.) the DIA allocation region is larger under the fixed asset allocation strategy due to it becoming a proxy for fixed-income allocation; and (iii.) when the investor is allowed to change asset-allocation, DIA allocation becomes less appealing. However, a case for higher DIA allocation can be made for those individuals who perceive their longevity to be higher than the population.

Habib, F., Huaxiong, H., and Milevsky, M. A. (2017). "Approximate solutions to retirement spending problems and the optimality of ruin." In: *SSRN e-Print*.
Milevsky and Huang (2011) investigated the optimal retirement spending policy for a utility-maximizing retiree facing a stochastic lifetime but assuming deterministic investment returns. They solved the problem using techniques from the calculus of variations and derived analytic expressions for the optimal spending rate and wealth depletion time under the Gompertz law of mortality. Of course, in the real world financial returns are stochastic as well as lifetimes, raising the question of whether their qualitative insights and approximations are generalizable or practical.

Hagelstein, P., Lackner, I., Otto, J., Perona, A., and Piziak, R. (2019a). "Fixed and Dynamic Asset Allocation in the Accumulation Phase." In: *Journal of Finance and Investment Analysis* 8(1), pp. 1–12.
In this paper, we consider the historical real returns of fixed and dynamic allocation portfolios consisting of equities and short term bonds over thirty year time horizons, where fixed real contributions are made to the portfolios annually. In particular, we consider both the scenario where the investor annually rebalances a portfolio to a fixed ratio as well as the scenario where the investor's annual contribution has a fixed ratio but the portfolio is never subsequently rebalanced. These results provide investors in the accumulation phase historical data that may provide a useful guide to asset allocation decisions. Of particular interest is that, over the 88 thirty-year time intervals considered, dynamic allocation portfolios had a better overall performance than fixed allocation portfolios, and that both fixed and dynamic allocation portfolios strongly benefited from a heavy equity exposure.

Hagelstein, P., Lackner, I., Otto, J., Perona, A., and Piziak, R. (2019b). "Markowitz Portfolios with Graham Bands in the Accumulation Phase." In: *The Journal of Wealth Management* 22(3), pp. 41–48.
In this article, the authors consider historical real returns of tax-exempt portfolios consisting of equities and short-term bonds over 90 different 30-year time periods from 1900 to 2018, in which fixed real contributions were made to the portfolios annually. Two main types of portfolios are considered. The first is a portfolio in which the investor annually contributed evenly between equities and bonds and never rebalanced the portfolio. The second is a portfolio with Graham bands, in which the investor annually contributed evenly between equities and bonds but rebalanced the portfolio to an overall 50/50 allocation whenever the equity or the bond portion of the portfolio exceeded 75% of the overall portfolio value. The authors also consider analogous results when short-term bonds were replaced by less liquid guaranteed income instruments, such as TIAA Traditional, providing higher yields. These results are intended to provide useful historical data to assist investors in the accumulation phase and their advisors in asset allocation decisions.

Halen, N., Faust, K., and Taylor, T. (2020). "Understanding the True Cost of Health Care in Retirement." In: *Retirement Management Journal*.
After years of working hard and diligently saving to prepare for retirement, many individuals expect they'll finally be able to slow down, pursue hobbies, and enjoy the next phase of their lives. However, the truth facing retirees today is that numerous financial risks and uncertainties threaten their ability to spend their hard-earned

money and maintain the quality of life they desire. Perhaps the greatest worry for those in, or near, retirement is whether they will be able to afford rising healthcare costs, particularly unplanned out-of-pocket expenses. Retirees' concerns around health care are not without merit, yet a closer look reveals a different picture. Our research concludes that healthcare expenses for many retirees are a small percentage of total spending and are far less variable than most people think, making them easier to plan for properly than conventional wisdom suggests. Two healthcare-related risks increase spending variability in retirement dramatically: long-term care events and longevity. An appropriate strategy for managing healthcare expenses in retirement is to plan for known or diversifiable risks and insure the unknown or undiversifiable risks. This study has three primary goals. First, to assess and quantify the variability of out-of-pocket healthcare costs to determine whether retiree concerns around funding these expenses are warranted. Second, to identify what factors drive variability in spending and quantify their potential financial impact. Third, to identify solutions financial professionals can utilize with clients to reduce spending variability and achieve better outcomes in retirement.

Haley, M. R. (2017). "K-fold cross validation performance comparisons of six naive portfolio selection rules: how naive can you be and still have successful out-of-sample portfolio performance?" In: *Annals of Finance* 13, pp. 341–353.

Recent research reports that optimal portfolio selection models often perform worse than equal-weight naive diversification in out-of-sample testing. This paper extends this line of inquiry by comparing the out-of-sample performance of the equal-weight naive strategy to the out-of-sample performance of five alternative naive strategies, each of which derives from a simple heuristic that does not require any optimization. Out-of-sample portfolio performance is assessed by mean, standard deviation, skewness, and Sharpe ratio; k-fold cross validation is used as the out-of-sample testing mechanism. The results indicate that the proposed naive heuristic rules exhibit strong out-of-sample performance, in most cases superior to the equal-weight naive strategy. These findings are consequential for at least two reasons: first, if these simple heuristic-based rules outperform the equal-weight naive strategy, then by transitivity they can outperform the mean-variance- and shortfall-optimal portfolio rules that have been shown in the literature to be inferior to the equal-weight naive rule, which further emphasizes the out-of-sample fragility of "optimal"methods; and second, among naive diversification strategies, some appear more robust in out-of-sample testing than others, hence the proposed methods may be useful when forming mixed portfolio selection models wherein a naive strategy is combined with an optimal strategy to improve performance.

Han, N.-w. and Hung, M.-w. (2012). "Optimal asset allocation for DC pension plans under inflation." In: *Insurance: Mathematics and Economics* 51(1), pp. 172–181.

In this paper, the stochastic dynamic programming approach is used to investigate the optimal asset allocation for a defined-contribution pension plan with downside protection under stochastic inflation. The plan participant invests the fund wealth and the stochastic interim contribution flows into the financial market. The nominal interest rate model is described by the Cox Ingersoll Ross (Cox et al., 1985) dynamics. To cope with the inflation risk, the inflation indexed bond is included in the asset menu. The retired individuals receive an annuity that is indexed by inflation and a downside protection on the amount of this annuity is considered. The closed-form solution is derived under the CRRA utility function. Finally, a numerical application is presented to characterize the dynamic behavior of the optimal investment strategy. We model the optimal asset allocation for DC pension plan under inflation. A minimum guarantee on the amount of retirement annuity is considered. We emphasize the importance of the inflation-indexed bond in the pension portfolio. The investments in indexed bonds would increase with the risk-aversion. The investments in indexed bonds would increase with the guarantee provided.

Hanna, S. and Kim, K. T. (2017). "Treatment of Inflation in Retirement Planning Calculations: An Improved Method." In: *Journal of Financial Planning* 30(1), pp. 44–53.

The typical treatment of inflation in retirement planning textbooks is complex and often not reasonable in terms of the amount to contribute, the first year being dependent on the inflation rate assumption. The typical financial planning approach requires an assumption about the future inflation rate, which can be difficult to justify based on the economic history in the United States and other countries. Economists typically put all amounts and interest rates in inflation-adjusted terms, which is simpler and can be a more rational approach to long-term planning. It can be difficult to comprehend the meaning of future amounts in nominal terms; therefore, doing projections in inflation-adjusted terms may help clients better understand projections. In calculating the capital needs for retirement, inflation-adjusted rates of return for each step should be chosen based on portfolio choices appropriate for before and after retirement.

Hao, H. (2019). "A Regime-Aware Agent-Based Framework for Financial Planning." PhD thesis. Princeton University.
The vulnerability of individuals planning for retirement has been growing due to the conversion from defined-benefit plans to defined-contribution plans, the steady increase in life longevity, and the uncertainty of asset returns under an ever-changing global environment. A serious problem is the lack of appropriate planning for retirement. How much should an individual save beyond the Social Security tax in order to maintain a reasonable lifestyle after retirement? This paper designs a framework to facilitate the process of setting realistic goals for financial planning, featuring the concept of agent-based simulations. The framework also provides policy-rule guidelines for the agent to search for an optimal strategy. Additionally, a micro-macro analysis enables us to analyze a cohort of representative agents and aggregate the individual results on the macro-level. The simulation module employs a regime-based Monte Carlo simulation of multiple asset categories, a factor-based diversifying asset allocation approach, and a collection of dynamic policy-rule-based investment strategies. Empirical results, consisting of a downside risk simulation for university endowments, a sustainability assessment for the Social Security fund, and a personal goal-based retirement planning, demonstrate stylized applications of the planning framework.

Harbron, G. L., Bloore, W., and Zorn, J. (2019). *Withdrawal order: making the most of retirement assets*. Tech. rep. Vanguard.
Retirees in the UK face more challenges today than ever before when it comes to retirement planning. The decline of defined benefit pensions in favour of less expensive (for the employer) defined contribution plans, pension freedoms, increased longevity and other factors have made retirement planning considerably more complex. Arguably foremost among these challenges is how to convert the retiree's accumulated retirement savings into a sustainable income that may need to last 30 years or more.While reducing spending and working longer are the most effective ways of extending the life of a retirement portfolio, increasing the portfolio's net after-tax return can also have apositive impact. One way to accomplish this is to select the proper withdrawal order when deciding which accounts to spend from. This paper looks at three withdrawal orders across three crystallisation strategies. Using our Vanguard Capital Markets Model (VCMM), we simulate the impact of withdrawal order and crystallisation strategy on a number of success metrics over a 30-year time horizon. Our analysis shows that, for most investors, withdrawing from taxable accounts first provides the best results.

Harris, M. (2019). "Should a Retiree File for Social Security at 62 or 70?" In: *The Journal of Retirement* 7(2), pp. 51–59.
Retirees can file for Social Security benefits as soon as they are 62 years old, but payments will become larger if they delay up to a maximum of 70 years of age. Many people believe the best strategy is to wait until later to file in order to maximize benefit payments. In this article, we reexamine that advice and add two considerations. First, we discuss how the claiming decision is made and recognize that in many cases the individual's goal is to support new lifestyle options, not to maximize Social Security withdrawals. Next, we consider alternative sources of income and how using those resources affects the claiming decision. The article concludes with example scenarios that demonstrate how a financial advisor might advise clients on the best strategy for their particular situation.

Harvey, C. R., Liu, Y., and Saretto, A. (2020). "An Evaluation of Alternative Multiple Testing Methods for Finance Applications." In: *The Review of Asset Pricing Studies* 10(2), pp. 199–248.
In almost every area of empirical finance, researchers confront multiple tests. One high-profile example is the identification of outperforming investment managers, many of whom beat their benchmarks purely by luck. Multiple testing methods are designed to control for luck. Factor selection is another glaring case in which multiple tests are performed, but numerous other applications do not receive as much attention. One important example is a simple regression model testing five variables. In this case, because five variables are tried, a t-statistic of 2.0 is not enough to establish significance. Our paper provides a guide to various multiple testing methods and details a number of applications. We provide simulation evidence on the relative performance of different methods across a variety of testing environments. The goal of our paper is to provide a menu that researchers can choose from to improve inference in financial economics.

Hejazi, S. A., Jackson, K. R., and Gan, G. (2017). "A Spatial Interpolation Framework for Efficient Valuation of Large Portfolios of Variable Annuities." In: *arXiv e-Print*.
Variable Annuity (VA) products expose insurance companies to considerable risk because of the guarantees they provide to buyers of these products. Managing and hedging these risks requires insurers to find the value of key risk metrics for a large portfolio of VA products. In practice, many companies rely on nested Monte Carlo (MC)

simulations to find key risk metrics. MC simulations are computationally demanding, forcing insurance companies to invest hundreds of thousands of dollars in computational infrastructure per year. Moreover, existing academic methodologies are focused on fair valuation of a single VA contract, exploiting ideas in option theory and regression. In most cases, the computational complexity of these methods surpasses the computational requirements of MC simulations. Therefore, academic methodologies cannot scale well to large portfolios of VA contracts. In this paper, we present a framework for valuing such portfolios based on spatial interpolation. We provide a comprehensive study of this framework and compare existing interpolation schemes. Our numerical results show superior performance, in terms of both computational efficiency and accuracy, for these methods compared to nested MC simulations. We also present insights into the challenge of finding an effective interpolation scheme in this framework, and suggest guidelines that help us build a fully automated scheme that is efficient and accurate.

Hens, T., Schenk-Hoppe, K. R., and Woesthoff, M.-H. (2020). "Escaping the backtesting illusion." In: *The Journal of Portfolio Management* 46(4), pp. 81–93.
Two tests can help asset managers to develop more robust investment strategies: an impact test and a survival test. Both tests complement the backtest, in which one checks how a proposed investment strategy would have performed in the past. The impact test considers the performance of the strategy when assets under management grow (crowdedness), and it checks the impact that growth in assets under management in competing strategies has on the proposed strategy (cross impact). The survival test considers the effect of the long-term evolution of assets under management in competition for market capital. Using Shiller S&P 500 index and bond market data, we show that time-series momentum (relative strength) performs best in the backtest and the impact test but that an expected relative cash-flow rule (relative dividend yield) has the best long-term survival properties.

Hicks, W. (2019). *How Much Is An Adviser Worth?* Tech. rep. Sapling Wealth Management.
Often clients try to quantify what good financial advice is worth. After all, how do you determine if the value of the service is greater than its cost? However, this is a tricky proposition, as it depends on how you define "good". Nevertheless, several reports have broached this topic and I have attempted to give a brief overview of the strengths and weaknesses of a few that I believe are quite substantive. Clients should aim to understand these reports in the context of their own personal situation. Identify your own personal areas that require the most support and think about if you are are getting all of the focus that you need.

Hofmann, P. (2021a). "Quantifying the Tax Benefit of Retirement Accounts for Better Client Decisions (Part 1)." In: *Advisor Perspectives*.
How should clients use tax-advantaged accounts? By quantifying the tax benefits of those accounts, we can refine recommendations on asset location and the traditional-versus-Roth decision. Specifically, low-return assets, even if otherwise taxed at high rates, may not benefit much from being placed in tax-advantaged accounts. Also, the choice between traditional and Roth contributions is asymmetric: Traditional accounts provide an opportunity for a higher tax benefit, but risk incurring a tax cost. Roth contributions provide greater certainty of a positive tax benefit.

Hofmann, P. (2021b). "Quantifying the Tax Benefit of Retirement Accounts for Better Client Decisions (Part 2)." In: *Advisor Perspectives*.
In my previous article, I showed how the value of tax-advantaged accounts depends on the tax-free investment income earned in those accounts. This article digs deeper into this dynamic to further demonstrate why the conventional wisdom of "stocks in taxable, bonds in tax advantaged" is not reliable. Compounding of tax-free investment income, protecting against tax policy changes, and avoiding taxes on inflation are all potential reasons for holding stocks in tax-advantaged accounts.

Hopkins, J. P., Ragatz, J. A., and Galli, C. (2016). "Ethical issues in retirement income planning: an advisor perspective." In: *The Journal of Retirement* 4(1), pp. 112–130.
The Ethical Issues in Retirement Income Planning study gathered the perceptions of expert retirement income planners. The good news is that the retirement income planners surveyed expressed a high level of satisfaction with the retirement planning profession ethical climate, as 64% of respondents believed that the overall ethical culture in the retirement income industry was solid as compared to 36% that expressed some concern. However, advisors were not without concerns. When asked to rank those concerns, a dominant theme emerged through the survey quantitative and qualitative findings: Respondents believed that ethical violations were the result of a lack of education and not purposeful malfeasance. The number one ethical concern respondents identified was financial elder abuse. Respondents were not overly concerned that financial advisors were the primary perpetrators but were worried about the industry ability to identify cases of financial elder abuse and exploitation, noting that

most retirement advisors had not been adequately trained to recognize the signs of abuse, which are often subtle and ambiguous. Respondents reported that they saw very few acts of outright fraud or deceit. Respondents generally believed that retirement advisors, while well-intentioned, often lacked sufficient knowledge in three main areas: Social Security, Medicare, and tax planning. They feared that these knowledge limitations might lead practitioners to make recommendations that were not in the best interest of their clients. An additional and related concern was that consumers would not be able to act as a check since in many cases they lacked the financial literacy to understand the complex products or plans offered to them. In some cases, clients simply trusted their advisor and remained disengaged from the planning process. Respondents also perceived that this level of client dependence reflected a general lack of understanding of the financial advisor pivotal role in the process of planning for retirement.

Horan, S. (2005). *Tax-Advantaged Savings Accounts and Tax-Efficient Wealth Accumulation*. Tech. rep. CFA Institute Research Foundation.

Until recently, the issue of tax-efficient investing has been largely overlooked by the mainstream literature. And simple heuristics to guide investors and their advisors are not always as obvious as they might initially seem. This monograph explores central issues surrounding the use of tax-deferred investment accounts as a means of accumulating wealth and presents a useful framework, grounded in basic time-value-of-money concepts, that can be readily implemented by investment professionals (U.S. as well as non-U.S. based) in various tax environments (current as well as those resulting from changes in the tax code).

Horneff, V., Maurer, R., and Mitchell, O. (2017). "How Persistent Low Expected Returns Alter Optimal Life Cycle Saving, Investment, and Retirement Behavior." In: *SSRN e-Print*.

This Chapter explores how an environment of persistent low returns influences saving, investing, and retirement behaviors, as compared to what in the past had been thought of as more "normal" financial conditions. Our calibrated lifecycle dynamic model with realistic tax, minimum distribution, and Social Security benefit rules produces results that agree with observed saving, work, and claiming age behavior of U.S. households. In particular, our model generates a large peak at the earliest claiming age at 62, as in the data. Also in line with the evidence, our baseline results show a smaller second peak at the (system-defined) Full Retirement Age of 66. In the context of a zero-return environment, we show that workers will optimally devote more of their savings to non-retirement accounts and less to 401(k) accounts, since the relative appeal of investing in taxable versus tax-qualified retirement accounts is lower in a low return setting. Finally, we show that people claim Social Security benefits later in a low interest rate environment.

Horneff, V., Maurer, R., and Mitchell, O. S. (2018a). "How Persistent Low Expected Returns Alter Optimal Life Cycle Saving, Investment, and Retirement Behavior." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.

This chapter explores how an environment of persistent low returns influences saving, investing, and retirement behaviors, compared to what in the past had been conceived of as "normal" financial conditions. Using a calibrated life cycle dynamic model with realistic tax, minimum distribution, and social security benefit rules, we can mimic the large peak at the earliest claiming age at 62 that is seen in the data. Also in line with the evidence, our baseline results show a smaller second peak at the (system-defined) Full Retirement Age of 66. In the context of a zero-return environment, we show that workers will optimally devote more of their savings to non-retirement accounts and less to 401(k) accounts, since the relative appeal of investing in taxable versus tax-qualified retirement accounts is lower in a low return setting. Finally, we show that people claim social security benefits later in a low interest rate environment.

Horneff, V., Maurer, R., and Mitchell, O. S. (2018b). "Putting the Pension Back in 401(k) Retirement Plans: Optimal versus Default Longevity Income Annuities." In: *SSRN e-Print*.

A recent US Treasury regulation allowed deferred longevity income annuities to be included in pension plan menus as a default payout solution, yet little research has investigated whether more people should convert some of the 15 trillion they hold in employer-based defined contribution plans into lifelong income streams. We investigate this innovation using a calibrated lifecycle consumption and portfolio choice model embodying realistic institutional considerations. Our welfare analysis shows that defaulting a small portion of retirees 401(k) assets (over a threshold) is an attractive way to enhance retirement security, enhancing welfare by up to 20% of retiree plan accruals.

Horneff, V., Maurer, R., and Mitchell, O. S. (2021). "Do Required Minimum Distribution 401(K) Rules Matter, and for Whom? Insights from a Lifecycle Model." In: *SSRN e-Print*.

Tax-qualified vehicles helped U.S. private-sector workers accumulate $25Tr in retirement assets. An often-overlooked important institutional feature shaping decumulations from these retirement plans is the "Required Minimum Distribution" (RMD) regulation, requiring retirees to withdraw a minimum fraction from their retirement accounts or pay excise taxes on withdrawal shortfalls. Our calibrated lifecycle model measures the impact of RMD rules on financial behavior of heterogeneous households during their worklives and retirement. We show that proposed reforms to delay or eliminate the RMD rules should have little effects on consumption profiles but more impact on withdrawals and tax payments for households with bequest motives.

Horneff, W. J., Maurer, R. H., Mitchell, O. S., and Dus, I. (2008). "Following the rules: Integrating asset allocation and annuitization in retirement portfolios." In: *Insurance: Mathematics and Economics* 42(1), pp. 396–408.
Financial advisers have developed standardized payout strategies to help Baby Boomers manage their money in their golden years. Prominent among these are phased withdrawal plans offered by mutual funds including the self-annuitization or default rules encouraged under US tax law, and fixed payout annuities offered by insurers. Using a utility-based framework, and taking account of stochastic capital markets and uncertain lifetimes, we first evaluate these rules on a stand-alone basis for a wide range of risk aversion. Next, we permit the consumer to integrate these standardized payout strategies at retirement and compare the results. We show that integrated strategies can enhance retirees' well-being by 25 percent 50 percent for low/moderate levels of risk aversion when compared to full annuitization at retirement. Finally, we examine how welfare changes if the consumer is permitted to switch to a fixed annuity at an optimal point after retirement. This affords the retiree the chance to benefit from the equity premium when younger, and exploit the mortality credit in later life. For moderately risk-averse retirees, the optimal switching age lies between 80 and 85.

Hosseini, R., Kopecky, K. A., and Zhao, K. (2022). "The evolution of health over the life cycle." In: *Review of Economic Dynamics* 45, pp. 237–253.
We construct a unified objective measure of health status: the frailty index, defined as the cumulative sum of all adverse health indicators observed for an individual. Using this index, we document four stylized facts on health dynamics over the life cycle and show that they are robust to other ways of constructing the index. We also compare the frailty index with self-reported health status and find significant differences in their dynamics and ability to predict health-related outcomes. Finally, we propose and estimate a stochastic process for frailty dynamics over the life cycle accounting for mortality bias. Our frailty measure and dynamic process can be used to study the evolution of health over the life cycle and its economic implications.

Hou, W. (2020). *How Accurate Are Retirees' Assessments of Their Retirement Risk?* Tech. rep. Center for Retirement Research at Boston College.
Retirees with limited financial resources face numerous risks, including out-living their money (longevity risk), investment losses (market risk), unexpected health expenses (health risk), the unforeseen needs of family members (family risk), and even retirement benefit cuts (policy risk). This study systematically values and ranks the financial impacts of these risks from both the objective and subjective perspectives and then compares them to show the gaps between retirees' actual risks and their perceptions of the risks in a unified framework. It finds that 1) under the empirical analysis, the greatest risk is longevity risk, followed by health risk; 2) under the subjective analysis, retirees perceive market risk as the highest-ranking risk due to their exaggeration of market volatility; and 3) the longevity risk and health risk are valued less in the subjective ranking than in the objective ranking, because retirees underestimate their life spans and their health costs in late life.

Hsu, Y.-C., Lin, H.-W., and Vincent, K. (2017). *Do Cross-Sectional Stock Return Predictors Pass the Test without Data-Snooping Bias?* Tech. rep. Institute of Economics Academia Sinica.
This study examines the possible data-snooping bias as a competing explanation for the anomalies in the cross-section of stock returns. We posit that the exhaustive standalone searches for profitable strategies could lead to recommending spuriously predictive variables. In order to explore the severity of this problem, we use a multiple testing method to evaluate the profitability of portfolios constructed by these predictors. Our empirical analyses suggest that over half of the findings based on individual testing method are no longer statistically significant after we adjust for data-snooping bias. Excluding the micro-cap stocks before portfolios construction and applying the notion of economic significance in this study further weaken the evidence for predictability.

Hsu, P.-H., Han, Q., Wu, W., and Cao, Z. (2018). "Asset allocation strategies, data snooping, and the 1 / N rule." In: *Journal of Banking & Finance* 97, pp. 257–269.
Using a series of advanced tests from White's (2000) Check to correct for data-snooping bias, we assess the out-of-sample performance of various portfolio strategies relative to the naive 1/N rule. When we analyze 16 basic portfolio strategies, 126 learning strategies, and nearly 2,000 extended strategies, we find that some strategies

outperform the 1/N rule in conventional tests that do not account for data-snooping bias. However, after we use the new tests that control for such bias, we find that none or very few of these strategies outperform the 1/N rule. Thus, our finding underscores the necessity to control for data-snooping bias when making asset allocation decisions.

Hu, W. (2021). "A Comprehensive Study of Guaranteed Minimum Maturity Benefit and Guaranteed Minimum Death Benefit under Regime-switching Models." PhD thesis. North Carolina State University.
In this dissertation, we investigate the pricing and hedging of Guaranteed Minimum Benefits (GMBs) in life annuity products, and we extend the existing framework by assuming the underlying asset dynamics evolve under a regime-switching jump-diffusion environment. First, two most popular GMB contracts, Guaranteed Minimum Maturity Benefit (GMMB) and Guaranteed Minimum Death Benefit (GMDB), are characterized by comprehensive quantitative models that assess both the market risk on the liability side and the revenue risk on the asset side from the viewpoint of insurers. Then, five stochastic mortality models (Cox-Ingersoll-Ross process model, Vasicek process model, Ornstein-Uhlenbeck process model, Feller process model and a two-factor mortality model) are utilized to describe the mortality risk embedded in GMBs, and their parameters are estimated by calibrating to the mortality data of the United States male population. After comparing the Akaike information criterion (AIC) and Bayesian information criterion (BIC) criteria, we find that the Feller process model fits the real data best among the five models. In addition, numeric solutions for net liabilities, fair rate of fees and Greeks (sensitivities of net liabilities with respect to model parameters) of GMMB and GMDB are derived by an accurate and fast Fourier Space Timestepping (FST) algorithm. Numerical results based on Monte Carlo simulations are also provided for comparative purpose. Moreover, an unhedged and three statically hedged portfolios which are constructed by different methods (linear algebra method, optimization method and regularization method) are introduced, and their performances are assessed by comparing the short-term and the long-term volatility, Value-at-risk (VaR) and expected shortfall (CVaR). The results indicate that, in the short-term, the portfolio constructed by optimization method performs the best and all three hedged portfolios perform better than unhedged portfolio in the long-term. Finally, these hedging results provide insurers some guidelines to hedge according to their risk tolerances. For example, if an insurer is more concerned about the short-term volatility, a linear algebra method is preferred although it requires frequent rebalancing. In addition, if one is more concerned about the transaction cost, a optimization method is more preferable and it does not need to be rebalanced every year. The regularization method is suitable for an incomplete market which only have few instruments for hedging.

Huang, H. and Milevsky, M. A. (2016). "Longevity risk and retirement income tax efficiency: A location spending rate puzzle." In: *Insurance: Mathematics and Economics* 71, pp. 50–62.
In this paper we model and solve a retirement consumption problem with differentially taxed accounts, parameterized by longevity risk aversion. The work is motivated by some observations on how Canadians de-accumulate financial wealth during retirement which seem rather puzzling. While the Modigliani lifecycle model can justify a variety of (pre-tax) de-accumulation or draw down rates depending on risk preferences, the existence of asymmetric taxes implies that certain financial accounts should be depleted faster than others. Our analysis of data from the Survey of Financial Security indicates that Canadian retirees maintain approximately two-thirds of their financial wealth in tax-sheltered accounts and a third in taxable accounts regardless of age. The ratio of taxable to tax-sheltered wealth increases slightly or remains relatively constant depending on household income which is not what one would expect from the lifecycle model. Indeed, using our model we cannot locate a plausible tax function that justifies a constant - account ratio - regardless of age. For example under flat rates taxable accounts should be depleted well before tax-sheltered accounts are ever touched. The account ratio should go to zero quite rapidly in the absence of government mandated withdrawals. We also demonstrate that under progressive income taxes withdrawals are made from both accounts but at different rates depending on account size, pension income and longevity risk preferences. Again, the - account ratio - should eventually decline. We postulate that this sort of behavior is likely due to irrational considerations linked to mental accounting, etc. It remains to be seen whether this will persist over time and under a more careful analysis of Canadian cohorts or if retirees in other countries exhibit the same behavior.

Huang, M. and Yu, S. (2020). "A new procedure for resampled portfolio with shrinkaged covariance matrix." In: *Journal of Applied Statistics* 47(44), pp. 642–652.
Dealing with estimation error is an important issue when we implement the mean-variance paradigm for portfolio construction. To tackle the problem, two approaches are proposed in literature, the portfolio resampling technique introduced by Michuad and the well-known shrinkaged covariance matrix method. There are certain evidences on

the advantages of shrinkaged covariance over portfolio resampling, however, it is unclear whether a combination of the two approaches could produce a better performance compared with using shrinkaged covariance alone. In this paper, we propose a new algorithm to integrated linear or nonlinear shrinkage estimation with resampled portfolio to achieve a further improvement. Our method are demonstrated via extensive simulation and application in active portfolio management process.

Huang, N., Li, J., and Ross, A. (2020). *Housing Wealth Shocks, Home Equity Withdrawal, and the Claiming of Social Security Retirement Benefits*. Tech. rep. Centre For Research On Successful Ageing, Singapore Management University.

This paper examines the impact of changes in house prices on when eligible individuals start receiving Social Security benefits. If house prices increase, financially constrained households may draw upon the additional home equity to finance expenses and delay receipt of Social Security in order to have increased lifetime monthly benefits. To address concerns that house price changes are correlated with unobserved local demand shocks, we use a control function approach and employ two different instrumental variables. We find that individuals delay Social Security claiming when house prices increase during the housing boom. The probability of claiming within two years after becoming eligible decreases by 8.67-8.81 percent for every 10 percent increase in house prices. We also find that the total home loan amount increases in response to the price appreciation, indicating households are drawing upon their home equity to finance consumption and delay receiving Social Security.

Huaxiong, H., Milevsky, M. A., and Salisbury, T. S. (2017). "Retirement Spending and Biological Age." In: *SSRN e-Print*.

We solve a retirement lifecycle model in which the consumer's age does not move in lockstep with calendar time. Instead, biological age increases at a stochastic non-linear rate in chronological age, which one can think of as working with a clock that occasionally moves backwards in time. Our paper is inspired by the growing body of medical literature that has identified biomarkers of aging which – practically speaking – offer better estimates of expected remaining lifetime and future mortality rates. It isn't farfetched to argue that in the not-too-distant future of wearable technology, personal age will be more closely associated with biological time vs. calendar age or time. Thus, after introducing our stochastic mortality model we derive optimal consumption rates in a classic Yaari (1965) framework adjusted to our proper clock and time. In addition to the normative implications of having access to biological age, our positive objective is to partially explain the cross-sectional heterogeneity in retirement spending rates at any given chronological age. In sum, we argue that biological age is not a sufficient statistic for making economic decisions and you need information about both your ages to behave rationally.

Hubble, A. and Grable, J. E. (2019). "The efficient frontuzzle: what investment risk profiling still fails to solve." In: *The Journal of Investing* 28(6), pp. 55–72.

A global study of over 200 professional financial advisors is undertaken to test how risk profile factors are used to make investment portfolio allocation recommendations. When presented with identical risk profile information, similar to what a competent financial advisor would normally collect from a prospective client, respondents are asked to recommend a portfolio allocation among equity, fixed income, and cash for five hypothetical client scenarios. The results find that financial advisors, using their professional judgment, inconsistently puzzled together the presented risk profile factors into portfolio recommendations, on average doing little more than applying the heuristic 100-minus-age rule to recommendations. These troubling results highlight the regulatory need for uniform risk profile evaluation guidance in fiduciary contexts.

Hugonnier, J., Pelgrin, F., and St-Amour, P. (2017). "Closing down the shop: optimal health and wealth dynamics near the end of life." In: *SSRN e-Print*.

Near the end of life, health declines, mortality risk increases and curative is replaced by uninsured long-term care, accelerating the fall in wealth. Whereas standard explanations emphasize inevitable aging processes, we propose a com- plementary closing down the shop justification where agents decisions affect their health and the timing of death. Despite preferring to live, individuals optimally deplete their health and wealth towards levels associated with high death risk and indifference between life and death. Reinstating exogenous aging processes reinforces the relevance of closing down. Using HRS data for elders, a structural estimation of the closed-form decisions identifies and tests conditions for these strategies to be optimal and confirm their economic relevance. We also discuss why policy intervention to reduce the incidence of closing down, although feasible, is not warranted.

Hunt, A. and Blake, D. P. (2020). "Basis Risk and Pensions Schemes: A Relative Modelling Approach." In: *SSRN e-Print*.

For many pension schemes, a shortage of data limits their ability to use sophisticated stochastic mortality models to assess and manage their longevity risk. In this study, we develop a relative model for mortality,

which compares the evolution of mortality rates in a sub-population with that observed in a larger reference population. We apply this relative approach to data from the CMI Self-Administered Pension Scheme study, using UK population data as a reference. We then use the relative approach to investigate the potential differences in the evolution of mortality rates between these two populations and find that, in many practical situations, basis risk is much less of a problem than is commonly believed.

Hwang, I., Xu, S., and In, F. (2018). "Naive versus optimal diversification: Tail risk and performance." In: *European Journal of Operational Research* 265(1), pp. 372–388.
It is well documented in portfolio optimization that naive diversification outperforms optimal mean-variance diversification because the latter is subject to severe estimation error. Our study provides an alternative explanation for the outperformance of naive diversification by examining the tail risk of naive diversification relative to optimal mean-variance diversification. We utilize a rolling-sample approach and compare the out-of-sample performance and tail risk of various optimal strategies to that of the naive diversification strategy. Using portfolios consisting of individual stocks, we show that for portfolios containing relatively small number of stocks, naive diversification outperforms optimal mean-variance diversification and is less exposed to tail risk. However, for relatively large number of stocks in the portfolio, naive diversification maintains its superior performance but increases tail risk and results in more concave portfolio returns. These results imply that the outperformance of naive diversification acts as compensation for the increase in tail risk and concavity.

Hyams, S. D., Smith, A. E., Squirrell, C. M., Warren, G. J., Warren, O. H., and Willetts, P. J. (2020). "Saving for retirement: rules of thumb." In: *British Actuarial Journal* 25 (e7).
Rules of thumb (RoTs) are proposed as a means of promoting higher levels of Defined Contribution (DC) pension saving and to help stimulate debate about the high and uncertain cost of pension provision, leading to the development of solutions. The Lifetime Pension Contribution (LPC) tells young people what pension contribution is required over a full working life to achieve a decent retirement income, calculated as 23% of average UK earnings. Another RoT is that each 1% of earnings provides a pension of 1.5% of earnings. Other RoTs show how costs vary by retirement age and if the saver's retirement planning is on track. The current high cost of pensions is partly due to low interest rates and the inefficiencies of the DC market, with inadequate bulk purchasing power and risk sharing. RoTs might help encourage higher employer contributions, either through automatic enrolment or on a voluntary basis.

Iannarone, N. G. (2018). "Rethinking automated investment adviser disclosure." In: *SSRN e-Print*.
Today, investment advisers, including robo-advisers, push dense disclosures to consumer investors who may not be able to fully comprehend their import. Tomorrow, robo-advisers and their innovative technology, if embraced by regulators and consumers, can facilitate and cultivate a trusted collaborative relationship between investor and adviser where the adviser learns from the investor and provides education tailored to the investor, helping to solve vexing consumer protection issues and lessen the impending retirement crisis. More and more of us are comfortable making decisions without human involvement and entirely through a digital interface. Despite consumers increasing comfort relying upon machines to direct and guide their decisions, most Americans nevertheless lack basic foundational knowledge in finance and investing. Even those Americans who are able to save for retirement lack financial sophistication and may not be able to understanding the complex products in which they are invested. Robo advice may not be the solution for these problems, as robo-advisers present a potential black box problem where the digital investment advisers may not be capable of explanation beyond the information input and advice output. In such a world, it is not only the new technology that should be questioned. The existing standards through which the new technology is regulated should similarly be explored. An understanding of potential benefits and potential harms may provide lessons that better allow us to protect consumer investors and better prepare for the so-called retirement crisis. This essay explores the parameters of the pre-existing regulatory regime as applied to a new landscape of investing with a focus on the key regulatory tool of disclosure. This short essay examines the challenges of delivering appropriate disclosures in a new world of robo-advisory advice. It suggests a reconceptualization of the aim of disclosure to capitalize on robo-advisers ability to truly know their clients and shift the burden of investor education from the customer to a fiduciary level adviser capable of tailoring advice and education to each investor at a level she is capable of understanding. The essay concludes with a recommendation: robo-advisers and investor collaboration via an active and iterative disclosure regime that encourages investor comprehension, understanding, and learning.

Idzorek, T., Stempien, J., and Voris, N. (2013). "Bait and Switch: Glide Path Instability." In: *The Journal of Investing* 22(1), pp. 74–82.

With target date funds widely expected to account for more than 50 percent of all defined contribution assets in the near future, it is difficult to overstate their importance on the retirement security of America. Yet there are very few tools and statistics for monitoring and evaluating target date funds. This article highlights a previously unrecognized characteristic of target date fund families: Glide paths aren't necessarily stable over time, and in some cases change dramatically. We document the stability, or lack of stability, of the glide paths for the largest target date fund families and introduce quantitative measures of glide path stability, including our preferred measure, the Glide Path Stability Score.

Ielpo, F., Merhy, C., and Simon, G. (2017). *Engineering Investment Process: Making Value Creation Repeatable*. Elsevier. 430 pp.
The book explores the quantitative steps of a financial investment process. The authors study how these steps are articulated in order to make any value creation, whatever the asset class, consistent and robust. The discussion includes factors, portfolio allocation, statistical and economic backtesting, but also the influence of negative rates, dynamical trading, state-space models, stylized facts, liquidity issues, or data biases. Besides the quantitative concepts detailed here, the reader will find useful references to other works to develop an in-depth understanding of an investment process.

Ilmanen, A. and Rauseo, M. (2018). "Intelligent Risk Taking." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.
Peoples' ability to consume in retirement is a function of how much they save, how they invest, and what those investments return over the lifecycle. This chapter explores what rate of return is needed to deliver a comfortable retirement based on current savings rates, as well as intelligent ways to construct portfolios to achieve this rate of return. Based on reasonable long-term return assumptions, defined contribution portfolios as frequently constructed today are unlikely to achieve this required rate of return. By relaxing existing constraints and taking advantage of well-known and broadly accepted investment themes, this required rate of return can be achieved with a well-diversified portfolio, which may lead to a more consistent portfolio across different economic environments.

Irlam, G. (2020a). "Lifetime Portfolio Selection: Using Machine Learning." In: *SSRN e-Print*.
The first half of this paper provides a primer on machine learning and relates it to problems in financial planning. Machine learning proves capable of handling real world complications and details which classical financial planning approaches cannot handle; e.g. taxes, reversion to the mean, and time varying yield curves. The second half of this paper provides a case study of the use of reinforcement learning for making asset allocation, annuitization, and consumption decisions in retirement planning. Machine learning typically delivers within a few percent of the theoretical optimal solution on highly abstract problems. Machine learning is found to outperform other approaches for more complex financial planning problems whose optimal solution is not known. The value of SPIAs and inflationindexed bonds is highlighted by the machine learning approach.

Irlam, G. (2020b). "Machine learning for retirement planning." In: *The Journal of Retirement* 8(1), pp. 32–29.
Machine learning provides a new approach to solving problems in many fields. This article explores the use of machine learning to solve the retirement portfolio problem: deciding how much wealth to consume and how to allocate the remainder. After first reviewing existing approaches to the portfolio problem, this article looks in detail at the use of reinforcement learning. For simple financial scenarios where the optimal solution is known, reinforcement learning is found to deliver to within a few percent of the optimal solution. For more complicated financial scenarios, with no known optimal solution, reinforcement learning outperforms other common approaches. Reinforcement learning proves capable of optimizing highly complex financial models, including the effects of income taxes, mean-reverting asset classes, and time-varying bond yield curves, all of which other approaches cannot handle. Reinforcement learning appears to be the first fundamentally new approach to the portfolio problem in over 50 years.

Irlam, G. (2020c). "Multi Scenario Financial Planning via Deep Reinforcement Learning AI." In: *SSRN e-Print*.
Financial planning via deep reinforcement learning holds much promise. One implementation, AIPlanner, delivered near optimal financial results, but had a major shortcoming. It required training a separate neural network model for each financial scenario. This paper describes extending AIPlanner so that a small family of trained neural network models are capable of rapidly producing financial plans for a wide variety of financial scenarios. Additionally AIPlanner is extended to produce results over the lifecycle, both pre and post retirement, and for couples, as well as individuals. A reasonably realistic income tax model is incorporated. And finally, a more realistic stock model is used. Over the lifecycle, compared to the best discovered alternative strategy, reinforcement learning was found to effectively deliver 14% more retirement consumption.

Irlam, G. and Tomlinson, J. (2014). "Retirement Income Research: What Can We Learn from Economics?" In: *The Journal of Retirement* 1(4), pp. 118–128.
Research on retirement income planning has, for many years, followed two separate tracks. Financial planning practitioners have developed guidelines for withdrawals from savings and asset allocation by testing and fine-tuning various rules of thumb. Economists have applied a different approach, based on life-cycle finance, aimed at maximizing the utility of lifetime consumption, using dynamic programming techniques to optimize retirement withdrawals and asset allocations. This article seeks to use a non-mathematical explanation to help financial planners and others who are not familiar with the economics approach to develop a conceptual understanding of how life-cycle finance and dynamic programming can be applied to retirement income planning. It also compares the performance of optimized recommendations produced by dynamic programming with various rule-of-thumb strategies that have been popular in the planning literature. It attempts to demonstrate the power that the economics approach can bring to improving retirement income planning and argues that more communication between economists and practitioners can help open up new approaches to research and improved practical applications.

