Vehicle scheduling problem for petrol buses and electric buses

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1 Abstract

This paper solves a bus scheduling problem with four locations and a depot for the buses. The goal is to minimize the variable costs, depending on the number of buses and their shifts. Next to combustion engine buses also electric buses are considered. For the electric buses, we consider buses with long charging times, buses with short charging times as well as buses that can be recharged at every stop. The problem is solved using the Single-Commodity Flow formulation and the Set Covering formulation. In our research, we extend the existing literature by looking at electric vehicles. The main findings are that replacing regular buses with electric buses will come at a great cost. Additionally, electric buses with a shorter charging time lower the costs compared to buses with longer charging times, and buses that can be recharged at every stop reduce the overall total costs of the electric bus problem.

2 Introduction

Currently, a lot of attention is drawn to reducing greenhouse gas emissions. The transport sector accounted for 37 percent of CO2 emissions by the end of 2021 (International Energy Agency, n.d.). As a result, electric vehicles are increasing in demand as their CO2 emission are significantly less. Research has shown that the use of electric cars reduces CO2 emissions by 35 percent (Gijlswijk et al., 2014), having a significant effect in reducing CO2 emissions. Consequently, regional transportation companies in the Netherlands are implementing electric buses in their planning to be more climate neutral and sustainable for the future.

Through our research, we want to aim to paint a clearer image of adding electric vehicles to the planning of buses. Therefore, our research focuses on the extent to which the necessity of charging these buses affects bus scheduling and cost. This is an important question to answer as electric buses are increasing in demand but still are in a large minority. Perhaps the costs are a reason why public transport companies are being held back. If this is the case maybe subsidies can be offered to companies to switch to electric buses. We will analyze how different scheduling problems with electric vehicles can affect costs, including different charging speeds and charging locations. This will show how different technologies of charging speed can affect the costs of electric bus planning.

Previous research that has been done on the Single Depot Scheduling problem, like from Freling, Wagelmans, and Paixao (2001), is relatively outdated so they did not take electric buses into consideration. We will adapt the models that are previously defined i.e. the Single Commodity-Flow formulation and the Set Covering formulation to include constraints regarding electric buses. First, we will test the previously defined formulations for regular petrol buses, and later on, we will add constraints to the models for the electric buses. We consider three types of electric buses: buses with long charging times, buses with short charging times, and buses that can be charged at every stop with average charging times. The paper is structured in several sections. Firstly, in section 3 we will go over the exact problem description. Secondly, in section 4 we will explain the methodology. Thirdly, section 5 shows the results and finally in section 6 we present the conclusion of our research.

3 Problem Description

In this paper, we will evaluate three different bus schedules and minimize the total costs of assigning buses to bus lines such that all trips on the bus schedules are fulfilled. We will evaluate the bus schedules on Monday-Friday, Saturday, and Sunday, which have respectively 156, 104, and 182 trips. Each trip has a different start and end location. In total there are 4 different locations: A, B, C, and D. The buses can make multiple trips per day and can also go to a different location without any passengers, we refer to this as the deadhead between two locations. Moreover, there is only one depot where all the buses need to start from and go back to after fulfilling all the trips in their ride. During the night, all buses are expected to be parked in the depot. Previous literature (Freling et al., 2001) refers to this problem as the Single-Depot Vehicle Scheduling problem.

This paper consists of two main cases. For the first case, we consider regular buses that directly emit carbon dioxide, whereas, for the second case, we only consider three sub-cases for electric buses with varying charging times.

Regarding the first case we can only make use of non-electric buses. Furthermore, there must be at most 50 minutes between the arrival of the bus and its next departure. Leaving out the possibility for potential deadhead between two trips. Hereinafter, we will refer this problem as *the regular bus case*. We show the results of this problem in subsection 5.1.