Iskhakov, F., Thorp, S., and Bateman, H. (2015). "Optimal Annuity Purchases for Australian Retirees." In: *Economic Record* 91(293), pp. 139–154.
We develop and simulate a stochastic lifecycle model to investigate optimal annuity purchases at retirement. Retirees can invest in risky assets, purchase fairly priced immediate or deferred lifetime annuities, and are eligible for a targeted safety net pension. We match baseline parameters to current Australian settings and conduct scenario analyses over a wide range of individual preferences and financial market outcomes. Except where individuals need to insure a consumption floor, both immediate and deferred annuity purchases are largely crowded out by the means-tested public pension. Welfare losses caused by zero annuitisation are small compared with the losses caused by completely annuitising all savings, particularly if wealth at retirement is low. Decumulation policy should ensure individuals are well informed of the insurance value of annuities and accommodate diverse choices.

Israelsen, C. L. (2017). "Retirement Portfolio Realities: The Mathematics of Survival." In: *International Journal of Trade, Economics and Finance* 8(4), pp. 194–197.
This paper deals with several important retirement questions: "How much money do I need in my investment portfolio at the start of retirement?" and "How much can I safely withdraw from my investment portfolio during the retirement years?" This paper introduces the novel concept of "RAM" or Retirement Account Multiple. The mathematics of income replacement in retirement are demonstrated and the historical survival of two different retirement portfolios are reviewed over a 90-year period from 1926-2015.

Jaconetti, C. M., DiJoseph, M. A., Jr., F. M. K., Pakula, D., and Lobel, H. (2020). *From assets to income: A goals-based approach to retirement spending*. Tech. rep. Vanguard.
Although the population and life expectancies of U.S. retirees are increasing, portfolio yields remain at historically low levels. As defined benefit income becomes less commonly available, the need for informed retirement portfolio spending strategies is more critical, and yet more complex, than ever. The stakes are high, and the impact of subpar decisions can be severe. Because every investor's financial situation is unique, there is no one-size-fits-all solution. But developing and implementing a personal spending strategy can reduce anxiety and stress about the ability to meet retirement income goals. Retirees who hold the majority of their assets in tax-deferred accounts can turn thoseassets into income by setting up an automatic withdrawal plan. They can also purchase an investment specifically designed to provide regular distributions. Those whose portfolios contain a significant portion of taxable assets can add value by working with an advisor to develop a goals-based strategy. Whatever spending strategy you choose, the complexity and consequences of the process underscore the need for and value of skillful guidance.

Jaeger, M., Krugel, S., Marinelli, D., Papenbrock, J., and Schwendner, P. (2020). "Understanding machine learning for diversified portfolio construction by explainable AI." In: *SSRN e-Print*.
In this paper, we construct a pipeline to investigate heuristic diversification strategies in asset allocation. We use machine learning concepts ("explainable AI") to compare the robustness of different strategies and back out implicit rules for decision making.In a first step, we augment the asset universe (the empirical dataset) with a range of scenarios generated with a block bootstrap from the empirical dataset.Second, we backtest the candidate strategies over a long period of time, checking their performance variability. Third, we use XGBoost as a regression model to connect the difference between the measured performances between two strategies to a pool of statistical features of the portfolio universe tailored to the investigated strategy. Finally, we employ

the concept of Shapley values to extract the relationships that the model could identify between the portfolio characteristics and the statistical properties of the asset universe. We test this pipeline for studying risk-parity strategies with a volatility target, and in particular, comparing the machine learning-driven Hierarchical Risk Parity (HRP) to the classical Equal Risk Contribution (ERC) strategy.In the augmented dataset built from a multi-asset investment universe of commodities, equities and fixed income futures, we find that HRP better matches the volatility target, and shows better risk-adjusted performances. Finally, we train XGBoost to learn the difference between the realized Calmar ratios of HRP and ERC and extract explanations.The explanations provide fruitful ex-post indications of the connection between the statistical properties of the universe and the strategy performance in the training set. For example, the model confirms that features addressing the hierarchical properties of the universe are connected to the relative performance of HRP respect to ERC.

Jang, C., Clare, A., and Owadally, I. (2022). "Glide paths for a retirement plan with deferred annuities." In: *Journal of Pension Economics and Finance*, pp. 1–17.
We construct investment glide paths for a retirement plan using both traditional asset classes and deferred annuities (DAs). The glide paths are approximated by averaging the asset proportions of stochastic optimal investment solutions. The objective function consists of power utility in terms of secured retirement income from purchased DAs, as well as a bequest that can be withdrawn before retirement. Compared with conventional glide paths and investment strategies, our DA-enhanced glide paths provide the investor with higher welfare gains, more efficient investment portfolios and more responsive retirement income patterns and bequest levels to different fee structures and personal preferences.

Janssen, R., Kramer, B., and Boender, G. (2013). "Life Cycle Investing: From Target-Date to Goal-Based Investing." In: *The Journal of Wealth Management* 16(1), pp. 23–32.
The authors propose the use of goal based investing- or private ALM, as they prefer to call it- to tailor a dynamic investment strategy to the needs of individual clients. They argue that this approach is superior to the - one-size-fits-all,- target-date-oriented static allocation path used in most current life cycle funds. They present the two pillars of their approach: the methodology for obtaining financial and economic scenarios, and the methodology of the goal-oriented dynamic allocation strategy. This approach reduces the risks and improves the feasibility of meeting the clients' goals.

Jennings, W. W. and Reichenstein, W. R. (2001). "Estimating the Value of Social Security Retirement Benefits." In: *The Journal of Wealth Management* 4(3), pp. 14–29.
The authors start with a simple question: what is the value of an individual's assets that can be used to satisfy retirement income needs, focusing more specifically on Social Security benefits? Consistent with an approach introduced in earlier works published in The Journal of Wealth Management, they then ask how these benefits affect the individual's current asset mix. In particular, they delve further into earlier conclusions that the present value of projected Social Security payments be included as a 'bond' in personal portfolios. They note that individuals' portfolios are usually substantially different when the value of Social Security is included than when it is excluded, and demonstrate that the profession has been miscalculating individuals' "true"portfolios by excluding Social Security. If individuals optimize their traditional portfolios, which exclude Social Security, then they have excessively conservative, sub-optimal true -portfolios.

Jeon, J., Kwak, M., and Park, K. (2021). "Horizon effect on optimal retirement decision." In: *SSRN e-Print*.
We study an optimal consumption, investment, life insurance, and retirement decision of an economic agent who has an option to retire early any time before the mandatory retirement date. We conduct a thorough theoretical analysis for the optimal retirement problem with general utility function in the presence of mandatory retirement date, which leads to the optimal stopping problem in finite horizon. Furthermore, different marginal utility of consumption before and after retirement is considered, which can provide an explanation for the retirement-consumption puzzle, while it makes the problem technically more challenging. Based on the theory of partial differential equation, we analyze the variational inequality arising from the dual problem and establish the duality theorem. We show that the optimal retirement decision is determined by the time-varying optimal retirement wealth boundary, and we provide an integral equation representation for the optimal retirement wealth boundary, which can be solved accurately and efficiently by using recursive integration method. As an extension, the case with stochastic labor income is also considered. The properties of the optimal strategies are provided with emphasis on the role of the mandatory retirement date and the impact of having a retirement option on the optimal financial decisions.

Johnston, K., Hatem, J., Carnes, T., and Kosedag, A. (2019). "An empirical evaluation of dynamic vs static withdrawal strategies." In: *Managerial Finance* 45(12), pp. 1509–1525.

Purpose The purpose of this paper is to compare simple dynamic withdrawal strategies with the static withdrawal method, examining not only failure rates and ending wealth but also spending. All withdrawal strategies are adjusted for the Internal Revenue Service's (IRS) required minimum distribution (RMD). In addition, this study investigates the use of small company stocks (SCS) in place of large company stocks (LCS). Results indicate SCS portfolios are superior to large. When returns are poor, some dynamic strategies will not ensure income for life. This study demonstrates that the simplest dynamic strategy is superior to two popular dynamic strategies. Design/methodology/approach Using historical overlapping periods, different withdrawal strategies are examined. Previous studies focused on failure rates and ending wealth. As discussed in Milevsky (2016) different statistical distributions can have similar tail properties (prob of failure) but dissimilar risk and return profile. The detailed examination of both spending and use of small stocks advances the literature in this area. Findings Results indicate that use of small stocks is superior to using large stocks in the portfolios. When US historical stock returns are adjusted downward, there is the potential that some dynamic strategies will not ensure income for life. This study demonstrates that the simplest dynamic strategy is superior to two popular dynamic strategies. Originality/value This paper is the first to examine, in detail, annual spending results for the retiree. Second, it is shown that, overall, SCS are superior to LCS for all stock/bond allocations. Even though absolute downside risk increases slightly, this increase in downside risk is dominated by the upside potential. In other words, the positive skewness of small stock returns along with the cumulative effects of compounding at a higher rate increases both the available wealth for spending and ending wealth. Third, IRS's RMDs are taken into account for every withdrawal strategy examined. Lastly, it demonstrates that the simplest dynamic strategy is superior to two popular dynamic strategies.

JP Morgan Asset Management Research (2021). *Guide to Retirement*. Tech. rep. JP Morgan Asset Management.
Updated annually, the Guide to Retirement provides an effective framework for supporting retirement planning conversations with clients. It includes charts and graphs to help you explain complex topics in a clear and concise manner. A description and audio commentary are available for every slide.

Jung, B. (2017). *Your advisor...worth more than 1%?* Tech. rep. Russell Investments.
Value of an advisor is still more than 1% As a follow up to my previous blog post on the value of an advisor, I wanted to provide you with even more evidence that suggests your advisory fee is a very good deal for the value you deliver to your clients.

Jung, J. (2020). *Estimating Markov Transition Probabilities Between Health States Using U.S. Longitudinal Survey Data*. Tech. rep. Towson University.
We use data from two representative U.S. household surveys, the Medical Expenditure Panel Survey (MEPS) and the Health and Retirement Study (Rand-HRS) to estimate Markov transition probability matrices between health states over the lifecycle from age 20-95. We use non-parametric and parametric methods and control for individual characteristics such as age, gender, race, education, income as well as cohort effects. We align two year transition probabilities from HRS with one year transition probabilities in MEPS using a stochastic root method. We find that the non-parametric counting method and the regression specifications based on ordered logit models produce similar results over the lifecycle. However, the counting method overestimates the probabilities of transitioning into bad health states. In addition, we find that young women have worse health prospects than their male counterparts but once individuals get older, being female is associated with transitioning into better health states with higher probabilities than men. We do not find significant differences of the conditional health transition probabilities between African Americans and the rest of the population. We also find that the lifecycle patterns are stable over time. Finally, we discuss issues with controlling for time effects, sample attrition, and other modeling issues that can arise with categorical outcome variables.

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Kahler, J. R., Clarke, A., and Bruno, M. A. (2020). *HSAs: An off-label prescription for retirement saving*. Tech. rep. Vanguard.

Choosing a health insurance plan involves a complex analysis of premiums, deductibles, out-of-pocket maximums, and tax costs. The right choice depends on an individual's policy options, budget, and expected health care needs.The rapid growth of high-deductible health plans (HDHPs) and health savings accounts (HSAs) adds more complexity to the decision but also creates a unique opportunity to save for retirement and other long-term goals. We explore these underappreciated savings opportunities by reviewing the what, why, and how of HSAs. 1) The what HSAs are tax-sheltered savings accounts available to those enrolled in a high-deductible health plan (HDHP). An HDHP charges lower premiums than traditional insurance plans but comes with higher deductibles and out-of-pocket maximums. An HSA provides tax benefits to help defray these higher costs. 2) The why In tax-advantaged accounts such as IRAs, 401(k) plans, and 529 college savings plans, you pay taxes now or later. With an HSA, the "now" or "later" can become "never." These tax savings can compound to produce higher returns than those available from other accounts 3) The how The how depends on how much you can afford to save. For those with enough savings capacity, the best strategy is to treat the HSA as a long-term investment account by paying for current medical expenses out of pocket. If you have less capacity to save, the decision is more complex, involving both investment and behavioral considerations.

Kaplan, P. D. (2006). "Asset Allocation with Annuities for Retirement Income Management." In: *The Journal of Wealth Management* 8(4), pp. 27–40.

The author starts with a statement of the problem: at, in, or near retirement, investors need to make specific decisions about how they are going to use their savings to generate income during retirement and meet their estate goals when they die. If they withdraw too much each year for spending needs, they face the risk of running out of assets before they die. This is often called risk. If they withdraw too little, they may not be able to meet their immediate expenses and support a certain lifestyle. Investors also need to decide how to invest their savings among asset classes and annuities. The author explores retirement income solutions in a simple setting to illustrate the trade-offs that retired investors face regarding how much income they can generate, how much short-term risk they are exposed to, how large an estate they can expect to leave, and how likely they are not to run out of assets before dying (the probability).

Kazak, E. and Pohlmeier, W. (2019). "Testing out-of-sample portfolio performance." In: *International Journal of Forecasting* 35(2), pp. 540–554.

This paper studies the quality of portfolio performance tests based on out-of-sample returns. By disentangling the components of the out-of-sample performance, we show that the observed differences are driven largely by the differences in estimation risk. Our Monte Carlo study reveals that the puzzling empirical findings of inferior performances of theoretically superior strategies result mainly from the low power of these tests. Thus, our results provide an explanation as to why the null hypothesis of equal performance of the simple equally-weighted portfolio compared to many theoretically-superior alternative strategies cannot be rejected in many out-of-sample horse races. Our findings turn out to be robust with respect to different designs and the implementation strategies of the tests. For the applied researcher, we provide some guidance as to how to cope with the problem of low power. In particular, we make use of a novel pretest-based portfolio strategy to show how the information regarding performance tests can be used optimally.

Kazak, E. and Pohlmeier, W. (2020). *Portfolio Pretesting with Machine Learning*. Tech. rep. University of Lancaster.

This paper exploits the idea of pretesting to choose between competing portfolio strategies. We propose an estimator for a portfolio weight vector, which optimally trades off between Type I and Type II errors when choosing the best investment strategy. Furthermore we accommodate the idea of bagging in the portfolio testing problems, which helps to avoid sharp thresholding and reduces the amount of portfolio turnover. Our approach borrows from both shrinkage and forecast combination literature. The portfolio weights of our strategy are weighted averages of the portfolio weights from a set of stand-alone strategies. More specifically, the weights are generated from a pseudo out-of-sample portfolio pretesting, such that they reflect the probability that a given strategy will be overall best performing. Contrary to previous approaches, the shrinkage intensity is continuously updated to incorporate the most recent information in the rolling window forecasting set-up. We show that the bagged pretest estimator performs exceptionally well, especially when combined with adaptive smoothing. The

resulting strategy allows for a flexible and smooth switch between the underlying strategies and is shown to outperform the corresponding stand-alone strategies.

Kelly, D. P. and Roy, K. (2021). *Annuities: An essential slice of the retirement pie*. Tech. rep. J.P. Morgan Asset Management.

When it comes to retirement planning, the greatest investment tools investors can exploit are diversification and annuitization. Annuities have become an essential slice of the retirement pie for Americans approaching and living in retirement. As part of a retirement income solution, annuities also can be an ideal complement to other portfolios. We believe they are very well positioned in today's uncertain market environment. Unlike other financial instruments, these contracts issued by insurance companies combine the features of investing and insurance in a single solution.

They offer a range of flexible benefits, including:

1) Lifetime income: Annuities offer a variety of guaranteed income options that can start immediately or at a future date.
2) Growth potential: The earlier your clients invest, the more opportunity they'll have for long-term growth potential from a broad range of investment choices. Growth will vary depending on the performance of the investment options you choose.
3) Tax-deferral: Taxes can have a big impact on long-term investment returns – making the benefit of a tax-deferred variable annuity especially attractive for high net worth and affluent investors
4) Legacy options: Many variable annuity contracts offer special riders (for a fee) to provide death benefit protection for loved ones.

We believe annuities are very well positioned in today's uncertain market environment.

Kenigsberg, M. B., Mazumdar, P. D., and Feinschreiber, S. (2014). "Return Sequence and Volatility: Their Impact on Sustainable Withdrawal Rates." In: *The Journal of Retirement* 2(2), pp. 81–98.

This article explores estimates for a sustainable withdrawal rate (SWR) for a typical retiree and examines two important factors that challenge sustainability: sequence of returns and volatility of returns. Using historical analysis, the article explores the implications of sequence of returns risk and suggests strategies for countering it, including annuitization and adaptive withdrawal strategies. The article also explores volatility risk, separated as much as possible from sequence risk, and finds that the relationship between volatility, return, and SWR is not linear. In fact, for a given horizon and degree of confidence, the relationship between the Sharp ratio and the rate of return displays an inflection point, where the level of return required per unit of risk reaches a minimum.

Khang, K. and Clarke, A. S. (2020). *Safeguarding retirement in a bear market*. Tech. rep. Vanguard.

What is the effect of "sequence-of-return" risk-the risk of receiving a concentrated series of particularly poor returns-on retirees who depend on a financial portfolio to generate income? We provide a quantitative answer to this question by examining the cohorts that would have retired during or near six major U.S. bear markets since 1926. Compared to otherwise similar investors retiring during the same periods, and assuming constant real-dollar withdrawals, the unlucky ones with a poor sequence of returns were 31% more likely to outlive their wealth, had 11% lower retirement income streams,and left 37% smaller bequests. These adverse effects can be mitigated with an adaptive withdrawal strategy. By countering a decline in portfolio value with an incremental decrease in planned withdrawal amounts, even those bearing the worst sequence-of-return risk could have eliminated the possibility of premature portfolio depletion and increased their bequests by 20%. These improvements would have required a manageable reduction in retirement income on the order of a 5% decrease in the first five years and effectively no change over the whole 35 years of retirement.

Khanna, K. and Chauhan, V. (2017). "A study of risk profiling and investment choices of retail investor." In: *SSRN e-Print*.

This research is an attempt to explore the association between various demographical factors, risk profile and investment decision of retail investors. All measures were tested for reliability through computation of Cronbach Alpha. The Alpha coefficient value was found 0.741 for variables like wealth, risk profile asset class, fixed return, mutual fund return, equity, real estate and gold commodity. Further, the chi-Square and crammer-v statistics has been used to interpreting the association between these factors. And it revealed that the investors do not always behave rationally and their choice of investments is decided by their risk profile and other demographical factors such as age, gender, income, wealth etc. This research is also useful for portfolio managers to construct the right portfolio for the investors according to their needs and preferences.

Kieren, P. and Weber, M. (2022). "When saving is not enough – wealth decumulation in retirement." In: *Journal of Pension Economics and Finance* 21(3), pp. 446–473.
We field a large online survey to study preferences and hypothetical product choices for phased withdrawal accounts and compare their demand to the demand of annuities. We find that most individuals prefer phased withdrawal accounts with dynamic withdrawal rates and equity-based asset allocation. Additionally, when offered the opportunity to exchange the phased withdrawal account with an annuity, most individuals decline to annuitize. Our results suggest that policymakers should consider offering combined solutions of phased withdrawals and annuities. Retirees who are averse to full annuitization could preserve some of their accumulated wealth while also acquiring protection against longevity risk.

Kim, W. C., Kwon, D.-G., Lee, Y., Kim, J. H., and Lin, C. (2020). "Personalized goal-based investing via multi-stage stochastic goal programming." In: *Quantitative Finance* 20(3) (3), pp. 515–526.
In this paper, we propose a goal-based investment model that is suitable for personalized wealth management. The model only requires a few intuitive inputs such as size of wealth, investment amount, and consumption goals from individual investors. In particular, a priority level can be assigned to each consumption goal and the model provides a holistic solution based on a sequential approach starting with the highest priority. This allows strict prioritization by maximizing the probability of achieving higher priority goals that are not affected by goals with lower priorities. Furthermore, the proposed model is formulated as a linear program that efficiently finds the optimal financial plan. With its simplicity, flexibility, and computational efficiency, the proposed goal-based investment model provides a new framework for automated investment management services.

Kinniry Jr., F. M., Jaconetti, C. M., DiJoseph, M. A., Zilbering, Y., and Bennyhoff, D. G. (2016). *Putting a value on your value: quantifying Vanguard Advisor's Alpha*. Tech. rep. Vanguard.
The value proposition of advice is changing. The nature of what investors expect from advisors is changing. And fortunately, the resources available to advisors are evolving as well In creating the Vanguard Advisor's Alpha concept in 2001, we outlined how advisors could add value, or alpha, through relationship-oriented services such as providing cogent wealth management via financial planning, discipline, and guidance, rather than by trying to outperform the market. Since then, our work in support of the concept has continued. This paper takes the Advisor's Alpha framework further by attempting to quantify the benefits that advisors can add relative to others who are not using such strategies. Each of these can be used individually or in combination, depending on the strategy. We believe implementing the Vanguard Advisor's Alpha framework can add about 3% in net returns for your clients and also allow you to differentiate your skills and practice. Like any approximation, the actual amount of value added may vary significantly, depending on clients' circumstances.

Kintzel, D. (2020). "Income Sustainability in Retirement: A Case Study of the Life-Cycle Account." In: *The Journal of Retirement* 7(3), pp. 31–45.
This article uses Monte Carlo analysis to simulate returns for a popular retirement portfolio strategy and the income derived from it in retirement in order to calculate the chance of it being depleted while the retiree is still alive. I analyze and contrast several withdrawal strategies from the account to illustrate the important trade-offs between income needs and account sustainability. Furthermore, the simulations are compared with payouts from an annuity to show the important differences between each vehicle ability to provide income. The primary findings are that portfolios have a higher chance of account instability with advanced age and income needs, whereas annuities may provide less income earlier in retirement, but the income is sustainable for life.

Kitces, M. (2015). "Sequence of return risk and safe withdrawal rates." In: *Retirement Income Conference*.
Watching a portfolio experience market volatility in the first few years of retirement can be terrifying to a new retiree, raising legitimate questions of whether there's a danger that early declines plus ongoing withdrawals could lead to a retirement spending shortfall. And as the safe withdrawal rate research has shown, that danger is real - in fact, it's been dubbed the "sequence of return" risk to retirement spending, a recognition of the reality that even if returns average out in the long run, it doesn't matter if ongoing withdrawals deplete the portfolio before the "good" returns finally show up. Yet the caveat is that while sequence of return risk is real, it's not necessarily just about the danger of getting a severe bear market on the eve of retirement. In fact, a deeper look at the data reveals that there is remarkably little relationship between returns in the first year or two of retirement, and the safe withdrawal rate that can be sustained in the portfolio, even if retirement starts out with a market crash. Instead, it turns out that the true driver of sequence of return risk and safe withdrawal rates are the returns that the retiree earns over the first decade - and specifically, the real returns over the first decade, that provide an indication of whether the retirement portfolio will have produced enough real growth to keep up with inflation-adjusted spending for the rest of retirement. Fortunately, though, bad decades of returns are

not entirely random, and instead can be reasonably predicted by long-term market valuation trends, providing retirees with at least a few tools to manage the dangers of sequence of return risk through adjusting asset allocation in retirement and setting a reasonable initial withdrawal rate in light of the market conditions that exist - and the potential for a bad decade of returns - when their retirement begins.

Kitces, M. and Richards, C. (2019). *Kitces & Carl Ep 01: How To Value The Value Of Financial Planning*. URL: https://www.kitces.com/blog/kitces-carl-richards-talk-value-of-financial-planning-intangible/.

One of the biggest challenges comprehensive financial planners face today is that "everyone" says they do financial planning. Yet in practice, what they actually do, whether it's really comprehensive financial planning, or even what constitutes "financial planning" as opposed to just ad hoc financial advice, has no clear and consistent definition within the industry. And if we can't figure out what the differences are within the industry, then it's almost impossible for the public to understand the differences between the services offered (including any potential conflicts) by a whole bunch of people who do different stuff (frequently playing by different sets of rules) yet say they all do the same thing! Not to mention trying to help the public understand the value (both financially tangible and intangible) of the financial planning choices available and being offered to them. Fortunately, both Morningstar, Vanguard, and Envestnet have done some research to try and quantify an answer to this very question. Their studies suggest that, even when we ignore any potential excess investment return, the advice that advisors provide around the dollars that they are managing can amount to an additional 1.5-3.0% over what a client would have likely gotten had they "done it themselves" But, outside of "helping clients avoid big mistakes" by re-allocating their portfolios or talking them off the proverbial "ledge" when volatility ticks up, there's arguably another aspect to helping clients that transcends the "quantifiable" aspect of the profession. Because most clients don't have anywhere else they can go and feel safe talking about this topic of money, that is so important yet remains a taboo topic of discussion, and for many carries an enormous amount of baggage. And those hurdles only expand geometrically when it's a couple that's sitting across the desk. In fact, perhaps the greatest problem of trying to explain the value of financial advice, especially while the industry is in the midst of such rapid change, is that we are still tying all our value conversations back to the portfolio-based model... to the extent that, even when trying to quantify the behavioral aspects of planning, industry studies still scale them to a percentage of assets. Even though an increasingly amount of the value of financial planning advice is specifically about the advice that occurs outside of the portfolio! Alternatively, perhaps instead of trying to put a dollar value on the advice we provide as advisors at all, we might just consider framing it in terms of the real-world outcomes for clients. For instance, what if we as advisors simply said to a couple having stressful arguments about money that, "We help couples have better conversations about money, so they're not as awkward"? And then let them figure out if it's valuable enough to them to pay for it? Because, if an advisor can help reduce marital stress and reduce the number of marriages that end out in divorce because of money issues, then clients might consider that a whole lot more valuable that we might have ever imagined anyway! Ultimately, then, the key point is to recognize that as financial planning continues to expand beyond just building and managing diversified asset allocated portfolios, the key task as an industry is to figure out how we can frame that value beyond the context of "investable assets". Because if we want to better communicate our value to prospects and clients, especially beyond the portfolio, we must first be willing to fully embrace and acknowledge that value ourselves.

Kitces, M. E. and Pfau, W. D. (2014). "Retirement Risk, Rising Equity Glidepaths, and Valuation-Based Asset Allocation." In: *SSRN e-Print*.

This research investigates two types of dynamic asset allocation strategies (predetermined equity glidepaths and valuation-based asset allocation) for retirees using U.S. historical data. We analyze fixed asset allocations, traditional declining equity glidepaths, rising equity glidepaths, accelerated traditional and rising glidepaths, valuation-based allocations tethered around a fixed allocation, and glidepaths with valuation-based overlays. With U.S. historical data, it is difficult to beat a strategy which maintains a consistently high allocation to stocks (especially as measured by terminal median wealth), to the extent that a retiree's risk tolerance allows for this, and subject to the caveat that high stock allocations cannot always be expected to do as well in the future. However, when we consider retirements beginning in varying valuation environments (as defined by the level of Robert Shiller's cyclically-adjusted price-earnings ratio relative to its then-current historical median), we find the potential for different dynamic allocation strategies to help retirees sustain higher spending levels with lower average stock allocations in certain situations. When retirements begin in overvalued market environments (which reflects the situation for new retirees today), an accelerated rising equity glidepath has shown much potential to

provide downside risk protection for retirees by minimizing equity exposure when an adverse market event would have the greatest impact. In other valuation environments, historical worst-case scenario sustainable withdrawal rates were highest with valuation-based asset allocation strategies, which maintain a midrange average stock allocation but adjust higher or lower when markets are deemed undervalued or overvalued, respectively.

Klement, J. (2015). "Investor Risk Profiling: An Overview." In: *SSRN e-Print*.
In this discussion of investor risk profiling, current risk-profiling practice is reviewed and contrasted with regulatory demands and recent research findings.

Klement, J. (2018). "Risk Profiling and Tolerance: Insights for the Private Wealth Manager." In: *Research Foundation Publications of CFA Institute*.
If risk aversion and willingness to take on risk are driven by emotions and we as humans are bad at correctly identifying them, the finance profession has a serious challenge at hand – how to reliably identify the individual risk profile of a retail investor or high-net-worth individual. In this series of CFA Institute Research Foundation briefs, we have asked academics and practitioners to summarize the current state of knowledge about risk profiling in different key areas.

Kobor, A. and Muralidhar, A. (2020). "Targeting Retirement Security with a Dynamic Asset Allocation Strategy." In: *Financial Analysts Journal* 76(3), pp. 38–55.
The goal of investing for retirement is to secure a target level of income that maintains the individual's preretirement lifestyle. Current "safe harbor" glide-path products shift investments from stocks to bonds on the basis of the individual's age. This approach is unlikely to secure a target retirement income because the glide path is focused on the wrong goal. We tested a dynamic asset allocation strategy that takes no view of future market performance and is based on a retirement income goal. We show how this dynamic strategy could dominate standard portfolio choices. The article introduces a new way to think about intermediate retirement targets and explores the implications of the dynamic asset allocation strategy for the level of savings required to achieve a retirement goal.

Kohlscheen, E. (2021). "What does machine learning say about the drivers of inflation?" In: *SSRN e-Print*.
This paper examines the drivers of CPI inflation through the lens of a simple, but computationally intensive machine learning technique. More specifically, it predicts inflation across 20 advanced countries between 2000 and 2021, relying on 1,000 regression trees that are constructed based on six key macroeconomic variables. This agnostic, purely data driven method delivers (relatively) good outcome prediction performance. It's out of sample root mean square errors (RMSE) systematically beat even the in-sample benchmark econometric models, with a 28% RMSE reduction relative to a naive AR(1) model and a 8% RMSE reduction relative to OLS. Overall, the results highlight the role of expectations for inflation outcomes in advanced economies, even though their importance appears to have declined somewhat during the last 10 years.

Koijen, R. S. J., Nijman, T. E., and Werker, B. J. M. (2011). "Optimal Annuity Risk Management." In: *Review of Finance* 15(4), pp. 799–833.
This paper studies the life-cycle consumption and portfolio choice problem taking account of annuity risk at retirement. The study allows for government-provided annuity income. Optimally, households allocate retirement wealth to nominal, inflation-linked and variable annuities, and condition this choice on the state of the economy. The case in which there are limitations in the types of annuities that are available is also considered and the costs of annuity market incompleteness are quantified. Subsequently, the paper determines how investors optimally anticipate annuitization before retirement. The conclusion is that ignoring annuity risk before and at retirement can be economically costly.

Konicz, A. K., Pisinger, D., and Weissensteiner, A. (2016). "Optimal retirement planning with a focus on single and joint life annuities." In: *Quantitative Finance* 16(2), pp. 275–295.
We optimize the asset allocation, consumption and bequest decisions of a couple with an uncertain lifetime. The asset menu consists of zero coupon bonds and pure endowments with different maturities, whole life annuities and stocks. The pure endowments pay either fixed or variable benefits, and, similarly to the whole life annuities, are contingent on either a single or a joint lifetime. We model the stock returns and the parameters for the term structure with a vector autoregressive model, thus considering time-varying investment opportunities. To find the optimal solution we use a multi-stage stochastic programming approach, which allows for including complex surrender charges on pure endowments and annuities, as well as transaction costs on stocks and bonds. Our findings indicate that despite high surrender charges, households should invest in a wide combination of life contingent products with different maturities and underlying financial risk.

Konicz, A. K., Pisinger, D., and Weissensteiner, A. (2015). "Optimal annuity portfolio under inflation risk." In: *Computational Management Science* 12(3), pp. 461–488.
The paper investigates the importance of inflation-linked annuities to individuals facing inflation risk. Given the investment opportunities in nominal, real, and variable annuities, as well as cash and stocks, we investigate the consumption and investment decisions under two different objective functions: (1) maximization of the expected CRRA utility function (2) minimization of squared deviations from an inflation-adjusted target. To find the optimal decisions we apply a multi-stage stochastic programming approach. Our findings indicate that independently of the considered ob- jective function and risk aversion, real annuities are a crucial asset in every portfolio. In addition, without investing in real annuities, the retiree has to rebalance the portfolio more frequently, and still obtains the lower and more volatile real consumption.

Koo, B., Pantelous, A. A., and Wang, Y. (2020). "Novel Utility-Based Life Cycle Models to Optimize Income in Retirement in the Presence of Heterogeneous Preferences." In: *SSRN e-Print*.
The global shift towards defined-contribution pension schemes has been accompanied by asymmetric risks and new responsibilities for households to plan and fund effectively their own retirement over the years. In this study, expressing and combining preferences for consumption, investment, bequest, public pension entitlement and the choice of reverse mortgage products, we develop several utility-based life cycle models to facilitate the complex decision-making process that retired households are required to follow to optimize their retirement income. This optimal policy is given in the form of either an analytical or a numerical solution using stochastic dynamic programming. The timing of this paper coincides with the launch of a reverse mortgage style loan, offered by the Australian federal government and allowing retired households to receive an income stream by taking out a loan against the equity in their home. Calibration is performed using real Australian household data.

Kopa, M., Moriggia, V., and Vitali, S. (2018). "Individual optimal pension allocation under stochastic dominance constraints." In: *Annals of Operations Research* 260(1-2), pp. 255–291.
An individual investor has to decide how to allocate his/her savings from a retirement perspective. This problem covers a long-term horizon. In this paper we consider a 40-year horizon formulating a multi-criteria multistage program with stochastic dominance constraints in an intermediate stage and in the final stage. As we are dealing with a real problem and we have formulated the model in cooperation with a commercial Italian bank, the intermediate stage corresponds to a possible withdrawal allowed by the Italian pension system. The sources of uncertainty considered are: the financial returns, the interest rate evolution, the investor's salary process and a considerable withdrawal event. We include a set of portfolio constraints according to the pension plan regulation. The objective of the model is to minimize the Average Value at Risk Deviation measure and to satisfy wealth goals. Three different wealth target formulations are considered: a deterministic wealth target (i.e. a comparison between the accumulated average wealth and a fixed threshold) and two stochastic dominance relations-the first order and the second order-introducing a benchmark portfolio and then requiring the optimal portfolio to dominate the benchmark. In particular, we prove that solutions obtained under stochastic dominance constraints ensure a safer allocation while still guaranteeing good returns. Moreover, we show how the withdrawal event affects the solution in terms of allocation in each of the three frameworks. Finally, the sensitivity and convergence of the stochastic solutions and computational issues are investigated.

Krasnopolsky, M. and Ashton, M. (2018). "Why Pairing LDI with De-Risking Glide Paths Produces Inferior Pension Fund Outcomes." In: *The Journal of Investing* 27(supplement), pp. 58–64.
Combining traditional Liability Driven Investment (LDI) with funded status responsive de-risking strategies involves inconsistent treatment of risks in these two elements of what has become a popular pension strategy. This inconsistency causes irreconcilable conflicts in their execution and imperils the positive pension fund outcome. This article provides a critique of the combined LDI/De-risking Glide Path strategy as currently implemented by many pension plan managers and also provides an example of an alternative solution that improves pension plan outcomes. Our prescription for the pension de-risking glide path approach differs from conventional wisdom, resulting in faster de-risking, without undesirable market betas that are unrelated to the liability. It also avoids illiquid assets that pension funds often gravitate toward in their quest for returns, takes fewer credit risks, and seeks more alpha risks.

Kritzman, M. (2017). "Target-Date Funds: A Regime-Based Approach." In: *The Journal of Retirement* 5(1), pp. 96–105.
Target-date funds automatically shift the asset mix at regular intervals toward safer assets as the fund's target date, which may be several decades away, draws near. A target-date fund is designed to reduce exposure to loss as the investor moves closer to retirement and has fewer years to offset potential losses. The implicit assumption

of a target-date fund is that exposure to loss from risky assets increases as time decreases; hence the shift to safer assets as the investment horizon shrinks. However, another implicit assumption of target-date funds is that the standard deviation of assets is relatively stationary. The typical approach for determining the glide path assumes implicitly that the asset's annualized standard deviation will not change much during any subperiod within the longer horizon. This second assumption is contrary to experience. Subperiod volatility is highly nonstationary, although it is not entirely unpredictable. Investors would be better served by the strategy this author describes, of not only managing the effect of time on exposure to loss but also accounting for the nonstationarity of risk.

Kuntz, L.-C. (2018). "Portfolio Strategies with Classical and Alternative Benchmarks." PhD thesis. Georg August University of Gottingen.

This dissertation addresses different key elements in portfolio management. It intends to improve and analyze influences on portfolio strategies and their performance. Likewise, it aims at the systematization and extension of benchmark specifications as well as their effect on portfolio strategies. Each chapter focuses on a different aspect of developing and implementing portfolio strategies. The dissertation seeks to contribute to the advancement of portfolio strategies by making the performance generating process and influences on it more comprehensible and transparent. In doing so, it attempts to strengthen the awareness of the impact of the exact design of portfolio strategies and benchmarks on the resulting portfolio and its performance. The key findings of this dissertation can be summarized as follows: The benchmark specification, especially in terms of the investible universe and the inherent risk conception, has substantial influence on the explicit design and performance of portfolio strategies. In general, the specification of the benchmark and design of portfolio strategies should be carefully considered and the implementation should be well thought out. Alternative risk conceptions, such as regret risk, can be applied to portfolio selection and lead to clearly different portfolio compositions. Moreover, timing strategies can be improved by choosing a careful investment approach on the basis of distributional regressions. All empirical work 3 of this thesis has in common that it pursues different ideas to set up portfolio strategies while explicitly addressing the benchmark specification used for the implementation and evaluation of said strategies.

Kuselias, S., Perreault, S. J., and Shafer, M. (2021). "The Financial and Tax Considerations of Social Security and Early Retirement." In: *The Journal of Wealth Management*.

While individuals who delay the receipt of social security benefits are entitled to larger monthly payments, 57% of recipients elect to receive benefits early. This article compares the present value of social security benefits when taken early to the present value of benefits taken at the full or maximum retirement ages, taking into account factors such as average life expectancy and the time value of money. Furthermore, we expand on prior research by investigating how tax rates may affect these comparisons. Overall, our analyses suggest that taking social security early may be advantageous for many individuals.

Lalive, R., Magesan, A., and Staubli, S. (2020). *The Impact of Social Security on Pension Claiming and Retirement: Active vs. Passive Decisions*. Tech. rep. NBER.

We exploit a unique Swiss reform to identify the importance of passivity, claiming social security benefits at the Full Retirement Age (FRA). Sharp discontinuities generated by the reform reveal that raising the FRA while imposing small early claiming penalties significantly delays pension claiming and retirement, but imposing large penalties and holding the FRA fixed does not. The nature of the reform allows us to identify that between 47 and 69% of individuals are passive, while imposing additional structure point identifies the fraction at 67%. An original survey of Swiss pensioners reveals that reference-dependent preferences is the main source of passivity.

Larson, S. (2022). "Required Minimum Distributions as a Retirement Strategy: The Tradeoff Between RMD Volatility and the Expected Number of Dollars Paid Out." In: *Journal of Financial Planning* 35(1), pp. 60–67.

The intent of this paper is to help financial advisers prepare for a client discussion about the required minimum distributions (RMD) withdrawal strategy, which many Americans follow.

Clients who plan to follow this strategy need to be advised that the asset allocation they choose affects the volatility in the RMDs.

It also affects the expected total number of dollars paid out over a client's retirement life expectancy.

Based on historic returns, the tradeoff between the volatility in RMDs and the expected number of dollars paid out is examined.

Larson, S. J. (2018). "Strategy: Assessing the Impact of Required Minimum Distributions on the 4 Percent Rule." In: *Journal of Financial Service Professionals* 72(1), pp. 55–63.

This paper assesses the impact of required minimum distributions on the 4 percent rule. The main finding pertains to ending retirement account balances. They are likely to be overstated when required minimum distributions

are ignored in an analysis, and this would create a false sense of security with regard to longevity risk and legacy goals.

Laster, D., Suri, A., and Vrdoljak, N. (2012). "Systematic Withdrawal Strategies for Retirees." In: *The Journal of Wealth Management* 15(3), pp. 36–49.
The high-profile debate on spending in retirement often misses a crucial point: No single spending rate suits all retirees. This article develops guidance on spending rates for retirees. Taking into account uncertain asset returns, inflation, and longevity, the authors identify spending rates and asset allocations that minimize the expected lifetime shortfall for a systematic withdrawal plan. The analysis shows that sustainable spending rates depend critically on a client's age, gender, and risk tolerance. It also demonstrates that retirees can reduce the risk of outliving their wealth and increase their expected future bequest by allocating at least some of their portfolios to equities. Moreover, the younger the retiree and the greater her spending needs, the more she should allocate to equities. Finally, because their wealth must last longer, couples should allocate more to equities and spend at a more moderate rate than those who are single.

Laster, D., Vrdoljak, N., and Suri, A. (2014). "Pitfalls in Retirement." In: *The Journal of Retirement* 1(1), pp. 91–99.
The market turmoil of recent years has left many who are in or near retirement uncertain about how best to protect their nest eggs. This article offers insights into the key financial pitfalls to which retirees are prone. Investors can achieve their retirement goals by spending sustainably, maintaining equity exposure, addressing longevity and inflation risks, and devising a sound plan with which they feel comfortable.

Laster, D., Vrdoljak, N., and Suri, A. (2016). "Strategies for Managing Retirement Risks." In: *The Journal of Retirement* 4(1), pp. 11–18.
Retirees commonly face four risks that can threaten to derail their retirement: longevity, health care, sequence of returns, and inflation. These risks pose a multifaceted challenge that requires a combination of strategies, such as carefully choosing when to claim Social Security, allocating assets to lifetime income annuities, adopting a systematic withdrawal strategy, and planning for long-term care. This article offers a range of ideas and insights to help retirees achieve a more secure retirement.

Latham, L. (2021). *Bridging the Gap Between Accumulation and Decumulation for Participants*. Tech. rep. T. Rowe Price.
Defined contribution plan participants are increasingly keeping retirement balances in plan, and a growing number of plan sponsors are interested in retaining these balances.
Information gleaned from focus groups suggests that participants have misperceptions about the value of staying in plan. Some participants do not even know that staying in plan is an option after retirement.
Participants approaching retirement age are inundated with information and even "advice" from sources outside their plan, but the quality of this information and advice can be uneven, misguided, or ill-informed. Plan sponsors can position themselves as a reputable resource and guide.
If plan sponsors want to maintain retirees in plan, they should not keep it a secret. It's crucial that they engage with participants early and often.

Le Guenedal, T. and Roncalli, T. (2022). *Portfolio Construction with Climate Risk Measures*. Tech. rep. Amundi Asset Management.
Because of the 2015 Paris Agreement, the development of ESG investing and the emergence of net zero emission policies, climate risk is certainly the most important topic and challenge for asset owners and managers now and will remain so over the next five years. It considerably changes portfolio allocation and the investment framework of both passive and active investors. The goal of this paper is to conduct a survey of the various climate risk measures that are available in the asset management industry and the practices of portfolio construction that use these metrics.
Therefore, the first part of this paper lists the different climate risk metrics - e.g., carbon footprint, carbon transition pathway, carbon transition and physical risks. The second part is dedicated to portfolio optimization, in particular portfolio decarbonization and portfolio alignment (Paris-based benchmarks and net zero carbon objective). Among the different findings, two are of great importance for investors. First, portfolio decarbonization is more difficult when we include scope 3 carbon emissions. Indeed, optimizing using the sum of scopes 1, 2 and 3 emissions leads to a portfolio with more tracking error risk than using direct plus first tier indirect carbon emissions. Second, portfolio alignment is more complex than portfolio decarbonization. Since aligning portfolios with scope 3 is becoming the standard approach to climate portfolio construction, the impact on portfolio management may be substantial, and the divergence between carbon investing and traditional investing will increase.

Lefrancois, R., Mamidipudi, P., and Li, J. (2020). "Expectation Risk: A Novel Short-Term Risk Measure for Long-Term Financial Projections." In: *SSRN e-Print*.
Presenting the results of long-term financial projections, particularly those of common interest in retirement planning such as portfolio value and retirement income, can be difficult using traditional risk measures . Long-term projected distributions tend to be very wide and offer limited insight to practitioners into what kind of adjustments might be required along the way to avoid undesirable outcomes. Short-term projections, where available, lack information about final, long-term outcomes. As a complement to the pure long-term and short-term risk measures associated with these distributions, we propose a third measure, expectation risk, which reflects the short-term risk in long-term outcomes. This measure is derived from the short-term distribution of long-term expectations. In the following, we demonstrate how to calculate these distributions for simple cases of portfolio value and retirement income. Further, we show how expectation risk can be used to estimate the magnitude of short-term adjustments that may be required to obtain a desired long-term income.

Levine, J. (2021). "No Investment Fee Is Small, Long Term." In: *arXiv e-Print*.
This note presents a simple "back of the envelope" formula estimating that an annual investment fee expense $\epsilon\%$, compounding for N years, consumes almost $N\epsilon\%$ of an investment's value. For example, an investment with 1% annual fee compounding for 30 years pays almost 30% of its final value to fees over this period. This approximation captures how annual fees can compound over a decade or more to cumulative costs an order of magnitude higher. The formula is simple enough for rapid hand calculation.

Lewis, N. D. (2009). "Is There a Role for Commodities in Long-Term Wealth Accumulation?" In: *The Journal of Wealth Management* 12(2), pp. 130–137.
The first three years of the 21st century brought one of the worst bear markets in U.S. history, with equity markets around the world falling around 40% in real terms. This was followed in 2008 and 2009 by a global slump in economic activity and stock market price declines not seen since the great depression. The poor performance of traditional asset classes and concern over their future prospects have raised interest in the inclusion of alternative assets as strategic sources of long-term wealth creation. In sharp contrast to the recommendations of modern portfolio theory, a vast majority of high-net-worth investors are not well diversified. The article outlines the role of commodities in long-term wealth maximization. It demonstrates an alternative approach to measuring risk and diversification by addressing the question: How much can I realistically expect to gain by strategically investing in commodities over my working life? The method is easy to understand and can be easily implemented. Even individual investors with small portfolios can use the approach to gauge their own diversification benefits and risk exposure.

Lewis, W. C. and Caliendo, F. N. (2006). "Tax-Deferred Retirement Saving." In: *The Journal of Wealth Management* 8(4), pp. 12–16.
The authors first observe that both the widespread popularity of saving in tax-deferred retirement vehicles such as IRAs, Keogh plans, and 401(k) programs and the magnitude of the dollar flows into such saving are almost prima facie evidence that at the margin they provide significant gains over saving in traditional taxable accounts, at least for those wanting to defer some part of lifetime consumption to their retirement years. They focus on the gains arising solely from shifting taxable income from the working (i.e., accumulation) years to the retirement (i.e., distribution) years. They turn to the advantages of tax-deferred investing and to the motivation for focusing on the intertemporal shifting of taxable income. Finally, they propose an informal model of the gains from such income shifting and illustrate the magnitude of the gains for levels of annual wage income ranging from 50,000 to 200,000. They show that the gain (as measured by the difference in after-tax retirement income from tax-deferred and taxable accounts) increases with wage income and ranges from 1.7 percent to 11.4 percent. A formal theoretical model is provided in the appendix to the article.

Li, H. and Hyndman, R. J. (2021). "Assessing mortality inequality in the U.S.: What can be said about the future?" In: *Insurance: Mathematics and Economics* 99, pp. 152–162.
This paper investigates mortality inequality across U.S. states by modeling and forecasting mortality rates via a forecast reconciliation approach. Understanding the heterogeneity in state-level mortality experience is of fundamental importance, as it can assist decision making for policymakers, health authorities, as well as local communities who are seeking to reduce inequalities and disparities in life expectancy. A key challenge of multi-population mortality modeling is high dimensionality, and the resulting complex dependence structures across sub-populations. Moreover, when projecting future mortality rates, it is important to ensure that the state-level forecasts are coherent with the national-level forecasts. We address these issues by first obtaining independent state-level forecasts based on classical stochastic mortality models, and then incorporating the

dependence structure in the forecast reconciliation process. Both traditional bottom-up reconciliation and the cutting-edge trace minimization reconciliation methods are considered. Based on the U.S. total mortality data for the period 1969-2017, we project the 10-year-ahead mortality rates at both national-level and state-level up to 2027. We find that the geographical inequality in the longevity levels is likely to continue in the future, and the mortality improvement rates will tend to slow down in the coming decades.

Li, H. and Shi, Y. (2021). "Mortality Forecasting with an Age-Coherent Sparse VAR Model." In: *Risks* 9(2), p. 35.
This paper proposes an age-coherent sparse Vector Autoregression mortality model, which combines the appealing features of existing VAR-based mortality models, to forecast future mortality rates. In particular, the proposed model utilizes a data-driven method to determine the autoregressive coefficient matrix, and then employs a rotation algorithm in the projection phase to generate age-coherent mortality forecasts. In the estimation phase, the age-specific mortality improvement rates are fitted to a VAR model with dimension reduction algorithms such as the elastic net. In the projection phase, the projected mortality improvement rates are assumed to follow a short-term fluctuation component and a long-term force of decay, and will eventually converge to an age-invariant mean in expectation. The age-invariance of the long-term mean guarantees age-coherent mortality projections. The proposed model is generalized to multi-population context in a computationally efficient manner. Using single-age, uni-sex mortality data of the UK and France, we show that the proposed model is able to generate more reasonable long-term projections, as well as more accurate short-term out-of-sample forecasts than popular existing mortality models under various settings. Therefore, the proposed model is expected to be an appealing alternative to existing mortality models in insurance and demographic analyses.

Li, Y. and Forsyth, P. A. (2019). "A data-driven neural network approach to optimal asset allocation for target based defined contribution pension plans." In: *Insurance: Mathematics and Economics* 86, pp. 189–204.
A data-driven Neural Network (NN) optimization framework is proposed to determine optimal asset allocation during the accumulation phase of a defined contribution pension scheme. In contrast to parametric model based solutions computed by a partial differential equation approach, the proposed computational framework can scale to high dimensional multi-asset problems. More importantly, the proposed approach can determine the optimal NN control directly from market returns, without assuming a particular parametric model for the return process. We validate the proposed NN learning solution by comparing the NN control to the optimal control determined by solution of the Hamilton-Jacobi-Bellman (HJB) equation. The HJB equation solution is based on a double exponential jump model calibrated to the historical market data. The NN control achieves nearly optimal performance. An alternative data-driven approach (without the need of a parametric model) is based on using the historic bootstrap resampling data sets. Robustness is checked by training with a blocksize different from the test data. In both two and three asset cases, we compare performance of the NN controls directly learned from the market return sample paths and demonstrate that they always significantly outperform constant proportion strategies.

Linnainmaa, J. T., Melzer, B., and Previtero, A. (2021). "The Misguided Beliefs of Financial Advisors." In: *SSRN e-Print*.
A common view of retail finance is that conflicts of interest contribute to the high cost of advice. Within a large sample of Canadian financial advisors and their clients, however, we show that advisors typically invest personally just as they advise their clients. Advisors trade frequently, chase returns, prefer expensive, actively managed funds, and underdiversify. Advisors' net returns of -3% per year are similar to their clients' net returns. Advisors do not strategically hold expensive portfolios only to convince clients to do the same; they continue to do so after they leave the industry.

Lobel, H., Jaconetti, C. M., and Cuff, R. (2019). *The replacement ratio: Making it personal*. Tech. rep. Vanguard.
A replacement ratio is a rule of thumb that estimates what percentage of a person's pre-retirement income will be needed to maintain their lifestyle at retirement. Most studies suggest aiming for a target of between 70 and 85 percent of pre-retirement income. Knowing which end of that range would be more appropriate, however, is an important step in developing a retirement plan. With our approach to calculating the replacement ratio, investors begin with their current annual consumption and then factor in the changes in taxes and healthcare costs. The replacement ratio is that total amount expressed as a percentage of the investor's pre-retirement income. Because people-and thus their retirement goals-are unique, otherwise-similar investors can have different replacement ratios. Variables affecting the desired ratio include broad demographic differences (marital status and income level, for example) as well as more subtle, personalized influences (homeownership, health status, the type of accounts dedicated for retirement, for example).

Lohre, H., Rother, C., and Schafer, K. A. (2020). "Hierarchical Risk Parity: Accounting for Tail Dependencies

in Multi-asset Multi-factor Allocations." In: *Machine Learning for Asset Management: New Developments and Financial Applications*. Ed. by E. Jurczenko. Wiley, pp. 329–368.
This chapter examines the use and merits of hierarchical clustering techniques in the context of multi-asset multi-factor investing. In particular, it contrasts these techniques with several competing risk-based allocation paradigms, such as 1/N, minimum-variance, standard risk parity and diversified risk parity. The chapter introduces hierarchical risk parity (HRP) strategies based on the Pearson correlation coefficient and also introduces hierarchical clustering based on the lower tail dependence coefficient. The chapter provides an overview of traditional risk-based allocation strategies and outlines a framework to measure and manage portfolio diversification. It examines the performance of the introduced HRP strategies relative to the traditional alternatives. The chapter discusses Meucci's approach to managing diversification, which serves to construct a diversified risk parity strategy based on economic factors.

Lopez de Prado, M. (2019). "A Data Science Solution to the Multiple-Testing Crisis in Financial Research." In: *The Journal of Financial Data Science* 1(1), pp. 99–110.
Most discoveries in empirical finance are false, as a consequence of selection bias under multiple testing. Although many researchers are aware of this problem, the solutions proposed in the literature tend to be complex and hard to implement. In this article, the author reduces the problem of selection bias in the context of investment strategy development to two sub-problems: determining the number of essentially independent trials and determining the variance across those trials. The author explains what data researchers need to report to allow others to evaluate the effect that multiple testing has had on reported performance. He applies his method to a real case of strategy development and estimates the probability that a discovered strategy is false.

Lopez de Prado, M. and Lewis, M. J. (2019). "Detection of false investment strategies using unsupervised learning methods." In: *Quantitative Finance* 19(9), pp. 1555–1565.
In this paper we address the problem of selection bias under multiple testing in the context of investment strategies. We introduce an unsupervised learning algorithm that determines the number of effectively uncorrelated trials carried out in the context of a discovery. This estimate is critical for computing the familywise false positive probability, and for filtering out false investment strategies.

Love, D. and Phelan, G. (2015). "Hyperbolic discounting and life-cycle portfolio choice." In: *Journal of Pension Economics and Finance* 14, pp. 492–524.
This paper studies how hyperbolic discounting affects stock market participation, asset allocation, and saving decisions over the life cycle in an economy with Epstein Zin preferences. Hyperbolic discounting affects saving and portfolio decisions through at least two channels: (1) it lowers desired saving, which decreases financial wealth relative to future earnings; and (2) it lowers the incentive to pay a fixed cost to enter the stock market. We find that hyperbolic discounters accumulate less wealth relative to their geometric counterparts and that they participate in the stock market at a later age. Because they have lower levels of financial wealth relative to future earnings, hyperbolic discounters who do participate in the stock market tend to hold a higher share of equities, particularly in the retirement years. We find that increasing the elasticity of intertemporal substitution, holding risk aversion constant, greatly magnifies the impact of hyperbolic discounting on all of the model's decision rules and simulated levels of participation, allocation, and wealth. Finally, we introduce endogenous financial knowledge accumulation and find that hyperbolic discounting leads to lower financial literacy and inefficient stock market investment.

Lozada, G. A. (2018a). *Financing Retirement using U.S. Treasury Bonds: Safe Withdrawal Rates, Mean/Standard-Deviation Frontiers,and Endpoint-Dependence of the Safest Maturity*. Tech. rep. University of Utah.
Calculating, for a fixed asset allocation, completely-SafeWithdrawal Rates over various historical periods of one fixed lengthgenerates a "SWR" distribution. Combining its minimum, or meanand standard deviation, with those of other allocations implies amaximin portfolio, or a SWR Mean/Standard-Deviation Frontier.Considering portfolios only of Treasurys of one constant maturity, the lowest-withdrawal-variance asset was sometimes not theshortest-maturity, and sometimes was the longest. Constructingmost subperiods, these anomalies were very rare for real SWRs andoccurred about one-fifth of the time for nominal SWRs, happeningwhen money-market yields changed greatly.

Lozada, G. A. (2018b). *Fixed income for retirement saving: TIAA traditional lessons on quality, duration, risk, and gradual withdrawals*. Tech. rep. University of Utah.
TIAA Traditional annuity has supported retirements for a century. It resembles a stable-value fund. The author investigates the holdings supporting it, constructs readily available alternatives resembling those holdings, compares the returns of those alternatives with Traditional, and constructs a new measure of risk to compare

Traditional risk with that of its alternatives in a more appropriate way than by using short-term standard deviation of returns. Under this new measure of risk, which is appropriate for retirement investors, some alternatives exhibited second-degree stochastic dominance over Traditional using 1987-2015 data. However, Traditional may still be a better choice for unsophisticated investors.

Lozada, G. A. (2020). "Fixed income for retirement saving: TIAA traditional lessons on quality, duration, risk, and gradual withdrawals." In: *The Journal of Retirement* 7(4), pp. 39–59.
TIAA Traditional annuity has supported retirements for a century. It resembles a stable-value fund. The author investigates the holdings supporting it, constructs readily available alternatives resembling those holdings, compares the returns of those alternatives with Traditional, and constructs a new measure of risk to compare Traditional risk with that of its alternatives in a more appropriate way than by using short-term standard deviation of returns. Under this new measure of risk, which is appropriate for retirement investors, some alternatives exhibited second-degree stochastic dominance over Traditional using 1987-2015 data. However, Traditional may still be a better choice for unsophisticated investors.

Lumby, J. (2017). "Three Essays on Managing Risk in Retirement." PhD thesis. Texas Tech University.
Individuals face a number of risks when entering retirement, such as unknown life expectancy, variable medical expenses, and uncertain financial market returns. Understanding these risks in advance will allow retirees to pursue appropriate risk management strategies to reduce the likelihood of experiencing negative events that could jeopardize the retirement period. Unfortunately, many retirees either do not understand the risks that they face, or are ill-prepared to handle these risks when they inevitably arise. In this dissertation, I provide new insights into the dynamic relationship between consumer knowledge, risk exposure, and insurance coverage. The first essay provides evidence that many elderly individuals do not understand the risks associated with unknown late-in-life medical expenses, such as nursing care. When educated, many retirees indicate an increased willingness to purchase a suitable insurance policy as a hedge against the risk. The second essay finds that a small percentage of retirees are insured against multiple retirement risks, and that this behavior is associated with increased financial wealth and successful retirement outcomes. Although insurance can provide a hedge against a multitude of financial risks, the third essay suggests that life satisfaction in retirement is about more than material wealth.

Lung, E., Roodt, C., Ryan, L., Warren, G. J., and Wymer, K. (2021). "A Framework for Designing Investment Strategies for Default Retirement Plans." In: *The Journal of Retirement* 8(3), pp. 40–60.
We identify and discuss four key elements to address when designing the investmentstrategy for default retirement plans: whether to cater for member needs or their wants, objectives, the member for which the default is being designed, and risk appetite. Addressing these design elements requires making assumptions about the member, of which the literature provides limited guidance to plan sponsors. We outline the main assumptions and demonstrate the potential impact on retirement experience through illustrative models. We find that mismatches between the member and the way that they are characterized can adversely impact on welfare.

Lurtz, M. R., Archuleta, K., Kothakota, M., and Jorgensen, T. J. (2021). "A deeper dive: A mixed methods approach to risk tolerance." In: *Financial Planning Review* 4(2) (e112).
Most risk tolerance studies are quantitative, even though many factors that may affect the manifestation of risk tolerance are qualitative. This study employed a mixed-methods approach to investigate how individuals consider risk tolerance as it relates to their financial situation. Fuzzy-trace theory, a psycholinguistic theory of risk processing rooted in prospect theory that is becoming increasingly popular in the medical field, guided the study. Quantitative results indicate that stated versus revealed measures of risk tolerance are not consistent for most people. However, higher risk literacy increases the likelihood of consistency. Qualitative results reveal that individuals perceive risk tolerance through various lenses, including knowledge, values, emotions, and personal experience.

Lussier, J. (2019). "Secure retirement: connecting financial theory and human behavior." In: *CFA Institute Research Foundation*.
Investors fear return uncertainty and drawdowns associated with owning relatively risky asset classes, such as equity. The fact that greater risk is associated with greater expected return does not preclude the possibility that realized returns may be far less than a low-risk asset could provide, even with horizons as long as 5 to 10 years. Fear prompts the average investor to sometimes act against his own best interest. Therefore, the average investor portfolio often underperforms a static benchmark, even before fees. The average investor tends to increase allocation to riskier assets after the market has already significantly risen and decrease allocation after a significant decline.Given that financial planning in the context of retirement is a multidecade endeavor that

can last 50 years or more, investors face an additional challenge. Financial planners cannot be concerned solely with the behavior of investors facing short-term return uncertainty, which remains an important challenge, but must also be concerned with how to address longer-term uncertainties. For example, there is significant uncertainty about the assumption for long-term expected returns, real as well as nominal. Some lucky investors may accumulate much of their wealth before retirement during an exceptionally strong bull market (e.g., 1982 to 2000), whereas less fortunate investors may have been planning to retire in 2008 or early 2009, just after the most dramatic global liquidity crisis since the Great Depression and a significant decline in interest rates. Moreover, average expected returns represent only part of the story. Some investors may have recorded above-average returns at a time when their accumulated wealth was already significant, whereas others may have recorded above-average returns when their accumulated wealth was low. The implication of the interactions among savings patterns (in accumulation), withdrawal patterns (in decumulation), and timing of above- and below-average portfolio returns is important to understand, particularly with respect to how the interactions should affect the allocation policy.Furthermore, investors face significant risks in the transition period - say, the last 5 to 10 years - from accumulation to decumulation. Considering the uncertainty in long-term expected returns, patterns of returns, and patterns of savings, it is unlikely that a financial plan established when an individual is 30 years old can remain static thereafter. The plan must be revaluated periodically, which requires an effective feedback mechanism or tool. The investor has many more options available before retirement, however, although some may not necessarily be pleasant. In the event of lower than expected accumulated wealth, the investor could decide to save more, postpone retirement, and/or adjust retirement plans. These options may be unavailable or may be harder to implement once the decision to retire has been made. The investor must implement a transition strategy that reduces the likelihood and/or significance of the unplanned adjustments that may be necessary as the targeted or desired retirement date draws near.In addition, longevity remains uncertain, and retirement plans often are based on expected longevity. Even though the median life expectancy for a 65-year-old individual in the United States is 83.3 years for men and 85.9 years for women, a significant percentage of people will live past age 90. Furthermore, as someone ages, the older she is expected to live. A dynamic issue, life expectancy becomes even more complicated in the context of a couple - one or both individuals could live a very long time. This possibility should also influence the allocation policy over time and the potential need for longevity insurance.Finally, governments in many countries have implemented policies and programs to support the retirement effort. The most important program in the United States is Social Security, but its purpose is to provide only a minimum level of inflation-adjusted income, not to sustain the standard of living that an individual had before retirement. Other programs seek to encourage savings and facilitate wealth accumulation, such as 401(k)s, traditional and Roth IRAs (Registered Retirement Savings Plans and Tax-Free Savings Accounts in Canada), and health savings accounts (HSAs). Most investors, however, underestimate the savings effort required to maintain the standard of living to which they are accustomed, cannot implement a comprehensive and coherent adaptive retirement plan, cannot optimize across all relevant parameters, and lack access to the feedback mechanism needed to make the appropriate adjustments over time.

MacDonald, B.-J., Jones, B., Morrison, R. J., Brown, R. L., and Hardy, M. (2013). "Research and Reality: A Literature Review on Drawing Down Retirement Financial Savings." In: *North American Actuarial Journal* 17(3), pp. 181–215.

How do, could, and should retirees draw down their financial savings? This article reviews over 100 papers on this topic from the perspective of individuals, families, governments, and financial institutions.

Three significant conceptual/methodological weaknesses in the existing literature are identified:

1) analysts have examined a limited range of self-managed drawdown strategies;
2) nearly all have ignored home ownership, pensions, debt, and government taxes and transfers when quantitatively evaluating alternative drawdown strategies;
3) there is a well-acknowledged gap between the behavior implied by economic models and that of real-life individuals, particularly when it comes to voluntary annuitization.

Expanding the set of drawdown strategies evaluated (e.g., including larger payouts when life expectancy is reduced after the onset of a significant health condition, or using savings as bridge income to delay the take-up of Social Security payments), refining the income concept used, and more exact modeling of the trade-offs underlying individual decision-making will likely increase the appeal of self-managed drawdown strategies and help resolve the "annuity puzzle" that has long dominated this line of research. It may also lead to advice and financial products that will better meet the needs of retirees.

MacDonald, B.-J., Morrison, R. J., Avery, M., and Osberg, L. (2018). "Drawing Down Retirement Savings - Do Pensions, Taxes and Government Transfers Matter Much for Optimal Decisions?" In: *ASTIN Bulletin* 48(3), pp. 1277–1306.
This paper examines the importance of pensions (employment and social security), taxes and government transfers for alternative retirement savings drawdown strategies (DS), compared to the conventional approach in published literature of using a gross income concept obtainable from retirement savings alone. Using a lifetime utility framework, our longitudinal dynamic micro-simulation model incorporates risk aversion, stochastic markets, stochastic mortality and the interactions among sources of retirement income within the complex Canadian tax and social benefit system, enabling us to rank commonly advocated DS and to ask whether incorporating pensions, taxes and transfers alters those rankings. Our findings show the importance of treating the evaluation of alternative DS as a comprehensive and integrated problem by including all sources of income - including pensions, taxes and government transfers. Using restricted income measures can potentially lead to simplistic, and possibly misleading, conclusions.

MacLean, L. and Zhao, Y. (2017). "Asset Price Dynamics: Shocks and Regimes." In: *Optimal Financial Decision Making under Uncertainty*. Ed. by G. Consigli, D. Kuhn, and P. Brandimarte. Vol. 245. Springer International Publishing, pp. 35–53.
Security prices changes are known to have a non-normal distribution, with heavy tails. There are modifications to the standard geometric Brownian motion model which accomodate heavy tails, most notably (1) adding point processes to the Brownian motion or (2) classifying time into regimes. With regimes the prices follow Brownian motion dynamics within regime, but the parameters vary by regime. The unconditional distribution of returns is a mixture of normals, with the mixing coefficients being Markov transition probabilities. The contrasting approaches have a common link-risk factors. In the case of the point processes, the intensity of "shocks" depends on a set of factors, eg. bond-stock yield differential, credit spread, implied volatility, exchange rates. The factors drive shocks, which are a component of the returns. With regimes, the economic state is hidden (latent) and is determined by the period by period observations on factors. The characterization of regimes follows from description in terms of the set of risk factors. In this paper the link between the shocks and regimes is explored. The shocks times defined by risk factors are an alternative method of determining regimes and the classifications by shocks and by the Expectation-Maximization algorithm are examined. The connections factors -> regimes -> shocks further justifies a classification of financial markets into homogeneous epochs. The regime structure leads to improved estimates for distribution parameters. The methods are applied to the prediction of returns on Sector Exchange Traded Funds (ETFs).The allocation of investment capital to funds based on predicted returns generates favorable wealth accumulation over a planning horizon.

Maclean, Ziemba, W. T., and Li, Y. (2005). "Time to wealth goals in capital accumulation." In: *Quantitative Finance* 5(4), pp. 343–355.
This paper considers the problem of investment of capital in risky assets in a dynamic capital market in continuous time. The model controls risk, and in particular the risk associated with errors in the estimation of asset returns. The framework for investment risk is a geometric Brownian motion model for asset prices, with random rates of return. The information filtration process and the capital allocation decisions are considered separately. The filtration is based on a Bayesian model for asset prices, and an (empirical) Bayes estimator for current price dynamics is developed from the price history. Given the conditional price dynamics, investors allocate wealth to achieve their financial goals efficiently over time. The price updating and wealth reallocations occur when control limits on the wealth process are attained. A Bayesian fractional Kelly strategy is optimal at each rebalancing, assuming that the risky assets are jointly lognormal distributed. The strategy minimizes the expected time to the upper wealth limit while maintaining a high probability of reaching that goal before falling to a lower wealth limit. The fractional Kelly strategy is a blend of the log-optimal portfolio and cash and is equivalently represented by a negative power utility function, under the multivariate lognormal distribution assumption. By rebalancing when control limits are reached, the wealth goals approach provides greater control over downside risk and upside growth. The wealth goals approach with random rebalancing times is compared to the expected utility approach with fixed rebalancing times in an asset allocation problem involving stocks, bonds, and cash.

Mahaney, J. (2020). *Innovative Strategies to Help Maximize Social Security Benefits*. Tech. rep. Prudential.
Developing a Social Security claiming strategy is an important component to helping enhance retirement security. This updated 2020 edition incorporates changes made to certain claiming strategies as a result of the Bipartisan Budget Act of 2015. In addition, it highlights ways in which withdrawals from retirement savings vehicles, such

as 401(k) plans and IRAs, can be integrated with a Social Security claiming strategy to maximize retirement income.

Mahayni, A. and Schneider, J. C. (2012). "Variable annuities and the option to seek risk: Why should you diversify?" In: *Journal of Banking and Finance* 36(9), pp. 2417–2428.
We analyze the impacts of an additional rider incorporated in recent retirement products. The payoff is linked to the performance of a multi asset investment strategy and includes a minimum interest rate guarantee. In addition, the buyer receives the option to decide on the investments dynamically. Prominent examples are so called variable annuities. Due to the embedded guarantee, these products are interesting for risk averse investors who benefit from diversification. However, the price setting of the provider takes into account the most risky strategy. This implies that the investor mitigates optimally between the diversification and the worst case strategy. We analyze the distortion and utility effects caused by the additional rider in the presence of background risk and borrowing constraints. A simulation analysis sheds light on the question if the additional rider is worth its costs. Analysis of impacts of the rider to decide on investment decisions dynamically. Illustration of investment and utility effects under background risk. Simulation study shedding light on the question if the rider is worth its costs.

Malavasi, M., Lozza, S. O., and Truck, S. (2021). "Second order of stochastic dominance efficiency vs mean variance efficiency." In: *European Journal of Operational Research* 290(3), pp. 1192–1206.
In this paper, we compare two of the main paradigms of portfolio theory: mean variance analysis and expected utility. In particular, we show empirically that mean variance efficient portfolios are typically sub-optimal for non satiable and risk averse investors. We illustrate that the second order stochastic dominance (SSD) efficient set is the solution of a multi-objective optimization problem. We further show that the market portfolio is not necessarily a solution to this optimization problem. We also conduct an empirical analysis, examining the ex ante and ex post performance of SSD and mean variance efficient portfolios, using a bootstrap approach. In an ex ante analysis, we compare empirical moments, the level of diversification and set distances of mean variance and SSD efficient sets. We also show that the global minimum variance (GMV) portfolio and the part of the mean variance efficient frontier (MVEF) composed of highly diversified portfolios is second order stochastically dominated. This result also provides a possible alternative explanation for the diversification puzzle. Conducting an ex post analysis, we construct second order stochastic dominating strategies that outperform the GMV portfolio in terms of wealth and various other performance measures, producing a positive ex post opportunity cost.

Malhotra, M. (2012). "A Framework for Finding an Appropriate Retirement Income Strategy." In: *Journal of Financial Planning*.
Even though retirement distribution is the most actively researched area in the world of financial planning, the profession still lacks a comprehensive analysis framework for comparing retirement distribution strategies. Such a framework should do an apples-to-apples comparison among various retirement distribution strategies such as the appropriate time to claim Social Security or the use of systematic portfolio withdrawals in combination with fixed annuities, variable annuities, and bond ladders. The proposed framework in this paper includes two reward and three risk metrics. The widely used metric of probability of success is complemented with two additional risk metrics: a view into what happens when the plan fails and the amount of income generated from fixed sources of cash flow. The proposed reward metrics are retirement income and average legacy. A general outline of the effect of various income strategies on the metrics is provided, which can help planners practice the art of building an income plan mixing a systematic withdrawal portfolio with a fixed source of cash flow such that the plan provides acceptable confidence and reasonable protection in unfavorable markets with a potential to leave a legacy (if so desired) in favorable markets.

Mantilla-Garcia, D., Martinez-Carrasco, M., Garcia Huitron, M. E., and Muralidhar, A. (2020). "From Defined-Contribution Towards Target-Income Retirement Systems." In: *SSRN e-Print*.
The current trend towards Defined Contribution (DC) retirement systems around the world has rendered the risk management of pension funds crucial for the financial health of millions of people. Severe market downturns have put in evidence the need for more effective practices to control losses of retirement income for pension funds' investors. Although the move from the static allocation of policy portfolios toward target-date funds (TDFs) encouraged by regulators features significant improvements, the latter strategies completely ignore the variations in the cost of financing future consumption, which lessens the strategies' ability to control losses in retirement income. While optimal long-term portfolio selection literature emphasizes the importance of hedging against changes in the discount rates that determine the cost of financing future consumption, TDFs strategies, regulatory incentives, and reporting to pension funds' investors disregard the variations in long-term discount

rates by focusing in absolute returns. We address these shortcomings by (i) developing performance metrics, such as DC funding-ratios, that foster appropriate incentives for pension fund managers and more sensible and simple reporting to their investors, (ii) introducing a series of asset allocation rules designed to secure a minimum level of target-income in retirement regardless of the returns of the risky assets in the portfolio. The strategies are consistent with insights from long-term portfolio theory and have the critical advantage of being free of any model or parameter estimation risks. We illustrate its advantages relative to a standard TDF strategy in terms of retirement security.

Marekwica, M., Schaefer, A., and Sebastian, S. (2013). "Life cycle asset allocation in the presence of housing and tax-deferred investing." In: *Journal of Economic Dynamics and Control* 37(6), pp. 1110–1125.
We study the dynamic consumption-portfolio problem over the life cycle, with respect to tax-deferred investing for investors who acquire housing services by either renting or owning a home. The joint existence of these two investment vehicles creates potential for tax arbitrage. Specifically, investors can deduct mortgage interest payments from taxable income, while simultaneously earning interest in tax-deferred accounts tax-free. Matching empirical evidence, our model predicts that investors with higher retirement savings choose higher loan-to-value ratios to exploit this tax arbitrage opportunity. However, many households could benefit from more effectively taking advantage of tax arbitrage.

Markwat, T., Molenaar, R., and Rodriguez, J. C. (2016). "Purchasing an Annuity: Now or Later? The Role of Interest Rates." In: *Bankers, Markets and Investors* (142), pp. 4–17.
This paper investigates whether the option to delay annuitization in times of low interest rates has value for retirees. The retiree who chooses to wait bets on a fall in the annuity price brought about by a rise in the interest rates; the cost of such a bet is the loss of the mortality credit during the waiting period. We show that an investor who can only invest in bonds during the waiting period will never find waiting ex-ante profitable if annuities are fairly priced, because waiting is costly and buying a fairly priced annuity is a zero-npv project. In contrast, an investor allowed to invest part of her wealth in the stock market will be able to attain more consumption on average, but at the cost of a sharp increase in risk. A retiree may choose to wait, however, if she believes her views are not priced in the term structure. For such a retiree, waiting is optimal if the expected increase in the interest rate is larger than the square of the hazard rate.

Martel, R. and Sharon, A. (2019a). *An Untimely Retirement: The Dangers of 'Sequence Risk' for Retirees – Introducing "Income to Outcome" retirement framework*. Tech. rep. PIMCO.
A major market downturn at the beginning of one's retirement journey is among the most potentially destructive scenarios investors face. Sharp declines can lead to de-risking and withdrawals from portfolios – both of which tend to crystallize losses and make recovery harder or even impossible.
Fortunately, investors are not completely at the mercy of the markets when it comes to sequence risk and retirement.
By setting up two dedicated portfolios – one, a bond portfolio designed to deliver necessary income in future years ("paycheck replacement"), the other composed of higher-risk long-term growth-oriented assets such as equities – retirees may more easily avoid costly, knee-jerk reactions that could undermine long-term growth.
This is the essence of PIMCO's "Income to Outcome" retirement framework. It's an approach that addresses both the hard issues of asset allocation and the softer, but no less significant, challenges of behavioral finance.

Martel, R. and Sharon, A. (2019b). *Financial Advisors and Retirement: The Decumulation Dilemma*. Tech. rep. PIMCO.
The retirement of the baby boomer generation represents a secular money-in-motion opportunity for financial advisors - one which may transform the asset management industry and individual advisory practices. But the withdrawal of assets over an uncertain remaining lifetime – decumulation – remains both the solution most in demand and the problem the industry has been least able to solve.
Our Income to Outcome decumulation framework incorporates behavioral insights that have become central to how the industry understands investor decisions.
It seeks to help investors set and automate regular withdrawals, protect against sequence-of-returns risk, embed better advice around optimal strategies for claiming Social Security, address longevity risk and pursue long-term growth in an endowment-like portfolio to support unknown expenditures and maximize potential for bequests.

Martellini, L. and Milhau, V. (2020). *Advances in Retirement Investing*. Cambridge University Press. 167 pp.
To supplement replacement income provided by Social Security and employer-sponsored pension plans, individuals need to rely on their own saving and investment choices during accumulation. Once retired, they must also decide at which rate to spend their savings, with the usual dilemma between present and future consump-

tion in mind. This Element explains how financial engineering and risk management techniques can help them in these complex decisions. First, it introduces 'retirement bonds', or retirement bond replicating portfolios, that provide stable and predictable replacement income during the decumulation period. Second, it describes investment strategies that combine the retirement bond with an efficient performance-seeking portfolio so as to reduce uncertainty over the future amount of income while offering upside potential. Finally, strategies using risk insurance techniques are proposed to secure minimum levels of replacement income while giving the possibility of reaching higher levels of income.

Martellini, L., Milhau, V., and Mulvey, J. (2019). "'Flexicure' Retirement Solutions: A Part of the Answer to the Pension Crisis?" In: *The Journal of Portfolio Management* 45(5), pp. 136–151.
Individuals preparing for retirement are currently left with an unsatisfactory choice between security with no flexibility with annuity products and flexibility without security with investment products such as balanced funds or target date funds. To get out of this impasse, the authors introduce a range of flexicure retirement goal-based investing strategies that offer both security and flexibility with respect to the objective of generating replacement income in decumulation. Recent advances in financial engineering and digital technologies make it possible to apply goal-based investing principles to a much broader population of investors than the few traditional clients who can afford customized mandates or private banking services, which suggests that these flexicure retirement solutions can be used as part of the solution to the global pension crisis.

Martellini, L., Milhau, V., and Mulvey, J. (2020). "Securing Replacement Income with Goal-Based Retirement Investing Strategies." In: *The Journal of Retirement* 7(4), pp. 8–26.
Individuals preparing for retirement are currently left with an unsatisfactory choice between security with no flexibility with annuity products and flexibility without security with investment products such as balanced funds or target date funds. To get out of this impasse, the authors introduce a range of flexicure retirement goal-based investing strategies that offer both security and flexibility with respect to the objective of generating replacement income in decumulation. Recent advances in financial engineering and digital technologies make it possible to apply goal-based investing principles to a much broader population of investors than the few traditional clients who can afford customized mandates or private banking services, which suggests that these flexicure retirement solutions can be used as part of the solution to the global pension crisis.

Martin, P. P. and Kintzel, D. (2016). "A Comparison of Free Online Tools for Individuals Deciding When to Claim Social Security Benefits." In: *SSRN e-Print*.
When to claim Social Security retirement benefits is one of the most important financial decisions many people make. The Social Security Administration (SSA) provides a variety of online tools and publications to help individuals decide on their own when to claim benefits, but maintains a neutral stance on when a person should claim. Because SSA remains neutral, other government, nonprofit, academic, and for-profit entities have developed tools to assist the public with their claiming decision. This note analyzes the advantages and limitations of six online benefit calculators. Users of these online tools should consider the source of their information and understand that the benefit estimates they produce are based on different underlying assumptions, which can result in different estimated benefit amounts.

Marwood, D. and Minnen, D. (2020). "Safely Boosting Retirement Income by Harmonizing Drawdown Paths." In: *Journal of Financial Planning* 33(11), pp. 46–60.
Building on Bengen's famous 4 percent rule (Bengen 1994), this paper introduces the DMSWR, a technique for calculating withdrawal rates and retirement income levels that are safe, steady, and adjusted for inflation.
The DMSWR (named after the authors' initials, plus the acronym for safe withdrawal rate) addresses two commonly cited shortcomings of the 4 percent rule. First, it addresses the starting point paradox (Kitces 2008) by ensuring that initial retirement incomes are stable regardless of the precise retirement date and short-term fluctuations in the stock market. Second, the DMSWR significantly reduces the risk of unnecessarily low retirement income that leads to large, unspent portfolios at the end retirement.
The DMSWR provides withdrawal rates that are often substantially higher, and never lower, than the 4 percent rule, which allows financial planners to recommend higher retirement incomes that are still safe.
The core insight underpinning the DMSWR is that a retiree can safely follow the "drawdown path" of an earlier retiree who follows the 4 percent rule and planned for a longer retirement. Since the earlier retiree's withdrawals are safe, the withdrawals of the new retiree must also be safe.
The DMSWR is safe in the same way the 4 percent rule is safe: the withdrawal rates have never failed in the past and, were DMSWR to fail in the future, the 4 percent rule would also fail. Unlike many similar efforts to boost safe withdrawal rates, the DMSWR does not include additional parameters or heuristics that might fail

at a time when following the 4 percent rule would succeed.
Historically, the mean DMSWR for 30-year retirements was 5.48 percent, the highest DMSWR was 13.3 percent, and the DMSWR exceeded the baseline withdrawal rate by more than 100 basis points over 55 percent of the time.
"Re-retirement" using the DMSWR often allows incomes to increase during retirement. These higher incomes succeed whenever the 4 percent rule succeeds. The software implementation is at `github.com/minnend/dmswr`.

Massa, M., Moussawi, R., and Simonov, A. (2021). "To Target a Date or Not to Target a Date? That is the Question: The Unintended Consequences of Investing for the Long Run." In: *SSRN e-Print*.
We study how managers of funds created to invest for the long run behave when shielded from liquidity constraints and their investors' short-term needs. Using the universe of US target-date funds (TDFs), we document that asset managers exploit lower investor attention to deliver lower performance. This results in a hypothetical cumulative return loss of 21% for the average investor holding the fund for 50 years. This underperformance is driven by fund families using the TDFs to smoothen the flow shocks of the affiliated open-ended funds. It is also due to higher fees arising from investing in the affiliated expensive share classes. We use the Pension Protection Act of 2006 as an exogenous shock that made TDFs the default investment options within 401(k) retirement plans.

McLean, R. (2021). "Rebalancing Frequency and Safe Withdrawal Rates." In: *Advisor Perspectives*.
Safe-withdrawal rate (SWR) research has traditionally assumed annual portfolio rebalancing, and this is the frequency suggested by many advisors (although some suggest quarterly, monthly, or even daily rebalancing!). But is such frequent fiddling necessary? Rebalancing generates costs in fees and commissions and, depending on the type of account, will trigger tax liabilities. I used the "Big Picture" client education software to examine the impact of rebalancing frequency on historical SWRs and other retirement outcomes. The results argue against the notion to rebalance often.