Our case for electric buses consists of three sub-cases. First, we consider electric buses with one additional constraints: after 8 hours, the bus needs to be recharged at the depot and the driver needs to stop working. The electric bus can then only be used again the next day. The results of this case will be shown in subsection 5.2. Also, to simplify the problem we relax the constraint that there can only be 50 minutes between the arrival of the bus and its departure. Secondly, in the second sub-case, we look at how many electric buses can be reduced when the recharging time of electric buses is only 1 hour. The results of this particular case will be discussed in subsection 5.3. Thirdly, we will engage in an extension to the first sub-case. In the extension electric bus drivers will have the opportunity to charge their buses at locations A, B, C, and D instead of only being able to recharge the depot. Every bus that is too early for its next trip can now recharge its electric bus while waiting to start its next trip. The charging speed of the charging stations at the locations is as follows: one minute of charging time results in two minutes of driving time. This is equivalent to the bus needing 4 hours to charge to get an extra 8 hours of driving time. This charging speed would be in between the two previous sub-cases for the electric buses. Now we can analyze if an improvement in charging speed and extra charging stations can drastically lower the costs for the electric bus problem. lastly, we impose the restriction that there can be at most 120 minutes between the arrival time of the bus at a location and its next departure time excluding any deadhead travelling.

We will refer to this sub cases for electric buses respectively as *the slow charging electric bus case*, *the fast charging electric bus case* and *the multiple charging location bus case*.

The relevant costs for both scenarios are as follows: the variable costs of the bus drivers' wage of €30 per hour, the variable bus cost of €2 per km, and the fixed cost of €100 for each bus that is deployed. We assume that a bus and a driver stay together for the whole day. When minimizing the total costs of our problem, we only consider the costs between trips as the cost of the trips themselves do not add any value in deriving our optimal solution.

4 Methodology

In this methodology, we will start tackling the problem with two formulations: the Single-Commodity Flow formulation and the Set Covering formulation. We will introduce the corresponding mathematical models in which the relevant variables and constraints are defined and explained. Finally, we will explain how these formulations are implemented for *the regular bus case*, *the slow charging electric bus case*, *the fast charging electric bus case* and *the multiple charging location bus case.*

4.1 Mathematical formulations

4.1.1 The Single-Commodity Flow formulation

For the single-commodity problem, we use the "straightforward formulation" from Freling, Wagelmans, and Paixao (2001)[1] as a reference. We introduce the set $\mathcal{N} = \{1,2,...,n\}$ containing all the n nodes, where each node corresponds to a trip. We also introduce a set $\mathcal{A}$ containing all the possible arcs between the nodes i, j, d_1 and d_2, where the latter two represent the depot at location d. Also, the decision variable y_{ij} is introduced which equals 1 if a bus covers trip j directly after trip i and equals 0 otherwise. The cost parameter c_{ij}, corresponding to each arc y_{ij}, denotes the cost for going from node i to node j. Our mathematical model will be as follows:

$$\min \sum_{j=1}^{n} c_{ij} y_{ij} \quad (1)$$

$$s.t. \sum_{j:(i,j)\in\mathcal{A}} y_{ij} = 1 \quad \forall i \in \mathcal{N}, \quad (2)$$

$$\sum_{i:(i,j)\in\mathcal{A}} y_{ij} = 1 \quad \forall j \in \mathcal{N}, \quad (3)$$

$$y_{i,j} \in \mathbb{B} \quad \forall (i,j) \in \mathcal{A}. \quad (4)$$

In this model, the constraints (2) and (3) make sure that every trip has one successor and one predecessor. Constraint (4) represents the binary variable y_{ij}.

Furthermore, for our problem we create a new set of arcs, $\mathcal{F}$, that will prove to be useful when defining the costs. This new set $\mathcal{F}$ consists of arcs between trips that are feasible. Arcs are feasible if and only if the following conditions are satisfied:

$$0 \leq y_{ij}(bt_j - et_i) \leq 50 \quad \forall i,j \in \mathcal{N} \quad (5)$$

$$y_{ij} b_j = y_{ij} e_i \quad \forall i,j \in \mathcal{N} \quad (6)$$

Here b_j is the starting location of trip j, e_i is the ending location of trip i, et_i is the time of arrival at the ending location of trip i, and bt_j is the time of departure at the starting location of trip j. The function $trav(e_i, b_j)$ gives the deadhead time (in hours) from the ending location of trip i to the starting location of trip j. Constraint (5) models the condition that there must be be at most 50 minutes, i.e., $\frac{5}{6}$ hours, between the arrival of the bus and its next departure. Constraint (6) is in turn required as it is only possible for a bus to take on another trip if the ending location of trip