McQuarrie, E. F. (2022a). "When and for Whom are Roth Conversions Most Beneficial? A New Set of Guidelines, Cautions and Caveats." In: *SSRN e-Print*.
Much has changed since penalty-free Roth conversions were inaugurated in 2010. Tax rates have gone up and down. The re-characterization provision went away. Heirs can no longer stretch out inherited Roth accounts over a lifetime. Medicare surcharges were expanded and began to adjust for inflation. The age to begin Required Minimum Distributions was pushed out to age 72 and the IRS changed the RMD divisor tables to further slow the pace of distribution.
Given these developments it seemed worthwhile to re-examine the rationale for Roth conversions. That effort exposed multiple flaws in conventional wisdom:

1) Future tax rates need not be higher for a conversion to pay off;
2) Nor is it all that helpful to pay the tax on conversion from outside funds;
3) Nor are Roth conversions especially beneficial for top bracket taxpayers as compared to middle class taxpayers;
4) Rather, the greatest benefit accrues to taxpayers who can make the conversion partly in the zero percent tax bracket, i.e., during a year with no other taxable income.

While the benefits from a Roth conversion are often small and slow to arrive, a Roth conversion will almost always pay off if given enough time, i.e., for life spans that extend past 90 and so long as annual distributions from converted amounts are not taken. Roth conversions work because of compounding, which requires the conversion to be left undisturbed for a long time. The paper elucidates the role played by the mathematics of compounding in underwriting the success of Roth conversions.

McQuarrie, E. F. (2022b). "Will Required Minimum Distributions Exhaust My Savings and Leave Me in Penury?" In: *SSRN e-Print*.
This paper probes the question of how long retirement savings can be sustained under mandated withdrawals whose rate increases with age. It begins with idealized, constant rates of return typical of balanced funds. These laboratory analyses show that funds are unlikely to be exhausted before age 100, and that exhaustion, when it comes, occurs because of Bengen's test, i.e., the eventual need to withdraw more than the minimum to maintain the initial age 72 income in constant dollars. Next, the paper examines sequence-of-returns risk using selected historical data. The worst decade for balanced fund returns in the US was located, but even with this bad start the crudest sort of balanced portfolio could still be made to sustain income to age 101. The paper proceeds to examine a variety of worst-case scenarios drawn from the two-century historical record, in the US and globally,

with the worst cases ex-USA limited to markets not suffering a national disaster. In the US, it was always possible to sustain income to age 100. Globally, this could not always be achieved, with the worst shortfalls explained by either inflation or prolonged periods of depressed asset returns. In the 20th century US, the analyses find a 30/70 allocation to be inferior to a 60/40 allocation when the goal is to sustain inflation-adjusted income for as long as possible; globally and historically, that generalization did not hold. Next, the paper considers whether small changes to the balanced mix, consistent with those seen in Target Date Retirement funds, could improve outcomes; in all the cases examined, small alterations were able to improve the longevity of savings, sometimes dramatically. The paper ends on an optimistic note: two centuries of global market history indicate that exhaustion of tax-sheltered retirement savings before the age of 100 is unlikely to occur for retirees who model their asset mix after those seen in Target Date funds.

Medeiros, M. C., Vasconcelos, G. F. R., Veiga, A., and Zilberman, E. (2021). "Forecasting Inflation in a Data-Rich Environment: The Benefits of Machine Learning Methods." In: *Journal of Business & Economic Statistics* 39, pp. 98–119.

Inflation forecasting is an important but difficult task. Here, we explore advances in machine learning (ML) methods and the availability of new datasets to forecast U.S. inflation. Despite the skepticism in the previous literature, we show that ML models with a large number of covariates are systematically more accurate than the benchmarks. The ML method that deserves more attention is the random forest model, which dominates all other models. Its good performance is due not only to its specific method of variable selection but also the potential nonlinearities between past key macroeconomic variables and inflation. Supplementary materials for this article are available online.

Mendu, H. (2021). "A critical review of 'optimal' annuitization strategies." MA thesis. University of Waterloo.

This paper is an analysis of different self-annuitization strategies advised to a retiree. At the time of retirement, an individual has the choice between annuitizing immediately with their wealth or delaying until a future date while making the most from returns earned through financial markets. This is a similar structure to how a Defined Contribution (DC) pension plan works. Various researchers have assisted retirees on this dilemma of whether or not to annuitize. They designed strategies that work under general market settings and any individual preferences that maximize the income generation. However, these studies assume that individuals are making decisions as if they are maximizing or behaving optimally. In reality, what might be "optimal" in a general sense may not match the optimality defined in a normative sense of giving advice.

Merton, R. C. (2014). "The Crisis in Retirement Planning." In: *Harvard Business Review*.

Corporate America began to really take notice of the looming retirement crisis in the wake of the dot-com crash, when companies in major industries went bankrupt in large part because of their inability to meet their pension obligations. The result was an acceleration of America's shift away from employer-sponsored pension plans toward defined-contribution plans – epitomized by the ubiquitous 401(k) – which transfer the investment risk from the company to the employee.

With that transfer has come a dangerous shift in investment focus, argues Nobel Laureate Robert C. Merton. Traditional pension plans were conceived and managed to provide members with a guaranteed income. And because that objective filtered right through the scheme, members thought of their benefits in those terms. Ask a member what her pension is worth and she'll reply with an income figure: "two-thirds of my final salary," for example. Most DC schemes, however, are designed and managed as investment accounts with the goal of accumulating the largest possible pot of savings. Communication with savers is framed entirely in terms of assets and returns. Ask a saver what his 401(k) is worth and you'll hear a cash amount and perhaps a lament to the value lost in the financial crisis.

The trouble is that investment value and asset volatility are simply the wrong measures if your goal is to secure a particular future income. In this article, Merton explains a liability-driven investment strategy whose aim is to improve the probability of achieving a desired retirement income rather than to maximize the capital value of the savings.

Merton, R. C. and Muralidhar, A. (2020a). "A Six-Component Integrated Approach to Addressing the Retirement Funding Challenge." In: *Journal Of Investment Management* 18(4), pp. 28–54.

This paper offers an integrated approach to addressing the global retirement funding challenge, especially in light of the coronavirus shock that has created an unanticipated and unprecedented impact on lifetime income/consumption. It frames the problem in a six-component approach to the funding challenge with an integrated package presented in a transparent, detailed modular fashion, so that any one module can be replaced with a different version and the rest of the system works. This also means that all six components need not be employed

simultaneously, but can be done in a secular fashion. Finally, it develops and proposes in detail a new financial instrument, SeLFIES (Standard-of-Living indexed, Forward-starting, Income-only Securities)-a single financial innovation that provides greatly improved efficiency of implementation to four of the six components. SeLFIES can help complete financial markets and could be a timely innovation given the coronavirus crisis because they are beneficial to governments that seek long-term, local currency debt financing.

Merton, R. C. and Muralidhar, A. (2020b). "Selfies: A new pension bond and currency for retirement." In: *Journal of Financial Transformation* 51.

There is a looming retirement crisis, as individuals are increasingly being asked to take responsibility for their own retirement planning and a majority of these individuals are financially unsophisticated. They cannot perform basic compounding calculations and do not understand the impact of inflation, both critical aspects of retirement planning. Yet, these individuals are being tasked with the responsibility for three complex, interconnected decisions: how much to save, how to invest (with many additional decisions), and how to decumulate one portfolio at retirement. Compounding these challenges, current financial instruments and products (e.g. T-Bills, TIPs, or Target Date Funds) are risky because they focus on the wrong goal - wealth at retirement, as opposed to how much retirement income can be guaranteed to support pre-retirement standard-of-living. Moreover, annuities are complex, costly, and illiquid and seldom used. Without financial innovation and a change in the metric for measuring retirement success, many individuals will retire poor - a financially and socially undesirable outcome for any country. This paper presents an easy, quick and efficient solution for countries to address all these challenges and improve retirement security by creating and issuing an innovative new bond - SeLFIES (Standard-of-Living indexed, Forward-starting, Income-only Securities). The SeLFIES bond is a single, liquid, low-cost, low-risk instrument, easy-to-understand for even the most financially unsophisticated individual, because it embeds accumulation, decumulation, compounding and inflation-adjustments. SeLFIES is good for governments too, as the bond lowers the risk of individuals retiring poor, improves balance sheet management, and funds infrastructure. The paper also discusses key design aspects of SeLFIES to show how they can ensure longevity risk protection and hedge standard-of-living risk, a key unmanaged risk globally today. Additionally, the paper by concludes by demonstrating the universality of the SeLFIES design as well as by showing how it serves a useful purpose by becoming the currency of retirement.

Meyer, W. and Reichenstein, W. (2013). "Adding Longevity through Tax-Efficient Withdrawal Strategies." In: *The Journal of Wealth Management* 16(1), pp. 57–64.

Suppose a client just retired and has funds in a tax-deferred account like a 401(k), a tax-exempt account like a Roth IRA, and a taxable account. She needs to withdraw sufficient funds to finance her spending plans in retirement. This study explains how, just using the tax code, she can tax-efficiently withdraw funds from her financial portfolio to meet her spending goal while allowing her financial portfolio to last several years longer.

Milevsky, M. A. (2006). *Calculus of Retirement Income: Financial Models for Pension Annuities and Life Insurance*. Cambridge University Press.

This 2006 book introduces and develops the basic actuarial models and underlying pricing of life-contingent pension annuities and life insurance from a unique financial perspective. The ideas and techniques are then applied to the real-world problem of generating sustainable retirement income towards the end of the human life-cycle. The role of lifetime income, longevity insurance, and systematic withdrawal plans are investigated in a parsimonious framework. The underlying technology and terminology of the book are based on continuous-time financial economics by merging analytic laws of mortality with the dynamics of equity markets and interest rates. Nonetheless, the book requires a minimal background in mathematics and emphasizes applications and examples more than proofs and theorems. It can serve as an ideal textbook for an applied course on wealth management and retirement planning in addition to being a reference for quantitatively-inclined financial planners.

Milevsky, M. A. (2020a). "Calibrating Gompertz in reverse: What is your longevity-risk-adjusted global age?" In: *Insurance: Mathematics and Economics* 92, pp. 147–161.

This paper develops a computational framework for inverting Gompertz-Makeham mortality hazard rates, consistent with compensation laws of mortality for heterogeneous populations, to define a longevity-risk-adjusted global (L-RaG) age. To illustrate its salience and possible applications, the paper calibrates and presents L-RaG values using country data from the Human Mortality Database (HMD). Among other things, the author demonstrates that when properly benchmarked, the longevity-risk-adjusted global age of a 55-year-old Swedish male is 48, whereas a 55-year-old Russian male is closer in age to 67. The paper also discusses the connection between the proposed L-RaG age and the related concept of Biological age, from the medical and gerontology literature. Practically speaking, in a world of growing mortality heterogeneity, the L-RaG age could be used

for pension and retirement policy. In the language of behavioral finance and economics, a salient metric that adjusts chronological age for longevity risk might help capture the public attention, educate them about lifetime uncertainty and induce many of them to take action —such as working longer and/or retiring later.

Milevsky, M. A., Huang, H., and Young, V. R. (2015). "A Glide Path for Target Date Fund Annuitization." In: *The Journal of Retirement* 3(1), pp. 27–37.
We describe a recursive algorithm that computes the timing and quantity of purchase of deferred income annuities (DIAs) within target-date funds (TDF) in defined contribution (DC) plans, although the algorithm could also be applied within any retirement account. We map a relatively small number of statistical parameters into a rule that conveys the dollar amount of DIAs to be purchased at any given age and time. Our model is of particular relevance given the recent announcement by the U.S. Treasury Department approving the inclusion of life annuities in 401(k) plans and in TDFs in particular. Note that to qualify as a TDF requires a methodology based on generally accepted investment theories using a consistent investment strategy. This article offers one possible such theory in the context of DIAs.

Milevsky, M. A. and Posner, S. E. (2014). "Can collars reduce retirement sequencing risk? analysis of portfolio longevity extension overlays (leo)." In: *The Journal of Retirement* 1(4), pp. 46–56.
Practitioners are well aware of the pernicious effect of the of investment returns on retirement income sustainability. Poor markets early in the withdrawal phase increase the ruin probability and reduce the longevity of a portfolio. In this article, the authors investigate how and when traded equity options can be used to extend the life of a retiree investments. They label this class of strategies longevity extension overlays (LEOs) and use simulation techniques to analyze the strategy theoretical properties. They also provide evidence on the efficacy of simple LEOs during the 2007-2013 period. Our results are encouraging and offer a justifiable alternative for wealth managers who want to avoid using (more complex and opaque) insurance-product solutions.

Milevsky, M. A., Salisbury, T. S., and Chigodaev, A. (2018). "The implied longevity curve: How long does the market think you are going to live?" In: *arXiv e-Print*.
We use life annuity prices to extract information about human longevity using a framework that links the term structure of mortality and interest rates. We invert the model and perform nonlinear least squares to obtain implied longevity forecasts. Methodologically, we assume a Cox-Ingersoll-Ross (CIR) model for the underlying yield curve, and for mortality, a Gompertz-Makeham (GM) law that varies with the year of annuity purchase. Our main result is that over the last decade markets implied an improvement in longevity of of 6-7 weeks per year for males and 1-3 weeks for females. In the year 2004 market prices implied a 40.1% probability of survival to the age 90 for a 75-year old male (51.2% for a female) annuitant. By the year 2013 the implied survival probability had increased to 46.1% (and 53.1%). The corresponding implied life expectancy has increased (at the age of 75) from 13.09 years for males (15.08 years for females) to 14.28 years (and 15.61 years.) Although these values are implied directly from markets, they are consistent with demographic projections. Similar to implied volatility in option pricing, we believe that our implied survival probabilities (ISP) and implied life expectancy (ILE) are relevant for the financial management of assets post-retirement and very important for the optimal timing and allocation to annuities; procrastinators are swimming against an uncertain but rather strong longevity trend.

Milevsky, M. A. (2018). "Swimming with Wealthy Sharks: Longevity, Volatility and the Value of Risk Pooling." In: *SSRN e-Print*.
Who values life annuities more? Is it the healthy retiree who expects to live long and might become a centenarian, or is the unhealthy retiree with a short life expectancy more likely to appreciate the pooling of longevity risk? What if the unhealthy retiree is pooled with someone who is much healthier and thus forced to pay an implicit loading? To answer these and related questions this paper examines the empirical conditions under which retirees benefit (or may not) from longevity risk pooling by linking the economics of annuity equivalent wealth (AEW) to actuarially models of aging. I focus attention on the Compensation Law of Mortality which implies that individuals with higher relative mortality (e.g. lower income) age more slowly and experience greater longevity uncertainty. Ergo, they place higher utility value on the annuity. The impetus for this research today is the increasing evidence on the growing disparity in longevity expectations between rich and poor.

Milevsky, M. A. (2020b). "Biological (and Other) Ages." In: *Retirement Income Recipes in R*. Springer International Publishing, pp. 259–279.
This chapter examines the implications of mortality heterogeneity, that is the dispersion of longevity prospects within the population. It begins by discussing the extended Gompertz-Makeham model, as well as the compensation law of mortality, linking moments of the remaining lifetime random variable. It then introduces non-chronological measures of age, such as biological age and (especially) longevity risk-adjusted age to illustrate its

dispersion. This chapter illustrates how true age can differ around the world and even within countries, based on wealth and income. The main computational implication of this chapter is that the human longevity random variable Tx, depends on (much) more than just chronological age x. Such heterogeneity must be accounted for in any intelligent drawdown methodology or pensionization scheme.

Milevsky, M. A. (2020c). "Intelligent Drawdown Rates." In: *Retirement Income Recipes in R*. Springer International Publishing, pp. 209–232.
This chapter, which could have been called rational decumulation, introduces dynamic risk-adjusted approaches to spending during retirement. The intelligent drawdown philosophy is contrasted with static approaches, such as the 4% rule and its variants, the focus of prior chapters. The material begins with a light-hearted game that develops an intuition for how longevity uncertainty should affect retirement spending as well as a discussion of the benefits from risk pooling. Moving on to the technical content, after a brief crash course on utility theory, the chapter develops algorithms for (1) computing optimal withdrawal rates based on risk aversion preferences and (2) adjusting ongoing spending based on realized financial and longevity variables. One of the surprising aspects of an intelligent approach to retirement spending is that planning to deplete one's liquid wealth at some advanced age is "rationale" if you have ample pension annuity income.

Milevsky, M. A. (2020d). "Life Annuities: From Immediate to Deferred." In: *Retirement Income Recipes in R*. Springer International Publishing, pp. 181–208.
This chapter develops a methodology for valuing simple cash-flow streams that last a lifetime, which are part of most Defined Benefit (DB) pensions. The focus is on the longevity-contingent building blocks of: (1) immediate, (2) temporary, and (3) deferred income annuities. The chapter begins with a discussion of the value of a longevity-contingent claim and how it differs from the market price versus the manufacturing cost of the product. The algorithms and user-defined R functions are mostly based on the Gompertz law of mortality, although a number of alternative continuous and discrete mortality models are discussed as well. The chapter concludes with a mathematical derivation and implementation of a closed-form expression for the Gompertz Annuity Valuation Model.

Milevsky, M. A. (2020e). "Modeling Human Longevity and Life Tables." In: *Retirement Income Recipes in R*. Springer International Publishing, pp. 111–130.
Up to this point in the book, I have assumed a great fiction, namely that human longevity is known and finite. The success or failure of a retirement income plan was monitored and measured until a finite, e.g. 30 year, horizon. This chapter is the first to focus on the uncertainty or randomness in human longevity versus portfolio longevity. It begins with a detailed description and analysis of (historical) cohort life tables from the Human Mortality Database. The cohort life tables are used to extract population survival and death rates, which are then used to reconstruct life tables. The chapter concludes with a high-level discussion of mortality projections and improvements for future birth cohorts. The main emphasis is on gaining familiarity with the basic atomic structure (qx) of actuarial life-science.

Milevsky, M. A. (2020f). "Modeling the Risk of Sequence-of-Returns." In: *Retirement Income Recipes in R*. Springer International Publishing, pp. 85–109.
This chapter is focused on a phenomenon known by professionals in the retirement income business, as the sequence-of-returns effect. Broadly speaking – and using terminology introduced in the previous chapter – this relates to the disproportionate sensitivity of portfolio longevity to realized investment returns in the early stages of retirement withdrawals. More specifically this chapter proposes some formal metrics that measure the extent and magnitude of the risk using statistical correlation and regression methodologies. The chapter concludes by analyzing some derivative-based strategies, using put and call options, that can be used to mitigate the risk of sequence-of-returns.

Milevsky, M. A. (2020g). *Retirement Income Recipes in R: From Ruin Probabilities to Intelligent Drawdowns*. Springer International Publishing. 302 pp.
This book provides computational tools that readers can use to flourish in the retirement income industry. Each chapter describes recipe-like algorithms and explains how to implement them via simple scripts in the freely available R coding language. Students can use those skills to generate quantitative answers to the most common questions in retirement income planning, as well as to develop a deeper understanding of the finance and economics underlying the field itself. The book will be an excellent asset for experienced students who are interested in advanced wealth management, and specifically within courses that focus on holistic modeling of the retirement income process. The material will also be useful to current and future wealth management professionals within

the financial services industry. Readers should have a solid understanding of financial principles, as well as a rudimentary background in economics and accounting.

Miller, A. K. (2015). "Improving Withdrawal Rates in a Low Yield World." In: *SSRN e-Print*.
Decreased real bond yields substantially increase failure rates for portfolios with an initial 4% withdrawal rate. One way to increase the safe withdrawal rate of a portfolio is to decrease the allocation to bonds and to increase the allocation to stocks. Unfortunately, increasing the allocation to stocks dramatically changes the risk profile of the portfolio. I propose that a solution to this conundrum is to use a low volatility stock portfolio as the primary asset class in the portfolio. A portfolio constructed primarily of low volatility stocks allows an investor to decrease the allocation to lower yielding bonds while still maintaining a similar risk profile to a traditional 50% stock and 50% bond portfolio.

Millossovich, P., Bacinello, A. R., Olivieri, A., and Pitacco, E. (2011). "Variable Annuities: A Unifying Valuation Approach." In: *SSRN e-Print*.
Life annuities and pension products usually involve a number of guarantees, such as minimum accumulation rates, minimum annual payments or a minimum total payout. Packaging different types of guarantees is the feature of so-called variable annuities. Basically, these products are unit-linked investment policies providing a post-retirement income. The guarantees, commonly referred to as GMxBs (namely, Guaranteed Minimum Benefits of type 'x'), include minimum benefits both in case of death and survival. In this paper we propose a unifying framework for the valuation of variable annuities under quite general model assumptions. We compute and compare contract values and fair fee rates under 'static' and 'mixed' valuation approaches, via ordinary and least squares Monte Carlo methods, respectively.

Mislinski, J. (2021). "How to Illustrate Planning Risks to Clients." In: *Advisor Perspectives*.
Clients face three big risks in retirement from the sequence of returns, volatility and asset shortfalls. Michael Hirthler, the founder and chief investment officer at Pennsylvania-based Jacobi Capital Management, explained to me how he uses the Big Picture app to explain those risks to his clients.

Mitchell, J. B. (2009a). "A Mean-Variance Approach to Withdrawal Rate Management: Theory and Simulation." In: *SSRN e-Print*.
Withdrawal rate management of retirement portfolios (decumulation) is explored in a traditional mean-variance context. The paper demonstrates that improvements in the composition of portfolios and in the management of withdrawal rates can both be thought of as increasing the opportunity cost price of annuitizing and reducing the optimal risk of portfolios. Increases in age are shown to have three separate components: decreasing remaining expected lifespan; decreasing variability of remaining lifespan; and increasing variability of returns over remaining holding period. The difficulty of employing a mean-variance approach when a portion of the variation in returns is absorbed by variation in the ending portfolio is noted.
The second portion of the paper demonstrates these principles with simulation analysis. A zero-tolerance (MAXSAFE) approach, as proposed by Bengen, is combined with a multiple-factor withdrawal management strategy, as developed by Stout and Mitchell, to generate a series of risk-return frontiers based on data from 1926-2008.
Improvements in risk-return relationships are found using more restrictive withdrawal rate management approaches than previously reported in the literature. Restricting withdrawal rates when retirees no longer have 1.45 times the present value of their expected withdrawals combined with limitations on withdrawal rate increases allow most retirees to experience average withdrawal rates in excess of 5.5% with average withdrawal rates in excess of 6% and with only a .1% chance of ruin before death.

Mitchell, J. B. (2009b). "Withdrawal Rate Strategies for Retirement Portfolios: Preventive Reductions and Risk Management." In: *SSRN e-Print*.
This paper builds on the work of Stout and Mitchell (2006), Stout (2008), and Blanchett and Frank (2009) by creating a preventive approach to withdrawal management. Proactive strategies, which reduce the withdrawal rate before there are insufficient funds, are shown to significantly reduce the probability of ruin (shortfall) while maintaining the average withdrawal rate. The paper also explores the micro effects of strategy changes by dividing the simulation iterations into groups which have been positively or negatively affected by any particular change, and demonstrates that conventional reporting of the effectiveness of withdrawal rate management techniques can be improved by examining additional moments of the distribution. Data covers 1926-2008 and the mortality table is extended to 108 years.

Mitchell, J. B. (2016). "Migrating with Black Swans: Climate Risk and Retirement Planning." In: *The Journal of Retirement* 4(2), pp. 24–36.

This article combines the thoughts of several authors to describe the migration of retirees through the retirement phase of their lives during a time of climate change. Taleb [2007] introduced the concept of the Black Swan, the (hopefully) infrequent but significant disruption of investment returns. Bernstein [2013] discussed sources of risk in portfolio design and the concept of deep risk, the long-term loss of real capital. Frank, Mitchell, and Blanchett [2012], among others, demonstrated how retirees move through retirement, frequently adjusting to changes in market performance and personal expectations of longevity. Nordhaus [2013] investigated the potential effects of climate change on economic performance and equity returns. The Intergovernmental Panel on Climate Change reports outlined many of the possible underlying climate changes that are likely to drive these risks and necessitate adjustment on the part of retirees. These sources, and others, motivate this article's exploration of climate change, its economic implications, and how retirees might cope as they migrate through their final years. Suggestions are made for how financial planners should prepare themselves and their clients for climate change-related risks.

Mitchell, J. B. and Felton, Z. S. (2015). "Same-Sex Couples and Sustainable Retirement Withdrawal Rates." In: *SSRN e-Print*.

This paper compares sustainable retirement withdrawal rates for same-sex versus heterosexual couples. Due to convergence of expected longevity for males and females at typical retirement ages, systematic differences between same-sex versus heterosexual couples are slight, ranging from .03% at age 50 to less than .4% at age 85. Therefore, same-sex couples can employ conventional withdrawal rate recommendations with, at most, slight modification. Due to a lack of public data regarding mortality differences based on sexual orientation, differences in mortality expectations between individual members of same-sex and heterosexual couples cannot be explored. Members of same-sex couples, like all retirees, should adjust withdrawal rate expectations periodically to reflect individual conditions.

Mitchell, O. S., Clark, R., and Maurer, R. (2018). "How Persistent Low Returns Will Shape Saving and Retirement." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.

Financial market developments over the past decade have undermined what was once thought to be conventional wisdom about saving, investment, and retirement spending. How Persistent Low Returns Will Shape Saving and Retirement explores how the weak capital market performance predicted for the next several years will shape pension saving, investment, and decumulation plans. Academics, policymakers, and industry leaders debate alternative strategies to cope with these challenges globally, as economic growth remains slow and low returns become the 'new normal.' This volume includes contributions from plan sponsors, benefit specialists, actuaries, academics, regulators, and others working to design resilient pensions for the next decades. Together, they identify several new tools for retirement savers and pension managers.

Mitchell, O. S. and Utkus, S. P. (2021). "Target-date funds and portfolio choice in 401(k) plans." In: *Journal of Pension Economics and Finance*, pp. 1–18.

Target-date funds in corporate retirement plans grew from $5 billion in 2000 to $734 billion in 2018, partly because federal regulation sanctioned these as default investments in automatic enrollment plans. We show that adopters delegated pension investment decisions to fund managers selected by plan sponsors. Inclusion of these funds in retirement saving menus raised equity shares, boosted bond exposures, curtailed cash/company stock holdings, and reduced idiosyncratic risk. The adoption of low-cost target-date funds may enhance retirement wealth by as much as 50% over a 30-year horizon.

Mitra, G. and Medova, E. (2010). "Asset and liability management/liability-driven investment for pension funds." In: *Journal of Asset Management* 11(2-3), pp. 71–72.

The demographics of the global community of working people are moving in a clear direction, essentially we are living longer. The requirement of financial care through pensions for this ageing population is becoming an important problem not only in the advanced economies but also in the emerging economies. This special issue brings together a collection of papers by a number of experts who have studied different facets of the pension problem. In recent years the pension fund industry has adopted tailor-made asset and liability management (ALM) strategies, also called liability-driven investment (LDI). The focus of the special issue is on quantitative methods for ALM/LDI for pensions funds. The aim of LDI strategies is to match and outperform a pension fund's liability stream and, at the same time, take into account country-specific regulations. The decision models as well as simulation/evaluation models which take into consideration stochastic asset price dynamics and stochastic behaviour of the liabilities are covered. Inflation risk, interest rate risk, contribution risk of the pension plan's sponsor and no doubt the longevity risk of its members, are examples of additional risk some or all of which are measured and managed by these models. The papers in this special issue address a number of inter-related themes

including pricing of assets, models for ex ante financial decision making and strategies of asset allocation including overlays. Schwaiger, Lucas and Mitra have a set of four alternative stochastic programming models addressing the question of optimum fund allocation in the face of future uncertainties. Their models include PV01 sensitivities and trade-off between cash input required and PV01 mismatch. Iyengar and Ma have applied robust optimisation for the asset allocation model where the uncertainty set is ellipsoidal. The authors formulate their model with second order cone programming constraint and report that the corresponding model is computationally tractable. Yang, Gondzio and Grothey also investigate the issue of computational tractability of an asset allocation model, which is based on second-order stochastic dominance for the purpose of risk control. Their computational study underlines how exploitation of structure and use of interior point optimisation can lead to efficient solution of the computational decision models. Asset pricing is the focus of the paper by Dempster, Medova and Villaverde. The authors present a three-factor term structure interest rate model for consol bond valuation. They discuss a detailed computational study and simulation results and present a detailed analysis of the yield curves. Given that almost all pension funds include fixed income assets in one form or other, this paper clearly makes a very important contribution to the valuation of this class of assets. Mulvey, Kim and Ma present a paper in which they first highlight the present situation that large corporate and public funds have become underfunded and thereby mismatched during the recent prolonged recessionary period. Their contribution is in proposing a strategy of incorporating a duration enhancing overlay to the pension fund assets, thereby controlling the underlying risk. Berkelaar and Kouwenberg address the question of asset allocation from an extremely novel perspective. They state that the goal of the investment policy should be to maximise expected excess returns over liabilities subject to an acceptable level of risk expressed relative to liabilities. Their asset allocation is based on drawdown risk optimisation and through simulation studies they show that their strategy leads to better downside protection. Ferstl and Weissensteiner study short-term treasury management strategies. They develop a multistage stochastic programming model, which in turns uses a scenario generator with no arbitrage interest rate model with market price estimation and change of measure. Finally, they carry out extensive back testing to validate their model. Overall, we believe we have here an exciting collection of research papers which truly reflect the state of the art. We would like to take this opportunity to thank the editor for suggesting this special issue; grateful thanks are also due to Michael Dempster and Katharina Schwaiger for assistance in preparing it.

Mladina, P. (2014). "Dynamic Asset Allocation with Horizon Risk: Revisiting Glide Path Construction." In: *The Journal of Wealth Management* 16(4), pp. 18–26.

We compare the empirical distributions of equity and fixed-income returns at different time horizons and find that the risk of equities relative to fixed income is more acute at short time horizons than long time horizons, confirming previous research. This creates the opportunity to develop a dynamic asset allocation process that exploits the reduced horizon risk of equities relative to fixed income. We highlight key data on changing relative risk with time and leverage this information to introduce methods and concepts that inform glide path construction-the building blocks for a dynamic asset allocation process that can support lifecycle, target date retirement, and goals-based investing frameworks.

Mladina, P. (2016). "Optimal Lifetime Asset Allocation with Goals-Based, Lifecycle Glide Paths." In: *The Journal of Wealth Management* 19(1), pp. 10–22.

Assets should serve a purpose: to fund a lifetime of financial goals. If assets serve the purpose of funding lifetime goals, it naturally follows that optimal lifetime asset allocation should be goals-based and multi-period, requiring customization according to goals, human capital, and risk preference. Ideally, risk preference is defined intuitively for private investors. We present a dynamic asset allocation method based on an intertemporal capital asset pricing model that incorporates these features to produce a goals-based lifecycle glide path – an asset allocation roadmap that optimally funds lifetime consumption goals while adapting to evolving conditions.

Mladina, P. (2020). "Refining After-Tax Return and Risk Parameters." In: *The Journal of Wealth Management* 23(2), pp. 8–17.

Taxes introduce certain complexities, requiring proper adjustments to return and risk parameters. The author offers a refined set of after-tax return and risk equations for use in practice and validates them with a stochastic future value cash flow model. The refined after-tax return and risk parameters can be used in portfolio optimization, Monte Carlo simulation, and deterministic present/future value portfolio modeling with internally consistent results. The refinements improve the discovery of the optimal after-tax portfolio and enhance long-term wealth planning in the presence of risk.

Mladina, P. and Grant, C. (2019). "Glide Paths Based on a Retirement Goal and Depleting Human Capital." In: *The Journal of Investing* 28(1), pp. 8–16.

The retirement goals of many Americans are underfunded. The problem is compounded by the complexity of self-managing distribution portfolios, particularly as DC plans replace DB plans. We believe most retirement glide paths are satisfactory but suboptimal solutions. We introduce a glide path of financial assets over the life cycle based on a retirement goal and depleting human capital. The method is anchored to the foundational principles of intertemporal portfolio theory while borrowing heavily from goals-based asset allocation. The result is a dynamic asset allocation over the life cycle that is a function of critical input variables relevant to retirement planning such as retirement savings, retirement consumption and risk aversion. The glide path can be customized to individuals, or semi-customized to discrete subpopulations of DC plan participants.

Moehle, N., Kochenderfer, M. J., Boyd, S., and Ang, A. (2021). "Tax-Aware Portfolio Construction via Convex Optimization." In: *arXiv e-Print*.

We describe an optimization-based tax-aware portfolio construction method that adds tax liability to standard Markowitz-based portfolio construction. Our method produces a trade list that specifies the number of shares to buy of each asset and the number of shares to sell from each tax lot held. To avoid wash sales (in which some realized capital losses are disallowed), we assume that we trade monthly, and cannot simultaneously buy and sell the same asset. The tax-aware portfolio construction problem is not convex, but it becomes convex when we specify, for each asset, whether we buy or sell it. It can be solved using standard mixed-integer convex optimization methods at the cost of very long solve times for some problem instances. We present a custom convex relaxation of the problem that borrows curvature from the risk model. This relaxation can provide a good approximation of the true tax liability, while greatly enhancing computational tractability. This method requires the solution of only two convex optimization problems: the first determines whether we buy or sell each asset, and the second generates the final trade list. In our numerical experiments, our method almost always solves the nonconvex problem to optimality, and when it does not, it produces a trade list very close to optimal. Backtests show that the performance of our method is indistinguishable from that obtained using a globally optimal solution, but with significantly reduced computational effort.

Moenig, T. (2021). "It's RILA Time: An Introduction to Registered Index-Linked Annuities." In: *SSRN e-Print*.

Registered index-linked annuities (RILAs) are increasingly popular equity-based retirement savings products offered by U.S. life insurance companies. They combine features of fixed-index annuities and traditional variable annuities (TVAs), offering investors equity exposure with downside protection in a tax-deferred setting. This article introduces RILAs to the academic literature by describing the products' key features, developing a general pricing model, and deriving the providers' hedging strategy by decomposing their liabilities into short-term European options.

Numerical illustrations show that RILAs offer investors similar risk profiles (in the long run) as TVAs with maturity guarantees, and that many products currently sold appear to be priced quite favorably for investors. For providers, RILAs may be a preferable alternative or complement to TVAs as they greatly simplify the management of the embedded equity risk and can naturally reduce the TVA capital requirements. These features position RILAs as a viable long-term solution for this product space.

Momen, O., Esfahanipour, A., and Seifi, A. (2020). "A robust behavioral portfolio selection: model with investor attitudes and biases." In: *Operational Research* 20(1), pp. 427–446.

This study develops a behavioral portfolio selection model that uses a robust estimator for expected returns in order to produce portfolios with less need to change over consecutive periods. We also consider investor attitudes toward risk through spectral risk measure as well as investor expectations on future returns by means of the Black-Litterman model, and finally, our model includes a varying risk aversion depending on investor behavioral biases and his latest realized return. In order to evaluate the proposed model and make comparisons possible, we conducted a survey on investor biases and attitudes along with market data of Tehran Stock Exchange. The results reveal that although our model is not mean-variance efficient, it recommends portfolios that are robust, well diversified, and have less utility loss compared to a famous behavioral portfolio model.

Mooney, T., Rapaka, R., and Vera, T. (2020). "Dynamic Regime Strategy for Stress Testing and Optimizing Institutional Investor Portfolios." In: *SSRN e-Print*.

Our work aims to develop a stand-alone trading system to construct portfolios that show the benefits of value and momentum style integration and presents the effectiveness of alternative integration methods for long-only absolute return funds, which seeks absolute returns that are not highly correlated to traditional assets such as stocks and bonds. Our approach uses the CRoss Industry Standard Process for Data Mining (CRISP-DM) model to guide the necessary steps, processes, and workflows for executing our project.

Mulvey, J. M. and Hao, H. (2020). "Setting Realistic Goals for Personal Retirement Planning via Micro-Macro Analyses." In: *The Journal of Retirement* 8(2), pp. 23–38.
The vulnerability of individuals planning for retirement has been growing as a result of the conversion from defined benefit plans to defined contribution plans, the steady increase in life longevity, and the uncertainty of asset returns under an ever-changing global environment. A serious problem is the lack of appropriate planning for retirement. How much should an individual in the United States save beyond the Social Security tax to maintain a reasonable lifestyle after retirement? The article designs a framework to facilitate the process of setting realistic goals for retirement planning, featuring the concept of agent-based simulations. Focusing on policy-rule-based investment strategies, the simulation framework includes multiple investable asset categories and explores dynamic allocation based on the investor's age, current salary, and Social Security accumulation situation. Empirical results demonstrate a stylized application of the planning framework.

Munnell, A. H., Sanzenbacher, G., Webb, A., and Gillis, C. M. (2016). "Are Early Claimers Making a Mistake?" In: *SSRN e-Print*.
Using Health and Retirement Study (HRS) data and Latent Class Analysis for three cohorts (those born in 1931-1936, 1937-1941, and 1942-1947), this paper explores: 1) who claims Social Security benefits at age 62; 2) what percentage of households claiming at 62 are unprepared for retirement; and 3) whether the unprepared early claimers were pushed into claiming through job shocks and/or poor health or simply decided to take benefits early. Looking across three cohorts makes it possible to see whether these patterns have changed as the average claim age has increased and pension coverage has shifted away from defined benefit (DB) plans. That is, have those who have moved out of age-62 claiming been educated, financially prepared households or unprepared households that have recognized the need to delay claiming?
The paper found that:

- Consistent with previous research, the HRS shows a decline in those claiming at 62.
- Age-62 claimers are less well off than "postponers" in some ways and better off in others.
- Latent class analysis shows that this mixed picture reflects the average of: 1) those with little education and poor job prospects (disadvantaged); and 2) those with at least some college and sufficient resources to claim early (advantaged).
- The percentage of the age-62 claimers in each of these groups has remained virtually constant over the three cohorts.
- Comparing the calculated household replacement rates with target rates from previous research shows that, overall, roughly 65 percent of households claiming at 62 are not prepared; the rate for the disadvantaged group is twice the rate of the advantaged group.
- The percentage unprepared at 62 has increased over time, reflecting an overall trend toward less preparedness.
- A simple probit regression suggests that health and employment shocks and the absence of a DB pension are related to the lack of preparedness for both the disadvantaged and advantaged.

The policy implications of the findings are:

- Given the increasing trend in unpreparedness, further cuts to Social Security benefits would exacerbate this problem.
- Workers claiming at 62 with DB plans were especially likely to be prepared; these plans are not coming back, so the challenge is whether the 401(k) system can be enhanced.

Srtudy whether Early Claimers Making a Mistake.

Munnell, A. H. and Webb, A. (2021). "The Impact of Leakages from 401(k)s and IRAs." In: *The Journal of Retirement* 9(3), pp. 32–54.
This article summarizes what is known about leakages from existing studies and relates these results to detailed data on leakages in 2013 provided by Vanguard's How America Saves. It then uses two data sets - the Survey of Consumer Finances and the Survey of Income and Program Participation - to estimate the impact of leakages on wealth at retirement. The Vanguard data are a critical component because they provide a comprehensive picture of assets and participant flows, whereas the surveys on which earlier studies were based tended to focus on one component, such as loans. A key limitation is that Vanguard's population is probably wealthier than the general population.

Munnell, A. H., Wettstein, G., and Hou, W. (2019). "How Best to Annuitize Defined Contribution Assets?" In: *SSRN e-Print*.

Unlike defined benefit pensions that provide participants with steady benefits for as long as they live, 401(k) plans and Individual Retirement Accounts (IRAs) provide little guidance on how to turn accumulated assets into income. As a result, retirees have to decide how much to withdraw each year and face the risk of either spending too quickly and outliving their resources or spending too conservatively and consuming too little. Surveys of individuals' plans and several recent studies suggest that people will not draw down their accumulations for fear that they will exhaust their money and be unable to cover end-of-life health care costs. They also must consider how to invest their savings after retirement. These are difficult decisions.

Better strategies are possible that will ensure a higher level of lifetime income, reduce the likelihood that people will outlive their resources, and alleviate some of the anxiety associated with post-retirement investing. Workers could use a portion of their 401(k) and IRA assets to purchase an immediate annuity that pays a fixed amount throughout their lives, typically starting at age 65. Or they could purchase an advanced life deferred annuity (ALDA) that requires a smaller share of accumulated assets and begins payments at a later age like 85. Alternatively, they could use their assets to delay claiming Social Security - essentially purchasing an inflation-indexed annuity. Right now, none of these three options is commonly used. Very few workers choose to purchase immediate or deferred annuities (the first two options). And few retirees appear to be deferring claiming in order to receive the maximum annuity income from Social Security - most people simply retire earlier and claim immediately.

Increasing annuitization in a meaningful way would require embedding annuities in 401(k) plans, with annuitization as the default. Recent proposed federal legislation, such as the SECURE Act (Setting Every Community Up for Retirement Enhancement), encourages plan sponsors to offer annuities in their plans by establishing a fiduciary safe harbor when specific statutory conditions are followed in selecting an insurance company. This legislation does not address, however, the question of defaults or the possibility of using 401(k) assets to purchase additional Social Security benefits. Moving forward on these fronts would require some consensus about the appropriate share of 401(k) assets to be annuitized and the best method for annuitizing them.

To address these issues, this paper compares the level of lifetime utility generated by alternative annuitization approaches - immediate annuities, deferred annuities, and additional Social Security through delayed claiming. The analysis also tests different assumptions for the share of initial wealth that participants use to purchase these products.