[1]Freling, R., Wagelmans, A., Paixão, J. A. (2001). Models and Algorithms for Single-Depot Vehicle Scheduling. Transportation Science, 35(2), 165–180. https://doi.org/10.1287/trsc.35.2.165.10135

i is equal to the starting location of trip j. This constraint is needed as the deadhead times from any ending location to any starting location is longer than 50 minutes.

Set $\mathcal{F}$ only consist of the arcs $(i,j) \in \mathcal{A}$ that satisfy constraints (5) and (6). Therefore $\mathcal{F}$ can be formulated in the following way: $\mathcal{F} = \{(i,j) \ | \ i,j \ \ compatible, \ \ i,j \in \mathcal{N}\}$.
It is only left for us to define the cost parameter c_{ij}. Four cases are considered. Firstly, we consider the costs corresponding to the arc that moves from the depot to a particular trip j. We will assign extra fixed costs of 100 as this case implies an extra bus to be assigned. Secondly, the costs of feasible arcs $(i,j) \in \mathcal{F}$ are considered. Thirdly, we focus on the arcs $(i,j) \in \mathcal{A} \setminus \mathcal{F}$. As this arcs are not compatible, we will assign a cost of 1 billion euros to this arcs. Lastly, we consider the arcs that moves from the last trip to the depot. The costs c_{ij} are calculated as follows:

$$c_{ij} = \begin{cases} 2traveldistance(d_1, b_j) + 30trav(d_1, b_j) + 100 \quad if \quad i = d_1 \quad (d_1 := depot) \quad (7) \\ 30(bt_j - te_i) + 2traveldistance(e_i, b_j) \quad if \quad (i,j) \in \mathcal{F}, \quad i \notin d_1, \quad j \notin d_2 \quad (8) \\ 1,000,000,000 \quad if \quad (i,j) \in \mathcal{A} \setminus \mathcal{F}, \quad \mathrm{i} \notin d_1, \quad j \notin d_2 \quad (9) \\ 2traveldistance(e_j, d_2) + 30trav(e_j, d_2) \quad if \quad j = d_2 \quad (d_2 := depot) \quad (10) \end{cases}$$

The distance in km between the ending location of trip i and the starting location of trip j is denoted by $traveldistance(e_i, b_j)$. The other definitions are the same as for the constraints (5) and (6).

These costs c_{ij} are calculated using the programming language Python. We first make a (n + 1) x (n + 1) matrix to store all the costs in, where the last row and column represent the depot. Then we make a nested loop where we loop over all possible trips twice and calculate all the costs between trips. After that we calculate the costs from the depot to all the trips and from all the trips to the depot.
The pseudocode for this is as follows:

Algorithm 1 Algorithm for computing elements of cost matrix in the Flow formulation

```
create the (n + 1) x (n + 1) cost matrix
for all trips i ∈ N do
    for all trips j ∈ N do
        if trips i and j are compatible then
            Add the cost for travelling between these two to the matrix
        else
            Set the cost equal to 1,000,000,000
        end if
    end for
end for

for all trips i ∈ N do
    add the cost between for travelling from the depot to trip i to the matrix
end for

for all trips i ∈ N do
    add the cost between for travelling from the trip i to the depot to the matrix
end for
```