Muralidhar, A., Shin, S. H., and Ohashi, K. (2016). "The Most Basic Missing Instrument in Financial Markets: The Case for Forward Starting Bonds." In: *The Journal of Investment Consulting* 17(2).

There is a looming retirement crisis globally with the three pillars of retirement threatened because of insufficient funding, improper investment decisions, and transferring risk to individuals who are least capable of bearing such risk. This paper argues that the introduction of a unique financial instrument, basically an inflation linked bond which pays coupons when you need it, might help ameliorate this crisis.

The Life Cycle Hypothesis (LCH) demonstrated why people save; namely, they try to set aside resources during their working lives, to be able to tap into them to ensure retirement income when labor income stops. Modern Portfolio Theory (MPT) attempted to help individuals make optimal investment decisions on these savings by investing in stocks, bonds and other assets to ensure sufficient retirement wealth. There are three problems with using MPT approaches for retirement planning: (a) MPT ignores the uses of funds; (b) focuses on wealth as opposed to retirement income maximization; and (b) none of these assets is an ideal hedge for a desired retirement income. As a result, seemingly safe MPT assets are risky from a retirement income perspective (because the Capital Asset Pricing Model (CAPM) is a specific case of a more general Relative Asset Pricing Model (RAPM)).

The need for our bond is simple: the riskless retirement asset is an inflation-indexed, coupon-only bond that defers payment until retirement and pays till death. All attempts to recreate this profile through traditional stocks and bonds, or purchase such a profile through annuities are sub-optimal, risky, complex or expensive thereby threatening retirement security. The paper goes further to demonstrate that there is a potentially willing supplier of such bonds, and that this bond offers the perfect offsetting cash flow profile to infrastructure projects, thereby completing the market. It also addresses challenges, issues and opportunities surrounding such an instrument and examines issues relating to the creation of a market for FSBs.

Murguia, A. and Pfau, W. D. (2021a). "A Model Approach to Selecting a Personalized Retirement Income Strategy." In: *SSRN e-Print*.

This study identifies and validates a set of scorable retirement income factors to define preferences for an overall retirement income style. This investigation further attempts to create a workable model for retirement income planning by showing how the factors connect to four main retirement income strategies: systematic withdrawals with total return investing, risk wrap with deferred annuities, protected income with immediate annuities, and time segmentation or bucketing. Approaching retirement income agnostically and matching retirement income strategies based on an individual's personal retirement income style may lead to improved outcomes that achieve greater "buy in" and comfort.

Murguia, A. and Pfau, W. D. (2021b). "Retirement Income Beliefs and Financial Advice Seeking Behaviors." In: *SSRN e-Print*.

This investigation identifies and validates a series of salient behavioral finance and psychological constructs that influence retirement income planning. We show how these scales are related to each other as well as retirement income concerns and investment behaviors. We also describe how four investment personas can be linked with the Advisor Usefulness and Retirement Income Self-Efficacy scales to successfully identify preferred financial implementation methods. This can assist individuals in more readily recognizing their relative strengths and weaknesses when implementing a retirement income strategy, and financial professionals can present advice in a manner that addresses a client's concerns and preferred implementation.

Murphy, R. O., Lamas, S., and Sin, R. (2020). "Identifying What Investors Value in a Financial Adviser: Uncovering Opportunities and Pitfalls." In: *Journal of Financial Planning* 33(7), pp. 44–52.

Understanding what investors value and look for in a financial adviser is critical to both the client's and adviser's success. This research identified the alignment and discordance between what non-retired investors value, and what financial advisers think their clients value. These insights show opportunities for advisers to better articulate their value and, in doing so, improve attraction, retention, and promote client satisfaction. Using an online platform, investors were asked to rank a set of 15 adviser attributes from most important to least important. Advisers were asked to rank the same attributes, predicting what they thought investors valued. Results show that both groups recognized that reaching financial goals is a top priority, with notable disagreements on how to achieve this. Investors underestimated the importance of behavioral coaching in helping them stay on course. Advisers underestimated the importance of tax-efficient strategies to their clients. A follow-up study tested different ways of describing behavioral coaching to potentially find better wording that more naturally resonates with investors, and results suggest that simple changes in phrasing and communication can improve how investors perceive the value behaviorally attuned advisers bring to the relationship.

Neville, H., Draaisma, T., Funnell, B., Harvey, C. R., and Hemert, O. van (2021). "The Best Strategies for Inflationary Times." In: *SSRN e-Print*.

Over the past three decades, a sustained surge in inflation has been absent in developed markets. As a result, investors face the challenge of having limited experience and no recent data to guide the repositioning of their portfolios in the face of heighted inflation risk. We provide some insight by analyzing both passive and active strategies across a variety of asset classes for the U.S., U.K., and Japan over the past 95 years. Unexpected inflation is bad news for traditional assets, such as bonds and equities, with local inflation having the greatest effect. Commodities have positive returns during inflation surges but there is considerable variation within the commodity complex. Among the dynamic strategies, we find that trend-following provides the most reliable protection during important inflation shocks. Active equity factor strategies also provide some degree of hedging ability. We also provide analysis of alternative asset classes such as fine art and discuss the economic rationale for including cryptocurrencies as part of a strategy to protect against inflation.

Ngai, A. and Sherris, M. (2011). "Longevity risk management for life and variable annuities: The effectiveness of static hedging using longevity bonds and derivatives." In: *Insurance: Mathematics and Economics* 49(1), pp. 100–114.

For many years, the longevity risk of individuals has been underestimated, as survival probabilities have improved across the developed world. The uncertainty and volatility of future longevity has posed significant risk issues for both individuals and product providers of annuities and pensions. This paper investigates the effectiveness of static hedging strategies for longevity risk management using longevity bonds and derivatives (qq-forwards) for the retail products: life annuity, deferred life annuity, indexed life annuity, and variable annuity with guaranteed lifetime benefits. Improved market and mortality models are developed for the underlying risks in annuities. The market model is a regime-switching vector error correction model for GDP, inflation, interest rates, and share prices. The mortality model is a discrete-time logit model for mortality rates with age dependence. Models were estimated using Australian data. The basis risk between annuitant portfolios and population mortality was

based on UK experience. Results show that static hedging using qq-forwards or longevity bonds reduces the longevity risk substantially for life annuities, but significantly less for deferred annuities. For inflation-indexed annuities, static hedging of longevity is less effective because of the inflation risk. Variable annuities provide limited longevity protection compared to life annuities and indexed annuities, and as a result longevity risk hedging adds little value for these products.

Nicolas, F. (2019). *Investment Strategies for Retirement*. World Scientific. 416 pp.
The issue of pension financing is evolving everywhere, becoming more of a corporate or individual matter rather than a state one. Demographic changes are making sharing mechanisms hard to control, and social deficits often lead governments to pull back from their obligations. This raises many questions for the individual: 1) At what age should I start saving? When should I increase or reduce my savings? 2) How do I secure my income? 3) What products should I choose to supplement my pension and savings? 4) What degree of risk should I take on? Despite the burden for securing one's retirement increasingly placed on individuals, many are often badly prepared to tackle this very long savings process, which is often complicated by the specific characteristics of a pension plan. This publication, intended for investment professionals, customer advisors, and individuals interested in personal finance and asset management, looks at some of the fundamental elements of investment strategies and techniques for retirement.

Novotny, L. E. and Mansson, A. (2020). "Finding the Optimal Withdrawal Rate on a Retirement Portfolio." MA thesis. Copenhagen Business School.
This thesis seeks to illuminate how investors may withdraw optimally from their retirement portfolio, considering personal attributes, to maximize their lifetime utility. It thereby challenges the popular existing research conducted by Cooley et al. (1999)which investigates a sustainable withdrawal rate, mainly by using the overlapping period method. The main finding of the thesis is, that the personal attributes of the investors heavily impacts their recommended payment plans, suggesting that the generic sustainable withdrawal rate presented by Cooley et al. (1999) hardly can be used as a basis for planning retirement. To do this, this study specifies a utility framework, which is being used as a tool for assessing investor utility by given payout plans, considering a series of personal traits, mainly including subjective discount rate, constant relative risk aversion, time horizon, habit consumption and the loss aversion of the investor. The thesis then assesses optimal investor payout plans with an exogenously given rate of return, thus excluding risk. Subsequently the study introduces uncertainty on the investment assets, making the optimization problem more complex. To assess risk, the thesis considers two assets with their simulated returns being provided by Monte Carlo simulation and bootstrapping simulation, over a 30-year time horizon. These are then indexed and used for providing optimal payment plans considering lower returns given by the utilization of tail risk measures such as Value at Risk. The thesis finally attempts to show the diversity in optimal withdrawals by applying the utility framework to the fictious case examples, and by using the data from the simulations. Despite only being a fictious case, the stereotypical investors each represent qualities which are relatable to real life investors.

Owadally, I., Jang, C., and Clare, A. (2021a). "Optimal investment for a retirement plan with deferred annuities." In: *Insurance: Mathematics and Economics* 98, pp. 51–62.
We construct an optimal investment portfolio model with deferred annuities for an individual investor saving in a retirement plan. The objective function consists of power utility in terms of consumption of all secured retirement income from the deferred annuity purchases, as well as bequest from remaining wealth invested in equity, bond, and cash funds. The asset universe is governed by a vector autoregressive model incorporating the Nelson-Siegel term structure and equity returns. We use multi-stage stochastic programming to solve the optimization problem numerically. Deferred annuity purchases are made continuously over the working lifetime of the investor, increasing particularly in the years before retirement. The investment strategy hedges price changes in deferred annuities, and bond holding and deferred annuity purchases increase when interest rates are high. Optimal investment and deferred annuity choices depend on realized and expected values of state variables. The optimal strategy is also compared with typical retirement plan strategies such as glide paths. Our results provide support for deferred annuities as a major source of retirement income.

Owadally, I., Mwizere, J.-R., Kalidas, N., Murugesu, K., and Kashif, M. (2021b). "Long-Term Sustainable Investment for Retirement." In: *Sustainability* 13(9), p. 5000.
We consider whether sustainable investment can deliver performance comparable to conventional investment in investors' long-term retirement plans. On the capital markets, sustainable investment can be achieved through various instruments and strategies, one of them being investment in mutual funds that subscribe to ESG (environmental, social, and governance) principles. First, we compare the investment performance of ESG funds with

matched conventional funds over the period 1994-2020, in Europe and the U.S. We find no significant evidence of differing performance (at 5% level) despite using a number of investment performance metrics. Second, we perform a historical backtest to model a UK personal retirement plan from 2000 till 2020, taking full account of investment management fees and transaction costs. We find that investing in an index-tracker fund overlaid with ESG screening delivers a pension which is 10.4% larger than is achieved if the index-tracker fund is used without screening. This is also 20.2% larger than is achieved by investing in a collection of actively managed funds with a sustainable purpose. We conclude that an ESG-screened long-term passive investment approach for retirement plans is likely to be successful in satisfying the twin objectives of a secure retirement income and of sustainability.

Pae, Y. and Atra, R. (2020). "Rules of Thumb versus Industry Glide Paths; Some Bootstrapping Evidence." In: *The Journal of Investing* 29(3), pp. 23–27.
We compare the performance of retirement portfolios using the average glide path of five popular target date funds to general rules of thumb for asset allocation. Surprisingly, the industry average target date fund has similar return and risk as the "120 minus your age rule". In addition, a simple "140 minus your age rule" produces greater expected savings at retirement and a lower failure rate for average US investors retiring in their early 60s. A naive approach such as the "120 minus your age" rule or the "140 minus your age" can benefit average US employees by reducing transaction costs, improving retirement balances and increasing the probability of a comfortable retirement through an easy-to-understand investing rule.

Pakula, D. (2020). *Guiding your clients through stormy weather: Sustainable withdrawal rates in times of crisis*. Tech. rep. Vanguard.
Many investors are concerned that a severe market crisis can dramatically affect their portfolio wealth and spending needs.
This research note uses simulated market scenarios to explore the interaction among market crises, expected returns, and sustainable withdrawal rates.
This analysis yields two insights. First, the inverse relationship between market downturns and expected returns may limit, but not eliminate, a downturn's impact on sustainable spending. And second, modest spending adjustments in response to a downturn can preserve much of a portfolio's long-term spending power.

Paradise, T. and Kahler, J. R. (2020). *What to do with your next dollar: A quantitative framework*. Tech. rep. Vanguard.
Our financial lives can quickly become complex. Various tools and account types are available to help us save for an uncertain future, including employer plans, IRAs, 529s, and HSAs. But in addition to managing their assets, most households are also juggling various liabilities, such as a mortgage or student loans. How should investors prioritize cash flows on the household balance sheet? Should they invest more in a 401(k) or focus on paying off their mortgage early? The goal is to maximize long-term after-tax net worth-assets minus debts-while also meeting shorter-term goals and spending needs. This paper introduces a framework for evaluating the best use of your next discretionary dollar. Key to success is smart management of both sides of the balance sheet-your assets as well as your liabilities. Our framework has three components 1) Understanding your investment options: It is important to understand the costs, tax benefits, and rules of the investment opportunities available to you. We quantify the relative return potential of several types of accounts, showing the benefits of prioritizing tax-advantaged vehicles. 2) Debt: the other side of the balance sheet What are the relative benefits and constraints of prioritizing debt prepayment over further investment? We show that prioritizing the prepayment of high-interest debt can lead to better outcomes and explore the behavioral, liquidity, and risk tolerance considerations involved in the trade-off. 3) Balancing wealth maximization with other financial objectives Finally, we analyze the cash allocation decision and suggest the following steps. First, prioritize matched savings and high-cost debt. Next, make sure you have adequate emergency savings. Last, select the highest-returning opportunities appropriate for your goals.

Park, H. (2022). "Influence of risk tolerance on long-term investments: a Malliavin calculus approach." In: *Stochastics*, pp. 1–32.
This study investigates the influence of risk tolerance on the expected utility in the long run. We estimate the extent to which the expected utility of optimal portfolios is affected by small changes in the risk tolerance. For this purpose, we adopt the Malliavin calculus method and the Hansen-Scheinkman decomposition, through which the expected utility is expressed in terms of the eigenvalues and eigenfunctions of an operator. We conclude that the influence of risk aversion on the expected utility is determined by these eigenvalues and eigenfunctions in the long run.

Parker, F. J. (2016a). "Goal-Based Portfolio Optimization." In: *The Journal of Wealth Management* 19(3), pp. 22–30.
The article presents a goal-based portfolio optimization approach that is truly native to the goal-based environment. It begins by redefining risk as the probability of failing to attain a specified goal and redefining reward as the excess wealth over and above what is required by the goal. It then presents an optimization procedure that seeks to minimize goal failure and maximize excess return. In preliminary tests, it finds that this goal-based procedure lowers the probability of failing to achieve a specified goal while delivering higher excess wealth than the procedures currently available.

Parker, F. J. (2016b). "Portfolio Selection in a Goal-Based Setting." In: *The Journal of Wealth Management* 19(2), pp. 41–46.
Using different portfolio ratios, we illustrate the deficiencies of using only modern portfolio theory (MPT) metrics and assumptions when selecting portfolios in a goal-based setting. Through the lenses of the ratios, we show how MPT can choose the wrong, albeit efficient, portfolio to accomplish a goal. These facts illustrate the need for goal-based practitioners to factor in other variables, such as time to a goal and maximum loss thresholds, when managing a portfolio to a goal-oriented mandate.

Parker, F. J. (2020). "Allocation of wealth both within and across goals: a practitioner guide." In: *The Journal of Wealth Management* 23(1), pp. 8–21.
Although the goals-based investment literature has grown, there remain two unsolved problems. First, there is no cohesive theory for the allocation of wealth across goals. If, for example, a client wants to retire in thirty years, send a child to university in eight years, and buy a vacation home in four, how she should divvy her wealth across those goals has been an open question. Restating the same problem: The vesting of shorter-dated goals carries a loss of achievement probability for longer-dated ones. How much probability loss is acceptable? Second, mean-variance portfolios yield lower probabilities of goal achievement than goals-based portfolios. I demonstrate use of the goals-based method. Parker (2020) introduced a cohesive goals-based allocation model that solves these problems. The approach, however, carries some practical challenges that are addressed in this discussion. Finally, I discuss some possible implications of the approach on the structure of firms, the regulatory environment, and the industry as a whole.

Parker, F. J. (2021a). "Achieving Goals While Making an Impact: Balancing Financial Goals with Impact Investing." In: *The Journal of Impact and ESG Investing* 1(3), pp. 27–38.
For investors who wish to engage in impact investing and who have specific goals to achieve, there exists the potential for a trade-off. When impact investments yield lower returns than nonimpact portfolios, how much return should an investor be willing to give up to incorporate it? Using recent advances in goals-based utility theory, this article explores an answer to that question and offers practical and concrete advice for advisors to individual investors and fiduciaries of trusts. Using the goals-based framework, the author shows how an investor's willingness to sacrifice return for an impact investing mandate changes in response to market and portfolio conditions.

Parker, F. J. (2021b). "Multi-Period Goals-Based Portfolio Optimization." In: *SSRN e-Print*.
Portfolio managers regularly have views on capital markets that span multiple periods. A portfolio manager, for example, may expect a recession with high probability over the coming period, followed by a recovery in the subsequent period. Currently, there are few methods for optimal portfolio allocation across these multiple periods that match how practitioners operate in the real world. Herein, we present a multi-period optimization method using a goals-based framework that allows practitioners to develop multi-period capital market expectations and optimize their portfolios across those periods.

Peijnenburg, K., Nijman, T., and Werker, B. J. M. (2017). "Health Cost Risk: A Potential Solution To the Annuity Puzzle." In: *The Economic Journal* 127(603), pp. 1598–1625.
We find that health cost risk lowers optimal annuity demand at retirement. If medical expenses can be sizeable early in retirement, full annuitisation at retirement is no longer optimal because agents do not have enough time to build a liquid wealth buffer. Furthermore, large deviations from optimal annuitisation levels lead to small utility differences. Our results suggest that health cost risk can explain a large proportion of empirically observed annuity choices. Finally, allowing additional annuitisation after retirement results in welfare gains of at most 2.5% when facing health cost risk, and negligible gains without this risk.

Pellerin, M. (2021). "Investing for Retirement Income: A Comparison of Asset Allocations and Spending Strategies." In: *SSRN e-Print*.

We study the performance of different investment and spending strategies for retirement. Investment strategies include wealth-focused glide paths that combine equities with short-term, high-quality fixed income. We also consider an income-focused glide path that combines a moderate equity allocation at retirement and an inflation-protected bond portfolio that uses liability-driven investing. Spending rules include fixed spending (similar to the 4% rule), flexible spending, as well as nominal and real annuitization. We examine simulated lifetimes with either stochastic longevity or fixed longevity of 30 years in retirement.

We find that, for all spending strategies, an income-focused asset allocation delivers similar retirement income to the wealth-focused allocations we study while offering better protection against market, interest rate, and inflation risk. We also find that a glide path with an LDI portfolio offers a better tradeoff between income growth and income risk management. Finally, our results suggest that high equity exposure in retirement is an inadequate tool to manage longevity risk.

Pepperell, R., Greenwood, D., and Alsharman, M. (2020). "Death or Bust? The Risk with Post-Retirement Income Models." In: *SSRN e-Print*.

This paper reviews the key models used to forecast and evaluate sustainable withdrawal rates from pension portfolios and enumerates their key risks so that financial advisors are better able to judge model limitations and thus the risks associated with the recommendations they are making.

We focus on the model risk in isolation to determine the single model class or group of models best suited to retirement income planning from the wider set of candidates. We also look at the implementation issues surrounding these choices. We conclude by discussing the industry and policy making implications with reference to the UK.

Pfau, W. (2021). *Retirement Planning Guidebook: Navigating the Important Decisions for Retirement Success*. Retirement Researcher Media. 476 pp.

From the author: Writing this book has been a nine-year long passion project. This is the book I always intended to write, and the other three books in the Retirement Researcher Guide Series were spun-off along the way as my writing on some of these topics became so long that they constituted separate books on their own. Three of the chapters in this book summarize other books in this series, and ten of the chapters present new and original content.

Let me show with Retirement Planning Guidebook's Table of Contents how the other books in this series fit in:

1) Chapter 1: Retirement Income Styles and Decisions
2) Chapter 2: Retirement Risks
3) Chapter 3: Quantifying Goals and Assessing Preparedness
4) Chapter 4: Sustainable Spending from Investments [Summarizes How Much Can I Spend in Retirement?]
5) Chapter 5: Annuities and Risk Pooling [Summarizes Safety-First Retirement Planning]
6) Chapter 6: Social Security
7) Chapter 7: Medicare and Health Insurance
8) Chapter 8: Long-Term Care Planning
9) Chapter 9: Housing Decisions in Retirement [Summarizes Reverse Mortgages and adds new content]
10) Chapter 10: Tax Planning for Efficient Retirement Distributions
11) Chapter 11: Legacy and Incapacity Planning
12) Chapter 12: The Non-Financial Aspects of Retirement Success
13) Chapter 13: Putting it All Together

Retirement Planning Guidebook: Navigating the Important Decisions for Retirement Success.

Pfau, W. D. (2015a). "Choosing a Retirement Income Strategy: A New Evaluation Framework." In: *SSRN e-Print*.

This article presents the initial stages of a new evaluation framework for choosing among retirement income strategies. The investigation includes eight retirement income strategies: constant inflation-adjusted withdrawal amounts, a constant withdrawal percentage of remaining assets, a withdrawal percentage based on remaining life expectancy, a more aggressive hybrid withdrawal percentage, inflation-adjusted and fixed single premium immediate annuities, a variable annuity with a guaranteed living withdrawal benefit rider, and a strategy which annuitizes the flooring level to meet basic needs and uses the hybrid withdrawal percentage for remaining assets. These eight strategies will be analyzed with six retirement outcome measures over a 30-year retirement period: the average amount whereby spending falls below the minimally acceptable level, the average spending amount, the remaining bequest at the end of the retirement period, the minimum spending amount for any year in the retirement period, a measure of whether spending increases or decreases over time defined as spending in the first year divided by spending in the 30th year, and the value of total spending after accounting for diminishing returns from increased spending for a client with somewhat inflexible spending needs. The model is applied to three client scenarios representing a cross-section of RIIA's client segmentation matrix. It is built using Monte Carlo simulations which reflect current market conditions, so that systematic withdrawals and guaranteed products share compatible underlying assumptions.

Pfau, W. D. (2015b). "How to Link Retirement Strategies to Sustainable-Spending Rates." In: *Advisor Perspectives*.
-Sustainable-spending rates for retirement depend on many factors: asset- and product-allocation, market valuations at the start of retirement (particularly, current interest rates), the desired spending pattern over retirement, the degree of flexibility to adjust spending in response to market performance and the length of the client's planning horizon. Last week, my article introduced the Retirement Accumulation and Retirement Affordability indices, which help clients determine if they are retiring at a good time. In this article, I will present my new Retirement Dashboard. More specifically, I will explain the section of the dashboard on "The Cost of Retiring Today - Sustainable Spending Rates for Retirement," with these factors in mind. This analysis is conducted for a couple who both turn 65 on January 1, 2015. I estimated the sustainable-spending rates for two very different types of retirement-income strategies: those based on dedicated-income sources that can match client spending needs without exposing the client to market volatility, and those based on investment portfolios in which market volatility will play a much larger role in determining retirement sustainability.

Pfau, W. D. (2015c). "Making Sense Out of Variable Spending Strategies for Retirees." In: *SSRN e-Print*.
Variable spending strategies can be situated on a continuum between two extremes: spending a constant amount from the portfolio each year without regard for the remaining portfolio balance, and spending a fixed percentage of the remaining portfolio balance. Variable spending strategies seek compromise between these extremes by avoiding too many spending cuts while also protecting against the risk that spending must subsequently fall to uncomfortably low levels. Two basic categories for variable spending rules explored include decision rule methods and actuarial methods. Ten strategies will be compared using a consistent set of portfolio return and fee assumptions, and using an XYZ formula to calibrate initial spending: the client willingly accepts an X percent probability that spending falls below a threshold of Y dollars (in inflation-adjusted terms) by year Z of retirement. Presenting the distribution of spending and wealth outcomes for different strategies in which the initial spending rate is calibrated with the XYZ formula will allow for a more meaningful comparison of strategies. The article provides a framework for identifying appropriate spending strategies based on client preferences.

Pfau, W. D. (2018). "An Overview of Retirement Income Planning." In: *Journal of Financial Counseling and Planning* 29(1), pp. 114–120.
Retirement income planning has emerged as a distinct field within financial planning with the realization that risks change dramatically in retirement. The combination of longevity risk, increasing market risk triggered by taking distributions from assets, and spending shocks create new challenges. Wealth management has traditionally focused on accumulating assets without applying further thought to these differences happening after retirement. Retirees experience reduced capacity to bear financial market risk once they have retired. This calls for different thought processes from those typically included in traditional investment management. Risk pooling becomes an important retirement income tool combined with a traditional investment portfolio. Retirement income challenges and a framework for helping individuals develop more efficient and successful retirement income plans are discussed.

Pfau, W. D. (2019). "The four approaches to managing retirement income risk." In: *SSRN e-Print*.
Sequence-of-returns risk amplifies the impacts of investment volatility when taking distributions from a volatile investment portfolio in retirement. There are four techniques for managing this sequence risk: reduce the spending rate, adjust spending to portfolio performance, reduce portfolio volatility in the early retirement years, and draw

from a buffer asset outside the portfolio to support spending when the portfolio is underperforming. We examine the peculiarities of sequence risk using an example with U.S. market returns which leads to wealth depletion after a thirty-year retirement. We show how minor spending adjustments can lead to dramatically different outcomes for remaining investment wealth at the end of the thirty-year retirement period. We provide examples for how each of these risk management techniques can be implemented in practice by integrating a wider range of tools to create the potential for greater retirement efficiency in terms of supporting the retirement financial goals with the available asset base. Examples include delaying Social Security, using annuities with lifetime spending protections, employing a rising equity glidepath in retirement, using a time segmentation or bucketing strategy, and creating a asset with a reverse mortgage or whole life insurance.

Pfau, W. D. and Dokken, W. (2015). "Rethinking Retirement: Sustainable Withdrawal Rates for New Retirees in 2015." In: *Financial Advisor Magazine*.

Recent research that Dr. Wade Pfau and I have conducted finds that investors retiring as of January 1, 2015, who pursue a traditional 4% withdrawal rate from their savings to fund retirement have a more than 50% chance of outliving their savings. Last month in Financial Advisor, we detailed our research on safe withdrawal rates for retirees and the findings that could disrupt the current idea about what savings can safely be generated during a person's retirement. Taking into account the person's investment decisions, the real safe withdrawal rate is much closer to 2% and could be lower. But we also believe that understanding investor behavior is key to understanding the withdrawal figure.

Pfau, W. D., Tomlinson, J., and Vernon, S. (2016a). "A Portfolio Approach to Retirement Income Security." In: *The Journal of Retirement* 4(2), pp. 11–22.

Workers with defined contribution retirement plans face a daunting array of choices as they enter retirement. They need to decide how to allocate their savings among asset classes and how much to withdraw from savings to support retirement. They also need to make choices about when to claim Social Security and whether or not to purchase annuities (and what type to purchase) to provide guaranteed lifetime income in addition to Social Security. In this article, we report on a recent collaboration between the Stanford Center on Longevity (SCL) and the Society of Actuaries (SOA) aimed at analyzing the various alternatives and finding optimal combinations of asset and product allocations, and strategies for withdrawing funds from savings. An important objective in this work has been finding solutions that are sufficiently straightforward to be implemented by retiring workers in employer-sponsored retirement plans or by utilizing low-cost advice options. Solutions that require substantial adviser involvement may be too expensive for typical middle-income workers. While the report focuses on solutions for defined contribution retirement plans, its framework can also be applied to out-of-plan solutions.

Pfau, W. D., Tomlinson, J., and Vernon, S. (2016b). *Optimizing Retirement Income Solutions in Defined Contribution Retirement Plans: A Framework for Building Retirement Income Portfolios*. Tech. rep. Stanford Center on Longevity.

This report helps plan sponsors, advisors, and retirees achieve these goals by demonstrating an analytical framework and criteria for helping them evaluate and compare a variety of possible retirement income solutions. Our goal is to further understanding of how to use various retirement income generators (RIGs) to meet specific retirement planning goals.

Pfau, W. D., Tomlinson, J., and Vernon, S. (2017). *Optimizing Retirement Income by Integrating Retirement Plans, IRAs, and Home Equity: A Framework for Evaluating Retirement Income Decisions*. Tech. rep. Stanford Center on Longevity.

In this report, the authors present a framework of analyses and methods that financial advisers, financial institutions, plan sponsors, and retirees can use to compare and assess strategies for developing lifetime retirement income. We recommend that financial advisers, plan sponsors, and financial institutions use disciplined analyses to demonstrate they are acting in the best interests of their clients who are approaching and entering retirement.

Pittman, S. (2014). "Is a Portfolio Built to Produce Yield a Sensible Retirement Income Portfolio?" In: *Journal of Financial Planning* 27(8), pp. 52–60.

This paper focuses on the efficiency of allowing yield payments to dictate retirement spending, as well as the efficiency of tilting a portfolio away from an "efficient" portfolio for the sake of producing more yield.

Much of the literature suggests that a total return objective is a better choice than focusing on portfolio yield for retirees. This analysis furthers this literature by revisiting the cash flow and estate objectives of a retiree.

Results suggest that (1) using a yield portfolio forces a tradeoff between retirement spending and estate goals; (2) current yields may not be high enough to meet retirement income needs; (3) yields change over time, leading

to inconsistent spending; (4) drawing down an efficient portfolio with the same risk as a portfolio emphasizing yield has higher expected principal accumulation; and (5) drawing down a portfolio is more tax-efficient than using yield to generate retirement income.

Pittman, S. (2015). "Use Your Client's Funded Ratio to Simplify and Improve Retirement Planning Decisions." In: *The Journal of Retirement* 3(2), pp. 93–104.
This article shows investment advisors how to compute a funded ratio and, subsequently, how to use this metric to assess retirement readiness for individual clients. The funded ratio allows the advisor to measure the relative size of assets and liabilities of an individual's retirement plan, providing an alternate perspective to that obtained from Monte Carlo simulation. Upon discussing how to calculate and use a funded ratio, a simple graphic is presented and described to show advisors how to quickly determine if their clients are on track to retire given their current retirement plan.

Platanakis, E., Sutcliffe, C. M., and Ye, X. (2021). "Horses for Courses: Mean-Variance for Asset Allocation and 1/N for Stock Selection." In: *European Journal of Operational Research* 288(1), pp. 302–317.
For various organizational reasons, large investors typically split their portfolio decision into two stages - asset allocation and stock selection. We hypothesise that mean-variance models are superior to equal weighting for asset allocation, while the reverse applies for stock selection, as estimation errors are less of a problem for mean-variance models when used for asset allocation than for stock selection. We confirm this hypothesis in separate analyses of US and international equities using four different types of mean-variance model (Bayes-Stein, Black-Litterman, Bayesian diffuse prior and Markowitz), a range of parameter settings, and a simulation analysis calibrated to US data.

Podkaminer, E., Tollette, W., and Siegel, L. (2022). "Protecting Portfolios Against Inflation." In: *The Journal of Investing* 31(2), pp. 23–44.
Inflation is a perennial threat to the real value of portfolios, even though current inflation rates are low. To protect portfolios against inflation, cash, inflation-indexed bonds, equities, real estate, and commodities are the usual candidates. We examine each, plus other assets and, importantly, various kinds of liabilities, to examine their historical and prospective responses to expected and unexpected inflation. Our article is integrative, bringing together ideas and data from many different sources in one place.

Prendergast, J. R. (2021). "Replicating Maximum-Yield Annuities with US Treasury Funds." In: *The Journal of Fixed Income* 30(4), pp. 81–99.
Many individuals rely on annuity purchases to provide a steady stream of income during their post-retirement years. Life insurance companies provide a variety of annuity contracts, but they often come with sizable fees that are extremely detrimental to the purchaser in a low-interest-rate environment. This article develops a methodology that enables retail investors to structure annuities using commonly available US Treasury exchange-traded funds (ETFs) or mutual funds. Only historical price or NAV data and Treasury yield data are required to implement the strategy. The funds compose a dynamically managed portfolio requiring only an initial investment. Each period, the portfolio is rebalanced to match the modified duration of a reference annuity. This minimizes interest rate risk from parallel shifts in future yield curves. The weights are optimized to provide the highest yield possible. The objective is for the portfolio to provide the required periodic annuity payments and have a zero balance at annuity maturity under a variety of interest rate scenarios, while providing the maximum possible yield. Scenario tests show that the strategy is effective under parallel yield curve shifts, but may have shortfalls for curve steepening and gains for curve flattening. Investors may choose to add to their initial investment to reduce the risk of shortfalls if the curve steepens. The article concludes with an implementation of the strategy using actual Treasury ETFs.

Pruser, J. (2021). "Forecasting US inflation using Markov dimension switching." In: *Journal of Forecasting* 40(3), pp. 481–499.
This study considers Bayesian variable selection in the Phillips curve context by using the Bernoulli approach of Korobilis (Journal of Applied Econometrics, 2013, 28(2), 204-230). The Bernoulli model, however, is unable to account for model change over time, which is important if the set of relevant predictors changes. To tackle this problem, this paper extends the Bernoulli model by introducing a novel modeling approach called Markov dimension switching (MDS). MDS allows the set of predictors to change over time. It turns out that only a small set of predictors is relevant and that the relevant predictors exhibit a sizable degree of time variation for which the Bernoulli approach is not able to account, stressing the importance and benefit of the MDS approach. In addition, this paper provides empirical evidence that allowing for changing predictors over time is crucial for forecasting inflation.

Quinn, J. F. and Cahill, K. E. (2018). "Challenges and Opportunities of Living and Working Longer." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.
This chapter describes the challenges and opportunities that older Americans face, with a focus on retirement income security and the role of continued work later in life. We first overview the new world of retirement income security including a discussion of how a low return environment (e.g. low interest rates) exacerbates existing retirement income security challenges. We then document how older people have responded to the evolving retirement income landscape, especially when and how they exit the labor force, and we explore how continued work later in life can help mitigate some of the anticipated retirement security challenges. We then pose some important outstanding questions. The implications of societal aging depend in large part on how we harness or squander the labor resources of older individuals.

Rabitti, G. and Borgonovo, E. (2020). "Is mortality or interest rate the most important risk in annuity models? A comparison of sensitivity analysis methods." In: *Insurance: Mathematics and Economics* 95, pp. 48–58.
Demographic and financial factors are key risk-drivers for insurance companies and pension funds. This paper proposes a systematic investigation for deepening our understanding how these risk drivers affect the annuity cost. We employ local and global sensitivity methods. For local sensitivity, we derive closed form expressions for the differential importance measures of perturbed annuities and connect them to the entropy of the annuity cost. For global sensitivity, we compare variance-based, moment-independent sensitivity measures and Shapley effects. In particular, moment-independent sensitivity measures and Shapley effects are compared for the first time in the case of dependent risk factors. Our framework encompasses and extends several previous results on the sensitivity analysis of annuity models. From a methodological viewpoint, the techniques compared in this paper can support analysts in building annuity models and in verifying the impact of risk drivers in their models. Numerical results using the U.S. 1990 and the U.K. 1990-1994 mortality tables show that the demographic factor is the most important risk source in low-interest rate contexts. However, when uncertainty on the two risk sources is taken into account, the financial factor becomes the global key-driver of risk. Also, interactions among the two factors appear quantitatively significant.

Radovanov, B. and Marcikic, A. (2014). "Testing The Performance Of The Investment Portfolio Using Block Bootstrap Method." In: *Economic Themes* 52(2).
The aim of this paper is to create a stable model of investment portfolio optimization through a high degree of diversification and reduction of sudden changes in the allocation with monitoring of the dynamics of the impact factor. In this sense, there is bootstrap application procedure, which, without an excessive number of constraints involved in the optimization process provides solutions based on uncertain information. Thus defined, the optimization method has been patented by Michaud (1999) entitled re-sampled efficiency. Accordingly, this paper offers a comparison of the performance block bootstrap optimization models and traditional Markowitz's model inside and outside of the sample by applying the most frequently traded stocks on the BSE. The results show a better performance out of the sample and the presence of a larger number of shares forming the portfolio through bootstrap methodology. However, only through the traditional optimization process could be attained optimum according to the required limits. Such effects can be observed by comparing the limits of efficiency obtained through these optimization models. However, optimization-based methods bootstrap finds its place in reducing errors of assessment resulting from the limited sample size.

Raskie, S. (2017). "Navigating Uncertainties in Accumulation and Decumulation of Retirement Portfolios." In: *Journal of Economics and Public Finance* 3(3), p. 470.
Individuals face many challenges when developing a retirement plan. Hurdles arise at different stages of the retirement planning lifecycle. In the pre-retirement period, a significant obstacle arises when individuals must save for retirement to maximize their utility in retirement. The question of how much to save along with where to save impacts the amount the individual has in retirement. Post-retirement individuals must overcome the obstacle of how to optimally withdraw from their retirement savings to mitigate sequence risk and longevity risk to reduce the chance of portfolio failure. Individuals in post-retirement must develop strategies that not only mitigate these risks but also allow them to enjoy the retirement they have envisioned.

Rebonato, R. (2019). "A financially justifiable and practically implementable approach to coherent stress testing." In: *Quantitative Finance* 19(5), pp. 827–842.
We present an approach to stress testing that is both practically implementable and solidly rooted in well-established financial theory. We present our results in a Bayesian-net context, but the approach can be extended to different settings. We show (i) how the consistency and continuity conditions are satisfied; (ii) how the result of a scenario can be consistently cascaded from a small number of macrofinancial variables to the constituents

of a granular portfolio; and (iii) how an approximate but robust estimate of the likelihood of a given scenario can be estimated. This is particularly important for regulatory and capital-adequacy applications.

Reichenstein, W. (2006). "After-Tax Asset Allocation." In: *Financial Analysts Journal* 62(4), pp. 14–19.
Several studies have found fundamental flaws in the traditional approach to managing individual investors' portfolios, including a failure to distinguish between 1USD of pretax funds in a 401(k) and 1USD of after-tax funds in either a taxable account or Roth IRA. This study recommends that an individual's asset values be converted to after-tax values and the asset allocation be based on the after-tax values. In general, within the target asset allocation, individuals should hold bonds and other assets subject to ordinary income tax rates in retirement accounts and hold stocks, especially passively managed stocks, in taxable accounts.

Reichenstein, W. (2020). "Saving in Roth Accounts and Making Roth Conversions before Retirement in Today's Low Tax Rates." In: *Journal of Financial Planning*.
There is a strong possibility that tax rates will rise in the next few years by more than the tax rate increases that are already scheduled to occur in 2026 according to the Tax Cuts and Jobs Act (TCJA). And, due to the taxation of Social Security benefits and income-based Medicare premiums, many retirees will have substantially higher marginal tax rates in retirement than their then-current tax brackets. As a result, many investors can expect to have higher marginal tax rates in their retirement years than in their pre-retirement years. This column recommends advice financial advisers can provide this year and perhaps the next few years to help clients-especially pre-retirement-age clients-take advantage of what my colleagues and I believe are today's temporarily relatively low tax rates. In the next few years, while tax rates are relatively low, pre-retirement-age investors should consider: (1) saving in tax-exempt accounts like a Roth 401(k) instead of tax-deferred accounts like a 401(k); and (2) making Roth conversions to fill today's relatively low tax brackets.

Reichenstein, W. and Meyer, W. (2016a). "Redo Strategies: When Can You Redo a Prior Social Security Claiming Decision?" In: *Journal of Financial Planning*.
Many financial planners are aware that a client has the one-time right to withdraw his or her application for retirement benefits if made within 12 months after benefits began. This article explains two other redo strategies that planners may not be aware of. One strategy is the ability to suspend retirement benefits at full retirement age (FRA) or later in order to earn delayed retirement credits, and then to reinstate suspended benefits at a later date. The second strategy applies to disabled individuals where the disabled person begins disability benefits before attaining FRA then suspends disability benefits at FRA in order to earn delayed retirement credits. Prior to the passage of the Bipartisan Budget Act of 2015, other redo strategies were available, however changes embedded in this Act eliminated these other strategies as of April 29, 2016. This article discusses these recent rule changes, explaining that there are now three sets of rules, noting which set of rules apply to individuals, depending upon when they were born.

Reichenstein, W. and Meyer, W. (2016b). "Social security claiming strategies for widows and widowers." In: *The Journal of Retirement* 3(4), pp. 77–86.
This study presents rules governing survivor benefits and provides several cases to illustrate these rules. It then presents guidelines to help financial advisors quickly determine the best strategy for a widow or widower who is eligible for survivor benefits. The authors present separate advice for four groups of clients who differ in terms of the age of the surviving spouse. In addition, they present examples that illustrate the logic of the recommended strategies for each group of survivors. As seen in several examples, there are many times when a widow or widower should not simply take the larger of his or her own retirement benefit or his or her survivor benefits.

Reichenstein, W. and Meyer, W. (2017). "Valuing Roth Conversion and Recharacterization Options." In: *Journal of Financial Planning* 30(11), pp. 48–56.
This paper explains the options provided by the tax code for Roth conversions and recharacterizations.
Models are presented of the after-tax value of Roth conversion strategies to the strategy of retaining funds in a TDA and withdrawing these funds in a later year.
Four reasonable cases illustrate how the Roth conversion and recharacterization options in the tax code could allow a taxpayer to increase the after-tax value of the funds converted to a Roth account by 5.56 percent, compared to the strategy of retaining these same funds in a tax-deferred account until later in retirement. One particular case, in which the Roth conversion increased the after-tax value by 62.79 percent, reflects the situation many taxpayers face. An example explains what a financial adviser should do to take advantage of the Roth conversion/recharacterization options available in the tax code.

Reichenstein, W. and Meyer, W. (2018). "Understanding the Tax Torpedo and Its Implications for Various Retirees." In: *Journal of Financial Planning* 31(7), pp. 38–45.

The "tax torpedo" refers to the sharp rise and then sharp fall in marginal tax rates caused by the taxation of Social Security benefits. For many middle-income households, this tax torpedo causes their marginal tax rate within a wide range of income to be 150 percent or 185 percent of their tax bracket. Many middle-income retirees can have a federal-alone marginal tax rate of 40.7 percent in 2018. If we return to the 2017 tax brackets in 2026, as called for in the Tax Cuts and Jobs Act, then many middle-income retirees will face a federal marginal tax rate of 46.25 percent. This paper provides income ranges in 2018 within which the tax torpedo likely will apply for singles, qualifying widow(er)s, married couples filing separately that lived apart for the entire year, and married couples filing jointly. Examples illustrate how singles and couples younger than age 70 may be able to reduce the adverse effects of the tax torpedo and thus extend the longevity of their financial portfolio by delaying Social Security benefits until 70. This paper also explains how singles and couples already subject to required minimum distributions may be able to reduce the adverse effects of the tax torpedo and extend the longevity of their financial portfolio using Roth conversions.

Reichenstein, W. and Meyer, W. (2019a). "Medicare and Tax Planning for Higher-Income Households." In: *The Journal of Wealth Management* 22(3), pp. 28–40.
This article explains how Medicare Part B and D premiums vary with a household Modified Adjusted Gross Income. Each time MAGI increases by 1 above several income threshold levels, a couple Medicare premiums two years hence can rise by more than 2,400. These premium hikes represent spikes in the marginal tax rate exceeding 240,000 %. It then explains how certain life-changing events can affect a household level of Medicare premiums. Next, it presents two cases that illustrate the value that financial advisors can add to clients portfolios by helping them coordinate a smart Social Security claiming strategy with a tax-efficient withdrawal strategy. These cases demonstrate that higher-income households should consider making Roth conversions from 2019 through 2025, when tax rates are scheduled to be temporarily lower, and before required minimum distributions begin. These Roth conversions may allow these households to greatly reduce the size of both their lifetime income taxes and their lifetime Medicare premiums. Finally, it explains the advantages for households with someone at least age 70.5 of making charitable contributions through Qualified Charitable Distributions from Individual Retirement Accounts.

Reichenstein, W. and Meyer, W. (2019b). "Optimizing Social Security Benefits Is Still Complicated." In: *The Journal of Retirement* 6(3), pp. 69–79.
The Bipartisan Budget Act of 2015 made significant changes to the rules affecting Social Security benefits. Some people think that these rule changes made it relatively easy to determine when someone should claim their Social Security benefits. In this article, the authors explain the rule changes and provide 10 reasons why making an optimal Social Security claiming decision is still complicated, even if we knew how long the single claimant or each partner in a married couple will live. Deciding when to claim Social Security benefits remains a complex decision.

Reichenstein, W. and Meyer, W. (2020). "Investment implications of the rising and falling pattern of marginal tax rates for retirees." In: *The Journal of Retirement* 8(1), pp. 53–64.
This article begins by explaining how income-based increases in Medicare premiums produce dramatic increases in marginal tax rates for retirees with relatively high levels of income. It then explains how the taxation of Social Security benefits causes most lower- and middle-income households to have marginal tax rates for a wide range of income after Social Security benefits begin that are either 150% or 185% of their tax bracket. By combining these two factors, the authors show that most retirees will have marginal tax rates in retirement that rise and fall frequently and sharply as their income increases. This article then explains how this rising and falling pattern of marginal tax rates should affect retired households withdrawal strategies from their financial portfolio, where withdrawal strategy is defined broadly to include Roth conversions.

Reichenstein, W. and Meyer, W. (2021a). "Advice for Married Couples When One Spouse Will Die Year(s) Before the Other Spouse." In: *Journal of Financial Planning* 34(1).
In a recent article, we explained how the taxation of Social Security benefits causes many retirees' marginal tax rate to be 150 percent or 185 percent of their tax bracket. We further explained that a $0.01 increase in 2018 income above any of three income-threshold levels can cause a married couple's 2020 Medicare annual premiums to increase by more than $2,540. Each of these increases in annual premiums represents a spike in marginal tax rates exceeding 254,000 percent. In this column, we discuss the implications of the taxation of Social Security benefits and income-based Medicare premiums for retired married couples filing jointly, when one spouse dies at least one calendar year before the other spouse. Thus, this column applies to most retired married couples.

Reichenstein, W. and Meyer, W. (2021b). "How Social Security Coordination Can Add Value to a Tax-Efficient Withdrawal Strategy." In: *The Journal of Retirement* 9(2), pp. 37–57.
This study describes a tax-efficient withdrawal strategy that can add substantial value to many clients of financial advisors with financial portfolios worth up to $2 million. In early retirement years, these households can delay the start of their Social Security benefits and make Roth conversions to fill relatively low tax brackets, which are generally also their marginal tax rates. Once Social Security benefits begin, they can make tax-free Roth withdrawals to minimize the amount of tax-deferred account (e.g., 401(k)) withdrawals that are taxed at 185% of their tax bracket, due to the taxation of Social Security benefits. With a series of cases, we show that a financial advisor can add substantial value to many of their clients' financial portfolios by recommending such a withdrawal strategy.

Reichenstein, W. and Meyer, W. (2021c). "Social Security Claiming Strategies for Singles and Their Implications for Couples." In: *Journal of Financial Planning* 34(5).
This report presents important insights related to the optimal Social Security claiming age for a single individual. The first section demonstrates that in order to maximize expected real (inflation-adjusted) lifetime Social Security benefits, it is seldom optimal for a single retiree to claim benefits in any month near either of two dates. The first date is 36 months prior to the full retirement age (FRA), while the second date is the FRA. The second section discusses four other factors that the single retiree should consider when selecting their claiming age. Each of these factors should encourage single individuals to delay claiming Social Security beyond this lifetime maximizing age. The report then explains when the lessons from the first section related to maximizing a single retiree's expected real lifetime benefits also apply to the claiming strategy for each partner of a married couple. As the reader will see, the lessons for single retirees apply to the claiming strategy for some married couples, but not for others.

Reichenstein, W. R. (2007). "Note on Applying After-Tax Asset Allocation." In: *The Journal of Wealth Management* 10(2), pp. 94–97.
The author discusses an area within the field of after-tax asset allocation in which he and his co-author William Jennings disagree with the point proposed in the preceding article by Stephen Horan. But, more importantly, the author expresses the importance of those areas in which Horan, Jennings and he, as well as other scholars, agree. In particular, there appears to be wide agreement among scholars that, when calculating an individual's asset allocation, one should distinguish between pretax funds and after-tax funds. In short, taxes matter! He concludes with a crucial message: if it is indeed right to argue that funds in tax-deferred or tax-exempt portfolios should be accounted differently than those held through fully taxable structures, then the profession is currently miscalculating individuals' asset allocations, and the measurement errors can be substantial.

Reilly, C. and Byrne, A. (2018). "Investing for Retirement in a Low Returns Environment." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.
Low returns on financial assets and increasing longevity mean saving for retirement is becoming more challenging than it has been in the past. Generations retiring in the near term face increased longevity but have lived through periods with strong market returns boosting their assets, and many also have defined benefit plan entitlements. Younger generations, who also face increasing longevity, are unlikely to earn historical investment returns on their retirement portfolios, and few have traditional pensions. We model the likely outcomes for different cohorts under scenarios for savings behavior, investment returns, and longevity. While younger generations do face substantial challenges, we show that plausible courses of action involve increased contributions and delayed or partial retirement, which can provide reasonable income replacement rates in retirement. We map out the steps that the retirement industry (government, employers, and financial services providers) must take to support people in following these courses of action, such as providing more flexibility over social security.

Reznik, G., Alleva, B., Song, J., Sarney, M., and Olsen, A. (2020). *Analysis of Benefit Estimates Shown in the Social Security Statement*. Tech. rep. Social Security Administration.
The Social Security Statement is one of the Social Security Administration's (SSA's) most important ways to communicate with the public. Because a worker's complete lifetime earnings are unknown at the time his or her Statement is prepared, SSA estimates his or her future benefits by using the worker's historical earnings to project future earnings until retirement. Prior literature has examined the accuracy of benefit estimates shown in the Statement; however, there have not been efforts to test the accuracy of alternative methods of estimating retirement benefits. This paper documents a study by SSA's Office of Retirement and Disability Policy of the accuracy of the current Statement estimation method, the current method's assumption that 2 years of zero earnings predict no future earnings, and the accuracy of potential alternative methods of projecting earnings

and benefits. Using administrative data from the Continuous Work History Sample (CWHS), the paper finds that the Statement's current estimation method performs as well as or better than any of the other methods tested. The paper concludes with recommendations for the Social Security Statement.

Rietz, R., Blumenschein, T., Crough, S., and Cohen, A. (2018). "Analyzing Retirement Withdrawal Strategies." In: *Preprints*.

An optimal withdrawal strategy beginning at age 65 provides a lifetime income from a portfolio, adjusted annually for inflation, while reducing the probability of living in financial ruin to an ac-ceptable level. This paper analyzes the probability of living in financial ruin, potentially for multiple years, rather than just the event of ruin. A stochastic Excel model was developed to simulate the effect of varying investment returns on a portfolio with two asset classes; large company stocks and long-term government bonds. A stochastic model is also applied to retiree mortality. The following variables were analyzed to determine their relative impact on withdrawal strategies: Withdrawing a constant percentage of the portfolio, Gender, Initial asset allocation, Asset allocation rebalancing methods, and Low investment return environments. For both genders and most withdrawal rates, an approximately equal initial asset allocation of stocks and bonds, combined with a level rebalancing function, provided the lowest probability of living in financial ruin. Because each investment return followed its own probability distribution function, some retirees experienced financial ruin even in the most conservative simulations.

Rietz, R. J., Blumenschein, T., Crough, S., Cohen, A., and Coleman, J. (2020). "A Simulation for Minimizing both the Probability and the Length of Financial Ruin in Retirement." In: *SSRN e-Print*.

Retirees worry about depleting their portfolio, but a greater concern could be how long they might live without income from that portfolio. A retiree may accept a 4% probability of portfolio depletion, but object to the possibility of living five or six years afterwards without that income. Thus, a retirement portfolio's exhaustion is not a terminal event, but rather is only the beginning of a retiree's living in poverty. This analysis simulates not only the event of financial ruin, but also its duration during the retiree's remaining lifetime. This paper analyzes withdrawing a constant percentage of the portfolio, gender, initial asset allocation, asset allocation rebalancing methods, and low investment return environments to determine their relative impact on withdrawal strategies.

Rogalla, R. (2021). "Optimal Portfolio Choice in Retirement With Participating Life Annuities." In: *North American Actuarial Journal* 25(sup1), S182–S195.

This article derives optimal consumption, investment, and annuitization patterns for retired households that have access to German-style participating payout life annuities (PLAs), allowing for capital market risks as well as idiosyncratic and systematic longevity risks. PLAs provide guaranteed minimum benefits in combination with participation in insurers' surpluses. Minimum benefits are calculated based on conservative assumptions regarding capital market and mortality developments, while surpluses distributed to annuitants bridge the gap between the insurers' actual investment and mortality experiences and the projections used in pricing. Through the participation scheme, systematic longevity risk is shared between insurers and annuitants, as unanticipated longevity shocks result in benefit adjustments via the surplus mechanism. We show that the retiree draws substantial utility from access to PLAs, equivalent to 20% of initial wealth in the presence of systematic longevity risk. We also find that stochasticity in mortality rates only has minor impact on the appeal of PLAs to the retiree. Even if the interest rate guarantee is reduced to zero in adverse capital market environments, PLAs prove to provide substantial utility for retirees. Overall, the participating life annuity design produces substantial welfare gains over a no-annuity world, while being an effective setup that helps providers manage long-term risks that are difficult to hedge otherwise, such as systematic longevity risks.

Rook, C. J. (2015a). "Minimizing the Probability of Ruin in Retirement." In: *arXiv e-Print*.

Retirees who exhaust their savings while still alive are said to experience financial ruin. These savings are typically grown during the accumulation phase then spent during the retirement decumulation phase. Extensive research into invest-and-harvest decumulation strategies has been conducted, but recommendations differ markedly. This has likely been a source of concern and confusion for the retiree. Our goal is to find what has heretofore been elusive, namely an optimal decumulation strategy. Optimality implies that no alternate strategy exists or can be constructed that delivers a lower probability of ruin, given a fixed inflation-adjusted withdrawal rate.

Rook, C. J. (2015b). "Optimal Equity Glidepaths in Retirement." In: *SSRN e-Print*.

Dynamic retirement glidepaths evolve over time based on some measure such as the retiree's funded status or current market valuations. Conversely, static glidepaths are fixed at a starting point and selected under the assumption that they will not change. In practice, new static glidepaths may be derived periodically making them more flexible. The optimal static retirement glidepath would be the one that performs better than all others

with respect to some metric. When systematic withdrawals are made from a retirement portfolio, glidepaths are often assessed via the probability of ruin (or success). Our goal here is to derive the optimal static glidepath with respect to this metric. It is a result new to the literature and the shape will be of special interest to retirees, financial advisors, retirement researchers, and target-date fund providers.

Rook, C. J. (2017). "Multivariate Density Modeling for Retirement Finance." In: *arXiv e-Print*.

Prior to the financial crisis mortgage securitization models increased in sophistication as did products built to insure against losses. Layers of complexity formed upon a foundation that could not support it and as the foundation crumbled the housing market followed. That foundation was the Gaussian copula which failed to correctly model failure-time correlations of derivative securities in duress. In retirement, surveys suggest the greatest fear is running out of money and as retirement decumulation models become increasingly sophisticated, large financial firms and robo-advisors may guarantee their success. Similar to an investment bank failure the event of retirement ruin is driven by outliers and correlations in times of stress. It would be desirable to have a foundation able to support the increased complexity before it forms however the industry currently relies upon similar Gaussian (or lognormal) dependence structures. We propose a multivariate density model having fixed marginals that is tractable and fits data which are skewed, heavy-tailed, multimodal, i.e., of arbitrary complexity allowing for a rich correlation structure. It is also ideal for stress-testing a retirement plan by fitting historical data seeded with black swan events. A preliminary section reviews all concepts before they are used and fully documented C/C++ source code is attached making the research self-contained. Lastly, we take the opportunity to challenge existing retirement finance dogma and also review some recent criticisms of retirement ruin probabilities and their suggested replacement metrics.

Root, E., Mallett, C., Zwerling, N., and Toale, T. (2013). "Strategic Liability Management: Building a Glide Path Strategy to Manage Interest Rate and Longevity Risks." In: *Institutional Investor Guides: Special Issues*, pp. 56–60.

Glide Path Strategy (GPS) may be thought of as a term for a transition from a traditional market-based asset allocation and returns model to one in which a defined-benefit (DB) plan's benefit liabilities are the primary reference point for asset allocations and returns assessment as a DB plan's funded status improves. This article discusses how the GPS provides a durable framework to support holistic and strategic DB plan management as the sponsor's understanding of plan liabilities deepens, how a GPS can serve to facilitate managing risks that are important today, such as interest rate risk, and those that are expected to become more apparent in the future, such as longevity risk, and the ways in which major risk reduction methods work in building and sustaining a GPS over time.

Rosenthal, D. (2018). "Joint effect of random years of longevity and mean reversion in equity returns on the safe withdrawal rate in retirement." In: *SSRN e-Print*.

Using historical data on inflation-adjusted total equity returns (price change plus dividend) from the S&P 500 from 1926 to 2017, this paper develops a simulation-based model to determine the safe, inflation-adjusted withdrawal rate from a portfolio of assets. The model, named the Realistic Retirement Simulator (RRS), improves upon other simulation models by directly addressing two factors that significantly affect the safe withdrawal rate: (1) uncertainty about the number of years of retirement, i.e., at what age will the retiree die; and (2) mean reversion in equity returns. RRS models the number of years in retirement as a random factor based on the Social Security Administration 2015 Actuarial Life Table. The mean-reverting stock return model within RRS is statically calibrated to the 1926 to 2017 S&P 500 data. With these two key factors addressed and assuming future equity returns follow the historical record, RRS shows that a 65-year-old male retiree can withdraw 6% of the starting portfolio balance each year from a 100% stock portfolio with a 90% success rate; a 4% withdrawal rate is 99% successful. A simulation model that does not address these two key factors the author is not aware of single model that addresses both factors that a 4% withdrawal rate results in a 90% success rate for a retirement lasting 30 years. At the 90% success level, about half of the increase from 4% to 6% comes from treating the length of the retirement as a random factor and other half comes from the mean-reverting model. Many scenarios are run to show how the success rate changes when RRS input assumptions are changed, e.g., age of retiree or stock/bond mix of retiree portfolio. Of particular importance is the assumption that future equity returns will repeat the historical record. If the future, long-run trend for equity returns is a 4% compound annual growth rate (CAGR) instead of the 6.93% observed in the historical data, RRS shows that a withdrawal rate of 4% has a success rate of 95%. Regardless of the assumption about future equity returns, directly modeling uncertainty in the length of retirement and mean reversion in equity returns results in more accurate and higher estimates of the safe withdrawal rates compared to models that do not directly address these factors.

Rossi, A. G. and Utkus, S. P. (2021). "The Needs and Wants in Financial Advice: Human versus Robo-advising." In: *SSRN e-Print*.
We use a broad survey to elicit investor needs and their satisfaction in the context of financial advice. We provide evidence that traditionally-advised individuals do not hire financial advisors mainly to maximize portfolio returns. They instead hire financial advisors to satisfy a broader set of needs. These needs include acquiring "peace of mind," having access to the opinions of an expert and delegating financial decisions. We also show the majority of investors do not know how much they pay for financial advice, but the miscalculation of the costs of financial advice are unlikely to drive the decision to hire financial advisors. Robo-advised investors are more interested in the financial performance of their portfolio. They also view robo-advising as an empowering tool that can contribute to their own self-improvement. Even robo-advised investors, however, value greatly the possibility to reach out and interact with humans. Our results provide novel evidence on why individual investors choose to hire financial advisors. They also inform on the optimal design of robo-advisers.

Roux, E.-M. and de Villiers, J. (2020). "A simplified approach to estimate the sustainable lifestyle level for retirement planning." In: *Investment Analysts Journal* 49, pp. 232–242.
In this article we offer a simplified version of the alternative retirement planning model we originally proposed (De Villiers and Roux, 2019). Our method focuses on determining the sustainable lifestyle level (SLL) that an individual can currently afford while still saving enough towards retirement to sustain this lifestyle level up to retirement and beyond. The model is simplified by assuming that the real rate of return on retirement savings before retirement will be the same as the withdrawal rate of income from the accumulated savings during retirement. This method yields a much simpler SLL relationship in that it is more generally applicable albeit possibly less accurate. This approach should improve communication of the extent of the retirement savings challenge, possibly leading to better savings outcomes.

Roy, K. and Kim-Steiner, Y. (2019). *Three retirement spending surprises*. Tech. rep. J.P. Morgan Asset Management.
We analyzed the spending patterns of more than 5 million households to evaluate real-world retirement behaviors. Our research uncovered three surprising patterns: a lifetime spending curve, a retirement spending surge and a high degree of retirement spending volatility.
These new insights indicate helping investors make the most of their retirement assets may require a more dynamic approach to withdrawals than the static rules of the past.

Rundle, J. (2018). "A Social Security Purchase: Is Delaying Social Security More Effective than Purchasing a Deferred Income Annuity?" In: *The Journal of Retirement* 6(1), pp. 8–21.
This article analyzes a retirement strategy of delaying Social Security commencement past Social Security retirement age to effectively increase annuity income.
Using Monte Carlo simulations for returns on a pre-tax retirement portfolio and simplified tax, marriage, and Social Security benefit assumptions, I compare three retirement strategies:

1) following the 4% rule with retirement at Social Security normal commencement age (66),
2) delayed commencement to age 70 with forgone Social Security income replaced by portfolio withdrawals
3) following the 4% rule with normal retirement age commencement and purchase of deferred life annuity of equivalent expected actuarial value to the increase in Social Security that would occur if commencement were delayed to age 70.

Results for the scenarios analyzed generally support delaying Social Security as a preferred method to annuitize assets over the direct purchase of a deferred annuity, particularly under the married scenario. Compared with Strategy 1, delaying Social Security increases portfolio fail rates by year 30 (age 96) by 10 percentage points (to about 20%) and results in reduced portfolio values across outcomes.
The trade-off is increased inflation-protected income under portfolio failure scenarios by 32% of the primary benefit.

Ruthbah, U. (2020). "The Retirement Puzzle." In: *SSRN e-Print*.
The present-day retirees may not be as well off as they expect to be during their retired life. Given the current state of the world - higher life expectancy, close to zero real interest rate and the economic turmoil caused by the Covid-19 pandemic, a superannuation fund as large as 545,000 USD may not be enough to support a comfortable lifestyle. The rules of means-tested age pension provide incentives for a broad spectrum of people to deplete their superannuation in the early years of retirement, with significant negative consequences for government finances. Spending at retirement does not increase proportionally with assets. For some range of superannuation balances, the net present value of spending could increase more than the increase in assets, as the net present value of

the age pension does not decline at par with increases in assets. In other asset levels, the opposite happens. The distributional impact of the age pension is unequal across different asset levels and is regressive to some extent.

Ryack, K. (2016). "Incorporating Financial Risk Tolerance Research into the Financial Planning Process." In: *Journal of Financial Planning* 29(10), pp. 54–61.
An individual's financial risk tolerance ultimately drives his or her financial decisions and is therefore of critical importance in personal financial planning. Two psychometrically valid measures of an individual's financial risk tolerance are highlighted, and academic research findings on the nine determinants of an individual's financial risk tolerance are discussed. For example, financial risk tolerance tended to increase with the level of the client's education, income and wealth, and financial knowledge and experience. Financial risk tolerance also appeared to vary with gender, age, and market conditions. A framework was developed to organize the nine discrete factors that influence a client's financial risk tolerance on the basis of foreseeability and manageability to make it easier for financial planners to integrate financial risk tolerance into the financial planning process.

Sahm, C. (2017). "How Much Does Risk Tolerance Change?" In: *SSRN e-Print*.
Stability of preferences is central to how economists study behavior. This paper uses panel data on hypothetical gambles over lifetime income in the Health and Retirement Study to quantify changes in risk tolerance over time and differences across individuals. The maximum-likelihood estimation of a correlated random effects model utilizes information from 12,000 respondents in the 1992-2002 HRS. The results are consistent with constant relative risk aversion and career selection based on preferences. While risk tolerance changes with age and macroeconomic conditions, persistent differences across individuals account for 73% of the systematic variation.

Salampasis, D., Mention, A.-L., and Kaiser, A. O. (2019). "Wealth Management in Times of Robo: Towards Hybrid Human-Machine Interactions." In: *SSRN e-Print*.
Breakthrough technological developments are grounded to disruptive business models that are leading foundational changes within the financial services industry. In this frame of reference, wealth management is experiencing transformational forces reaching the fundamental core function of the business, redefining existing narratives and practices. Advisory services in management wealth are deeply embedded within a human-to-human interaction and robo-advising is going to augment the existing value chain leading to the creation of a hybrid advisory model for complex investment portfolios. Therefore, the fundamental question is not about 'either or' but 'together for'.

Sapra, S. and Pedersen, N. (2017). "The Role of Long-Maturity TIPS in Retirement Portfolios." In: *The Journal of Retirement* 4(4), pp. 95–106.
This article argues that long-duration Treasury Inflation-Protected Securities (TIPS) should play an important role in the portfolios of workers who are within 10 to 20 years of retirement. Long TIPS effectively help hedge retirees against fluctuations in the real wealth required to sustain a given level of consumption in their retirement years. As such, long-duration TIPS act as the "risk-free asset"in the context of retirement savings, and, for investors on a reasonable savings path, an allocation to this asset class may increase the likelihood of a successful retirement outcome.

Sato, M. (2016). "Optimal Withdrawal Rate Under Longevity Risks: A Criterion for Life Planning after Retirement." In: *SSRN e-Print*.
We discuss the optimal withdrawal rate in the existence of longevity risks. The longevity is realized as the factor of conditional survival rate in the model. We solve a optimal consumption problem under longevity risks, while managing the assets. The objective of the current work is giving a rational criterion that can be practically used in planning the life after retirement.

Scherer, B. and Lehner, S. (2021). "What Drives Robo-Advice?" In: *SSRN e-Print*.
The promise of robo-advisory firms is to provide low cost access to diversified portfolios built in accordance with the academic literature on normative portfolio choice. We investigate the latter claim. How much normative advice does robo-advice contain? For this purpose we web-scrap portfolio recommendations for 151200 investor types (input combinations from an online questionnaire) for one of the largest US robo-advisors. Our results show that the type of investment goal and the length of time horizon are dominating inputs with significant influence on recommended equity allocations. Normative advice in the form of Merton type hedging demands plays no role at all.

Schumann, E. (2019). "Backtesting." In: *SSRN e-Print*.
We discuss the backtesting of investment and trading strategies. We start with the challenges and pitfalls: overfitting, data preparation, and the effects of randomness. Then, we introduce and describe R software for backtesting. We demonstrate how to use the software for univariate and multivariate strategies (i.e. portfolio

strategies) for two equity data sets. Specifically, we discuss the implementation and testing of momentum and portfolio optimization models. Throughout, we stress the analysis of sensitivity and robustness checks. Since such analyses require to run many backtests, we also discuss how backtests can be run in parallel.

Scruggs, J. T. (2019). "Asset Allocation and Withdrawal Strategies: Three Levers for Managing Retirement Outcomes." In: *Journal of Financial Planning*.
This paper demonstrates how moving three choice levers (asset allocation, initial withdrawal rate, and withdrawal flexibility) affects the joint distribution of retirement outcomes in terms of future consumption and terminal wealth. Using Monte Carlo simulation, the joint effects of the three choices on retirement outcomes in terms of income and terminal wealth were investigated. Results indicate that withdrawal flexibility effectively transferred some portfolio risk from a household's wealth to its income, and allowing for even modest withdrawal flexibility dramatically reduced the probability of running out of money. Because retiree goals in terms of lifestyle and legacy can be related to income and terminal wealth, a case may be made for featuring regular Monte Carlo simulation in the retiree decision-making process.

Šestanović, T. and Arnerić, J. (2021). "Neural network structure identification in inflation forecasting." In: *Journal of Forecasting* 40(1), pp. 62–79.
Neural networks (NNs) are appropriate to use in time series analysis under conditions of unfulfilled assumptions, i.e., non-normality and nonlinearity. The aim of this paper is to propose means of addressing identified shortcomings with the objective of identifying the NN structure for inflation forecasting. The research is based on a theoretical model that includes the characteristics of demand-pull and cost-push inflation; i.e., it uses the labor market, financial and external factors, and lagged inflation variables. It is conducted at the aggregate level of euro area countries from January 1999 to January 2017. Based on the estimated 90 feedforward NNs (FNNs) and 450 Jordan NNs (JNNs), which differ in variable parameters (number of iterations, learning rate, initial weight value intervals, number of hidden neurons, and weight value of the context unit), the mean square error (MSE), and the Akaike Infor- mation Criterion (AIC) are calculated for two periods: in-the-sample and out- of-sample. Ranking NNs simultaneously on both periods according to either MSE or AIC does not lead to the selection of the 'best' NN because the optimal NN in-the-sample, based on MSE and/or AIC criteria, often has high out-of- sample values of both indicators. To achieve the best compromise solution, i.e., to select an optimal NN, the preference ranking organization method for enrichment of evaluations (PROMETHEE) is used. Comparing the optimal FNN and JNN, i.e., FNN(4,5,1) and JNN(4,3,1), it is concluded that under approximately equal conditions, fewer hidden layer neurons are required in JNN than in FNN, confirming that JNN is parsimonious compared to FNN. Moreover, JNN has a better forecasting performance than FNN.

Sestok, B. (2021). "Implementing Advice based on "A Comparison of the Tax Efficiency of Decumulation Strategies"." In: *Journal of Financial Planning* 34(11).
This paper revisits research published in March 2021 to provide guidance on effective decumulation strategies that go contrary to the conventional wisdom of which accounts to pull from first in retirement.

Sexauer, S. C., Peskin, M. W., and Cassidy, D. (2015). "Making Retirement Income Last a Lifetime." In: *Financial Analysts Journal* 71(1), pp. 79–89.
To enable investors to spend down the assets in their defined contribution accounts more easily, the authors propose a decumulation benchmark comprising a laddered portfolio of TIPS for the first 20 years (consuming 88 percent of available capital) and a deferred life annuity purchased with the remaining 12 percent. This portfolio can be used directly by the investor (akin to indexing) or as a benchmark for evaluating the performance of a more aggressive strategy.

Seymour, A., Flint, E. J., and Chikurunhe, F. (2018). "Dynamic portfolio management strategies: A framework for historical analysis." In: *SSRN e-Print*.
The performance of dynamic trading and investment strategies can be difficult to predict. Although not without its problems, analysis of the historical performance of a strategy can provide valuable insight into its general risk and return properties. Furthermore, historical analysis allows one to compare variations of a strategy and examine the impact of various parameter choices and implementation rules. Dynamic strategy applications in three areas are considered, namely derivatives, asset allocation and equity factor portfolios. Firstly, the analysis of a strategy involving single-stock derivatives is examined in which call options on certain constituents of an index portfolio are sold as an alternative method of under-weighting the underlying. Secondly, the historical performance of an optimization-based asset allocation strategy is considered. The assumed aim of the strategy is to outperform a benchmark of CPI 5 via dynamic trading in a portfolio of domestic equities, bonds, property and cash, as well as international equities and bonds. Finally, the effects of portfolio construction on factor performance are studied

via an historical analysis in which portfolios corresponding to a selection of fundamental factors are managed according to a range of weighting schemes, rebalance frequencies and portfolio sizes.

Shang, H. L. and Haberman, S. (2020). "Forecasting age distribution of death counts: an application to annuity pricing." In: *Annals of Actuarial Science* 14(1), pp. 150–169.
We consider a compositional data analysis approach to forecasting the age distribution of death counts. Using the age-specific period life-table death counts in Australia obtained from the Human Mortality Database, the compositional data analysis approach produces more accurate 1- to 20-step-ahead point and interval forecasts than Lee-Carter method, Hyndman-Ullah method and two naive random walk methods. The improved forecast accuracy of period life-table death counts is of great interest to demographers for estimating survival probabilities and life expectancy, and to actuaries for determining temporary annuity prices for various ages and maturities. Although we focus on temporary annuity prices, we consider long-term contracts that make the annuity almost lifetime, in particular when the age at entry is sufficiently high.

Sharpe, W. (2019). *Retirement Income Analysis with scenario matrices*. Stanford University. 200 pp.
financial economics textbook covering the various subject areas related to retirement planning. The textbook is titled RISMAT, which stands for Retirement Income Analysis (with scenario matrices). It covers subjects within the scope of financial economics such as utility theory and the valuation of assets, and others from outside such as demographics and building mortality tables. Incorporated in the text is a comprehensive system for retirement planning developed by Sharpe using Matlab software. In modeling investments, Sharpe uses an approach based on the capital asset pricing model (CAPM), which posits that the optimum relationship between return and volatility can be achieved with portfolios that combine a risk-free asset and a highly diversified market portfolio that includes proportionate shares of all publicly-traded investible assets. Sharpe develops a proxy for the market portfolio based on a mix of four Vanguard funds: 1) Total stock market index 2) Total bond market index 3) Total international stock market index 4) Total international bond market index He uses the Vanguard inflation-protected securities fund to represent the risk-free asset.

Sherwood, M. W. (2021). "Risk Capacity Portfolio Construction." In: *The Journal of Investing* 30(2), pp. 31–52.
This paper introduces a portfolio construction framework for risk-averse investors that aim to meet or exceed a client's capital accumulation needs for a future event, such as retirement. Risk Capacity Portfolio Construction (RCPC) presents a risk-optimized alternative to Target Date Funds (TDFs). Risk Capacity Portfolio Construction is an economic sciences innovation that is validated by Skew-Risk Modeling, which was first presented in Targeted Return Portfolio Construction (Sherwood, 2019) and is detailed within this paper in its application to retirement-focused investors. Risk Capacity Portfolio Construction factors skewness and kurtosis into the risk management and asset allocation of an individual's life cycle and can optimize risk beyond simply accounting for the equity risk premium and human capital.

Shi, Y. (2021). "Forecasting Mortality Rates with the Adaptive Spatial Temporal Autoregressive Model." In: *Journal of Forecasting* 40(3), pp. 528–546.
Accurate forecasts of mortality rates are essential to various demographic research like population projection, and the pricing of insurance products such as pensions and annuities. Recent studies have considered a spatial-temporal autoregressive (STAR) model for the mortality surface, where mortality rates of each age depend on the historical values of itself (temporarily) and the neighboring cohorts ages (spatiality). This model has sound statistical properties including co-integrated dependent variables and existence of closed-form solutions. Despite its improved forecasting performance over the famous Lee-Carter (LC) model, the constraint that only the effects of the same and neighboring cohorts are significant can be too restrictive. In this study, we adopt a data-driven adaptive weighted structure and propose the adaptive STAR (ASTAR) model. Retaining all desirable features of the STAR, our model uniformly outperforms the LC and STAR counterparties for forecasting accuracy, when mortality data aged 0–100 of the United Kingdom, France, Italy, Spain and Japan over 1950–2016 are considered. Two sensitivity tests and additional simulation results also lead to robust conclusions. The proposed ASTAR model is therefore a widely useful tool in modelling and forecasting mortality rates in other contents, and may be extensible to multi-population modelling.

Shoven, J. and Slavov, S. (2012). "The Decision to Delay Social Security Benefits: Theory and Evidence." In: *SSRN e-Print*, pp. 121–144.
Social Security benefits may be commenced at any time between age 62 and age 70. As individuals who claim later can, on average, expect to receive benefits for a shorter period, an actuarial adjustment is made to the monthly benefit amount to reflect the age at which benefits are claimed. We investigate the actuarial fairness of this adjustment. Our simulations suggest that delaying is actuarially advantageous for a large subset of people,

particularly for real interest rates of 3.5 percent or below. The gains from delaying are greater at lower interest rates, for married couples relative to singles, for single women relative to single men, and for two-earner couples relative to one-earner couples. In a two-earner couple, the gains from deferring the primary earner's benefit are greater than the gains from deferring the secondary earner's benefit. We then use panel data from the Health and Retirement Study to investigate whether individuals' actual claiming behavior appears to be influenced by the degree of actuarial advantage to delaying. We find no evidence of a consistent relationship between claiming behavior and factors that influence the actuarial advantage of delay, including gender and marital status, interest rates, subjective discount rates, or subjective assessments of life expectancy.

Shoven, J. B. and Sialm, C. (2004). "Asset location in tax-deferred and conventional savings accounts." In: *Journal of Public Economics* 88(1-2), pp. 23–38.
This paper derives optimal asset allocations (which assets to hold) and asset locations (in which accounts to hold them) for a risk-averse investor saving for retirement. The investor can hold taxable corporate bonds, tax-exempt municipal bonds, and stocks either in a tax-deferred or a conventional taxable savings account. Taxable bonds have a preferred location in the tax-deferred account and tax-exempt bonds have a preferred location in the taxable account for investors in sufficiently high tax brackets. Tax-efficient stock portfolios (e.g. passively-managed mutual funds) should be held in the taxable account and tax-inefficient stock portfolios (e.g. actively-managed mutual funds) should be held in the tax-deferred account. We show that locating assets optimally can significantly improve the risk-adjusted performance of retirement saving.

Shoven, J. B. and Walton, D. B. (2021). "An Analysis of the Performance of Target Date Funds." In: *The Journal of Retirement* 8(4), pp. 43–65.
This article presents a thorough evaluation of target date funds (TDFs) for the period 2010-2020. TDFs have grown enormously in assets, reaching USD1.4 trillion at the end of 2019, and account for approximately 24% of all assets in 401(k) accounts. We report on the results of a style analysis evaluation of TDFs that determines their effective asset allocation. It examines both the constant in the style analysis regressions and resulting Sharpe ratios, which reflect the over- or under-performance of the funds relative to a passive benchmark with the same asset allocation. Lower cost TDFs tend to match the benchmark returns, while higher cost TDFs deviate from them considerably. We examine how TDFs performed in the stock market crash between February 19 and March 23, 2020, during which five-week period broad market averages fell by about one-third. We find that the value of long-dated TDFs (those with a target date of 2045 and beyond) fell by between 30% and 35%, while the 2025 funds, designed for people roughly 60 years old, lost between 20% and 25% of their value. We find that past performance only weakly influences future expected performance. As with equity funds in general in this period, TDFs with actively managed ingredient funds, on average, trailed the performance of their cheaper passively managed counterparts.

Simsek, K. D., Jeong Kim, M., Chang Kim, W., and Mulvey, J. M. (2018). "Optimal Longevity Risk Management in the Retirement Stage of the Life Cycle." In: *The Journal of Retirement* 5(3), pp. 73–92.
The uncertainty in life expectancy plays a critical role in individual financial planning. Its impact is magnified during the retirement years (the wealth distribution stage of the life cycle), as new sources of income typically are not available to retired persons. Utilizing a multi-stage stochastic program, the authors model and solve the optimal asset allocation problem of a retired couple with uncertain life expectancy in the presence of a term life insurance policy. In the base case, they find optimal policies assuming no longevity risk (i.e., lifetime scenarios are uncertain although life expectancy is fixed on the retirement date). Next, they introduce longevity risk in the scenario generation stage through either a shift in the expected lifetimes or an unexpected cut in periodic retirement income. The authors find that the optimal asset allocation policy depends on the presence and the type of these risks as well as the relative price of insurance and the size of any cut in pension benefits.

Siskos, C. (2021). "What's Your Strategy for Maximizing Your Social Security Benefits?" In: *Kiplinger*.
Deciding when to start taking your Social Security benefits can have significant financial consequences. Here are some of the factors you should think through.

Sivarajan, S. and De Bruijn, O. (2021). "Risk Tolerance, Return Expectations and Other Factors Impacting Investment Decisions." In: *The Journal of Wealth Management* 23(4).
Do risk tolerance questionnaires predict risk-taking? Traditionally, investor preferences have been identified using risk tolerance questionnaires. However, there has been little focus as to whether the questionnaires are actually useful in predicting risk-taking decisions of investors and advisers. The purpose of this research was to review the existing literature and, using an online analytical survey, conduct a statistical analysis to determine the relationship between risk tolerance and risk-taking. The resulting quantitative analysis was complemented with

qualitative interviews. The main findings are that return expectations and demographic variables are important predictors of risk-taking decisions, whereas risk tolerance questionnaires are not. A framework of two models was developed to better understand investment decisions. It forms the basis for a general discussion of the implications of these findings for practitioners and other stakeholders.

Smith, G. and Gould, D. P. (2007). "Measuring and controlling shortfall risk in retirement." In: *The Journal of Investing* 16(1), pp. 82–95.
A key challenge for retired investors is determining the stock-bond asset allocation that, for a given spending rate, provides an acceptable probability of shortfall – having real wealth drop below a specified floor during the investor's lifetime. Standard portfolio analysis yields the well known tradeoff between risk and return described by the Markowitz frontier. For retirement planning, we reconceptualize this as a tradeoff between shortfall probability (risk) and the median value of terminal wealth (return). For specified assumptions, there is a stock-bond asset allocation that minimizes shortfall risk. Portfolios with more stocks increase the median values of terminal wealth, but at the expense of higher shortfall risk. Portfolios with fewer stocks are inferior in that they decrease the median value of terminal wealth and increase shortfall risk. We find that for a variety of plausible assumptions about asset returns, investment strategies, and what constitutes a shortfall, the minimum-risk portfolio generally has between 50 and 70% stocks.

Smith, L. (2021). *The Best Age for Social Security Retirement Benefits*. Tech. rep. SmartAsset.
Let's take a look at how Social Security works, and what you need to know when deciding the best age for your retirement.

Sneddon, T., Bao, C., and Zhu, Z. (2015). "Optimal Asset Liability Management for Post-retirement stage with Income Protection Product." In: *21st International Congress on Modelling and Simulation*.
Australia has a compulsory defined contribution retirement provision system, whereby employers must contribute a proportion of the pre-tax salary of their employees towards an individual account which cannot be accessed until retirement except in extraordinary circumstances. These funds are generally invested in a portfolio of financial assets from which the retiree may draw throughout retirement. Retirees under this system face two key problems when making investment and withdrawal decisions regarding this portfolio. Firstly, retirees must manage their superannuation investment portfolio to maximise their riskadjusted returns and thereby best financially provide for their own retirement. Secondly, retirees must optimise their withdrawal pattern from the superannuation account throughout retirement so as to maximise their post-retirement lifetime utility given the need to minimise the risk of portfolio ruin prior to death. We model this issue as a dynamic stochastic optimisation problem with constraints. The market value of the portfolio is a function of the annual contributions invested by the individual throughout their career and the returns derived from their investment in uncertain financial markets. The post-retirement lifetime utility function is a function of discounted annual income throughout retirement and is therefore subject to market and inflation risk. We model the financial market uncertainties as correlated stochastic processes as projected by a variant on the Wilkie stochastic investment model developed within CSIRO, the SUPA (Simulation of Uncertainty for Pension Analysis) model. We also define an income protection asset (an inflation index linked annuity) which is available to the individual as a tool to hedge inflation risk. We then solve the dynamic superannuation/pension portfolio optimisation problem using a numerical approach that is based on the stochastic control algorithm to calculate the conditional value functions of investors for a sequence of discrete decision dates. The algorithm provides optimal decisions for portfolio asset allocation in financial markets and the optimal amount to shift towards the annuity product on an annual basis to achieve maximum post-retirement lifetime utility whilst minimising the risk of portfolio ruin prior to death.