4.1.2 The Set Covering formulation

The second formulation for our problem is the Set Covering formulation. We define a fixed set $\mathcal{N} = \{1, ..., n\}$ consisting of duties, e.g., feasible sequences of trips assigned to one bus and one driver, and we define a fixed set $\mathcal{M}$ containing all the possible trips for $1 \leq i \leq m$. A duty is feasible if every two successive trips in the sequence satisfy constraint (5) and (6).
A duty always starts and ends with the depot, and in between is at least one trip of the total m trips. However, for this problem formulation, we define a duty as only the sequence of trips right after and before the depot. Later on when defining the costs we will take the depot in consideration. This implies that each element of $\mathcal{N}$ is a set of all the trips of the duty in between the depots. We choose to let $\mathcal{N}$ consist of only feasible duties. We achieve this by first making a list called feasibleDutiesList in which we add all feasible duties as their own list, so each list represents a duty. Then we loop over all possible trips and add them to their own list. After that we loop over all duties in feasibleDutieList, including duties we add in the loop, and give each duty to a method that finds all feasible trips to concatenate to a given duty.
This pseudocode for this is the following:

Algorithm 2 Algorithm for finding feasible duties

```
create the list feasibleDutiesList
for all trips do
    add the trip to a new list
    add this new list to feasibleDutiesList
end for

for every duty in feasibleDutiesList do
    give this duty and feasibleDutiesList to a method to find feasible trips to add
    update feasibleDutiesList
end for
```

This method then loops over all trips. At the start of the loop it makes a copy of the duty it was given. After that it adds a trip to the copy of the duty if it is feasible to travel from the last trip in that duty to the current trip in the loop. It then adds this duty to feasibleDutiesList. Finally, when the method looped over all the trips it returns feasibleDutiesList.
This pseudocode looks as follows:

```
for all trips do
    make a copy of the duty passed on to the method
    if if it is feasible to travel from last trip in the duty to the current trip in the loop then
        add this trip to the copy of the duty
        add this new duty to feasibleDutiesList
    end if
end for
return feasibleDutiesList
```

Selecting a duty $j \in \mathcal{N}$ is a yes/no problem. Therefore we introduce for every duty j, the following binary decision variable that corresponds to constraint (13):

$$x_j = \begin{cases} 1 & \text{if duty } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

To determine whether for every duty j belonging to the set $\mathcal{N}$ every trip is included in only one of the selected duties, we introduce the following binary variable:

$$a_{ij} = \begin{cases} 1 & \text{if trip } i \text{ is included in duty } j \\ 0 & \text{otherwise} \end{cases}$$

The variable a_{ij} corresponds to the elements of the matrix A, from which the rows i correspond to the m trips, and the columns j correspond to the n feasible duties. As each selected duty corresponds to the route a bus and bus driver will take, we will assign every trip to exactly one bus and bus driver, each trip can only be on one of the selected duties. Therefore, our problem formulation should satisfy constraint (12).

To calculate the cost for duty j, c_j, first take the depot and the first stop of the first trip in the list and calculate the travel and wage costs, also add 100 for the fixed cost. This is done using formula (7). Then you calculate the travel and wage costs between every trip in the list with formula (8) and add this. In the end you have to add the travel and wage costs between the last stop of the last trip and the depot, which is done with formula (10). For this minimising problem we do not have to take into account the fixed costs of the trips themselves because those will always be made and do not change the problem. The costs c_j are computed using the program language Python. The pseudo code for this is similar to algorithm, which is stated at the end of section 4.1. However, in this case, to compute a cost c_j, we do not loop over all possible combinations of trips, but we loop over the sequence of trips in a duty j and immediately add the cost from the depot to the first trip and from the last trip to the depot.

Now that the costs corresponding to every duty j is known, we now define our objective function (11). We try to find the duties that satisfy constraint (12), e.g., each trip is only included in one of the selected duties, and minimize total costs. Our mathematical model is therefore as follows:

$$min \quad \sum_{j=1}^{n} c_j x_j \quad (11)$$

$$s.t. \quad \sum_{j=1}^{n} a_{ij} x_j = 1 \quad 1 \leq i \leq m \quad (12)$$

$$x_j \in \mathbb{B} \quad 1 \leq j \leq n \quad (13)$$

In the next section, the regular base case, the slow charging electric bus case, the fast charging electric case and the multiple charging location bus case will be tackled. We will explain which formulation is used in doing so, and will give details on the consequences of changing assumptions in either case.