Sneddon, T., Zhu, Z., and O'Hare, C. (2016). "Modelling defined contribution retirement outcomes: A stochastic approach using Australia as a case study." In: *Australian Journal of Actuarial Practice* 4, pp. 5–19.
In this paper we present a stochastic forecast model in a defined contribution pension system for projecting the accumulation and decumulation phases from an individual fund perspective. We use the Australian superannuation system as the context to demonstrate this "SUPA"(Simulation of Uncertainty for Pension Analysis) model. The SUPA model can be used to simulate the evolution of superannuation fund balances across time during the accumulation and decumulation phases. The model comprises four elements: (1) a stochastic projection of investment returns; (2) a stochastic projection of income levels (upon which contributions to the fund are based); (3) a projection of levels of withdrawal in retirement; and (4) a stochastic projection of increasing longevity (life table). The combination of these four elements within the SUPA model is described in detail in this paper. One application of the model is demonstrated through a case study involving recent Australian

legislative amendments. In this example, we show how the model can be used to forecast likely outcomes (i.e. whether individuals will have sufficient funds in retirement), under the current superannuation structure and a previous structure. This will demonstrate how the SUPA model can be used to model the potential impact of any changes to a superannuation system.

Society of Actuaries Research (2017). *Deciding When to Claim Social Security*. Tech. rep. Society of Actuaries.
The decision to claim Social Security benefits is one of the most important retirement decisions a person will make. Many people decide to claim their Social Security benefits early without considering other options. Perhaps they do not know that they have options or they may think that one option isn't much different from another. Options do exist, however, and some of them can significantly improve retirement security. This Decision Brief discusses many of those options.

Sornwanee, M. (2020). "Multi-Regime, Goal-Based Retirement Solutions: Sensitivity Analysis and Post-Retirement Performance Comparison of Dynamic Strategies." MA thesis. Princeton University.
The shift from defined-benefit to defined-contribution pension plans worldwide has left many individuals who lack access to private banking services without proper retirement financial planning. As individuals have more responsibilities for their retirement planning, there is a growing concern of choosing asset allocation strategies that are not suitable for their goals. Besides, in the light of the current situation regarding the COVID-19 pandemic, individuals must be able to prepare for a possible prolonged "crash" economic regime. Understanding the impact of such a crash regime is important for individuals to make optimal financial decisions. These aspects will be considered in the paper. The objective of this study is to model and comprehend a fully automated retirement planning system for individual investors. By understanding the characteristics of each strategy through various risk measures and analyzing its sensitivity to prolonged crash economic regimes, health risks, savings rate, and replacement income ratio (spending after retirement, divided by the last salary before retirement), we can use the results to recommend the most appropriate asset allocation and savings rate strategy for an individual investor. In this study, we implement multi-regime Monte Carlo simulations to simulate asset returns throughout the accumulation phase (pre-retirement) and decumulation phase (post-retirement). We consider different asset allocation strategies: standard glide path (SGP: linearly reduce equity weight over time), reversed glide path (RGP: linearly increase equity weight over time), "Catch-Up" strategy (CGP: follow SGP and increase equity weight when the wealth falls behind a benchmark portfolio, called "goal portfolio"), "Play-Safe" strategy (PGP: follow SGP and decrease equity weight when the wealth exceeds the goal portfolio), and "Dynamic Zone" glide path (DGP: combine CGP and PGP). We analyze the performances concerning non-goal-based and goal-based risk measures while incorporating the longevity risk and its corresponding risk measures, such as the probability of outliving retirement savings, to further comprehend the characteristics of each policy. We evaluate a goal-based, dynamic strategy that alters the savings rate based on its performance relative to the goal portfolio. That is, it increases the savings rate during a crash regime and decreases the savings rate during a normal regime. Moreover, we conduct sensitivity analyses on 4 parameters: prolonged crash economic regime, health risks, savings rate, and replacement income ratio. The results are analyzed to fully comprehend the strengths and weaknesses of each strategy. First, we recommend PGP for individual investors, especially when there is a prolonged crash regime. Generally, it outperforms other strategies in average maximum drawdown ratio, Goal-at-Risk (GaR), Conditional Goal-at-Risk (CGaR), and the probability of outliving retirement savings. If a 10-year crash regime starts after the middle of the pre-retirement phase, PGP best protects the portfolio from a huge loss during the crisis as it has the lowest probability of outliving retirement savings, which is 18 percentage points lower than that of SGP. With PGP, individuals need to increase their savings rate by 8-31 percentage points, depending on the timing of a 10-year crash regime, to narrow the probability of outliving savings to 10%. However, it is less effective to increase the savings rate only during the crisis or to decrease the replacement income ratio. Besides, for any allocation strategy, the worst time to have a prolonged crash regime is in the middle of the pre-retirement phase, leading to an increase of 69 percentage points in the probability of outliving savings. In terms of savings rate, shifting more savings to an earlier part in the pre-retirement phase can result in a reduction in both the average maximum drawdown and the probability of outliving retirement savings for CGP, PGP, and DGP. Also, the result of the goal-based, dynamic savings rate strategy, in terms of the probability of outliving savings, is equivalent to that of increasing the savings rate for the entire pre-retirement period by 12%.

Sosner, N., Liberman, J., and Liu, S. (2021). "Integration of Income and Estate Tax Planning." In: *The Journal of Wealth Management* 24(1), pp. 78–104.
The preservation and transfer of wealth to future generations are among the central financial goals for most high-net-worth families. Using a stylized theoretical model and Monte Carlo simulations, this article quantifies

the benefits of income and estate tax planning for growing wealth over generations. The article shows that a family that invests with income and estate tax efficiency in mind can achieve substantially higher wealth levels than a family oblivious to taxes. More important, the article demonstrates that a significant value accrues from integrating income tax efficiency and estate tax planning: Becoming efficient with respect to one tax should make the family even more eager to become efficient with respect to the other.

Steinorth, P. and Mitchell, O. S. (2015). "Valuing variable annuities with guaranteed minimum lifetime withdrawal benefits." In: *Insurance: Mathematics and Economics* 64, pp. 246–258.
Variable annuities with guaranteed minimum lifetime withdrawal benefits (VA/GLWB) offer retirees longevity protection, exposure to equity markets, and access to flexible withdrawals in emergencies. We model how risk-averse retirees optimally withdraw from the products, balancing returns and the embedded longevity protection. Assuming reasonable individual preferences, the resulting cash flow generates a Money's Worth Ratio of around 0.9 for our stylized VA/GLWB in the post-crisis product, considerably lower than what was offered prior to 2008. Sensitivity analyses with respect to portfolio choice, mortality, fees, and guaranteed withdrawal rates show that MWRs range from 0.80 to 1.0, with the portfolio choice making the biggest difference. For most parameter choices, the utility value of the VA/GLWB exceeds that of a similar mutual fund, but it is less than for a fixed annuity. Interestingly, VA/GLWB withdrawals in early retirement can optimally exceed contract maximum withdrawals, despite the fact that this reduces future withdrawal guarantees.

Suarez, E. D. (2020). "The perfect withdrawal amount over the historical record." In: *Financial Services Review* 28(2), pp. 96–132.
What has been the perfect withdrawal amount (PWA) from retirement savings accounts in longterm historical data? The PWA is that which, if taken out in the first year of retirement and used again every year adjusted by inflation, leaves exactly the desired final balance on the account. We present the formula for obtaining this measure and evaluate the values it has taken in the past under varying combinations of the relevant parameters. We find that safety-minded investors should enter retirement with a higher stock allocation than what is currently used in most investment funds designed to provide income during retirement.

Suarez, E. D., Suarez, A., and Walz, D. T. (2015). "The Perfect Withdrawal Amount: A Methodology for Creating Retirement Account Distribution Strategies." In: *Financial Services Review* 24(4).
We present a new way to develop withdrawal strategies from retirement portfolios. It is derived analytically, instead of from empirical testing, and it iterates always in the same manner. It is based on a new measure we develop, the Perfect Withdrawal Amount, for which we discuss how to construct a probability distribution and how to apply it sequentially. We also derive a new measure of sequencing risk. We present new strategies built with this framework.

Suhonen, A., Lennkh, M., and Perez, F. (2017). "Quantifying Backtest Overfitting in Alternative Beta Strategies." In: *The Journal of Portfolio Management* 43 (2), pp. 90–104.
The authors investigate the biases in the backtested performance of "alternative beta"strategies using a unique sample of 215 trading strategies developed and promoted by global investment banks. Their results lend support to the cautions in the recent literature regarding backtest overfitting and lack of robustness in trading strategy performance during the "live"period (out of sample). The authors report a median 73 percent deterioration in Sharpe ratios between backtested and live performance periods for the strategies, and they establish a link between performance deterioration and strategy complexity, with the realized reduction in live versus backtested Sharpe ratios of the most complex strategies exceeding those of the simplest ones by over 30 percentage points. The robustness of strategy exposure to risk factors varies between asset classes and strategies; it appears reasonable in equity volatility and FX carry strategies but quite weak in the equity value strategy in particular.

Sun, Q. (2019). "Dynamic Retirement Financial Planning Model." PhD thesis. University of Connecticut.
We developed a retirement financial planning strategy based on Markov chain modeling of retirement health conditions and Geometric Brownian Motion modeling of asset values. The annual living expenses of a retiree are modeled as basic expenses plus discretionary expenses. Our goal is to solve for the maximum discretionary expenses while healthy, which was obtained using a closed-form solution and quantile optimization technique. The highlight of this model is the use of Kalman Filter for annual recalibration. It allows the model to automatically adjust the suggested amount of discretionary expenses by looking at daily fund values from previous year. After running a lot of simulations and testings, we showed that our dynamic model beats other static models and a naive recalibration model in the sense that it virtually eliminates ruin and is able to let a retiree withdraw the largest possible amount.

Sun, W., Triest, R. K., and Webb, A. (2008). "Optimal Retirement Asset Decumulation Strategies: The Impact of Housing Wealth." In: *Asia-Pacific Journal of Risk and Insurance* 3(1).
A considerable literature examines the optimal decumulation of financial wealth in retirement. We extend this research to incorporate housing, which comprises the majority of most households' non-pension wealth.We estimate the relationship between the returns on housing, stocks, and bonds, and simulate a variety of decumulation strategies incorporating reverse mortgages. We show that homeowner's reversionary interest, the amount that can be borrowed through a reverse mortgage, is a surprisingly risky asset. Under our baseline assumptions, we find that the average household would be as much as 24 percent better off taking a reverse mortgage as a lifetime income relative to what appears to be the most common strategy: delaying tapping housing wealth until financial wealth is exhausted and then taking a line of credit. In addition, we show that housing wealth displaces bonds in optimal portfolios, making the low rate of participation in the stock market even more of a puzzle.

Sutcliffe, C. (2015). "Trading death: The implications of annuity replication for the annuity puzzle, arbitrage, speculation and portfolios." In: *International Review of Financial Analysis* 38, pp. 163–174.
Annuities are perceived as illiquid financial securities. Long positions in annuities can be offset using life insurance and debt. The ability to offset annuities may help to reduce the annuity puzzle. Offsetting annuities may also improve the efficiency of the annuities market. Annuities are perceived as being illiquid financial instruments, and this has limited their attractiveness to consumers and their inclusion in financial models. However, short positions in annuities can be replicated using life insurance and debt, permitting long positions in annuities to be offset, or short annuity positions to be created. The implications of this result for the annuity puzzle, arbitrage between the annuity and life insurance markets, and speculation on expected longevity are investigated. It is argued that annuity replication could help reduce the annuity puzzle, improve the price efficiency of annuity markets and promote the inclusion of annuities in household portfolios.

Swedroe, L. (2021). "The Role of Financial Risk Tolerance in Investment Policy." In: *Advisor Perspectives*.
Evaluating an investor's ability, willingness and need to take risk, and then designing his or her portfolio accordingly, are the most crucial functions of investment/financial planning and the "fintech" software that supports the profession. Thus, the ability to estimate an investor's financial risk tolerance (FRT) is key to the success of a financial plan. Financial advisors know that psychological factors, such as financial anxiety, financial satisfaction (the knowledge that one has "enough"), obsession with money, personality type, self-esteem and sensation-seeking behavior, are all important contributors in determining an investor's FRT - the willingness to accept financial risk.

Taljaard, B. H. and Maré, E. (2021). "Why has the equal weight portfolio underperformed and what can we do about it?" In: *Quantitative Finance* 21(11), pp. 1855–1868.
It is widely noted that market capitalisation weighted portfolios are inefficient and underperform an equal weighted portfolio over the long-term. However, at least since 2016, an equal weighted portfolio of stocks in the S&P500 has significantly underperformed the market capitalisation weighted portfolio. In this paper, we analyse this underperformance using stochastic portfolio theory. We show that the equal weighted portfolio does appear to outperform the market capitalisation weighted portfolio over the long-term but with periods of significant short-term underperformance. In addition, we find that concentration in the market capitalisation weighted portfolio has increased in recent years and has contributed to the recent underperformance together with a significantly lower level of diversification benefits. Furthermore, we highlight an approach to improve the performance of a portfolio by dynamically selecting a market cap or an equal weighting using a rudimentary linear regression model.

Tayali, S. T. (2020). "A novel backtesting methodology for clustering in mean–variance portfolio optimization." In: *Knowledge-Based Systems* 209, p. 106454.
The decisions of asset selection and allocation lie at the heart of financial portfolio management. For these challenging tasks, the mathematical programming model of the mean-variance optimization problem proposes to use the concept of diversification. The novel methodology in this article is a representation of the accumulated knowledge of this model from the modern portfolio theory. It is a practical application for portfolio managers to help synthesize the available historical data and to infer rational decisions. The state-of-the-art backtesting methodology integrates the unsupervised machine learning method of clustering analysis into the mean-variance portfolio optimization model. The test results from the proposed novel methodology show that clustering with Euclidean distance measures outperform the results of the benchmark and other specified clustering methods for different datasets, backtesting periods, and temporal scales of major stock indices.

Templin, N. (2021). "When to Tap Social Security: The Sometimes Surprising Answer From Online Calculators." In: *Wall Street Journal*.
The conventional wisdom is to wait as long as possible. But that isn't always the best advice.

Tergersen, A. (2018). "Forget the 4% Rule: Rethinking Common Retirement Beliefs." In: *Wall Street Journal*.
Three retirement-savings rules come into question as stocks and bonds get more expensive and retirements last longer.

Tertilt, M. and Scholz, P. (2018). "To Advise, or Not to Advise Robo-Advisors Evaluate the Risk Preferences of Private Investors." In: *The Journal of Wealth Management* 21(2), pp. 70–84.
Robo-advisors promise efficient, rational, and transparent investment advisory. The authors analyze how robo-advisors determine their users risk tolerance and which equity exposure is derived from the individual risk profile. Findings indicate significant differences in the quality of offered investment advice. Robo-advisors usually ask relatively few questions in the assessment of their users risk profile, and it is particularly surprising that some of the questions do not seem to have any impact on the risk categorization. Moreover, the recommended equity exposure is relatively conservative.

Thanki, H., Karani, A., and Goyal, A. K. (2020). "Psychological antecedents of financial risk tolerance." In: *The Journal of Wealth Management* 23(2), pp. 36–51.
Research indicates that financial risk tolerance (FRT) is a subjective and complex phenomenon and may diverge from individual to individual based on their demographics, genetic makeup, socioeconomic profiles, personality types, and psychological constructs. Past researchers have mainly focused on relating the socioeconomic profile, demographics, and personality type of investors to FRT. This article uses a novel approach that relates other critical behavioral and psychological factors, such as financial anxiety, financial satisfaction, obsession with money, personality type, self-esteem, and sensation-seeking behavior, as equally important contributors for determining the FRT. Confirmatory factor analysis and structural equation modeling techniques were used to test the variables and hypotheses. The results show that financial satisfaction is negatively correlated with FRT, whereas financial anxiety, obsession with money, personality type, self-esteem, and sensation-seeking behavior are positively correlated with FRT. All these variables act as antecedents of FRT.

Tharp, D. T. and Kitces, M. E. (2018). "Life-Cycle Earnings Curves and Safe Savings Rates." In: *The Journal of Retirement* 5(3), pp. 109–120.
Analyses of savings rates needed for succcessful retirement rates (SSRs) typically assume constant real earnings growth throughout one's career. However, data on the life-cycle earnings patterns of millions of U.S. workers suggest that earnings growth does not occur at a constant rate that matches inflation. Instead, earnings tend to increase at a decreasing rate during the early years of one's career and decrease at an increasing rate in the later years. Utilizing simulations of saving and dissaving throughout the life cycle based on both historical market returns and forecasted returns, the authors examine the impact of assuming more realistic earnings growth relative to constant inflation-adjusted growth. Results indicate that failing to account for more realistic earnings curves throughout the life cycle may overstate SSRs for lower-income households while understating SSRs for higher-income households, and understate SSRs for younger households while overstating SSRs for older households. Furthermore, historical SSRs of 10% or less are found for all but the highest-income households after accounting for more realistic earnings curves and Social Security benefits-though variations in specific effects may exist based on the simulation methods utilized.

Thompson, J. R., Feng, L., Reesor, R. M., Grace, C., and Metzler, A. (2021). "Measuring Financial Advice: Aligning Client Elicited and Revealed Risk." In: *SSRN e-Print*.
Financial advisors use questionnaires and discussions with clients to determine a suitable portfolio of assets that will allow clients to reach their investment objectives. Financial institutions assign risk ratings to each security they offer, and those ratings are used to guide clients and advisors to choose an investment portfolio risk that suits their stated risk tolerance. This paper compares client Know Your Client (KYC) profile risk allocations to their investment portfolio risk selections using a value-at-risk discrepancy methodology. Value-at-risk is used to measure elicited and revealed risk to show whether clients are over-risked or under-risked, changes in KYC risk lead to changes in portfolio configuration, and cash flow affects a client's portfolio risk. We demonstrate the effectiveness of value-at-risk at measuring clients' elicited and revealed risk on a dataset provided by a private Canadian financial dealership of over 50,000 accounts for over 27,000 clients and 300 advisors. By measuring both elicited and revealed risk using the same measure, we can determine how well a client's portfolio aligns with their stated goals. We believe that using value-at-risk to measure client risk provides valuable insight to advisors to ensure that their practice is KYC compliant, to better tailor their client portfolios to stated goals,

communicate advice to clients to either align their portfolios to stated goals or refresh their goals, and to monitor changes to the clients' risk positions across their practice.

Thorp, S., Bateman, H., Dobrescu, L., Newell, B., and Ortmann, A. (2020). "Flicking the switch: Simplifying disclosure to improve retirement plan choices." In: *Journal of Banking & Finance* 121, p. 105955.
Standardized information disclosures aim to help people compare complex financial products and make better choices. We investigate the extent to which information shown in a regulator-mandated dashboard helps retirement savers choose between alternative pension plans. We conduct incentivized experiments that collect participants' repeated choices between two pension plans using the mandatory dashboard, and subsequently test whether an even simpler dashboard improves choices, and by how much. Participants switch quickly from a high- to a low-fee pension plan when they see explicit nominal fees but are significantly more confused by percentage fees and adjust slower. When differences between plan performance arise from gross returns, not fees, we find that complex information formats can seriously hinder participants' recognition and reactions. We present a Bayesian updating model which estimates the relative noisiness of the signals from fees and gross returns across different treatments and use this model to show how better information presentation raises retirement savings.

Tian, W. and Zhu, Z. (2020). "Optimal Investing after Retirement Under Time-Varying Risk Capacity Constraint." In: *arXiv e-Print*.
This paper explores an optimal investing problem for a retiree facing longevity risk and living standard risk. We formulate the optimal investing problem as an optimal portfolio choice problem under a time-varying risk capacity constraint. Under the specific condition on model parameters, we show that the value function is a C*C solution of the HJB equation and derive the optimal investment strategy in terms of second-order ordinary differential equations. The optimal portfolio is nearly neutral to the stock market movement if the portfolio's value is at a sufficiently high level; but, if the portfolio is not worth enough to sustain the retirement spending, the retiree actively invests in the stock market for the higher expected return. In addition, we solve an optimal portfolio choice problem under a leverage constraint and show that the optimal portfolio would lose significantly in stressed markets. This paper shows that the time-varying risk capacity constraint has important implications for asset allocation in retirement.

Tomlinson, J. (2017). "Coping with Sequence Risk: How Variable Withdrawal and Annuitization Improve Retirement Outcomes." In: *Advisor Perspectives*.
Both the level and the sequence of investment returns will have a big impact on retirement outcomes. Poor returns during the early years of retirement are bad news. However, the particular withdrawal strategy used affects sequence risk, and an approach where withdrawals are variable and respond to portfolio performance can improve retirement outcomes. I'll examine the evidence and then use my own modeling to show how a strategy that combines variable withdrawals with partial annuitization using a single-premium immediate annuity (SPIA) maximizes the cash available for consumption.

Tomlinson, J. (2018). "Retirement Strategies in Pictures." In: *Advisor Perspectives*.
Advisors providing retirement recommendations need to evaluate strategies and present options to clients. Communication is key and it can be a challenge to come up with the best way to compare alternatives. One way involves presenting metrics such as expected bequests and plan failure probabilities. However, it may be more helpful to use a graphical approach to show the year-by-year progression of funds available during retirement. I'll demonstrate a graphical approach by comparing a variety of strategies.

Tomlinson, J. (2020a). "New Estimates of the Need for Long-Term Care." In: *Advisor Perspectives*.
When the subject of long-term care (LTC) comes up, advisors need to be able to discuss with clients the likelihood of needing such care and its potential duration. These discussions feed into decisions about whether to purchase LTC insurance. In this February 2020 article (and APViewpoint conversation), Allan Roth got the discussion going in terms of both the probabilities of needing care and an evaluation of insurance options. Recently I have analyzed additional data that improves our understanding of LTC needs and informs the decision-making about insurance.

Tomlinson, J. (2020b). "The Unimportance of Asset Allocation in Retirement Planning." In: *Advisor Perspectives*.
Earlier this year, Jeremy Siegel said that, "75/25 is the new 60/40," a recommendation to raise stock allocations to make up for lower bond yields. However, what matters for investors saving for retirement is not the asset class performance, but how those returns translate into retirement consumption. To examine the impact of increasing stock allocations, I'll focus on retirement planning and show impacts on both the average and variability of retirement withdrawals. Spoiler alert - as the title hints, 75/25 versus 60/40 doesn't matter that much.

Traccucci, P., Dumontier, L., Garchery, G., and Jacot, B. (2019). "A Triptych Approach for Reverse Stress Testing of Complex Portfolios." In: *Risk (Cutting Edge)*.
Pascal Traccucci, Luc Dumontier, Guillaume Garchery and Benjamin Jacot present an extended reverse stress test (ERST) triptych approach with three variables: level of plausibility, level of loss and scenario. Any two of these variables can be derived, provided the third is given as input. A new version of the Levenberg-Marquardt optimisation algorithm is introduced to derive the ERST in certain complex cases.

Tretiakova, I. and Yamada, M. S. (2017). "Autonomous portfolio:a decumulation investment strategy that will get you there." In: *The Journal of Retirement* 5(2), pp. 83–95.
The article primary contribution is applying a dynamic decision theory approach to investment management and adding spending flexibility to improve incomes for retirees for whom saving more and working longer are no longer options. We investigate whether a self-driving portfolio, engineered to protect against market risk, can deliver more income than other investment approaches while minimizing the risk of ruin. We consider whether a dynamic strategy, based on a stochastically dominant decision theory algorithm and tested with fixed and variable spending policies, can minimize the chance of running out of money before dying. Outcomes are compared with a number of static approaches. We find that the dynamic strategy provides an average of 51% more income than a target date fund through retirement when combined with a hybrid spending rule over a relevant probability of ruin range of 1% through 25%. Furthermore, when rebalanced to constant risk rather than to fixed asset mixes, sequence of return risk can be reduced. Variable spending policies add effectiveness and should be considered regardless of investment strategy. Importantly, dynamic decision theory establishes a framework for goal-seeking solutions that may have application in other phases of the investor life cycle.

Tsai, C. C.-L. and Cheng, E. S. (2021). "Incorporating statistical clustering methods into mortality models to improve forecasting performances." In: *Insurance: Mathematics and Economics* 99, pp. 42–62.
Statistical clustering is a procedure of classifying a set of objects such that objects in the same class (called cluster) are more homogeneous, with respect to some features or characteristics of objects, to each other than to those in any other classes. In this paper, we apply four clustering approaches to improving forecasting performances of the Lee-Carter and CBD models. First, each of four clustering methods (Ward's hierarchical clustering, divisive hierarchical clustering, K-means clustering, and Gaussian mixture model clustering) is adopted to determine, based on some characteristics of mortality rates, the number and partition of age clusters from the whole study ages 25-84. Next, we forecast 10-year and 20-year mortality rates for each of the age clusters using the Lee-Carter and CBD models, respectively. Finally, numerical illustrations are given with two R packages "NbClust" and "mclust" for clustering. Mortality data for both genders of the US and the UK are obtained from the Human Mortality Database, and the MAPE (mean absolute percentage error) measure is adopted to evaluate forecasting performance. Comparisons of MAPE values are made with and without clustering, which demonstrate that all the proposed clustering methods can improve forecasting performances of the Lee-Carter and CBD models.

Turvey, P. A., Basu, A. K., and Verhoeven, P. (2013). "Embedded Tax Liabilities and Portfolio Choice." In: *The Journal of Portfolio Management* 39(3), pp. 93–101.
Taxes play an important role in determining after-tax investment risk and returns, but many practitioners still make investment decisions based on pre-tax values. The apparent complexity of dealing with deferred capital gains and the question of how these implied future tax liabilities should be valued are central to this problem. The authors use a simple arbitrage argument to show that a risk-free discount rate is appropriate for calculating the present value of future tax liabilities. This lets analysts adjust risk and returns for effective tax rates and present a more accurate picture to the investor. The results show a taxation-induced preference for holding equities over bonds and a location preference for holding equities in a taxable account and bonds in retirement accounts. These important findings contrast with traditional investment advice that suggests a greater capacity for risk in retirement accounts.

Valentine, K. D., Buchanan, E. M., Scofield, J. E., and Beauchamp, M. T. (2019). "Beyond p values: utilizing multiple methods to evaluate evidence." In: *Behaviormetrika* 46(1), pp. 121–144.
Null hypothesis significance testing is cited as a threat to validity and reproducibility. While many individuals suggest that we focus on altering the p value at which we deem an effect significant, we believe this suggestion is short-sighted. Alternative procedures (i.e., Bayesian analyses and observation-oriented modeling: OOM) can be more powerful and meaningful to our discipline. However, these methodologies are less frequently utilized and are rarely discussed in combination with NHST. Herein, we discuss three methodologies (NHST, Bayesian Model comparison, and OOM), then compare the possible interpretations of three analyses (ANOVA, Bayes Factor, and an Ordinal Pattern Analysis) in various data environments using a frequentist simulation study.

We found that changing significance thresholds had little effect on conclusions. Furthermore, we suggest that evaluating multiple estimates as evidence of an effect allows for more robust and nuanced interpretations of results and implies the need to redefine evidentiary value and reporting practices. Recent events in psychological science have prompted concerns within the discipline regarding research practices and ultimately, the validity and reproducibility of published reports (Etz and Vandekerckhove 2016; Lindsay 2015, Open Science Collaboration 2015; van Elk et al. 2015). One often discussed matter is over-reliance, abuse, and potential hacking of p values produced by frequentist null hypothesis significance testing (NHST), as well as misinterpretations of NHST results (Gigerenzer 2004; Ioannidis 2005; Simmons et al. 2011). We agree with these concerns and believe that many before us have voiced sound, generally accepted opinions on potential remedies, such as an increased focus on effect sizes (Cumming 2008; Lakens 2013; Maxwell et al. 2015; Nosek et al. 2012). However, other suggestions have been met with less enthusiasm, including an article by Benjamin et al. (2018) advocating that researchers should begin thinking only of p values less than .005 as "statistically significant", thus changing alpha levels to control Type I error rates. Alternatively, Pericchi and Pereira (2016) promote the use of fluctuating alpha levels as a function of sample size to assist with these errors. Trafimow et al. (2018) critiques this suggestion to broadly lower the alpha level to .005 and suggested that findings should be weighted on the basis of evidence accumulation from multiple studies. We argue that alpha should not be the sole focus of our attention, but rather, we should wonder if a p value should be utilized at all, and, if so, what that p value can tell us in relation with other indicators. While NHST and p values may have merit, researchers have a wealth of other statistical tools available to them. We believe that improvements may be made to the sciences as a whole when individuals become aware of these tools and how these methods may be used, either alone or in combination, to strengthen understanding of data and conclusions. These sentiments have been shared by the American Statistical Association who recently held a conference focusing on going beyond NHST, expanding their previous stance on p values (Wasserstein and Lazar 2016). Therefore, the main goal of this project was to show researchers how two alternative paradigms compare to NHST in terms of their methodological design, statistical interpretations, and comparative robustness. Herein, we will discuss the following methodologies: NHST, Bayes factor comparisons, and observation-oriented modeling. To compare their methodological designs, we first provide historical backgrounds, procedural steps, and limitations for each paradigm. We then simulated data using a three timepoint repeated measures design with a Likert-type scale as the outcome variable to be able to compare the statistical interpretations and comparative robustness. By simulating possible data sets and analyzing them with each of the three paradigms, we will be able to discuss the conclusions these three methods reach given the same data and to compare how often these methodologies agree within different data environments (i.e., given varying sample sizes and effect sizes). Beyond simply comparing methodologies, we also sought to identify how changing the alpha criteria within the NHST framework may alter conclusions. Although previous work has already compared Frequentist NHST to Bayesian approaches (Goodman 1999; Rouder et al. 2012; Wetzels et al. 2011), this manuscript adds a novel contribution: observation-oriented modeling. By introducing social scientists to observation-oriented modeling (OOM), a relatively new paradigm that is readily interpretable, we will show both how useful this paradigm can be in these contexts, and how it compares to two well-known methods. We hope that by discussing these methodologies in terms of a simple statistical analysis researchers will be able to easily compare and contrast methodologies.

van Bilsen, S. and Bovenberg, A. L. (2020). "The decumulation period of a personal pension with risk sharing: investment approach versus consumption approach." In: *Journal of Pension Economics and Finance* 19(2), pp. 262–291.

This paper models the decumulation period of a Personal Pension with Risk sharing (PPR). We derive several relationships between the contract parameters. Individuals can adopt two approaches to the decumulation period of a PPR: the investment approach and the consumption approach. In the investment approach, individuals specify how to invest wealth and how much wealth to withdraw. Retirement consumption follows endogenously. In the consumption approach, in contrast, individuals specify retirement consumption exogenously. Investment and withdrawal policies follow endogenously. We explore these two approaches in detail. Consistent with habit formation, we allow for excess smoothness and excess sensitivity in retirement consumption.

van Bilsen, S. and Linders, D. (2019). "Affordable and adequate annuities with stable payouts: Fantasy or reality?" In: *Insurance: Mathematics and Economics* 86, pp. 19–42.

This paper introduces a class of unit-linked annuities that extends existing annuities by allowing portfolio shocks to be gradually absorbed into the annuity payouts. Consequently, our new class enables insurers to offer an affordable and adequate annuity with a stable payout stream. We show how to price and adequately hedge

the annuity payouts in a general financial environment. In particular, our model accounts for various stylized facts of stock returns such as asymmetry and heavy-tailedness. Furthermore, the generality of our framework makes it possible to explore the impact of a parameter misspecification on the annuity price and the hedging performance.

Van Harlow, H. V. and Brown, P. (2016a). "Market Risk, Mortality Risk, And Sustainable Retirement Asset Allocation: A Downside Risk Perspective." In: *Journal of Investment Management* 14(2).
Despite its clear importance, there is no consensus on the optimal asset allocation strategy for retirement investors of varying age, gender, and risk tolerance. This study analyzes the allocation question by focusing on the downside risks that result from the joint uncertainty over investment returns and life expectancy. Using a new analytical approach, we show that concentrating on the severity of retirement funding shortfalls, rather than just the probability of ruin, markedly increases the sustainability of a retirement portfolio. We demonstrate that for retirement investors attempting to minimize downside risk while sustaining future withdrawals, appropriate equity allocations range between five and 25 percent, levels that are strikingly low compared to those typically found in life-cycle funds. Further, these optimal portfolio constructions appear to vary little with alternative capital market assumptions.We also show that more aggressive investors having substantial bequest motives should still be relatively conservative in their stock allocations. We conclude that the higher equity allocations commonly employed in practice significantly underestimate the risks that these higher-volatility portfolios pose to the sustainability of retirement savings and incomes.

Van Harlow, W. (2014). *Optimal Asset Allocation in Retirement: A Downside Risk Perspective*. Tech. rep. Putnam Institute.
Once an individual has retired, asset allocation becomes a critical investment decision. Unfortunately, there is no consensus on what the optimal allocation should be for retirees of varying age, gender and risk tolerance. This study analyzes the allocation question through a focus on the downside risks created by uncertainty over investment returns and life expectancy. We find that the range of appropriate equity asset allocations in retirement is strikingly low compared with those of typical lifecycle and retirement funds now in the marketplace. In fact, for retirement portfolios whose primary goal is to minimize the risk of depletion and sustain withdrawals, optimal equity allocations range between 5 percent and 25 percent. This quite conservative level of equity holdings changes little even when we significantly change our assumptions on capital market returns. We even find that more aggressive equity allocations, those that still retain some focus on depletion risk but also seek to provide substantial bequests to heirs, are also relatively conservative. The study suggests, in short, that the higher equity allocations used in many popular retirement investment products today significantly underestimate the risks that these higher-volatility portfolios pose to the sustainability of retirees' savings and to the incomes on which retirees depend.

Van Harlow, W. and Brown, K. C. (2016b). "Improving the Outlook for a Successful Retirement: A Case for Using Downside Hedging." In: *The Journal of Retirement* 3(3), pp. 35–50.
One of the biggest risks to a successful retirement is the exposure of savings to poor investment returns in the early stages of the retirement. Mitigating this "sequence-of-returns"risk is in consequence an important investment question. In this study, we conduct extensive simulation analysis to show that for sustainable withdrawal rates, hedging with costless collars or with put options can eliminate or significantly reduce funding shortfall risk for a retirement portfolio. In addition, we demonstrate with a few examples that, for a given level of shortfall risk, hedging can increase the income generated by retirement savings by almost 40 percent. Thus, downside hedging strategies within retirement portfolios appear to offer attractive benefits to retirees worried about outliving their income resources.

Van Harlow, W. and Brown, K. C. (2017). "Health State and the Savings Required for a Sustainable Retirement." In: *The Journal of Retirement* 4(4), pp. 25–38.
Life expectancy varies greatly with health conditions, but very rarely is this relationship incorporated into retirement investment planning. This is surprising given that the vast majority of older households have one or more adverse health conditions. The authors show that the savings required to fund a successful retirement for someone with one of several diseases whose impact on life expectancy has been estimated can be reduced by as much as 26 percent for females and 33 percent for males relative to the savings required for a healthy individual. Similarly, the savings required to fund healthcare expenses in retirement can be reduced by 29 percent to 39 percent. The interaction of projected health costs and the effect of disease on life expectancy have counterintuitive effects on retirement planning. For example, the higher retiree healthcare expenses associated

with conditions such as diabetes and tobacco use are offset by reduced life expectancies. The net effect is that, in certain health states, less savings are required for healthcare than would be necessary for a healthy individual.

Van Harlow, W., Brown, K. C., and Jenks, S. E. (2020). "The Use and Value of Financial Advice for Retirement Planning." In: *The Journal of Retirement* 7(3), pp. 46–79.
Offering professional advice around the retirement planning process represents an important component of the financial services industry. The authors examine the demographic, investment, and behavioral characteristics of individuals who obtain this advice as well as the economic value that it ultimately adds. Using a survey of more than 4,000 working households, they find that wealth and income levels are positively correlated with the decision to engage a professional advisor, as are factors such as marital status, age, and education level. To assess the value added by this advice, the authors develop a unique metric of retirement income replacement that incorporates health-based life expectancy and household-specific financial circumstances. The approach estimates the percentage of annual pre-retirement income that a household will be able to spend each year in retirement. The authors establish the unconditional finding that advised households generate significantly larger proportions of post-employment spending (both gross and net of Social Security benefits) than do nonadvised households. Controlling for additional explanatory factors, we find that an advisor adds more than 15 percentage points of income replacement in retirement. These findings support the conclusion that obtaining and implementing financial advice in the retirement planning process leads to a demonstrable increase in the level of sustainable retirement spending.

Vandenbroucke, J. and Fortuna, G. (2019). "Loss Aversion Implied by a Risk-Based Questionnaire." In: *The Journal of Wealth Management* 22(1), pp. 39–48.
This article uses data from an existing classical risk-based questionnaire to define subgroups of investors that are expected to exhibit a different attitude towards loss. Field research confirms that differences in revealed loss attitude match the model prediction even when selecting investors with the same classical risk profile. The study should motivate to define investor profiles based on two coordinates rather than just one, meaning a combination of risk and latitude vis-a-vis losses. Such behavioral investor profiles improve customer centricity, contribute to a long-term relationship and simply increase the likelihood of right-selling individual products.

Vanguard Research (2020). *Dynamic spending: A better way to budget in retirement*. Tech. rep. Vanguard.
Financial markets are unpredictable – they can bring tremendous gains one day and post painful declines the next. If you're close to retirement or already retired, this can pose a significant concern – but it doesn't have to. Investors use a number of methods to figure out how much they can spend in retirement, but one, in particular, stands out: a dynamic spending approach. Dynamic means it's flexible – by making minor changes to your spending in any given year in response to the markets' performance, you can help sustain your portfolio during market downturns (while sacrificing little) and have more to spend following high-performing years.

Vaughan, C. (2017). "RMD Arbitrage: Strategies for Legally Delaying and Reducing RMDs." In: *Journal of Financial Planning* 30(6), pp. 49–57.
Most American households have an age gap of greater than one year between the spouses. For couples with an age gap, required minimum distributions (RMDs) will start in different years. This disparity creates an opportunity to maximize late-life portfolio values by maximizing the assets subject to the younger spouse's RMD schedule. This paper identifies numerous strategies to maximize the assets in the younger spouse's account. The primary benefit of these strategies is that they allow the couple to maximize the tax deferral of the assets by delaying and reducing RMDs. Although a surviving younger spouse would typically want to roll the assets into their own IRA, a surviving older spouse may benefit from keeping the assets in an inherited spousal IRA in certain situations.

Veres, B. (2020). "A Comparison of Risk Tolerance Products." In: *Advisor Perspectives*.
Bob Veres compares Risk Tolerance Products from Andes Wealth Tech, Finametrica, Riskalyze and Totum Risk. Bear markets beget portfolio losses, unhappy clients and, sadly, lawsuits against advisors. If you relied on any of the popular risk tolerance products to construct that portfolio, here's how you are likely to fare under the careful scrutiny of an arbitrator.

Vincent, K., Hsu, Y.-C., and Lin, H.-W. (2018). "Analyzing the Performance of Multifactor Investment Strategies under a Multiple Testing Framework." In: *The Journal of Portfolio Management* 44(4), pp. 113–126.
Evaluating portfolios based on numerous combinations of factors using the individual backtesting method could suffer from serious data mining bias and lead to spurious significant findings. Accordingly, the authors employ a multiple hypothesis testing method to examine the multifactor portfolio performance. Their empirical results show that even after they adjust for the multiple comparisons bias, stock-picking strategies with certain combined

firm characteristics could generate significantly better liquidity risk-adjusted returns. In addition, the outperforming multifactor strategies that the authors report are robust to alternative definitions of factors. However, they observe that the number of significantly profitable multifactor portfolios has decreased substantially in the era of increased liquidity and trading activity in the U.S. stock market.

Vovk, V. and Wang, R. (2020). "True and false discoveries with e-values." In: *arXiv e-Print*.
The topic of this paper is multiple hypothesis testing based on e-values, which are Bayes factors stripped of their Bayesian content. Using e-values instead of p-values, which are standard in this area, leads to simple and efficient procedures that control the number of false discoveries under arbitrary dependence of the base e-values. We prove an optimality result for our main procedure and demonstrate advantages of our methods over standard methods using simulated and real-world datasets.

Vovk, V. and Wang, R. (2021). "E-values: Calibration, combination, and applications." In: *Annals of Statistics* 49(3), pp. 1736–1753.
Multiple testing of a single hypothesis and testing multiple hypotheses are usually done in terms of p-values. In this paper we replace p-values with their natural competitor, e-values, which are closely related to betting, Bayes factors, and likelihood ratios. We demonstrate that e-values are often mathematically more tractable; in particular, in multiple testing of a single hypothesis, e-values can be merged simply by averaging them. This allows us to develop efficient procedures using e-values for testing multiple hypotheses.