4.2 Implementing methodology for different bus cases

For the regular bus case we will use both the Single-Flow formulation and the Set Covering formulation. The mathematical models explained in section 4.1 will be used precisely, as well as the derivation of the costs. Both of the formulations are used to compare which one is more efficient and to find out which one is more adaptable to three electric bus cases.

From now on we will implement the next three cases, i.e., slow charging electric bus case, fast charging electric bus case and the multiple charging location bus case, with the Set Covering formulation. By using the Set Covering problem we can more easily control if trips are compatible and if the constraints for maximum driving time are met when using the method to find feasible duties, i.e., implementing it in the pre-processing of the data before entering it in AIMMS. In the contrary, this is very hard to control when using the Single-Commodity Flow formulation and implementing it in AIMMS.

For the slow charging electric bus case we adjust the formulation as follows: with respect to computing the costs, constraint (5) and (6) are irrelevant now for feasibility purposes. A duty j is only feasible now if a bus can return to its depot in 8 hours. We implement this 8 hour constraint in the method that finds all feasible duties. Other than these changes, the exact mathematical formulation described in section 4.1.2 will be used.

Regarding the fast charging electric bus case, we give the buses the option to get back to work after ending in a depot. As for this problem the constraints and mathematical formulation is exactly the same as the slow charging electric bus case, so we use the solution of the latter case as a heuristic for finding the optimal bus schedule. The pseudocode for the heuristic is as follows:

Algorithm 3 Algorithm for finding a feasible solution for the fast charging case

```
Make a new list chosenDuties that contains all duties from optimal solution of slow charging case
Make a list newDuties that is a copy of chosenDuties
for each dutyA in chosenDuties do
    for each dutyB in chosenDuties do
        if dutyA and dutyB can be combined then
            if dutyA is not in newDuties or dutyB is not in newDuties then
                continue
            end if
            Add them together, as a list, to newDuties and remove dutyA and dutyB from newDuties
        end if
    end for
end for
```

Finally, for the extension of the electric bus problem it is possible to recharge a bus at every stop. Here the maximum charging time at a bus stop is 120 minutes, so in this case the maximum time a bus can drive before returning to the depot depends on the time it recharges between trips. We implement the method to find feasible duties such that the new constraints are taken into account. These constraint will replace constraint (5) and (6). The remainder of the formulation described in 4.1.2 will then be used to solve this problem.

5 Results

Firstly, we will discuss the results of the regular bus case and explain the differences in computation times between the Single-Commodity Flow and the Set Covering formulation. Afther this, we will show the results of respectively the the slow charging electric bus case, the fast charging electric bus case and the multiple charging location bus case. It is important to note that there are also fixed costs involved that are not considered when minimizing the problems but are relevant to the total costs of the transport company. The fixed costs on the weekdays are equal to €16,653.00, on Saturday the fixed costs are €9,692.80 and on Sunday they are equal to €17,625.40. To obtain the total costs these fixed costs can be added to the variable costs shown in the results below.

5.1 The regular bus case

Table 1

Optimal costs for the regular bus problem

	Flow			Set covering		
Variable	Weekdays	Saturday	Sunday	Weekdays	Saturday	Sunday
Buses	45	23	38	45	23	38
Optimal Costs (€)	12,377.58	6769.68	10,362.90	12,377.58	6769.68	10,362.90
Computation time	0.49	0.08	0.26	0.03	0.03	0.20

Table 1 shows the optimal costs for both formulations for regular buses as well as the total number of buses needed. The table shows that the costs for Saturday are the lowest which we would expect since it has the least trips to be fulfilled. However we see that the costs for Sunday are lower than during weekdays even though there are more trips to be fulfilled. In the table, we can see also that the computation times for the Set Covering formulation are always more optimal. The difference in computation times between the two formulations is most likely due to the difference in variables and constraints in the two problems. In the flow formulation, there are always more constraints and variables than in the set-covering formulation. Because the set-covering problems have both fewer variables and fewer constraints, it is easier for Aimms to compute the solution and it, therefore, has a lower computation time. This is also an extra benefit of using the Set Covering Formulation.

5.2 The slow charging electric bus case

Table 2

Optimal costs for the slow charging electric bus problem

Variable	Weekdays	Saturday	Sunday
Buses	130	78	143
Optimal Costs (€)	32,083.49	19,600.52	35,774.79

In table 2 the optimal costs for the set covering formulation for electric buses as well as the total number of buses needed are displayed. On weekdays 85 extra buses and bus drivers are needed, on Saturdays 55 and on Sundays 105. Now that electric buses are used, there is an increase in amount of buses needed to fulfill all the trips making the optimal costs rise drastically.

5.3 The fast charging electric bus case

Table 3

Buses used for the fast charging electric bus problem

Variable	Weekdays	Saturday	Sunday
Buses	86	51	93

Because of the reduced charging time, 44 electric buses can be saved on weekdays, 27 on Saturdays, and 50 on Sundays. Relative to the slow charging bus problem only the amount of buses saved will reduce the variable costs, since the travel time and travel distance of the duties will not change. So this respectively deducts the total costs with the following amounts: €4400, €2700 and €5000. Note that the used heuristic creates a feasible solution and this is not necessarily the optimal one. This in turn does make the costs for using electric buses lower relative to the slow charging case but still higher than the case for regular buses.

5.4 The multiple charging location bus case

Table 4

Optimal costs for the multiple charging location bus problem

Variable	Weekdays	Saturday	Sunday
Buses	82	42	78
Optimal Costs (€)	23,473.21	13,748.93	24,483.73

In table 4 you can observe the optimal costs and the optimal amount of buses needed to cover all trips when it is possible to recharge buses at the stations. In this case, there are fewer buses deployed on Sunday but the costs are higher. This is probably caused by the wage costs of bus drivers that are waiting for their bus to be recharged. However, compared to the results displayed in table 2 and 3, it becomes clear that a lot of costs can be saved every week when charging stations at every stop are installed.

6 Conclusion

In this paper, we have looked into the vehicle scheduling problem for regular buses and electric buses. The formulations that were used to solve this problem are the Single-Commodity Flow formulation and the Set Covering formulation. We have considered four different cases of bus plannings, and found their respective optimal costs. It is clear that using petrol buses are still the least costly way to operate the planning of buses. In the different cases that we analyzed for the electric bus problems, it seems that charging time and the amount of charging stations play a very important role in reducing the costs for electric bus planning. If charging technology for batteries of electric vehicles is relatively less advanced as in our slow charging problem, we can see that variable costs almost triple which will have substantial impact on the expenses of bus companies. In our results for the fast charging problem, we see that costs can already be lowered by a significant amount. It is in our extension, where we implement multiple charging stations with a more realistic charging speed that we see the costs at the lowest for electric buses making it more possible for bus companies to implement these in their planning. In conclusion, by having more charging stations and investing in technology to increase the charging time of electric vehicles, bus companies can accelerate their process into being climate neutral.

As the use of electric buses are becoming inevitable, we strongly recommend for logistics planners to implement multiple charging stations to routes where their electric buses are operating. As the costs for electric buses are still significantly higher relative to petrol buses, we think it is necessary for governments to subsidize transportation companies into transitioning into a more environmentally friendly bus schedule. Using these subsidies, bus companies can transition faster into electric vehicles, making a great impact on climate change. As discussed in the introduction this is of great importance as transportation is one of the main contributors to carbon dioxide emission.

For future research, it is important to analyze the costs of installing multiple charging stations as we did not take this into account in our research. It is clear that increasing the amount of charging stations will decrease costs, however it is also needed to take into account the installment costs for the bus company when implementing this. Furthermore, it would be interesting to look into bus planning where part of the vehicles are electric and another part still petrol fueled. In our research we have only focused in the extreme cases where only petrol buses are used or only electric buses are used. As bus companies are still using petrol buses this would be important to analyze for the slow transition of fully petrol fueled buses to bus planning with only electric buses.

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