Vrdoljak, N., Laster, D., and Suri, A. (2014). "The Role of Variable Annuities in Addressing Retirement Risks." In: *The Journal of Retirement* 2(2), pp. 55–66.
Investors commonly fund their retirement through portfolios of stocks and bonds or mutual funds, which offer scant protection against the risk of a market downturn or of outliving their wealth. This article examines how adding a variable annuity with a guarantee lifetime withdrawal benefit (VA + GLWB) to a balanced portfolio can help reduce the risk of running out of money in retirement. It quantifies, through simulation analysis, the degree to which an allocation to a VA + GLWB can help mitigate two key retirement risks: longevity and sequence of returns risk. This risk reduction comes at the cost of a decrease in the expected future bequest. The article also explores how the timing of withdrawals and the asset allocation within a VA + GLWB impacts retirement outcomes.

Walker, P., Sacks, B. H., and Sacks, S. R. (2021). "To Reduce the Risk of Retirement Portfolio Exhaustion, Include Home Equity as a Non-Correlated Asset in the Portfolio." In: *Journal of Financial Planning* 34(12), pp. 82–97.
The conventional wisdom dictates that you should take distributions from your portfolio first and only from home equity as a backup. But including home equity as part of the portfolio can greatly increase the probability of maintaining constant purchasing power throughout retirement.

Wallick, D. W., Berkowitz, D. B., Clarke, A. S., DiCiurcio, K. J., and Stockton, K. A. (2018). "Getting More from Less in Defined Benefit Plans." In: *How Persistent Low Returns Will Shape Saving and Retirement*. Oxford University Press.
As global interest rates hover near historic lows, defined benefit pension plan sponsors must grapple with the prospect of lower investment returns. We examine three levers that can enhance portfolio outcomes in a low-return world: increased contributions; reduced investment costs; and increased portfolio risk. We use portfolio simulations based on a stochastic asset class forecasting model to evaluate each lever according to two criteria: the magnitude of impact and the certainty that this impact will be realized. We show that increased contributions have the greatest and most certain impact. Reduced costs have a more modest, but equally certain impact. Increased risk can deliver a significant impact, but with the least certainty.

Wang, J., Meric, G., Liu, Z., and Meric, I. (2011). "The Determinants of Stock Returns in the October 9, 2007 March 9, 2009 Bear Market." In: *The Journal of Investing* 20(3), pp. 18–24.
The bear market of October 9, 2007 March 9, 2009, was the worst in U.S. history since the Great Depression. During this period, U.S. stocks lost about 58 percent of their value in just 16 months. Because of declining real estate prices, foreclosures, and the large amount of mortgage-backed assets held by banks, the amount of bank credit available to business firms was sharply reduced, creating a serious liquidity shortage. The authors test the hypothesis that technical insolvency and bankruptcy risks were significant determinants of stock returns in the October 9, 2007 March 9, 2009 bear market. They also test several hypotheses related to the effects of beta, firm size, market-to-book ratio, and volatility on stock returns in bear markets and stock market crashes.

Waring, M. B. and Siegel, L. B. (2018). "What Investment Risk Means to You, Illustrated Strategic Asset Allocation, the Budget Constraint, and the Volatility of Spending During Retirement." In: *The Journal of Retirement* 6(2), pp. 7–26.

In our experience, many investment professionals don articulate risk well to clients. This research uniquely and graphically reveals the nature of strategic asset allocation investment risk not only for single-period investors, but also for multi-period investors such as those whose savings are held for retirement. Using Monte Carlo simulations, we evaluate and picture the nature of that multi-period consumption risk with spending rules that are subject to a budget constraint and those that aren (such as the 4% rule). Risk in a multi-period context means that realized spending may increase with greater SAA risk, but it may instead be worse.

Warren, G. J. (2019). "Design of Investment Options using Utility Functions: A Demonstration for Products." In: *SSRN e-Print*.

Utility functions can assist in designing a menu of investment options by encoding the objectives and preferences for various investor types with the aim of developing suitably tailored products. This paper demonstrates the process through the construction of an illustrative menu of Australian MyRetirement products. The demonstration shows how utility-based analysis can be combined with more traditional metrics to compare strategies and convey their implications; and provides direction on how product options might be presented to a retiring member. Central to the process is characterizing investor types by selected attributes. This both facilitates specifying utility functions for use in product development, and supports communication based around the type of investor for which a product is designed along with its key features and the outcomes it may deliver. Utility functions thus provide the mechanics that drive the analysis, without becoming an explicit component of investor engagement.

Warshawsky, M. (2018). "Reforming Retirement Income: Annuitization, Combination Strategies, and Required Minimum Distributions." In: *SSRN e-Print*.

Laddered immediate life annuity purchase and asset withdrawal combination strategies represent an excellent way for retirees to manage their retirement assets in order to get lifetime income in a flexible manner while still maintaining growth, liquidity, and bequest potentials. Simulations show that even by age 95, a retiree using this strategy will get higher income, in inflation-adjusted terms, on average and across most scenarios than by using full and complete annuitization or just using the common 4 percent withdrawal rule. Moreover, a significant fund balance remains to the retiree throughout life, on average, but with no risk of running out of assets. The minimum distribution requirements that govern tax-qualified retirement accounts for older retirees should be reformed to encourage the use of these partial annuitization combination strategies. This broad reform of the required minimum distribution rules needs to be done for the same reason the Obama administration made a narrow exception to the required minimum distribution rules for longevity insurance to achieve the reasonable public policy goal of encouraging the use of partial annuitization by retirees.

Weber, E. U. and Klement, J. (2018). "Risk tolerance and circumstances." In: *CFA Institute Research Foundation Briefs* 4(2).

An investor risk attitude is a stable characteristic, like a personality trait, but risk-taking behavior can change based on the investor age, recent market events, and life experiences. These factors change investors perceptions of the risks. Differences in risk tolerance between men and women or in different circumstances trace back to emotional as much as rational considerations. Financial advisers should consider all of these factors when advising clients and can use four simple steps to incorporate best practices: be aware, educate, nudge, and hand hold. The term risk tolerance is defined and used in different ways. Whether risk tolerance is a stable characteristic of a given investor or also takes into account external circumstances (e.g., economic shocks or the domain of the decision) depends on how it is defined and measured. This brief focuses on a definition of risk tolerance prevalent in the practitioner community, an investor willingness to take perceived risk or the trade-off an investor is willing to make between the perceived risk and expected return of different investment choices. This definition derives from a psychological interpretation of the risk-return framework of classical portfolio theory. It treats risk tolerance as an attitude toward risk and decouples this pure attitudinal variable from the perceptions of risks and returns variables in their own right and distinct from the expected value and variance of the distribution of possible outcomes. Defined in this way, risk tolerance may differ among investors as a function of socioeconomic and biological differences but (with the exception of a brief boost during adolescence) shows stability across an investor lifespan, financial shocks, and other circumstances. Risk tolerance, in this sense, is the mediator that translates perceptions of risk and situational needs and constraints into decision and action. The variables that change with market conditions and other circumstances are investors perceptions of investment risks and expectations of return. In contrast to risk tolerance, which attaches to an individual and her biological makeup and personality, these variables change over time in response to changing external conditions. Therefore, an investor risk-taking behavior (as revealed by her investment decisions) can look like it has changed, despite the stability in her risk tolerance. Perceived risks and expected returns are influenced by hopes and fears as

much as by past returns and rational expectations and thus need to be assessed in their own right and possibly corrected. Managing investor emotions through the ups and downs of financial markets is arguably a financial adviser most important task. Calm times provide an opportunity to discuss and formulate an investment policy for each client that can be consulted when emotions are running high. Managing risk perceptions requires the financial adviser to act more like a therapist than a mechanic. It is above all about managing expectations and emotions and helping clients to better deal with emotions when it comes to financial decisions. The end result of this process might be a portfolio that is not in the sense of modern portfolio theory, with its assumption of econs, but rather a portfolio that the human need for investments that can be handled in the presence of changing emotions and changing risk perceptions.

Wiafe, O. (2015). "Investment strategies in retirees' decumulation phase." PhD thesis. Queensland University of Technology.

This thesis consists of three studies on investment strategies for Australian retirees. Specifically, it investigates retirees' preference between alternative drawdown strategies in the presence of government pensions, appropriate management of longevity risk through the use of deferred annuities and asset allocation in retirement. It finds drawdown strategies linked to life expectancy to be the best performers. Deferred annuities are found to improve retirement incomes for risk averse retirees. For retirees who want to meet certain wealth thresholds in retirement, equity dominated portfolios provide superior outcomes for higher threshold levels.

Wiafe, O. K., Basu, A. K., and Chen, E. T. (2020). "Portfolio choice after retirement: Should self-annuitisation strategies hold more equities?" In: *Economic Analysis and Policy* 65, pp. 241–255.

We compare the performance of alternative investment strategies in the decumulation phase for retirees who self-annuitise. We find that portfolios with constant high exposure to equities, as well as portfolios that increase exposures to equities over time, consistently outperform conservative portfolios which avoid investment in equities or the conventional lifecycle portfolios which reduce allocation to equities over time. While an increasing equity glidepath improves the performance of an investment strategy, the allocations to equities at the start of retirement is critical. Using a "utility of terminal wealth" approach that allows for loss aversion as in prospect theory of Kahneman and Tversky (1979), we find a growth portfolio with very high (but not total) exposure to equities to dominate the alternative strategies at low and moderate thresholds. With further increases in wealth threshold levels, a strategy with an all equity allocation becomes dominant. The lifecycle portfolio is dominated by the "reverse lifecycle" portfolio at all threshold levels. Finally, for retirees who defer annuitisation, we find strategies with a higher equity component have greater likelihood of outperforming an immediate annuitisation strategy in terms of generating a guaranteed income late in retirement.

Wiafe, O. K., Basu, A. K., and Chen, E.-T. ((2016). "Asset Allocation in Retirement: Does Glide Path Matter?" In: *SSRN e-Print*.

We compare the performance of the commonly nominated default retirement investment option, the lifecycle fund, to alternative investment strategies during retirees' decumulation phase. Under different shortfall risk measures, we find balanced portfolios with constant exposure to equities, equity dominated portfolios as well as 'reverse lifecycle' portfolios that increase exposures to equities over time to consistently outperform the conventional lifecycle portfolio. While an increasing equity glidepath improves the performance of an investment strategy, the starting asset allocations are equally important. Using a utility-of-terminal wealth approach which allows for loss aversion as discussed in prospect theory by Kahneman and Tversky (1979), we find the Growth portfolio to dominate the alternative strategies at low and moderate thresholds. With increasing wealth threshold levels, a strategy with all equity allocations becomes dominant. The lifecycle portfolio is dominated by the 'reverse lifecycle' portfolio at all threshold levels.

Wiafe, O. K., Basu, A. K., and Chen, E.-T. J. (2014). "Portfolio Strategies in Decumulation Phase: Does Lifecycling Fail?" In: *SSRN e-Print*.

We compare the performance of the commonly nominated default retirement investment option, the lifecycle fund, to alternative investment strategies during retirees' decumulation phase. Under different shortfall risk measures, we find balanced portfolios with constant exposure to equities, equity dominated portfolios as well as 'reverse lifecycle' portfolios that increase exposures to equities over time to consistently outperform the conventional lifecycle portfolio. While an increasing equity glidepath improves the performance of an investment strategy, the starting asset allocations are equally important. Using a utility-of-terminal wealth approach which allows for loss aversion as discussed in prospect theory by Kahneman and Tversky (1979), we find the Growth portfolio to dominate the alternative strategies at low and moderate thresholds. With increasing wealth threshold

levels, a strategy with all equity allocations become dominant. The lifecycle portfolio is dominated by the 'reverse lifecycle' portfolio at all threshold levels.

Wiecki, T., Campbell, A., Lent, J., and Stauth, J. (2016). "All That Glitters Is Not Gold: Comparing Backtest and Out-of-Sample Performance on a Large Cohort of Trading Algorithms." In: *The Journal of Investing* 25(3), pp. 69–80.

When automated trading strategies are developed and evaluated using backtests on historical pricing data, there exists a tendency to overfit to the past. Using a unique dataset of 888 algorithmic trading strategies developed and backtested on the Quantopian platform, with at least six months of out-of-sample performance, this article studies the prevalence and impact of backtest overfitting. Specifically, the authors find that commonly reported backtest evaluation metrics, such as the Sharpe ratio, offer little value in predicting out-of-sample performance ($R2 < 0.025$). In contrast, higher-order moments, such as volatility and maximum drawdown, as well as portfolio construction features (e.g., hedging), show significant predictive value of relevance to quantitative finance practitioners. Moreover, in line with prior theoretical considerations, the authors find empirical evidence of overfitting-the more backtesting a quant has done for a strategy, the larger the discrepancy between backtest and out-of-sample performance. Finally, they show that by training nonlinear, machine-learning classifiers on a variety of features that describe backtest behavior, out-of-sample performance can be predicted with much greater accuracy ($R2 = 0.17$) on hold-out data than when using linear, univariate features. A portfolio constructed by using predictions on hold-out data performed significantly better out-of-sample than one constructed from algorithms with the highest backtest Sharpe ratios.

Williams, P. D. (2021). "Inflation Expectations in the U.S.: Linking Markets, Households, and Businesses." In: *SSRN e-Print*.

Inflation has been below the Federal Reserve's target for much of the past 20 years, creating worries that inflation may be deanchoring from the FOMC's target. This paper uses a factor model that incorporates information from professional forecasters, household and business surveys, and the market for Treasury inflation protected securities (TIPS) to estimate long-run inflation expectations. These have fallen notably in the past few years (to roughly 1.9 percent for CPI inflation, well below the FOMC's target). It appears that, even before the covid recession, the private sector viewed the economy as likely to suffer from persistent headwinds to inflation.

Winter, P. and Planchet, F. (2022). "Modern tontines as a pension solution: a practical overview." In: *European Actuarial Journal* 11.

In the context of global aging population, improved longevity and ultra-low interest rates, the question of pension plan under-funding and adequate elderly financial planning is gaining awareness worldwide, both among experts, regulatory bodies, and popular media. Additional emergence of societal changes-Peer to Peer business model and Financial Disintermediation-have contributed to the resurgence of the concept of "Tontines" in various papers and the proposal of further models. These generalizations can offer efficient decumulation schemes with high longevity protection which is particularly well adapted for retirement needs-both for its members and carriers. In this paper, we revisit the mechanism proposed by Fullmer and Sabin (Journal of Accounting and Finance, 2019.)-which allows the pooling of Modern Tontines through a self-insured community. This "Tontine" generalization retains the flexibility of an individual design: open contribution for a heterogeneous population, individualized asset allocation and predesigned annuitization plan. The actuarial fairness is achieved by allocating the deceased proceedings to survivors using a specific individual pool share which is a function of the prospective expected payouts for the period considered. After a brief introduction, this article provides a formalization of the mathematical framework with prospective analysis, characterizes the inherent bias, generalizes the mechanism to joint lives, and analyses simulated outcomes based on various assumptions. A reverse moral hazard limit is exposed and discussed (the "Term Dilemma"). Some solutions are then proposed to overcome scheme shortcomings and some requirements for practical implementation are discussed.

Woerheide, W. J. and Nanigian, D. (2011). "Sustainable Withdrawal Rates from Retirement Portfolios: The Historical Evidence on Buffer Zone Strategies." In: *SSRN e-Print*.

This paper evaluates the performance of portfolios that dynamically adjust one's mix of risky and risk-free assets during retirement based on the performance of risky assets. We analyze these "buffer zone" portfolio decumulation strategies over various time periods from 1926 to 2009. We find that they are generally inferior to a simpler static asset allocation at minimizing longevity risk.

Wolfe, B. and Brazier, R. (2018). *Spending retirement assets ... or not?* Tech. rep. BlackRock.

Something unexpected has been the shared experience for our most recent generation of retirees. The vast majority haven't been spending their retirement savings - leaving nest eggs mostly untouched and living on

ready sources of income instead. However, future retirees may be less fortunate.
While on the surface this is indeed good news - and appears to support the argument that fears of a future retirement crisis are overstated, the conditions that supported this spending and savings behavior are unlikely to persist. Future retirees will face a much different retirement landscape and will need to adopt new sets of skills – behavioral and financial – that will help them systematically tap into retirement savings to support future spending.
Financial industry norms and academic theories have always assumed assets accumulated for retirement would be systematically withdrawn - following the "4% rule" or some other rule of thumb or system - by retirees in order to maintain a consistent standard of living. Technically, this is referred to as "consumption smoothing" whereby individuals seek to have consistent spending on par with pre-retirement levels. With concerns that retirement savings for individuals may be dangerously low,1 the fear has been that withdrawals for such smoothing could leave retirees running out of funds well short of their passing away.
This research conducted by the BlackRock Retirement Institute (BRI) in conjunction with the Employee Benefit Research Institute (EBRI) found that on average across all wealth levels, most current retirees still have 80% of their pre-retirement savings after almost two decades in retirement.
This is significant because:

- These findings begin to challenge industry norms and academic theories about lifecycle consumption especially during the retirement phase
- Across all wealth levels measured, more than one third of current retirees grew their assets – leaving considerable potential consumption on the table
- Late in life out-of-pocket medical expenses – a major reason to retain assets – do not appear to be warranted except for a very small portion of the population
- The financial landscape for future retirees will most likely be more challenging, requiring different savings and spending behaviors

This paper sets to lay the foundation for how retirees have managed their sources of cash– assets and income– against their spending behaviors. The resulting "husbanding" of assets over the past two decades may be due to a host of favorable environmental factors current retirees benefited from during their working, accumulation years. These included beneficial changes to Social Security and Medicare, a relatively high percentage of jobs that offered defined benefit pensions, strong real estate appreciation and an investment market that generally delivered strong returns and high interest rates.
Has the confluence of these factors created a situation whereby retirees may not have felt the pressure to draw down principal from retirement savings in order to maintain a reasonable standard of living? Perhaps retirees had other plans for their assets beyond themselves – bequests or charitable donations come to mind. Possibly they would have preferred to spend more freely but lacked the financial confidence or tools to efficiently decumulate their assets or were worried about end-of-life healthcare expenses? Looking further, perhaps there were strong emotional biases at play – with fear of outliving retirement assets at the top of the list.
Methodology
The objective of the study was to analyze the "how" and some possible "whys" spending and liquid assets change during retirement, taking into account (non-housing) assets, income, spending, out-of-pocket medical expenses and bequests. Data was collected from the bi-annual Health and Retirement Study (HRS, 1992-2014) and the Consumption and Activities Mail Survey (CAMS, 2005-2015). A sample of 7,148 retiree households provided self-reported asset data and out-of-pocket medical expenditure and a subsample of 1,660 households provided the household expenditure data. Retirees were segmented into three groups based on pre-retirement non-housing retirement assets – $0 to less than $200,000 (lowest wealth), $200,000 to less than $500,000 (medium wealth) and $500,000 and above (highest wealth).

Wolfe, B. and Ferraro, M. (2022). *Decumulation challenges and potential solutions – Helping build a pathway towards retirement spending and income confidence*. Tech. rep. BlackRock.
Something unexpected has been the shared experience for many of our most recent generation of retirees that will reverberate across the investment and insurance industries. The vast majority haven't been spending their retirement savin – leaving nest eggs mostly untouched and living on ready sources of income instead. However, future retirees may be less fortunate, and may need to spend down principal in order to fund their spending needs. In this paper, we summarize the findings of The BlackRock Retirement Institute's research and white

paper, Spending retirement assets ... or not? and provide a framework for advisors to engage with their clients around the challenges in store as they transition from asset accumulation to retirement income.
Key findings:

1) On average across all wealth levels, most current retirees still have 80% of their pre-retirement savings after almost two decades in retirement
2) More than one third of current retirees actually grew their assets – leaving considerable potential consumption on the table
3) The financial landscape for future retirees will most likely be more challenging, requiring different savings and spending behaviors
4) Future retirees are going to need retirement income solutions that can provide spending confidence – for both essential spending needs and more discretionary "wants" – and insurance solutions can play an important role within an integrated retirement income and investment strategy
5) The potential role of insurance can be better understood with the use of illustrative tools that show investors how well insurance products can be integrated into investment plans to fully optimize hard-earned savings in retirement and spend with greater certainty

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Wu, H., Wang, X., Liu, Y., and Zeng, L. (2020). "Multi-period optimal investment choice post-retirement with inter-temporal restrictions in a defined contribution pension plan." In: *Journal of Industrial & Management Optimization* 16(6), pp. 2857–2890.
This paper studies a multi-period portfolio selection problem during the post-retirement phase of a defined contribution pension plan. The retiree is allowed to defer the purchase of the annuity until the time of compulsory annuitization. A series of investment targets over time are set, and restrictions on the inter-temporal expected values of the portfolio are considered. We aim to minimize the accumulated variances from the time of retirement to the time of compulsory annuitization. Using the Lagrange multiplier technique and dynamic programming, we study in detail the existence of the optimal strategy and derive its closed-form expression. For comparison purposes, the explicit solution of the classical target-based model is also provided. The properties of the optimal investment strategy, the probabilities of achieving a worse or better pension at the time of compulsory annuitization and the bankruptcy probability are compared in detail under two models. The comparison shows that our model can greatly decrease the probability of achieving a worse pension at the compulsory time and can significantly increase the probability of achieving a better pension.

Wu, S. (2021). "Assessing the relationship between health and household portfolio allocation." In: *Financial Planning Review* 4(4).
This paper surveys the literature on the relationship between health and household portfolio allocation and provides updated empirical analysis based on recent data. Prior research finds robust evidence for cross-sectional correlations between measures of health status and portfolio decisions, but establishing the causal pathways and underlying mechanisms has proven more difficult and complex. Analysis from the most recently available 2016 and 2018 waves of the Health and Retirement Study yields results that are consistent with existing literature. Households with worse self-reported health have a lower probability of holding various types of financial assets and invest a higher share of their portfolios in safe assets, relative to other asset categories. However, there is only weak evidence that new health shocks to a household change portfolio holdings. The paper concludes with a discussion of the implications of this research and directions for future work.

Xu, G. (2015). "The risk profiles of 401(k) accounts." In: *The Journal of Retirement* 2(3), pp. 67–77.
This article examines the relationship between the riskiness of the portfolios of 401(k) plan participants, as gauged by the total risk, global beta, total equity, and U.S. and foreign equity exposures of their portfolios, and the participant age, gender, approach to investment, and whether he or she participates in a pension plan. The author finds evidence not only of behavioral biases affecting the investment decisions of investors with brokerage accounts, but also of two new apparent behavioral biases. First, participants with a defined-benefit pension plan do not take more risk, other things being equal, than participants without such a plan. Second, young active participants (participants who have made an active choice of their investments) take less risk compared with young passive participants, while older active participants take more risk than older passive participants.

Xu, G. (2018). "Static and Dynamic Tax Diversification of Withdrawals from Multiple Individual Retirement Accounts." In: *The Journal of Retirement* 6(2), pp. 75–87.

This article shows why diversified simultaneous retirement funding drawn from accounts with differing tax treatments will save more on taxes than sequential withdrawal. We utilize dynamic programming to quantify the optimal funding under the assumption that the goal is to maximize the total discounted after-tax consumption and bequest amounts. The dynamic programming setting uses the actual tax schedule, considers the required minimum distribution and life expectancy, and constrains consumption with upper and lower bounds. We also find the optimal funding through static diversification, a simple optimization schema. We compare the static and full dynamic programming solutions in two cases, one for a single filer and another for a married joint filer. The static and the full dynamic programming solution differ more in the single filer case than in the married joint filer case. The static optimal withdrawal is close to sequential withdrawal. The full dynamic programming optimal withdrawal is close the sequential withdrawal when both types of accounts have a substantial balance.

Xu, G. and Anichini, T. (2016). "Mean-Variance Analysis in Post-Retirement Planning." In: *The Journal of Retirement* 3(3), pp. 62–76.
In this study we analyze the variability and average level ofspending in retirement under two strategies: the popular 4 percent rule and what we will term the self-funded variable annuity strategy (SVA strategy). The 4 percent rule stipulates that each year a retiree should spend 4 percent of her initial wealthat retirement; the value of spending grows with the rateof inflation. Under the SVA strategy, each year the retiree determines her spending level as equivalent to the income she could theoretically obtain by purchasing an annuity(but without actually purchasing one). The 4 percent rule entailslower average spending and little variability, while theSVA strategy entails higher average spending and highervariability. The SVA strategy is based on a theoretical foundation of expected utility maximization. We demonstrate how a retiree can make an informed decision about allocating among stocks, bonds, and annuities, and whether to combine the 4 percent spending rule with the SVA strategy to achieve higher average spending. In particular, if future returns turnout to be lower than historical returns, as many experts predict,combining the 4 percent rule and SVA strategy may achieve spending of more than 4 percent of initial wealth on average while reducing the failure rate to less than 10 percent. We also conside rhow mortality might affect a mixed strategy that combinesa purchased annuity with the SVA strategy.

Xu, G., Markowitz, H., and Guerard, J. B. (2019). "Shortfall risk and shortfall duration for portfolio choice in decumulation." In: *The Journal of Retirement* 7(1), pp. 24–34.
The article studies the portfolio selection problem in the retirement phase by using the habit formation utility function in the context of traditional utility maximization. The habit formation utility can be further simplified to a linear combination of shortfall risk and shortfall duration. A retiree who can easily adapt to a new spending level should emphasize shortfall duration whereas a retiree who is rigid in spending should emphasize shortfall risk. The article provides the conditions in which current practitioners favorite choices of shortfall risk, as a criterion to choose retirement portfolios, are consistent with utility maximization.

Xu, J. and Hoesch, A. (2018). "Predicting longevity: an analysis of potential alternatives to life expectancy reports." In: *The Journal of Retirement* 5(4), pp. 9–24.
Retirees, pension funds, and the insurance industry have all been negatively affected by the wrongful estimation of longevity. The inaccuracies in current life expectancy (LE) reports primarily result from misinterpretations of the influence of resilience factors on longevity. This study examines different and more accurate measurement metrics to minimize the risks related to biased LE calculations. By using both qualitative and quantitative research approaches, this research develops a new conceptual model: a two-factor-LE-analysis model with a telomere test as a medical basis (physiological factors) and a big data approach to filter the psychological factors to longevity. The authors suggest that the new model, together with the insights of the existing LE-projection methodologies, has considerable potential to improve LE predictions.

Yang, B. (2017). "Longevity and statistical modelling." PhD thesis. Nanyang Technological University Singapore.
This dissertation consists of two studies on the modelling aspects of mortality (or longevity). In the first paper, we examine cohort extensions of the Poisson common factor model for modelling mortality of both genders jointly. Several alternatives are specified and applied to datasets from five developed regions. We find that direct parameterisation of cohort effect could improve model fitting, reduce the need for additional period factors, and produce consistent mortality forecasts for females and males. Furthermore, we find that the cohort effect appears to be gender indifferent for the populations examined and has an interaction effect with age in certain cases. The second paper explores the prediction error in mortality projection. This is important given the increasing longevity risk and the rising demand for longevity-linked products. Insofar, only parameter error and process error have been considered jointly while the issue of model error has been little studied. Here, we propose a method to account for process error, parameter error and model error in an integrated manner

by modifying the semi-parametric bootstrapping technique. We apply the method to two datasets from the Continuous Mortality Investigation (CMI) and use the simulated scenarios to price the q-forward contracts with a risk-neutral approach. We find that model selection has a significant impact on the valuation results and thus it is crucial to incorporate model error in mortality projection. The third part of the dissertation surveys the current landscape of the longevity market and discusses some open issues related to the pricing of longevity products in the context of the broader literature.

Yoon, Y. (2010). "Glide path and dynamic asset allocation of target date funds." In: *Journal of Asset Management* 11(5), pp. 346–360.
The glide path of typical target date funds is based on the relatively simple assumption of risk. If an explicit term structure of risk is present or risk is time-varying, the conventional glide path may not be adequate to fulfil the purpose of target date funds. We introduce a new approach to define the glide path of target date funds. Our starting point is to determine the level of risk budget for each target date. According to the pre-defined risk budget, we derive the asset allocation of target date funds by explicitly incorporating the current term structure of risk. As risk does change through different market phases, we implement a dynamic asset allocation strategy for target date funds that considers simultaneously both the pre-defined risk budget and the prevailing market risks. The main difference is that at any given time our risk-controlled dynamically rebalanced target date funds would not exceed the pre-defined risk budget regardless of market movements.

Young, R. (2020a). *Tax-Efficient Withdrawal Strategies*. Tech. rep. T. Rowe Price.
There are alternatives to the conventional strategy of drawing on a taxable account first, followed by tax-deferred and then Roth accounts. Many people can take advantage of income in a low tax bracket or tax-free capital gains. If planning to leave an estate to heirs, consider which assets will ultimately maximize the after-tax value.

Young, R. (2020b). "The Roth/Pretax Decision in Late Career Years: The Increasing Importance of Accumulated Assets in Light of the SECURE Act." In: *Journal of Financial Planning* 33(12), pp. 59–68.
A late-career worker with a choice between pretax and Roth retirement contributions should consider the level of tax-deferred assets already accumulated. That asset level affects the worker's tax rates over the course of retirement, due to required minimum distributions (which may not be needed to support spending), and also affects taxes on beneficiaries. This study analyzes the decision for a variety of scenarios by calculating breakeven asset levels (as a multiple of income) where Roth and pretax contribution strategies yield the same after-tax value for beneficiaries. Contrary to the conventional view that people in peak earning years should save using pretax contributions, the analysis shows that many people who are well on track for retirement would help their beneficiaries-without sacrificing retirement spending-by choosing the Roth strategy. The SECURE Act, passed in late 2019, requires most non-spouse beneficiaries of retirement accounts to withdraw all of the funds within 10 calendar years. That requirement can increase beneficiaries' tax rates, compared with a "stretch" strategy allowable under prior law. This study shows that breakeven asset levels are significantly lower after passage of the SECURE Act, indicating that more people would benefit from a Roth strategy now. Among the scenarios evaluated were different beneficiary characteristics, which showed that beneficiaries' future incomes and tax situations can significantly affect the breakeven asset levels.

Yu, L. (2021). "Comparing Classical Portfolio Optimization and Robust Portfolio Optimization on Black Swan Events." MA thesis. University of Waterloo.
Black swan events, such as natural catastrophes and manmade market crashes, historically have a drastic negative influence on investments; and there is a discrepancy on losses caused by these two types of disasters. In general, there is a recovery and it is of interest to understand what type of investment strategies lead to better performance for investors. In this thesis we study classical portfolio optimization, robust portfolio optimization and some historical black swan events. We compare two main strategies: mean variance optimization vs robust portfolio optimization on two types of black swan events: natural vs anthropogenic. The comparison illustrates that robust portfolio optimization is much more conservative, and has a shorter recovery time than classical portfolio optimization. Moreover, the losses in the stock investment resulted from a natural disaster are very minor compared to the losses resulted from an anthropogenic market crash.

Yu, S., Chen, Y., and Dong, C. (2021a). "Learning Time Varying Risk Preferences from Investment Portfolios using Inverse Optimization with Applications on Mutual Funds." In: *arXiv e-Print*.
The fundamental principle in Modern Portfolio Theory (MPT) is based on the quantification of the portfolio's risk related to performance. Although MPT has made huge impacts on the investment world and prompted the success and prevalence of passive investing, it still has shortcomings in real-world applications. One of the main challenges is that the level of risk an investor can endure, known as *risk-preference*, is a subjective

choice that is tightly related to psychology and behavioral science in decision making. This paper presents a novel approach of measuring risk preference from existing portfolios using inverse optimization on the mean-variance portfolio allocation framework. Our approach allows the learner to continuously estimate real-time risk preferences using concurrent observed portfolios and market price data. We demonstrate our methods on real market data that consists of 20 years of asset pricing and 10 years of mutual fund portfolio holdings. Moreover, the quantified risk preference parameters are validated with two well-known risk measurements currently applied in the field. The proposed methods could lead to practical and fruitful innovations in automated/personalized portfolio management, such as Robo-advising, to augment financial advisors' decision intelligence in a long-term investment horizon.

Yu, S., Wang, H., and Dong, C. (2021b). "Learning Risk Preferences from Investment Portfolios Using Inverse Optimization." In: *arXiv e-Print*.

The fundamental principle in Modern Portfolio Theory (MPT) is based on the quantification of the portfolio's risk related to performance. Although MPT has made huge impacts on the investment world and prompted the success and prevalence of passive investing, it still has shortcomings in real-world applications. One of the main challenges is that the level of risk an investor can endure, known as *risk-preference*, is a subjective choice that is tightly related to psychology and behavioral science in decision making. This paper presents a novel approach of measuring risk preference from existing portfolios using inverse optimization on the mean-variance portfolio allocation framework. Our approach allows the learner to continuously estimate real-time risk preferences using concurrent observed portfolios and market price data. We demonstrate our methods on real market data that consists of 20 years of asset pricing and 10 years of mutual fund portfolio holdings. Moreover, the quantified risk preference parameters are validated with two well-known risk measurements currently applied in the field. The proposed methods could lead to practical and fruitful innovations in automated/personalized portfolio management, such as Robo-advising, to augment financial advisors' decision intelligence in a long-term investment horizon.

Zhang, C., Li, Y., Chen, X., Jin, Y., Tang, P., and Li, J. (2020a). "DoubleEnsemble: A New Ensemble Method Based on Sample Reweighting and Feature Selection for Financial Data Analysis." In: *IEEE International Conference on Data Mining (ICDM)*. IEEE.

Modern machine learning models (such as deep neural networks and boosting decision tree models) have become increasingly popular in financial market prediction, due to their superior capacity to extract complex non-linear patterns. However, since financial datasets have very low signal-to-noise ratio and are non-stationary, complex models are often very prone to overfitting and suffer from instability issues. Moreover, as various machine learning and data mining tools become more widely used in quantitative trading, many trading firms have been producing an increasing number of features (aka factors). Therefore, how to automatically select effective features becomes an imminent problem. To address these issues, we propose DoubleEnsemble, an ensemble framework leveraging learning trajectory based sample reweighting and shuffling based feature selection. Specifically, we identify the key samples based on the training dynamics on each sample and elicit key features based on the ablation impact of each feature via shuffling. Our model is applicable to a wide range of base models, capable of extracting complex patterns, while mitigating the overfitting and instability issues for financial market prediction. We conduct extensive experiments, including price prediction for cryptocurrencies and stock trading, using both DNN and gradient boosting decision tree as base models. Our experiment results demonstrate that DoubleEnsemble achieves a superior performance compared with several baseline methods.

Zhang, F., Guo, R., and Cao, H. (2020b). "Information Coefficient as a Performance Measure of Stock Selection Models." In: *arXiv e-Print*.

Information coefficient (IC) is a widely used metric for measuring investment managers' skills in selecting stocks. However, its adequacy and effectiveness for evaluating stock selection models has not been clearly understood, as IC from a realistic stock selection model can hardly be materially different from zero and is often accompanies with high volatility. In this paper, we investigate the behavior of IC as a performance measure of stick selection models. Through simulation and simple statistical modeling, we examine the IC behavior both statically and dynamically. The examination helps us propose two practical procedures that one may use for IC-based ongoing performance monitoring of stock selection models.

Zhang, S. (2018). "Optimal Retirement Planning: Scenario Generation, Preferences, and Objectives." PhD thesis. University of Waterloo.

The global trend of shifting from defined benefit (DB) to defined contribution (DC) workplace pension plans is putting growing pressure on individuals to take more ownership in retirement planning and financial decision-

making. The essence of the DB is the life-long income guarantee, which requires limited financial planning decisions to be made, either in the accumulation or decumulation phase. The DC on the other hand, is significantly more complex. The lump sum payment at retirement burdens individuals with the task of income generation, in the presence of challenges stemming from an uncertain future lifetime, economic conditions, and evolving consumption needs. The average retiree has limited competency to navigate these challenges, due to low financial literacy, lack of willpower, or deteriorating cognitive abilities with older ages. The high stake of these challenges calls for a normative solution to be proposed - a solution that considers the intricacy of risks, preferences, and normative objective formulations. The objective of this thesis is to explore such a solution. This thesis comprises three inter-related research directions: long-term economic scenario generators (ESGs), recursive preferences in life-cycle portfolio selection, and retirement objective formulation. A brief description of the subsequent chapters will now follow. The first chapter conducts a review of Wilkie's ESG, with analysis restricted to series pertinent to retirement planning. Our main findings indicate that there exist challenges in modelling long-term economic series due to the presence of multiple structural shifts in the historical time series. Consequently, certain assumptions of stationarity are violated, and parameters are sensitive to the calibration period. A backtest based on 30-year out-of-sample data indicated that over that period the model had tended to overestimate inflation, underestimate total return on stocks, and performed relatively well for long-term interest rates. Additionally, Wilkie's ESG can be under-representative of the risk in long-term stock investment, particularly in the tails. The second chapter provides an introductory discussion of Epstein-Zin preferences, which are adopted in the succeeding chapter as a normative preference model. The purpose is to first investigate the implied optimal behaviour and its plausibility. We pay particular attention to whether the output leads to plausible behaviour given the context of retirement planning. Specifically, analytical solutions for a simple consumption problem are derived, isolating the impact of relative risk aversion (RRA), elasticity of intertemporal substitution (EIS), time discounting, and risks stemming from mortality, investment, and inflation. We investigate three Epstein-Zin models employed in the literature, which differ in their treatment of mortality risk, and find that some lead to normatively implausible solutions. Importantly, we find that the EIS is not always monotone in its effect on consumption volatility over time, meaning that its interpretation can be ambiguous when considering an uncertain future lifetime. This has been misinterpreted in the literature to date. We also show that one particular Epstein-Zin specification is not necessarily a generalization of expected utility maximization under constant relative risk aversion, as many works wrongly claim. The third chapter investigates the normative validity of the optimal consumption and investment strategies of a discrete-time Epstein-Zin utility maximizing DC retiree who wishes to benefit from stock investment, longevity insurance, and inflation protection. A comparison of three Epstein-Zin specifications is conducted. We use a combination of qualitative and quantitative criteria to evaluate the adequacy of the optimal consumption profile, with special attention paid to the downside risk at extreme old ages. We find that it remains optimal to fully annuitize, but agents with high relative risk aversion hold precautionary savings, the level of which is impacted by the EIS and the preference specification. As discussed in the preceding chapter, the interpretation of EIS on consumption volatility is found to be ambiguous. Investigations of the optimal consumption profile reveal that agents are exposed to relatively high levels of downside risk in the long run. This is partially attributed to a time discounting factor less than 1, which implicitly (and contradictorily) assumes myopia in normative decision-making. An investigation of zero time discounting is conducted, with downside risk found be to significantly reduced in the long run. The fourth chapter focuses on retirement objective formulation. This chapter is motivated by the unsatisfactory normative solutions found in the preceding chapter under mathematically convenient objective functions. In order to develop more actionable prescriptive solutions, we seek to holistically explore actual retirement decision-making. To this end, we conduct a survey study of 1,000 Canadian (pre-)retirees age 50 to 80, on topics of retirement consumption, wealth, income, risk perception, decision making, and planning objectives. Additionally, we investigate the descriptive validity of the expected lifetime discounted utility maximization framework in predicting optimal planning behaviours. Overall, there is overwhelming evidence of heterogeneity in wealth, income, concerns, and objectives. We find a prevalence of low retirement assets, a severe underestimation of survival probabilities to an extreme old age of 95, and a strong aversion toward life annuities. Pre-retirees appear to have reasonable expectations regarding income and assets in retirement, with the median retiree relying heavily on public pension sources. (Pre-)retirees are primarily concerned with liquidity needs, consumption smoothing, inflation, and longevity in retirement, and are least concerned with bequests. We elicited risk and time preferences, and found an average RRA parameter between 1.74 to 2.48 for pre-retirees and 2.48 to 3.74 for retirees, and a median subjective time discount factor of 0.997. A study of decision-making under risky scenarios reveals dramatic differences between

the actual and implied choices under the expected utility maximization framework. Particularly, in the presence of inflation risk, agents lack the understanding of the long-term cumulative impact of inflation on the cost of living. In the presence of investment risk, the upside gain drives decision-making, and the presence of minimum income protection effectively provided by public pension income induces more risk-taking behaviour. The last chapter concludes the thesis, and proposes general directions for future work in retirement planning research.

Ziemba, W. T. (2016). "An Approach to Financial Planning of Retirement Pensions with Scenario-Dependent Correlation Matrixes and Convex Risk Measures." In: *The Journal of Retirement* 4(1), pp. 99–111.
The article describes an approach to asset-liability modeling using discrete time stochastic linear programming. The model uses future scenarios and optimizes the asset-liability mix subject to various constraints and is applicable to insurance companies, bank trading departments, overall bank asset-liability management, and other financial institutions. An application to the Siemens Austria pension fund, where the model has been in use since 2000, is described. The model has had considerable success and has been used by regulators to determine the effect of various possible pension fund policy changes. It has also been used by pension fund advisors who deal with uncertain assets and liabilities subject to various legal and policy constraints.

Zuss, N. (2022). *Participants Will Need Support to Understand Lifetime Income Projections*. Tech. rep. PlanSponsor.
Recordkeepers are bolstering education, projection modeling tools and tailored advice capabilities to support plan participants and encourage them to remain on track for retirement income planning.
Plan sponsors and advisers are preparing for defined contribution (DC) plan participants' reactions to the lifetime income estimates that will be coming on their plan statements this year with custom tools and planning advice, as well as investment menu evaluations.
In 2019's Setting Every Community Up for Retirement Enhancement (SECURE) Act, Congress required that DC plans provide retirement plan participants with projections illustrating monthly retirement income amounts generated from their accumulated retirement savings. The Department of Labor (DOL)'s interim final rule on the matter, published in September, requires that participants' projected income be illustrated as an account balance conversion to a lifetime income annuity.